	Carátula para entrega de prácticas
Facultad de Ingeniería	Laboratorios de docencia

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Asignatura:	Estructuras de Datos y Algoritmos II
Grupo:	2
No de Práctica(s):	2
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Observaciones:	

CALIFICACIÓN: ________________

Sorting Algorithms II.

Objective: Learn about the structure of sorting algorithms “Quicksort” and “Heapsort”.

Activities:

- Implement the Quicksort algorithm in Python language for sorting a data sequence.
- Implement the Heapsort algorithm in Python language for sorting a data sequence.

Instructions:

- Implement in Python the ascending-order sorting algorithms (Quicksort and Heapsort) for sorting numbers or nodes.
- Parting from the Python algorithms, obtain the polynomial of best, average and worst case of time complexity for Quicksort and Heapsort.
- Parting from the Python algorithms, generate the graph for different time instances (lists from 1 to 1000 elements for the next cases).
  - Best case
  - Worst case
  - Average case

The practice must be done individually.

The practice is checked during the laboratory session and must be uploaded with all the code and the report in a compressed file, once qualified, to the SiCCAAD platform.

Heapsort

Heap Sort

Heap sort is a comparison-based sorting algorithm that uses the heap data structure for finding the maximum number or the minimum number for pushing it into the array until it is sorted.

Heap sort was invented by J. W. J. Williams in 1964, and heap data structure was also invented by him but earlier in that year.

Heap

Heap data structure is a binary tree structure that satisfies one of the following conditions:

- For any subtree of the heap, the parent node is greater or equal to the child nodes (MAX HEAP)

- For any subtree of the heap, the parent node is minor or equal to the child nodes (MIN HEAP)

Heapsort implementation and analysis using RAM model

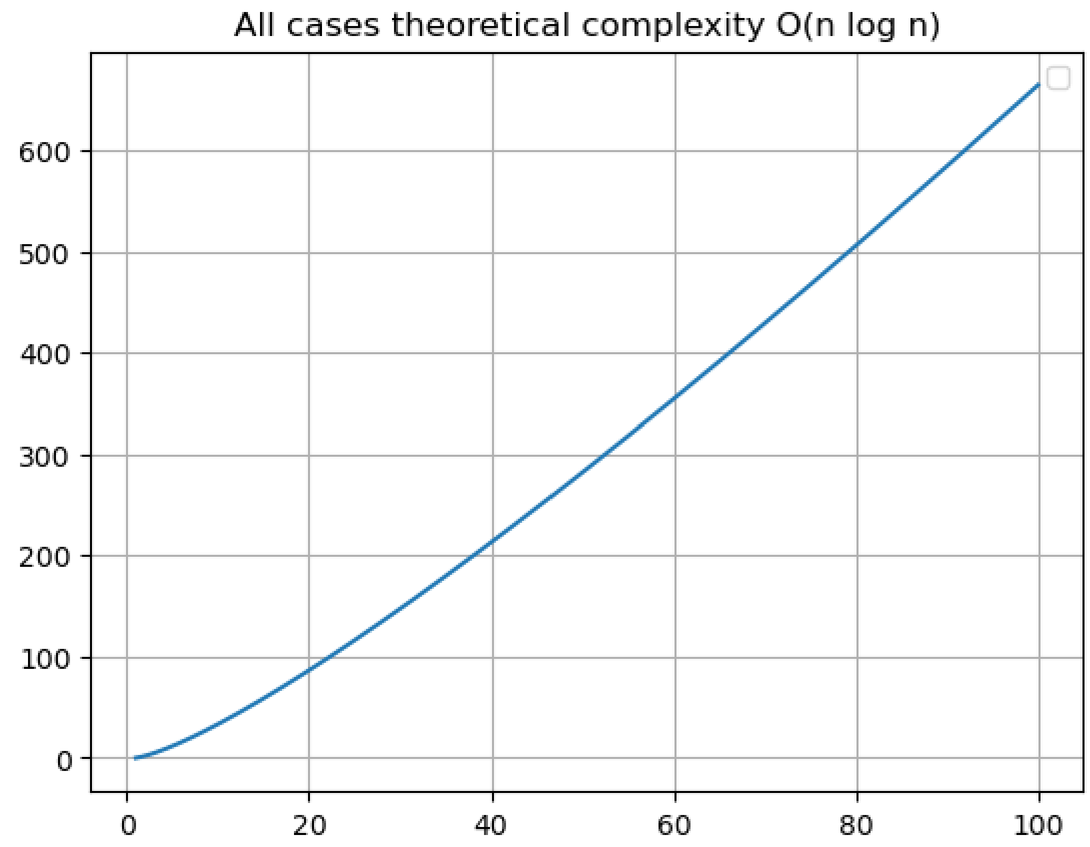
```
#        Worst Case        ||        Average case          ||        Best case


def maxHeap(arr, n):

    for i in range(n // 2 - 1, -1, -1): #4 * (n/2)

        heapify(arr, n, i) #45 (log n) * (n/2)

    # maxHeap polyomial: 45 (n log n) + 4 (n/2) = O(n log n)



def heapify(arr, n, i):

    largest = i # 3

    left_child = 2 * i + 1 # 5

    right_child = 2 * i + 2 # 5


    if left_child < n and arr[left_child] > arr[largest]: # 8

        largest = left_child # 3


    if right_child < n and arr[right_child] > arr[largest]: # 8
```

```
        largest = right_child # 3


     # After this we've been completed a subtree transversal, dividing by
two the set, making the recursive call

    # logarithmic


    if largest != i: # 4

        arr[i], arr[largest] = arr[largest], arr[i]  # 6

          heapify(arr, n, largest)  # Recursive call, we go down the tree
until we get to the leaves (log n)


    # Heapify polynomial: 45 (log n) = O(log n)


def heapSort(arr):

    n = len(arr) # 4


    maxHeap(arr,n) # 45(n log n) + 4(n/2)


    for i in range(n - 1, 0, -1): # 5(n+1)

        arr[0], arr[i] = arr[i], arr[0]  # 6(n)

        heapify(arr, i, 0)  # 45(log n) * (n)


    # heapSort polyomial: 90 (n log n) + 11(n) + 4(n/2) + 9 = O(n log n)


# Example usage

arr = [12, 11, 13, 5, 6, 7, 10, 15, 90, 117, 95, 80, 77, -100, 0]
```

```
heapSort(arr)

print("Sorted array:", arr)
```

Heapsort all cases complexity explanation.

Due to the nature of the heap we can prove that for any case, time complexity will always be n log n, creating the heap takes n log n time no matter the order of the numbers, and is proven that always we will traverse the array in linear time, multiplied by the heapify complexity that is log n, generating n log n complexity.

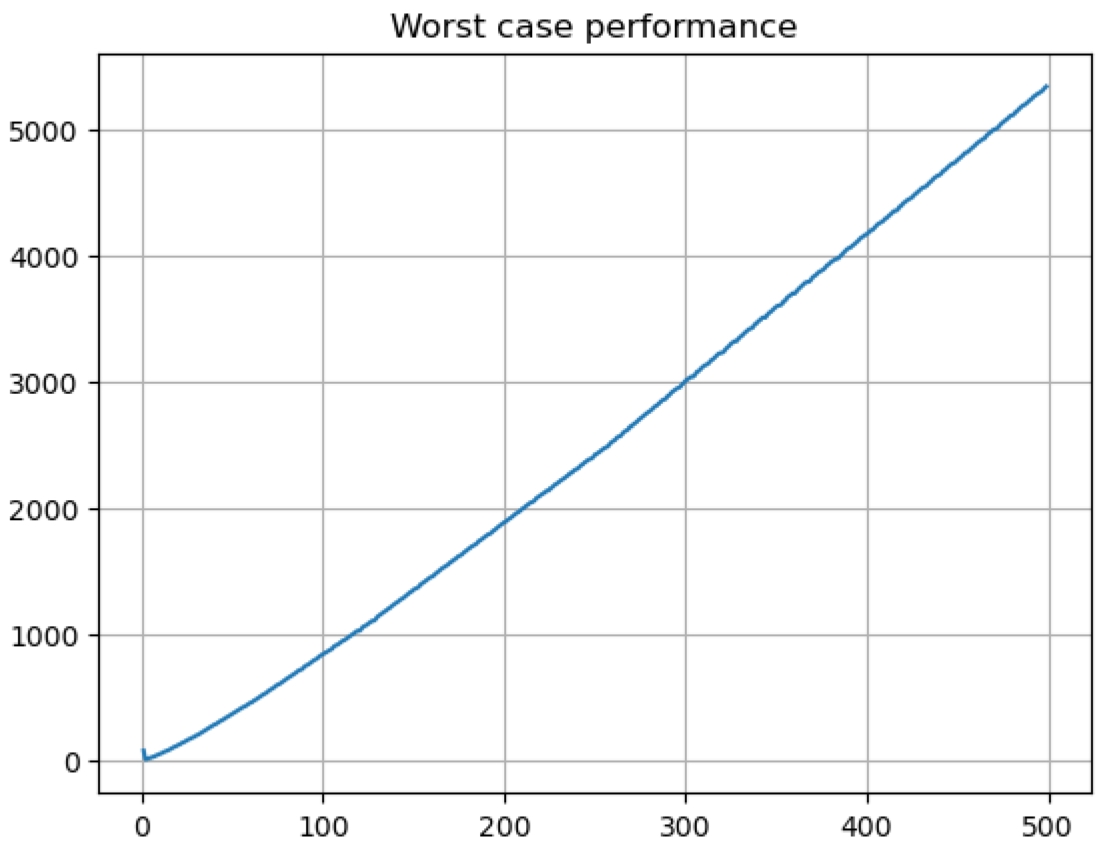


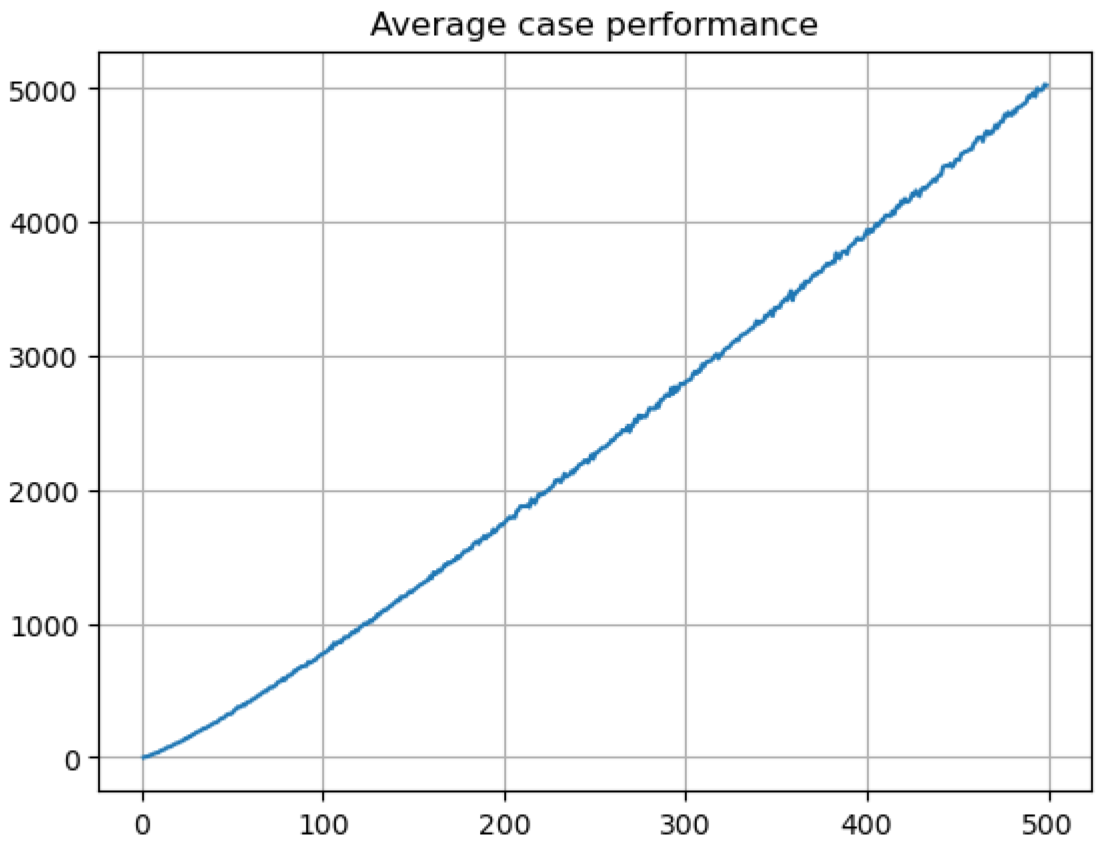
Graph code.

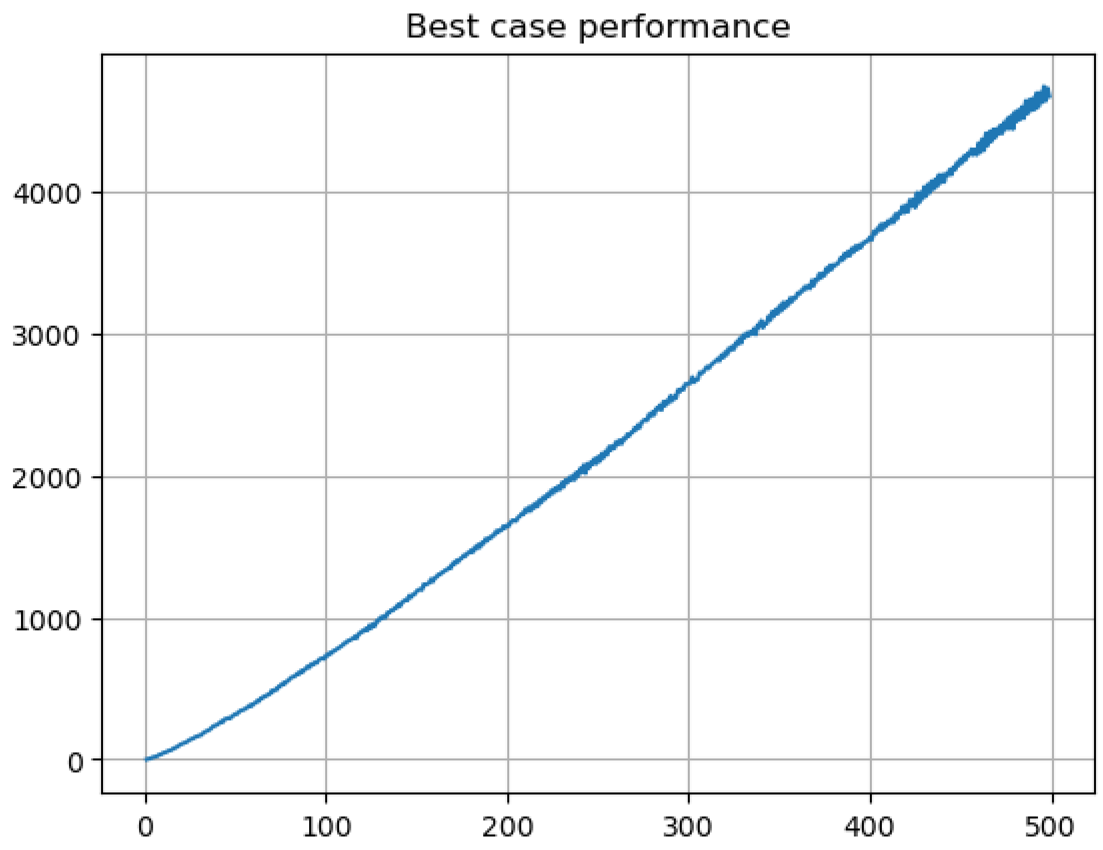
```
import matplotlib.pyplot as plt

import random

times = 0
```

```python
def maxHeap(arr, n):

    global times

    for i in range(n // 2 - 1, -1, -1):

        times += 1

        heapify(arr, n, i)


def heapify(arr, n, i):

    global times

    times += 1

    largest = i

    left_child = 2 * i + 1

    right_child = 2 * i + 2


    if left_child < n and arr[left_child] > arr[largest]:

        largest = left_child


    if right_child < n and arr[right_child] > arr[largest]:

        largest = right_child


    if largest != i:

        arr[i], arr[largest] = arr[largest], arr[i]

        heapify(arr, n, largest)


def heapSort(arr):
    global times
    n = len(arr)

    maxHeap(arr,n)

    for i in range(n - 1, 0, -1):
        times += 1
        arr[0], arr[i] = arr[i], arr[0]
        heapify(arr, i, 0)

arr = [12, 11, 13, 5, 6, 7, 10, 15, 90, 117, 95, 80, 77, -100, 0]
heapSort(arr)
print("Sorted array:", arr)

x = []
y = []
for tam in range(1,500):
    l = []
    x.append(tam)
    for value in range(tam):
        l.append(value)
    heapSort(l)
    y.append(times)
    times = 0
```

```
plt.plot(x,y)

plt.title('Worst case performance')

plt.grid(True)

plt.show()


x = []

y = []

for tam in range(1,500):

    l = []

    x.append(tam)

    for value in range(tam):

        l.append(random.randint(-500,500))

    heapSort(l)

    y.append(times)

    times = 0


plt.plot(x,y)

plt.title('Average case performance')

plt.grid(True)

plt.show()


x = []

y = []

for tam in range(1,500):

    l = []
    x.append(tam)

    for value in range(tam):

        l.append(value)

        l.reverse()

    heapSort(l)

    y.append(times)

    times = 0


plt.plot(x,y)

plt.title('Best case performance')

plt.grid(True)

plt.show()
```


Worst case performance
0
1000
2000
3000
4000
5000
0
100
200
300
400
500


Average case performance
0
1000
2000
3000
4000
5000
0
100
200
300
400
500



Quicksort.

Quicksort is a comparison based sorting algorithm created by Tony Hoare in 1959 and published in 1961.

Uses a divide and conquer approach, selecting a pivot for partitioning the data into sub-arrays in a recursive way.

Implementation.

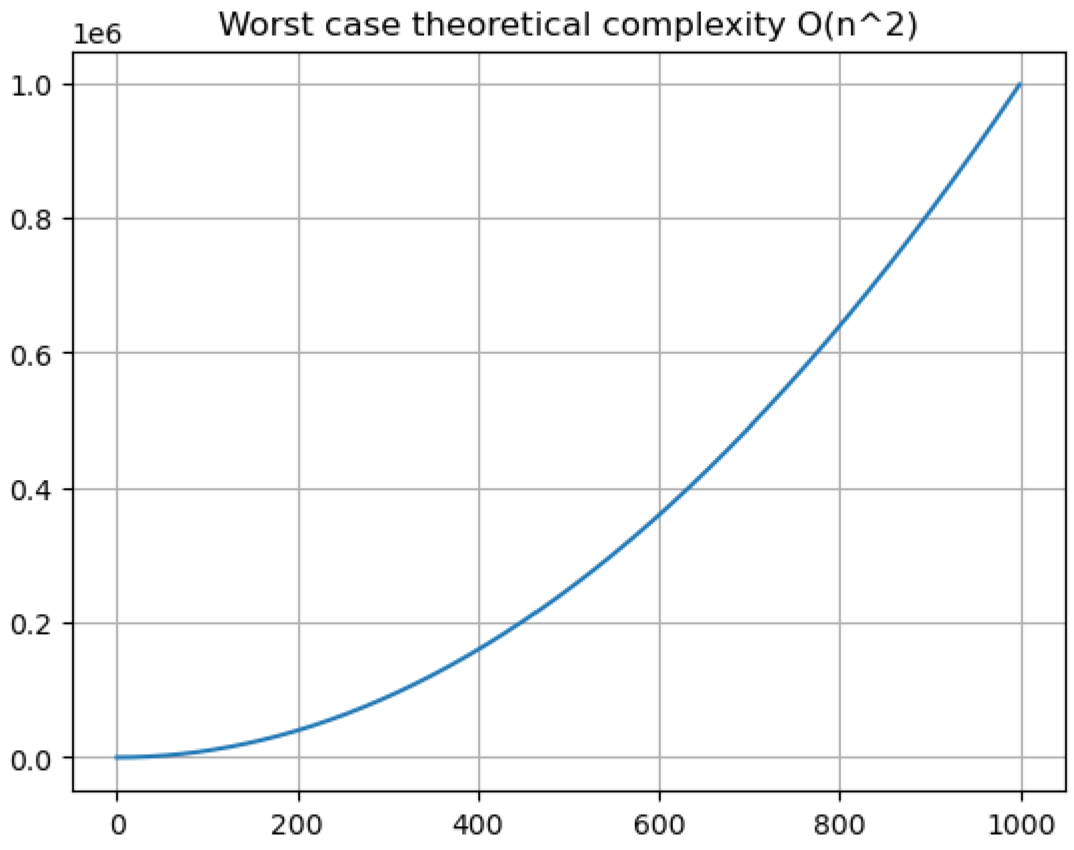
```
# Function to find the partition position

def partition(array, low, high):



    # Choose the rightmost element as pivot

    pivot = array[high] # 4
    # Pointer for greater element

    i = low - 1 # 5


    for j in range(low, high): # 5 (n+1)

        if array[j] <= pivot: # 5(n)


            i = i + 1 # 5(n) (Gets the greater element)


            (array[i], array[j]) = (array[j], array[i]) # 6(n)


    (array[i + 1], array[high]) = (array[high], array[i + 1]) # 6


    # Return the position from where partition is done

    return i + 1 # 1


    # Partition polynomial: 16n + 21 = O(n)

     # This is the function that decides the partition, is always the same
complexity.




def quicksort(array, low, high):

    if low < high: # 4


         pi = partition(array, low, high) # 4 (16n + 21) // This is called
in loop recursively by quicksort, so
```

```
                                                              #                    // it is
multiplied by the quicksort complexity


          # Recursive call on the left of pivot

           quicksort(array, low, pi - 1) # (n/2) * (16n + 21) || (2 log n) *
(16n + 21) || (log n) * (16 + 21)


          # Recursive call on the right of pivot

          quicksort(array, pi + 1, high) # (n/2) * (16n + 21) || (2 log n) *
(16n + 21) || (log n) * (16 + 21)


     # Quicksort polynomial (Worst case): 16(n^2) + 37 * 4 (n) + 84

      # Quicksort polynomial (Average case): 16 (n log n) + 64 (n) + 42 (log
n) + 84

      # Quicksort polynomial (Best case): 8 (n log n) + 64 (n) + 21 (log n)
+ 42




# Driver code

if __name__ == '__main__':

     array = [10, 7, 8, 9, 1, 5, -8, -5, 0, 49]

     N = len(array)


     # Function call

     quicksort(array, 0, N - 1)

     print('Sorted array:')

     for x in array:
        print(x, end=" ")
```

Complexity cases explanation.

- Worst case:

The worst is in which we have an array and in each partition, the element moves only +1 place, making n^2/2 comparisons just like a bubble sort.

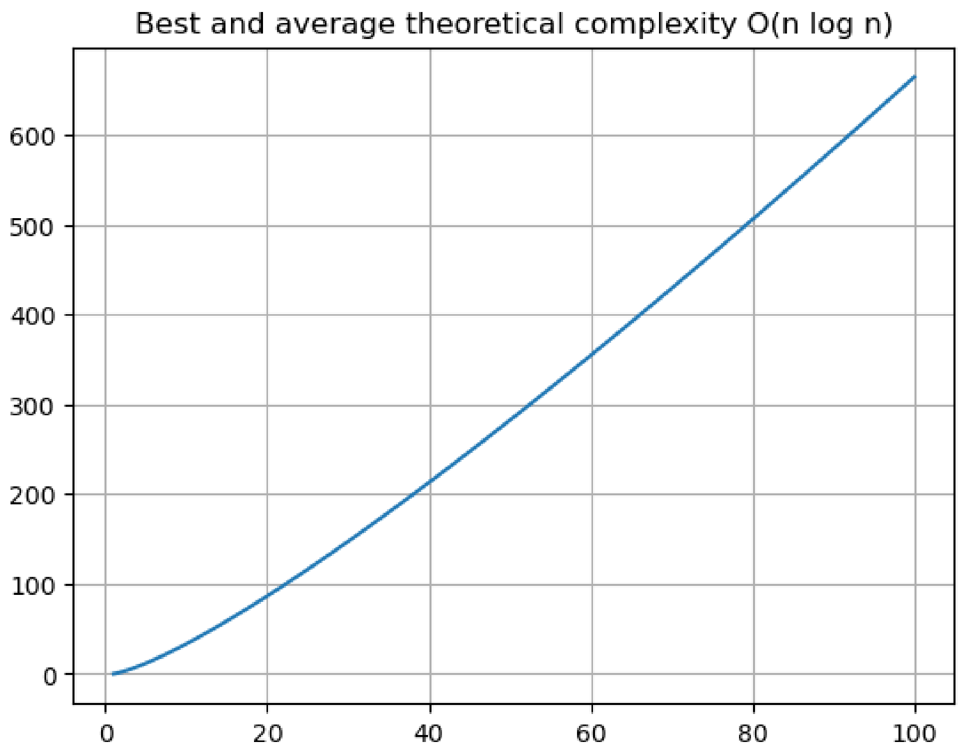


- Average case:

The average case is in which we have an array and in each partition, the element moves approximately 1/4 of n places, dividing the arrays into two but a.size = 1/4 and b.size = 3/4 respectively of the array on each partition

- Best case:

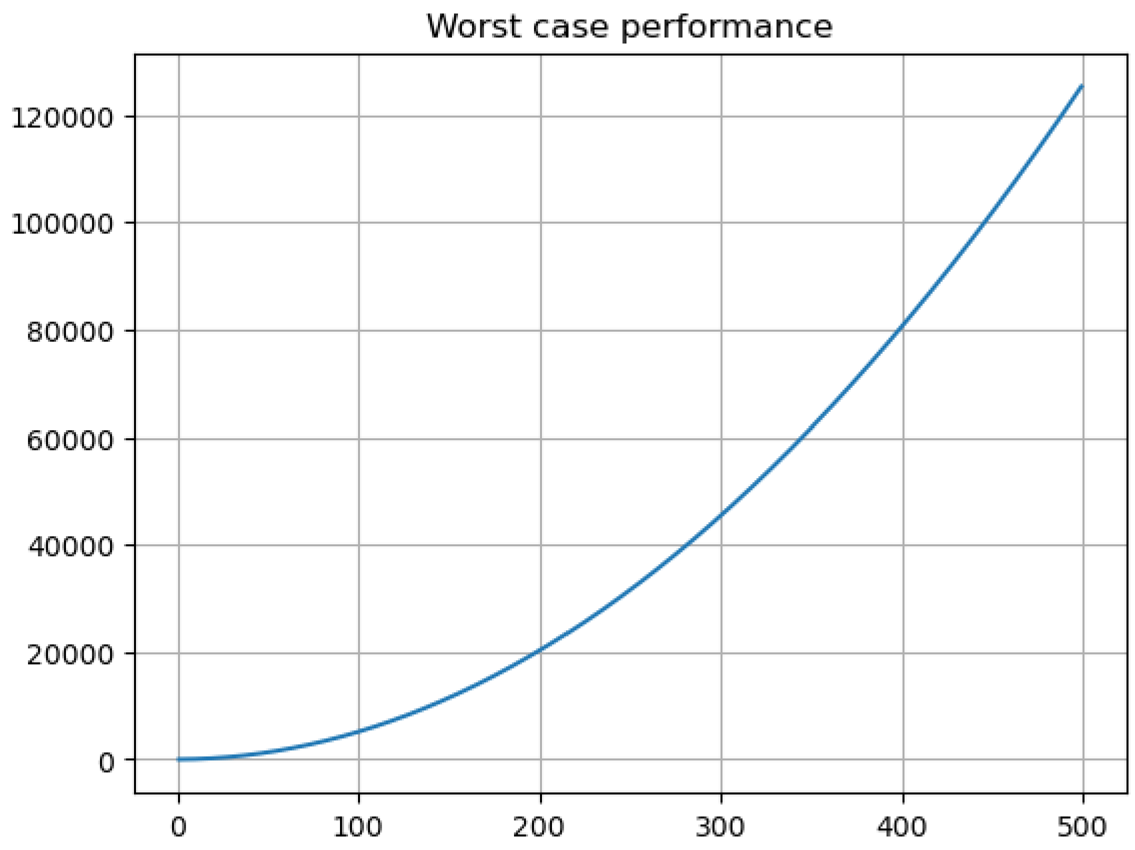
The best case is in which we have an array and in each partition, the element moves 1/2 of n places, and the length of the array is odd, making it just like merge sort but with the two pointers making the process of sorting

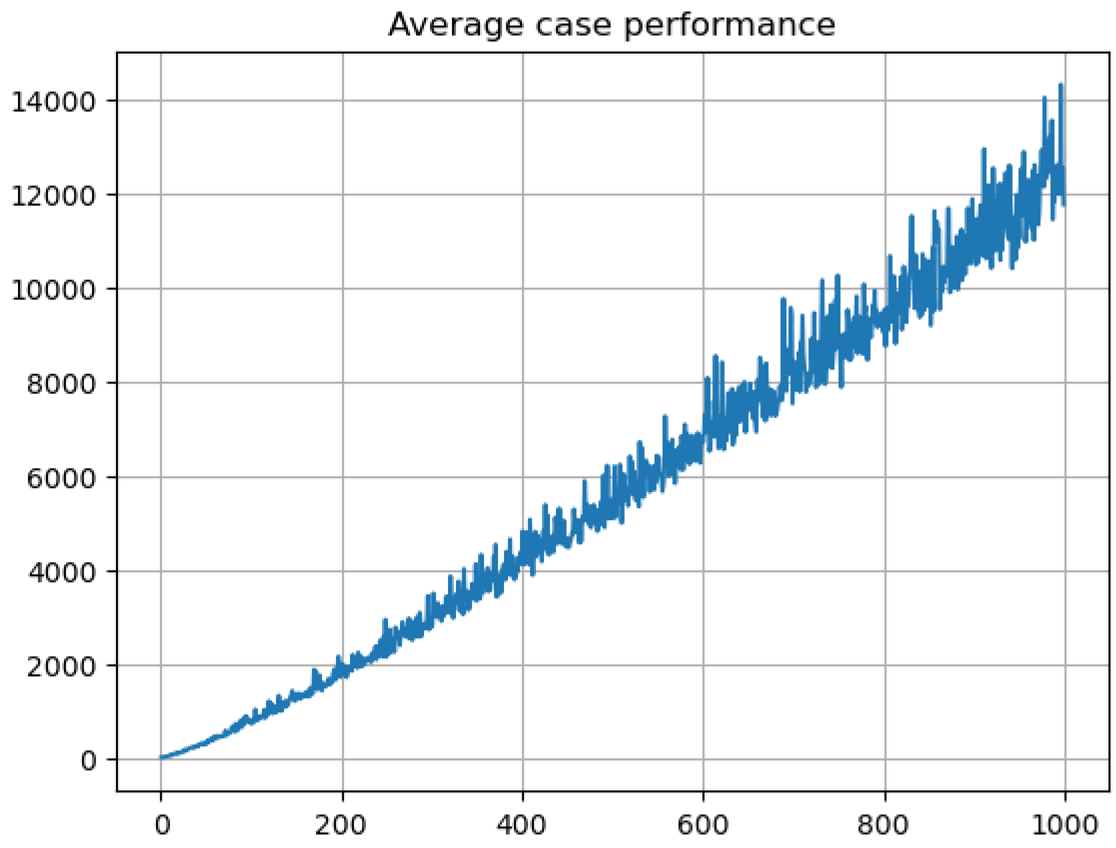


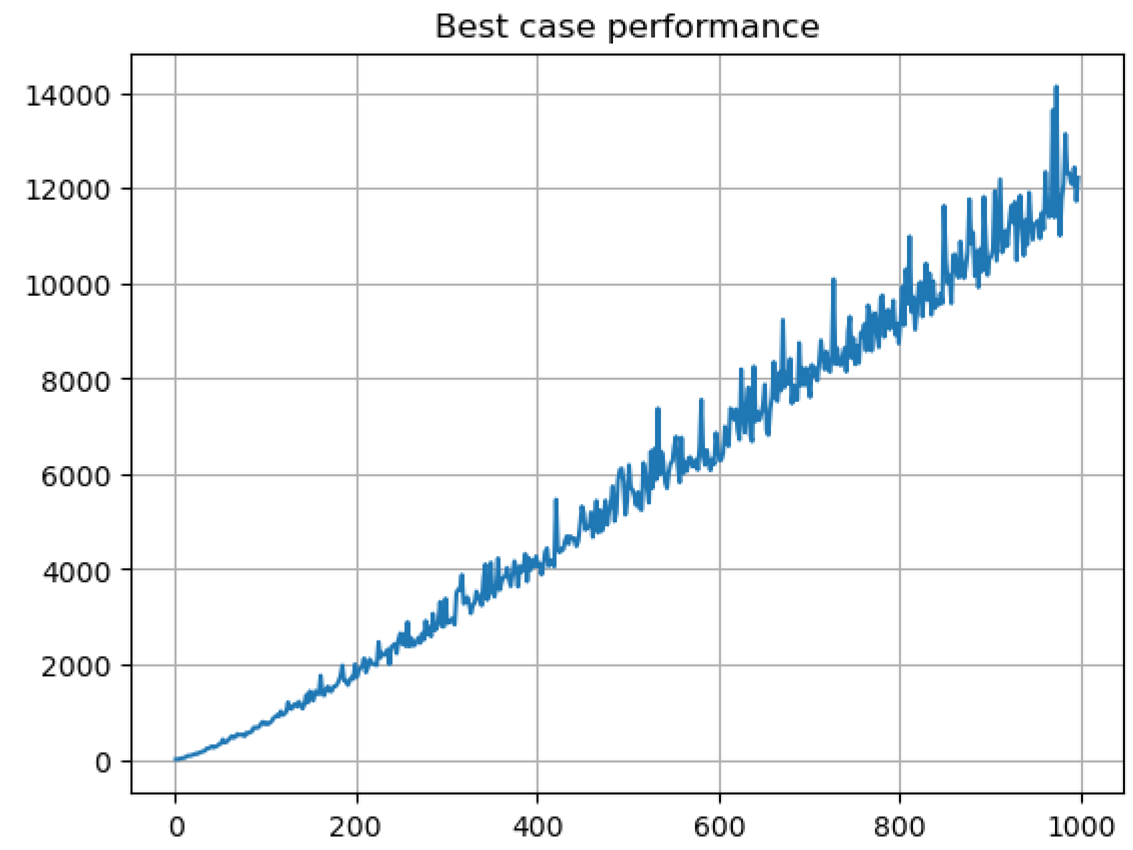
Graph code.

```
import matplotlib.pyplot as plt

import random

times = 0


def partition(array, low, high):

    global times

    pivot = array[high]

    i = low - 1


    for j in range(low, high):
        times += 1

        if array[j] <= pivot:

            i = i + 1

            (array[i], array[j]) = (array[j], array[i])

    (array[i + 1], array[high]) = (array[high], array[i + 1])


    return i + 1


def quicksort(array, low, high):

    global times

    times += 1

    if low < high:

        pi = partition(array, low, high)

        quicksort(array, low, pi - 1)

        quicksort(array, pi + 1, high)


x = []

y = []

for tam in range(1,500):

    l = []

    x.append(tam)

    for value in range(tam):

        l.append(value)

    quicksort(l,0,tam-1)

    y.append(times)
    times = 0


plt.plot(x,y)

plt.title('Worst case performance')

plt.grid(True)

plt.show()


x = []

y = []

for tam in range(1,1000):

    l = []

    x.append(tam)

    for value in range(tam):

        l.append(random.randint(-500,500))

    quicksort(l,0,tam-1)

    y.append(times)

    times = 0


plt.plot(x,y)

plt.title('Average case performance')

plt.grid(True)

plt.show()


x = []

y = []
```

```python
for tam in range(1,999,+2):

    l = []

    x.append(tam)

    count = 1

    num = 0

    mid = tam//2

    for value in range(tam):

        l.append(random.randint(-500,500))

    mid = tam // 2

    l[mid], l[tam - 1] = l[tam - 1], l[mid]

    #for i in range(tam - mid):

        #l[i] = mid + i

    #print(l)

    quicksort(l,0,tam-1)

    y.append(times)

    times = 0


plt.plot(x,y)

plt.title('Best case performance')

plt.grid(True)

plt.show()
```


Worst case performance
120000
100000
80000
60000
40000
20000
0
0
100
200
300
400
500


Average case performance
14000
12000
10000
8000
6000
4000
2000
0
0
200
400
600
800
1000



Conclusion: It was difficult to realize which input gives us the complexity cases for Quicksort, it was complex to think of the permutations of the array that forces the algorithm to the distinct cases.

For heapsort, it was very interesting to implement the heap, because it works for many other applications like a priority queue. I've missed using data structures since DSA I the last semester.

I’ve enjoyed this practice so much and I think the next one will be a challenge for ending this topic of sorting algorithms.