**Selected topics of lattice gauge theory**

CHRISTIAN HÖLBLING, LUKAS VARNHORST

Sheet 2 in WiSe 19/20

BERGISCHE UNIVERSITÄT WUPPERTAL

https://moodle.uni-wuppertal.de/course/view.php?id=18653

## 1. Energy eigenstates of the harmonic oscillator

In this task we measure the energy gap between the ground state and the first excited state of the harmonic oscillator. For this task we use the code from the last exercise sheet.

a) Measure the correlation function

$$C(\hat{\tau}) = \langle x(0)x(\hat{\tau})\rangle \tag{1}$$

for $\hat{\tau} \in \{0, \Delta\tau, 2\Delta\tau, \ldots, (N-1)\Delta\tau\}$ for several values of the parameters $\hat{\omega}$ and $N$.

b) Calculate with your measurement the effective energy by calculating

$$E_{\text{eff}}^{\text{exp}}\left(\hat{\tau} + \frac{1}{2}\Delta\tau\right) = \frac{1}{\Delta\tau}\log\frac{C(\hat{\tau})}{C(\hat{\tau}+\Delta\tau)}. \tag{2}$$

c) Estimate the statistical errors of your effective energies.

d) Compare your results with the prediction obtained from the analytical solution of the harmonic oscillator.

e) Show that

$$E_{\text{eff}}^{\text{3pt}}(\hat{\tau}) = \operatorname{acosh}\left[\frac{C(\hat{\tau}+\Delta\tau)+C(\hat{\tau}-\Delta\tau)}{2C(\hat{\tau})}\right] \tag{3}$$

is equal to $E_1 - E_0$ if

$$C(\hat{\tau}) \propto \cosh\left(-\left(\hat{\tau} - \frac{\tau}{2}\right)(E_1 - E_0)\right). \tag{4}$$

f) Repeat the analysis but use $E_{\text{eff}}^{\text{3pt}}(\hat{t})$ instead of $E_{\text{eff}}^{\text{exp}}(\hat{\tau} + \frac{1}{2}\Delta\tau)$. What are the differences, what is similar?