Selected topics of lattice gauge theory

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1. Dispersion relation in scalar field theory

In this task we will simulate a free, real scalar field theory in 1+1 dimensions. We will use units in which $a = 1$ and therefore the euclidean action is

$$S_E = \sum_{\vec{n}\in\Lambda} \left(\frac{1}{2} \sum_{\mu=1}^{2} \left(\phi(\vec{n}+\hat{\mu}) - \phi(\vec{n})\right)^2 + \frac{m^2}{2}\phi^2(\vec{n}) \right). \tag{1}$$

In our calculation, the number of lattice sites in the spatial direction will be $N_s = 8$ and the number of lattice sites in the temporal direction will be $N_t = 8$. Furthermore, the mass will be $m = 0.4$. The exercise is planned to be discussed in the next two exercise sessions. The focus in the first week should be to implement your simulation code. In the second week you should concentrate on calculating observables.

— first week —

a) Write a code that generates field configurations according to this action, using the Metropolis algorithm.

— second week —

b) If $\phi(x,\hat{\tau})$ is your field configuration at a given update, defined at discrete values of x and τ, the discrete Fourier transform in position space is defined by

$$\Phi(p,\hat{\tau}) := \sum_x \exp(-\mathrm{i}px)\,\phi(x,\hat{\tau}). \tag{2}$$

where p must fulfill $p = (2\pi n)/N_s$ with $n \in \{0, 1, N_s/2\}$. Perform this discrete Fourier transform on each configuration where you want to measure the correlation function.

Hint: *You can either implement the Fourier transform yourself or used a fast Fourier transform (FFT) function from a library available in your programming language.*

c) With the Fourier transform applied, calculate the temporal correlation functions

$$C_p(\hat{\tau}) = \langle \Phi^*(p,0)\Phi(p,\hat{\tau})\rangle \tag{3}$$

on these configurations and write them to a file.

d) With the data collected, calculate the energy gaps between the vacuum and the one particle states with definite momentum p by using the effective energy on $C_p(\hat{\tau})$.

e) Compare the results to the continuum dispersion relation

$$E^2 = m^2 + p^2 \tag{4}$$

as well as with the lattice dispersion relation

$$\left(2\sinh\left(\frac{E}{2}\right)\right)^2 = m^2 + \left(2\sin\left(\frac{p}{2}\right)\right)^2 \tag{5}$$

Hint: *To calculate the effective energies will require some statistics. Be prepared to run your simulation code for some time before you can extract the effective energies from your files.*