Communication Systems - Assignment 1

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Matlab Assignment A1 - exercise 1.1

Bit Rate vs. key parameters

In this exercise we consider:

- The Shannon theorem to estimate the maximum achievable Bit Rate
- The Friis equation to model free space attenuation (Line-of-Sight link)
- Additive White Gaussian noise
- Realistic values for 5G base-stations

and we analyze the behaviour of the Bit Rate against

- Band
- Power
- Range
- Frequency

Shannon theorem

For any channel we can define a capacity C. For any bit rate R_b smaller than C the probability of error can be made arbitrarily small by properly design encoders and decoders. (Conversely, if R_b is greater we cannot make the error probability arbitrarily small). Meaning: disturbance does not impose limits on accuracy, but on the bit rate.

For the AWGN (Additive White Gaussian Noise) channel we have:

$$R_b < C = B \log_2 \left(1 + \frac{P_{RX}}{P_N}\right)$$

where B is the band (amplitude of the frequency slot containing the transmitted signal spectrum), P_{RX} is the received power and P_N the noise power.

Friis equation

We focus on a LOS (Line of Sight) link between due antennas (no obstacles, no reflections, only a direct path). The received power P_{RX} is linked to the transmitted power P_{TX} by

$$P_{RX} = P_{TX} \frac{G_{TX} G_{RX}}{\left(\frac{4\pi d}{\lambda}\right)^2} = P_{TX} \frac{G_{TX} G_{RX}}{\left(\frac{4\pi df}{c}\right)^2}$$

where:

- G_{TX} and G_{RX} are the transmitter and receiver antenna gains
- d is the distance between the antennas
- $\lambda = c/f$ is the wavelength
- f is the transmission frequency
- c is the light speed

AWGN channel

We model the channel as AWGN. The noise power spectral density is constant and equal to $N_0/2$ with $N_0 = kT$, where

- k is the Boltzmann constant $k = 1.038e - 23$
- T the operative temperature of the receiver in Kelvin degrees

The noise power is computed as

$$P_N = kTBF$$

where the noise figure F takes into account of SNR decrease due to the receiver.

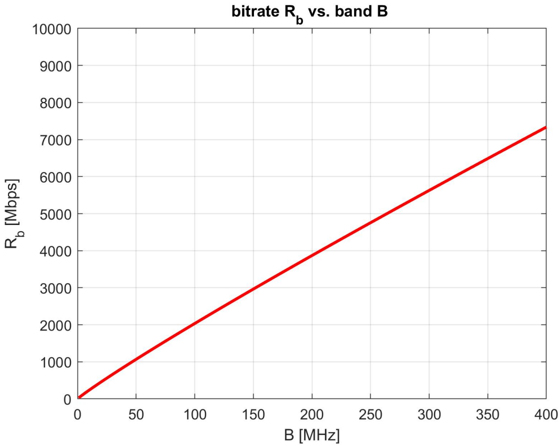
Realistic values for 5G base-stations

As an example, we start from these values

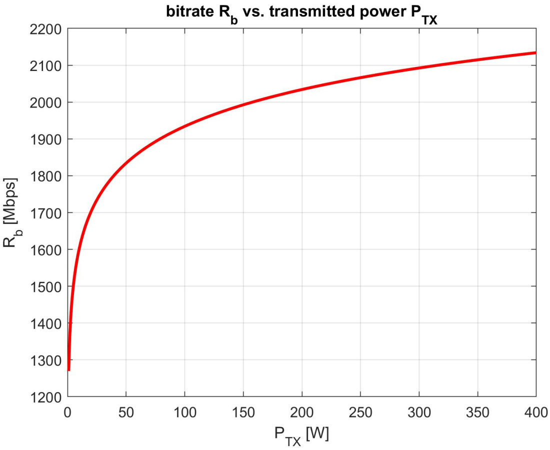
- $P_{TX} = 200W(55dBm)$ (typical transmitted power for large 5G band)
- $B = 100MHz$ (typical 5G band)
- $G_{TX} = 10dBi$ (base station antenna gain for a 120^o degree antenna covering a sector of the cell)
- $G_{RX} = 0dBi$ (user omni-directional antenna gain)
- $f = 3.6GHz$ (typical 5G transmission frequency)
- $d = 200m$ (typical 5G cell radius)
- $F = 6dB$ (typical user noise figure)
- $T = 300K$

Investigate the behavior of the bit rate vs. band, transmitted power, covered distance, frequency.

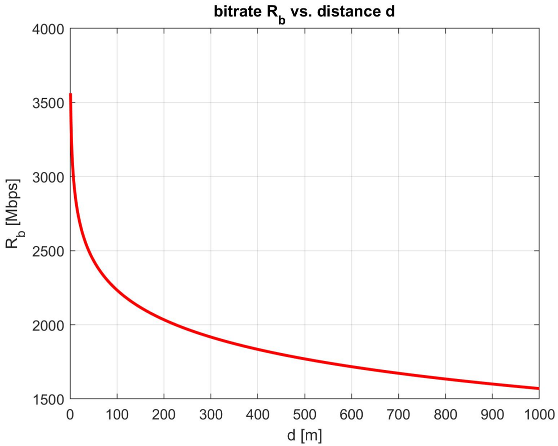
Bit rate vs. band



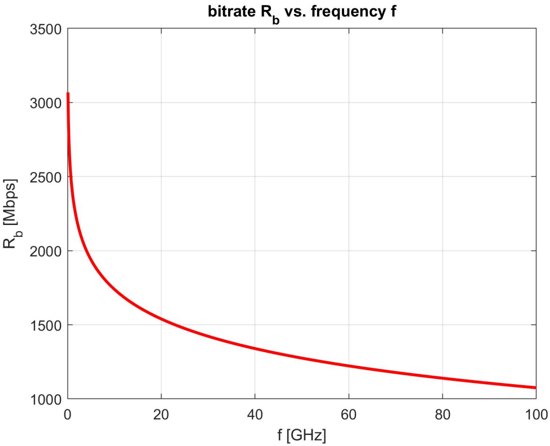
Bit rate vs. transmitted power



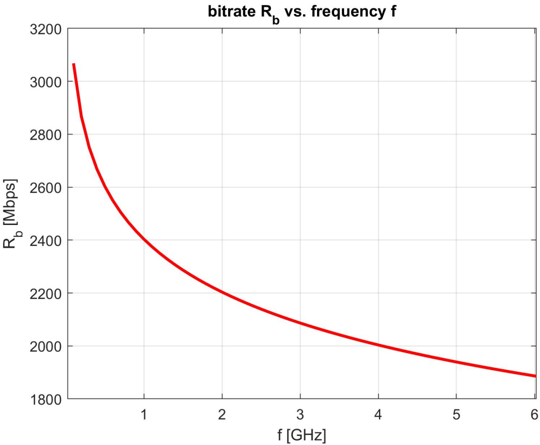
Bit rate vs. distance



Bit rate vs. frequency (zoom)



Bit rate vs. frequency



Report

Present all the figures and discuss the corresponding bit rate behavior.

Matlab Assignment A1 - exercise 1.2

Link budget

In this exercise we study a realistic link budget for a space communication systems.

Link Budget 1

System parameters

Slant range	m	384e6
Telemetry info bit rate	bit/s	1e6
Coding rate		0.5
Symbol rate	symbol/s	2e6
RF Bandwidth/Symbol rate		1.5
RF Bandwidth	Hz	3e6

$C(n, k)$ $\quad n > k$

k INFO BITS $\rightarrow$ n CODED BITS

ACTUALLY TX

OVER THE CHANNEL

$$R_b^* = R_b \frac{n}{k} > R_b$$

R_b — INFO BIT RATE

R_b^* — CODED BIT RATE

$$\frac{k}{n} = \frac{1024}{2048} = \frac{1}{2} = 0.5$$

IN OUR LINK BUDGET

$R_b = 1$ Mbps $\longrightarrow$ $R_b^* = 2$ Mbps

CONSTELLATION

2-PSK

1 bit/symbol

2^m-PSK/QAM $\rightarrow$ m bit/symbol

SYMBOL RATE

$$R_S = \frac{R_b^*}{m}$$

IN OUR LINK BUDGET

$m = 1 \rightarrow \qquad R_S = R_b^*$

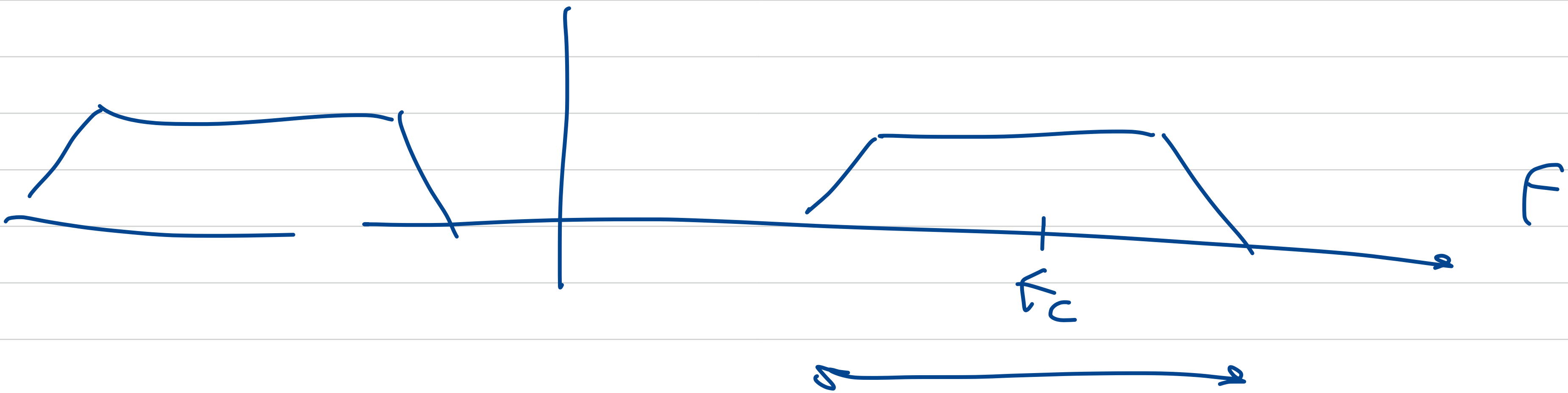
CARRIER FREQ.

$p(t)\cos(2\pi f_c t)$

SHAPING FILTER

$p(t) \equiv$ SQUARE ROOT RAISED COSINE FILTER

$\alpha = 0.5$



$B = R_s(1+\alpha)$

Link Budget 2

Spacecraft parameters - Transmitter

S/C TX power	W	3
S/C TX power	dBW (or dBm)	compute
TX loss	dB	1.5
TX antenna gain (S band)	dBi	???
Pointing loss	dB	1
S/C EIRP	dBW (or dBm)	compute

EIRP = Effective Isotropic Radiated Power = power that a hypothetical isotropic radiator should emit to observe the same received power at a receiver located in the direction of the antenna's main lobe

- WRITE A REALISTIC VALUE AND IN THE REPORT WRITE THE SOURCE

$$P\,[\mathrm{dBW}] = 10 \log_{10} \underset{[\mathrm{W}]}{P}$$

$$P\,[\mathrm{dBm}] = 10 \log_{10} \frac{\overset{[\mathrm{w}]}{P}}{1\mathrm{e}{-3}}$$

Link Budget 3

Channel parameters

Frequency	GHz	2.113
Free space path loss	dB	compute
Atmospheric and Ionospheric loss	dB	2.5
Total propagation loss	dB	compute
Power at G/S	dBW	compute

Link Budget 4

Ground Station parameters - Receiver

FT

RX antenna gain	dBi	77
Pointing loss	dB	1
RX power	dBW	compute
RX system temperature (FT)	K	26
Noise power	dBW	compute
RX SNR	dB	compute
RX E_b/N_o	dB	compute

Link Budget 5

Final computation

Required E_b/N_o at $\boxed{\text{FER=1e-6}}$	dB	???
E_b/N_o margin	dB	compute

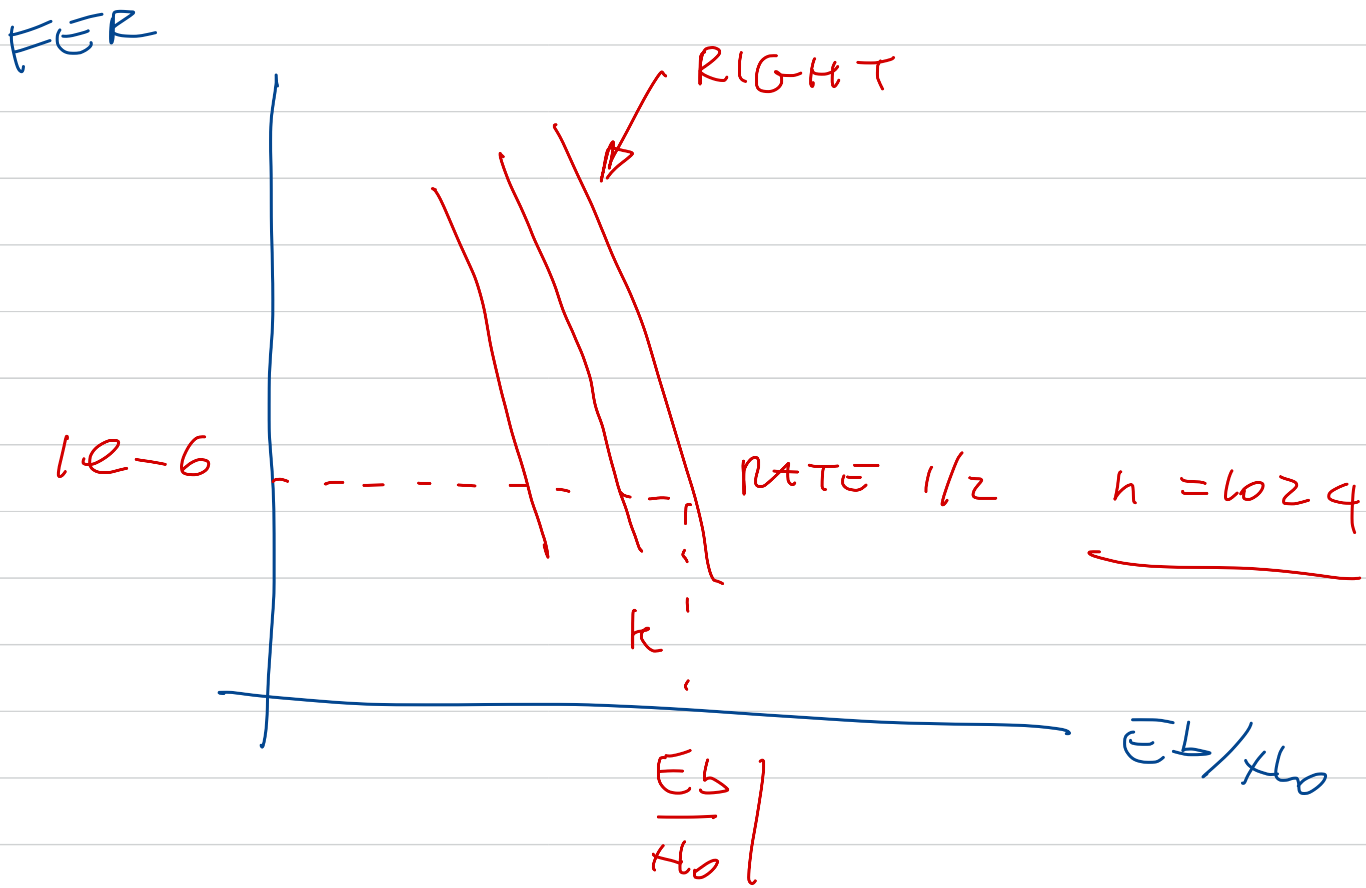
Coding Scheme

LDPC(2048,1024) (Code Rate = 1/2) - CCSDS TM Synchronization and Channel Coding - Green Book CCSDS 130.1-G-3

WWW. CCSDS. ORG

PUBLICATIONS

BLUE BOOKS - GREEN BOOKS


FER
RIGHT
1e-6
RATE 1/2
n = 1024
k
Eb/No
Eb/No

Questions

E_b/N_0 [dB]

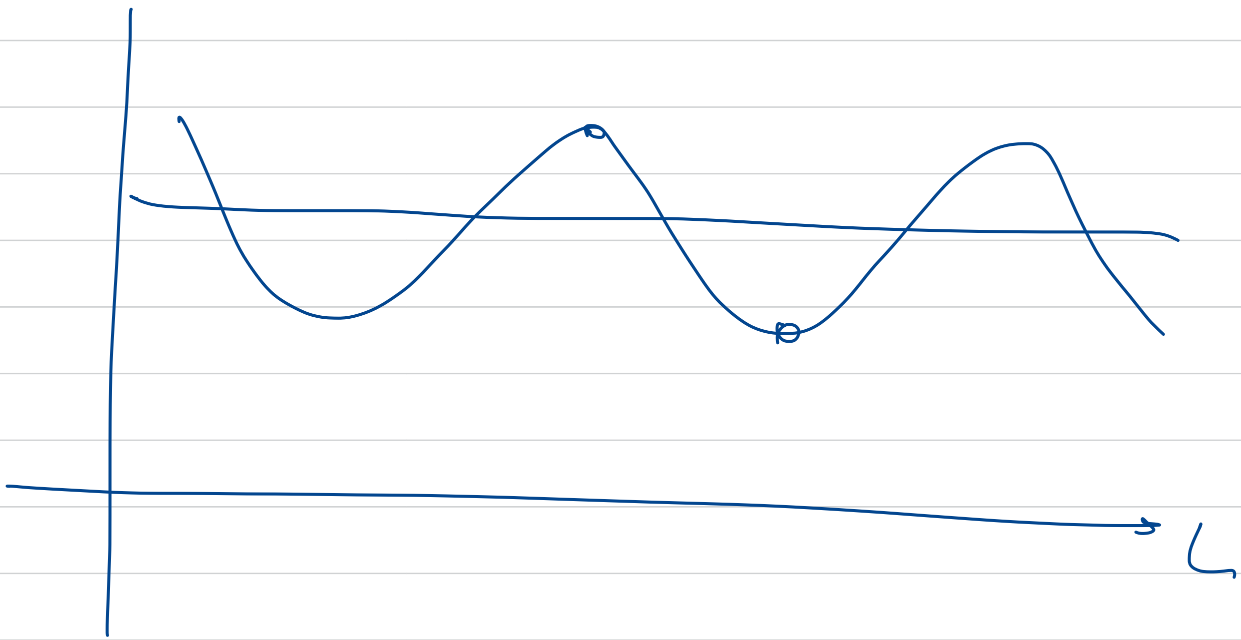
1. Which is the margin for a transmission from the Moon at bit rate 1 Mbit/s?
2. If we transmit from Mars and we reduce the margin to 10 dB, which is the maximum information bit rate we can transmit?

THIS WAY WE INVESTIGATE THE TRADE-OFF BTW BIT RATE AND DISTANCE

Multipath fading and Rayleigh pdf

In this exercise we study the statistical characterization of multipath fading.

TWO-RAY



Problem

We have a cell, served by a base-station. At a given distance d (for example, the edge of the cell), we expect to receive a power $\overline{P}$ (that, for example, we can compute by considering the EIRP, the large scale path loss and the log-normal shadowing). Instead, due to multipath fading, sometimes we receive more power and sometimes less power. We want to statistically characterize this problem.

Problem

We suppose that the user is reached by N rays (no direct path, large N), with length $l_1, \cdots, l_N$. The impulse response is given by

$$\boxed{h(t) = \sum_{i=1}^{N} \alpha_i \delta(t - \underline{D_i})}$$

where each delay is given by

$$D_i = \frac{l_1}{c}$$

Received signal

We transmit a signal

$$s(t) = \cos(2\pi f_o t)$$

and we receive a signal

$$r(t) = \sum_{i=1}^{N} \alpha_i \cos(2\pi f_o (t - D_i))$$

Phases

We have

$$r(t) = \sum_{i=1}^{N} \alpha_i \cos(2\pi f_o (t - D_i)) = \sum_{i=1}^{N} \alpha_i \cos(2\pi f_o t - 2\pi f_o D_i) =$$

$$\longrightarrow \quad = \sum_{i=1}^{N} \alpha_i \cos(2\pi f_o t - \phi_i)$$

where the generic phase $\phi_i = f_o D_i$ depends on the carrier frequency (deterministic) and the delay D_i (random). Then it is a random variable and we suppose it has a uniform pdf inside the interval $[0, 2\pi]$.

Received signal

We have

$$r(t) = \sum_{i=1}^{N} \alpha_i \cos(2\pi f_o t - \phi_i) =$$

$$= \boxed{\sum_{i=1}^{N} \alpha_i \cos(\phi_i)} \cos(2\pi f_o t) + \boxed{\sum_{i=1}^{N} \alpha_i \sin(\phi_i)} \sin(2\pi f_o t)$$

We introduce

$$X_1 = \sum_{i=1}^{N} \alpha_i \cos(\phi_i)$$

$$X_2 = \sum_{i=1}^{N} \alpha_i \sin(\phi_i)$$

Power

We have

$$r(t) = X_1 \cos(\phi_i) + X_2 \sin(\phi_i) = R\cos(2\pi f_o t - \theta)$$

with

$$R = \sqrt{X_1^2 + X_2^2}$$

The received power is then given by

$$P = \frac{R^2}{2}$$

We want to characterize when P is more or less than the expected value.

Gaussian pdfs

$$X_1 = \sum_{i=1}^{N} \alpha_i \cos \phi_i$$

is a random variable with a Gaussian pdf (large N) with mean

$$E[X_1] = \sum_{i=1}^{N} \alpha_i E[\cos \phi] = 0$$

and variance

$$E[X_1^2] = E\left[\sum_i \alpha_i \cos \phi_i \sum_j \alpha_j \cos \phi_j\right] = \sum_i \sum_j \alpha_i \alpha_j E[\cos \phi_i \cos \phi_j] =$$

$$= \sum_i \alpha_i^2 E[\cos^2 \phi_i] = \frac{\sum_i \alpha_i^2}{2}$$

and the same results hold for

$$X_2 = \sum_{i=1}^{N} \alpha_i \sin \phi_i$$

Statistical independence

We show that

$$E[X_1 X_2] = E[X_1]\,E[X_2]$$

then they are uncorrelated and ,since they are Gaussian, statistically independent. In fact

$i \neq j$

$$E[X_1 X_2] = E\left[\sum_i \alpha_i \cos\phi_i \sum_j \alpha_j \sin\phi_j\right] = \sum_i \sum_j \alpha_i \alpha_j E[\cos\phi_i \sin\phi_j] =$$

$$= \sum_i \alpha_i^2 E[\cos\phi_i \sin\phi_i] = \sum_i \alpha_i^2 E[\sin 2\phi_i]\,\frac{1}{2} = 0$$

Property

X_1 and X_2 are two Gaussian random variables, with zero mean, variance σ^2 and statistically independent.
Their joint pdf is

$$f_{X_1X_2}(x_1x_2) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{x_1^2}{2\sigma^2}} \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{x_2^2}{2\sigma^2}} = \frac{1}{2\pi\sigma^2} e^{-\frac{x_1^2+x_2^2}{2\sigma^2}}$$

Radius

What are the cdf and the pdf of the amplitude $\boxed{R = \sqrt{X_1^2 + X_2^2}}$ of the received signal?
The cdf is

$$F_R(r) = Pr\left(R \leq r\right) = \int\int \frac{1}{2\pi\sigma^2} e^{-\frac{x_1^2+x_2^2}{2\sigma^2}} dx_1 dx_2 = \int_0^r \int_0^{2\pi} \frac{1}{2\pi\sigma^2} e^{-\frac{\rho^2}{2\sigma^2}} \rho d\rho d\psi$$

where we have used

$$dx_1 dx_2 = \rho d\rho d\psi$$

Radius

We remember that

$$\frac{d}{dr}\int_0^r f(x)dx = f(r)$$

Then the pdf is given by

$$f_R(r) = \frac{d}{dr}F_R(r) = \frac{d}{dr}\int_0^r \int_0^{2\pi} \frac{1}{2\pi\sigma^2} e^{-\frac{\rho^2}{2\sigma^2}} \rho d\rho d\psi = \int_0^{2\pi} \frac{1}{2\pi\sigma^2} e^{-\frac{r^2}{2\sigma^2}} r d\psi =$$

$$= \frac{1}{2\pi\sigma^2} e^{-\frac{r^2}{2\sigma^2}} r \int_0^{2\pi} d\psi = \frac{1}{2\pi\sigma^2} e^{-\frac{r^2}{2\sigma^2}} r 2\pi = \boxed{\frac{r}{\sigma^2} e^{-\frac{r^2}{2\sigma^2}}}$$

which is the **Rayleigh pdf**.

Power

We have

$$P = \frac{R^2}{2}$$

where R has pdf

$$f_R(r) = \frac{r}{\sigma^2} e^{-\frac{r^2}{2\sigma^2}}$$

since $R = \sqrt{2P}$ we have

$$F_P(p) = Pr\left(P \leq p\right) = \int_0^{\sqrt{2p}} \frac{r}{\sigma^2} e^{-\frac{r^2}{2\sigma^2}} dr$$

Power

We set

$$x = \frac{r^2}{2\sigma^2}$$

then

$$dx = \frac{r}{\sigma^2} dr$$

$$F_P(p) = Pr\left(P \leq p\right) = \int_0^{\sqrt{2p}} \frac{r}{\sigma^2} e^{-\frac{r^2}{2\sigma^2}} dr =$$

$$= \int_0^{\frac{p}{\sigma^2}} \frac{r}{\sigma^2} e^{-x} \frac{\sigma^2}{r} dx = \boxed{1 - e^{-\frac{p}{\sigma^2}}}$$

Result

The power is a random variable with exponential pdf. The cdf is

$$F_P(p) = Pr\,(P \leq p) = 1 - e^{-\frac{p}{\sigma^2}}$$

The pdf is

$$f_P(p) = \frac{1}{\sigma^2} e^{-\frac{p}{\sigma^2}}$$

The mean value is σ^2.

Suggestion: to solve

$$\int p \frac{1}{\sigma^2} e^{-\frac{p}{\sigma^2}}\, dp$$

use

$$\int_0^\infty x e^{-x} dx = 1$$

Question 1

Generate Z (very large number) samples of X_1 and X_2, which are two statistically independent Gaussian random variables with zero mean and variance $\sigma^2 = 1$.
Compute the amplitude

$$R = \sqrt{X_1^2 + X_2^2}$$

and the power

$$P = \frac{R^2}{2}$$

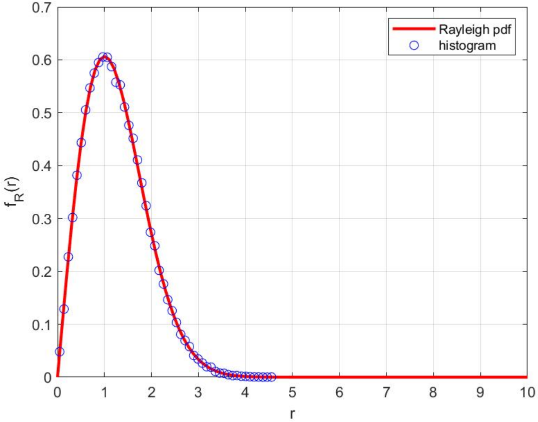
The amplitude has a Rayleigh pdf:

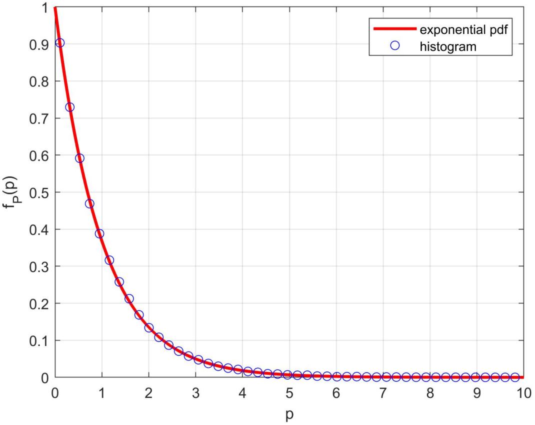
$$f_R(r) = \frac{r}{\sigma^2} e^{-\frac{r^2}{2\sigma^2}}$$

The power has an exponential pdf:

$$f_P(p) = \frac{1}{\sigma^2} e^{-\frac{p}{\sigma^2}}$$

1. Compare the analytic pdf and the histogram for R and P


0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
$f_R(r)$
0 1 2 3 4 5 6 7 8 9 10
r
Rayleigh pdf
histogram


1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0 1 2 3 4 5 6 7 8 9 10
p
f_P(p)
exponential pdf
histogram

Question 2

By using the simulated data, compute the probability that the received power is less than

- the mean value
- 10 dB below the mean value
- 20 dB below the mean value
- 30 dB below the mean value

Given the exponential cdf of the power:

$$F_P(p) = Pr\left(P \leq p\right) = 1 - e^{-\frac{p}{\sigma^2}}$$

provide a simple explanation of the -10/-20/-30 dB results.

DISCUSSION OF THIS EXPRESSION WHEN THE POWER IS SMALL

EARLY DELIVERY $\dot{\equiv}$ +2 POINTS
FOR THIS
ASSIGNMENT

24/10/2021