Calculus Exams 3

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1 Question 1: 2pt

Compute the limit of the following sequence

$$s_n = \frac{3n^4 + \sin(n)}{2n^4 + 5n^2 + 2} \tag{1}$$

1.1 Solution

$$\frac{3n^4 + \sin(n)}{2n^4 + 5n^2 + 2} \sim \frac{3n^4}{2n^4} = \frac{3}{2}$$

2 Question 1: 1pt

Show that these two functions are asymptotic to each other for $x \to 0$.

$$f(x) = 1 - \cos(x) \tag{2}$$
$$g(x) = x(e^x - 1)/2 \tag{3}$$

2.1 Solution

$$\frac{1 - \cos(x)}{x(e^x - 1)/2} \sim \frac{1/2x^2}{x^2/2} = 1$$

3 Question 3: 1pt

Write a Python script to approximate the following derivative at $x_0 = 1$ with dx equal to 0.1.

$$\frac{\mathrm{d}\left(xe^{x^2}\right)}{\mathrm{d}x} \tag{4}$$

3.1 Solution

```
f = lambda x: x*np.exp(x**2)
    a = 0
    b = 1
    dx = 0.1
    x0 = 1.
    result = (f(x0 + dx) - f(x0))/dx
```

4 Question 4: 2pt

Compute derivative of the following function

$$f(x) = \phi(x^2 + \sin(\omega(x)^2)) \tag{5}$$

where $\omega(x)$ and $\phi(x)$ are arbitrary functions and ϕ_0 is a real number.

4.1 Solution

$$\frac{\mathrm{d}f(x)}{\mathrm{d}x} = \phi'(x^2 + \sin(\omega(x)^2))(2x + \cos(\omega(x)^2))2\omega(x)\omega'(x)$$

5 Question 4: 2pt

Compute the following integral:

$$\int_0^1 e^{2x+e^x}\mathrm{d}x \tag{6}$$

Hint: You need to use both substitution and integration by parts.

5.1 Solution

You start with the substitution $u(x) = e^x$, $du(x) = e^x\mathrm{d}x$.

$$\int_0^1 e^{2x+e^x}\mathrm{d}x = \int_0^1 e^x e^{e^x} e^x \mathrm{d}x \tag{7}$$

$$= \int_0^{\pi/2} u(x)e^{u(x)}\mathrm{d}u(x) \tag{8}$$

$$\tag{9}$$

you can now use the integration by parts formula:

$$\int_0^1 u(x)e^{u(x)}\mathrm{d}u(x) + \int_0^1 e^{u(x)}\mathrm{d}u(x) = u(1)e^{u(1)} - u(0)e^{u(0)} \tag{10}$$

Hence:

$$\int_0^1 u(x)e^{u(x)}\mathrm{d}u(x) = -e^{e^1} + e^1 + ee^{e^1} - e^1 \tag{11}$$
$$= e^e(e-1) \tag{12}$$

6 Question 5: 2pt

Compute the integral

$$\int_0^1 x\mathrm{d}x$$

using only the definition of integral.

Hint: you need to compute the limit of a sequence of approximations with rectangles.

6.1 Solution

In the lecture notes