HW3- Theory

A 1-D stride-2-convolution layer is applied twice sequentially to an input 1-D tensor, with the same filter denoted as k_2. The input is padded with zeros, only once before the sequence of two convolution layers is applied, so that the final output size is exactly $^1/_4$ of the input size. Find an equivalent filter k_4 that when applied just once with a stride-4 convolution layer and the same padding, yields the exact same result.

<u>Guidance</u>: Recall that convolution layer (with appropriate padding) actually performs cross-correlation- discrete non-circular convolution with the **flipped** filter. Also note that a convolution with stride is equivalent to a convolution followed by subsampling. Thus, k_4 is defined by:

$$x * \tilde{k_4} \downarrow_4 = (x * \tilde{k_2} \downarrow_2) * \tilde{k_2} \downarrow_2$$

where $\tilde{k}$ is flipped k and $\downarrow_s$ means subsampling by a factor of s (e.g., subsampling by 2 means taking every second value).

<u>Hint</u>: use the notion of dilated convolution.

<u>Note</u>: Perfect score will be obtained by a full formal proof. However, correct final answer with a reasonable explanation will be considered too.