Selected topics of lattice gauge theory
CHRISTIAN HÖLBLING, LUKAS VARNHORST
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BERGISCHE UNIVERSITÄT WUPPERTAL

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1. Metropolis update for $U(1)$ gauge theories in 1+1 dimensions

In this task your a going to implement a Metroplis update of the $U(1)$ gauge field on a $N_s \times N_t$ lattice.

a) In the lecture you have seen that the gauge action changes by

$$\Delta S_E^{\text{gauge}} = -\beta\,\mathfrak{Re}((U'_\mu(\vec{n}) - U_\mu(\vec{n}))A^\dagger_\mu(\vec{n})) \tag{1}$$

when the link $U_\mu(\vec{n})$ is changed to $U'_\mu(\vec{n})$. Here, the staple $A_\mu(\vec{n})$ is given by

$$A_\mu(\vec{n}) = \sum_{\nu\neq\mu} \left(U_\nu(\vec{n})U_\mu(\vec{n}+\hat{\nu})U^\dagger_\nu(\vec{n}+\hat{\mu}) + U^\dagger_\nu(\vec{n}-\hat{\nu})U_\mu(\vec{n}-\hat{\nu})U_\nu(\vec{n}-\hat{\nu}+\hat{\mu})\right). \tag{2}$$

Write a routine that calculates $A^\dagger_\mu(\vec{n})$ directly.

b) Implement the Metropolis update using eq. (1) to calculate the change in the action in he update step.

c) Perform a Monte-Carlo simulation on a $N_s \times N_t = 12 \times 12$ lattice with $\beta = 1.8$. During the simulation, measure the average $P_{\text{avg}} = \frac{1}{N_s N_t}\sum_{\vec{n}\in\Lambda} P_{xt}(\vec{n})$. Verify that the result is complatible with

$$\langle P_{\text{avg}}\rangle = 0.665 \pm 0.001 \tag{3}$$