1.第一步骤校正时的Jacobians推导.

$$e_{rot_{i,i+1}} = \mathrm{Log}\{(\Delta\bar{R}_{i,i+1}\mathrm{Exp}(J^{g}_{\Delta R}\delta b_g))^T R_c^b \cdot \mathrm{Exp}(-w_{c_i} t_d) R_w^{c_i} R_{c_{i+1}}^w \mathrm{Exp}(w_{c_{i+1}} t_d) R_b^c\}$$

需要求解：$\frac{\partial e_{rot_{i,i+1}}}{\partial \delta b_g}$、$\frac{\partial e_{rot_{i,i+1}}}{\partial \delta \phi_c^b}$、$\frac{\partial e_{rot_{i,i+1}}}{\partial \delta t_d}$　　$\frac{f(x+\Delta x)-f(x)}{\Delta x}$

为了推导方便，设 $i+1=j$，令

$\phi_1' = \mathrm{Log}\{(\Delta\bar{R}_{ij}\mathrm{Exp}(J^{g}_{\Delta\bar{R}}\delta b_g))^T\}$　　只与 δb_g 相关

$\phi_2' = \mathrm{Log}\{\mathrm{Exp}(-w_{c_i} t_d) R_w^{c_i} R_{c_j}^w \mathrm{Exp}(w_{c_j} t_d)\} = \mathrm{Log}\{R_2'\}$　　只与 t_d 相关

$\phi_3' = \mathrm{Log}\{\Delta R_{ij}^T R_c^b R_2' R_b^c\}$　　只与 R_c^b 相关

1.按照扰动的方式求Jacobian，先求对 R_c^b 的Jacobian

根据指数映射的Adjoint性质：

$$e_{rot}(R_c^b) = \mathrm{Log}\{\mathrm{Exp}(\phi_1') R_c^b \mathrm{Exp}(\phi_2') R_b^c\} = \mathrm{Log}\{\mathrm{Exp}(\phi_1')\mathrm{Exp}(R_c^b\phi_2')\}$$

$R \cdot \mathrm{Exp}(\vec{\phi}) R^T = \mathrm{Exp}(R\vec{\phi})$

$$e_{rot}(R_c^b\mathrm{Exp}(\delta\phi_c^b)) = \mathrm{Log}\{\mathrm{Exp}(\phi_1')\mathrm{Exp}(R_c^b\mathrm{Exp}(\delta\phi_c^b)\phi_2')\}$$

$\mathrm{Exp}(\vec{\phi}) \simeq I + (\vec{\phi})^\wedge$

$$\simeq \mathrm{Log}\{\mathrm{Exp}(\phi_1')\mathrm{Exp}(R_c^b\phi_2' + R_c^b\delta\phi_c^{b\wedge}\phi_2')\}$$

BCH近似：$\mathrm{Exp}(\vec{\phi}+\delta\vec{\phi}) \simeq \mathrm{Exp}(\vec{\phi})\cdot\mathrm{Exp}(J_r(\vec{\phi})\delta\vec{\phi})$
$\simeq \mathrm{Exp}(J_l(\vec{\phi})\delta\vec{\phi})\mathrm{Exp}(\vec{\phi})$

$$\simeq \mathrm{Log}\{\mathrm{Exp}(\phi_1')\mathrm{Exp}(R_c^b\phi_2')\mathrm{Exp}(J_r(R_c^b\phi_2')R_c^b\delta\phi_c^{b\wedge}\phi_2')\}$$

$$= \mathrm{Log}\{\mathrm{Exp}(e_{rot}(R_c^b))\mathrm{Exp}(-J_r(R_c^b\phi_2')R_c^b\phi_2'^\wedge\delta\phi_c^b)\}$$

BCH近似：$\mathrm{Log}(\mathrm{Exp}(\vec{\phi})\cdot\mathrm{Exp}(\delta\vec{\phi})) \simeq \vec{\phi} + J_r^{-1}(\vec{\phi})\cdot\delta\vec{\phi}$
$\mathrm{Log}(\mathrm{Exp}(\delta\vec{\phi})\mathrm{Exp}(\vec{\phi})) \simeq \vec{\phi} + J_l^{-1}(\vec{\phi})\delta\vec{\phi}$

$$\simeq e_{rot}(R_c^b) - J_r^{-1}(e_{rot}(R_c^b))J_r(R_c^b\phi_2')R_c^b\phi_2'^\wedge\delta\phi_c^b$$

所以 $$\frac{\partial e_{rot_{i,j}}}{\partial\delta\phi_c^b} = \frac{e_{rot}(R_c^b\mathrm{Exp}(\delta\phi_c^b)) - e_{rot}(R_c^b)}{\delta\phi_c^b} = -J_r^{-1}(e_{rot}(R_c^b))J_r(R_c^b\phi_2')R_c^b\phi_2'^\wedge$$

2. 按照扰动的方式求对 δb_g 的Jacobian.

$$e_{rot}(\delta b_g) = \mathrm{Log}\{\mathrm{Exp}(-J^{g}_{\Delta R}\delta b_g)\mathrm{Exp}(\phi_3')\}$$

$$e_{rot}(\delta b_g + \widetilde{\delta b_g}) = \mathrm{Log}\{\mathrm{Exp}(-J^{g}_{\Delta R}(\delta b_g + \widetilde{\delta b_g}))\mathrm{Exp}(\phi_3')\}$$

$$\simeq \mathrm{Log}\{\mathrm{Exp}(-J_l(-J^{g}_{\Delta R}\delta b_g)J^{g}_{\Delta R}\widetilde{\delta b_g})\mathrm{Exp}(-J^{g}_{\Delta R}\delta b_g)\mathrm{Exp}(\phi_3')\}$$

$$= \mathrm{Log}\{\mathrm{Exp}(-J_l(-J^{g}_{\Delta R}\delta b_g)J^{g}_{\Delta R}\widetilde{\delta b_g})\mathrm{Exp}(e_{rot}(\delta b_g))\}$$

$$\simeq e_{rot}(\delta b_g) - J_l^{-1}(e_{rot}(\delta b_g))\cdot J_l(-J^{g}_{\Delta R}\delta b_g)J^{g}_{\Delta R}\cdot\widetilde{\delta b_g}$$

所以 $$\frac{\partial e_{rot,ij}}{\partial \widetilde{\delta b_g}} = \frac{\partial e_{rot}(\delta b_g + \widetilde{\delta b_g}) - \partial e_{rot}(\delta b_g)}{\widetilde{\delta b_g}} = -J_l^{-1}(e_{rot}(\delta b_g)) \cdot J_l(-J^g_{\Delta R}\delta b_g) J^g_{\Delta R}$$

3. 按照扰动的方式求对 t_d 的 Jacobian.

令 $R_1'' = (\Delta \bar{R}_{ij} \mathrm{Exp}(J^g_{\Delta R}\delta b_g))^T R_C^b$

$R_2'' = R_W^{C_i} R_{C_j}^W$

$R_3'' = R_b^C$

$$e_{rot}(t_d) = \mathrm{Log}\{R_1'' \mathrm{Exp}(-w_{ci} t_d) R_2'' \mathrm{Exp}(w_{cj} t_d) R_3''\}$$

$$e_{rot}(t_d + \delta t_d) = \mathrm{Log}\{R_1'' \mathrm{Exp}(-w_{ci}(t_d + \delta t_d)) R_2'' \mathrm{Exp}(w_{cj}(t_d + \delta t_d)) R_3''\}$$

$$\approx \mathrm{Log}\{R_1'' \mathrm{Exp}(-J_l(-w_{ci} t_d) w_{ci} \delta t_d) \mathrm{Exp}(-w_{ci} t_d) R_2'' \cdot \mathrm{Exp}(w_{cj} t_d) \mathrm{Exp}(J_r(w_{cj} t_d) w_{cj} \delta t_d) R_3''\}$$

$$= \mathrm{Log}\{\mathrm{Exp}(-R_1'' J_l(-w_{ci} t_d) w_{ci} \delta t_d) R_1'' \mathrm{Exp}(-w_{ci} t_d) R_2'' \mathrm{Exp}(w_{cj} t_d) R_3'' \mathrm{Exp}(R_3''^T J_r(w_{cj} t_d) w_{cj} \delta t_d)\}$$

$$= \mathrm{Log}\{\mathrm{Exp}(-R_1'' J_l(-w_{ci} t_d) w_{ci} \delta t_d) \mathrm{Exp}(e_{rot}(t_d)) \mathrm{Exp}(R_3''^T J_r(w_{cj} t_d) w_{cj} \delta t_d)\}$$

$\mathrm{Exp}(\phi) R = R\,\mathrm{Exp}(R^T \phi)$

$$= \mathrm{Log}\{\mathrm{Exp}(e_{rot}(t_d)) \mathrm{Exp}(-\mathrm{Exp}(e_{rot}(t_d))^T R_1'' J_l(-w_{ci} t_d) w_{ci} \delta t_d) \cdot \mathrm{Exp}(R_3''^T J_r(w_{cj} t_d) w_{cj} \delta t_d)\}$$

$$= \mathrm{Log}\{\mathrm{Exp}(e_{rot}(t_d)) \cdot \mathrm{Exp}(D \cdot \delta t_d) \cdot \mathrm{Exp}(E \cdot \delta t_d)\}$$

$$\approx \mathrm{Log}\{\mathrm{Exp}(e_{rot}(t_d)) (I + (D+E)^\wedge \delta t_d)\}$$

$$\approx \mathrm{Log}\{\mathrm{Exp}(e_{rot}(t_d)) \mathrm{Exp}((D+E)\delta t_d)\}$$

$$\approx e_{rot}(t_d) + J_r^{-1}(e_{rot}(t_d)) (D+E) \delta t_d$$

所以: $$\frac{\partial e_{rot,ij}}{\partial t_d} = \frac{e_{rot}(t_d + \delta t_d) - e_{rot}(t_d)}{\delta t_d} = J_r^{-1}(e_{rot}(t_d))(D+E)$$