

Puntos clave para resolver una recurrencia

Cada $T(N)$ se debe reemplazar por la expresión en la definición

Cada ocurrencia de N se debe reemplazar por el nuevo valor

$$T(n) = \begin{cases} 1, n = 1 \\ 8T\left(\frac{n}{2}\right) + n^3, n \geq 2 \end{cases}$$

$$T(n) = 8T\left(\frac{n}{2}\right) + n^3, n \geq 2$$

$$T(n) = 8 \left[8T\left(\frac{\frac{n}{2}}{2}\right) + \left(\frac{n}{2}\right)^3 \right] + n^3, n \geq 4$$

Paso 2.a

$$T(n) = \begin{cases} 1, n = 1 \\ 8T\left(\frac{n}{2}\right) + n^3, n \geq 2 \end{cases}$$

$$T(n) = 8T\left(\frac{n}{2}\right) + n^3, n \geq 2$$

$$T(n) = 8 \left[8T\left(\frac{\frac{n}{2}}{2}\right) + \left(\frac{\frac{n}{2}}{2}\right)^3 \right] + n^3, n \geq 4$$

$$T(n) = 8 \left[8T\left(\frac{n}{4}\right) + \frac{n^3}{2^3} \right] + n^3, n \geq 4$$

Paso 2.b

$$T(n) = 8^2 T\left(\frac{n}{4}\right) + 8 \frac{n^3}{2^3} + n^3, n \geq 4$$

$$T(n) = 8^2 T\left(\frac{n}{2^2}\right) + n^3 + n^3, n \geq 4$$

$$T(n) = 8^2 T\left(\frac{n}{2^2}\right) + 2n^3, n \geq 2^2$$

Paso 3.a

$$T(n) = 8^2 T\left(\frac{n}{2^2}\right) + 2n^3, n \geq 2^2$$

$$T(n) = 8^2 \left[8T\left(\frac{n}{2^2}\right) + \left(\frac{n}{2^2}\right)^3 \right] + 2n^3, n \geq 2^3$$

$$T(n) = 8^2 \left[8T\left(\frac{n}{2^3}\right) + \frac{n^3}{2^6} \right] + 2n^3, n \geq 2^3$$

Paso 3.b

$$T(n) = 8^2 \left[8T\left(\frac{n}{2^3}\right) + \frac{n^3}{2^6} \right] + 2n^3, n \geq 2^3$$

$$T(n) = 8^3 T\left(\frac{n}{2^3}\right) + 8^2 \frac{n^3}{2^6} + 2n^3, n \geq 2^3$$

$$T(n) = 8^3 T\left(\frac{n}{2^3}\right) + n^3 + 2n^3, n \geq 2^3$$

$$T(n) = 8^3 T\left(\frac{n}{2^3}\right) + 3n^3, n \geq 2^3$$

$$T(n) = 8^i T\left(\frac{n}{2^i}\right) + in^3, n \geq 2^i$$

Paso genérico

$$T(n) = 8^i T\left(\frac{n}{2^i}\right) + in^3, n \geq 2^i$$

$$\frac{n}{2^i} = 1$$

$$n = 2^i$$

$$i = \text{Log}_2(n)$$

Reemplazando paso genérico

$$T(n) = 8^{\text{Log}_2(n)} T\left(\frac{n}{2^{\text{Log}_2(n)}}\right) + \text{Log}_2(n)n^3$$

$$a^{\text{Log}_b(c)} = c^{\text{Log}_b(a)}$$

$$T(n) = n^{\text{Log}_2(8)} T\left(\frac{n}{n}\right) + \text{Log}_2(n)n^3$$

$$8^{\text{Log}_2(n)} = n^{\text{Log}_2(8)}$$

$$T(n) = n^3 T(1) + \text{Log}_2(n)n^3$$

$$T(n) = n^3 + \text{Log}_2(n)n^3$$