Calculus 1 - Exercise 8

All exercises should be submitted by December 27th by 23:00. Delays won't be accepted aside for special cases which will be approved beforehand. These are the submission regulations (also available on the Moodle):

1. Exercises are personal and cannot be submitted in groups.
2. Write your name, ID and tutorial group in the header of the exercise.
3. They should be written clearly on A4 pages. Hard-to-read exercises will not be graded.
4. Serious effort has to be shown by the student. Unreadable or extremely partial answers will be disregarded.
5. Exercises submitted late without the TA's approval will not be accepted.

Questions:

1. Compute each of the following limits or prove that it doesn't exist:

   (a) $\lim_{x\to 0} \lfloor \tan x \rfloor \cdot \cos x$

   (b) $\lim_{x\to 2} \frac{4-x^2\cdot\cos(x-2)}{x-2}$

   (c) $\lim_{x\to 0^+} \frac{\sin(6x)}{\sqrt{\sin(2x)}}$

   (d) $\lim_{x\to 0} \frac{\cos(3x)-\cos(2x)}{x^2}$

   (e) $\lim_{x\to 0} \frac{1-\cos(1-\cos x)}{x^4}$

2. Compute each of the following limits, or prove that it doesn't exist:

   (a) $\lim_{x\to 5} (6-x)^{\frac{1}{x-5}}$

   (b) $\lim_{x\to 0^+} (1+3x)^{\frac{-2}{3x^3}}$

3. Prove or disprove each of the following statements:

   (a) There exists a function $f : \mathbb{R} \to \mathbb{R}$ such that

   $$f(\cos x) = \sin x + sign(x)$$

   for every $x \in \mathbb{R}$.

   (b) Let $f(x) = \ln\left(\sqrt{1+x^2} - x\right)$ for every $x \in \mathbb{R}$. Then f is well defined and odd.