Selected topics of lattice gauge theory

CHRISTIAN HÖLBLING, LUKAS VARNHORST

Sheet 5 in WiSe 19/20

BERGISCHE UNIVERSITÄT WUPPERTAL

https://moodle.uni-wuppertal.de/course/view.php?id=18653

1. Gauge transformation and the plaquette

Consider a gauge field $U_\mu(\vec{n}) \in U(1)$ on an $N_s \times N_t$ lattice. Every element in $U(1)$ has an absolute square of 1 and can be represented by a complex phase, that is one real number. Therefore, the gauge field can be stored as one real number per lattice site $\vec{n}$ and direction μ. A gauge configuration is given by all the values of $U_\mu(\vec{n})$ for every $\vec{n}$ and μ on the lattice.

a) Write a routine that takes as input a gauge configuration and calculates the plaquette

$$P_{\mu\nu}(\vec{n}) = U_\mu(\vec{n})U_\nu(\vec{n}+\hat{\mu})U_\mu^\dagger(\vec{n}+\hat{\nu})U_\nu^\dagger(\vec{n}). \tag{1}$$

b) Write a routine that takes as input a gauge configuration and a gauge transformation $\Omega(\vec{n})$ and outputs the transformed gauge configuration

$$U_\mu(\vec{n}) \to U'_\mu(\vec{n}) = \Omega(\vec{n})U_\mu(\vec{n})\Omega^\dagger(\vec{n}+\hat{\mu}). \tag{2}$$

c) Write a routine that calculates for a given gauge configuration the action

$$S_E^{\text{gauge}} = \beta \sum_{\vec{n}\in\Lambda} \sum_{\mu<\nu} \mathfrak{Re}\left(1 - P_{\mu\nu}(\vec{n})\right) \tag{3}$$

d) Start with an initial gauge configuration where every link is equal 1 and calculate S_E^{gauge} for that configuration. Construct a gauge transformation where the phase of $\Omega(\vec{n})$ is a random number between 0 and 2π for each lattice site $\vec{n}$. Apply that transformation to the initial configuration and verify that S_E^{gauge} does not change.

e) Generate a random gauge configuration where the phase of $U_\mu(\vec{n})$ is a random number between 0 and 2π for each lattice site $\vec{n}$ and each μ. Calculate the action. Verify that, while it is different from the action in d), it does not change, when one applies, again, random gauge transformations.