Selected topics of lattice gauge theory

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1. Solutions to fermionic path integrals

The path integrals for fermions discussed in the lecture have euclidean action that are quadratic in the fermion fields. Therefore, they can be written as

$$Z = \int \left\{ \prod_{k=0}^{N} \mathrm{d}\bar{\psi}_k \mathrm{d}\psi_k \right\} \exp(-\bar{\Psi} M \Psi) \tag{1}$$

where $\bar{\Psi}$ and Ψ are vectors containing all the $\bar{\psi}_k$ and ψ_k. Proove that

$$\int \left\{ \prod_{k=0}^{N} \mathrm{d}\bar{\psi}_k \mathrm{d}\psi_k \right\} \exp(-\bar{\Psi} M \Psi + \bar{\eta}\Psi + \bar{\Psi}\eta) = (-1)^N \det M \exp(\bar{\eta} M^{-1} \eta) \tag{2}$$

where $\bar{\eta}$ and η are Grassmann-vectors of the same dimensions that $\bar{\Psi}$ and Ψ.

Hint: *First find a trnasformation rule for a linear change of Grassmann variables. Use this rule to simplify your integral.*