# ICPC MX 2025 Reference (Date 3 version)

Falsificamos el INE de Wicho

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## Table of Contents

1	C++	- Functions		4.7	BFS Iterative Flood Fill	. [
	1.1	Common STL Algorithms		4.8	DFS Flood Fill	١
2		ry search in the answer		4.9	Lava Flow (Multi-source BFS)	ا
3	Data	Structures		4.10	Dijkstra	.(
	3.1	Fenwick Tree			Bellman Ford (With path restoring)	
	3.2	Fenwick Minimum			SPFA Bellman Ford	
	3.3	1-Indexed Fenwick Tree			Floyd-Warshall	
	3.4	Fenwick 2D (Sum query)			Prim's Algorithm (MST)	
	3.5	Fenwick 2D (Counting in range)			Kruskal's Algorithm (MST)	
	3.6	Fenwick Tree Range Update - Point Query 8			<u> </u>	
	3.7	Fenwick Tree - Range update and query 8			Another Kruskal	
	3.8	Segment Tree (Iterative)			Kosaraju Algorithm (SCC)	
	3.9	Segment Tree (Sum query)			SCC	
	3.10	Segment Tree (Minimum query)			Tarjan algorithm (SCC) $\dots \dots \dots$	
	3.11	Segment Tree Lazy Propagation			Finding Articulation Points	
	3.12	Segment Tree 2D		4.21	Finding bridges $\ldots$ $\ldots$ $\ldots$ $\ldots$ $\ldots$ $\ldots$ $\ldots$ 3	(
	3.13	Segment tree with Index Compression		4.22	Finding Bridges Online	,]
	3.14	Segment Tree Preffix-Suffix-Biggest		4.23	Bridge Tree	12
	3.15	Persistent Array		4.24	2-SAT	,
	3.16	Path Copying - Persistent Array		4.25	Hierholzer's Algorithm (Eulerian Path)	, 2
	3.17 3.18	Persistent Segment Tree			Gale-Shapley Algorithm (Stable marriage)	
	3.18	Policy Ordered Set	5			
	3.19 $3.20$	DSU to detect cycles	Ŭ		Succesor	
	3.20 $3.21$	DSU to check online bipartitness			Euler Tour	
	3.21	DSU with rollback			Lowest Common Ancestor	
	3.23	Dynamic connectivity			Binary Lifting	
	3.24	Trie			· ·	
	3.25	Palindromic Tree			Cartesian Tree	
	3.26	Implicit Treap			Heavy-Light Decomposition	
	3.27	Treap			Centroid Decomposition	
4		bh Theory			Tree Distances	
	4.1	Bipartite Check BFS	6		s	
	4.2	Cycle Detection DFS			Ford-Fulkerson Maximum Flow	
	4.3	Topological Sort		6.2	Dinic's Max Flow	(
	4.4	Kahn's Algorithm		6.3	Min-cost Flow	):
	4.5	Lexicographically Min. TopoSort		6.4	Hungarian Algorithm	. ]
	4.6	BFS Flood Fill		6.5	Kuhn's Algorithm	. 1

Falsificamos el INE de Wicho
Page 2 of 67

7	Dyn	amic Programming	10 Linear Algebra
	7.1	Coin Problem (Count ways)	10.1 Determinant of a Matrix
	7.2	Coin Problem (Count sorted ways)	10.2 Rank of a Matrix
	7.3	Coin Problem (Minimum)	10.3 Gauss-Jordan
	7.4	Counting paths	10.4 Matrix Exponentiation
	7.5	Longest Increasing Subsequence	11 Geometry
	7.6	Length of LIS	11.1 Line Segment Intersection
	7.7	Longest Common Subsequence	11.2 Minimum Euclidian Distance
	7.8	Edit Distance	11.3 Point in polygon
	7.9	Bitmask DP	11.4 Point Location Test
	7.10	Digit DP	11.5 Polygon Area
		Double DP	11.6 Convex Hull
			11.7 Complex point
8		h	11.8 Polar sort
	8.1	Miller Rabin	12 Strings
	8.2	Sieve of Erathostenes	12.1 Marranadas de Quique
	8.3	Sieve of Eratosthenes (count primes)	12.3 Rolling Hash
	8.4	Segmented Sieve	12.4 Hash marrano
	8.5	Linear sieve	12.5 Suffix Array
	8.6	Sum of divisors	12.6 LCP
	8.7	Finding the divisors of a number (Trial Division) 46	12.7 Z Function
	8.8	Factorials	12.8 Longest Palindrome
	8.9	Binpow	12.9 String Hashing
	8.10	Modulo Inverse	12.10 Manacher Algorithm
	8.11	BinPow Modulo Inv	12.11 Suffix Automaton
	8.12	Binomial Coefficients	13 Formulas
	8.13	Newton Method (Sqrt and iSqrt)	13.1 Sums
	8.14	Integration with Simpson Method	13.2 Catalan numbers
	8.15	Ternary Search	13.3 Cayley's Formula
	8.16	DP Pascal triangle 1D	13.4 Geometric series
	8.17	DP Pascal triangle 2D	13.5 Divisors
	8.18	Euler's Totient	13.6 Number of primes between 1 and n $\dots \dots $
	8.19	Diophantine equations	13.7 Pythagorean triplets
	8.20	Discrete Log	13.8 Derangments
		~	14 Miscellaneous
		rnomials	14.1 Random number generator
			14.2 Custom comparators
	9.2	NTT	14.3 Kadane's Algorithm
	9.3	Berlekamp Messey	14.4 Moore's Voting Algorithm

Falsificamos el INE de Wicho

.5	Mar	ranadas de $C++$								. (	67
	15.1	Compilation								. (	67
	15.2	Compiler optimizations.								. (	67
	15.3	Decimal printing								. (	6
	15.4	Bit tricks								. (	67

## 1 C++ Functions

## 1.1 Common STL Algorithms

## Sorting Algorithms

Function	Parameters	Description
sort	begin, end, [comp]	Standard unstable sort (O(n log
		n))
stable_sort	begin, end, [comp]	Stable sort preserves element order
is_sorted	begin, end, [comp]	Checks if range is sorted (returns
		bool)
$nth\_element$	begin, nth, end, [comp]	Partitions around nth element

## **Searching Functions**

Function	Parameters	Description
lower_bound	begin, end, val, [comp]	First element $\leq$ value
upper_bound	begin, end, val, [comp]	First element ¿ value
binary_search	begin, end, val, [comp]	Existence check in sorted range
find	begin, end, val	Linear search for value
find_if	begin, end, pred	Find first matching predicate

## **Sequence Operations**

Function	Parameters	Description
reverse	begin, end	Reverse elements in-place
rotate	begin, mid, end	Rotate elements left
next_permutation	begin, end	Generate next permutation
unique	begin, end, [pred]	Remove consecutive duplicates
remove	begin, end, val	Remove elements equal to value

## **Numerical Functions**

Function	Parameters	Description
accumulate	begin, end, init, [op]	Sum/accumulate elements
partial_sum	begin, end, dest, [op]	Compute prefix sums
gcd	a, b	Greatest common divisor (C++17)
lcm	a, b	Least common multiple (C++17)
iota	begin, end, val	Fill with consecutive values

## Memory/Array Operations

Function	Parameters	Description
memset	ptr, value, count	Fill memory with byte value
fill	begin, end, value	Fill range with value
fill_n	begin, count, value	Fill N elements with value
copy	src_b, src_e, dest	Copy range to destination
copy_if	src_b, src_e, dest, pred	Copy elements matching predicate

## **Utility Functions**

Function	Parameters	Description
swap	a, b	Swap two values
max_element	begin, end, [comp]	Find maximum element
min_element	begin, end, [comp]	Find minimum element
count	begin, end, val	Count element occurrences
all_of	begin, end, pred	Check all elements satisfy condition

## Mathematical / Bitwise Builtins

Function	Parameters	Description
_builtin_popcount	x (int)	Count number of 1-bits
_builtin_popcountll	x (long long)	Count number of 1-bits (64-
		bit)
_builtin_clz	x (unsigned int)	Count leading zeros
_builtin_clzll	x (unsigned long long)	Leading zeros (64-bit)
_builtin_ctz	x (unsigned int)	Count trailing zeros
_builtin_ctzll	x (unsigned long long)	Trailing zeros (64-bit)
_builtin_parity	X	Return 1 if #bits is odd
_builtin_ffs	x	Position of least significant
		1-bit (1-indexed)
lg	X	Floor of $\log_2(x)$ (index of
		highest bit)

## Priority Queues and Heaps

Function	Parameters	Description
priority_queue	[type], [container], [comp]	Max-heap by default
make_heap	begin, end, [comp]	Turn range into heap
push_heap	begin, end, [comp]	Push element into heap
pop_heap	begin, end, [comp]	Pop max element into end
sort_heap	begin, end, [comp]	Heap sort

## Set / Map Utilities

Operation	Usage	Description
$s.lower\_bound(x)$	set/map	First element $\geq x$
$s.upper\_bound(x)$	set/map	First element $> x$
$s.equal\_range(x)$	multiset/map	Pair of lower/upper bound
s.erase(it)	iterator	Erase element at iterator
s.find(x)	key	Iterator to key or end

## **String Functions**

Function	Parameters	Description
stoi, stol, stoll	string, [pos], [base]	Convert string $\rightarrow$ int/long/ll
stoul, stoull	string, [pos], [base]	Convert string $\rightarrow$ unsigned
stod, stof, stold	string	Convert string $\rightarrow$ double/float/-
		long double
to_string	value	Convert number $\rightarrow$ string
substr	pos, len	Substring
find	str, pos	Find first occurrence
rfind	str, pos	Find last occurrence

### Random Number Utilities

Type / Function	Usage	Description
mt19937 rng	chrono::steady_clock::now()	Fast random gen-
		erator
uniform_int_distribution	dist(a,b)(rng)	Random int in
		[a,b]
shuffle	begin, end, rng	Random shuffle

#### Other Useful Utilities

Function	Parameters	Description
chrono::high_resolution_clock	now()	Get precise current
		time
int128	value	128-bit integer (man-
		ual I/O needed)
bitset¡N¿	ops: &, —, ; jj, ¿¿	Fixed-size bitset ma-
		nipulation
tuple	get < i > (t)	Store and access het-
		erogenous data
pair	first, second	Store pair of values

## 2 Binary search in the answer

```
// Standard binary search (iterative)
      int binary_search(vector<int>& arr, int target) {
          int left = 0, right = arr.size() - 1;
          while (left <= right) {</pre>
              int mid = left + (right - left) / 2;
              if (arr[mid] == target) return mid;
              if (arr[mid] < target) left = mid + 1;</pre>
              else right = mid - 1;
10
          return -1;
13
      // Lower bound (first element >= target)
      int lower_bound(vector<int>& arr, int target) {
          int left = 0, right = arr.size();
15
          while (left < right) {</pre>
              int mid = left + (right - left) / 2;
              arr[mid] < target ? left = mid + 1</pre>
                                 : right = mid;
19
20
21
          return left;
22
23
      // Upper bound (first element > target)
      int upper_bound(vector<int>& arr, int target) {
26
          int left = 0, right = arr.size();
          while (left < right) {</pre>
              int mid = left + (right - left) / 2;
              arr[mid] <= target ? left = mid + 1
29
                                   : right = mid;
31
32
          return left;
```

```
}
33
34
      // Binary search on real numbers (e.g. sqrt)
      double sqrt_precision(double n, double eps=1e-6) {
36
          double left = 0, right = n;
37
          for (int i = 0; i < 100; ++i) { // or while (right-left >
38
              eps)
              double mid = (left + right) / 2;
39
              if (mid*mid < n) left = mid;</pre>
40
              else right = mid;
41
          return left;
43
44
45
      // Binary search on answer space (monotonic condition)
46
47
      int find min valid(vector<int>& nums. int k) {
          auto is_valid = [&](int x) {
48
              /* condition check */
49
          }:
50
51
          int left = 0, right = 1e9; // adjust bounds
52
53
          while (left < right) {</pre>
              int mid = left + (right - left) / 2;
54
              is_valid(mid) ? right = mid
55
                            : left = mid + 1;
57
          return left;
58
      } z
```

## 3 Data Structures

#### 3.1 Fenwick Tree

```
16 }
17
18
      FenwickTree(vector<int> const &a) : FenwickTree(a.size()) {
          for (size_t i = 0; i < a.size(); i++)</pre>
19
               add(i, a[i]);
20
      }
21
22
      int sum(int r) {
23
24
          int ret = 0;
          for (: r \ge 0: r = (r \& (r + 1)) - 1)
25
26
               ret += bit[r]:
27
          return ret;
      }
28
29
      int sum(int 1, int r) {
30
          return sum(r) - sum(l - 1):
31
32
33
34
      void add(int idx, int delta) {
35
          for (; idx < n; idx = idx | (idx + 1))
36
               bit[idx] += delta:
      }
37
38 };
```

#### 3.2 Fenwick Minimum

```
struct FenwickTreeMin {
      vector < int > bit;
      int n:
      const int INF = (int)1e9;
      FenwickTreeMin(int n) {
           this \rightarrow n = n;
           bit.assign(n, INF);
10
      FenwickTreeMin(vector<int> a) : FenwickTreeMin(a.size()) {
11
           for (size_t i = 0; i < a.size(); i++)</pre>
12
               update(i, a[i]);
13
14
16
      int getmin(int r) {
17
          int ret = INF;
          for (; r \ge 0; r = (r & (r + 1)) - 1)
18
               ret = min(ret, bit[r]);
19
20
          return ret;
      }
21
22
      void update(int idx, int val) {
```

#### 3.3 1-Indexed Fenwick Tree

```
struct FenwickTreeOneBasedIndexing {
      vector<int> bit; // binary indexed tree
      int n:
      FenwickTreeOneBasedIndexing(int n) {
          this \rightarrow n = n + 1:
          bit.assign(n + 1, 0);
      FenwickTreeOneBasedIndexing(vector<int> a)
10
          : FenwickTreeOneBasedIndexing(a.size()) {
11
          for (size_t i = 0; i < a.size(); i++)</pre>
12
              add(i, a[i]);
      }
14
      int sum(int idx) {
          int ret = 0;
17
          for (++idx; idx > 0; idx -= idx & -idx)
18
              ret += bit[idx]:
20
          return ret;
      }
21
22
      int sum(int 1, int r) {
23
          return sum(r) - sum(1 - 1);
24
25
26
      void add(int idx, int delta) {
27
          for (++idx; idx < n; idx += idx & -idx)
              bit[idx] += delta:
29
30
31 };
```

#### 3.4 Fenwick 2D (Sum query)

```
struct Fenwick2D {
    vector<vector<int>> tree;
    int rows, cols;

Fenwick2D(int r, int c) : rows(r), cols(c),
    tree(r + 1, vector<int>(c + 1)) {}
```

```
// Update: add delta to (x, y) (1-based)
      void update(int x, int y, int delta) {
9
10
          for(int i = x; i <= rows; i += lsb(i))</pre>
              for(int j = y; j \le cols; j += lsb(j))
                  tree[i][j] += delta;
12
      }
13
14
      // Query sum from (1,1) to (x,y)
15
      int query(int x, int y) {
16
          int sum = 0:
          for(int i = x; i > 0; i -= lsb(i))
18
              for(int j = y; j > 0; j -= lsb(j))
19
                  sum += tree[i][i]:
21
          return sum;
22
23
      // Range sum from (x1,y1) to (x2,y2)
24
25
      int range_query(int x1, int y1, int x2, int y2) {
          return query(x2, y2) - query(x1-1, y2)
26
27
               - query (x2, y1-1) + query (x1-1, y1-1);
28
29
30
      int lsb(int i) { return i & -i; }
31 };
```

## 3.5 Fenwick 2D (Counting in range)

```
struct Fenwick2DPerType {
      int rows, cols;
      unordered_map < int , Fenwick2D > trees; // Map from type to 2D
          Fenwick Tree
      Fenwick2DPerType(int r, int c) : rows(r), cols(c) {}
      // Update: add 'delta' objects of type 't' at position (x, y)
      void update(int t, int x, int y, int delta) {
          if (trees.find(t) == trees.end()) {
              trees[t] = Fenwick2D(rows, cols);
11
12
          trees[t].update(x, y, delta);
      }
13
14
      // Query: count of type 't' in rectangle [x1,y1] to [x2,y2]
      int query(int t, int x1, int y1, int x2, int y2) {
16
          if (trees.find(t) == trees.end()) return 0:
17
18
          return trees[t].range_query(x1, y1, x2, y2);
19
20 };
21
```

```
22 // Requires the base Fenwick2D implementation from previous answer
23 struct Fenwick2D {
      vector < vector < int >> tree;
      int rows, cols;
25
26
      Fenwick2D(int r, int c) : rows(r), cols(c),
27
          tree(r + 1, vector\langle int \rangle(c + 1)) {}
28
29
      void update(int x, int y, int delta) { /* same as before */ }
30
31
      int query(int x, int y) { /* same as before */ }
33
      int range_query(int x1, int y1, int x2, int y2) { /* same as
34
          before */ }
35
36
      int lsb(int i) { return i & -i: }
37 };
```

### 3.6 Fenwick Tree Range Update - Point Query

```
1 // Range Update - Point Query (1-based indexing)
2 struct FenwickRUQ {
      int n:
      std::vector<int> bit;
      FenwickRUQ(int size) : n(size + 1), bit(size + 2) {}
      // Add val to range [1, r] (1-based)
      void range_add(int 1, int r, int val) {
          add(1, val):
          add(r + 1, -val);
11
12
      // Get value at position idx (1-based)
14
      int point_query(int idx) {
          int res = 0:
16
          for(; idx > 0; idx -= idx & -idx)
17
              res += bit[idx]:
18
          return res;
19
20
21
      void add(int idx. int val) {
          for (: idx < n: idx += idx & -idx)
24
              bit[idx] += val;
27 };
```

#### 3.7 Fenwick Tree - Range update and query

```
1 // Range Update - Range Query (1-based indexing)
2 struct FenwickRURQ {
      int n:
      std::vector<int> B1, B2;
      FenwickRURQ(int size) : n(size + 1), B1(size + 2), B2(size +
      // Add val to range [1, r] (1-based)
      void range_add(int 1, int r, int val) {
          add(B1, 1, val):
          add(B1, r + 1, -val);
12
          add(B2, 1, val * (1 - 1));
          add(B2, r + 1, -val * r):
13
14
15
16
      // Get sum of range [1, r] (1-based)
      int range_sum(int 1, int r) {
17
          return prefix_sum(r) - prefix_sum(l - 1);
19
20
21 private:
      void add(std::vector<int>& b, int idx, int val) {
          for(: idx < n: idx += idx & -idx)</pre>
24
              b[idx] += val;
25
      }
26
27
      int sum(const std::vector<int>& b, int idx) {
28
          int total = 0:
          for(; idx > 0; idx -= idx & -idx)
29
              total += b[idx];
30
31
          return total:
      }
32
33
34
      int prefix_sum(int idx) {
          return sum(B1, idx) * idx - sum(B2, idx);
35
36
37 };
```

#### 3.8 Segment Tree (Iterative)

```
int segtree [2*100000 + 5];

void build(vector<int> &arr, int n){
    for(int i=0; i<n; i++)
        segtree[n+i] = arr[i];
</pre>
```

```
for(int i=n-1; i>=1; i--)
               segtree[i] = max(segtree[2*i], segtree[2*i+1]);
10
      void update(int pos, int value, int n){
11
          pos+=n;
          segtree[pos] = value;
13
14
          while(pos>1){
15
16
               segtree[pos] = max(segtree[2*pos],segtree[2*pos+1]);
17
18
      }
19
20
      int query(int 1, int r, int n){
21
22
          1 += n:
          r += n;
23
24
          int mx = INT MIN:
25
26
          while(1 <= r){
27
28
              if(1 % 2 == 1) mx = max(mx, segtree[1++]);
              if (r \% 2 == 0) mx = max(mx, segtree[r--]);
29
              1 >>= 1:
30
               r >>= 1;
31
          }
32
33
          return mx;
```

## 3.9 Segment Tree (Sum query)

```
1 11 t[4*MAX];
3 // Shout-out to CP algo for the SegTree implementation: https://cp
      -algorithms.com/data_structures/segment_tree.html#memory-
      efficient-implementation
void buildSegTree(vector<11> &a, int v, int tl, int tr) {
      if (t1 == tr) {
          t[v] = a[t1];
      } else {
          int tm = (tl + tr) / 2;
          buildSegTree(a, v*2, t1, tm);
          buildSegTree(a, v*2+1, tm+1, tr):
11
          t[v] = t[v*2] + t[v*2+1];
12
13
14 }
15
```

```
16
17 ll sum(int v, int tl, int tr, int l, int r) {
      if (1 > r)
19
          return 0;
      if (1 == t1 && r == tr) {
          return t[v];
21
22
      int tm = (t1 + tr) / 2:
23
24
      return sum(v*2, tl, tm, l, min(r, tm))
             + sum(v*2+1, tm+1, tr, max(1, tm+1), r);
25
26 }
27
28 void update(int v, int tl, int tr, int pos, ll new_val) {
      if (t1 == tr) {
          t[v] = new_val;
31
      } else {
          int tm = (tl + tr) / 2;
32
33
          if (pos <= tm)
34
              update(v*2, tl, tm, pos, new_val);
35
          else
              update(v*2+1, tm+1, tr, pos, new_val);
36
          t[v] = t[v*2] + t[v*2+1];
37
38
39 }
```

## 3.10 Segment Tree (Minimum query)

```
1 11 t[4*MAX];
3 // Shout-out to CP algo for the SegTree implementation: https://cp
      -algorithms.com/data_structures/segment_tree.html#memory-
      efficient-implementation
void buildSegTree(vector<11> &a, int v, int tl, int tr) {
     if (t1 == tr) {
          t[v] = a[t1];
      } else {
          int tm = (t1 + tr) / 2;
          buildSegTree(a, v*2, t1, tm);
10
          buildSegTree(a, v*2+1, tm+1, tr);
          t[v] = min(t[v*2], t[v*2+1]); // Change to minimum
12
     }
14 }
15
17 ll query(int v, int tl, int tr, int l, int r) {
     if (1 > r)
19
         return LLONG_MAX; // Return maximum possible value for
              empty range
```

```
if (1 == t1 && r == tr) {
20
          return t[v];
21
22
      int tm = (tl + tr) / 2;
23
      return min(query(v*2, t1, tm, 1, min(r, tm)),
24
                  query(v*2+1, tm+1, tr, max(1, tm+1), r));
25
26 }
27
void update(int v, int tl, int tr, int pos, ll new_val) {
      if (t1 == tr) {
29
          t[v] = new val:
30
      } else {
31
          int tm = (tl + tr) / 2:
32
          if (pos <= tm)</pre>
33
               update(v*2, t1, tm, pos, new_val);
34
35
               update(v*2+1, tm+1, tr, pos, new_val);
36
37
          t[v] = min(t[v*2], t[v*2+1]); // Change to minimum
38
39 }
```

## 3.11 Segment Tree Lazy Propagation

```
1 typedef long long 11;
2 typedef vector <int> vec;
3 typedef vector < pair < int , int >> vpii;
4 const 11 mod=1e9+7;
5 const int MAX=1e5+3;
6 const int limit=2e5+3:
const int TAM=2e5+1;
8 11 t[4*TAM];
9 11 op [4*TAM];
10 int type [4*TAM];
//ascii https://elcodigoascii.com.ar/
void propagate(int root, int 1, int r)
14 {
      if (type[root] == 1)
16
          t[root]+=op[root]*(r+1-1);
17
          if(1!=r){
18
               op[2*root]+=op[root];
19
               op [2*root+1]+=op [root];
20
               type [2*root+1] = max(1, type [2*root+1]);
21
               type[2*root] = max(1, type[2*root]);
22
23
      }
24
25
      else
      {
```

```
if (type[root] == 2) {
28
               t[root] = op[root] * (r+1-1);
29
               if(1!=r){
                   op[2*root]=op[root];
30
                   op [2*root+1] = op [root];
31
                   type [2*root+1]=2;
32
                   type[2*root]=2;
33
               }
34
35
          }
36
37
      op[root]=0;
38
      type[root]=0;
40
41 void build(int root, int 1, int r, vector<11> &arr)
43
      if(1==r)
44
45
           t[root]=arr[1]:
46
           op[root]=0;
47
           type[root]=0;
48
           return;
49
50
      int mid=(1+r)/2:
51
      build(2*root,1,mid,arr);
      build(2*root+1,mid+1,r,arr);
53
      t[root]=t[2*root]+t[2*root+1];
54
      op[root]=0;
55
      type[root]=0;
56 }
58 void sum(int root.int l.int r.int a.int b.ll val)
59 {
      propagate(root,1,r);
      if(a>b) return;
61
      if(l==a && r==b)
62
63
           op[root]=val;
64
           type[root]=1;
65
           propagate(root,1,r);
66
67
           return;
68
69
      int mid=(1+r)/2;
      sum(2*root,1,mid,a,min(b,mid),val);
71
      sum(2*root+1,mid+1,r,max(mid+1,a),b,val);
72
      t[root] = t[2*root] + t[2*root+1];
73 }
void setR(int root, int l, int r, int a, int b, ll val)
76 {
      propagate(root,1,r);
```

```
if(a>b) return;
       if(l==a && r==b)
79
           op[root]=val;
81
           type[root]=2;
82
           propagate(root,1,r);
           return;
84
85
       int mid=(1+r)/2;
       setR(2*root,1,mid,a,min(b,mid),val);
87
       setR(2*root+1, mid+1, r, max(mid+1, a), b, val);
       t[root] = t[2*root] + t[2*root+1];
89
90 }
91
92 11 consult(int root, int 1, int r, int a, int b)
93 {
       propagate(root,1,r);
94
95
       if(a>b) return 0;
       if(l==a && r==b){
96
           return t[root];
97
98
       int mid=(1+r)/2;
       return consult(2*root,1,mid,a,min(b,mid))+
100
       consult(2*root+1, mid+1, r, max(mid+1, a), b);
101
102 }
```

## 3.12 Segment Tree 2D

```
1 typedef long long 11;
2 typedef vector <int> vec;
3 const ll mod=1e9+7;
4 const int TAM=1e3+1;
5 //ascii https://elcodigoascii.com.ar/
6 vector < vector < int >> forest(TAM, vector < int > (TAM));
7 11 t[4*TAM][4*TAM];
s int n;
10 void buildNode(int root, int 1, int r, int node, vector < int & arr) {
      if(l==r)
12
          t[node][root]=arr[1];
13
          return:
14
      int mid=(1+r)/2;
      buildNode(2*root.1.mid.node.arr):
17
18
      buildNode(2*root+1,mid+1,r,node,arr);
      t[node][root]=t[node][2*root]+t[node][2*root+1];
19
20 }
21
```

```
void build(int root,int 1,int r,vector<vector<int>> &arr)
23 {
24
      if(l==r)
      {
25
           buildNode(1,0,n-1,root,arr[1]);
26
          return;
27
28
      int mid=(1+r)/2:
29
      build(2*root,1,mid,arr);
30
      build(2*root+1.mid+1.r.arr):
31
      FO(i,4*TAM) t[root][i]=t[2*root][i]+t[2*root+1][i];
32
33
34 }
35
36 void updateNode(int root, int 1, int r, int y, int node, int val)
38
      if(1==r)
39
40
          t[node][root]=val:
41
           return;
42
43
      int mid=(1+r)/2;
      if(v>mid)
44
45
46
           updateNode(2*root+1,mid+1,r,y,node,val);
47
      }
48
      else{
49
           updateNode(2*root,1,mid,y,node,val);
50
      t[node][root]=t[node][2*root]+t[node][2*root+1];
52 }
53
54 void update(int root, int l, int r, int x, int y, int val)
55 {
      if(l==r)
56
57
58
           updateNode(1,0,n-1,y,root,val);
59
           return:
      }
60
      int mid=(1+r)/2:
61
      if(x>mid)
62
63
64
           update(2*root+1,mid+1,r,x,y,val);
65
66
      else{
67
           update(2*root,1,mid,x,y,val);
68
69
      int i=0,j=n-1,Ndt=1,mid_aux;
70
      while(i!=j)
71
72
           mid_aux=(i+j)/2;
```

```
t[root][Ndt]=t[2*root][Ndt]+t[2*root+1][Ndt];
           if (v>mid_aux) {
74
               i=mid_aux+1;
               Ndt = 2 * Ndt + 1;
           }
77
           else{
                j=mid_aux;
               Ndt *=2;
80
81
82
       t[root][Ndt]=t[2*root][Ndt]+t[2*root+1][Ndt];
83
84 }
85
86 11 consultNode(int root, int 1, int r, int node, int y1, int y2)
87 {
88
       if(v1>v2) return 0:
       if(l==y1 && r==y2) return t[node][root];
89
90
       int mid=(1+r)/2;
       return consultNode(2*root,1,mid,node,y1,min(y2,mid))+
91
       consultNode(2*root+1, mid+1, r, node, max(mid+1, y1), y2);
92
93 }
95 ll consult(int root, int l, int r, int x1, int x2, int y1, int y2)
96 {
97
       if(x1>x2) return 0;
       if (1==x1 \& x r==x2) return consultNode (1,0,n-1,root,y1,y2);
98
99
       return consult(2*root,1,mid,x1,min(x2,mid),y1,y2)+
       consult(2*root+1, mid+1, r, max(mid+1, x1), x2, y1, y2);
101
102 }
```

## 3.13 Segment tree with Index Compression

```
1 typedef long long 11;
typedef vector <int> vec;
3 typedef vector < pair < int , int >> vpii;
4 const ll mod=1e9+7:
5 const int MAX=4e5+3;
6 const int limit=2e5+3;
const int TAM=2e5+1;
8 11 t[4*MAX];
9 //ascii https://elcodigoascii.com.ar/
10
void update(int root, int l, int r, int pos, int val)
13 {
      if(l==r)
14
15
      {
16
          t[root]+=val;
```

```
return;
18
19
      int mid=(1+r)/2;
      if (pos>mid)
20
21
           update(2*root+1,mid+1,r,pos,val);
22
      }
23
      elsef
24
           update(2*root,1,mid,pos,val);
25
26
27
      t[root] = t[2*root] + t[2*root+1];
28 }
29
30 ll consult(int root, int l, int r, int a, int b)
31 {
32
      if(a>b) return 0:
33
      if(l==a && r==b) return t[root];
34
      int mid=(1+r)/2:
      return consult(2*root.1.mid.a.min(b.mid))+
35
36
      consult(2*root+1, mid+1, r, max(mid+1, a), b);
37 }
38
39 inline void solve()
40 {
41
     int n,m,index;
42
     cin>>n>>m:
     vector<ll> arr(n):
     vector < tuple < char , ll , ll >> queries (m);
44
     set < 11 > salary;
45
46
     memset(t,0,sizeof(t));
47
     FO(i,n){
           11 aux: cin>>aux:
49
           arr[i]=aux;
50
           salary.insert(aux);
     }
51
52
     FO(i,m)
53
54
           char a;
55
          11 b,c;
56
           cin>>a>>b>>c:
57
           queries[i]=make_tuple(a,b,c);
           if(a=='!') salary.insert(c);
58
59
     }
60
61
     vector<ll> coord(all(salary));
     int tn=coord.size();
62
     //FO(i,tn) cout << coord[i] << " ";
     //cout << endl:
64
65
     FO(i,n)
66
     {
67
           index=lower_bound(all(coord),arr[i])-coord.begin();
```

```
update(1,0,tn-1,index,1);
68
     }
69
     FO(i,m)
70
71
     {
          char a=get<0>(queries[i]);
72
          11 b=get<1>(queries[i]);
73
          11 c=get<2>(queries[i]);
74
          if(a=='?'){
75
               b=lower_bound(all(coord),b)-coord.begin();
76
               c=(upper_bound(all(coord),c)-coord.begin())-1;
77
               if(b==tn || c==tn ){
                   cout << 0 << end1:
80
               else cout << consult(1,0,tn-1,b,c) << endl;</pre>
81
          }
82
83
          else{
               index=lower_bound(all(coord),arr[b-1])-coord.begin();
84
85
               update(1,0,tn-1,index,-1);
               arr[b-1]=c:
86
               index=lower_bound(all(coord),arr[b-1])-coord.begin();
87
               update(1,0,tn-1,index,1);
88
          }
90
     }
91
92
93 }
```

## 3.14 Segment Tree Preffix-Suffix-Biggest

```
1 typedef long long 11;
2 typedef vector <int> vec;
3 typedef vector < pair < int , int >> vpii;
4 const 11 mod=1e9+7;
5 const int MAX=1e5+3;
6 const int limit=2e5+3;
7 const int TAM=2e5+1;
8 11 t[4*TAM];
9 ll prefix[4*TAM], suffix[4*TAM], biggest[4*TAM];
10 //ascii https://elcodigoascii.com.ar/
void build(int root,int 1,int r,vector<11> &arr)
13 {
      if(1==r)
14
          t[root]=arr[1]:
16
17
           suffix[root] = max(t[root], cero);
          prefix[root] = max(t[root], cero);
18
19
          biggest[root] = max(t[root], cero);
20
          return:
```

```
21
22
      int mid=(1+r)/2;
23
      build(2*root,1,mid,arr);
      build(2*root+1,mid+1,r,arr);
24
25
      t[root] = t[2*root] + t[2*root+1];
26
      biggest[root]=max(biggest[2*root],
27
      max(biggest[2*root+1], suffix[2*root]+prefix[2*root+1]));
      prefix[root] = max(prefix[2*root], t[2*root] + prefix[2*root+1]);
28
29
      suffix[root] = max(suffix[2*root+1],t[2*root+1]+suffix[2*root]);
30
31 }
32
33 void update(int root, int 1, int r, int pos, 11 val)
34 {
35
      if(l==r)
36
37
           t[root]=val;
38
           suffix[root] = max(cero,t[root]);
           prefix[root]=max(cero.t[root]):
39
40
           biggest[root] = max(t[root], cero);
41
           return:
42
      int mid=(1+r)/2;
43
44
      if (pos>mid)
45
      {
46
           update(2*root+1,mid+1,r,pos,val);
47
      }
48
      else{
           update(2*root,1,mid,pos,val);
49
50
51
      t[root] = t[2*root] + t[2*root+1];
52
      biggest[root]=max(biggest[2*root],
      max(biggest[2*root+1], suffix[2*root]+prefix[2*root+1]));
53
      prefix[root] = max(prefix[2*root], t[2*root] + prefix[2*root+1]);
54
55
      suffix[root] = max(suffix[2*root+1],t[2*root+1]+suffix[2*root]);
56 }
57
58 ll consult(int root, int l, int r, int a, int b)
59 {
      if(a>b) return 0;
60
      if(l==a && r==b) return t[root];
      int mid=(1+r)/2:
63
      return consult(2*root,1,mid,a,min(b,mid))+
      consult(2*root+1, mid+1, r, max(mid+1, a), b);
65 }
```

#### 3.15 Persistent Array

vector<pair<int, int>> arr[100001]; // The persistent array

```
3 int get_item(int index, int time) {
     // Gets the array item at a given index and time
          upper_bound(arr[index].begin(), arr[index].end(),
              make_pair(time, INT_MAX));
      return prev(ub)->second;
8 }
void update_item(int index, int value, int time) {
     // Updates the array item at a given index and time
      // Note that this only works if the time is later than all
12
     // update times
13
      assert(arr[index].back().first < time);</pre>
14
      arr[index].push back({time. value}):
16 }
18 void init arr(int n. int *init) {
     // Initializes the persistent array, given an input array
      for (int i = 0; i < n; i++) arr[i].push_back({0, init[i]});</pre>
21 }
```

#### 3.16 Path Copying - Persistent Array

```
struct Node {
     int val;
     Node *1. *r:
     Node(ll x) : val(x), l(nullptr), r(nullptr) {}
     Node(Node *11, Node *rr) : val(0), 1(11), r(rr) {}
9 int n, a[100001]; // The initial array and its size
Node *roots[100001]; // The persistent array's roots
11
Node *build(int l = 0, int r = n - 1) {
     if (1 == r) return new Node(a[1]):
     int mid = (1 + r) / 2;
14
     return new Node(build(1, mid), build(mid + 1, r));
15
16 }
17
18 Node *update(Node *node, int val, int pos, int l = 0, int r = n -
19
     if (1 == r) return new Node(val);
     int mid = (1 + r) / 2;
20
     if (pos > mid) return new Node(node->1, update(node->r, val,
21
         pos, mid +1, r));
```

```
else return new Node(update(node->1, val, pos, 1, mid), node->
23 }
24
int query (Node *node, int pos, int 1 = 0, int r = n - 1) {
      if (1 == r) return node->val;
     int mid = (1 + r) / 2;
27
     if (pos > mid) return query(node->r, pos, mid + 1, r);
28
29
      return query(node->1, pos, 1, mid);
30 }
31
32 int get_item(int index, int time) {
      // Gets the array item at a given index and time
34
      return query(roots[time], index);
35 }
37 void update_item(int index, int value, int prev_time, int
      curr time) {
     // Updates the array item at a given index and time
39
      roots[curr_time] = update(roots[prev_time], index, value);
40 }
41
42 void init_arr(int nn, int *init) {
     // Initializes the persistent array, given an input array
     for (int i = 0: i < n: i++) a[i] = init[i]:</pre>
      roots[0] = build():
46
47 }
```

## 3.17 Persistent Segment Tree

```
using 11 = long long;
3 class PersistentSegtree {
   private:
      struct Node {
          11 sum = 0:
          int 1 = 0, r = 0;
      }:
      const int n;
11
      vector < Node > tree:
12
      int timer = 1;
13
      Node join(int 1. int r) { return Node { tree [ 1 ] . sum + tree [ r ] .
14
          sum, 1, r}; }
15
16
      int build(int tl, int tr, const vector<int> &arr) {
          if (tl == tr) {
```

```
tree[timer] = {arr[t1], 0, 0};
18
               return timer++;
19
21
          int mid = (t1 + tr) / 2;
          tree[timer] = join(build(tl, mid, arr), build(mid + 1, tr,
23
               arr)):
24
          return timer++;
25
26
27
      int set(int v, int pos, int val, int tl, int tr) {
28
          if (t1 == tr) {
29
               tree[timer] = {val, 0, 0};
30
               return timer++;
31
32
          }
33
34
          int mid = (tl + tr) / 2;
          if (pos <= mid) {</pre>
35
              tree[timer] = join(set(tree[v].1, pos, val, tl, mid),
                   tree[v].r):
               tree[timer] = join(tree[v].1, set(tree[v].r, pos, val,
38
                    mid + 1, tr));
40
          return timer++:
41
43
      11 range_sum(int v, int ql, int qr, int tl, int tr) {
44
          if (qr < tl || tr < ql) { return 011; }</pre>
45
          if (al <= tl && tr <= ar) { return tree[v].sum: }
46
47
          int mid = (t1 + tr) / 2:
48
          return range_sum(tree[v].1, q1, qr, t1, mid) +
49
                 range_sum(tree[v].r, ql, qr, mid + 1, tr);
50
51
52
53
    public:
      PersistentSegtree(int n, int MX_NODES) : n(n), tree(MX_NODES)
55
56
      int build(const vector<int> &arr) { return build(0, n - 1, arr
57
      int set(int root, int pos, int val) { return set(root, pos,
58
          val. 0. n - 1): }
59
      11 range_sum(int root, int 1, int r) { return range_sum(root,
60
          1. r. 0. n - 1): }
61
```

```
int add_copy(int root) {
    tree[timer] = tree[root];
    return timer++;
}
```

#### 3.18 Policy Ordered Set

```
#include <ext/pb_ds/assoc_container.hpp> // Common file
#include <ext/pb_ds/tree_policy.hpp>
3 #include <functional> // for less
using namespace __gnu_pbds;
6 // To allow repetitions
typedef tree<int, null_type, less<int>, rb_tree_tag,
               tree_order_statistics_node_update>
      ordered_set;
11 // To not allow repetitions
typedef tree < pair < int , int > , null_type ,
               less<pair<int, int> >, rb_tree_tag,
14
               tree_order_statistics_node_update>
15
      ordered_multiset;
ordered_set pt; // Definition
19 pt.order_of_key(x); // Number of items strictly smaller than x
pt.find_by_order(k); // Iterator to the kth element
```

## 3.19 Disjoint Set Union

```
bool unite(int x, int y){
15
               int x_root = find(x);
16
              int y_root = find(y);
17
18
              if(x_root == y_root) {return false;}
19
               if(sizes[x_root] < sizes[y_root]) {swap(x_root,y_root)</pre>
21
               sizes[x_root] += sizes[y_root];
22
               parents[y_root] = x_root;
23
               components --;
24
25
               return true;
26
27
          vector<int> getAllComponentSizes(){
28
               map < int , int > component_sizes;
29
               for (int i = 0; i < parents.size(); ++i){</pre>
30
31
                   int root = find(i);
                   if (component_sizes.find(root) == component_sizes.
32
                       end()){
                       component_sizes[root] = sizes[root];
33
                   }
34
              }
36
               vector<int> result;
37
               for (auto& [root, size] : component_sizes) {
38
                   result.push_back(size);
39
41
               return result;
42
          }
43
44
45
          bool connected(int x, int y) { return find(x) == find(y);}
          int getSize(int x) {return sizes[find(x)];}
47
          int getComponents() const {return components;}
49 };
```

#### 3.20 DSU to detect cycles

```
class CycleDetectionDSU {
    vector<int> parent;
    vector<int> size;

public:
    CycleDetectionDSU(int n) : parent(n), size(n, 1) {
        iota(parent.begin(), parent.end(), 0);
    }
}
```

```
int find(int x) {
          return parent[x] == x ? x : parent[x] = find(parent[x]);
12
13
      // Returns true if adding edge u-v creates a cycle
14
      bool add_edge(int u, int v) {
15
          int u_root = find(u);
16
          int v_root = find(v);
17
18
          if (u_root == v_root) return true;
19
          if (size[u_root] < size[v_root]) swap(u_root, v_root);</pre>
20
          parent[v_root] = u_root;
21
          size[u root] += size[v root]:
          return false;
23
      }
24
25 };
```

## 3.21 DSU to check online bipartitness

```
class BipartiteDSU {
      vector < int > parent;
      vector < int > size;
5 public:
      BipartiteDSU(int n) : parent(2*n), size(2*n, 1) {
          iota(parent.begin(), parent.end(), 0);
8
9
      int find(int x) {
          return parent[x] == x ? x : parent[x] = find(parent[x]);
11
12
13
14
      // Returns true if graph remains bipartite after adding u-v
15
      bool add_edge(int u, int v) {
16
          int u_orig = 2*u;
                                 // Original node
          int u_mirror = 2*u+1; // Mirror node
17
18
          int v_orig = 2*v;
          int v_mirror = 2*v+1;
19
20
          // Union u_orig <-> v_mirror and v_orig <-> u_mirror
21
          for(int i = 0; i < 2; i++) {
22
23
              int x = i ? v_orig : u_orig;
24
              int y = i ? u_mirror : v_mirror;
25
26
              int x_root = find(x);
27
              int v_root = find(v);
28
              if (x_root != y_root) {
29
                   if (size[x_root] < size[y_root]) swap(x_root,</pre>
                       v_root);
```

```
parent[y_root] = x_root;
30
                   size[x_root] += size[v_root];
31
              }
32
          }
33
34
          // Check if u is in both partitions
35
          return find(u_orig) != find(u_mirror);
36
37
38 }
39
40
     -- Other implementation --
42
43 void make_set(int v) {
      parent[v] = make_pair(v, 0);
      rank[v] = 0:
45
      bipartite[v] = true;
46
47 }
48
49 pair < int , int > find_set(int v) {
      if (v != parent[v].first) {
          int parity = parent[v].second;
51
          parent[v] = find_set(parent[v].first);
          parent[v].second ^= parity;
53
      return parent[v];
55
56 }
57
58 void add_edge(int a, int b) {
      pair < int , int > pa = find_set(a);
59
      a = pa.first;
60
      int x = pa.second;
61
62
      pair < int , int > pb = find_set(b);
63
64
      b = pb.first;
      int y = pb.second;
66
      if (a == b) {
67
          if (x == y)
68
               bipartite[a] = false;
69
70
      } else {
          if (rank[a] < rank[b])</pre>
71
72
               swap (a, b);
          parent[b] = make_pair(a, x^y^1);
73
          bipartite[a] &= bipartite[b];
74
          if (rank[a] == rank[b])
75
               ++rank[a]:
76
77
78 }
80 bool is_bipartite(int v) {
```

#### 3.22 DSU with rollback

```
1 class DSU {
private:
      vector < int > p, sz;
      // stores previous unites
      vector < pair < int &, int >> history;
    public:
      DSU(int n): p(n), sz(n, 1) { iota(p.begin(), p.end(), 0); }
      int get(int x) { return x == p[x] ? x : get(p[x]); }
12
      void unite(int a, int b) {
          a = get(a):
13
14
          b = get(b):
          if (a == b) { return; }
15
          if (sz[a] < sz[b]) { swap(a, b); }</pre>
16
17
          // save this unite operation
18
19
          history.push_back({sz[a], sz[a]});
          history.push_back({p[b], p[b]});
20
21
          p[b] = a;
22
23
          sz[a] += sz[b];
      }
24
25
      int snapshot() { return history.size(); }
26
27
28
      void rollback(int until) {
          while (snapshot() > until) {
29
              history.back().first = history.back().second;
30
31
              history.pop_back();
32
          }
33
      }
34 }:
```

## 3.23 Dynamic connectivity

```
struct dsu_save {
   int v, rnkv, u, rnku;

dsu_save() {}

dsu_save(int _v, int _rnkv, int _u, int _rnku)
```

```
: v(_v), rnkv(_rnkv), u(_u), rnku(_rnku) {}
8 };
10 struct dsu_with_rollbacks {
      vector < int > p, rnk;
      int comps;
12
      stack < dsu_save > op;
13
14
      dsu_with_rollbacks() {}
15
16
      dsu with rollbacks(int n) {
17
          p.resize(n);
18
          rnk.resize(n):
19
          for (int i = 0; i < n; i++) {</pre>
20
               p[i] = i;
21
22
               rnk[i] = 0:
23
24
          comps = n;
25
26
      int find set(int v) {
27
          return (v == p[v]) ? v : find_set(p[v]);
28
29
30
      bool unite(int v, int u) {
31
          v = find set(v):
32
          u = find set(u):
33
          if (v == u)
34
               return false;
35
           comps --;
36
          if (rnk[v] > rnk[u])
37
               swap(v. u):
38
          op.push(dsu_save(v, rnk[v], u, rnk[u]));
39
          p[v] = u;
40
           if (rnk[u] == rnk[v])
41
               rnk[u]++;
42
43
          return true;
      }
44
45
      void rollback() {
46
47
          if (op.empty())
               return;
48
49
          dsu_save x = op.top();
          op.pop();
50
          comps++;
51
          p[x.v] = x.v;
52
          rnk[x.v] = x.rnkv;
53
          p[x.u] = x.u;
54
           rnk[x.u] = x.rnku;
56
57 };
```

```
59 struct query {
      int v, u;
       bool united;
       query(int _v, int _u) : v(_v), u(_u) {
63
64 };
65
66 struct QueryTree {
       vector < vector < query >> t;
       dsu_with_rollbacks dsu;
69
       int T:
70
71
       QueryTree() {}
72
73
       QuervTree(int T. int n) : T(T) {
74
           dsu = dsu_with_rollbacks(n);
           t.resize(4 * T + 4);
75
       }
76
77
       void add_to_tree(int v, int l, int r, int ul, int ur, query& q
 78
          ) {
           if (ul > ur)
79
               return:
 80
           if (1 == ul && r == ur) {
81
82
               t[v].push_back(q);
 83
               return:
84
           int mid = (1 + r) / 2:
85
           add_to_tree(2 * v, 1, mid, ul, min(ur, mid), q);
 86
           add to tree(2 * v + 1, mid + 1, r, max(ul, mid + 1), ur, q)
87
               ):
88
      }
89
       void add_query(query q, int 1, int r) {
90
91
           add_to_tree(1, 0, T - 1, 1, r, q);
92
93
94
       void dfs(int v, int 1, int r, vector<int>& ans) {
           for (query& q : t[v]) {
95
96
               q.united = dsu.unite(q.v, q.u);
97
98
           if (1 == r)
               ans[1] = dsu.comps;
           else {
100
               int mid = (1 + r) / 2:
               dfs(2 * v, 1, mid, ans);
102
               dfs(2 * v + 1, mid + 1, r, ans);
103
104
           for (query q : t[v]) {
105
               if (q.united)
106
```

#### 3.24 Trie

```
1 class TrieNode
   public:
      // Array for children nodes of each node
      TrieNode *children[26]:
     // for end of word
     bool isLeaf:
     TrieNode()
11
          isLeaf = false;
          for (int i = 0: i < 26: i++)
13
14
              children[i] = nullptr;
16
      }
17
18
    // Method to insert a key into the Trie
void insert (TrieNode *root, const string &key)
22 {
23
      // Initialize the curr pointer with the root node
24
      TrieNode *curr = root;
25
26
      // Iterate across the length of the string
27
      for (char c : key)
28
29
30
          // Check if the node exists for the
31
          // current character in the Trie
32
33
          if (curr->children[c - 'a'] == nullptr)
34
35
              // If node for current character does
```

```
// not exist then make a new node
              TrieNode *newNode = new TrieNode();
38
39
              // Keep the reference for the newly
40
              // created node
41
              curr->children[c - 'a'] = newNode;
42
43
44
45
          // Move the curr pointer to the
          // newly created node
46
47
          curr = curr->children[c - 'a'];
48
50
      // Mark the end of the word
51
      curr->isLeaf = true:
52 }
54 // Method to search a key in the Trie
55 bool search (TrieNode *root, const string &key)
56 {
57
58
      if (root == nullptr)
59
60
          return false;
61
62
      // Initialize the curr pointer with the root node
63
      TrieNode *curr = root;
64
65
66
      // Iterate across the length of the string
      for (char c : kev)
67
68
69
          // Check if the node exists for the
70
          // current character in the Trie
71
72
          if (curr->children[c - 'a'] == nullptr)
              return false;
73
74
         // Move the curr pointer to the
75
          // already existing node for the
76
          // current character
77
          curr = curr->children[c - 'a'];
78
79
      // Return true if the word exists
82
      // and is marked as ending
83
      return curr->isLeaf:
84 }
86 // Method to check if a prefix exists in the Trie
87 bool isPrefix(TrieNode *root, const string &prefix)
```

```
88 {
      // Initialize the curr pointer with the root node
89
      TrieNode *curr = root;
91
      // Iterate across the length of the prefix string
92
      for (char c : prefix)
94
           // Check if the node exists for the current character in
95
          if (curr->children[c - 'a'] == nullptr)
96
              return false:
97
98
          // Move the curr pointer to the already existing node
99
          // for the current character
100
           curr = curr->children[c - 'a'];
102
      }
      // If we reach here, the prefix exists in the Trie
104
      return true:
105
106
```

#### 3.25 Palindromic Tree

```
const int MAXN = 105000:
3 struct node {
    int next[26];
     int len;
     int sufflink:
     int num;
8 };
10 int len;
char s[MAXN];
12 node tree[MAXN];
13 int num;
                      // node 1 - root with len -1, node 2 - root
     with len O
                     // max suffix palindrome
14 int suff;
15 long long ans;
16
17 bool addLetter(int pos) {
     int cur = suff, curlen = 0;
18
     int let = s[pos] - 'a';
19
     while (true) {
21
          curlen = tree[cur].len;
22
          if (pos - 1 - curlen >= 0 && s[pos - 1 - curlen] == s[pos
              1)
24
              break;
```

```
cur = tree[cur].sufflink;
26
27
     if (tree[cur].next[let]) {
          suff = tree[cur].next[let];
28
          return false;
29
30
31
32
      num++:
33
      suff = num;
      tree[num].len = tree[cur].len + 2;
35
      tree[cur].next[let] = num:
36
      if (tree[num].len == 1) {
37
38
          tree[num].sufflink = 2;
39
          tree[num].num = 1;
40
          return true:
41
     }
42
43
      while (true) {
44
          cur = tree[cur].sufflink;
          curlen = tree[cur].len;
45
          if (pos - 1 - curlen >= 0 && s[pos - 1 - curlen] == s[pos
46
              tree[num].sufflink = tree[cur].next[let]:
              break;
48
49
          }
      }
50
51
      tree[num].num = 1 + tree[tree[num].sufflink].num:
53
54
      return true:
55 }
57 void initTree() {
     num = 2; suff = 2;
      tree[1].len = -1; tree[1].sufflink = 1;
      tree[2].len = 0; tree[2].sufflink = 1;
60
61 }
63 // -- Other implementation --
65 const int maxn = 1e5, sigma = 26;
int s[maxn], len[maxn], link[maxn], to[maxn][sigma];
69 int n, last, sz;
71 void init()
72 {
     s[n++] = -1:
73
     link[0] = 1;
74
```

```
len[1] = -1;
      sz = 2;
76
77 }
79 int get_link(int v)
      while (s[n - len[v] - 2] != s[n - 1]) v = link[v];
81
      return v:
82
83 }
84
85 void add_letter(int c)
86 {
      s[n++] = c:
87
      last = get_link(last);
      if(!to[last][c])
89
90
          len [sz] = len[last] + 2;
91
92
          link[sz] = to[get_link(link[last])][c];
          to[last][c] = sz++:
93
94
      last = to[last][c];
95
```

## 3.26 Implicit Treap

```
using namespace std;
 3 #include < random >
 4 #include < chrono >
6 mt19937 rng(chrono::steady_clock::now().time_since_epoch().count()
      );
 8 #define ll long long
10 struct TreapNode {
      ll key, pr, sz;
12
      TreapNode *1, *r;
13 };
14
15 typedef TreapNode* Treap;
16
int getSize(Treap &t){
      return t ? t->sz : 0;
19 }
20
void updateSize(Treap &t){
      if (t) t\rightarrow sz = 1 + getSize(t\rightarrow 1) + getSize(t\rightarrow r);
23 }
```

```
void split(Treap& t, ll k, Treap &l, Treap &r){
      if(not t) l = r = nullptr;
27
      else if(k < t->key){
           split(t->1,k,l, t->1);
29
30
          r = t;
           updateSize(r);
31
      }else{
32
33
           split(t->r,k,t->r,r);
34
          1 = t:
           updateSize(1);
35
36
37 }
38
39 void insert (Treap& t, Treap a) {
      if(not t) t=a;
41
      else if(a->pr > t->pr){
           split(t, a->key, a->l, a->r);
43
          t = a;
44
      }else{
45
          if(a->key < t-> key) insert(t->1,a);
           else insert(t->r,a);
46
47
      updateSize(t);
48
49 }
50
void merge (Treap &t, Treap 1, Treap r){
      if(not 1) t = r:
53
      else if(not r) t = 1;
54
55
      else if(l \rightarrow pr > r \rightarrow pr){
56
           merge(1->r, 1->r,r);
57
          t=1;
58
           updateSize(t);
59
      }else{
60
           merge (r->1,1,r->1);
61
62
           updateSize(t);
      }
63
64 }
65
of void erase(Treap &t, ll k){
      if(not t) return;
      if(t->key == k) merge(t,t->1, t->r);
69
70
      else{
71
          if(k<t->key) erase(t->1,k);
72
           else erase(t->r, k);
73
74
      updateSize(t);
```

```
75 }
76
77 bool find(Treap& t, ll k){
      if (not t) return false;
      if(t->key == k) return true;
79
      if(k<t->key) return find(t->1,k);
      return find(t->r,k);
81
82 }
83
84 void insertValue(Treap &t, ll k){
      if(not find(t,k)){
85
          Treap new_node = new TreapNode {k,rng(), 0,nullptr,
86
          insert(t, new_node);
88
89 }
91 ll getKth(Treap &t, int k){
      if(!t || k<=0 || k>getSize(t)) return 0;
92
      int leftSize = getSize(t->1);
93
      if(k == leftSize+1) return t->key;
94
      if(k <= leftSize) return getKth(t->1,k);
      return getKth(t->r, k-leftSize-1);
97 }
```

### 3.27 Treap

```
typedef struct item * pitem;
2 struct item {
      int prior, value, cnt;
      bool rev;
      pitem 1, r;
6 };
8 int cnt (pitem it) {
      return it ? it->cnt : 0;
10 }
11
void upd_cnt (pitem it) {
13
          it \rightarrow cnt = cnt(it \rightarrow 1) + cnt(it \rightarrow r) + 1;
14
15 }
16
17 void push (pitem it) {
      if (it && it->rev) {
18
          it->rev = false;
19
           swap (it->1, it->r);
21
          if (it->1) it->1->rev ^= true;
           if (it->r) it->r->rev ^= true;
```

```
23
24 }
25
void merge (pitem & t, pitem 1, pitem r) {
      push (1);
      push (r);
29
      if (!1 || !r)
          t = 1 ? 1 : r;
      else if (1->prior > r->prior)
31
          merge (1->r, 1->r, r), t = 1;
32
33
          merge (r->1, 1, r->1), t = r;
34
35
      upd_cnt (t);
36 }
37
38 void split (pitem t, pitem & 1, pitem & r, int key, int add = 0) {
      if (!t)
40
          return void( 1 = r = 0 );
      push (t):
42
      int cur_key = add + cnt(t->1);
      if (key <= cur_key)</pre>
44
          split (t->1, 1, t->1, key, add), r = t;
45
46
          split (t-r, t-r, r, key, add + 1 + cnt(t-r)), l = t;
47
      upd_cnt (t);
50 void reverse (pitem t, int 1, int r) {
      pitem t1, t2, t3;
      split (t, t1, t2, 1);
      split (t2, t2, t3, r-l+1);
54
      t2->rev ^= true:
      merge (t, t1, t2);
56
      merge (t, t, t3);
57 }
59 void output (pitem t) {
     if (!t) return;
      push (t);
      output (t->1);
63
      printf ("%d ", t->value);
64
      output (t->r);
65 }
```

## 4 Graph Theory

## 4.1 Bipartite Check BFS

```
bool bfs(int s){
      queue < int > q;
      q.push(s);
      color[s] = 1; // Assign the initial color
      while(!q.empty()){
          int u = q.front();
          q.pop();
          // Check all adjacent vertices of u
          for(auto v : adj[u]){
11
              // If v is not colored yet
12
              if(color[v] == 0){
13
                   color[v] = (color[u] == 1) ? 2 : 1;
                   q.push(v);
15
16
              else if (color[v] == color[u]){
17
18
                  return false;
19
20
21
22
      return true;
```

## 4.2 Cycle Detection DFS

```
1 // Thanks CP-Algo for Cycle finding implementation: https://cp-
      algorithms.com/graph/finding-cycle.html
3 bool dfs(int v, int par) { // passing vertex and its parent vertex
      visited[v] = true;
      for (int u : adj[v]) {
          if (u == par) continue; // skipping edge to parent vertex
          if (visited[u]) {
              cycle_end = v;
              cycle_start = u;
              return true;
          parent[u] = v;
12
          if (dfs(u, parent[u]))
13
              return true;
14
      return false;
16
17 }
18
19 void find_cycle() {
      visited.assign(n+1, false);
      parent.assign(n+1, -1);
21
      cvcle_start = -1;
```

```
24
      for (int v = 0; v < n; v++) {
25
           if (!visited[v] && dfs(v, parent[v]))
26
27
28
      if (cycle_start == -1) {
29
           cout << "IMPOSSIBLE" << endl;</pre>
30
      } else {
31
32
           vector<int> cycle;
           cycle.push_back(cycle_start);
33
           for (int v = cycle_end; v != cycle_start; v = parent[v])
34
35
               cycle.push_back(v);
36
           cycle.push_back(cycle_start);
37
38
           cout << cycle.size() << endl;;</pre>
39
           for (int v : cvcle)
40
               cout << v << " ";
41
           cout << endl:</pre>
42
      }
43 }
```

#### 4.3 Topological Sort

```
vector<int> ans:
3 void dfs(int v) {
      visited[v] = true;
      for (int u : adj[v]) {
          if (!visited[u])
7
              dfs(u);
8
9
      ans.push_back(v);
10 }
void topological_sort() {
      visited.assign(n+1, false);
      ans.clear();
14
15
      for (int i = 1; i <= n; ++i) {</pre>
          if (!visited[i]) {
16
17
              dfs(i):
18
19
      reverse(ans.begin(), ans.end());
20
21 }
```

## 4.4 Kahn's Algorithm

```
def kahnTopoSort(self,adj: List[List[int]]) -> List[int]:
          #print(adi)
          in_deg = [0] * len(adj)
          for i in range(len(adj)):
              for u in adj[i]:
                  in_deg[u]+=1
          q = []
          for i in range(len(in_deg)):
              if in_deg[i] == 0:
                  q.append(i)
12
          arns = []
13
          while len(q)>0:
14
              u = q[0]
15
              q.pop(0)
16
              arns.append(u)
17
18
              for v in adj[u]:
19
                  in_deg[v]-=1
20
                  if in_deg[v] == 0:
21
22
                       q.append(v)
23
          print(str(len(arns))+" "+str(len(adj)))
24
          if(len(arns) != len(adj)):
25
              return []
26
27
          return arns
```

## 4.5 Lexicographically Min. TopoSort

```
vector < vector < int >> adj(MAX);
vector < int > in_degree (MAX);
4 vector < int > group_ids(MAX);
5 vector < int > ans;
7 //topological sort implementation: https://cp-algorithms.com/graph
      /topological-sort.html
9 void topological_sort() {
      priority_queue<pair<int, int>, vector<pair<int, int>>, greater
          <pair<int, int>>> pq;
11
      for(int i = 1; i <= n; i++) {</pre>
12
13
          if(in_degree[i] == 0) {
              pq.emplace(group_ids[i], i);
15
      }
16
```

```
18
      while(!pq.empty()) {
19
          int u = pq.top().second;
20
          pq.pop();
          ans.push_back(u);
21
22
          for(int v : adj[u]) {
23
               in_degree[v]--;
24
               if(in_degree[v] == 0) {
25
                   pq.emplace(group_ids[v], v);
26
27
28
          }
      }
29
30
31 }
```

#### 4.6 BFS Flood Fill

```
bool validate(int x, int y){
      if(vis[x][y]) return false;
      if(maze[x][y] == '#') return false;
3
      if(x<0 or x>=n or y<0 or y>=m) return false;
      return true:
5
6 }
8 bool solveMaze(int x, int y){
      queue < pii > q;
10
      q.push(mp(x,y));
11
      vis[x][y] = true;
12
13
      int dx[] = \{1, -1, 0, 0\};
      int dy[] = \{0, 0, 1, -1\};
14
      char move_dir[] = {'D', 'U', 'R', 'L'};
16
17
      while(!q.empty()){
          int u = q.front().fs;
18
          int v = q.front().sc;
19
20
          q.pop();
21
22
          if (maze[u][v] == 'B'){
              while(true){
23
24
                   res.push_back(path[u][v]);
25
26
                   if (res.back() == "U" && u + 1 < n) u++;
                   if(res.back() == 'D' && u - 1 >= 0) u--:
27
28
                   if(res.back() == 'L' && v + 1 < m) v++;
                   if(res.back() == 'R' && v - 1 >= 0) v--;
29
30
31
                   if (u == x and v == y) break;
```

```
32
33
               return true;
              for (int i = 0; i < 4; ++i) {
35
                   int new_u = u + dx[i];
36
                  int new_v = v + dy[i];
                   if (validate(new_u, new_v)) {
38
                       path[new_u][new_v] = move_dir[i];
39
                       vis[new_u][new_v] = true;
40
                       q.push(mp(new_u, new_v));
41
                  }
43
44
45
      return false;
46 }
```

#### 4.7 BFS Iterative Flood Fill

```
void floodFill(int x, int y, char color ,int r, int c) {
      if (maze[x][y] == color) return;
      queue < pii > q;
      q.push(pii(x, y));
      while (!q.empty()) {
          pii currentCoor = q.front();
          q.pop();
          x = currentCoor.fi;
          y = currentCoor.sc;
          if (x >= 0 \&\& x < r \&\& y >= 0 \&\& y < c \&\& maze[x][y] !=
              color) {
              maze[x][v] = color;
              q.push(pii(x + 1, y));
12
              q.push(pii(x - 1, y));
13
              q.push(pii(x, y + 1));
14
              q.push(pii(x, y - 1));
15
16
17
18 }
```

#### 4.8 DFS Flood Fill

```
floodFill(x,y-1,color,board);
8
```

#### 4.9 Lava Flow (Multi-source BFS)

```
struct Cell{
      int x,y,t;
3 };
5 const int MAX = 1005:
6 int n.m:
8 char maze[MAX][MAX];
9 int vis[MAX][MAX];
int player[MAX][MAX];
char path[MAX][MAX];
12 set < pii > isExit;
13 queue < Cell > q;
14 string res;
16 bool isValid(int x, int v){
      if (x < 0 \mid | x >= n \mid | y < 0 \mid | y >= m) return false;
      if(maze[x][y] == '#') return false;
      return true:
19
20 }
21
22 bool isSafe(int x, int y, int u, int v){
      return player[x][y] == -1 and maze[x][y] != 'M' and (vis[x][y]
           == -1 \text{ or player[u][v]} + 1 < vis[x][y]);
24 }
25
void restorePath(int u, int v, int x, int y){
       while (x != u || y != v) {
          res.push_back(path[u][v]);
29
30
          if (res.back() == 'U') u++;
31
          if (res.back() == 'D') u--;
32
          if (res.back() == 'L') v++;
33
          if (res.back() == 'R') v--;
34
35
36 }
37
bool lavaFlow(int x,int y){
          q.push(\{x,y,1\});
40
          player[x][y] = 0;
41
42
      while(!q.empty()){
          int u = q.front().x;
```

```
int v = q.front().y;
44
               int t = q.front().t;
45
          q.pop();
47
48
               vector \langle pii \rangle dir = {{1, 0}, {-1, 0}, {0, 1}, {0, -1}};
49
50
          for(auto it: dir){
51
                   int i = u+it.fs;
52
                   int j = v+it.sc;
53
54
                   if(isValid(i,j)){
55
                        if(t == 0){
56
                            if(vis[i][j] == -1){
57
                                 vis[i][j] = vis[u][v]+1;
58
59
                                 q.push(Cell{i,j,0});
                            }
61
                        }else{
                            if(isSafe(i,j,u,v)){
62
                                 path[i][j] = (it.fs == 1) ? 'D' : (it.
63
                                     fs == -1) ? 'U' : (it.sc == 1) ? '
                                     R': 'L':
                                 player[i][j] = player[u][v]+1;
                                 q.push(Cell{i,j,1});
65
                                 if (isExit.find({i,j}) != isExit.end()
                                     ) {
                                     if (player[i][j] < vis[i][j] ||</pre>
67
                                         vis[i][j] == -1) {
                                         restorePath(i, j, x, y);
68
                                          return true;
69
                                     }
71
                                }
                            }
72
73
                        }
                   }
74
75
76
          }
77
78
      return false;
79
80 }
```

## 4.10 Dijkstra

```
typedef pair<11, ll> pll;

vector<1l> dijkstra(int n, int source, vector<vector<pll>> &adj) {
    vector<1l> dist(n, INF);
    priority_queue<pll, vector<pll>, greater<pll>> pq;
```

```
dist[source] = 0;
      pq.push({0, source});
      while (!pq.empty()) {
          11 d = pq.top().first;
10
          11 u = pq.top().second;
11
12
          pq.pop();
13
          if (d > dist[u]) continue;
14
15
          for (auto &edge : adj[u]) {
16
               ll v = edge.first;
17
               11 weight = edge.second;
18
19
               if (dist[u] + weight < dist[v]) {</pre>
20
21
                   dist[v] = dist[u] + weight:
                   pq.push({dist[v], v});
22
23
24
          }
25
      }
26
27
      return dist;
28 }
```

## 4.11 Bellman Ford (With path restoring)

```
struct Edge {
      int src, dest, weight;
3 };
void bellmanFord(int V, int E, vector<Edge>& edges, int start) {
      vector < int > dist(V+1, INT_MAX);
      dist[start] = 0;
      for (int i = 1; i < V; i++) {</pre>
9
          for (int j = 0; j < E; j++) {
10
              int u = edges[j].src;
11
              int v = edges[j].dest;
12
              int weight = edges[j].weight;
13
               if (dist[u] != INT_MAX && dist[u] + weight < dist[v])</pre>
14
                   dist[v] = dist[u] + weight;
16
17
          }
      }
18
19
20
      for (int j = 0; j < E; j++) {
21
          int u = edges[j].src;
22
          int v = edges[j].dest;
```

```
int weight = edges[j].weight;
23
           if (dist[u] != INT_MAX && dist[u] + weight < dist[v]) {</pre>
24
                //cout << "Graph contains a negative weight cycle\n";</pre>
25
                return;
26
           }
27
      }
28
29
      for(int i=1; i<=V; i++){</pre>
30
           if(dist[i]!=INT_MAX){
31
                cout << dist[i] << " ";
32
           }else{
33
                cout << "30000 ";
34
35
36
       cout << endl;</pre>
37
38
39 }
40
41 void solve()
42 {
43
      vector < int > d(n, INF);
      d[v] = 0;
44
      vector < int > p(n, -1);
45
46
      for (;;) {
47
           bool any = false;
48
           for (Edge e : edges)
49
               if (d[e.a] < INF)
                    if (d[e.b] > d[e.a] + e.cost) {
51
                        d[e.b] = d[e.a] + e.cost;
52
                        p[e.b] = e.a;
53
                         any = true;
54
55
           if (!any)
56
57
               break;
      }
58
59
       if (d[t] == INF)
60
           cout << "No path from " << v << " to " << t << ".";</pre>
61
      else {
62
63
           vector < int > path;
           for (int cur = t; cur != -1; cur = p[cur])
64
65
                path.push_back(cur);
           reverse(path.begin(), path.end());
66
67
           cout << "Path from " << v << " to " << t << ": ";</pre>
68
           for (int u : path)
               cout << u << ' ';
71
72 }
```

#### 4.12 SPFA Bellman Ford

```
const int INF = 1000000000:
vector < vector < pair < int , int >>> adj;
4 bool spfa(int s, vector < int > & d) {
      int n = adj.size();
      d.assign(n, INF);
      vector < int > cnt(n, 0);
      vector < bool > inqueue(n, false);
      queue < int > q;
10
      d[s] = 0:
11
12
      q.push(s);
      inqueue[s] = true;
      while (!q.empty()) {
14
          int v = q.front();
16
           q.pop();
           inqueue[v] = false;
17
18
19
           for (auto edge : adj[v]) {
               int to = edge.first;
20
               int len = edge.second;
21
22
               if (d[v] + len < d[to]) {</pre>
23
                   d[to] = d[v] + len:
24
25
                   if (!inqueue[to]) {
                        q.push(to);
26
27
                        inqueue[to] = true;
                        cnt[to]++;
28
29
                        if (cnt[to] > n)
                            return false; // negative cycle
30
                   }
31
32
               }
          }
33
34
      return true;
35
36 }
```

## 4.13 Floyd-Warshall

9 }

## 4.14 Prim's Algorithm (MST)

```
1 ll prim(int V. int E. vector < vector < pll >> &adi) {
      priority_queue < pll, vector < pll>, greater < pll>> pq;
      vector < bool > visited(V, false);
      11 \text{ res} = 0;
      pq.push({0, 0});
      while(!pq.empty()){
11
           auto p = pq.top();
12
           pq.pop();
13
14
           int wt = p.first;
15
           int u = p.second;
17
18
           if(visited[u] == true){
               continue:
20
21
           res += wt;
22
           visited[u] = true;
23
24
           for(auto v : adj[u]){
25
               if(visited[v.first] == false){
26
                    pq.push({v.second, v.first});
28
           }
29
30
31
      for(int i=0: i<V: i++){</pre>
32
           if(!visited[i])
33
               return -1;
34
35
36
37
      return res;
```

## 4.15 Kruskal's Algorithm (MST)

```
struct Edge { int u, v, weight; };
int kruskal(vector<Edge>& edges, int n) {
```

```
sort(edges.begin(), edges.end(),
          [](Edge& a, Edge& b) { return a.weight < b.weight; });
      DisjointSets dsu(n);
      int total_weight = 0;
      for (Edge& e : edges) {
10
          if (!dsu.connected(e.u, e.v)) {
              dsu.unite(e.u, e.v);
12
              total_weight += e.weight;
13
         }
14
15
16
      return total_weight;
17 }
```

#### 4.16 Another Kruskal

```
struct Edge {
      int u, v, w;
      bool operator < (Edge const& other) {</pre>
          return w < other.w:</pre>
6 };
s int kruskal(int n, vector < Edge > & edges, DisjointSets & dsu, vector <
      Edge > & ans) {
      int cost = 0;
      sort(edges.begin(), edges.end());
      for (Edge e : edges) {
          if (ans.size() == n - 1) break;
12
          if(dsu.unite(e.u, e.v)){
               cost += e.w;
14
               ans.push_back(e);
16
      }
19
      if(ans.size()!=n-1) return -1;
20
      return cost;
21 }
```

## 4.17 Kosaraju Algorithm (SCC)

```
vector<br/>
vector<br/>
visited; // keeps track of which vertices are already<br/>
visited<br/>
// runs depth first search starting at vertex v.<br/>
// each visited vertex is appended to the output vector when dfs<br/>
leaves it.
```

```
5 void dfs(int v, vector<vector<int>> const& adj, vector<int> &
      output) {
      visited[v] = true;
      for (auto u : adj[v])
          if (!visited[u])
              dfs(u, adj, output);
      output.push_back(v);
11 }
12
13 // input: adj -- adjacency list of G
_{14} // output: components -- the strongy connected components in G
15 // output: adj_cond -- adjacency list of G^SCC (by root vertices)
16 void strongly_connected_components(vector<vector<int>> const& adi,
                                     vector < vector < int >> & components ,
17
                                     vector < vector < int >> & adj_cond) {
18
19
      int n = adi.size():
      components.clear(), adj_cond.clear();
20
      vector < int > order: // will be a sorted list of G's vertices by
22
           exit time
23
      visited.assign(n, false);
24
25
      // first series of depth first searches
26
      for (int i = 0; i < n; i++)
27
          if (!visited[i])
28
              dfs(i, adj, order);
29
30
      // create adjacency list of G^T
31
      vector < vector < int >> adj_rev(n);
32
      for (int v = 0; v < n; v++)
33
          for (int u : adi[v])
34
              adj_rev[u].push_back(v);
35
36
37
      visited.assign(n, false);
      reverse(order.begin(), order.end());
38
39
      vector<int> roots(n, 0); // gives the root vertex of a vertex'
40
          s SCC
41
42
      // second series of depth first searches
      for (auto v : order)
43
          if (!visited[v]) {
44
              std::vector<int> component;
45
              dfs(v, adj_rev, component);
46
              components.push_back(component);
47
              int root = *min_element(begin(component), end(
                   component));
              for (auto u : component)
                  roots[u] = root:
51
          }
```

#### 4.18 SCC

```
1 typedef long long ll;
typedef vector<int> vec;
3 const 11 mod=1e9+7;
4 const int MAX=1e5+3;
5 vector < vector < int >> g(MAX);
6 vector < vector < int >> r(MAX);
vector < int > id(MAX):
8 bool visitados[MAX]={false};
9 vector < int > 1:
10
void dfs(int s){
      visitados[s]=true;
      for(int c:g[s]){
          if(!visitados[c]) dfs(c);
14
15
      l.push_back(s);
16
17 }
19 void rdfs(int s, int d)
20 1
21
      visitados[s]=true;
      id[s]=d:
22
      for(int c:r[s])
23
24
           if(!visitados[c]) rdfs(c,d);
25
26
27 }
```

## 4.19 Tarjan algorithm (SCC)

```
/** Takes in an adjacency list and calculates the SCCs of the
    graph. */
class TarjanSolver {
    private:
    vector<vector<int>> rev_adj;
    vector<int> post;
    vector<int> comp;
```

```
vector < bool > visited;
      int timer = 0;
      int id = 0;
11
      void fill_post(int at) {
          visited[at] = true;
13
          for (int n : rev_adj[at]) {
14
               if (!visited[n]) { fill_post(n); }
15
16
          post[at] = timer++;
17
18
19
      void find_comp(int at) {
20
          visited[at] = true;
21
22
          comp[at] = id:
          for (int n : adj[at]) {
23
24
               if (!visited[n]) { find_comp(n); }
25
      }
26
27
    public:
28
      const vector < vector < int >> & adj;
30
      TarjanSolver(const vector<vector<int>> &adj)
31
           : adj(adj), rev_adj(adj.size()), post(adj.size()), comp(
32
               adi.size()).
            visited(adj.size()) {
           vector < int > nodes(adj.size());
34
          for (int n = 0; n < adj.size(); n++) {</pre>
35
               nodes[n] = n:
36
               for (int next : adj[n]) { rev_adj[next].push_back(n);
37
          }
38
39
          for (int n = 0; n < adj.size(); n++) {</pre>
40
               if (!visited[n]) { fill_post(n); }
41
42
           std::sort(nodes.begin(), nodes.end(),
43
                     [&](int n1, int n2) { return post[n1] > post[n2]
44
                         ]; });
45
           visited.assign(adj.size(), false);
46
           for (int n : nodes) {
47
               if (!visited[n]) {
48
                   find_comp(n);
49
                   id++:
51
52
53
54
```

```
int comp_num() const { return id; }
int get_comp(int n) const { return comp[n]; }
};
```

#### 4.20 Finding Articulation Points

```
1 // adi[u] = adiacent nodes of u
2 // ap = AP = articulation points
_3 // p = parent
4 // disc[u] = discovery time of u
5 // low[u] = 'low' node of u
int dfsAP(int u, int p) {
8 int children = 0;
   low[u] = disc[u] = ++Time;
  for (int& v : adj[u]) {
      if (v == p) continue; // we don't want to go back through the
          same path.
12
                             // if we go back is because we found
                                 another wav back
13
      if (!disc[v]) { // if V has not been discovered before
        children++:
14
        dfsAP(v, u); // recursive DFS call
        if (disc[u] <= low[v]) // condition #1</pre>
16
17
          ap[u] = 1;
        low[u] = min(low[u], low[v]); // low[v] might be an ancestor
18
      } else // if v was already discovered means that we found an
19
        low[u] = min(low[u], disc[v]); // finds the ancestor with
            the least discovery time
21
   return children;
23 }
25 void AP() {
   ap = low = disc = vector < int > (adj.size());
for (int u = 0; u < adj.size(); u++)
29
     if (!disc[u])
        ap[u] = dfsAP(u, u) > 1; // condition #2
30
31 }
```

## 4.21 Finding bridges

```
1 // br = bridges, p = parent
2
```

```
3 vector < pair < int , int >> br;
5 int dfsBR(int u, int p) {
   low[u] = disc[u] = ++Time;
   for (int& v : adj[u]) {
      if (v == p) continue; // we don't want to go back through the
          same path.
                             // if we go back is because we found
                                 another way back
      if (!disc[v]) { // if V has not been discovered before
        dfsBR(v, u); // recursive DFS call
11
        if (disc[u] < low[v]) // condition to find a bridge</pre>
12
          br.push_back({u, v});
13
        low[u] = min(low[u], low[v]); // low[v] might be an ancestor
14
      } else // if v was already discovered means that we found an
        low[u] = min(low[u], disc[v]); // finds the ancestor with
16
            the least discovery time
17
18 }
19
20 void BR() {
   low = disc = vector < int > (adj.size());
   Time = 0;
   for (int u = 0; u < adj.size(); u++)</pre>
23
      if (!disc[u])
24
        dfsBR(u, u)
25
26 }
```

## 4.22 Finding Bridges Online

```
vector <int > par, dsu_2ecc, dsu_cc, dsu_cc_size;
2 int bridges;
3 int lca_iteration;
4 vector <int > last_visit;
6 void init(int n) {
      par.resize(n);
      dsu_2ecc.resize(n);
      dsu_cc.resize(n);
      dsu_cc_size.resize(n);
      lca_iteration = 0;
      last_visit.assign(n, 0);
      for (int i=0: i<n: ++i) {</pre>
          dsu_2ecc[i] = i;
14
          dsu_cc[i] = i;
15
16
          dsu_cc_size[i] = 1;
          par[i] = -1;
17
```

```
18
19
      bridges = 0;
20 }
21
22 int find_2ecc(int v) {
      if (v == -1)
23
24
          return -1;
      return dsu_2ecc[v] == v ? v : dsu_2ecc[v] = find_2ecc(dsu_2ecc
25
           [v]):
26 }
27
28 int find_cc(int v) {
      v = find 2ecc(v):
      return dsu_cc[v] == v ? v : dsu_cc[v] = find_cc(dsu_cc[v]);
30
31 }
32
33 void make_root(int v) {
      int root = v:
35
      int child = -1:
36
      while (v != -1) {
          int p = find_2ecc(par[v]);
37
38
          par[v] = child;
          dsu_cc[v] = root;
39
          child = v;
40
41
          v = p;
42
43
      dsu cc size[root] = dsu cc size[child]:
44 }
45
46 void merge_path (int a, int b) {
      ++lca_iteration;
      vector < int > path_a, path_b;
      int lca = -1;
49
50
      while (lca == -1) {
          if (a != -1) {
51
              a = find_2ecc(a);
52
53
               path_a.push_back(a);
54
               if (last_visit[a] == lca_iteration){
                   lca = a;
55
56
                   break;
57
               last_visit[a] = lca_iteration;
58
59
               a = par[a];
60
          if (b != -1) {
61
              b = find_2ecc(b);
62
               path_b.push_back(b);
63
              if (last_visit[b] == lca_iteration){
64
65
                   lca = b;
                   break;
66
67
                   }
```

```
last_visit[b] = lca_iteration;
68
               b = par[b];
69
70
71
      }
72
73
      for (int v : path_a) {
74
           dsu_2ecc[v] = lca;
75
           if (v == lca)
76
               break:
77
           --bridges;
78
79
      for (int v : path_b) {
80
           dsu_2ecc[v] = lca;
81
           if (v == lca)
82
83
               break:
           --bridges;
84
85
86 }
87
88 void add_edge(int a, int b) {
       a = find_2ecc(a);
      b = find_2ecc(b);
90
      if (a == b)
91
92
           return;
93
       int ca = find cc(a):
94
      int cb = find_cc(b);
96
      if (ca != cb) {
97
           ++bridges;
98
           if (dsu_cc_size[ca] > dsu_cc_size[cb]) {
99
               swap(a, b);
100
               swap(ca, cb);
101
           }
102
           make_root(a);
           par[a] = dsu_cc[a] = b;
104
           dsu_cc_size[cb] += dsu_cc_size[a];
105
      } else {
106
           merge_path(a, b);
108
109 }
```

#### 4.23 Bridge Tree

```
vector<pair<int, int>> g[MAXN];
bool used[MAXN], isBridge[MAXM];
int comp[MAXN], tin[MAXN], minAncestor[MAXN];
4
```

```
5 | vector < int > tree [MAXN]; // Store 2-edge-connected component tree.(
      Bridge tree).
void dfs(int v, int p) {
      tin[v] = minAncestor[v] = ++timer;
      used[v] = 1;
      for(auto &e: g[v]) {
10
          int to, id;
          tie(to, id) = e;
12
          if(to == p) continue;
13
          if(used[to]) {
14
               minAncestor[v] = min(minAncestor[v], tin[to]);
15
          } else {
16
17
               dfs(to, v);
               minAncestor[v] = min(minAncestor[v], minAncestor[to]);
18
               if(minAncestor[to] > tin[v]) {
19
                   isBridge[id] = true;
20
21
22
          }
23
      }
24 }
25
26 void dfs1(int v, int p) {
      used[v] = 1;
      comp[v] = compid;
28
29
      for(auto &e: g[v]) {
          int to, id;
30
          tie(to, id) = e;
31
32
          if(isBridge[id]) { // avoid traversing from this edge. so
33
               we get full component.
34
               continue:
35
          if(used[to]) {
36
37
               continue;
38
39
          dfs1(to, v);
      }
40
41 }
42
43 vector <pair <int, int >> edges;
45 void addEdge(int from, int to, int id) {
      g[from].push_back({to, id});
47
      g[to].push_back({from, id});
48
      edges[id] = {from, to};
49 }
51 void initB() {
52
      for(int i = 0; i <= compid; ++i)</pre>
```

```
54
           tree[i].clear();
      for(int i = 1; i <= N; ++i)</pre>
           used[i] = false;
56
      for(int i = 1; i <= M; ++i)</pre>
57
          isBridge[i] = false;
58
59
      timer = 0;
60
      compid = 0;
61
62 }
63
64 void bridge_tree() {
65
      initB():
66
67
      dfs(1, -1); //Assuming graph is connected.
68
69
      for(int i = 1; i <= N; ++i)</pre>
70
          used[i] = 0:
71
72
73
      for(int i = 1; i <= N; ++i) {</pre>
           if(!used[i]) {
74
               dfs1(i, -1);
75
               ++compid;
76
77
           }
      }
78
79
      for(int i = 1; i <= M; ++i) {</pre>
80
           if(isBridge[i]) {
81
               int u. v:
82
               tie(u, v) = edges[i];
83
               // connect two componets using edge.
84
               tree[comp[u]].push_back(comp[v]);
85
               tree[comp[v]].push_back(comp[u]);
86
87
      }
88
89 }
90
91 void init() {
      edges.clear(); edges.resize(M + 1);
92
      for(int i = 1; i <= N; ++i)</pre>
93
           g[i].clear();
94
95 }
```

#### 4.24 2-SAT

```
struct TwoSatSolver {
   int n_vars;
   int n_vertices;
   vector<vector<int>> adj, adj_t;
```

```
vector < bool > used;
      vector<int> order, comp;
      vector < bool > assignment;
      TwoSatSolver(int _n_vars) : n_vars(_n_vars), n_vertices(2 *
          n_vars), adj(n_vertices), adj_t(n_vertices), used(
          n_vertices), order(), comp(n_vertices, -1), assignment(
          n vars) {
          order.reserve(n_vertices);
10
11
12
      void dfs1(int v) {
13
          used[v] = true;
          for (int u : adi[v]) {
14
               if (!used[u])
16
                   dfs1(u):
17
18
          order.push_back(v);
19
20
21
      void dfs2(int v, int cl) {
22
          comp[v] = c1;
23
          for (int u : adj_t[v]) {
               if (comp[u] == -1)
24
25
                   dfs2(u. cl):
26
          }
27
      }
28
29
      bool solve_2SAT() {
          order.clear():
30
          used.assign(n_vertices, false);
31
          for (int i = 0; i < n_vertices; ++i) {</pre>
32
               if (!used[i])
33
34
                   dfs1(i);
          }
35
36
          comp.assign(n_vertices, -1);
37
          for (int i = 0, j = 0; i < n_vertices; ++i) {</pre>
38
               int v = order[n_vertices - i - 1];
39
               if (comp[v] == -1)
40
                   dfs2(v, j++);
41
          }
42
43
44
           assignment.assign(n_vars, false);
          for (int i = 0; i < n_vertices; i += 2) {</pre>
45
46
               if (comp[i] == comp[i + 1])
47
                   return false;
48
               assignment[i / 2] = comp[i] > comp[i + 1];
49
50
          return true;
51
      }
52
```

```
void add_disjunction(int a, bool na, int b, bool nb) {
53
          // na and nb signify whether a and b are to be negated
54
          a = 2 * a ^na;
          b = 2 * b ^n b;
56
          int neg_a = a ^ 1;
57
          int neg_b = b ^ 1;
          adj[neg_a].push_back(b);
59
          adj[neg_b].push_back(a);
60
          adj_t[b].push_back(neg_a);
61
          adj_t[a].push_back(neg_b);
62
63
64
      static void example_usage() {
65
          TwoSatSolver solver(3); // a, b, c
66
          solver.add_disjunction(0, false, 1, true); //
67
          solver.add_disjunction(0, true, 1, true); // not a v
68
          solver.add_disjunction(1, false, 2, false); //
69
          solver.add_disjunction(0, false, 0, false); //
70
          assert(solver.solve_2SAT() == true);
          auto expected = vector < bool > (True, False, True);
72
          assert(solver.assignment == expected);
74
75 };
```

## 4.25 Hierholzer's Algorithm (Eulerian Path)

```
1 int n, m;
vector < vector < int >> g;
3 vector <int> in, out, path;
5 // Undirected
7 int n, m;
8 vector < vector < pair < int , int >>> g;
9 vector < int > path;
10 vector <bool> seen:
12 void dfs(int node) {
      while (!g[node].empty()) {
13
          auto [son, idx] = g[node].back();
          g[node].pop_back();
16
           if (seen[idx]) { continue; }
          seen[idx] = true;
18
           dfs(son);
      }
```

```
20
      path.push_back(node);
21 }
22
23 // Directed
24 void dfs(int node) {
      while (!g[node].empty()) {
          int son = g[node].back();
26
           g[node].pop_back();
27
           dfs(son);
28
29
30
      path.push_back(node);
31 }
```

#### 4.26 Gale-Shapley Algorithm (Stable marriage)

```
1 // Checks if woman 'w' prefers 'm1' over 'm'
bool wPrefersM1OverM(vector<vector<int>> &prefer, int w, int m,
      int m1)
3 {
      int N = prefer[0].size();
      for (int i = 0; i < N; i++)</pre>
5
6
          // If m1 comes before m, w prefers
7
          // her current engagement
8
          if (prefer[w][i] == m1)
10
              return true:
12
          // If m comes before m1, w prefers m
13
          if (prefer[w][i] == m)
14
              return false;
      }
16 }
18 // Implements the stable marriage algorithm
vector<int> stableMarriage(vector<vector<int>> &prefer)
20 {
      int N = prefer[0].size();
21
22
      // Stores women's partners
23
24
      vector < int > wPartner(N, -1);
25
26
      // Tracks free men
27
      vector < bool > mFree(N, false);
      int freeCount = N;
29
30
      while (freeCount > 0)
31
32
          int m;
33
          for (m = 0; m < N; m++)
```

```
if (!mFree[m])
34
                   break;
35
          // Process each woman in m's preference list
37
          for (int i = 0; i < N && !mFree[m]; i++)</pre>
38
39
               int w = prefer[m][i];
40
               if (wPartner[w - N] == -1)
41
42
                   // Engage m and w if w is free
43
                   wPartner[w - N] = m;
                   mFree[m] = true;
45
                   freeCount --:
46
               }
               else
48
49
                   int m1 = wPartner[w - N];
51
                   // If w prefers m over her current partner,
                       reassign
                   if (!wPrefersM1OverM(prefer, w, m, m1))
53
                       wPartner[w - N] = m;
54
                       mFree[m] = true;
                       mFree[m1] = false;
56
                   }
57
              }
58
59
      return wPartner;
61
```

#### 5 Trees

#### 5.1 Succesor

```
const ll mod=1e9+7;
const ll MAX=1e9+1;
const int limit=2e5+1;
const int m=30;
int succesorM[limit][m];
//ascii https://elcodigoascii.com.ar/

inline void solve()
{
   int n,q; cin>>n>>q;
   int res,aux;
   ll k;
   IFOR(i,n){
```

```
14
           cin>>succesorM[i][0];
       }
16
      FOR (j, 1, m)
17
           1FOR(i,n)
18
19
                succesorM[i][j]=succesorM[succesorM[i][j-1]][j-1];
20
21
      }
22
       FO(i,q)
23
24
25
           cin>>res>>k;
           aux=0:
26
27
           while(k)
28
29
                if(k%2){
                     res=succesorM[res][aux];
30
31
32
                k/=2:
33
                aux++;
34
35
            cout << res << endl;</pre>
36
37 }
```

#### 5.2 Euler Tour

```
1 \text{ const int MAXN} = 1e5 + 5;
3 vector < int > adj[MAXN];
4 int in_time[MAXN], out_time[MAXN];
5 int timer = 0;
struct FenwickTree {
      vector < int > bit;
      int n;
10
       FenwickTree(int n) {
           this \rightarrow n = n;
12
13
           bit.assign(n + 1, 0);
14
      }
15
16
       void update(int idx, int delta) {
           for (; idx <= n; idx += idx & -idx)</pre>
17
               bit[idx] += delta:
18
19
20
21
       int query(int idx) {
           int sum = 0;
22
```

```
for (; idx > 0; idx -= idx & -idx)
23
               sum += bit[idx];
24
          return sum;
25
26
27
      int range_query(int 1, int r) {
28
          return query(r) - query(1 - 1);
29
30
31 };
33 void euler tour(int root) {
      stack<tuple<int, int, bool>> st;
      st.push({root, -1, false});
35
36
      while (!st.empty()) {
37
          auto [u, parent, visited] = st.top();
38
          st.pop();
39
40
          if (!visited) {
41
              in_time[u] = ++timer;
42
              st.push({u, parent, true});
43
              for (auto it = adj[u].rbegin(); it != adj[u].rend();
45
                   ++it) {
                   if (*it != parent) {
                       st.push({*it, u, false});
47
48
              }
          } else {
50
              out_time[u] = ++timer;
51
52
53
54 }
```

### 5.3 Lowest Common Ancestor

```
struct LCA {
    vector < int > height, euler, first, segtree;
    vector < bool > visited;
    int n;

LCA(vector < vector < int >> &adj, int root = 0) {
        n = adj.size();
        height.resize(n);
        first.resize(n);
        euler.reserve(n * 2);
        visited.assign(n, false);
        dfs(adj, root);
    int m = euler.size();
```

```
14
           segtree.resize(m * 4);
           build(1, 0, m - 1);
      }
16
17
      void dfs(vector<vector<int>> &adj, int node, int h = 0) {
18
           visited[node] = true;
19
          height[node] = h;
20
          first[node] = euler.size();
21
22
           euler.push_back(node);
          for (auto to : adi[node]) {
               if (!visited[to]) {
24
25
                   dfs(adj, to, h + 1);
                   euler.push_back(node);
27
28
          }
      }
29
30
      void build(int node, int b, int e) {
31
          if (b == e) {
32
33
               segtree[node] = euler[b];
          } else {
34
35
               int mid = (b + e) / 2;
36
               build(node << 1, b, mid);</pre>
37
               build(node << 1 | 1, mid + 1, e);
               int 1 = segtree[node << 1], r = segtree[node << 1 |</pre>
38
                   11:
               segtree[node] = (height[1] < height[r]) ? 1 : r;</pre>
39
          }
40
      }
41
42
43
      int query(int node, int b, int e, int L, int R) {
44
          if (b > R \mid l \in L)
45
               return -1;
          if (b >= L && e <= R)
46
47
               return segtree[node];
          int mid = (b + e) >> 1;
48
49
50
           int left = query(node << 1, b, mid, L, R);</pre>
          int right = query(node << 1 | 1, mid + 1, e, L, R);</pre>
51
          if (left == -1) return right;
52
53
           if (right == -1) return left;
54
           return height[left] < height[right] ? left : right;</pre>
55
56
57
      int lca(int u. int v) {
58
          int left = first[u], right = first[v];
59
          if (left > right)
60
               swap(left, right);
61
          return query(1, 0, euler.size() - 1, left, right);
62
      }
63 };
```

### 5.4 Binary Lifting

```
1 int n, 1;
vector < vector < int >> adj;
4 int timer:
5 vector < int > tin, tout;
6 vector < vector < int >> up;
8 void dfs(int v, int p)
9 {
      tin[v] = ++timer;
      up[v][0] = p;
11
      for (int i = 1; i <= 1; ++i)
12
          up[v][i] = up[up[v][i-1]][i-1];
14
      for (int u : adj[v]) {
          if (u != p)
16
               dfs(u, v);
17
18
19
      tout[v] = ++timer:
20
21 }
23 bool is ancestor(int u. int v)
      return tin[u] <= tin[v] && tout[u] >= tout[v];
25
26 }
27
28 int lca(int u, int v)
29 {
      if (is_ancestor(u, v))
30
          return u:
31
32
      if (is_ancestor(v, u))
33
          return v;
      for (int i = 1; i >= 0; --i) {
34
          if (!is_ancestor(up[u][i], v))
35
               u = up[u][i];
36
37
38
      return up[u][0];
39 }
40
41 void preprocess(int root) {
      tin.resize(n):
42
43
      tout.resize(n);
      timer = 0;
      1 = ceil(log2(n));
45
      up.assign(n, vector<int>(1 + 1));
```

```
dfs(root, root);

{ 8 }
```

#### 5.5 Cartesian Tree

```
vector<int> parent(n, -1);
stack<int> s;
for (int i = 0; i < n; i++) {
   int last = -1;
   while (!s.empty() && A[s.top()] >= A[i]) {
      last = s.top();
      s.pop();
   }
   if (!s.empty())
      parent[i] = s.top();
   if (last >= 0)
      parent[last] = i;
   s.push(i);
}
```

### 5.6 Heavy-Light Decomposition

```
vector <int > parent, depth, heavy, head, pos;
1 int cur_pos;
4 int dfs(int v, vector < vector < int >> const& adj) {
      int size = 1;
      int max c size = 0:
      for (int c : adj[v]) {
          if (c != parent[v]) {
              parent[c] = v, depth[c] = depth[v] + 1;
              int c_size = dfs(c, adj);
              size += c size:
11
12
              if (c_size > max_c_size)
                  max_c_size = c_size, heavy[v] = c;
13
          }
14
      return size;
16
19 void decompose(int v, int h, vector < vector < int >> const& adj) {
      head[v] = h, pos[v] = cur_pos++;
      if (heavy[v] != -1)
          decompose(heavy[v], h, adj);
23
     for (int c : adj[v]) {
          if (c != parent[v] && c != heavy[v])
24
25
              decompose(c, c, adj);
```

```
27 }
28
29 void init(vector < vector < int >> const& adj) {
      int n = adj.size();
      parent = vector<int>(n);
31
      depth = vector < int > (n);
32
      heavy = vector \langle int \rangle (n, -1);
33
      head = vector < int > (n);
      pos = vector < int > (n);
      cur_pos = 0;
37
      dfs(0, adj);
38
      decompose(0, 0, adj);
39
40 }
41
42 int query(int a, int b) {
      int res = 0;
43
      for (; head[a] != head[b]; b = parent[head[b]]) {
44
           if (depth[head[a]] > depth[head[b]])
45
               swap(a, b);
46
           int cur_heavy_path_max = segment_tree_query(pos[head[b]],
47
           res = max(res, cur_heavy_path_max);
48
49
      if (depth[a] > depth[b])
           swap(a, b);
51
      int last_heavy_path_max = segment_tree_query(pos[a], pos[b]);
52
      res = max(res, last_heavy_path_max);
      return res:
54
```

## 5.7 Centroid Decomposition

```
vector < vector < int >> adj;
vector < bool > is_removed;
vector < int >> subtree_size;

/** DFS to calculate the size of the subtree rooted at 'node' */
int get_subtree_size(int node, int parent = -1) {
    subtree_size[node] = 1;
    for (int child : adj[node]) {
        if (child == parent || is_removed[child]) { continue; }
        subtree_size[node] += get_subtree_size(child, node);
}

return subtree_size[node];

/**
```

```
16 * Returns a centroid (a tree may have two centroids) of the
* containing node 'node' after node removals
* Oparam node current node
19 * @param tree_size size of current subtree after node removals
20 * Oparam parent parent of u
* @return first centroid found
23 int get_centroid(int node, int tree_size, int parent = -1) {
      for (int child : adj[node]) {
          if (child == parent || is_removed[child]) { continue; }
          if (subtree_size[child] * 2 > tree_size) {
26
              return get_centroid(child, tree_size, node);
28
29
30
      return node:
31 }
32
33 /** Build up the centroid decomposition recursively */
void build_centroid_decomp(int node = 0) {
      int centroid = get_centroid(node, get_subtree_size(node));
36
37
      // do something
38
      is_removed[centroid] = true;
39
40
      for (int child : adi[centroid]) {
41
          if (is_removed[child]) { continue; }
42
          build_centroid_decomp(child);
43
44
45 }
```

## 5.8 Tree Distances

```
vector < int > graph [200001];
3 int fir[200001], sec[200001], ans[200001];
5 void dfs1(int node = 1, int parent = 0) {
      for (int i : graph[node])
          if (i != parent) {
              dfs1(i. node):
              if (fir[i] + 1 > fir[node]) {
9
                  sec[node] = fir[node];
11
                  fir[node] = fir[i] + 1:
              } else if (fir[i] + 1 > sec[node]) {
12
                  sec[node] = fir[i] + 1;
13
14
          }
```

```
16 }
17
void dfs2(int node = 1, int parent = 0, int to_p = 0) {
      ans[node] = max(to_p, fir[node]);
19
      for (int i : graph[node])
20
          if (i != parent) {
21
              if (fir[i] + 1 == fir[node]) dfs2(i, node, max(to_p,
22
                  sec[node]) + 1);
              else dfs2(i, node, ans[node] + 1);
23
24
25 }
```

### 6 Flows

#### 6.1 Ford-Fulkerson Maximum Flow

```
vector < vector < int >> capacity;
vector < vector < int >> adj;
5 int bfs(int s, int t, vector<int>& parent) {
      fill(parent.begin(), parent.end(), -1);
      parent[s] = -2:
      queue < pair < int , int >> q;
      q.push({s, INF});
10
      while (!q.empty()) {
          int cur = q.front().first;
12
          int flow = q.front().second;
13
          q.pop();
14
15
          for (int next : adj[cur]) {
16
               if (parent[next] == -1 && capacity[cur][next]) {
17
                   parent[next] = cur;
18
                   int new_flow = min(flow, capacity[cur][next]);
19
                   if (next == t)
20
                       return new_flow;
21
                   q.push({next, new_flow});
22
23
24
          }
25
26
27
      return 0;
28 }
29
30 int maxflow(int s, int t) {
31
      int flow = 0;
      vector < int > parent(n);
```

```
int new_flow;
34
35
      while (new_flow = bfs(s, t, parent)) {
36
          flow += new_flow;
          int cur = t;
37
          while (cur != s) {
38
              int prev = parent[cur];
39
               capacity[prev][cur] -= new_flow;
40
41
               capacity[cur][prev] += new_flow;
42
               cur = prev:
          }
43
44
      }
45
46
      return flow;
47 }
```

#### 6.2 Dinic's Max Flow

```
struct FlowEdge {
      int v, u;
      long long cap, flow = 0;
      FlowEdge(int v, int u, long long cap) : v(v), u(u), cap(cap)
5 }:
7 struct Dinic {
      const long long flow_inf = 1e18;
      vector < FlowEdge > edges;
10
      vector < vector < int >> adi:
      int n, m = 0;
12
      int s, t;
      vector<int> level, ptr;
14
      queue < int > q;
15
      Dinic(int n, int s, int t) : n(n), s(s), t(t) {
16
17
          adj.resize(n);
          level.resize(n);
18
          ptr.resize(n);
19
      }
20
21
      void add_edge(int v, int u, long long cap) {
22
23
          edges.emplace_back(v, u, cap);
          edges.emplace_back(u, v, 0);
24
          adj[v].push_back(m);
25
          adj[u].push_back(m + 1);
26
27
          m += 2;
28
29
      bool bfs() {
```

```
while (!q.empty()) {
31
               int v = q.front();
32
               q.pop();
33
               for (int id : adj[v]) {
34
                   if (edges[id].cap == edges[id].flow)
35
                       continue;
                   if (level[edges[id].u] != -1)
37
                       continue:
38
                   level[edges[id].u] = level[v] + 1;
39
                   q.push(edges[id].u);
40
              }
41
42
          return level[t] != -1:
43
44
45
46
      long long dfs(int v, long long pushed) {
          if (pushed == 0)
47
48
               return 0:
          if (v == t)
49
               return pushed;
50
          for (int& cid = ptr[v]; cid < (int)adj[v].size(); cid++) {</pre>
51
52
               int id = adj[v][cid];
               int u = edges[id].u;
               if (level[v] + 1 != level[u])
54
                   continue;
               long long tr = dfs(u, min(pushed, edges[id].cap -
56
                   edges[id].flow));
               if (tr == 0)
                   continue:
58
               edges[id].flow += tr;
59
               edges[id ^ 1].flow -= tr;
60
               return tr:
61
62
          return 0;
63
64
65
66
      long long flow() {
          long long f = 0;
67
          while (true) {
68
               fill(level.begin(), level.end(), -1);
69
70
               level[s] = 0;
               q.push(s);
71
72
               if (!bfs())
                   break:
73
               fill(ptr.begin(), ptr.end(), 0);
74
               while (long long pushed = dfs(s, flow_inf)) {
75
                   f += pushed:
77
78
          return f;
```

```
81 };
```

#### 6.3 Min-cost Flow

```
1 struct Edge
2 {
3
      int from, to, capacity, cost;
4 };
6 vector < vector < int >> adj, cost, capacity;
8 const int INF = 1e9;
10 void shortest_paths(int n, int v0, vector<int>& d, vector<int>& p)
      d.assign(n, INF);
      d[v0] = 0;
12
      vector < bool > inq(n, false);
13
      queue < int > q;
      q.push(v0);
15
16
      p.assign(n, -1);
17
      while (!q.empty()) {
18
          int u = q.front();
19
20
          q.pop();
21
          ing[u] = false:
22
          for (int v : adj[u]) {
              if (capacity[u][v] > 0 && d[v] > d[u] + cost[u][v]) {
23
24
                   d[v] = d[u] + cost[u][v]:
                   p[v] = u;
25
26
                   if (!inq[v]) {
27
                       inq[v] = true;
28
                       q.push(v);
                   }
29
              }
30
          }
31
      }
32
33 }
int min_cost_flow(int N, vector<Edge> edges, int K, int s, int t)
36
      adj.assign(N, vector<int>());
37
      cost.assign(N, vector<int>(N, 0));
      capacity.assign(N, vector < int > (N, 0));
38
      for (Edge e : edges) {
39
40
          adj[e.from].push_back(e.to);
          adj[e.to].push_back(e.from);
41
42
          cost[e.from][e.to] = e.cost;
          cost[e.to][e.from] = -e.cost;
```

```
capacity[e.from][e.to] = e.capacity;
44
      }
45
      int flow = 0;
47
      int cost = 0;
48
      vector < int > d, p;
      while (flow < K) {</pre>
50
          shortest_paths(N, s, d, p);
51
          if (d[t] == INF)
52
               break:
53
54
          // find max flow on that path
55
          int f = K - flow:
56
          int cur = t;
57
          while (cur != s) {
58
59
               f = min(f, capacity[p[cur]][cur]);
               cur = p[cur];
61
62
          // apply flow
63
          flow += f;
64
          cost += f * d[t];
65
          cur = t;
66
          while (cur != s) {
67
               capacity[p[cur]][cur] -= f;
               capacity[cur][p[cur]] += f;
69
               cur = p[cur];
70
          }
71
      }
72
73
74
      if (flow < K)
75
          return -1;
76
      else
77
          return cost;
78 }
```

## 6.4 Hungarian Algorithm

```
vector<int> u (n+1), v (m+1), p (m+1), way (m+1);
for (int i=1; i<=n; ++i) {
    p[0] = i;
    int j0 = 0;
    vector<int> minv (m+1, INF);
    vector<bool> used (m+1, false);
    do {
        used[j0] = true;
        int i0 = p[j0], delta = INF, j1;
        for (int j=1; j<=m; ++j)
        if (!used[j]) {</pre>
```

```
int cur = A[i0][j]-u[i0]-v[j];
                   if (cur < minv[j])</pre>
13
                        minv[j] = cur, way[j] = j0;
14
                   if (minv[j] < delta)</pre>
15
                        delta = minv[j], j1 = j;
16
               }
17
          for (int j=0; j<=m; ++j)</pre>
18
               if (used[j])
19
                   u[p[j]] += delta, v[j] -= delta;
20
21
                    minv[j] -= delta;
22
23
           j0 = j1;
      } while (p[j0] != 0);
24
25
      do {
           int j1 = way[j0];
26
          p[j0] = p[j1];
27
           j0 = j1;
28
29
      } while (j0);
30 }
32 vector < int > ans (n+1);
33 for (int j=1; j<=m; ++j)
      ans[p[j]] = j;
36 | int cost = -v[0];
```

# 6.5 Kuhn's Algorithm

```
int n. k:
vector < vector < int >> g;
3 vector < int > mt;
4 vector < bool > used;
6 bool try_kuhn(int v) {
     if (used[v])
          return false;
      used[v] = true;
      for (int to : g[v]) {
10
          if (mt[to] == -1 || try_kuhn(mt[to])) {
11
               mt[to] = v;
12
13
               return true;
14
          }
      return false;
16
17 }
18
19 int main() {
      //... reading the graph ...
```

```
mt.assign(k, -1);
for (int v = 0; v < n; ++v) {
    used.assign(n, false);
    try_kuhn(v);
}

for (int i = 0; i < k; ++i)
    if (mt[i] != -1)
        printf("%d %d\n", mt[i] + 1, i + 1);
}</pre>
```

# 7 Dynamic Programming

### 7.1 Coin Problem (Count ways)

```
vector <11> coins(n);
1  for (int i=0; i<n; i++){</pre>
      cin>>coins[i];
4 }
6 vector <11> dp(x+1,0);
7 dp[0] = 1;
8 for(int i=0; i<=x; i++){</pre>
      for(int j=0; j<n; j++){</pre>
           if(i-coins[j]>=0){
10
                dp[i] = (dp[i] + dp[i-coins[j]]);
                dp[i]%=MOD;
12
13
      }
14
15 }
16
17
18 cout <<dp[x] << endl;</pre>
```

## 7.2 Coin Problem (Count sorted ways)

```
vector<ll> coins(n);
for(int i=0; i<n; i++){
        cin>>coins[i];
}

int dp[102][1000005];

dp[0][0] = 1;
for(int i=1; i<=n; i++){
        for(int j=0; j<=x; j++){
            dp[i][j] = dp[i-1][j];
            int l = j-coins[i-1];</pre>
```

# 7.3 Coin Problem (Minimum)

```
vector<1l> coins(n);
2 for(int i=0; i<n; i++){</pre>
       cin>>coins[i];
4 }
6 vector < 11 > dp(x+1, INT_MAX);
7 dp[0] = 0;
8 for(int i=0; i<=x; i++){</pre>
      for(int j=0; j<n; j++){</pre>
           if (i-coins[j]>=0){
                dp[i] = min(dp[i], dp[i-coins[j]]+1);
      }
13
14 }
16 if (dp[x] != INT_MAX) {
       cout << dp[x] << endl;</pre>
18 }else{
       cout << " -1 " << end1;
20 }
```

## 7.4 Counting paths

```
int n; cin>>n;
char grid[n][n];
int dp[n][n];

for(int i=0; i<n; i++){
    for(int j=0; j<n; j++){
        cin>>grid[i][j];
        dp[i][j] = 0;
}

if(grid[0][0] != '*')dp[0][0] = 1;
else dp[0][0] = 0;
for(int i=0; i<n; i++){</pre>
```

```
for(int j=0; j<n; j++){</pre>
14
           if(grid[i+1][j] == '.' and i+1 < n){</pre>
               dp[i+1][j] += dp[i][j]%MOD;
16
17
           if(grid[i][j+1] == '.' and j+1 < n){</pre>
18
               dp[i][j+1] += dp[i][j]%MOD;
20
21
           if(grid[i][j] == '*'){
22
               dp[i][j] = 0;
23
24
25
27 cout <<dp[n-1][n-1]%MOD <<endl;
```

### 7.5 Longest Increasing Subsequence

```
vector < int > lis(vector < int > const& a) {
      int n = a.size();
      vector \langle int \rangle d(n, 1), p(n, -1);
      for (int i = 0; i < n; i++) {</pre>
           for (int j = 0; j < i; j++) {</pre>
               if (a[j] < a[i] && d[i] < d[j] + 1) {</pre>
                    d[i] = d[j] + 1;
                    p[i] = j;
               }
           }
      }
11
12
      int ans = d[0], pos = 0;
14
      for (int i = 1; i < n; i++) {
           if (d[i] > ans) {
               ans = d[i];
16
17
               pos = i;
18
           }
19
20
      vector < int > subseq;
21
      while (pos != -1) {
22
           subseq.push_back(a[pos]);
23
           pos = p[pos];
24
25
      reverse(subseq.begin(), subseq.end());
26
      return subseq;
28 }
```

## 7.6 Length of LIS

```
int lis(vector<ll> const& a) {
      int n = a.size():
      const int INF = 1e9;
      vector < int > d(n+1, INF);
      d[0] = -INF;
      for (int i = 0; i < n; i++) {</pre>
          int 1 = upper_bound(d.begin(), d.end(), a[i]) - d.begin();
          if (d[l-1] < a[i] && a[i] < d[l])</pre>
               d[1] = a[i]:
10
      }
12
      int ans = 0;
      for (int 1 = 0; 1 <= n; 1++) {
15
          if (d[1] < INF)</pre>
16
               ans = 1:
18
      return ans;
19 }
```

### 7.7 Longest Common Subsequence

```
1 // Returns length of LCS for s1[0..m-1], s2[0..n-1]
int lcs(string &s1, string &s2) {
      int m = s1.size();
      int n = s2.size():
      // Initializing a matrix of size (m+1)*(n+1)
      vector < vector < int >> dp(m + 1, vector < int > (n + 1, 0));
      // Building dp[m+1][n+1] in bottom-up fashion
      for (int i = 1; i <= m; ++i) {</pre>
          for (int j = 1; j <= n; ++j) {</pre>
11
12
              if (s1[i - 1] == s2[j - 1])
                   dp[i][j] = dp[i - 1][j - 1] + 1;
13
14
                   dp[i][j] = max(dp[i - 1][j], dp[i][j - 1]);
16
          }
17
      }
      // dp[m][n] contains length of LCS for s1[0..m-1]
19
      // and s2[0..n-1]
20
      return dp[m][n];
21
22 }
```

### 7.8 Edit Distance

```
int editDistance(string &s1, string &s2) {
      int m = s1.length();
      int n = s2.length();
      // Create a table to store results of subproblems
      vector < vector < int >> dp(m + 1, vector < int > (n + 1));
      // Fill the known entries in dp[][]
      // If one string is empty, then answer
      // is length of the other string
11
      for (int i = 0; i <= m; i++)</pre>
12
          dp[i][0] = i:
13
      for (int j = 0; j \le n; j++)
14
          dp[0][j] = j;
15
16
      // Fill the rest of dp[][]
17
18
      for (int i = 1; i <= m; i++) {</pre>
          for (int j = 1; j <= n; j++) {</pre>
19
               if (s1[i - 1] == s2[j - 1])
20
                   dp[i][j] = dp[i - 1][j - 1];
21
22
                   dp[i][j] = 1 + min({dp[i][j - 1]},
23
                                     dp[i - 1][j],
24
                                      dp[i - 1][j - 1]});
25
26
      }
27
28
      return dp[m][n];
29
```

#### 7.9 Bitmask DP

```
1 typedef long long 11;
1 typedef vector int vec;
3 const ll mod=1e9+7;
4 const int limit=20;
5 vector < pair < 11, 11 >> dp((1 << limit));</pre>
6 //ascii https://elcodigoascii.com.ar/
8 inline void solve()
9 {
      int n; cin>>n;
10
      11 x; cin>>x;
11
      vector<ll> weight(n);
12
13
      dp[0]=\{1,0\};
      FO(i,n) cin>>weight[i];
15
      for(ll i=1;i<(1<<n);i++)</pre>
16
      {
```

```
dp[i] = \{n+1,0\};
             for(int j=0;j<n;j++)</pre>
18
19
                 if(i&(1<<j))</pre>
20
21
22
                       pair < 11, 11 > aux = dp[i^(1 < < j)];
                       if (aux.second+weight[j] <= x) {</pre>
23
                            aux.second+=weight[j];
24
                      }
25
                       else{
26
                            aux.first++;
27
                            aux.second=weight[j];
28
29
                       dp[i]=min(dp[i],aux);
30
31
32
            }
33
34
       cout << dp [(1 << n) -1] . first << endl;
35 }
```

### 7.10 Digit DP

```
typedef long long 11;
typedef vector<int> vec;
3 const ll mod=1e9+7;
4 11 dp [20] [10] [2] [2];
5 //ascii https://elcodigoascii.com.ar/
7 | 11 mem(int idx, int tight, int prev, int ld, string s)
8 {
      if(idx==0)
10
      {
          return 1;
12
      if (dp[idx][prev][ld][tight]!=-1){
13
           return dp[idx][prev][ld][tight];
14
15
      int k=9;
16
      if(tight) k=s[s.size()-idx]-'0';
17
18
      11 sum = 0;
      for(int i=0;i<=k;i++)</pre>
19
20
21
          if(ld || prev!=i)
22
23
               int new_ld,new_tight;
24
               if(i==0 && ld) new_ld=1;
25
               else new_ld=0;
26
               if(tight && k==i) new_tight=1;
27
               else new_tight=0;
```

#### 7.11 Double DP

```
1 typedef long long 11;
2 typedef vector <int> vec;
3 const ll mod=1e9+7;
4 const 11 MAX=1e6+3:
5 ll dp[MAX][2];
6 //ascii https://elcodigoascii.com.ar/
8 inline void solve()
      int n; cin>>n;
10
      dp[n][0]=1;
11
      dp[n][1]=1;
12
      for(int i=n-1;i>0;i--)
13
1.5
          dp[i][1]=4*dp[i+1][1]+dp[i+1][0];
          dp[i][0]=2*dp[i+1][0]+dp[i+1][1];
16
          dp[i][1]%=mod;
17
          dp[i][0]%=mod;
18
19
      cout <<(dp[1][1]+dp[1][0])%mod << endl;</pre>
20
21 }
```

## 8 Math

#### 8.1 Miller Rabin

```
bool MillerRabin(u64 n) { // returns true if n is prime, else
    returns false.
    if (n < 2)
        return false;

int r = 0;
    u64 d = n - 1;
    while ((d & 1) == 0) {
        d >>= 1;
        r++;
    }
}
```

```
for (int a : {2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37}) {
    if (n == a)
        return true;
    if (check_composite(n, a, d, r))
        return false;
}
return true;
}
```

#### 8.2 Sieve of Erathostenes

# 8.3 Sieve of Eratosthenes (count primes)

```
int count_primes(int n) {
      const int S = 10000;
3
      vector < int > primes;
      int nsqrt = sqrt(n);
      vector < char > is_prime(nsqrt + 2, true);
      for (int i = 2; i <= nsqrt; i++) {</pre>
          if (is_prime[i]) {
               primes.push_back(i);
               for (int j = i * i; j <= nsqrt; j += i)</pre>
10
11
                   is_prime[j] = false;
12
          }
      }
13
14
      int result = 0;
15
16
      vector < char > block(S);
17
      for (int k = 0; k * S <= n; k++) {
18
          fill(block.begin(), block.end(), true);
19
          int start = k * S;
20
          for (int p : primes) {
21
               int start_idx = (start + p - 1) / p;
22
               int j = max(start_idx, p) * p - start;
               for (; j < S; j += p)
23
                   block[j] = false;
24
          }
25
```

### 8.4 Segmented Sieve

```
vector < char > segmentedSieve(long long L, long long R) {
      // generate all primes up to sqrt(R)
      long long lim = sqrt(R);
      vector < char > mark(lim + 1, false);
      vector<long long> primes;
      for (long long i = 2; i <= lim; ++i) {</pre>
          if (!mark[i]) {
              primes.emplace_back(i);
              for (long long j = i * i; j <= lim; j += i)
                  mark[i] = true;
12
13
      vector < char > isPrime(R - L + 1, true);
      for (long long i : primes)
15
          for (long long j = \max(i * i, (L + i - 1) / i * i); i <= R
              ; j += i)
              isPrime[j - L] = false;
17
      if (L == 1)
18
          isPrime[0] = false;
19
20
      return isPrime;
```

#### 8.5 Linear sieve

```
const int N = 10000000;
vector<int> lp(N+1);
vector<int> pr;

for (int i=2; i <= N; ++i) {
    if (lp[i] == 0) {
        lp[i] = i;
        pr.push_back(i);
    }

for (int j = 0; i * pr[j] <= N; ++j) {
        lp[i * pr[j]] = pr[j];
}</pre>
```

### 8.6 Sum of divisors

```
1 long long SumOfDivisors(long long num) {
      long long total = 1;
      for (int i = 2; (long long)i * i <= num; i++) {</pre>
          if (num % i == 0) {
              int e = 0;
               do {
                   e++:
                   num /= i;
               } while (num % i == 0);
10
12
              long long sum = 0, pow = 1;
13
               do {
14
                   sum += pow;
                   pow *= i;
15
16
              } while (e-- > 0);
17
               total *= sum;
          }
18
19
      if (num > 1) {
          total *= (1 + num);
21
22
23
      return total;
24 }
```

## 8.7 Finding the divisors of a number (Trial Division)

```
factorization.push_back(n);
return factorization;
factorization;
```

#### 8.8 Factorials

```
1 // Precompute factorials and inverse factorials
void precompute(ll n = MAXN - 1) {
     factorial[0] = factorial[1] = 1;
     // Compute factorials
     for (11 i = 2; i <= n; i++) {</pre>
          factorial[i] = (factorial[i - 1] * i) % MOD;
     // Compute inverse factorials efficiently
10
     inv_factorial[n] = modInv(factorial[n]);
11
      for (11 i = n - 1; i \ge 0; i--) {
13
          inv factorial[i] =
              (inv factorial[i + 1] * (i + 1)) % MOD:
14
15
16 }
```

### 8.9 Binpow

```
long long binpow(long long a, long long b) {
    long long res = 1;
    while (b > 0) {
        if (b & 1)
            res = res * a;
            a = a * a;
        b >>= 1;
    }
    return res;
}
```

# 8.10 Modulo Inverse

```
int modInverse(int A, int M) {
   int m0 = M;
   int y = 0, x = 1;

if (M == 1)
   return 0;

while (A > 1) {
   // q is quotient
```

```
int q = A / M;
          int t = M;
12
          // m is remainder now, process same as
13
          // Euclid's algo
          M = A \% M, A = t;
15
16
          t = y;
17
18
          // Update y and x
19
          y = x - q * y;
20
          x = t;
21
22
      // Make x positive
23
      if (x < 0)
24
25
          x += m0:
26
27
      return x;
28 }
```

#### 8.11 BinPow Modulo Inv

```
1 11 modInv(11 a, 11 mod = MOD) {
2    return power(a, mod - 2, mod);
3 }
```

#### 8.12 Binomial Coefficients

```
long long binomial_coefficient(int n, int k) {
   return factorial[n] * inverse_factorial[k] % m *
        inverse_factorial[n - k] % m;
}
```

## 8.13 Newton Method (Sqrt and iSqrt)

```
double sqrt_newton(double n) {
    const double eps = 1E-15;
    double x = 1;
    for (;;) {
        double nx = (x + n / x) / 2;
        if (abs(x - nx) < eps)
            break;
        x = nx;
    }
    return x;
}</pre>
```

```
int isgrt_newton(int n) {
      int x = 1;
      bool decreased = false;
      for (;;) {
16
          int nx = (x + n / x) >> 1;
17
          if (x == nx \mid | nx > x \&\& decreased)
               hreak.
19
          decreased = nx < x;
20
21
          x = nx;
22
      return x;
24 }
```

## 8.14 Integration with Simpson Method

```
const int N = 1000 * 1000; // number of steps (already multiplied
    by 2)

double simpson_integration(double a, double b){
    double h = (b - a) / N;
    double s = f(a) + f(b); // a = x_0 and b = x_2n
    for (int i = 1; i <= N - 1; ++i) { // Refer to final Simpson's
        formula
        double x = a + h * i;
        s += f(x) * ((i & 1) ? 4 : 2);
    }
    s *= h / 3;
    return s;
}</pre>
```

## 8.15 Ternary Search

```
double ternary_search(double 1, double r) {
                                    //set the error limit here
     double eps = 1e-9;
      while (r - 1 > eps) {
          double m1 = 1 + (r - 1) / 3;
          double m2 = r - (r - 1) / 3;
          double f1 = f(m1):
                                //evaluates the function at m1
          double f2 = f(m2);
                                 //evaluates the function at m2
          if (f1 < f2)
             1 = m1:
          else
              r = m2:
11
12
     return f(1);
                                      //return the maximum of f(x)
          in [1, r]
```

## 8.16 DP Pascal triangle 1D

```
int binomialCoeff(int n, int k) {
      vector < int > dp(k + 1);
       // nCO is 1
      dp[0] = 1;
      for (int i = 1; i <= n; i++) {
          // Compute next row of pascal triangle using
10
          // the previous row
          for (int j = min(i, k); j > 0; j--)
11
12
              dp[j] = dp[j] + dp[j - 1];
13
      return dp[k];
14
15 }
```

### 8.17 DP Pascal triangle 2D

```
// Returns value of Binomial Coefficient C(n, k)
int binomialCoeff(int n. int k) {
        vector < vector < int >> dp(n + 1, vector < int > (k + 1));
      // Calculate value of Binomial Coefficient
      // in bottom up manner
      for (int i = 0: i <= n: i++) {
          for (int j = 0; j \le min(i, k); j++) {
              // Base Cases
              if (j == 0 || j == i)
11
                  dp[i][j] = 1;
12
13
14
              // Calculate value using previously
              // stored values
15
16
                  dp[i][j] = dp[i - 1][j - 1] + dp[i - 1][j];
          }
18
19
      }
20
21
      return dp[n][k];
22 }
```

### 8.18 Euler's Totient

```
void phi_1_to_n(int n) {
    vector < int > phi(n + 1);
    for (int i = 0; i <= n; i++)</pre>
```

```
phi[i] = i;
      for (int i = 2; i <= n; i++) {
          if (phi[i] == i) {
              for (int j = i; j <= n; j += i)</pre>
                   phi[j] -= phi[j] / i;
12 }
13
void phi_1_to_n(int n) {
      vector < int > phi(n + 1);
      phi[0] = 0:
16
      phi[1] = 1;
17
      for (int i = 2; i <= n; i++)</pre>
19
          phi[i] = i - 1;
20
21
      for (int i = 2; i <= n; i++)
          for (int j = 2 * i; j <= n; j += i)
                phi[i] -= phi[i];
23
24 }
```

### 8.19 Diophantine equations

```
void shift_solution(int & x, int & y, int a, int b, int cnt) {
      x += cnt * b:
      v -= cnt * a;
4 }
int find_all_solutions(int a, int b, int c, int minx, int maxx,
      int miny, int maxy) {
      int x, y, g;
      if (!find_anv_solution(a, b, c, x, y, g))
          return 0;
      a /= g;
      b /= g;
13
      int sign_a = a > 0 ? +1 : -1;
      int sign_b = b > 0 ? +1 : -1;
      shift_solution(x, y, a, b, (minx - x) / b);
16
      if (x < minx)
          shift_solution(x, y, a, b, sign_b);
18
      if (x > maxx)
         return 0:
20
21
      int lx1 = x;
22
23
      shift_solution(x, y, a, b, (maxx - x) / b);
      if (x > maxx)
```

```
shift_solution(x, y, a, b, -sign_b);
26
      int rx1 = x;
27
      shift_solution(x, y, a, b, -(miny - y) / a);
28
      if (y < miny)</pre>
30
          shift_solution(x, y, a, b, -sign_a);
      if (y > maxy)
31
          return 0;
32
33
      int 1x2 = x;
34
35
      shift_solution(x, y, a, b, -(maxy - y) / a);
      if (y > maxy)
36
37
          shift_solution(x, y, a, b, sign_a);
38
      int rx2 = x;
39
      if (1x2 > rx2)
41
          swap(1x2, rx2);
      int lx = max(lx1, lx2);
     int rx = min(rx1, rx2):
44
      if (lx > rx)
45
46
          return 0;
      return (rx - lx) / abs(b) + 1;
47
48 }
```

## 8.20 Discrete Log

```
1 // Returns minimum x for which a ^ x % m = b % m.
int solve(int a. int b. int m) {
      a \% = m, b \% = m;
      int k = 1, add = 0, g;
      while ((g = gcd(a, m)) > 1) {
          if (b == k)
6
              return add;
          if (b % g)
              return -1;
          b /= g, m /= g, ++add;
10
          k = (k * 111 * a / g) % m;
12
13
      int n = sqrt(m) + 1;
15
      int an = 1:
16
      for (int i = 0; i < n; ++i)
17
          an = (an * 111 * a) \% m;
18
19
      unordered_map <int, int > vals;
      for (int q = 0, cur = b; q \le n; ++q) {
20
21
          vals[cur] = q;
          cur = (cur * 111 * a) % m;
22
```

```
}
23
24
      for (int p = 1, cur = k; p \le n; ++p) {
          cur = (cur * 111 * an) % m;
26
          if (vals.count(cur)) {
27
              int ans = n * p - vals[cur] + add;
              return ans;
29
30
      }
31
      return -1;
32
```

# 9 Polynomials

#### 9.1 FFT

```
using cd = complex < double >;
const double PI = acos(-1);
4 int reverse(int num, int lg_n) {
      int res = 0;
      for (int i = 0; i < lg_n; i++) {</pre>
          if (num & (1 << i))
              res |= 1 << (lg_n - 1 - i);
      return res;
10
11 }
12
void fft(vector < cd> & a, bool invert) {
      int n = a.size();
      int lg_n = 0;
      while ((1 << lg_n) < n)
16
          lg_n++;
17
18
      for (int i = 0; i < n; i++) {</pre>
19
          if (i < reverse(i, lg_n))</pre>
20
               swap(a[i], a[reverse(i, lg_n)]);
21
      }
22
23
      for (int len = 2; len <= n; len <<= 1) {
24
          double ang = 2 * PI / len * (invert ? -1 : 1);
25
          cd wlen(cos(ang), sin(ang));
26
          for (int i = 0; i < n; i += len) {</pre>
27
               cd w(1):
28
29
               for (int j = 0; j < len / 2; j++) {
                   cd u = a[i+j], v = a[i+j+len/2] * w;
                   a[i+j] = u + v;
31
32
                   a[i+j+len/2] = u - v;
```

```
w *= wlen;
34
35
          }
      }
36
37
      if (invert) {
38
          for (cd & x : a)
39
              x /= n;
40
41
      }
42 }
43
44 vector < int > multiply (vector < int > const& a, vector < int > const& b) {
      vector < cd > fa(a.begin(), a.end()), fb(b.begin(), b.end());
      int n = 1;
      while (n < a.size() + b.size())</pre>
47
          n <<= 1:
49
      fa.resize(n);
50
      fb.resize(n);
51
52
      fft(fa, false);
53
      fft(fb, false);
54
      for (int i = 0; i < n; i++)
55
          fa[i] *= fb[i];
56
     fft(fa, true);
57
58
      vector < int > result(n);
      for (int i = 0: i < n: i++)
          result[i] = round(fa[i].real());
60
      return result:
61
62 }
64 // Normalization
66 int carry = 0;
67 for (int i = 0; i < n; i++){
     result[i] += carry;
      carry = result[i] / 10;
69
      result[i] %= 10;
70
71 }
```

#### 9.2 NTT

```
const int mod = 7340033;
const int root = 5;
const int root_1 = 4404020;
const int root_pw = 1 << 20;

void fft(vector<int> & a, bool invert) {
   int n = a.size();
}
```

12

13

14

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44 45

46

47

48

49

50

51

52

```
for (int i = 1, j = 0; i < n; i++) {
          int bit = n >> 1;
          for (; j & bit; bit >>= 1)
              j ^= bit;
12
          j ^= bit;
14
          if (i < j)</pre>
15
               swap(a[i], a[j]);
16
17
18
      for (int len = 2; len <= n; len <<= 1) {</pre>
19
          int wlen = invert ? root 1 : root:
20
          for (int i = len; i < root_pw; i <<= 1)</pre>
21
               wlen = (int)(1LL * wlen * wlen % mod);
22
23
          for (int i = 0; i < n; i += len) {</pre>
24
25
               int w = 1:
               for (int i = 0: i < len / 2: i++) {
26
                   int u = a[i+j], v = (int)(1LL * a[i+j+len/2] * w %
27
                   a[i+j] = u + v < mod ? u + v : u + v - mod;
                   a[i+j+len/2] = u - v >= 0 ? u - v : u - v + mod;
29
                   w = (int)(1LL * w * wlen % mod):
30
              }
31
          }
32
      }
33
      if (invert) {
35
          int n_1 = inverse(n, mod);
36
          for (int & x : a)
37
              x = (int)(1LL * x * n 1 \% mod):
38
39
40 }
```

## 9.3 Berlekamp Messey

```
for (int j=1; j<=(int)c.size(); j++)</pre>
    delta -= c[j-1] * s[i-j]; // c_j is one-indexed, so
        we actually need index j - 1 in the code
if (delta == 0)
    continue; // c(i) is correct, keep going
// now at this point, delta != 0, so we need to adjust it
if (f == -1) {
    // this is the first time we're updating c
    // s_i was the first non-zero element we encountered
    // we make c of length i + 1 so that s i is part of
        the base case
    c.resize(i + 1);
    mt19937 rng(chrono::steady_clock::now().
        time_since_epoch().count());
    for (T &x : c)
        x = rng(); // just to prove that the initial
            values don't matter in the first step, I will
            set to random values
    f = i:
} else {
   // we need to use a previous version of c to improve
        on this one
    // apply the 5 steps to build d
    // 1. set d equal to our chosen sequence
    vector <T> d = oldC;
    // 2. multiply the sequence by -1
    for (T &x : d)
        x = -x;
    // 3. insert a 1 on the left
    d.insert(d.begin(), 1);
    // 4. multiply the sequence by delta / d(f + 1)
    T df1 = 0: // d(f + 1)
    for (int j=1; j<=(int)d.size(); j++)</pre>
        df1 += d[j-1] * s[f+1-j];
    assert(df1 != 0);
    T coef = delta / df1; // storing this in outer
        variable so it's O(n^2) instead of O(n^2 \log MOD)
    for (T &x : d)
        x *= coef:
    // 5. insert i - f - 1 zeros on the left
    vector <T> zeros(i - f - 1);
    zeros.insert(zeros.end(), d.begin(), d.end());
    d = zeros:
    // now we have our new recurrence: c + d
    vector <T> temp = c; // save the last version of c
        because it might have a better left endpoint
    c.resize(max(c.size(), d.size()));
    for (int j=0; j<(int)d.size(); j++)</pre>
        c[i] += d[i];
    // finally, let's consider updating oldC
    if (i - (int) temp.size() > f - (int) oldC.size()) {
```

# 10 Linear Algebra

### 10.1 Determinant of a Matrix

```
const double EPS = 1E-9;
2 int n;
g vector < vector <double> > a (n, vector <double> (n));
5 double det = 1;
6 for (int i=0; i<n; ++i) {
      int k = i;
      for (int j=i+1; j<n; ++j)</pre>
           if (abs (a[j][i]) > abs (a[k][i]))
               k = j;
      if (abs (a[k][i]) < EPS) {</pre>
11
           det = 0;
12
13
           break;
14
      swap (a[i], a[k]);
      if (i != k)
16
17
           det = -det;
      det *= a[i][i];
18
      for (int j=i+1; j<n; ++j)</pre>
19
20
           a[i][j] /= a[i][i];
      for (int j=0; j<n; ++j)</pre>
21
          if (j != i && abs (a[j][i]) > EPS)
22
               for (int k=i+1; k<n; ++k)</pre>
23
                   a[j][k] -= a[i][k] * a[j][i];
25 }
26
27 cout << det;
```

#### 10.2 Rank of a Matrix

```
const double EPS = 1E-9;
int compute_rank(vector<vector<double>> A) {
  int n = A.size();
```

```
int m = A[0].size();
      int rank = 0;
      vector <bool > row_selected(n, false);
      for (int i = 0; i < m; ++i) {</pre>
          int j;
          for (j = 0; j < n; ++j) {
               if (!row_selected[j] && abs(A[j][i]) > EPS)
12
13
                   break:
14
          if (j != n) {
16
17
              ++rank:
               row_selected[j] = true;
18
              for (int p = i + 1; p < m; ++p)
19
20
                   A[i][p] /= A[i][i];
              for (int k = 0; k < n; ++k) {</pre>
21
22
                   if (k != j && abs(A[k][i]) > EPS) {
23
                       for (int p = i + 1; p < m; ++p)
24
                            A[k][p] -= A[j][p] * A[k][i];
25
26
              }
          }
27
28
29
      return rank;
30 }
```

#### 10.3 Gauss-Jordan

```
const double EPS = 1e-9;
const int INF = 2; // it doesn't actually have to be infinity or a
       big number
int gauss (vector < vector <double > > a, vector <double > & ans) {
      int n = (int) a.size();
      int m = (int) a[0].size() - 1;
      vector < int > where (m, -1);
      for (int col=0, row=0; col<m && row<n; ++col) {</pre>
           int sel = row;
10
          for (int i=row; i<n; ++i)</pre>
12
               if (abs (a[i][col]) > abs (a[sel][col]))
13
14
          if (abs (a[sel][col]) < EPS)</pre>
15
               continue:
16
          for (int i=col; i<=m; ++i)</pre>
17
               swap (a[sel][i], a[row][i]);
18
           where [col] = row;
```

```
for (int i=0; i<n; ++i)</pre>
20
                if (i != row) {
21
                    double c = a[i][col] / a[row][col];
                    for (int j=col; j<=m; ++j)</pre>
23
                         a[i][j] -= a[row][j] * c;
24
25
           ++row;
26
      }
27
28
      ans.assign (m, 0);
29
      for (int i=0; i<m; ++i)</pre>
30
           if (where[i] != -1)
31
                ans[i] = a[where[i]][m] / a[where[i]][i];
32
       for (int i=0; i<n; ++i) {</pre>
33
           double sum = 0;
34
35
           for (int j=0; j<m; ++j)</pre>
                sum += ans[j] * a[i][j];
36
37
           if (abs (sum - a[i][m]) > EPS)
                return 0:
38
39
40
41
      for (int i=0; i<m; ++i)</pre>
           if (where[i] == -1)
42
                return INF;
43
       return 1;
45 }
```

## 10.4 Matrix Exponentiation

```
#include <bits/stdc++.h>
2 using namespace std;
4 using 11 = long long;
6 \text{ const } 11 \text{ MOD} = 1e9 + 7;
s using Matrix = array<array<11, 2>, 2>;
10 Matrix mul(Matrix a, Matrix b) {
      Matrix res = \{\{\{0, 0\}, \{0, 0\}\}\};
      for (int i = 0; i < 2; i++) {</pre>
12
           for (int j = 0; j < 2; j++) {
               for (int k = 0; k < 2; k++) {
14
                    res[i][j] += a[i][k] * b[k][j];
                    res[i][j] %= MOD;
16
17
          }
      }
19
```

```
return res;
22 }
23
24 int main() {
       11 n;
       cin >> n;
26
27
       Matrix base = \{\{\{1, 0\}, \{0, 1\}\}\};
29
       Matrix m = \{\{\{1, 1\}, \{1, 0\}\}\};
       for (; n > 0; n /= 2, m = mul(m, m)) {
31
32
           if (n & 1) base = mul(base, m);
33
34
35
       cout << base[0][1];</pre>
36 }
```

# 11 Geometry

### 11.1 Line Segment Intersection

```
1 // BeginCodeSnip{Point Class}
2 struct Point {
      Point(int a = 0, int b = 0) : x(a), y(b) {}
      friend istream &operator>>(istream &in, Point &p) {
          int x, y;
          in >> p.x >> p.y;
9
          return in;
10
11 };
12 // EndCodeSnip
int sign(long long num) {
     if (num < 0) {
          return -1;
     } else if (num == 0) {
          return 0;
18
     } else {
20
          return 1;
21
22 }
24 long long trigonometric_sense(Point p, Point p1, Point p2) {
      return sign(1LL * (p1.x - p.x) * (p2.y - p.y) -
26
                  1LL * (p2.x - p.x) * (p1.y - p.y));
27 }
```

```
28
29 // Check if the rectangles with [P1, P2] and [P3, P4] as diagonals
       intersect
30 bool quick_check(Point p1, Point p2, Point p3, Point p4) {
      int x1, x2, x3, x4, y1, y2, y3, y4;
31
      x1 = min(p1.x, p2.x), x2 = max(p1.x, p2.x);
32
      y1 = min(p1.y, p2.y), y2 = max(p1.y, p2.y);
33
      x3 = min(p3.x, p4.x), x4 = max(p3.x, p4.x);
34
      y3 = min(p3.y, p4.y), y4 = max(p3.y, p4.y);
      return x2 < x3 || x4 < x1 || y2 < y3 || y4 < y1;
36
38
39 bool check(Point p1, Point p2, Point p3, Point p4) {
      if (trigonometric_sense(p1, p2, p3) * trigonometric_sense(p1,
          p2, p4) > 0) {
41
          return false:
42
43
      if (trigonometric_sense(p3, p4, p1) * trigonometric_sense(p3,
          p4. p2) > 0) {
          return false;
44
45
      return true;
46
47 }
48
49 int main() {
      int test num:
      cin >> test num:
51
      for (int t = 0; t < test_num; t++) {</pre>
          Point p1, p2, p3, p4;
53
          cin >> p1 >> p2 >> p3 >> p4;
54
55
          if (quick_check(p1, p2, p3, p4)) {
56
               cout << "NO" << endl;
57
          } else if (check(p1, p2, p3, p4)) {
58
59
               cout << "YES" << endl;</pre>
          } else {
60
               cout << "NO" << endl;</pre>
61
62
63
64 }
```

### 11.2 Minimum Euclidian Distance

```
const ll mod=1e9+7;
const ll MAX=8e18;
const ll limit=1e9+1;
//ascii https://elcodigoascii.com.ar/
toldistance(point a,point b){
```

```
return (a.X-b.X)*(a.X-b.X)+(a.Y-b.Y)*(a.Y-b.Y);
8 }
9
10 inline void solve()
11 |
12
      int n; cin>>n;
      vector < point > sortedX(n);
13
      set < point > sortedY;
14
      FO(i,n)
15
16
17
          11 x,y; cin>>x>>y;
           sortedX[i]=make_pair(x,y);
18
19
20
      sort(all(sortedX));
21
      sortedY.insert(make_pair(sortedX[0].Y,sortedX[0].X));
22
      11 d.minSquare=MAX:
23
      int j=0;
24
      FOR (i,1,n)
25
26
          d=ceil(sqrt(minSquare));
          while(sortedX[i].X-sortedX[j].X>d)
27
28
               sortedY.erase(make_pair(sortedX[j].Y,sortedX[j].X));
29
30
               j++;
31
          auto lower=sortedY.lower_bound(make_pair(sortedX[i].Y-d,0)
32
          auto upper=sortedY.upper_bound(make_pair(sortedX[i].Y+d,0)
33
          for(auto pointer=lower; pointer!=upper; pointer++)
34
35
36
               minSquare=min(minSquare, distance(*pointer, make_pair(
                   sortedX[i].Y,sortedX[i].X)));
37
           sortedY.insert(make_pair(sortedX[i].Y,sortedX[i].X));
38
39
40
      cout << minSquare << endl;</pre>
41 }
```

## 11.3 Point in polygon

```
struct point{
    ll x,y;
    void show(){
        cout<<x<<" "<<y<endl;
};

int sign(ll a){</pre>
```

```
if(a<0) return -1;
      if(a==0) return 0;
10
      if(a>0) return 1;
12|}
14 int signCP (point p, point p1, point p2)
15 {
      return sign(1LL*((p1.x-p.x)*(p2.y-p.y)-(p1.y-p.y)*(p2.x-p.x)))
16
17 }
18
19 bool intersect(point n, point m, point a, point b)
20 1
      if(signCP(n,a,b)*signCP(m,a,b)>0) return false;
21
      if(signCP(a,n,m)*signCP(b,n,m)>0) return false;
22
23
      return true:
24 }
25
26 bool inside(point a, point b, point c){
      return a.x>=min(b.x,c.x) && a.x<=max(b.x,c.x) && a.y>=min(b.y,
27
          c.y)
      && a.y <= max(b.y,c.y);
28
29 }
30
31 inline void solve()
32 4
      int n.m: cin>>n>>m:
33
      vector < point > vertices(n);
34
      FO(i,n)
35
36
          cin>>vertices[i].x>>vertices[i].y;
37
38
      point query,par,init,first,second;
39
      int counter;
41
      int resta=0;
      FO(i,m)
42
43
44
          resta=0;
45
          counter=0;
          cin>>query.x>>query.y;
46
47
          par.x=query.x;
          par.y=-MAX-1;
48
           init.x=vertices[0].x;
49
          init.y=vertices[0].y;
50
          first.x=init.x;
51
          first.y=init.y;
52
          bool ver=false;
53
          for(int j=1; j<=n; j++)</pre>
54
55
               second.x=vertices[j%n].x;
56
               second.y=vertices[j%n].y;
57
```

```
point AB,u;
59
                AB.x=second.x-first.x;
                AB.y=second.y-first.y;
60
61
                u.x=second.x-query.x;
62
                u.y=second.y-query.y;
                if ((AB.x*u.y-AB.y*u.x) == 0 && inside(query, first, second
63
                    cout << "BOUNDARY" << endl;</pre>
65
                    ver=true;
66
                    break:
               }
67
                if(intersect(query,par,first,second) && first.x<=query</pre>
68
                    .x && query.x<second.x)</pre>
                {
69
70
                    counter++;
71
                }
72
                if(intersect(query,par,first,second) && second.x<=</pre>
                    query.x && query.x<first.x){
                    counter++:
74
               }
75
                first.x=second.x:
76
                first.y=second.y;
           }
77
78
           point AB.u:
           AB.x=init.x-first.x;
79
80
           AB.y=init.y-first.y;
81
           u.x=init.x-query.x;
           u.y=init.y-query.y;
82
           if(!ver){
83
                //if(intersect(query,par,first,init)) counter++;
84
85
                if((counter)&1) cout << "INSIDE";</pre>
86
                else cout << "OUTSIDE":</pre>
87
                cout << end1;
88
           }
      }
89
90 }
```

# 11.4 Point Location Test

```
struct point{
    double x,y;
};

struct Vector{
    double a=0,b=0;
    void getVector(point p1,point p2){
        a=p2.x-p1.x;
        b=p2.y-p1.y;
}
```

```
double getModulo(){
12
           return pow(a*a+b*b,0.5);
14
15
      Vector getUnitarian(){
16
           Vector x;
17
           x.a=a/getModulo();
18
           x.b=b/getModulo();
19
           //cout << x.a << " " << x.b << endl;
20
21
           return x;
22
23
24 };
25
26 double dotProduct(Vector x, Vector y)
28
      return x.a*y.a+x.b*y.b;
29 }
31 double CrossProduct(Vector x, Vector y)
      return x.a*y.b-x.b*y.a;
33
34 }
36 inline void solve()
37 
38
      point p1,p2,p3,p4;
39
      cin>>p1.x>>p1.y>>p2.x>>p2.y>>p3.x>>p3.y;
40
      Vector u,v,t;
41
      u.getVector(p1,p3);
42
      //cout <<u.a<<" "<<u.b<<endl;
43
      v.getVector(p2,p3);
      if(CrossProduct(u,v)>0) cout<<"LEFT"<<endl;</pre>
      else if(CrossProduct(u,v)<0) cout<<"RIGHT"<<endl;</pre>
      else cout << "TOUCH" << endl;</pre>
47
48
49 }
```

## 11.5 Polygon Area

```
struct point{
    ll x,y;
};

ll CrossP(point a,point b){
    return a.x*b.y-a.y*b.x;
}
```

```
9 inline void solve()
10 {
      int n; cin>>n;
      ll res=0;
      point p1,p2,p3;
      cin>>p3.x>>p3.y;
14
      p1.x=p3.x;
      p1.y=p3.y;
16
      FO(i,n-1)
17
18
           cin>>p2.x>>p2.y;
19
          res+=CrossP(p1,p2);
20
21
          p1.x=p2.x;
22
           p1.y=p2.y;
23
24
      res+=CrossP(p1,p3);
25
      cout << abs(res) << endl;</pre>
26 }
```

#### 11.6 Convex Hull

```
const ll mod=1e9+7:
2 const ll limit=4e9:
3 //ascii https://elcodigoascii.com.ar/
5 int orientation(point a, point b, point c){
      ll ori=(b.y-c.y)*(b.x-a.x)-(b.y-a.y)*(b.x-c.x);
      if(ori==0) return 0:
      if(ori>0) return 1;
9
      return 2;
10 }
void getLastTwo(point &a, point &b, stack <point &s)
14
     a=s.top();
     s.pop();
      b=s.top();
16
17
      s.pop();
18 }
20 void show(point a){
      cout <<a.x<<" "<<a.y<<endl;
22 }
24 //Graham scan
26 void solve(){
      int n; cin>>n;
```

```
vector < point > puntos(n);
28
      FO(i,n){
29
           ll a,b; cin>>a>>b;
30
           puntos[i]=make_pair(a,b);
31
32
      sort(all(puntos));
33
      //Lower Part
34
      stack<point> lower;
35
      FO(i,n)
36
37
           if(lower.size()<2){</pre>
38
               lower.push(puntos[i]);
39
               continue:
40
           }
41
           point a,b;
42
43
           getLastTwo(a,b,lower);
           if(orientation(a,b,puntos[i])<2)</pre>
44
45
               lower.push(b):
46
               lower.push(a);
47
               lower.push(puntos[i]);
48
           }
           else{
50
               lower.push(b);
51
               i--;
52
           }
53
54
      stack < point > upper;
      for(int i=n-1:i>=0:i--)
56
57
           if(upper.size()<2){</pre>
58
               upper.push(puntos[i]);
59
               continue;
60
61
62
           point a,b;
           getLastTwo(a,b,upper);
63
           if(orientation(a,b,puntos[i])<2)</pre>
64
65
               upper.push(b);
66
               upper.push(a);
67
               upper.push(puntos[i]);
68
           }
69
70
           elsef
               upper.push(b);
71
72
               i++;
73
74
      }
75
      set < point > res;
76
77
      while(!lower.empty()){
```

```
res.insert(lower.top());
80
           lower.pop();
      }
81
       while(!upper.empty()){
82
           res.insert(upper.top());
83
84
           upper.pop();
85
86
       cout << res.size() << endl;</pre>
      for(auto c:res) show(c);
88 }
```

### 11.7 Complex point

```
1 typedef double T;
1 typedef complex <T> pt;
3 #define x real()
4 #define y imag()
6 typedef long long 11;
typedef vector<int> vec;
8 const 11 mod=1e9+7:
g const int MAX=2e5+3;
//ascii https://elcodigoascii.com.ar/
13 T norma(pt a) {return a.x*a.x+a.y*a.y;}
16 int sgn(T X) {
      return (T(0) < X) - (T(0) > X);
18 }
19
pt translate(pt a,pt v){return a+v;}
pt scale(pt p,pt c,T factor){return c+(p-c)*factor;}
pt rot(pt p,T a){return p*polar(1.0,a);}
24 pt perp(pt p){return pt({-p.y,p.x});}
pt linearFunc(pt p,pt q,pt r,pt fp,pt fq){
      return fp+(r-p)*(fq-fp)/(q-p);
28 T dot(pt v,pt w){ return v.x*w.x+v.y*w.y;}
29 T cross(pt v,pt w) { return v.x*w.y-v.y*w.x;}
bool isperp(pt a,pt b){return dot(a,b)==0;}
33 double angle(pt v,pt w){
      return acos(clamp(dot(v,w)/abs(v)/abs(w),-1.0,-1.0));
35 }
36
```

```
37 T orientation(pt a,pt b,pt c){return cross(b-a,c-a);}
38
39 bool inAngle(pt a,pt b,pt c,pt p){
      if(orientation(a,b,c)<0) swap(b,c);</pre>
       return sgn(orientation(a,b,p))*sgn(orientation(a,c,p)) <=0;</pre>
41
42 }
43
44 bool isconvex(vector<pt> p){
      bool hasPos=false,hasNeg=false;
      for(int i=0,n=p.size();i<n;i++){</pre>
46
           int o=orientation(p[i],p[(i+1)%n],p[(i+2)%n]);
47
           if(o>0) hasPos=true;
48
           if(o<0) hasNeg=true;</pre>
49
50
      return !(hasPos && hasNeg);
51
52 }
54 inline void solve()
55 {
      pt p{3,-4};
      p+=pt({1,2});
57
      cout <<p<<endl;</pre>
      cout << norma(p) << endl;</pre>
60 }
```

#### 11.8 Polar sort

```
#define x real()
2 #define v imag()
4 typedef long long 11;
5 typedef double T;
6 typedef complex <T> pt;
7 typedef vector <int> vec;
8 const ll mod=1e9+7;
9 const int MAX=2e5+3;
11 T cross(pt v,pt w){ return v.x*w.y-v.y*w.x;}
12 T norma(pt a) {return a.x*a.x+a.y*a.y;}
//ascii https://elcodigoascii.com.ar/
14
15 bool half(pt p){
      assert(p.x!=0 || p.y!=0);
      return p.y>0 || (p.y==0 && p.x<0);
18 }
19
20 void polarSort(vector<pt> &v){
      sort(all(v),[](pt v,pt w){
```

# 12 Strings

### 12.1 Marranadas de Quique

```
2 //To Upper and Lower
transform(s.begin(), s.end(), s.begin(), ::toupper);
transform(s.begin(), s.end(), s.begin(), ::tolower);
6 // From i to the end
7 string a = s.substr(i):
8 // From i to j
g string a = s.substr(i,j);
11 int a;
12 int b:
13 int c;
14 char comma;
15 char colon:
17 // Createa a stringstream object
18 stringstream ss(fullString);
19 // Extract the strings
20 ss >> a >> colon >> b >> comma >> c;
```

# 12.2 KMP Algorithm

```
1 // LPS for s, lps[i] could also be defined as the longest prefix
    which is also a proper suffix
2 vi computeLPS(string s){
3    size_t len = 0;
```

```
size_t M = s.size();
      vi lps(M, 0);
      size_t i = 1;
      while(i < M) {</pre>
          if( s[i] == s[len]){
              len++;
               lps[i] = len;
11
              i++;
12
          } else {
13
               if(len != 0){
14
                   len = lps[len-1];
15
               } else {
16
                   lps[i] = 0;
17
                   i++;
18
19
20
21
22
      return lps;
23
24 }
25
_{26}|// Get number of occurrences of a pattern in a text using KMP
27 // O(N+M)
28 size_t KMPOccurrences(string pattern, string text){
      vi lps = computeLPS(pattern); // LPS array
30
      size_t M = pattern.size();
31
      size_t N = text.size();
32
33
      size_t i = 0; // Index for text
34
      size_t j = 0; // Index for pattern
35
36
      size_t cnt = 0; // Counter
37
38
      while ((N - i) >= (M - j)) {
39
          // Watch for the pattern
40
          if (pattern[j] == text[i]) {
41
42
               j++;
43
               i++;
44
          }
45
          // If the full match found
46
          if (j == M) {
47
               cnt++;
48
               j = lps[j - 1];
49
51
          // Mismatch after j matches
52
          else if (i < N && pattern[j] != text[i]) {</pre>
               // Do not match lps[0..lps[j-1]] characters,
54
```

```
// they will match anyway
               if (j != 0)
56
                    j = lps[j - 1];
57
58
               else
59
                    i++;
60
          }
61
62
63
      return cnt;
64 }
```

### 12.3 Rolling Hash

```
1 // Rolling hash
2 struct Hash {
      // Prime number and modulo
      long long p = 31, m = 1e9 + 7;
      long long hash_value;
6
      Hash(const string& s)
          long long hash_so_far = 0;
          long long p_pow = 1;
          const long long n = s.length();
          for (long long i = 0; i < n; ++i) {</pre>
12
              hash_so_far
13
                   = (hash_so_far + (s[i] - 'a' + 1) * p_pow)
14
                     % m;
              p_pow = (p_pow * p) % m;
15
16
17
          hash_value = hash_so_far;
18
      bool operator == (const Hash& other)
19
20
           return (hash_value == other.hash_value);
21
22
23 }:
24
25 // Usage
26 int main(){
      string s = "hello";
28
29
      return 0;
30 }
```

#### 12.4 Hash marrano

```
vector < vector < int >> group_identical_strings(vector < string > const&
s) {
```

### 12.5 Suffix Array

```
1 // Structure to store information of a suffix
2 struct suffix
3 {
      int index:
      char *suff:
6 };
8 // A comparison function used by sort() to compare two suffixes
9 int cmp(struct suffix a, struct suffix b)
11
      return strcmp(a.suff, b.suff) < 0? 1 : 0:</pre>
12 }
14 // This is the main function that takes a string 'txt' of size n
15 // argument, builds and return the suffix array for the given
int *buildSuffixArray(char *txt, int n)
17 1
      // A structure to store suffixes and their indexes
18
      struct suffix suffixes[n]:
19
20
      // Store suffixes and their indexes in an array of structures.
21
      // The structure is needed to sort the suffixes alphabetically
22
      // and maintain their old indexes while sorting
23
      for (int i = 0; i < n; i++)</pre>
25
26
          suffixes[i].index = i;
          suffixes[i].suff = (txt+i);
27
28
```

```
// Sort the suffixes using the comparison function
31
      // defined above.
32
      sort(suffixes, suffixes+n, cmp);
33
      // Store indexes of all sorted suffixes in the suffix array
35
      int *suffixArr = new int[n];
      for (int i = 0; i < n; i++)</pre>
36
           suffixArr[i] = suffixes[i].index;
37
38
      // Return the suffix array
      return suffixArr:
40
41 }
42
43 // A utility function to print an array of given size
44 void printArr(int arr[], int n)
      for(int i = 0; i < n; i++)</pre>
          cout << arr[i] << " ";
47
      cout << endl:
48
49 }
```

### 12.6 LCP

```
1 // Structure to store information of a suffix
2 struct suffix
3 {
      int index; // To store original index
      int rank[2]; // To store ranks and next rank pair
6 }:
8 // A comparison function used by sort() to compare two suffixes
9 // Compares two pairs, returns 1 if first pair is smaller
int cmp(struct suffix a, struct suffix b)
11 |
      return (a.rank[0] == b.rank[0])? (a.rank[1] < b.rank[1] ?1: 0)</pre>
12
             (a.rank[0] < b.rank[0] ?1: 0):
13
14 }
16 // This is the main function that takes a string 'txt' of size n
_{
m 17}|// argument, builds and return the suffix array for the given
vector < int > buildSuffixArray(string txt, int n)
      // A structure to store suffixes and their indexes
      struct suffix suffixes[n]:
21
22
      // Store suffixes and their indexes in an array of structures.
```

```
// The structure is needed to sort the suffixes alphabetically
      // and maintain their old indexes while sorting
25
      for (int i = 0; i < n; i++)</pre>
27
          suffixes[i].index = i;
28
          suffixes[i].rank[0] = txt[i] - 'a';
29
          suffixes[i].rank[1] = ((i+1) < n)? (txt[i + 1] - 'a'): -1;
30
31
32
      // Sort the suffixes using the comparison function
33
      // defined above.
      sort(suffixes, suffixes+n, cmp);
35
36
      // At his point, all suffixes are sorted according to first
37
      // 2 characters. Let us sort suffixes according to first 4
      // characters, then first 8 and so on
      int ind[n]; // This array is needed to get the index in
          suffixes[]
      // from original index. This mapping is needed to get
41
      // next suffix.
42
      for (int k = 4: k < 2*n: k = k*2)
43
          // Assigning rank and index values to first suffix
45
          int rank = 0:
46
          int prev_rank = suffixes[0].rank[0];
          suffixes[0].rank[0] = rank;
48
          ind[suffixes[0].index] = 0:
49
50
          // Assigning rank to suffixes
51
          for (int i = 1; i < n; i++)
52
53
54
              // If first rank and next ranks are same as that of
                  previous
              // suffix in array, assign the same new rank to this
55
              if (suffixes[i].rank[0] == prev_rank &&
                       suffixes[i].rank[1] == suffixes[i-1].rank[1])
57
58
                  prev_rank = suffixes[i].rank[0];
59
                  suffixes[i].rank[0] = rank:
61
              else // Otherwise increment rank and assign
62
63
                  prev_rank = suffixes[i].rank[0];
64
                  suffixes[i].rank[0] = ++rank;
65
66
              ind[suffixes[i].index] = i;
67
          }
68
69
          // Assign next rank to every suffix
          for (int i = 0; i < n; i++)</pre>
71
```

```
int nextindex = suffixes[i].index + k/2;
 73
 74
                suffixes[i].rank[1] = (nextindex < n)?</pre>
                                        suffixes[ind[nextindex]].rank
 75
                                            [0]: -1:
           }
 76
 77
            // Sort the suffixes according to first k characters
 78
 79
            sort(suffixes, suffixes+n, cmp);
 80
 81
 82
       // Store indexes of all sorted suffixes in the suffix array
       vector < int > suffixArr:
 84
       for (int i = 0; i < n; i++)</pre>
 85
            suffixArr.push_back(suffixes[i].index);
 86
 87
       // Return the suffix array
       return suffixArr;
 88
 89 }
 91 /* To construct and return LCP */
 92 vector <int > kasai(string txt, vector <int > suffixArr)
       int n = suffixArr.size();
 95
 96
       // To store LCP array
       vector < int > lcp(n, 0):
 97
98
       // An auxiliary array to store inverse of suffix array
100
       // elements. For example if suffixArr[0] is 5, the
       // invSuff[5] would store 0. This is used to get next
101
       // suffix string from suffix array.
102
       vector < int > invSuff(n, 0);
103
104
       // Fill values in invSuff[]
105
       for (int i=0; i < n; i++)</pre>
106
           invSuff[suffixArr[i]] = i;
107
108
109
       // Initialize length of previous LCP
110
       int k = 0:
111
112
       // Process all suffixes one by one starting from
113
       // first suffix in txt[]
       for (int i=0; i<n; i++)</pre>
114
115
116
           /* If the current suffix is at n-1, then we dont
              have next substring to consider. So lcp is not
117
118
               defined for this substring, we put zero. */
119
           if (invSuff[i] == n-1)
120
121
               k = 0;
```

```
continue;
123
124
           /* j contains index of the next substring to
               be considered to compare with the present
126
               substring, i.e., next string in suffix array */
127
           int j = suffixArr[invSuff[i]+1];
128
129
           // Directly start matching from k'th index as
130
           // at-least k-1 characters will match
131
           while (i+k<n && j+k<n && txt[i+k] == txt[j+k])</pre>
132
               k++;
133
134
           lcp[invSuff[i]] = k; // lcp for the present suffix.
135
136
137
           // Deleting the starting character from the string.
           if (k>0)
138
                k--:
139
140
141
       // return the constructed lcp array
142
       return lcp;
143
144 }
145
146 // Utility function to print an array
void printArr(vector<int>arr, int n)
148 €
       for (int i = 0; i < n; i++)</pre>
149
           cout << arr[i] << " ":
150
       cout << endl;</pre>
152 }
```

### 12.7 Z Function

```
vector < int > z_function(string s) {
    int n = s.size();
    vector < int > z(n);
    int l = 0, r = 0;
    for(int i = 1; i < n; i++) {
        if(i < r) {
            z[i] = min(r - i, z[i - 1]);
        }
        while(i + z[i] < n && s[z[i]] == s[i + z[i]]) {
            z[i]++;
        }
        if(i + z[i] > r) {
            l = i;
            r = i + z[i];
        }
}
```

```
16 }
17 return z;
18 }
```

### 12.8 Longest Palindrome

```
1 typedef long long 11;
2 typedef vector < int > vec;
3 const ll mod=1e9+7;
4 const int MAX=1e6+3:
5 vector < int > lps(2*MAX);
6 int n;
7 string s;
9 //ascii https://elcodigoascii.com.ar/
11 void show(int idx)
12 {
      int start=(idx-lps[idx])/2;
13
      int end=start+lps[idx];
14
      for(int i=start:i<end:i++){</pre>
15
           cout << s[i];
16
17
18
19 }
21 inline void solve()
22 {
23
      cin>>s;
24
      n=s.size();
      lps[0]=0;
26
      int rightCenter,leftCenter,center,curRightCenter,curLeftCenter
28
      center=1;
29
      rightCenter=center+lps[center];
      leftCenter=center-lps[center];
30
      int maxLPScenter=1;
31
      int diff=-1:
32
33
      bool exp;
      for(curRightCenter=2; curRightCenter < 2*n+1; curRightCenter++)</pre>
34
35
36
           //Condicion de cambio de centro
37
           curLeftCenter = 2 * center - curRightCenter;
38
           diff=rightCenter-curRightCenter;
39
           exp=false;
           if (diff>=0) {
40
41
               if(lps[curLeftCenter] < diff) {</pre>
42
                    lps[curRightCenter] = lps[curLeftCenter];
```

```
43
               else if(lps[curLeftCenter] == diff && rightCenter == 2*n)
44
                   lps[curRightCenter] = lps[curLeftCenter];
46
47
               else if(lps[curLeftCenter] == diff && rightCenter < 2*n){</pre>
                   lps[curRightCenter] = lps[curLeftCenter];
49
                   exp=true;
50
51
               else if(lps[curLeftCenter]>diff){
52
                   lps[curRightCenter] = diff;
53
                   exp=true;
54
55
          }
56
           elsef
57
               lps[curRightCenter]=0;
58
               exp=true;
59
60
          if(exp)
61
62
               while(((curRightCenter+lps[curRightCenter]) < 2*n &&</pre>
63
                   curRightCenter-lps[curRightCenter]>0)
               && ((curRightCenter+lps[curRightCenter]+1)%2==0 || s[(
64
                   curRightCenter+lps[curRightCenter]+1)/2]==s[(
                   curRightCenter -lps[curRightCenter] -1)/2])){
                   lps[curRightCenter]++;
65
66
67
           if (lps[curRightCenter]>lps[maxLPScenter])
68
69
               maxLPScenter=curRightCenter;
70
71
          if (curRightCenter+lps[curRightCenter]>rightCenter) {
72
               center=curRightCenter;
73
               rightCenter = curRightCenter + lps [curRightCenter];
74
75
76
      show(maxLPScenter);
77
78
79 }
```

## 12.9 String Hashing

```
typedef long long l1;
typedef vector<int> vec;
const l1 mod=1e9+7;
const int MAX=1e6+3;
const l1 A=911382323;
const l1 B=972663749;
```

```
7 ll str[MAX];
8 11 pk[MAX];
9 bool prefix[MAX]={false};
12
13 ll subs(int i,int j)
14
15
      if(i)
           return ((str[j]-pk[j-i+1]*str[i-1])%B+B)%B;
16
17
      else
           return str[j];
18
19 }
20
21 //ascii https://elcodigoascii.com.ar/
23 inline void solve()
24 1
      string s; cin>>s;
26
      memset(prefix, true, sizeof(prefix));
      str[0]=s[0]:
27
      pk[0]=1;
28
29
      int n=s.size();
30
      for(int i=1:i<n:i++)</pre>
31
32
           str[i]=A*str[i-1]+s[i];
           pk[i]=pk[i-1]*A;
33
34
           pk[i]%=B;
           str[i]%=B:
35
36
37
      11 aux:
38
      bool ver:
39
      for (int i=1; i <= n; i++)</pre>
40
41
           aux=subs(0,i-1);
           for(int j=0; j+i<=n; j+=i)</pre>
42
43
44
               if (aux!=subs(j,j+i-1))
45
                    //cout <<aux << " "<<subs(j,j+i-1) << " "<<i<< " "<<j<<
46
                         endl;
                    prefix[i]=false;
48
                    break:
50
51
           if(!prefix[i]) continue;
52
           if(n\%i \&\& (subs(n-n\%i,n-1)!=subs(0,n\%i-1)))
53
           {
54
                continue;
55
56
           cout << i << " ";
```

```
57 } 58 }
```

### 12.10 Manacher Algorithm

```
vector<int> manacher(string s) {
    string t;
    for(auto c: s) {
        t += string("#") + c;
    }
    auto res = manacher_odd(t + "#");
    return vector<int>(begin(res) + 1, end(res) - 1);
}
```

## 12.11 Suffix Automaton

```
struct state {
      int len, link;
      map < char , int > next;
4 };
6 const int MAXLEN = 100000:
7 state st[MAXLEN * 2]:
8 int sz, last;
void sa_init() {
      st[0].len = 0;
      st[0].link = -1;
12
      sz++;
13
      last = 0;
14
15 }
16
void sa_extend(char c) {
      int cur = sz++;
18
      st[cur].len = st[last].len + 1;
19
      int p = last;
20
      while (p != -1 && !st[p].next.count(c)) {
21
          st[p].next[c] = cur;
22
          p = st[p].link;
23
24
25
      if (p == -1) {
          st[cur].link = 0;
26
27
      } else {
          int q = st[p].next[c];
28
29
          if (st[p].len + 1 == st[q].len) {
              st[cur].link = q;
31
          } else {
              int clone = sz++;
```

```
st[clone].len = st[p].len + 1;
              st[clone].next = st[q].next;
34
35
              st[clone].link = st[q].link;
              while (p != -1 && st[p].next[c] == q) {
36
                   st[p].next[c] = clone;
                   p = st[p].link;
39
              st[q].link = st[cur].link = clone;
40
41
42
43
      last = cur;
44 }
45
46 long long get_diff_strings(){
      long long tot = 0;
      for(int i = 1: i < sz: i++) {
49
          tot += st[i].len - st[st[i].link].len;
50
51
      return tot:
52 }
53
54 long long get_tot_len_diff_substings() {
      long long tot = 0;
      for(int i = 1; i < sz; i++) {</pre>
          long long shortest = st[st[i].link].len + 1;
57
58
          long long longest = st[i].len;
59
60
          long long num_strings = longest - shortest + 1;
          long long cur = num_strings * (longest + shortest) / 2;
61
          tot += cur:
62
63
      return tot;
64
65 }
67 string lcs (string S, string T) {
      sa_init();
      for (int i = 0; i < S.size(); i++)</pre>
69
70
          sa_extend(S[i]);
71
      int v = 0, l = 0, best = 0, bestpos = 0;
72
73
      for (int i = 0; i < T.size(); i++) {</pre>
          while (v && !st[v].next.count(T[i])) {
74
75
              v = st[v].link;
              1 = st[v].len;
76
77
          if (st[v].next.count(T[i])) {
78
              v = st [v].next[T[i]]:
79
              1++:
80
81
          if (1 > best) {
82
83
              best = 1;
```

### 13 Formulas

### 13.1 Sums

$$c^{a} + c^{a+1} + \dots + c^{b} = \frac{c^{b+1} - c^{a}}{c-1}, c \neq 1$$

Gauss

$$1 + 2 + 3 + \dots + n = \frac{n(n+1)}{2}$$

Gauss squares

$$1^{2} + 2^{2} + 3^{2} + \dots + n^{2} = \frac{n(2n+1)(n+1)}{6}$$

Cubes

$$1^3 + 2^3 + 3^3 + \dots + n^3 = \frac{n^2(n+1)^2}{4}$$

Powers of 4

$$1^4 + 2^4 + 3^4 + \dots + n^4 = \frac{n(2n+1)(n+1)(3n^2 + 3n - 1)}{30}$$

#### 13.2 Catalan numbers

$$C_0 = 1$$
,  $C_{n+1} = \sum_{i=0}^{n} C_i C_{n-i}$  (Recursive)
$$(2n) \qquad (2n) \qquad (2n)! \qquad (2n)!$$

$$C_n = \frac{1}{n+1} {2n \choose n} = {2n \choose n} - {2n \choose n+1} = \frac{(2n)!}{(n+1)!n!} \quad \text{(Closed-form)}$$

- Valid Parentheses: Count of balanced parentheses expressions with *n* pairs.
- Full Binary Trees: Structurally unique full binary trees with n+1 leaves.

- Polygon Triangulation: Ways to triangulate a convex (n+2)-gon.
- Dyck Paths: Paths from (0,0) to (2n,0) that never dip below the x-axis.
- Non-Crossing Partitions: Ways to connect 2n points on a circle without crossing chords.
- Stack Permutations: Valid stack-sortable permutations of length n.
- Mountain Ranges: Sequences of 2n up/down steps forming valid mountain ranges.
- Unique BSTs: Number of distinct binary search trees with n keys.
- **Diagonal-Avoiding Paths**: Paths in a grid from (0,0) to (n,n) without crossing the diagonal.

### 13.3 Cayley's Formula

Number of labeled trees of n vertices:  $n^{n-2}$ . Number of rooted forest of n vertices is:  $(n+1)^{n-1}$ 

### 13.4 Geometric series

Finite:

$$\sum_{k=0}^{n} ar^{k} = \begin{cases} a \frac{1 - r^{n+1}}{1 - r} & \text{if } r \neq 1, \\ a(n+1) & \text{if } r = 1. \end{cases}$$

Infinite:

$$\sum_{k=0}^{\infty} ar^k = \frac{a}{1-r} \quad \text{(converges iff } |r| < 1\text{)}$$

### 13.5 Divisors

The number of divisors of any number n is:

$$\begin{cases} \approx 100 & n < 5 \times 10^4 \\ \approx 500 & n < 1 \times 10^7 \\ \approx 2000 & n < 1 \times 10^10 \\ \approx 200000 & n < 1 \times 10^19 \end{cases}$$

# 13.6 Number of primes between 1 and n

$$\frac{n}{\ln(n)}$$

### 13.7 Pythagorean triplets

$$a = k \cdot (m^2 - n^2), \quad b = k \cdot (2mn), \quad c = k \cdot (m^2 + n^2)$$

With m > n > 0, k = 0,  $m \perp n$ , and either m or n even.

## 13.8 Derangments

Permutations of a set sush that none of the elements appear in their original position.

$$D(n) = (n-1)(D(n-1) + D(n-2)) = nD(n-1) + (-1)^n = \lfloor \frac{n!}{e} \rfloor$$

# 14 Miscellaneous

Gus, this is a reminder to add more stuff here

# 14.1 Random number generator

```
mt19937 rng(chrono::steady_clock::now().time_since_epoch().count()
    );

uniform_int_distribution <int>(0,n)
normal_distribution <> normal_dist(mean,2)
exponential_distribution
```

# 14.2 Custom comparators

```
bool cmp(const Edge &x, const Edge &y) {return x.w < y.w}
```

# 14.3 Kadane's Algorithm

```
inline void solve()
{
   int n; cin>>n;
   vector<int> normal(n);
   vector<int> rever(n);
```

```
FO(i,n){
           cin>>normal[i];
           rever[i]=-normal[i];
      11 \text{ sum} = 0, \text{ max\_sum} = -1e9;
      ll sumr=0;
12
      for (int i = 0; i < n; i++) {
           sum += normal[i];
           max_sum = max(max_sum, sum);
14
           sumr+= rever[i]:
15
           max_sum=max(max_sum, sumr);
16
           if(i%2==1){
17
                sum=max(sum.sumr):
18
               sumr=max(sum,sumr);
19
20
           if (sum < 0) sum = 0:
22
           if (sumr<0) sumr=0;</pre>
23
      cout << max sum << endl:
25
      //Geeks for geeks
      //https://www.geeksforgeeks.org/cses-solutions-maximum-
26
           subarray-sum/
27 }
```

## 14.4 Moore's Voting Algorithm

```
int majorityElement(vector<int>& nums) {
      int vote = 0, r = 0;
      for(int i=0: i<nums.size():i++){</pre>
           if(nums[i] == nums[r])
               vote++;
               vote --;
           if(vote == 0){
               r = i;
               vote = 1;
11
      }
12
13
14
      int goal = (nums.size())/2;
16
      for(int i=0; i<nums.size(); i++){</pre>
17
           if(nums[i] == nums[r]){
               cnt++;
19
               if(cnt > goal){
20
                   break:
21
22
          }
```

# 15 Marranadas de C++

# 15.1 Compilation

```
g++-13 -std=c++20 name.cpp
```

## 15.2 Compiler optimizations

```
// Makes bit operations faster

#pragma GCC target("popcnt")

//Auto vectorize for-loops and optimizes floating points (assumes associativity and turns off denormals)

#pragma GCC optimize("Ofast")

// Doubles performance of vectorized code, crashes in old computers

#pragma GCC target("avx2")

#pragma GCC optimize("03,unroll-loops")

#pragma GCC target("avx2,bmi,bmi2,lzcnt,popcnt")
```

# 15.3 Decimal printing

Friendly reminder to use printf() with decimals

```
cout << fixed << setprecision(n) << endl;
```

### 15.4 Bit tricks

```
x \& -x is the least bit in x = c = x \& -x, r = x + c, (((bin_pow(r,x)) >> 2)/c) OR r next number bigger than x same number of bits set.
```