Selected topics of lattice gauge theory

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1. The fermionic path integral

In this task you should derive, in close analogy to the lecture, the path integral representation for a single fermionic degree of freedom using coherent states.

a) Show that the trace $\mathrm{tr}[\boldsymbol{A}] = \langle 0 \mid \boldsymbol{A} \mid 0\rangle + \langle 1 \mid \boldsymbol{A} \mid 1\rangle$ for an operator fulfilling $[\boldsymbol{A}, \psi] = 0$ and $[\boldsymbol{A}, \bar{\psi}] = 0$ can be written as

$$\mathrm{tr}[\boldsymbol{A}] = \int \mathrm{d}\bar{\psi}\mathrm{d}\psi \, \langle -\bar{\psi} \mid \boldsymbol{A} \mid \psi\rangle e^{-\bar{\psi}\psi} \tag{1}$$

b) Derive the path integral representation

$$Z = \int \left\{ \prod_{k=0}^{N} \mathrm{d}\bar{\psi}_k \mathrm{d}\psi_k \right\} \prod_{k=0}^{N} \exp\left(- \left(-\bar{\psi}_k \frac{\psi_{k+1} - \psi_k}{\Delta t} + H(\bar{\psi}_k, \psi_{k+1}) \right) \Delta t \right) \tag{2}$$

for the partition function of one fermionic degree of freedom where $\bar{\psi}_{N+1} = -\bar{\psi}_0$ and $\psi_{N+1} = -\psi_0$. Note that $H(\bar{\psi}_k, \psi_{k+1})$ is the function that has the same functional form then the Hamilton operator but with each instance of $\boldsymbol{c}^\dagger$ replaced by $\bar{\psi}_k$ and each instance of $\boldsymbol{c}$ replaced by $\bar{\psi}_{k+1}$.