Selected topics of lattice gauge theory

CHRISTIAN HÖLBLING, LUKAS VARNHORST

Sheet 7 in WiSe 19/20

BERGISCHE UNIVERSITÄT WUPPERTAL

https://moodle.uni-wuppertal.de/course/view.php?id=18653

1. The Field strength trensor and the topolgical charge

a) Verify, for small lattice spacing a, that

$$Q = \frac{1}{2\pi} \Im\mathfrak{m} \left[\sum_{\vec{n} \in \Lambda} \log P_{xt}(\vec{n}) \right] \tag{1}$$

is, up to constant prefactor, the sum over the Field strength tensor $F_{xt}\left(\vec{n} + \frac{1}{2}\left(\hat{\mu} + \hat{\nu}\right)\right)$ over all lattice sites.

b) Simulate the $U(1)$ gauge theory on a 12×12 lattice with $\beta = 1.8$. Estimate the distributions of Q and S_E^{gauge} by creating two histograms. Do you notice a qualitative difference between the two histograms? Also, plot Q and S_E^{gauge} as a function of the Monte-Carlo update.

c) Repeat b) on a finer 32×32 lattice with $\beta = 12.8$. What is similar? What is different?

d) Imagine you would reduce the lattice spacing even further. What would be the problem in determining the distribution of Q? Does this difficulty also affect the histogram of S_E^{gauge}?