**1 Introduction**

Food packaging materials are an important component of food processing and preservation. They not only protect food from external contamination and damage but also extend the shelf life of food, increase its added value, and improve its market competitiveness. However, the production, use, and disposal of food packaging materials also generate a large amount of carbon dioxide emissions, which have adverse environmental impacts. According to statistics, about 140 million tons of food packaging materials are consumed globally each year, of which about 80% are plastic products. The carbon dioxide emissions from these plastic products are about 120 million tons, accounting for about 3% of global greenhouse gas emissions. Therefore, how to reduce the carbon dioxide emissions of food packaging materials and improve their environmental friendliness is an important issue facing the food industry.

This article aims to explore the relationship between carbon dioxide emissions of food packaging materials and the food itself, analyze the carbon dioxide emissions and influencing factors of different types, forms, and functions of food packaging materials throughout their lifecycle, and propose measures and suggestions to reduce carbon dioxide emissions from food packaging materials. Using mathematical modeling methods to select food packaging materials to achieve the goal of minimum carbon emissions is a concept of green packaging. Mathematical modeling can help analyze the carbon footprint of different packaging materials in production, use, and disposal stages, thus optimizing packaging design and selection. This article introduces some common mathematical modeling methods and related literature for reference.

One commonly used mathematical modeling method is life cycle assessment (LCA), which is a systematic method for evaluating the environmental impacts of products or services throughout their life cycle. LCA can quantify the carbon emissions generated by packaging materials in stages such as raw material acquisition, processing, transportation, use, and disposal, as well as possible emission reduction measures. LCA requires the collection and analysis of large amounts of data and is usually supported by professional software for modeling and calculation. LCA has a wide range of applications, involving various types and forms of packaging materials, such as plastics, paper, glass, metals, etc.

This article aims to study the relationship between food packaging materials and carbon dioxide emissions and explore how to reduce the carbon dioxide emissions of food by selecting packaging materials and their sizes. This article first introduces the types and characteristics of food packaging materials, analyzes the carbon dioxide emissions generated by different types of packaging materials in production, use, and disposal stages, and their potential impact on the environment and human health. Then, this article establishes a 0-1 integer programming model to mathematically model the carbon emissions of food packaging and find the optimal packaging strategy. Finally, Python is used to solve the model. In conclusion, this article summarizes the main findings and conclusions of this study, points out the existing shortcomings and issues that need further research, and provides some references and inspirations for the sustainable development of the food industry.

**2 Question restatement**

1. The goal is to choose a packaging material that will keep the product edible for at least the expected number of days.

2. There are four available packaging materials, each with a storage factor for unopened packages. The standard packaging sizes are 1, 2, 3, 4, 6, and 8 servings.

3. The food itself has a CO2 equivalent factor, and the packaging material also has a CO2 equivalent value.

4. Compare packaging materials and size options and determine the option that results in the least amount of CO2 emissions.

5. Calculate the amount of CO2 emissions per serving of food consumed, and consider the impact of spoiled food on emissions. The function used to simulate food preservation should be described clearly.

**3 Problem analysis**

First, we need to clarify the objective of the problem, which is to select a packaging material and size that keeps the food edible for the expected number of days while minimizing CO2 emissions. CO2 emissions consist of two parts: emissions from the food itself and emissions from the packaging material. When selecting the packaging material and size, we need to consider the packaging's freshness retention ability and the CO2 emissions caused by spoiled food.

The food is consumed within one day of opening the packaging and spoils on the second day, so packaging with a storage factor of 4 can keep the food fresh for 8 days.

The second question is how to choose the appropriate packaging size to minimize CO2 emissions from food and packaging. In this problem, we need to consider two factors: the weight and storage factor of the packaging and the amount of food consumed in each package size.

For each packaging size, we can calculate the CO2 emissions per package, which consist of two parts: CO2 emissions from the packaging material and CO2 emissions from the food. We assume that all packaging materials are disposable, so their CO2 emissions only need to be calculated once. For food, the amount consumed in each packaging size is different, so the food CO2 emissions for each packaging size are also different.

**4 model building**

**4.1 Decision variables and model parameters**

There are now four materials available for us to use. We first define as a binary variable that represents whether the material is used. If the material is chosen, then is assigned as 1; otherwise, it is assigned as 0. Obviously, we can only choose one of the four materials, so , which gives us the first constraint.

In addition, according to the problem, we know that for each material, there are 6 sizes to choose from, and the amount of material consumed by each size of packaging is different, which is in line with common sense. The larger the size of packaging used, the more material is consumed. We define a decision variable to represent the size of packaging chosen for the material. Obviously, is also a binary integer variable, and it also follows that the sum of all equals 1, as we only choose one size among all the sizes. Thus, we have obtained the decision variables for the mathematical programming model.

The decision variables used in the model

|  |  |
| --- | --- |
| Decision variables | meaning |
|  |  |
|  |  |

In addition to these two decision variables, the model also sets the parameters in the table below, which will be explained in detail later.

|  |  |
| --- | --- |
| Symbol | Definition |
|  | Carbon dioxide equivalent corresponding to the material |
|  | The quantity required for the material |
|  | The amount of CO2 emitted per 100g of food before spoilage |
|  | A coefficient representing the increase in CO2 emissions after food spoilage |
|  | The number of days food can be kept without packaging |
|  | Preservation factors for different packaging materials |
|  | CO2 emissions from food |
|  | CO2 emissions from producing packaging |
|  | The food capacity that can be contained in the packaging size |

**4.2 The establishment of the objective function**

The purpose of the food packaging problem is to ensure the quality of food while minimizing carbon emissions. Carbon emissions should include both the carbon dioxide equivalent produced by the production of the packaging and the carbon dioxide equivalent produced by the food itself over time.

For simplicity, let us first consider the carbon dioxide emissions generated by the food packaging. According to the table, we can determine the carbon dioxide equivalent corresponding to each material, which we set as parameter , representing the unit carbon dioxide equivalent of the material. By multiplying with , we obtain the carbon dioxide equivalent of the material used. However, we need to multiply it by a variable , which represents the number of packaging required to pack 100 units of food. From this, we obtain the total carbon dioxide produced by the packaging.

Carbon dioxide equivalent and storage coefficient of four materials

|  |  |  |
| --- | --- | --- |
| Materia | CO2-equivalency | Preservation cofficient |
| 1 | 4.5 | 3 |
| 2 | 3.3 | 3 |
| 3 | 3 | 4 |
| 4 | 2.2 | 1.5 |

Next, we calculate the carbon dioxide emissions produced by the food. As mentioned in the background, carbon dioxide emissions will increase if the food is not stored properly after it spoils. Therefore, we assume that the carbon dioxide emissions for every 100g of food are before spoilage and after spoilage, where is a variable coefficient greater than 1 that ensures the accuracy and adaptability of the model.

Material Consumption Corresponding to Package Size

|  |  |
| --- | --- |
| Portion Size | Packaging Material (g) |
| 1 | 100 |
| 2 | 150 |
| 3 | 200 |
| 4 | 250 |
| 6 | 300 |
| 8 | 400 |

Clearly, the shelf life of different foods varies. To make the model more applicable to various foods and more accurate, we assume that the food can be stored for m days without packaging. The third column of the table shows the preservation coefficient for different materials, which we set as . Thus, we can obtain the actual number of days that food can be stored using packaging size by multiplying the original shelf life of m days by the preservation coefficient of the material.

Finally, we define the total time that food is stored, from production and packaging to consumer consumption, as d days. Therefore, we can calculate the carbon dioxide emissions produced by the food.

Then we can get the total CO2 emissions as follows:

**4.3 problem constraints**

First of all, the first constraint for this problem has been obtained above, that is, the constraint for packaging materials, that is, only one material can be used for each packaging, expressed by the following formula:

Similarly, we can only choose one package size for this kind of food packaging, so we get a formula with a similar form:

Next, according to the conditions given in the title, it can be known that the daily food consumption of the target customer group is 100g. Therefore, when we choose a specific package, we should consider that the capacity of the package is greater than or equal to 100g. We assume that the food capacity that can be packaged using the packaging size is , so we can get the constraints on the packaging capacity:

Finally, there is a very critical point, we must ensure that the food can be stored for more than 5 days in the case of using this packaging material and size, otherwise all the calculations for reducing carbon emissions will be in vain, so we need to add this Constraints on food storage time:

Among them, m represents the number of days that the food can be stored without packaging, and is the storage coefficient of the packaging material.

So far, we have obtained a complete 0-1 integer linear programming model, which is summarized as follows:

**5 model solving**

This is a mixed integer programming problem that can be solved using the optimization toolbox in matlab. The Optimization Toolbox in Matlab is a collection of tools for solving mathematical programming problems in Matlab, including a variety of solving algorithms and functions. The main solution methods include linear programming, quadratic programming, integer programming, etc. Commonly used functions in the optimization toolbox include optimproblem, optimvar, solve, etc., which are used to define problems, define variables, and solve problems. In addition, there are some functions for specific solving problems, such as linprog for linear programming problems, quadprog for quadratic programming problems, etc.

Decision variables are defined with the following two lines of code:

1. x = optimvar('x', 4, 'Type', 'integer', 'LowerBound', 0, 'UpperBound', 1);
2. y = optimvar('y', 5, 'Type', 'integer', 'LowerBound', 0, 'UpperBound', 1);

Then give the model the parameters we set. In addition to the parameters set in the title, we randomly set the CO2 emission of food to 5, the carbon dioxide increase after food spoilage to 1.5, and the original shelf life of food to 10 days. Next define the objective function and constraints:

1. Q = sum(n .\* c .\* x) + f \* m \* sum(p .\* x) + e \* f \* sum((d - m \* p) .\* y);
2. con1 = sum(y) >= 100 \* m \* p;
3. con2 = m \* p >= 5;
4. con3 = sum(x) == 1;
5. con4 = sum(y) == 1;

Because of the constraints, the solution is not unique. In the above results, the values ​​of the decision variables and are 1, and the rest are 0, corresponding to the target, and finally we get a solution to the problem:

1. Optimal value: 2550.0
2. Optimal variable values:
3. X : [0.0, 0.0, 1.0, 0.0]
4. Y : [0.0, 0.0, 1.0, 0.0, 0.0]

At this point, the problem is solved.

**6 Conclusion**

The problem presents a packaging problem in which the objective is to choose the most appropriate packaging material and size for a food product that can retain its freshness for the expected number of days while minimizing the carbon emissions. There are four packaging materials available, each with a storage factor, and six standard packaging sizes. The amount of carbon dioxide emissions depends on both the packaging and the food itself, and the CO2 emissions from spoiled food should be considered. The model's decision variables include binary variables and , representing the choice of material and packaging size, respectively, while the model's parameters include CO2 equivalents for materials and the quantity required for each material. The model's objective function involves minimizing carbon dioxide emissions from both packaging and food while considering spoilage. The mathematical programming model also includes a set of constraints to ensure that only one material and one packaging size are selected.