

Part II: Practical Part Answers

Warmup Question 1: How many degrees of freedom does the joint have? i.e. how many parameters would you need to specify an arbitrary probability distribution over all possible 28×28 binary images?

As we have learned in class a joint probability function with n parameters, has $2^n - 1$ degrees of freedom. In our case we have 784 variables, and so we would have to specify $2^{784} - 1$ variables.

Warmup Question 2: How many degrees of freedom does the BN in fig. 1 have?

As we have seen in the recitation $\deg(X|Y) = \sum_y \deg(X|y) = (|Val(X)| - 1) \cdot |Val(Y)|$. And so in our case we have :

$$\deg(p_B(X)) = \left(\sum_{i=1}^{784} (|Val(X_i)| - 1) \cdot |Val(Z_1)| \cdot |Val(Z_2)| \right) + |Val(Z_1)| + |Val(Z_2)| = 784 \cdot 25 \cdot 25 + 24 + 24 = 490,048$$