

# COURSE STRUCTURE

19-9-23

## A NUMERICAL SOLUTION OF HYPERBOLIC EQUATIONS

### 1 ADVECTION EQUATION

- BASIC CONCEPTS OF NUMERICAL SIMULAT.
- ACCURACY AND STABILITY
- FEW FINITE DIFFERENCE SCHEMES

### 2 SYSTEMS OF 2 HYPERBOLIC EQUATIONS PRACTICAL EXAMPLES:

- ELASTIC WAVES IN SOLIDS
  - PRESSURE WAVES IN FLUIDS
  - ELECTROMAGNETIC WAVES IN CABLES
- #### NUMERICAL METHODS
- METHOD OF CHARACTERISTICS
  - FEW FINITE DIFFERENCE SCHEMES

## B FUNDAMENTAL EQUATIONS GOVERNING WAVES

- SIMPLE HARMONIC OSCILLATOR
- 2 COUPLED OSCILLATORS
- TRAVELLING WAVES

- DERIVE ALL EQUATIONS SOLVED IN PART A

## ASSESSMENT:

ASSIGNMENT	1	35%
"	2	65%

TIMETABLE: 2H LECTURES / WEEK  
2H COMPUTING CLASS

# INTRODUCTION

**WAVES:** DISTURBANCES OF A SYSTEM WHICH PROPAGATE THROUGH THE SYSTEM FROM ONE REGION TO ANOTHER

**EXAMPLES:** OCEAN WAVES, SOUND, LIGHT, SEISMIC WAVES, X-RAYS, RADIO WAVES

**TYPES OF WAVES REGARDING THE MEDIUM:**

- MECHANICAL (NEED A MEDIUM)
- ELECTROMAGNETIC (CAN TRAVEL IN A VACUUM)

**TYPES OF WAVES BY DIMENSIONALITY**

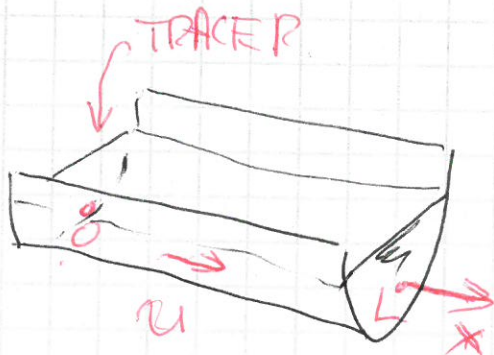
- 1D
- 2D
- 3D



## PART A NUMERICAL SOLUTIONS OF HYPERBOLIC EQUATIONS

### 1. ADVECTION EQUATION

MODEL HYPERBOLIC EQUATION. IT DESCRIBES THE MOVEMENT OF A TRACER BY A FLUID



$$\frac{\partial C}{\partial t} + u \frac{\partial C}{\partial x} = 0$$

C - TRACER CONCENTRATION  $[kg/m^3]$

u - FLUID VELOCITY  $[m/s]$

x, t - SPACE, TIME  
 $[m]$   $[s]$



## 1.1 NUMERICAL SOLUTION OF THE ADVECTION EQUATION (AE)

AE IS A FIRST ORDER (ONLY FIRST DERIVATIVE) PARTIAL DIFFERENTIAL EQUATION (PDE)

AE CONTAINS:

- 2 INDEPENDENT VARIABLES:  $x, t$
- 1 KNOWN PARAMETER:  $u$
- 1 UNKNOWN VARIABLE:  $C$

IN ORDER TO BE ABLE TO SOLVE IT WE NEED TO KNOW:

- SIMULATION DOMAIN:  $0 \leq x \leq L$
- SIMULATION TIME:  $0 \leq t \leq T$
- INITIAL CONDITION:  $C(0 \leq x \leq L, t=0)$
- BOUNDARY CONDITION:  $C(x=0, 0 \leq t \leq T)$