Communication Systems - Assignment 2

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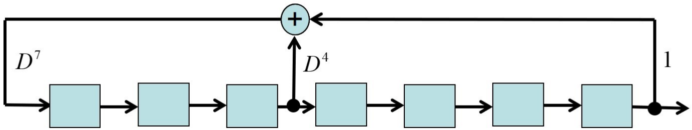
Matlab Assignment A2 - exercise 1.1

Properties of 5G PSS m-sequence

The Primary Synchronization Signal (PSS) of 5G New Radio uses an m-sequence generated by this Linear Feedback Shift Register (LFSR):



The LFSR is characterized by;

- m=7 cells
- polynomial description $D^7 + D^4 + 1$

(The association between the feedback links and the polynomial coefficients is not unique, the reverse order is used too. The proposed one is the association used by Matlab in its functions.)

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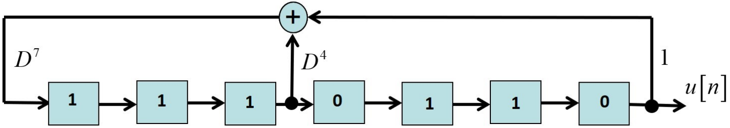
The starting seed is 1110110:



The polynomial is primitive then the LFSR generates an m-sequence with

- period $N = 2^m - 1 = 127$ bits
- first bits $= 0110111100\ldots$

Denote the binary sequence by $u(n)$ and the corresponding bipolar sequence ($0 \to -1$, $1 \to +1$) by $b(n)$, for $0 \leq n \leq N - 1$.
When needed, consider them as the principal periods of periodic sequences.

Matlab Assignment A2 - exercise 1.1

Write a Matlab program to:

1. Generate and plot the 127-symbol bipolar sequence $b(n)$
2. Write a table with
   - N_1 and N_0 (number of bits equal to 1 or 0 in $u(n)$)
   - N_T and N_{NT} (number of transitions and no-transitions in $u(n)$)
3. Check if the N_1/N_0 and N_T/N_{NT} properties are verified.

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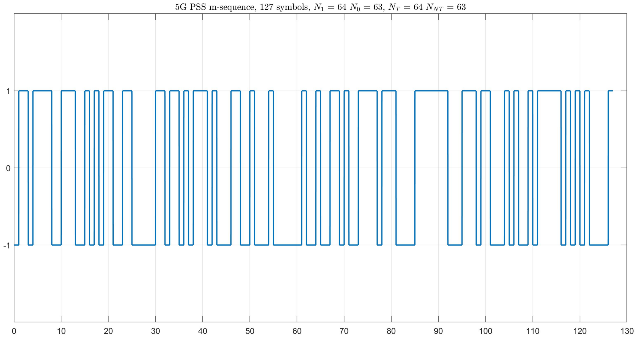


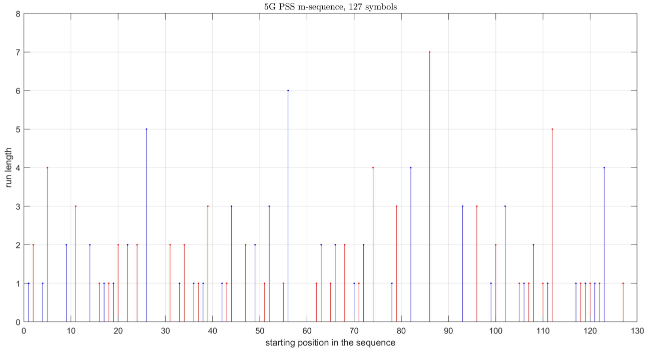
Matlab Assignment A2 - exercise 1.1

4. Write a table with the values of $NR_0(i)$ and $NR_1(i)$ (number of length i-runs of consecutive 0/1 symbols in $u(n)$)
5. Verify the property described in this table:

length	0-runs	1-runs
1	2^{m-3}	2^{m-3}
2	2^{m-4}	2^{m-4}
$\vdots$	$\vdots$	$\vdots$
r	2^{m-r-2}	2^{m-r-2}
$\vdots$	$\vdots$	$\vdots$
$m-2$	1	1
$m-1$	1	0
m	0	1
Totals:	2^{m-2}	2^{m-2}

6. Plot the run lengths vs. their starting points in the sequence (use different colors for 0 and 1 runs)

Matlab Assignment A2 - exercise 1.1



Matlab Assignment A2 - exercise 1.1

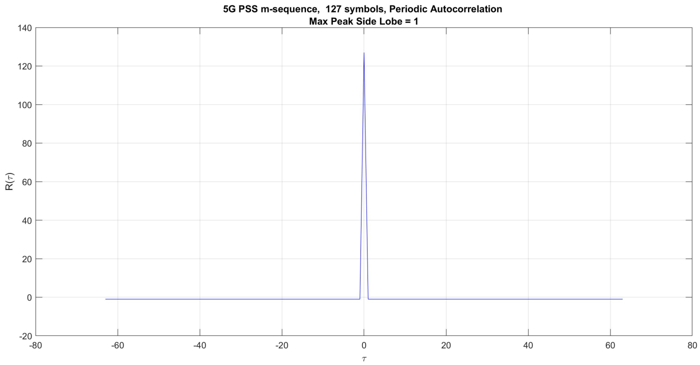
6. Compute and plot the periodic autocorrelation function

$$R(\tau) = \sum_{n=0}^{N-1} b(n)\, b(n-\tau) \qquad -(N-1) \leq \tau \leq N-1$$

   and verify the property:

   - $R(\tau) = N$ for $\tau = 0$
   - $R(\tau) = -1$ for $\tau \neq 0$

Matlab Assignment A2 - exercise 1.1



Matlab Assignment A2 - exercise 1.1

Matlab example

```
m=6; % number of cells
Nb=2∧m-1; % period
pnSequence = comm.PNSequence('Polynomial',[6 1 0], ...
'SamplesPerFrame',Nb,'InitialConditions',[1 1 1 1 1 1]);
x1 = pnSequence()';
```

Matlab Assignment A2 - exercise 1.1

Matlab example

x1b=2*x1-1; % bipolar version 0 $\rightarrow$ -1 1 $\rightarrow$ +1
R=ifft(fft(x1b).*conj(fft(x1b))); % non-normalized periodic autocorr.

Matlab Assignment A2 - exercise 1.2

Properties of truncated m-sequence

Cancel the last $i = 10$ bits and compute the MPSL.

1. Plot the autocorrelation function.
2. Compute the MPSL

$$MPSL = max_{\tau \neq 0} \left| R \left(\tau \right) \right|$$

3. Comment the result.
4. Repeat with another starting seed.
5. Comment the result.

Matlab Assignment A2 - exercise 1.3

Cross-correlation of m-sequence

Given the (entire, bipolar) 5GG m-sequences b_1 of period 127, consider these other 3 primitive polynomial of degree 7:

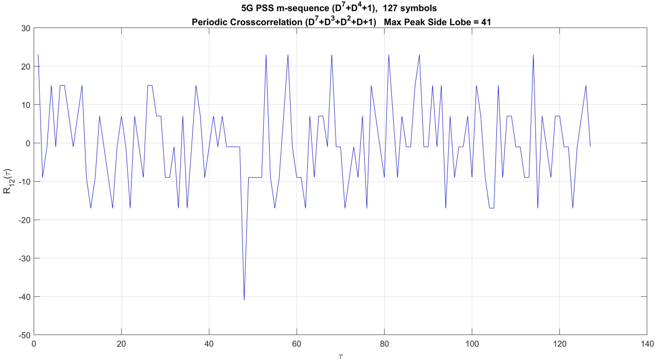
$$p_2(D) = D^7 + D^3 + D^2 + D + 1$$
$$p_3(D) = D^7 + D + 1$$

1. Consider the m-sequence b_2 generated by $p_2(D)$. Compute and plot the periodic cross-correlation between b_1 and b_2

$$R_{12}(\tau) = \sum_{n=0}^{N-1} b_1(n)\, b_2(n-\tau) \qquad -(N-1) \leq \tau \leq N-1$$

2. Repeat for p_3.
3. Comment the results.

Matlab Assignment A2 - exercise 1.3



Matlab Assignment A2 - exercise 2

Simulate one of the phases of 5G NR synchronization: detect the PSS.
NOTE: We are not considering OFDM, we are working only in the frequency domain (simplified system)

Matlab Assignment A2 - exercise 2

1. Initialize a sequence of zeros of length L, with $L = 240$ (the number of subcarriers for a SSB symbol), as the signal to be transmitted
2. Generate a PSS sequence, i.e. an m-sequence of 127 bits, not shifted (PSS with $N_{ID2} = 0$).
3. Apply BPSK modulation to the PSS. Map $0 \longrightarrow +1$ and $1 \longrightarrow -1$.
4. Insert the PSS inside the previously defined signal, in the position as in the structure of the SSB (subcarrier 57)

Matlab Assignment A2 - exercise 2

1. Insert Additive White Gaussian Noise

$$r(n) = s(n) + w(n)$$

Consider $E_S/N_0 = 0$ dB with noise variance $\sigma^2 = N_0/2$

Matlab Assignment A2 - exercise 2

1. Generate a local replica of the 5G PSS sequence associated to $N_{ID2} = 0$.
2. Generate the other two replicas shifting by 43 (PSS with $N_{ID2} = 1$) and 86 (PSS with $N_{ID2} = 2$) the first one (use Matlab *circshift*)
3. Detect the PSS sequence correlating the received signal with the PSS local replica. Plot the correlation for each shift (use Matlab *xcorr* and *abs*)

Matlab Assignment A2 - exercise 2