# Improving SNR with Optimized Kaiser Window FIR Filter Coefficients Via Genetic Algorithm

Guy Nguyen-Phuoc
*University of Hawai'i West O'ahu*
*Ewa Beach, HI USA*
nguyen46@hawaii.edu

***Abstract*—Wireless signal transmission inevitably contains unwanted elements (noise). A big component of signal processing is the elimination or reduction of these unwanted elements. Modern use of digital filters has become the most cost-effective way of removing noise, with Finite Impulse Response (FIR) filters being popular and used extensively in electronics, telecommunications, radio, television, image processing, and computer graphics. The most flexible FIR filter design is the Kaiser Window method, and in this paper is used with Genetic Algorithm (GA) to improve Signal-to-Noise-Ratio (SNR) and obtain a least noisy signal.**

***Index Terms*—Signal-to-Noise-Ratio, Coefficients, Finite Impulse Response Filter, Window Function, Kaiser Window, Genetic Algorithm**

## I. Introduction

In the modern world, data is increasingly being sent over to wireless devices via signals. We can receive strong and clear data with filters in signal processing, which is "a device or process that removes unwanted components or features from a signal". It is commonly used in electronics, telecommunications, radio, television, audio recording, radar, control systems, music synthesis, image processing, and computer graphics [6]. However, not all signals are equal and can contain unwanted components (ex. noise) from time to time. As a result, it can lead to a decrease in performance and sometimes become difficult to distinguish sounds. A few ways to mitigate these situations lies within power, antenna design, and noise filtering. Out of the three, noise filtering continues to remain a hot-spot for researchers in the field of signal processing [1] with Signal-to-Noise ratio (SNR) being a key identifier for quality signal strength.

Finite Impulse Response (FIR) filters have become a popular solution to digital filtering with lower overhead designs and lesser costs. A FIR filter is finite given that the duration of the impulse settles to zero over a finite time. Design of FIR filters requires calculations of filter transfer function coefficients $h_n, n = 0, 1, ..., N$ providing a target frequency. Methods proposed for FIR filtering include Window method, where parameters such as sampling frequency, cut off frequency, pass-band ripple, stop-band attenuation, order of filter, filter length, width of lobes, etc were used. Among the most commonly used windows methods, Kaiser Window, was used for its flexibility in design in this research.

I would like to thank Aleeya Barayuga, for love and support, and PhD. J. Burrell, for guidance and inspiration

For this paper, Kaiser Window was used with cut off frequency at 10 Hz. Using the Kaiser Window function, signals with unwanted noise can be filtered to increase its SNR. However, further SNR optimization can be achieved with Genetic Algorithms, a commonly used evolutionary search algorithm. However, further SNR optimization can be achieved with a commonly used evolutionary search algorithm called Generic Algorithms (which will be detailed later in this paper). Genetic Algorithms is detailed later in this paper. Applying the proposed algorithm outlined in [1], a least noisy signal was obtained for heartbeat monitoring. This paper proposes the use of this algorithm on a more general approach of improving SNR. Taking the coefficients generated by the Kaiser Window function of a noisy signal, an initial population of filtered signals will be used as the basis for the Genetic Algorithm to find an improved SNR.

## II. SIGNAL PROCESSING

"A signal is a function of an independent variable such as time, distance, position, or temperature" [7]. Signals can be, continuous when its domain is a set of real numbers $x \in \mathbb{R}$, and discrete when the domain consists of non real numbers $x \notin \mathbb{R}$. Analog signals are real-valued continuous signals with digital signals being discrete in time and value.

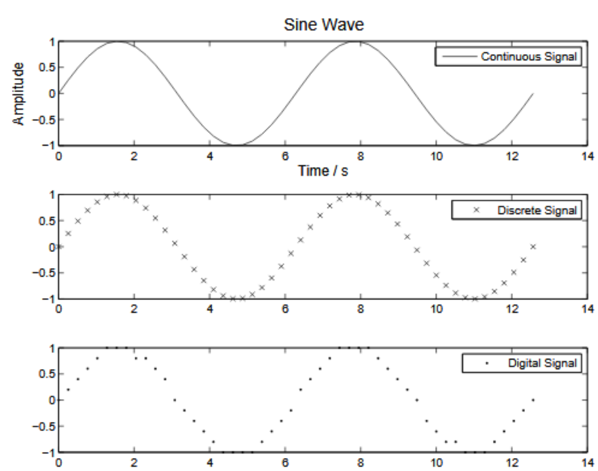


Fig. 1. Sine Waves in Continuous, Discrete and Digital Signals

Signal processing is a field of study that is used for signal generation, modifying signals, and extracting information from signals, having a large impact in areas of electrical engineering, applied mathematics, statistics, and so on. One such discipline of signal processing is removing unwanted elements (noise) from a signal; yet such elements are random and, in some cases, unavoidable. SNR is used to determine if the amount of noise is significant and is a critical part of any sensor [8]. Filters are used in a de-noising process, reducing, or eliminating noise from a signal, where SNR can be used to measure the effectiveness of the filters.

## III. SNR

Signal to Noise Ratio (SNR), is defined as the ratio of signal power to noise power. The following equations show SNR in power and Root Mean Square (RMS) with its accompanying logarithmic values:

### *A. Power Signal and RMS*

$$SNR = \frac{Psignal}{Pnoise} \tag{1}$$

Alternatively, can be defined as:

$$SNR = (\frac{Asignal}{Anoise})^2 \tag{2}$$

$A$ being the RMS of the signal's amplitude [8]. This can be expressed on a logarithmic scale in decibels (dB).

### *B. Logarithmic Scale*

$$SNR = 10log_{10}(\frac{Psignal}{Pnoise}) \tag{3}$$

and RMS:

$$SNR = 20log_{10}(\frac{Asignal}{Anoise}) \tag{4}$$

### *C. SNR to Power Ratio*

A SNR of 0 dB means the signal is indistinguishable from its noise whereas 3 dB means the signal is twice as strong as the noise counterpart, and at 30 dB the signal is 1000 times greater than its noise. The table below highlights this fact.

TABLE I
USEFUL RATIOS AND RESPECTIVE $dB$

| $P/P_{\text{ref}}$ | 1 | 2 | 4 | 10 | 100 | 1000 | 10000 | 100000 |
|---|---|---|---|---|---|---|---|---|
| dB | 0 | 3 | 6 | 10 | 20 | 30 | 40 | 50 |

## IV. FIR FILTER

Finite impulse response (FIR) filters are designed with a series of delays, multipliers, and adders to create an output. Known as feed forward or non-recursive, or transversal filters [6]. Common types of digital filters include Low-Pass filters, High-Pass filters, Band-Pass filters, and Band-Stop filters. The equation below represents a general class of FIR filters [9]:

$$y[n] = \textstyle\sum_{k=0}^{M} b_k x[n-k] \tag{5}$$

Where $x[n]$ is the input signal, $y[n]$ is the output signal, $n$ is the filter order; an $n$th-order filter has $n+1$ terms on the right-hand side, and $b_k$ is the value of the impulse response at the $i$th instant for $0 \leq i \leq n$ of an $n$th-order FIR filter.

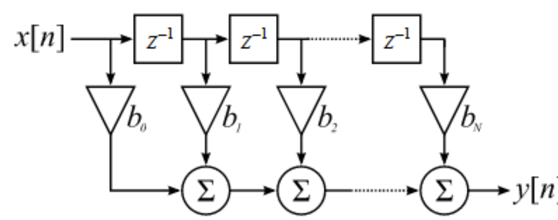


Fig. 2. Discrete-time FIR filter of order N

Common methods of designing FIR filters are: Window design method, Frequency Sampling method, Weighted Least Squares design, Parks-McClellan method and Equiripple method. This paper focuses on the Window design method, namely the Kaiser Window, however other designs include Rectangular, Barlett, Tukey, Taylor, Hamming, Hanning Gaussian, etc. [1][6].

## V. KAISER WINDOW

Kaiser Window, a Window function also known as a tapering function, is a mathematical function that is zero-valued outside of a chosen interval, usually symmetric around the middle of the interval. When you multiply a Window function with a signal, the product is zero-valued outside of the interval, leaving only the overlap or window [6]. The following equation represents the Kaiser Window function, a 0th-order modified Bessel function [1]:

$$w_k(n) = \{\frac{I_o\{[1-(\frac{n-\alpha}{\alpha})^2]^{\frac{1}{2}}\}}{I_o(\beta)}\} \tag{6}$$

for $0 \leq n \leq N$

Bessel functions are canonical solutions of Bessel's differential equation, where $\alpha$ is the order of the Bessel function:

$$x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + (x^2-\alpha^2)y = 0 \tag{7}$$

## VI. Genetic Algorithm

### A. Background

In the late 1950s, Genetic Algorithms (GA), under John Holland, were used to study adaptation in nature for the basis of applying the mechanisms in computer science, with the idea of solving engineering problems of optimization [2][3]. GA seeks solutions to optimization problems using ideas from natural evolution, such as inheritance, mutation, selection, and crossover [2]. In Charles Darwin's Theory of Evolution, natural selection shows that some species are more suited for certain environments than others. These species are considered more "fit" as they are more likely to survive and produce offspring, passing on their traits or genes. Loosely inspired by Darwin's Theory of Evolution, GA is "a heuristic, stochastic, randomized search optimization" algorithm, falling under a larger class of Evolutionary Algorithms (EA) [1][5].

### B. The basic GA procedure

1) Initial a random population of size $N$, such that $x_1, ..., x_N$.
2) For every N, evaluate the fitness value $f(x_1, ..., x_N)$ for a maximum function or $\frac{1}{f(x_1,...,x_N)}$ for minimum functions.
3) Select the best-fit values of $N$, in the population, for reproduction.
4) 4 New individuals are bred through crossover and mutation operations to give birth to offspring.
5) Evaluate fitness of new offspring.
6) Replace least-fit values of $N$, with new offspring
7) Repeat steps 2 through 6 $x$ times, $x \in \mathbb{Z}$, until desired results.

### C. Four Components of GA Generation Transitioning

1) **Selection:** Best-fit values of $N$, objective function $f(x_1, ..., x_N)$ or $\frac{1}{f(x_1,...,x_N)}$.
2) **Crossover:** Combining values of $N$, to produce offspring of desired qualities.
3) **Mutation:** Mutations in GA are random deformations of $N$, whose effect is "genetic diversity" and avoiding local maxima.
4) **Sampling:** Creates a new generation from the previous generations' offspring, based off values of $N$.

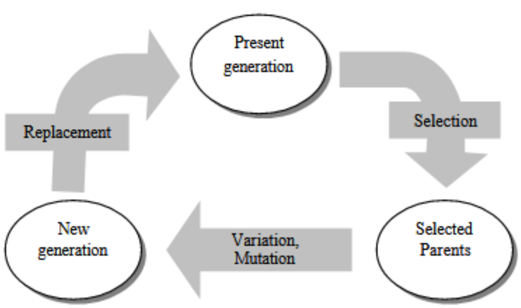


Fig. 3. Basic GA Cycle

### D. Proposed Genetic Algorithm

*Step 1:* Kaiser Window function is used as a FIR filter on a signal, resulting in coefficients with an improved SNR. These coefficients are used as the initial population in the GA.

*Step 2:* For fitness values the following equation is used:

$$\beta = \frac{CorruptedSignal - FilteredSignal}{FilteredSignal} \tag{8}$$

$$FitnessValue = 10log_{10}(\beta) \tag{9}$$

*Step 3:* Depending on the fitness values from the initial population, a set of coefficients are selected by using the Roulette Wheel Selection procedure also known as Fitness Proportionate Selection. Defined as the equation below:

$$p_i = \frac{f_i}{\sum_{j=1}^{N} f_i} \tag{10}$$

$f_i$ is the fitness of individual $i$ in the population, $N$ is the number of individuals in the population.

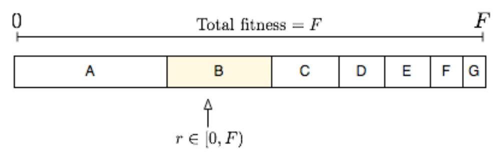


Fig. 4. Roulette Wheel Selection

*Step 4:* Single point Crossover is performed with 100% probability in between the selected set of chromosomes and off-springs are generated.

*Step 5:* Mutation is performed on the offspring chromosomes with a 25% probability.

*Step 6:* Replace parent coefficients with offspring coefficients with better fitness values than the parents.

*Step 7:* Repeat steps 4 through step 7 $N$ times, until desired results.

## VII. Results

The proposed algorithm is capable of improving SNR up to an estimate of 6 dB in some cases. However, due to time constraints, a proper comparison could not be attained with the proposed GA. Results below show theoretical estimates based on the best-case-scenario of the Kaiser Window function. In addition, the proposed GA would be in its infancy stage with limited consideration given to mutation, crossover, sample size and the initial population. A methodical approach and a better understanding of GA and FIR filters would potentially improve the Kaiser Window past previous estimates.

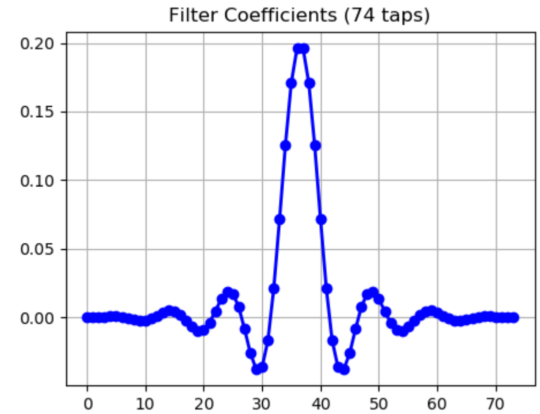


Fig. 5. Kaiser Window Coefficients

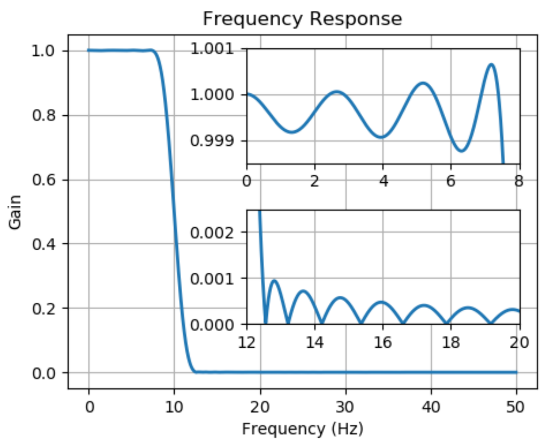


Fig. 6. Frequency Response

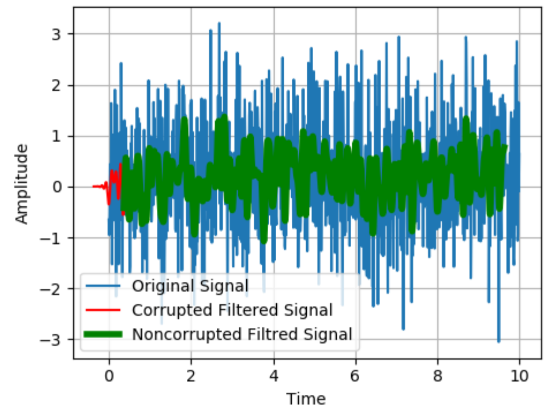


Fig. 7. Filtered Signal

## VIII. Conclusion

The paper explores SNR improvements via GA, introducing a modified algorithm proposed by "Patra et al". Implementation of the Kaiser Window function saw up to 6 dB SNR, approximately 4 times greater than its noise values. Optimizing the Kaiser Window coefficients using a simple GA produced inconclusive results, with a more methodical approach suggesting variant improvements. FIR filter window method selection, namely Kaiser, Hanning, and other common digital filters could be further looked into with a more structured GA to give a holistic comparison. The proposed algorithm shows promise, with more work required to validate.

## References

[1] S. Patra, P. Das, S. Panja, and S. Naskar, "An Approach for obtaining Least Noisy Signal using Kaiser Window and Genetic Algorithm," International Journal of Computer Applications, vol. 150, Sep. 2016, doi: 10.5120/ijca2016911512.

[2] P. K. Yadav and D. N. L. Prajapati, "An Overview of Genetic Algorithm and Modeling," vol. 2, no. 9, p. 4, 2012.

[3] M. Dianati, I. Song, and M. Treiber, "An Introduction to Genetic Algorithms and Evolution Strate…," p. 11.

[4] H. Dong, C. Yang, M. Ao, and Y. Liu, "Heterodyne phase measurement in low signal-to-noise ratios using genetic algorithm," Optics Communications, vol. 434, pp. 152–156, Mar. 2019, doi: 10.1016/j.optcom.2018.10.066.

[5] M. Pfeiffer and T. Pfeil, "Deep Learning With Spiking Neurons: Opportunities and Challenges," Front. Neurosci., vol. 12, p. 774, Oct. 2018, doi: 10.3389/fnins.2018.00774.

[6] N. M. Shehu, J. Baraya, and M. Sani, "Comparative Analysis of a High Pass Finite Impulse Response (FIR) Filter Designed and Simulated by Window Methods," vol. 23, pp. 721–725, May 2019.

[7] T. Puoliv ali, "A Short Introduction to Signal Processing," p. 17.

[8] J. M. D. Coey, "Lecture 3: Signal and Noise," p. 35.

[9] M. Wickert, "Introduction to Signals and Systems," Chapter 5.