67800 | Probabilistic Methods in Artificial Intelligence | Ex 1

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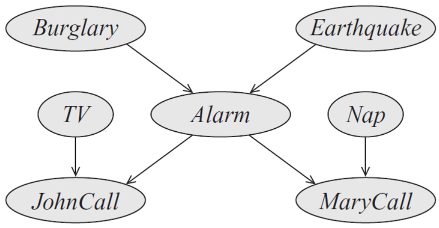
Question 3 - Removing a Variable From a Bayesian Network



1. Consider the Burglary Alarm network given above (It's a bit different than the example we saw in class - create a suitable story if you wish). Construct a Bayesian network over all nodes except the Alarm node that is a minimal I-Map for the marginal distribution over the remaining variables (B, E, N, T, J, M). Specifically, construct a new graph over the remaining variables which models all of the dependencies (active paths) from the original network in which A is unobserved (it's an I-map) and in which no edge can be removed without creating new independencies (it's minimal).

Solution :

The solution is as follows:

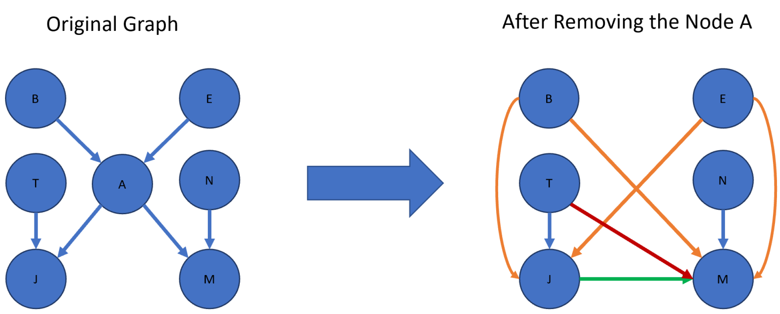


Figure 1: BN after removing "A"

Explanation: Firsly since we remove A we need to assume that A is indeed unobserved. Next, our goal here is to to preserve all independencies in the original graph:

$$asad$$

Firstly, nodes that are unrelated to A are untouched and therefore we leave them be (Blue edges). The most obvious connection we see is that with A gone, there exist active paths between A's parents and children. And so we need to connect A's parents - B, E to A's children - J, M (Orange edges).

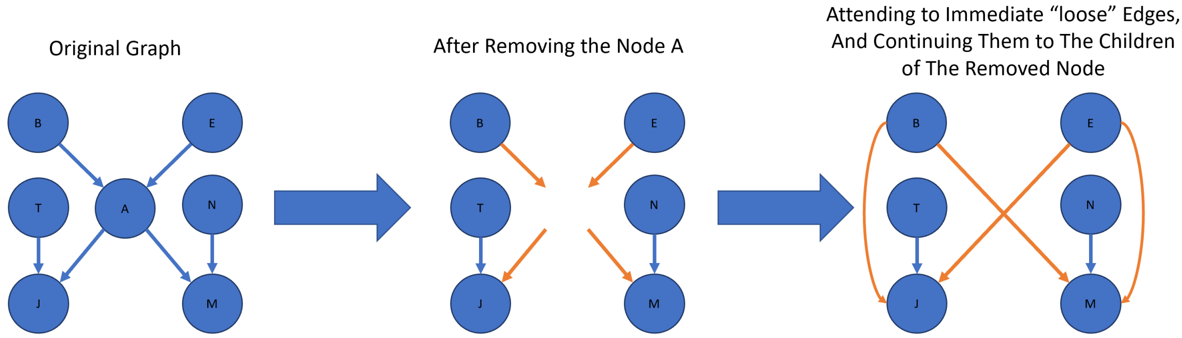


Figure 2: Step 1: Connecting The Parents of A to its Children (Orange Edges)

In addition we know that all children of A will now have an active path between them we must draw an edge between them (the order does not matter, and we could use the topological ordering of the graph to decide the direction of the edge), And so we draw an edge from J to M (Green edge).

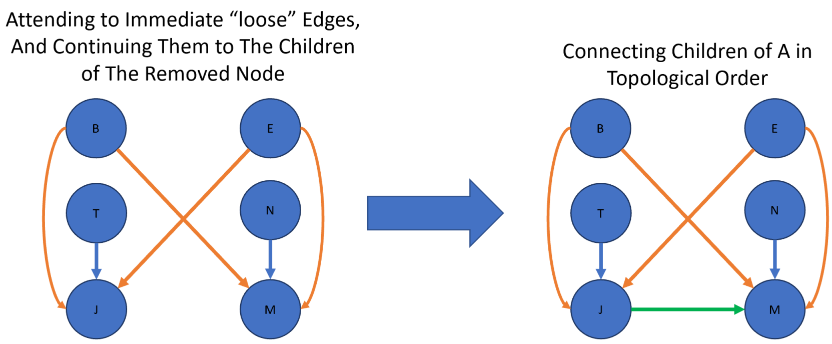


Figure 3: Step 2: Connecting Children of A in Topological Order (Green Edge)

Finally we need to attend to the v structure that was between $T \rightarrow J \leftarrow A$. We now see that there is an active path between T, M and so we need to connect an edge from T to M (Red edge).

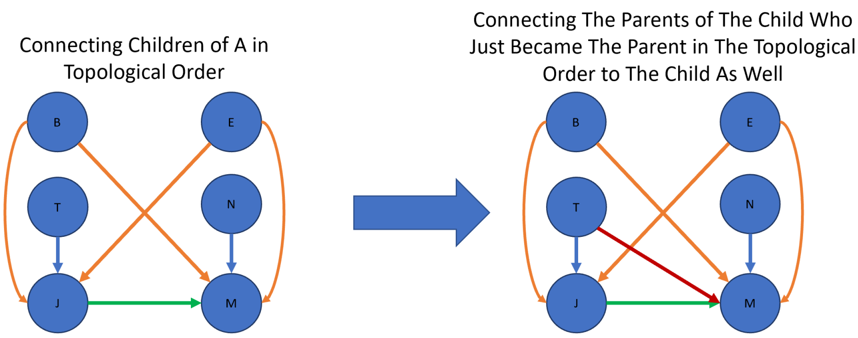


Figure 4: Step 3: Connecting The Parents of The Child Who Just Became The Parent in The Topological Order to The Child As Well (Red Edge)

As we can there are 3 classes of new edges we need to add (color coded in my answer), and we will generalize this form in the next question. Note: since we have chosen the edge from J to M arbitrarily, we could have had an edge from M to J and then we would have also had N to J from the same logic.

2. Generalize the procedure you used above to an arbitrary network. More precisely, assume we are given a BN $\mathcal{B}$, an ordering $X_1, \ldots, X_n$ that is consistent with the topological ordering of the variables in $\mathcal{B}$, and a node X_i to be removed. Specify a network $\mathcal{B}'$ which is consistent with this ordering and which is a minimal I-Map of $p_{\mathcal{B}}(X_{-i}) = p_{\mathcal{B}}(X_1, \ldots, X_{i-1}, X_{i+1}, \ldots, X_n)$. Your answer should be an explicit specification of the set of parents for each variable in $\mathcal{B}'$.

Solution :

Suppose a graph with a topological order on the variables: $V = \{X_i\}_{i=1}^{n}$ and suppose we want to remove the variable $R = X_j \ s.t \ j \in [n]$. Based on my answer to question 3.1, intuitively we need to keep all of the vertices that have to relation to R the same, and do the following 3 operation:

1. Connect the all of the parents of R to all of the children (directly connected to R) - meaning $\forall p \in pa(R), c \in V \ s.t \ pa(c) = R$ connect $p \to c$.
2. Connect all of the children of R together based on the topological order - meaning we will connect the children $c_1, ...c_m$ such that $\forall k, l \ c_k \to c_l \Leftrightarrow k < l$.
3. For each pair of children c_1, c_2 that were connected in step 2 such that $c_1 \to c_2$ we need to connect the parents of c_1 to c_2 - $\forall p \in pa(c_1)$, connect $p \to c_2$.

In conclusion the set of parents is:

For convinience sake let us mark:

$$A = \overbrace{pa(pa(R))}^{\text{Connecting to the parents of the removed node}}$$

$$B = \overbrace{\{c_j | pa(c_j) \ni R, \ j < i\}}^{\text{Connecting to my siblings in topological order}}$$

$$C = \overbrace{\{pa(c_j) | pa(c_j) \ni R, \ j < i\}}^{\text{Connecting to the paretns of those siblings im connected to in topological order}}$$

$$\forall x \in V :$$

$$pa(x_i) = \begin{cases} A \cup B \cup C & pa(x_i) \ni R \\ \overbrace{pa(x_i)}^{\text{If I'm not related to the removed node, do not change anything}} & pa(x_i) \not\ni R \end{cases}$$