EG501S Numerical Simulation of Waves **ASSIGNMENT 2**

Numerical solution of systems of two hyperbolic equations

You should choose ONE of the three topics.

Working on the Computing tasks 2-4 you have already done majority of the coding needed for the Assignment 2. You are expected to finalise the code, carry out a number of simulations and discuss their results. One of the input parameters (wave speed) is specified, but most of them are left for you to select, so that you can create the cases which you find interesting. You will also choose the number of cases that you want to include in the report. It is recommended that you start with a simplest possible case and then build up the complexity by varying ONE PARAMETER AT A TIME, otherwise it is difficult to interpret the results.

Present results in a short report. The report should have maximum 5000 words, excluding figures, equations, and the code (the code should be included as an appendix). Results and discussion are the most important part of the report. Make sure that you present the results as clearly and concisely as possible. The report should contain:

- A brief background to explain why the selected problem is important (up to 2 CGS points),
- A detailed explanation of the numerical method (up to 3 CGS points),
- List of simulation runs carried out and rationale behind choosing them (up to 2 CGS points),
- Results and discussion (up to 12 CGS points: 4 CGS points for each investigated parameter)
- Conclusions (up to 3 CGS points).

Topic 1

Transient pipe flow

Transient pipe flow can be described using the following simplified system of equations

$$\begin{aligned} &\frac{\partial H}{\partial t}+\frac{c^2}{gA}\frac{\partial Q}{\partial x}=0 \\ &\frac{\partial Q}{\partial t}+gA\frac{\partial H}{\partial x}=0 \end{aligned} \qquad (1)$$

where H is the hydraulic head in the pipe ($H=p/\rho g+z$, p-pressure, ρ-density, z-pipe elevation), Q is the discharge, c is speed of sound in the pipe and friction losses have been neglected.

Consider a pipe which consists of two sections with the following characteristics:

1: L_1= to be varied, D_1 = to be varied, c_1 = 1000 m/s

2: L_2= to be varied, D_2 = to be varied, c_2 = 1000 m/s

where L_1, L_2 are the lengths of the respective sections of the pipe, D_1, D_2 are their diameters, and c_1, c_2 is the wave speed for the liquid flowing through the sections.

The upstream end of the pipe is connected to a large tank with a constant level H_T. The downstream end has a valve that discharges into the atmosphere. The discharge coefficient for the valve, C_Q, does not change with the change of its area, A_v.

Initially the valve is partially open and flow is steady, with energy loss occurring only at the valve, so that hydraulic head and discharge in the pipe are both constant: $H(x,0)=H_0$, $Q(x,0)=Q_0$. The valve then closes during a certain time interval T_0 causing a surge of hydraulic head which propagates along the pipe, called water hammer. Frictional energy losses are negligible.

Project objective: *Investigate transient flow in the pipe described above.*

Write a Matlab routine that simulates water hammer in the pipe by solving the system of equations (1) numerically, without using the existing Matlab functions for solving PDEs.

Run a series of simulations in order to investigate how the pipe diameter in the first section and the duration of the valve closure influence water hammer.
Investigate the effect of another parameter or boundary condition of your choice.

Topic 2

Propagation of signals in a transmission cable

Change of voltage and current in cables is described by the following set of partial differential equations

$$\frac{\partial V}{\partial t} = -\frac{1}{C'}\frac{\partial I}{\partial x}$$
$$\frac{\partial I}{\partial t} = -\frac{1}{L'}\frac{\partial V}{\partial x} \qquad (1)$$

where *V*, *I* are the voltage and the current in the cable, respectively and *L'*, *C*' are unit capacitance and inductance of the cable, respectively.

Consider a long transmission cable. Initially both voltage and current are zero, $V(x,0)=0$; $I(x,0)=0$. A signal is generated at one end of the cable by a master transmitter. The signal consists of a single rectangular pulse of voltage. This change of voltage causes the change of current and this disturbances propagate along the cable. The cable consists of two sections with the following characteristics:

1: L_1= to be varied, Z_1 = to be varied, $c_1 = 200 \times 10^6$ m/s

2: L_2= to be varied, Z_2 = to be varied, $c_2 = 200 \times 10^6$ m/s

where L_1, L_2 are the lengths of the respective sections of the cable and Z_1, Z_2 are their impedances. The cable is terminated by a resistor with the resistance *R*.

Project objective: *Investigate propagation of the signal in the cable*.

Write a Matlab routine that simulates propagation of the signal along the cable by solving the system of equations (1) numerically, without using the existing Matlab functions for solving PDEs.

Run a series of simulations in order to investigate how the impedance of the first cable section and the resistance of the resistor influence quality of the signal.
Investigate the effect of another parameter or boundary condition of your choice.

Topic 3

Elastic waves in a thin bar

Longitudinal motion of particles in a thin elastic bar is governed by the system of equations

$$\begin{aligned} \frac{\partial p}{\partial t} &= -E\frac{\partial v}{\partial x} \\ \frac{\partial v}{\partial t} &= -\frac{1}{\rho}\frac{\partial p}{\partial x} \end{aligned} \qquad (1)$$

where p is the compressive stress ($p=-\sigma$) , v is the speed of the rod particles ($v = \partial y/\partial t$, y – displacement), E is the Young's modulus of elasticity and ρ is density.

Consider a very long thin elastic bar (diameter 35mm). Initially both the compressive stress and the velocity are zero, $p(x,0)=0; v(x,0)=0$. A disturbance is generated at one end of a bar by a load-cell. At the other end the bar is connected to a material which generates the compressive stress proportional to the speed, i.e. relationship between p and v is $p = K v$. The signal consists of a single rectangular pulse of the compressive force. The resulting disturbances propagate along the bar, which consists of two sections with the following characteristics:

1: L_1= to be varied, Z_1 = to be varied, c_1 = 5000 m/s

2: L_2= to be varied, Z_2 = to be varied, c_2 = 5000 m/s

where L_1, L_2 are the lengths of the respective sections of the bar, Z_1, Z_2 are their impedances ($Z = \sqrt{\rho E}$), and c_1, c_2 is the wave speed for the sections ($c = \sqrt{E/\rho}$).

Project objective: *Investigate propagation of the signal along the bar*.

Write a Matlab routine that simulates propagation of the signal along the bar by solving the system of equations (1) numerically, without using the existing Matlab functions for solving PDEs.

Run a series of simulations in order to investigate how the impedance of the first bar section and the resistance parameter K influence the propagation of the disturbance along the bar. Investigate the effect of another parameter or boundary condition of your choice.