Part II: Practical Part Answers

Warmup Question 1: How many degrees of freedom does the joint have? i.e. how many parameters would you need to specify an arbitrary probability distribution over all possible 28×28 binary images?

As we have learned in class a joint probability function with n parameters, has $2^n - 1$ degrees of freedom. In our case we have 784 variables, and so we would have to specify $2^{784} - 1$ variables.

Warmup Question 2: How many degrees of freedom does the BN in fig. 1 have?

As we have seen in the recitation \begin_inset Formula $deg(X|Y) = \sum_y deg(X|y) = (|Val(X) - 1| \cdot |Val(Y)|$ And so in our case we have :

$$deg(p_{\mathcal{B}}(X)) = (\sum_{i=1}^{784}(|Val(X_i|-1)\cdot|Val(Z_1)|\cdot|Val(Z_2)|)+Val(Z_1)|+|Val(Z_2)| = 784\cdot 25\cdot 25+24+24 = 490,048$$

Q1:

Figure 1: Q1 answer

Q2: What is the intuitive role of the Z1,Z2 variables in this model?

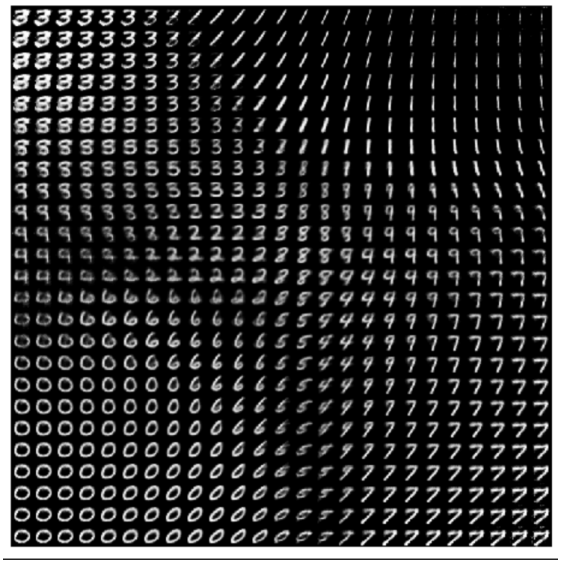

Figure 2: Q2 answer

What is the intuitive role of the Z1,Z2 variables in this model?

it seems as though they represent the dimensions of a lower dimension of the data. That is to say that if we took the data and tried to reduce its dimension to 2D, then Z1 Z2 would be the axes of this latent dimension. We can also see that the nunbers seem to cluster together, which supports this hypothesis since we would expect this king of behaviour in a well designed dimensionality reduction. To take an even bigger risk and to try and interpret these axes, from this figure, I would say that the first dimension is "skewness" to be diagonal, and he second dimension is "roundness".

Q3:

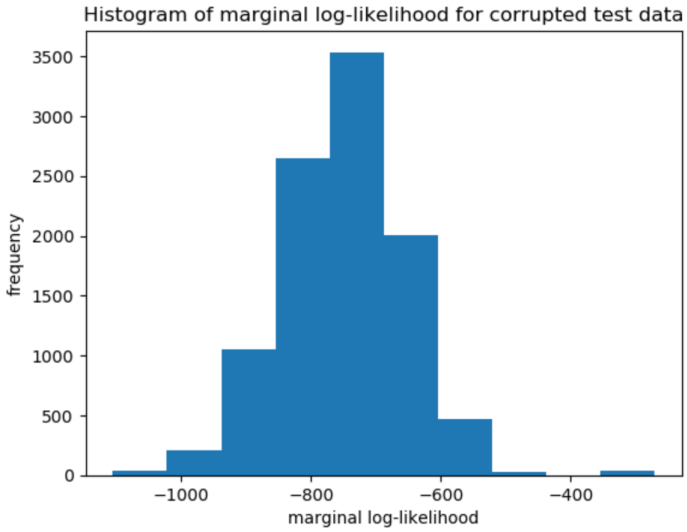

Figure 3: Q3 answer

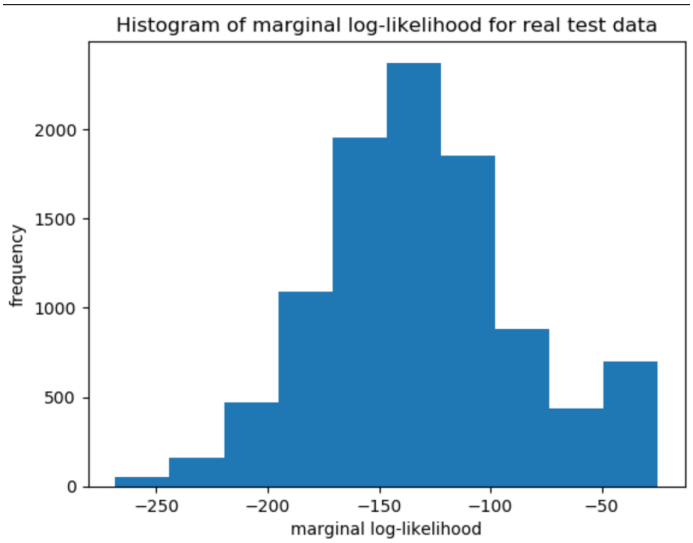

Figure 4: Q3 answer

Q4:

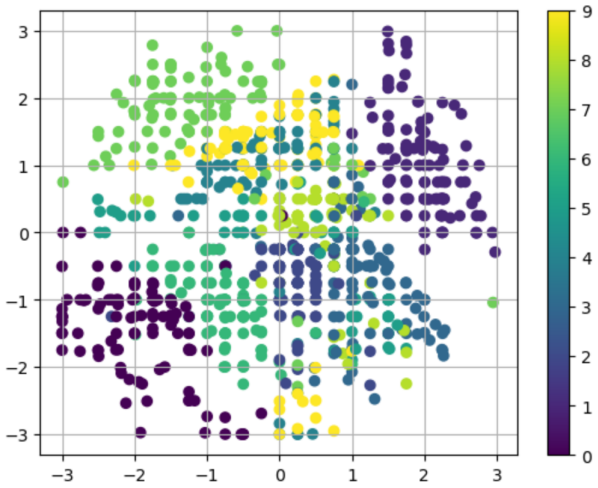

Figure 5: Q4 answer

What is the relationship with the figure you produced for Q 2?

We can see that this image is similar to the one we saw in Q2, since both project the space onto a 2D space and in both we see clustering of images based in the digit. In addition the realtive position of the digit cluster seem to correspond and be in the same places.