African Institute for Mathematical Sciences
CAMEROON

Revealing the functional connectivity underlying motor movement through multiplex networks

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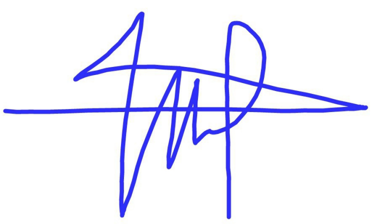
Nowadays, the need to understand the transition of brain activity to various behaviors has proven to be capital and crucial when it comes to accurately analyzing a variety of brain dysfunctions observed in many patients. In particular, refining these analyses by looking at interactions at other levels, such as distinct brain frequency bands in healthy people, has proven to be extremely helpful and vital in understanding and treating many patients with various brain dysfunctions and handicaps just by comparing different observations. Thus, in this research work, we have incorporated the multiplex network to untangle the relationships in electroencephalogram (EEG) signals reflecting the brain activity in 109 healthy patients during a motor movement, namely openings and closings of the left and right fist. Specifically, by using the short-time Fourier transform, we have defined in different frequency bands on which the brain activity are analyzed to understand the nature of the cerebral signals translating into movement and model the whole-head functional connectivity network during the execution of this movement in question. The obtained results through this analysis clearly showed that the Delta [0-4 Hz] and Theta [4-8 Hz] brain waves mainly characterize the brain activity and play a very crucial role in the interaction and communication between brain regions during the execution of these movements in question. Moreover, the use of the multiplex network was critical in revealing the existence of two major cerebral communities synchronized locally via the low-frequency bands Delta, Theta, and Alpha, as well as a weak cerebral connection via high frequencies during the execution of this motion.

Keywords: Multiplex Network, Electroencephalograph, Short Fourier Transform, Motor Movement.

Declaration

I, the undersigned, hereby declare that the work contained in this essay is my original work, and that any work done by others or by myself previously has been acknowledged and referenced accordingly.



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1. Introduction

The analysis of numerous real-world networks in biology, physics, chemistry, finance, and economics demonstrates that the vast majority of them have a hyper-complex dynamic behaviour and a multi-layered structure primarily showing the various relationships that exist between the components of these various networks [14, 39]. As a result, analyzing the underlying network architecture and linking variables involved in that system is crucial to our understanding of a range of behaviour and events stemming from these networks. Aside from the large number of variables that such systems contain, their dynamic behaviour also presents a hurdle when it comes to fully comprehend them.

We also face such a scenario when we consider the human brain, which turns out to be the most complex network discovered in nature [63] generating intricate behaviour. Understanding this system more accurately requires both the identification of different functional areas and their interaction structure. Thus, defining appropriate measures based on recordings reflecting such interactions is fundamental when we aim to understand the brain's information processing. Particularly, refining these analyses by examining interactions on different levels, such as different frequency bands that were demonstrated to play a significant role in generating behaviour [26], is considered as essential.

Particularly, when it comes to accurately treating brain diseases and dysfunction, the comprehension of the transition of brain activity into various behaviour [12] is proving to be critical in the analysis and treatment of a variety of brain dysfunctions seen in patients. Some of such diseases are for example epilepsy, cardiovascular diseases, autism, or Parkinson disease [71]. As an example, recent developments show that in patients with spinal cord injuries, where the brain functions are intact but where the flow of information to the limbs is disrupted, the comprehension of the mapping between brain activity and motor movement successfully constituted the basis of brain-machine interfaces aiming at conducting the desired movement based on the processing of brain activity.[8, 57]

Considering the brain anatomy and the association of different areas with different functions, it is meaningful to introduce advanced concepts from the field of network science to modelling brain activity. With the introduction of the multiplex network [10, 13, 40, 46], a new tool developed in the field of network science that aims to be the most appropriate tool for the study and analysis of multilayer networks, it is now possible to analyze the brain network and highlight the various properties and peculiarities specific to cerebral activity during the execution of a motor movement [31, 72]. In a multiplex network, each layer represents a different sort of interaction between a group of nodes, and hence, it aims at revealing the network features in complicated networks in which the same variables are known to have several types of connections. As mentioned above, interaction in the brain happens on different frequency bands which gives rise to the idea of using a multiplex network to understand the information processing in the brain more accurately.

1.1 Problem Statement

We have witnessed many patients who suffer from various brain disorders that cause the paralysis of many members of their body, resulting in the death of numerous organs of their body, for several decades now, and the rate of people suffering from this sort of brain sickness has not slowed over time. However, thanks to the advancements made possible by the computer brain interface [8, 57], some of them have been able to restore the use of their organs. With recent advances in the study of brain function, it has been possible to collect certain signals from the brain that control these movements and transmit them to the machine, allowing the patient to regain organ mobility. Despite the significant advances made in the treatment of this type of disease, we continue to witness an increase in these brain dysfunctions. Using a new tool developed in the field of network science called Multiplex networks, which has contributed to the progress of revealing network properties in complex networks where the same variables are known to have different types of connections, as is the case with the human brain, we aim to incorporate this multiplex networks to untangle the relationships in electroencephalogram (EEG) signals reflecting brain activity during a motor move. As a result, this novel technique may help us better understand the nature of brain signals that translate into behavioural behaviours, as well as open up new views on brain-computer interfaces that aim to execute desired activities based on the analysis and translation of brain signals. This will be extremely helpful in the treatment of numerous neurological disorders that cause paralysis of the body's various organs.

1.2 Research objectives

The objectives of this study will be to :

(i) Determine the characteristics of the brain signals during motor movement concerning its underlying network structure.

(ii) Make a comparative analysis based on which we aim to see stability in the revealed network structure for several subjects.

(iii) Improve our understanding of the nature of brain signals translated into behavioural actions.

(iv) Reveal new perspectives on brain-computer interfaces aiming at the execution of desired action based on the analysis and translation of brain signals.

1.3 Overview of the work

This work is organized as follows:

Chapter 2 will be devoted to the literature review. In this part, we will do a review of the background of our study, present the different techniques used in recording of brain activity, do a review on the multiplex networks and finally review previous work done on brain network.

Chapter 3 will present the methodology used in our work. In this part, we will do a description of the datasets, outline the process of constructing our multiplex network and finally describe the measures that we have applied in this multiplex network.

Chapter 4 will present our findings and discussion on them.

2. Literature Review

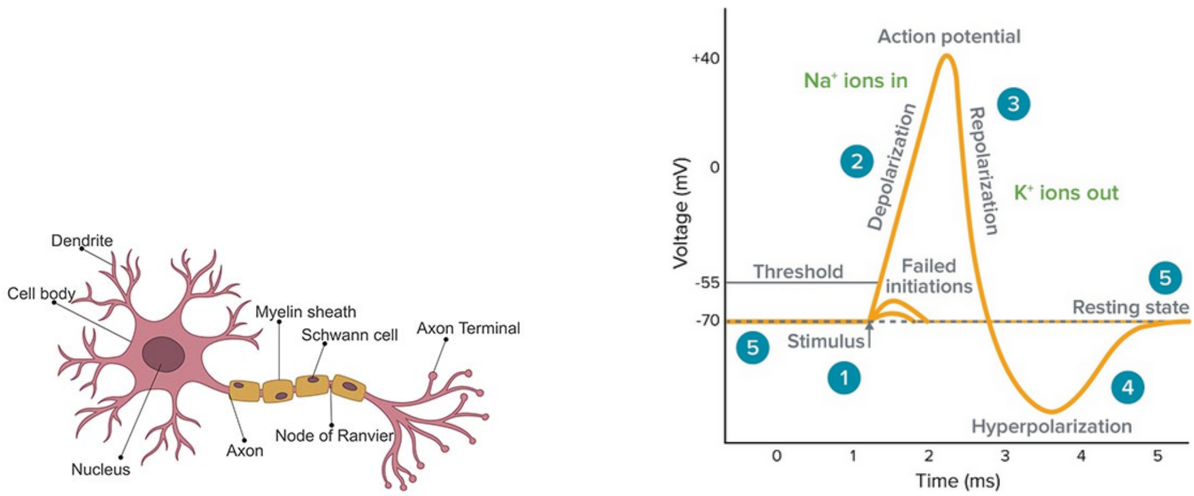
2.1 Background of the study

Over time, neuroscience has generated extraordinary breakthroughs, from non-invasive imaging of the human brain [68, 19] to uncovering the molecular mechanisms of certain complex processes and disease states [71, 19].

In fact, the brain is a complicated organ that regulates every action in our body, including motor skills, vision, respiration, memory, emotions, and thoughts. Together with the spinal cord, it forms the central nervous system. The brain is not a muscle in and of itself. It is made up of blood vessels and nerves, as well as neurons, and glial cells.

Information is sent and received by chemical and electrical impulses [43] . Different impulses govern various processes, which the brain decodes [43] . Some messages are stored in the brain, others are sent far away of the brain via the spine and the huge body nervous network [18] . The central nervous system relies on billions of neurons (nerve cells) to do this[43].

The neuron (figure 2.1a) is the cell that is in charge of information production, processing, and transmission [30] . It gets information from billions of neurons and delivers it to billions of neurons [43] . In fact, sensory receptors or other neurons send information to neurons' dendrites. This data is subsequently transmitted to the cell body and ultimately to the axon. When information reaches the axon, it is converted into an electrical signal known as action potential (2.1b)[5, 1]. Like current in electric lines, these impulses move along the extensions of the neurons. Between two neurons (synapse, area of exchange information between two neurons), the current is transformed into a chemical signal (neurotransmitters) [33] .



(a) Anatomy of Neuron [2] .

(b) Explanation of Action Potential [1] .

Figure 2.1: Production, Processing and Transmission of Information by Neurons

Indeed, the two neurons are not in contact with one another because they are separated by the synaptic clefts [33], and when an electrical signal is received, substances known as neurotransmitters are released into this synaptic cleft. These neurotransmitters connect to the post-synaptic neuron and induce the opening of channels in the membrane polarization, resulting in the production of an electrical signal [33] . The receipt of an electrical signal transmitted by the nerves causes the muscles to contract because each muscle receives signals from a specific nerve and can send signals to several muscles. In fact, when a nerve impulse is received, the neuron releases acetylcholine, which binds to receptors in muscle fibers, causing them to contract [30] .

In comparison to the microscopic level in the brain introduced above, when considering the brain on a larger scale, it can be divided into the cerebrum, brainstem, and cerebellum (figure 2.2) .

- **Cerebrum**

  Gray matter of the cerebral cortex and white matter make up the cerebrum (figure 2.2) [66] . The cerebrum is the largest region of the brain which controls temperature and initiates and directs movement [66] .

  - **Cerebral Cortex**

    The cerebral cortex is the gray matter covering that makes up about half of the weight of the brain [66] . It is separated into right and left hemispheres. The left hemisphere is in charge of the left side of the body, whereas the right hemisphere is in charge of the right side [66] . The corpus callosum connects the two hemispheres.

- **Brainstem**

  The brainstem (figure 2.2) connects the cerebrum with the spinal cord. The brainstem includes the midbrain, the pons, and the medulla [66] .

  - **Midbrain**

    The mesocephalus is a complex structure with numerous neuronal groups and pathways which is vital for hearing control and adaptation to changes in the environment [66, 67] .
  - **Pons**

    The pons is the link between the mesencephalon and the medulla. It controls functions such as tear production, blinking, visual concentration, balance, hearing, and facial expression [66, 67] .
  - **Medulla**

    At the bottom of the brainstem, the medulla is where the brain and the spinal cord meet [67] . It is the brain's organ whose existence is critical to its effective operation [66] . It regulates body activities including heart rate, breathing, blood flow, oxygen, and carbon dioxide levels on the one hand, and reflex movements like sneezing, vomiting and coughing on the other [66] .

  Moreover, The spinal cord is supported by vertebrae and delivers messages between the brain and the rest of the body [67] .

- **Cerebellum**

  The cerebellum (figure 2.2) is the brain area located behind the ears. It, like the cerebral cortex, is divided into two hemispheres [67] . The outer section includes neurons, whereas the inner segment connects with the cerebral cortex [66] . It ensures balance in humans by coordinating muscular motions [66] .

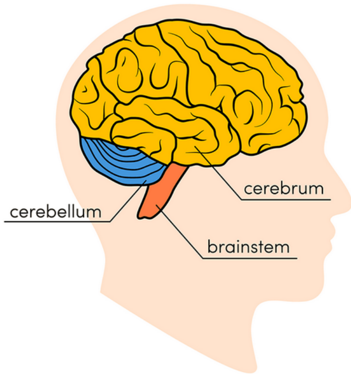


Figure 2.2: The three main parts of the brain [3]

Specifically, each brain hemisphere has four sections : the frontal, parietal, temporal, and occipital lobes. Each lobe controls specific functions:

(a) **The Frontal Lobe**

The frontal lobe (2.3) is the brain's largest lobe, positioned in the front of the head [67] . It's vital for functions including personality traits and decision-making [66] . In addition, some parts of this lobe are frequently implicated in odour recognition [66] .

(b) **The Occipital Lobe**

The occipital lobe(figure 2.3) is the brain region responsible for vision, image detection, and interpretation [66] .

(c) **The Temporal Lobe**

The temporal lobes (figure 2.3) on the sides of the brain, helps in short-term memory, speech, and musical rhythm [66] .

(d) **The Parietal Lobe**

Located in the middle of the head, the parietal lobe (figure 2.3) plays an important role in the object recognition process [66] . Moreover, it is also involved in the functions of pain and touch perception in the human body [66] .

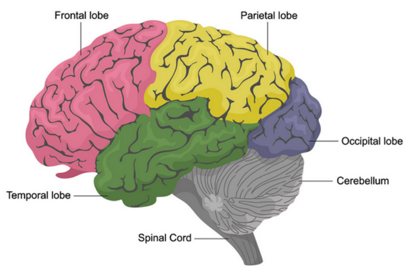


Figure 2.3: The different lobes of the brain [3]

A deeper exploration of the structure of the brain reveals that there are other critical areas of the brain that play vital roles in its structure and the process of brain activity:

(i_1) **Pituitary Gland**

The pituitary gland (figure 2.4) is deep gland within the brain organ located behind the bridge of the nose [67] . It controls the flow of hormones from the ovaries and testicles, as well as other glands in the body [28] . Through its stalk and blood supply, it receives chemical signals from the hypothalamus [28] .

(i_2) **Hypothalamus**

Situated above the pituitary gland (figure 2.4), the hypothalamus provides chemical instructions to the pituitary gland that govern its function [28] . It controls body temperature and sleep cycles [28] .

(i_3) **Amygdala**

Located under each hemisphere of the brain, the amygdala(figure 2.4) governs emotion and memory [28] .

(i_4) **Hippocampus**

The hippocampus (figure 2.4) is an important organ for memory and learning placed beneath each temporal lobe which receives information from the cerebral cortex [28] .

(i_5) **Ventricles and Cerebrospinal Fluid**

Cerebrospinal fluid, a watery fluid that flows between the meninges and in and around the ventricles and spinal cord, is produced by the ventricles [67] . They protect and cushion the spinal cord and brain while also removing waste and contaminants and providing nutrition [66] .

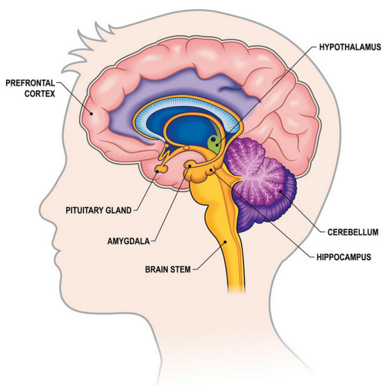


Figure 2.4: Deeper Structures within the brain [3]

Brain function can be described as chemical and electrical impulses that are sent and received by the brain throughout the body, to record this brain activity, we use the Electroencephalograph (EEG). Briefly, Electroencephalography (EEG) is a non-invasive method that was invented in 1924 [32] to record and identify the functioning of the brain. Indeed, changes in ion current flows in many neurons cause the electrical activity of the brain to fluctuate, giving an indication of the kind and degree of activation of different brain regions [48, 61] .

EEG is a term used in medicine to describe the recording of the brain's spontaneous electrical activity over time using several electrodes implanted on the scalp [61] . The analysis of the different neuronal oscillations, also called brain waves, recorded using EEG plays an important role in medical diagnosis and can be analyzed in the frequency domain using various methods.

Many studies on EEG have been conducted over time, with some extracting EEG energy from a small number of frequency bands [38, 26] and others extracting EEG energy from a time-locked average of the response [56]. The EEG has played a critical role in the diagnosis of epilepsy, proving that epilepsy is simply an overabundance of synchronization between neurons and brain areas. As a result, more complex signal processing algorithms sensitive to early synchrony change have been developed [24] .

In the study of EEG data, however, more powerful signal decomposition and feature extraction approaches have only recently evolved [17, 21, 55] . Among these recently powerful approaches used nowadays, we have the new tool developed in the network science called the multiplex network.

In fact, the main idea in our actual work is to use the short Fourier transform to construct the multiplex network of the brain, with the different typical frequency bands of EEG activity as interconnected layers where each layer is adjusted to a within-frequency synchronized network and also cross-frequency connection. To do that, we use electrical waves reflecting the cerebral activity of the brain of 109 patients recording using EEG times series when they perform cognitive tasks and repeated reflex movements for one to two minutes. By doing this, we aim to untangle and analyze the underlying structure and dynamics of brain activity of these patients when they are performing these tasks in question.

In order to have a good overview of the work we plan to do, let's start by doing a brief review of the techniques most used today to record brain activity.

2.2 Techniques for recording and analyzing brain activity

Different techniques (electro-physiological and brain imaging) have been developed over the last 40 years with the goal of viewing the various cerebral regions, studying their functioning, and observing their interactions. These techniques have proven to be essential and useful in the study of neurological illnesses over time. They have made it possible for scientists to link clinical indicators of disease (such as memory loss) to brain imaging (dysfunction of certain neurons). As a result, these techniques are proving to be an invaluable ally in determining the diagnosis of some illnesses and better comprehending their effects on the brain.

Some of these methods allow for accurate brain observation without the need to open the cranial box. Radiation (X-ray emissions, detection of injected radioactive chemicals), electrical activity, and, more recently, magnetic fields are all used. They have proven to be quite useful in medical diagnosis. Furthermore, they take into account the activity of brain zones performing specific tasks (speech, movement, etc.). So, here are a handful of these techniques.

2.2.1 Electroencephalography (EEG). It is a non-invasive method. It measures electrical waves that reflect brain activity [61, 38] . Indeed, several electrodes are placed on the scalp to collect not an image but traces of activity for each electrode which represents a combination of brain rhythms [16] , that is to say, each area of the brain studied [61] because different rhythms underlie functionally dissociated brain activity whose interaction forms a core of cognition [25] . And the analysis of those EEG recordings reflecting brain activity using diverse methods has been of major importance in understanding many brain phenomena.

Indeed, current research examining the interaction of distinct brain rhythms activity in different frequency bands [65, 64] has highlighted the causal importance of inter-frequency coupling in the understanding of visual working memory [65] as well as in tasks demanding continuous attention [59] .

In addition, additional recent research has found that the synchronization of slow theta brain rhythms

[4-8 Hz] governs the interplay of neuronal signals in human memory [35] and sensorimotor integration [27] . Furthermore, synchronization of alpha [8-12 Hz] and beta [15-30 Hz] brain rhythms has been demonstrated to be central and dominating in top-down communication, perceptual decision-making, and sensory processing [49, 42] .

2.2.2 Functional Magnetic Resonance Imaging (fMRI). It is a technique for recording fluctuations in blood flow in small locations of the brain. As a result, we may deduce the brain oxygen consumption and, as a result, the active brain areas. The use of fMRI became popular over the time and this brain activity measure led to great progress in brain activity interpretation and in attributing functional labels to different brain regions. [19]

Furthermore, the brain functional networks explored through fMRI investigations are the spatial scale of brain networks.These non-invasive investigations enable us to assess brain function in healthy people and people with mental illnesses while they are at rest or performing specific activities.

2.2.3 MagnetoEncephalography (MEG) . It is a technique for measuring magnetic fields caused by neuronal activity. The machine is ultrasensitive and insulated against natural ambient magnetism because these magnetic fields are relatively weak. MEG has a better ability to visualize the deep architecture of the brain than fMRI.

Moreover, since that we aim to incorporate multiplex network to untangle the relation in EEG signal reflecting brain activity during a motor move, let us first begin by reviewing this multiplex network.

2.3 Multiplex network

In order to better understand this specific network and why we specifically decided to use it, let us start by listing some concepts that are related to it.

2.3.1 Network. Network is an architectural representation of the various interaction peers that exist in a complex system between these many components, which are referred to as network nodes [11] . As a result, Network Science defines network topologies in order to better understand complex systems, as it is considered that a complex system underlying network structure stores information about its function[11] .

2.3.2 Single Network. A single network is a graph G = (N,L) constructed by a pair of sets N and L, with N representing nodes and L representing links. Networks are used to describe the real interaction within complex systems such as the brain or the Internet [11].

Undirected and directed networks are two types of single networks. Undirected interactions produce undirected networks, and if node **i** is linked to node **j** in these networks, node **j** is automatically linked to node **i**. However, in Directed networks, the interactions are direct, and just because node **i** points to node **j** does not mean that node**j** points to node **i**.

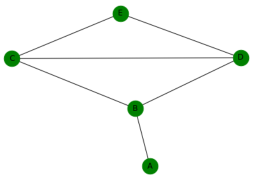


Figure 2.5: Undirected Graph

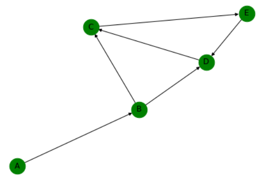


Figure 2.6: Directed Graph

Unweighted and weighted networks are also two types of single networks. Weighted networks, are networks in which each connection has a weight, which typically describes the 'strength' of the interaction, whereas in unweighted networks, each interaction can either exist or not. [11] .

2.3.3 Multilayer Network . The multilayer network approach is a relatively new discovery in the field of network research, and it focuses on describing the interactions of several interconnected networks. In fact, solitary, isolated networks with links of similar meaning and connotation rarely form complex systems.

The interactions in brain and biology networks can get a variety of valences, thus it's critical to study these systems using a multilayer framework that allows us to address the differences between different types of interactions.

Multilayer networks were initially presented in social science to describe the various forms of social ties that exist between the nodes of a social network [23, 51, 70]. However, the importance of understanding multilayer network topologies has only lately become more widely understood. Multilayer networks are now being researched in a variety of domains, including economics, neurology, climate change, transportation networks, ecology [11] .

A multilayer network is made up of many interconnected networks. A set of M networks detailing the interactions inside each layer form a particular multilayer made up of separate M layers. and $\frac{M(M-1)}{2}$ networks describing the interconnection between nodes in each pair of the various layers [11] .

Multiplex networks are the most basic type of multilayer network. The Multiplex Network are commonly employed When the same set of nodes is connected by links signaling different types of interactions [11] .

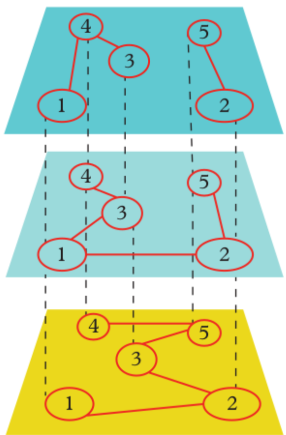


Figure 2.7: Multiplex network with M = 3 layers [11].

Something, we could use a single colour network to depict a Multiplex networks by assigning a different colour to each type of link. It is also conceivable to suppose that the layers of the multiplex network are made up of links of the same type.

As shown in the figure 2.7, the links inside each layer reflect various forms of interactions, whereas the connection between layers are only between same node in each layer.

However, the use of multiplex network to analyze and interpret the result obtained from that electrophysiological and imaging techniques for recordings reflecting brain activity explain in the section 2.2 has revealed a lot about the structure and dynamics of the brain network.

2.4 Brain Multiplex Network

The use of multilayer networks to characterise brain networks has recently gained traction, As opposed to single networks, multilayer networks allow for a more detailed portrayal of brain networks. In fact, the majority of brain processing activities are governed by intra- and inter-frequency connections, according to the new research mentioned above in the section (2.2.1) . This explains the widespread application of network theory, which provides a set of theoretical tools in the form of relevant graphs to properly characterize these various cerebral processes during the performance of a variety of tasks or rest, as explained for example in [50, 6] . Furthermore, the considerable multilayer feature of brain networks has been identified at both the neuronal and macroscopic levels of brain areas.

Indeed, Positive correlations between the degrees of the neurons in the different levels may be detected at the neuronal level in this multiplex network, indicating that hub neurons in one layer are probably to be hub neurons in another. [47] . In addition, the novel centrality measure known as Functional Multiplex PageRank has recently been shown to be able to analyse the diverse functions (cell types) of neurons starting from the multiplex network structure [34] .

On a macroscopic level, a vast body of work has underlined the need of employing network theory [15] methods to investigate the link between functional and anatomical brain networks. The brain can be regarded as a multiplex network in which distinct macroscopic parts of the brain communicate structurally, owing to brain fibers, and functionally, as evidenced by their associated activity, in these investigations.

A new article [20] proposes a clustering of brain areas that is related to the alignment of the two networks graphs. The proposed method allows us to find a common skeleton that is shared by structure and function, from which we can create a novel brain partition that is distinct from the commonly utilized anatomical or functional parcellations. These findings show a strong link between brain shape and resting-state dynamics, as well as the evolution of the human coherent brain organization.

2.4.1 Multiplex network using EEG recording. Very recently multiple works where a multilayer network, with each layer adjusted to a within-frequency synchronized network and cross-frequency connection specified by the multiplex connection, have been proved to be a useful tool construct for modeling neural communication [37].

Indeed, It is actually the case in [26] which demonstrate an approach to a frequency-based multilayer functional network constructed from non-stationary multivariate data by analyzing recurrences in application to electroencephalography (EEG). Using the recurrence-based index of synchronization in [26], they have constructed intralayer (within-frequency) and interlayer (cross-frequency) graph edges to model the evolution of a whole-head functional connectivity network during a prolonged stimuli classification task. By doing that, they have demonstrated that the graph edges weights increase during the experiment and negatively correlate with the response time. They have also shown in [26] that while high-frequency activity evolves towards synchronization of remote local areas, low-frequency connectivity tends to establish large scale coupling between them.

Following this line of reasoning, in [41], they have used wavelet phase coherence to reconstruct the functional multiplex network of the brain, with the different typical frequency bands of EEG activity acting as interconnected layers. They show that switching from a resting state to evaluating a cognitive task causes a large outflow of low-frequency shortest routes and a strengthening of high-frequency connectivity in the brain. Their findings are consistent with recent studies of cognitive activity and can be used in the design of brain-computer interfaces[8, 57] for the estimate of cognitive load or attention for the estimation of cognitive load or attention.

In [58] also, they have demonstrated how to convert a univariate time series into a graphical form while maintaining its temporal properties.[58] offer a new method for constructing a complex functional brain connection network employing clustering co-efficient based on reciprocal correlation between distinct electrodes based on the graphical representation of the converted EEG time series.

Furthermore, in-depth detection of multilayered communities is an effective way for assessing the flexibility of the network . In reality, it is a metric that may be used to assess the multilayered resilience of community [44] over the time. It became revealed that the functional brain networks of the learner executing the activity in issue are extremely flexible during learning.[7]. This is also actually what we will check in our EEG time series data recording for brain activity.

2.4.2 Multiplex network using fMRI. Despite the fact that fMRI recording is different from EEG recording, which is currently what concerns us, we decided to include it because the work done utilizing the multiplex on fMRI recording has shown many similar aspects of the brain that we can also see by using multiplex on EEG recording.

Indeed, by discriminating between layers created by strongly correlated brain regions interacting through weaker linkages, multilayer networks [54] have been extracted from functional brain networks derived from fMRI data. Surprisingly, it has been shown that the topology of these networks has a set of degree correlations that provide superior robustness features than random multilayer networks.

In [19] for example ,they attempt to address the problem of identifying hubs in networks made up of various frequency bands that are mainly unexplored. To do that, they first identify each frequency component using one layer of a multiplex network and then evaluate the underlying structure of the multiplex . They then show that hubs in the multiplex network, as opposed to those obtained by discarding or aggregating the measured signals as is usually done, provide a more accurate map of the brain's most important functional regions, allowing them to distinguish between healthy and schizophrenic populations (recording by fMRI) better than traditional network approaches.

Moreover,in [71], Independent component analysis was applied to extract spatially independent components from the fMRI. The functional connectivity between independent components was calculated across the entire time series and for dynamic brain states to investigate the distributed network properties in patients with advanced Parkinson Disease . Thus, the dynamic analysis identified two unique brain states: a relative hypoconnected state and a relative hyperconnected state and the graph theory analysis demonstrated significant relationships with the deep brain stimulation of the subthalamic nucleus(STN-DBS) response only during the hypoconnected state STN-DBS was negatively correlated.

3. Methodology

In this section, we present the methodology that we use to reveal the properties of the brain network during motor execution. We will begin by describing the content of our dataset [4].

3.1 Description of our dataset

In this section of the work, we present the content of our dataset. The dataset depicts the performance of tasks that we are accustomed to performing on a daily basis, and we believe that studying the cerebral activity that corresponds to these motions could be very useful in the treatment of a patient who is no longer able to perform these movements.

3.1.1 Acknowledgment of this dataset. Gerwin Schalk and his colleagues at the BCI R&D Program produced and contributed this dataset to PhysioBank. The Wadsworth Center, the New York State Department of Health, Albany, and NY collected the information. Sarnacki, W.A. Aditya Joshi was in charge of compiling and preparing the documentation. Experimental design and project oversight were handled by D.J. McFarland and J.R. Wolpaw, respectively [60, 29] .

3.1.2 Experimental protocol. The data set for this study is made up of over 1500 one -and two-minute EEG recordings collected from 109 volunteers. The subjects performed several motor and imagery activities while 64 EEG channels recorded the brain activity reflected as electrical potential difference, i.e., voltage. Each subject completed 14 experimental runs, including two one-minute baseline runs (one with eyes open, one closed) and three two-minute runs of each of the four activities listed below:

1. A target appears on either the left or the right side of the screen. The subject opens and closes the corresponding fist until the target disappears. Then the subject relaxes[4] .

2. A target appears on either the left or the right side of the screen. The subject imagines opening and closing the corresponding fist until the target disappears. Then the subject relaxes [4] .

3. A target appears on either the top or the bottom of the screen. The subject opens and closes either both fists (if the target is on top) or both feet (if the target is on the bottom) until the target disappears. Then the subject relaxes [4].

4. A target appears on either the top or the bottom of the screen. The subject imagines opening and closing either both fists (if the target is on top) or both feet (if the target is on the bottom) until the target disappears. Then the subject relaxes [4].

For this research work, we will only analyze the data of the 109 subjects executing the first task. Data analysis of the execution of other tasks is planned for future work to be conducted.

3.1.3 Montage. The picture (3.1) below depicts the locations of the 64 EEG electrodes on the scalps of each of the 109 volunteers. The numbers behind each electrode name in this figure show the sequence in which they appear in the records. It is worth noting that the signals in the records are numbered from 0 to 63, whilst the numbers on the figure are numbered from 1 to 64; this will come in handy later when interpreting the data.

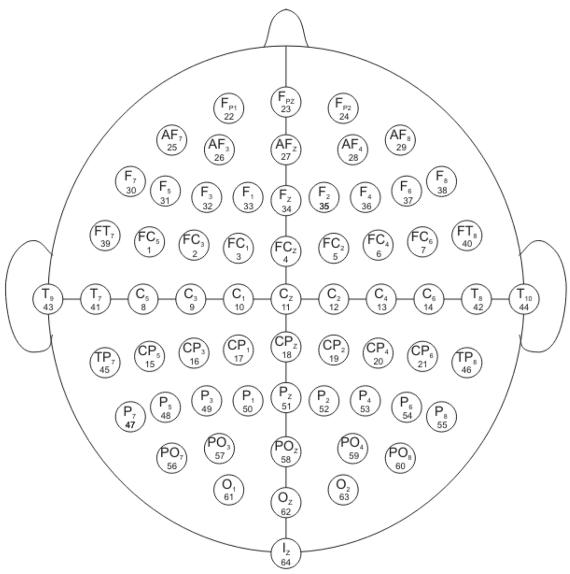


Figure 3.1: Montage of electrodes of the electroencephalograph (EEG)

3.2 Description of the methodology

A multiplex graph is supposed to be modeled with respect to the given EEG time-series data. In this part, we will define the nodes of our multiplex network and also the different frequency bands whereas each individual frequency band is assigned to one layer in this multiplex network. Let us start by explaining the method used for the extraction of the different frequencies from the brain signals.

3.2.1 Extraction of frequencies from EEG time-series data. Given that the brain signals collected from the 109 patients are time-dependent voltage changes, which can be denoted as time-series data, we use the short-time Fourier transform to investigate the evolution of the spectrum with time. In order to fully understand this extraction process, let us first start by presenting the Fourier transform.

1. **The Fourier transform**

   The Fourier transform (FT) is a mathematical transform that decomposes integrable functions depending on space or time into functions depending on the frequency, hence, it can be summarized as a mathematical operator used to analyze and represent an integrable, non-periodic, and stationary signal(function) in the frequency domain.

   Mathematically, there are several common conventions for defining the Fourier transform of an integrable function $f : \mathbb{R} \to \mathbb{C}$[36, 53], one of them is:

$$\hat{f}(w) = \int_{-\infty}^{\infty} f(x)\, e^{-i2\pi w x}\, dx, \quad \forall\, w \in \mathbb{R} \tag{3.2.1}$$

   Here, the Fourier transform of the integrable, non-periodic and stationary function $f(x)$ at frequency w is given by the complex number $\hat{f}(w)$. Evaluating (3.2.1) for all values of w produces the frequency-domain function.

   Concretely, the Fourier transform makes it possible to associate nonperiodic functions with their frequency spectrum. It does this by seeking to obtain the expression of the function as a sum of trigonometric functions.

Thus, when the independent variable represents time (often denoted by t instead of x), the transform variable represents frequency (often denoted by f instead of w). So, if time is measured in seconds, then frequency is in Hertz.

The properties of the Fourier transform are briefly reviewed in the following [52].

Let us assume that $f(x), g(x)$ and $h(x)$ are integrable functions(Lebesgue-measurable) which verify

$$\int_{-\infty}^{\infty} |f(x)|\, dx < \infty. \tag{3.2.2}$$

We denote the Fourier transforms of these functions as $\hat{f}(w)$, $\hat{g}(w)$ and $\hat{h}(w)$ respectively. Thus, we have the basic properties :

- **Linearity**

For any complex numbers a and b, if $h(x) = af(x) + bg(x)$, then $\hat{h}(w) = a.\hat{f}(w) + b.\hat{g}(w)$.

- **Translation / time shifting**

For any real number x_0 , if $h(x) = f(x - x_0)$, then $\hat{h}(w) = e^{-2\pi i x_0 w}\hat{f}(w)$.

- **Modulation /frequency shifting**

For any real number w_0 , if $h(x) = e^{2\pi i x w_0} f(x)$, then $\hat{h}(w) = \hat{f}(w - w_0)$.

- **Time scaling**

For a non-zero real number a, if $h(x) = f(ax)$, then

$$\hat{h}(w) = \frac{1}{|a|}\hat{f}(\frac{w}{a})$$

2. **The Short-Fourier Transform (STFT)**

The STFT is a natural extension of the Fourier transform consisting of applying a sequence of Fourier transforms to a non-stationary signal segmented using a fixed window. It provides the time-localized frequency information for situations in which frequency components of a signal vary over time (non-stationary signal) as is the case in figure 3.2 below.

Concretely, the Short-Fourier Transform is used to determine the sinusoidal frequency and phase content of local sections of a signal as it changes over time [62] . In practice, to compute STFT, we divide a longer time signal into shorter segments of equal length and then compute the Fourier transform separately on each shorter segment. This reveals the Fourier spectrum on each shorter segment.

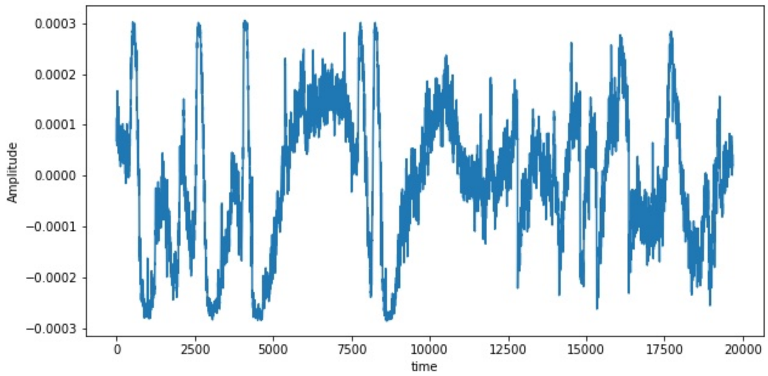


Figure 3.2: Brain activity recorded by electrode 1 from patient 109 doing run 3, where one time point corresponds to $\frac{1}{160}s$.

In this figure 3.2, the amplitude refers to the voltage the electrode record and which reflects the fluctuations of the cerebral activity of this patient. The sampling frequency of the EEG recording is $160Hz$, i.e., within one second, 160 samples voltage values were captured. Hence the depicted recording lasts for approximately 2 minutes.

Basically, the STFT can be applied in the continuous-time case and the discrete-time case. Let us present the formulation of the STFT in each of those cases.

- **Continuous-time STFT**

In the continuous-time case, the function to be transformed is multiplied by a window function which is nonzero for only a short period of time. The Fourier transform (a one-dimensional function) of the resulting signal is taken as the window is slid along the time axis, resulting in a two-dimensional representation of the signal. Mathematically, this is written as

$$\mathbf{STFT}\{x(t)\}(\tau,\omega) \equiv X(\tau,\omega) = \int_{-\infty}^{\infty} x(t)w(t-\tau)e^{-i\omega t}\,dt, \tag{3.2.3}$$

where $w(\tau)$ is the window function, commonly a Hann window or Gaussian window centered around zero, and $x(t)$ is the signal to be transformed. $X(\tau,\omega)$ is essentially the Fourier transform of $x(t)w(t-\tau)$, a complex function representing the phase and magnitude of the signal over time and frequency. The time index τ is normally considered to be slow time and usually not expressed in as high resolution as time t .

- **Discrete-time STFT**

In the discrete time case, the data to be transformed could be broken up into chunks or frames. Each chunk is Fourier transformed, and the complex result is added to a matrix, which records the magnitude and phase for each point in time and frequency. Mathematically, this can be expressed as:

$$\mathbf{STFT}\{x[n]\}(m,\omega) \equiv X(m,\omega) = \sum_{n=-\infty}^{\infty} x[n]w[n-m]e^{-j\omega n} \tag{3.2.4}$$

Likewise, with signal $x\,[n]$ and window $w\,[n]$ and in this case, m is discrete and ω is continuous.

More, the magnitude squared of the STFT yields the spectrogram representation of the Power Spectral Density of the function:

$$\text{spectrogram}\{x(t)\}(\tau, \omega) \equiv |X(\tau, \omega)|^2 \tag{3.2.5}$$

Moreover, in comparison with the standard Fourier transform, the STFT provides an advantage of revealing the frequency contents of the signal at each time point in the signal whereas the standard Fourier transform provides the frequency information averaged over the entire signal time interval as is the case in the figure 3.3 below.

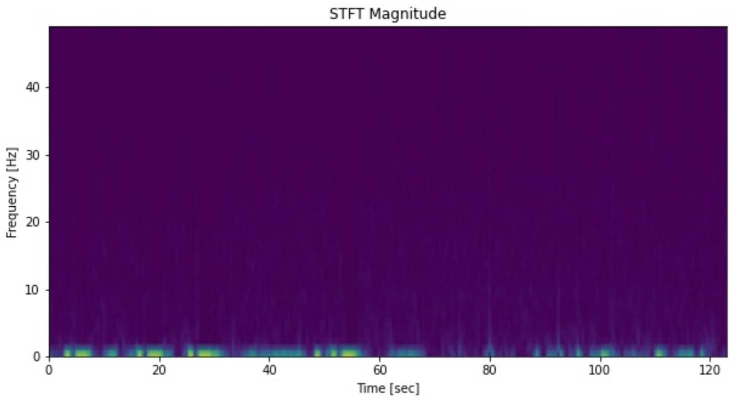


Figure 3.3: STFT of 3.2

This information will be useful to extract the different frequency bands that we will define for our multiplex network and also construct the nodes and the layers of this multiplex network. Moreover, this kind of information can be used to provide control and perform several tasks in a brain-computer interface system. For example, the paper in [22] describes the STFT analysis of EEG signals obtained during relaxing and writing, and the results of the STFT analysis showed that there are significant differences in the frequency components and patterns between the EEG signals obtained from relaxing and writing.

3.2.2 Definition of the frequencies bands. Since the discovery of the EEG in humans by Hans Berger in 1929, major brain rhythms studied over time in EEG in humans have revealed 5 different frequency bands present in the EEG which are given in the table 3.1:

Numbers	Frequencies Bands	Intervals
1	Delta	[0-4 Hz]
2	Theta	[4-8 Hz]
3	Alpha	[8-12 Hz]
4	Beta	[12-30 Hz]
5	Gamma	[30-100 Hz]

Table 3.1: Existing frequencies bands

Based on these same models, we decided to organize the different frequencies obtained following the application of the STDT of the different signals obtained from the different electrodes of the EEG times series into these different frequency bands.

As depicted in figure 3.3, the STFT of a signal results in a spectrum ranging from 0 to 50 Hz. Therefore, for better frequency analysis, we decided to organize these frequencies in 12 frequency bands delimited as we can see in the table 3.2:

Numbers	Frequencies Bands	Intervals
1	Delta	[0-4 Hz]
2	Theta	[4-8 Hz]
3	Alpha	[8-12 Hz]
4	Beta	[12-16 Hz]
5	Beta	[16-20 Hz]
6	Beta	[20-24 Hz]
7	Beta	[24-28 Hz]
8	Beta	[28-30 Hz]
9	Gamma	[30-36 Hz]
10	Gamma	[36-40 Hz]
11	Gamma	[40-44 Hz]
12	Gamma	[44-48 Hz]

Table 3.2: Frequencies bands

3.2.3 Nodes of the multiplex network. In our multiplex network, a node represents an electrode and contains a time-varying signal. The procedure to obtain this signal is as follows: First, the spectrum of the electrode is computed with the STFT. Second, the signals in this time-varying spectrum that belong to the same frequency band are averaged. Based on this procedure, the averaged signal over in a particular frequency band corresponds to a node in the respective layer in the multiplex network.

Hence, for each patient, we obtain a multiplex network with 12 layers, where each layer represents a frequency band and where each layer contains 64 nodes, each standing for one electrode.

3.2.4 Layers of our multiplex network. As explained above, in our multiplex network, each layer represent a frequency band as defined in section(3.2.2) with 64 nodes as explain previously in section (3.2.3) .

The connection between a pair of nodes is determined in each layer by the Pearson correlation coefficient [9] measuring the linear relationship between the signals of the nodes.

3.3 Characteristics of layers of the multiplex network

In this section, we present the definitions, characteristics, and general concepts of some of the measures that we will apply to our multiplex, questions for us to reach the objectives set for our study.

3.3.1 Degree distribution. The distribution function $p(k)$, which is the chance that a node picked uniformly at random has degree k, characterizes the scattering of node degrees over a network. The fraction of nodes in a network with degree k is defined as $p(k)$. In a network of size n, $p(k) = \frac{n(k)}{n}$, where $n(k)$ is the number of nodes with degree k. The probability distribution of node degrees over a network is referred to as the network's degree distribution.

3.3.2 Clustering coefficient. Clustering coefficient is a measure of the degree to which nodes tend to cluster together.

Consider the nodes l, j, and k in a network. If l is connected to both j and k (two neighbors of a node are neighbors), then the probability that j and k are also connected is known as the clustering coefficient. In other words, the density of triangles in a network is measured by the clustering coefficient. The clustering coefficient has a value between 0 and 1.

3.3.3 Average path length. The average path length of a network is the average number of steps along the shortest paths for all possible pairs of network nodes. Let G = (V, E) be a graph the average path length is defined as

$$L_G = \frac{1}{n(n-1)} \sum_{i,j \in V, i \neq j} d_{ij}, \tag{3.3.1}$$

where d_{ij} is the shortest path between node i and j and n is the total number of nodes in G. The size of a network is determined by the value of L, which also aids in determining the efficiency of information flow over a network [69].

3.3.4 Adjacency matrix. Any single network G = (V, E) is fully determined by its adjacency matrix. The adjacency matrix is an $N \times N$ matrix A(G), whose elements a_{ij} indicate whether node i is linked to node j. For unweighted and undirected networks the adjacency matrix elements A_{ij}(G) are given by

$$A_{ij}(G) = \begin{cases} 1 \text{if node i is linked to node j,} \\ 0 \text{ Otherwise} \end{cases} \tag{3.3.2}$$

The diagonal entries of A(G) are zeroes and if the network is undirected. Moreover, for these networks the adjacency matrix A(G) is symmetric [45]. Using the adjacency matrix we can represent a network [45], which is what we have used to represent the network of each sub-band.

Having computer for each frequency band the Pearson correlation coefficients between all pairs of nodes, we removed the weak correlations to investigate the network properties emerging due to strong connections. Hence, calculate for each frequency band the absolute value of all the Pearson correlations coefficients.

Then, we defined a threshold value of 0.5, where correlation coefficients above this value are considered as significant. If a correlation coefficient was above this threshold it was replaced with 1 and otherwise, it was replaced with 0. Through this procedure, we obtained adjacency matrices in each layer of the multiplex network.

3.3.5 Degree of a node . The degree of a node v is the number of nearest neighbors of v. Mathematically, it is given by

$$k_v = |N_G(v)| \tag{3.3.3}$$

In fact, the neighborhood of a vertex $v \in V$ is a set of all vertices that are adjacent to v [45] . Mathematically, it is given by

$$N_G(v) = \{u \in V | uv \in E\} \tag{3.3.4}$$

3.3.6 Centrality. The most central or 'important' node in a network must be identified during network analysis. In fact, The centrality of a node has a wide range of real-world applications, including preventing a computer network from failing, controlling the spread of disease, and more. It is actually the reason that we need to detect them in a network. To calculate the centrality of a node, we employ several methods. Some of the most Centralities include :

1. **Degree centrality**

   The number of edges occurring to a node is known as degree centrality. It is the most basic centrality metric, and it is particularly excellent at selecting the most important node. The degree centrality measure, on the other hand, has the flaw of only capturing information about the local structure around a node, leaving out the global structure. The degree centrality $C_D(i)$ of a node **i** is calculated mathematically as follows:

$$C_D(i) = \sum_j A_{ij} \tag{3.3.5}$$

   Where A_{ij}, is the adjacency matrix.

2. **Closeness centrality**

   Closeness centrality is a centrality metric that determines the importance of a node based on its proximity to its neighbors. In other words, how accessible a node is. Closeness centrality can provide information about how rapidly information can flow over a network by assessing how fast it is to go to other nodes from a given node. The information about the global network topology is captured by closeness centrality. It is defined as the inverse sum of the shortest distances from the focal node i to all other nodes in a network, that is,

$$C_C(i) = \frac{1}{\sum_j d_{ij}} \tag{3.3.6}$$

   where d_{ij} is the shortest distance between nodes i and j.The normalised closeness centrality is given by

$$C'_C(i) = (n-1)C_C(i) \tag{3.3.7}$$

3. **Betweenness centrality**

   The node that acts as a bridge between two nodes is determined by this form of centrality. In other words, it finds which node in a network is on the shortest path between two nodes. The Betweenness centrality, for example, in social networks evaluates an actor's power over communication between two other peers. Mathematically we define the betweenness centrality as

$$C_B(i) = \sum_{i \neq j} \sum_{j \neq i, j \neq h} \frac{L_{h,j}(i)}{L_{h,j}} \tag{3.3.8}$$

   where $L_{h,j}$ is the total number of shortest paths between nodes h and j, $L_{h,j}(i)$ is the number of shortest paths that pass through node i.

   The centrality of a node's betweenness provides information about a node's global position in a network and can be applied to unconnected components. The normalised betweenness centrality is given as

$$C'_B(i) = \frac{2}{(n-1)(n-2)} C_B(i) \tag{3.3.9}$$

4. Results and Discussion

In this section, we will show the primary findings of this study and explore the substance of these findings in order to untangle significant information on brain features during motor action in relation to the underlying network structure.

4.1 The inter-layer similarities

The purpose of evaluating the similarities between the different layers of a multiplex network in a multiplex network is to expose the inter-communication or interaction between the different levels of the multiplex network. Therefore, the inter-layer similarities between pairs of layers of our multiplex network might reveal crucial insights into the differences in cerebral configuration between different frequency bands during motor activity.

Indeed, we know from prior research 2.4 that the different areas of the cerebral communicate with one another via different cerebral rhythms cadences by different bands of frequencies (Delta, Theta, Alpha, Beta, Gamma), which represent the different layers of our multiplex network in our study. Thus, comparing the similarities between these sub-bands provides an adequate overview of the cerebral communication of the 109 patients executing the task of opening and closing the left and the right fits.

For the inter-layer similarity, we computed the pairwise similarity with the Pearson correlation coefficient between each pair of adjacency matrices for each patient. Having obtained one matrix for each patient, where each such matrix reflects the pairwise similarities between the frequency bands, we computed the average of these similarity matrices. Hence, this averaged matrix is thought to reflect the similarities between the frequency bands for all patients.

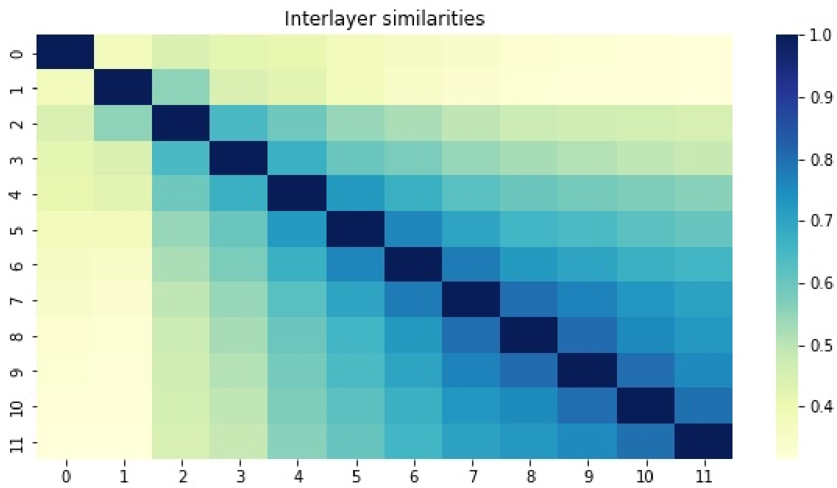


Figure 4.1: Inter-layer similarities between the 12 frequencies bands (averaged across all 109 patients)

We can see in the figure 4.1 that the first sub-band, which represents the Delta[0-4Hz] sub-band, solely interacts with itself and not with the other ten sub-bands. We can extrapolate from this that the regions communicating with this frequency band may play a specific role in the brain configuration during the execution of these motions by the various patients. The second sub-band, Theta[4-8 Hz], interacts primarily with itself in the same way but unlike the Delta[0-4Hz] sub-band, it has a very little interaction with the third sub-band, the Alpha[8-12] sub-band.

Furthermore, in figure 4.1, we can see that from sub-band 3 (Alpha [8-12 Hz]) to the last sub-band (Gamma [44-48 Hz]), the different sub-bands have a lot of interaction with their direct neighbors, but these interactions reduce as one gets away from the sub-band in question.

From a general point of view, we can deduce from these research results of inter-layer correlation that communication between the different regions of the brain, whether distant or close together, mainly occurs at high frequencies ranging from 8 to 48 Hz (Alpha to Gamma) during the simultaneous opening and closing their fists. Furthermore, the regions interacting with low frequencies ranging from 0 to 8 Hz (Delta to Theta) have a specific and decisive role in the patients' execution of the motions.

4.2 The intra-layer similarities

In this section, we examine the correlation between the different cerebral regions for all the 109 patients, i.e., we compute within each frequency band the Pearson correlation coefficient between each pair of nodes.

4.2.1 Layers Pairwise correlation multiplexity. Figure 4.2 reflects the Pearson correlation coefficient between the different brain regions of the last patient performing the task of opening and closing his/her fists.

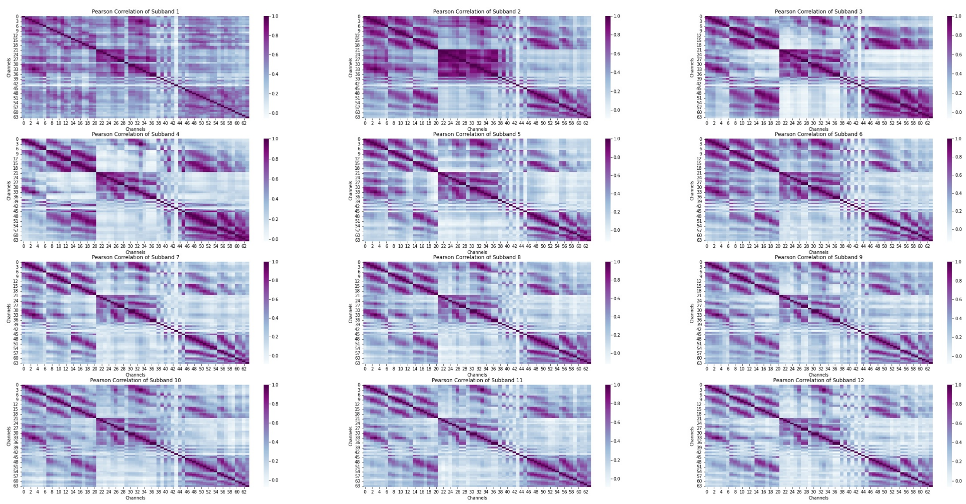


Figure 4.2: Pearson correlation within each frequency band for the last patient

We observe that the patterns of the frequency band Delta [0-4 Hz] are different from the patterns of the other frequency bands, which might reveal that the interaction between the different brain regions in this frequency band could be intended for a specific purpose when the patient opens and closes one of his/her fists.

We also observe structured assemblies of high correlation coefficients in the theta frequency band [4-8 Hz], which could mean that the communication between the different brain regions in this frequency band could be the dominant or preponderant communication between the different brain regions when this patient performs the task in question.

Moreover, apart from these 2 frequency bands, we notice some commonalities in the other frequency bands which could reflect the type of communication between the different cerebral regions for these frequency bands.

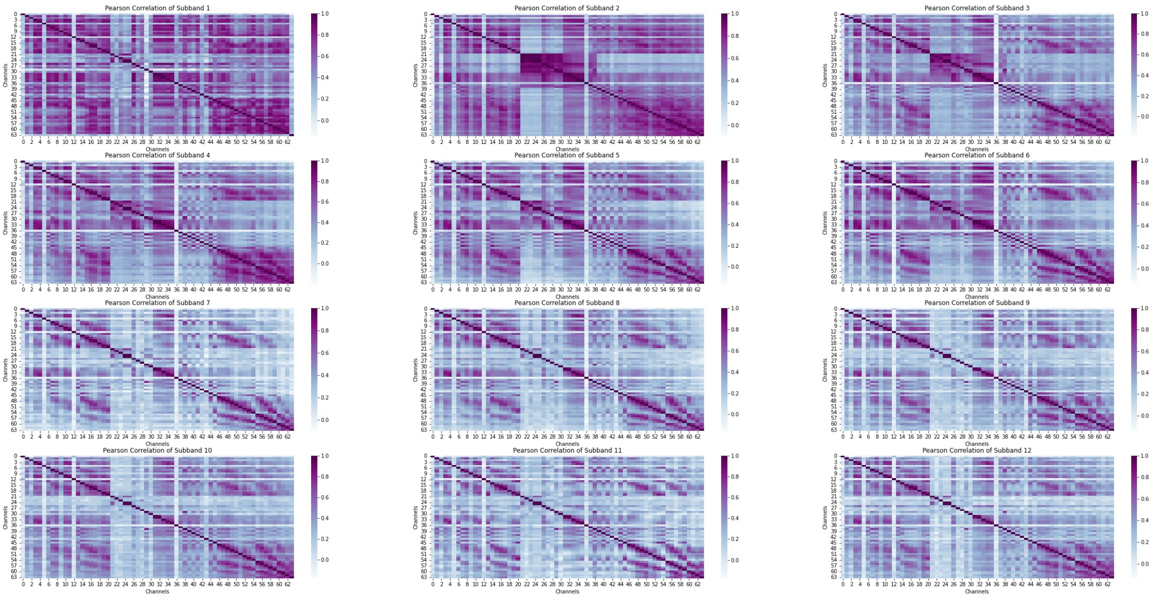


Figure 4.3: Similarity matrices for all frequency bands averaged across all 109 patients

Figure in 4.3 above reflects the Pearson correlations observed within each frequency band for all the different 109 patients. This figure is obtained by averaging the similarity matrices in each frequency band across all patients.

In this figure, we can see that the patterns of the first two frequency bands are radically different from the other 10 frequency bands. This might indicate the special character and the importance that the frequencies of these 2 frequency bands could have in the cerebral communication of these patients when they are in the process of performing this task in question.

We also note that the similarity matrices of these two frequency bands are characterized by notably high values, which might mean that the communication between the cerebral regions when these patients execute the movements plays a significant role in terms of encoding the execution of the motor movement.

Moreover, for frequency bands 3,4,5, and 6 we observe almost the same patterns in their communication structures. This could mean that the interactions of nodes in these frequency bands do not significantly differ from each other and hence, these frequency bands might be considered as contributing to the same function when the motor task is generated and executed, respectively.

4.2.2 Pearson correlation using adjacency Matrix. Having constructed the adjacency matrix of the different frequency bands as described in section 3.3.4, we can also evaluate the correlation for all 109 patients between the nodes within the frequency band by also computing the Pearson correlation between nodes within frequency bands.

This approach will give us a better view of cerebral communication because in the construction of the adjacency matrix we have only kept the significant interactions between the nodes, that is to say, those which are greater than the threshold value of 0.5 (section 3.3.4).

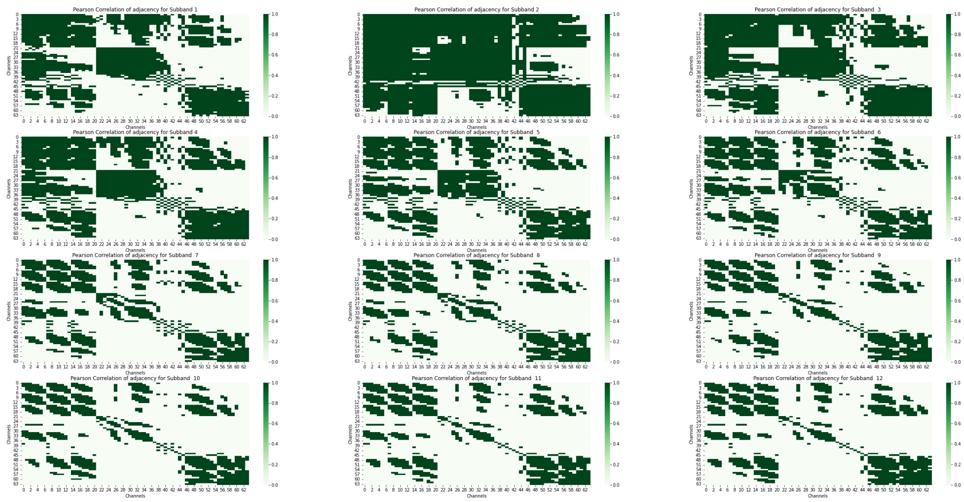


Figure 4.4: Layers pairwise similarity matrices using adjacency matrix for all the 109 patients

Figure (4.4) above was obtained by evaluating for the 109 patients the Pearson correlation within each frequency band on the adjacency matrix of each frequency band. It illustrates the different correlation patterns of the brain regions of the 109 patients in each of the frequency bands.

Through this figure, we observe that the frequency band Theta [4-8 Hz] is the frequency band containing the largest amount of strong correlations. Moreover, we note that this pattern is also very different from that of the other frequency bands. This might reveal that the strongest synchronization or communication is realized in this frequency band.

We also note that the frequency bands Delta [0-4 Hz], Alpha [8-12 Hz], and beta [12 -16 Hz] present approximately the same correlation patterns. This means that these frequency bands show similar correlation properties when the 109 patients perform the task in question. This means the different brain regions communicate in the same way via the different frequencies of these 3 frequency bands.

We also observe a weak correlation in the patterns of the last 7 frequency bands. This means that the brain regions communicate weakly via the frequencies of these regions when the 109 patients perform the task in question.

Ultimately, the intra-layers similarities reveal to us that the brain regions communicate strongly with each other via the frequencies of the frequency band Theta [4-8 Hz] when the 109 patients perform the motor tasks. Moreover, it also reveals that when the 109 perform the task in question, the communication of the brain regions is very weak via the frequencies between 16 and 48 Hz and presents a lot of similarity for the frequency bands Delta, Alpha and Beta.

4.3 Centrality measures within layers

In both multilayer and single-layer networks, it is frequently required to rank the nodes in order to summarize the information encoded in the network structure. The influence of nodes on the structure and communication of distinct networks of the multiplex network has been widely studied using centrality measures initially developed for single networks. These measurements can be extended in a variety of ways in multiplex networks, and they can provide important information about nodes in the network, which can be very useful in understanding the structure and interactions between components of the various layers of the multiplex network.

4.3.1 Degree centrality within layers. As the name implies, this metric allows us to access the network's center nodes (the most significant) by revealing the number of edges that occur at each node. Capturing local information and structure around the network's most significant node is quite beneficial in understanding the communication process of the various network components.

To compute the degree centrality we see in the figure 4.5 below, we just use the adjacency matrix of each frequency band for all the 109 patients and apply the mathematical formula shown the section 3.3.6.

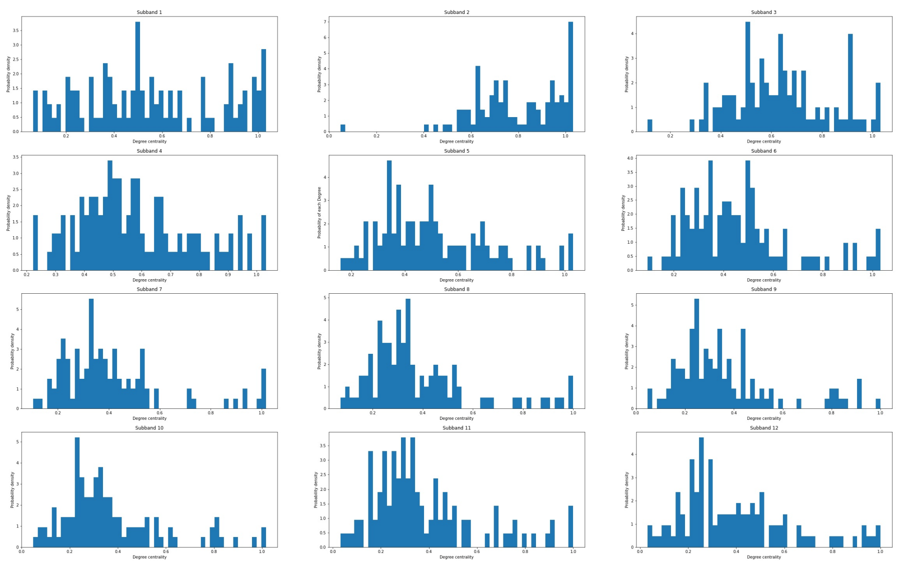


Figure 4.5: Probability density of degree centrality in each frequency band for all 109 patients executing the task to open and close one of his/her fists.

In this figure, we observe that the frequency band that has the greates shift to the right in the probability density of is the frequency band Theta. This means that it is the most important frequency band in brain communication when the 109 patients perform the task.

We also note that the Delta frequency band [0-4 Hz] also presents a great probability of having important nodes for cerebral communication when the 109 patients executing the task in question, which means the cerebral regions also communicate a lot via this frequency band but not as much as via the frequency band Theta.

However, we note that the probability densities are left-shifted in the last 7 frequency bands confirming the insights obtained from the analysis of intra-layer similarities.

4.3.2 Closeness centrality within layers. This metric assesses how quickly information can flow across a network by determining how fast information can move from one node to another. It offers information about each global network (layers) of the multiplex network.

To compute the closeness centrality that we see in the figure 4.6 below, we just also use the adjacency matrix of each frequency band for all the 109 patients and apply the mathematical formula shown in 3.3.6.

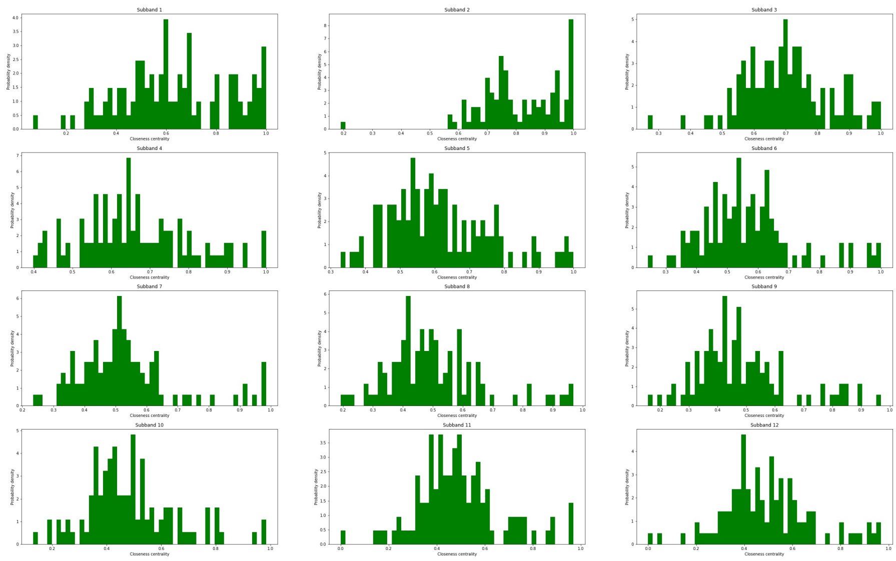


Figure 4.6: Probability density of closeness centrality in each frequency band for all the 109 executing the task to open and close one of his/her fists

According to the explanation of closeness centrality, this figure might indicate that the probability that the information can be communicated rapidly is higher in the Theta frequency band (4-8 Hz] which once again confirms the major importance that this frequency band has when the 109 patients perform the task in question.

We also see that the probability density functions are left-shifted in the other frequency bands, which shows that the cerebral communications in these bands are done more slowly.

4.3.3 Betweeness centrality within layers. In the multiplex, for each layer (network), this metric finds which node in a network is on the shortest path between two nodes. It is also helpful to understand how the interaction occurs between components in each network of the multiplex network.

To compute the betweenness centrality that we see in the figure 4.7 below, we just also use the adjacency matrix of each frequency band for all the 109 patients and apply the mathematical formula shown in 3.3.6.

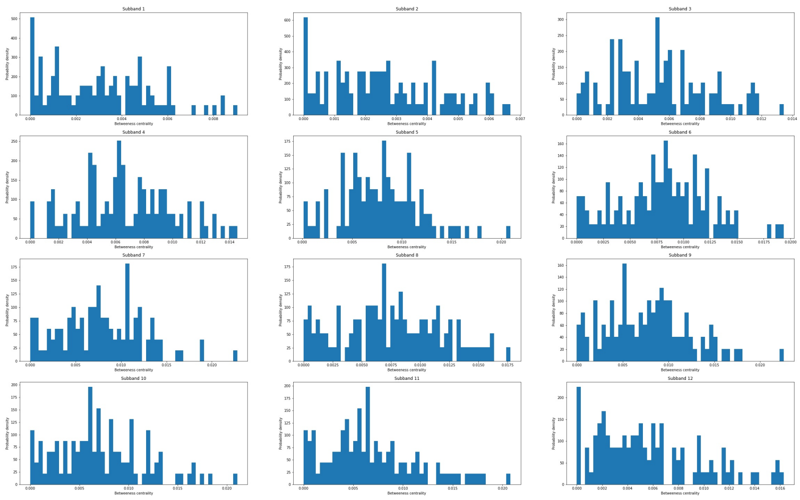


Figure 4.7: Probability density of betweeness centrality in each frequency band for all the 109 patients executing the task to open and close one of his/her fists.

This figure shows that the communication between neighboring brain regions within frequency band when the 109 patients perform the task in question has high variance in all frequency bands. Based on this measure, it is difficult to infer a property of the communication structure.

4.4 Cluster and community detection within layers

4.4.1 Clustering coefficient within layers. With this measure, we compute for each layer of the multiplex network the degree to which nodes tend to cluster together.

To compute the clustering coefficient that we see in the figure 4.8 below, we just also use the adjacency matrix of each frequency band for all the 109 patients.

This figure 4.8 below shows that the probability that the brain regions communicate in close proximity when the 109 patients perform the task in question for each frequency band is high in the Delta and Theta frequency bands. This means in other words that the probability which the cerebral regions having similarities between them communicate together when these 109 patients execute the task in question is more certain in the frequency bands Delta[0-4 Hz] and Theta[4-8Hz].

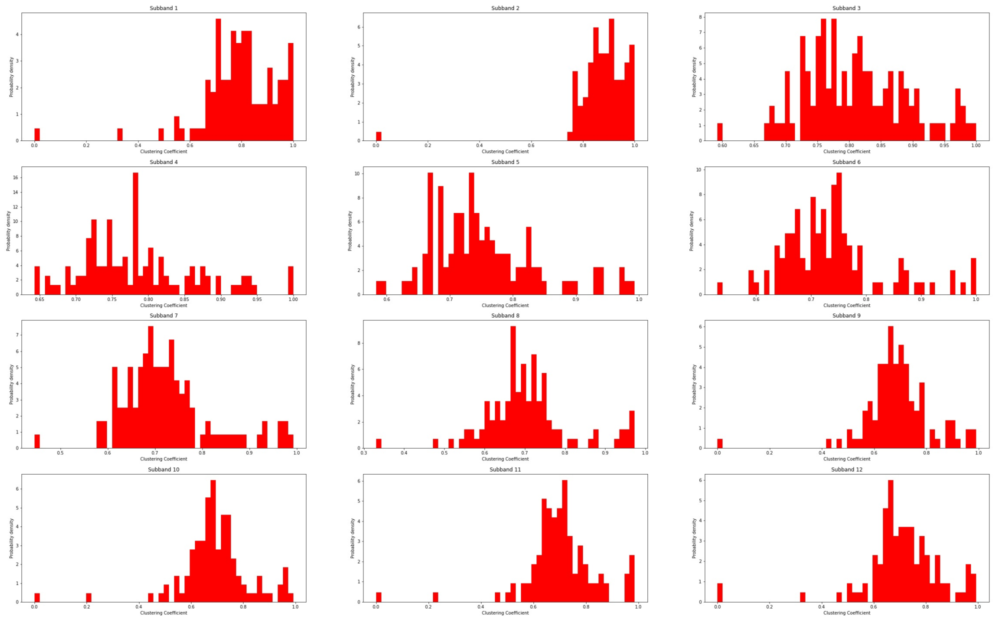


Figure 4.8: Probability density of Clustering coefficient in each frequency band for all the 109 patients executing the task to close and open both fists or both feet.

4.4.2 Community detection in the frequency band Delta[0-4 Hz] using the Louvain algorithm.

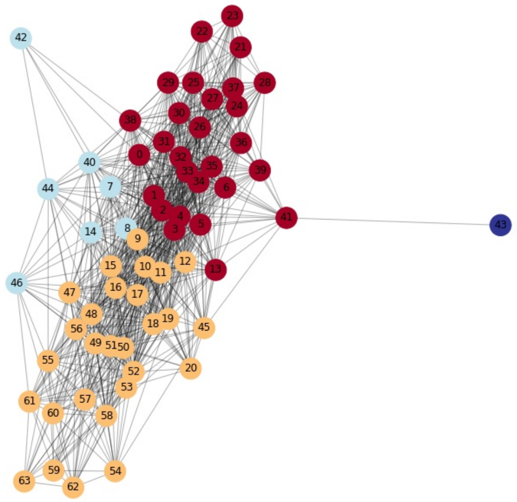
The Louvain method for community detection is a method to extract communities from large networks.

In the Louvain Method of community detection, Small communities are detected initially by optimizing modularity locally on all nodes in the Louvain Method of community detection, then each small community is grouped into one node and the first step is repeated. For the multiplex network, This method is an efficient method to detect communities in each layer (network) of the multiplex. It is a powerful tool to detect the different components on a network that interact together or have many similarities.

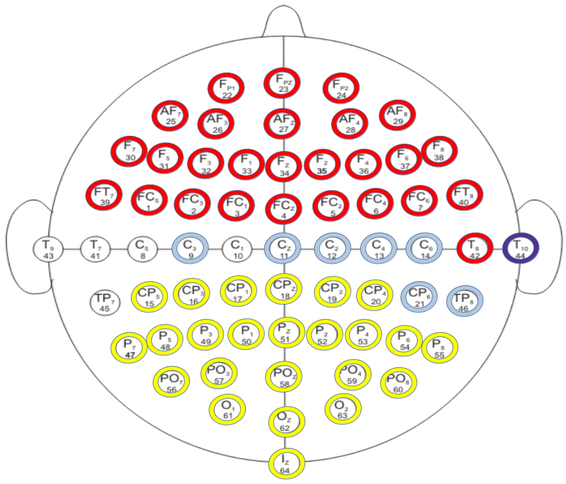
In figure 4.9a below, we have the different communities detected in the frequency band Delta [0-4 Hz] obtained by applying the Louvain algorithm on the adjacency matrices describing the frequency bands. Note that these adjacency matrices are averaged across all patients. We can detect five communities in this frequency band.

By referring to the different regions of the cerebral cortex corresponding to each channel (see annotations in figure 4.9b below), we can have a clear view of the different cerebral regions of the 109 patients who communicate with each other via the frequencies contained in this frequency band when they perform the task of opening and closing either the two fists or the two feet.

This result in the figure 4.9 confirms that in this sub-band of low frequencies, the connection between distinct parts of the brain is indeed localized between neighbors regions, implying that these different localized regions may play a specific and important role during the execution of these motions by the 109 patients.



(a) Detected Communities



(b) Illustration of the communities detected on brain regions

```
community 0
[7, 9, 40, 42, 44]
community 1
[43]
community 2
[8, 10, 11, 12, 13, 20, 45]
community 3
[0, 1, 2, 3, 4, 5, 6, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 41]
community 4
[14, 15, 16, 17, 18, 19, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63]
```

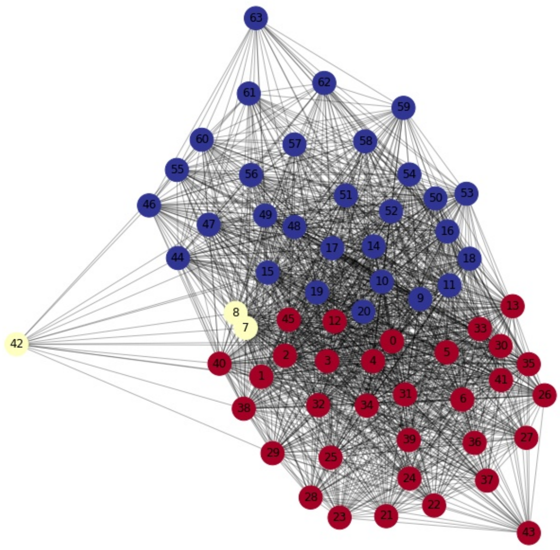
(c) Indices of nodes/electrodes belonging to same communities

Figure 4.9: Community detection in frequency band Delta [0-4 Hz] using the Louvain algorithm

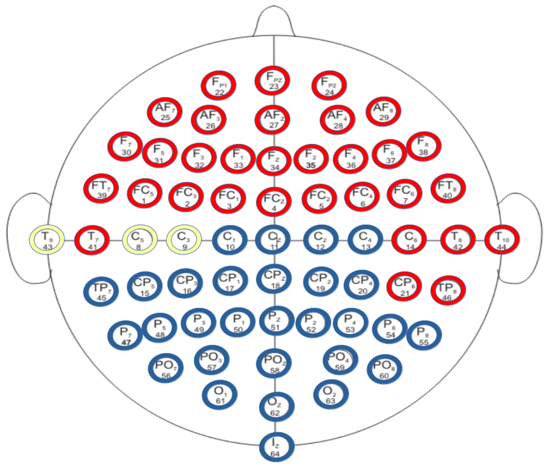
4.4.3 Community detection in the frequency band Theta [4-8 Hz] using the Louvain algorithm. Figure 4.10a below presents the communities obtained by applying the Louvain algorithm on the frequency band Theta[4-8 Hz]. In this figure, we can see 3 communities detected by using this algorithm.

By referring to the different regions of the cerebral cortex corresponding to each channel (see annotations in figure 4.10b), we can have a clear view of the different cerebral regions of the 109 patients who communicate with each other via the frequencies contained in this frequency band when they perform the task of opening their fists.

These results reveal the structure of the communication between the cerebral regions of the 109 patients during the execution of this task in question. In these results, we see that when the 109 patients execute the task communication between the cerebral region is more localized, and each region communicates more with its neighbors.



(a) Detected communities



(b) Illustration of the communities detected on brain regions

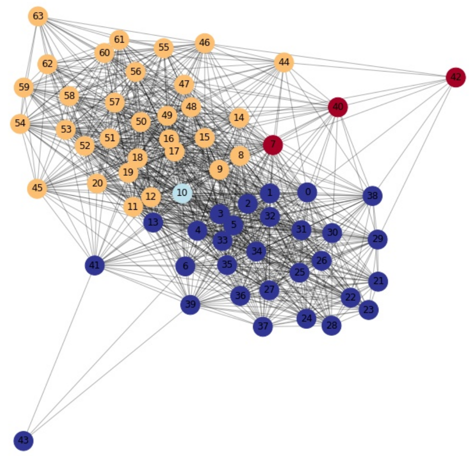
```
community 0
[0, 1, 2, 3, 4, 5, 6, 13, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 4
3, 45]
community 1
[7, 8, 42]
community 2
[9, 10, 11, 12, 14, 15, 16, 17, 18, 19, 44, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63]
```

(c) Table of communities

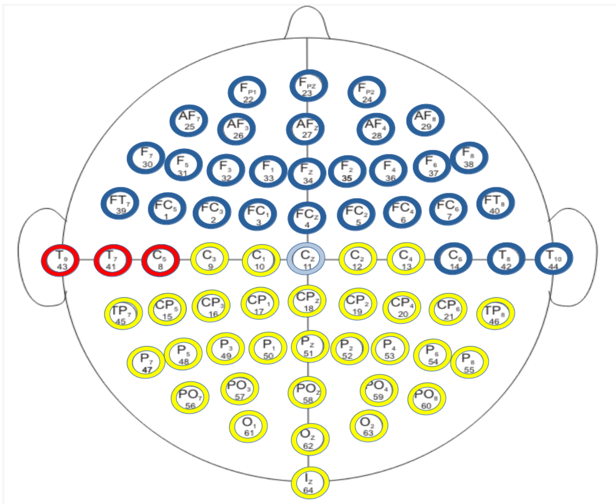
Figure 4.10: Indices of nodes/electrodes belonging to same communities

4.4.4 Community detection in the frequency band Theta[4-8 Hz] using the Louvain algorithm. The figure 4.11a below presents the communities obtained by applying the Louvain algorithm on the frequency band Theta[8-12 Hz]. In this figure, we can see 4 communities detected by using this algorithm.

By referring to the different regions of the cerebral cortex corresponding to each channel (see annotations in figure 4.10b), we can have a clear view of the different cerebral regions of the 109 patients who communicate with each other via the frequencies contained in this frequency band when they perform the motor task.



(a) Detected communities



(b) Illustration of the communities detected on brain regions

```
community 0
[7, 40, 42]
community 1
[10]
community 2
[0, 1, 2, 3, 4, 5, 6, 13, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 41, 43]
community 3
[8, 9, 11, 12, 14, 15, 16, 17, 18, 19, 20, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 6
2, 63]
```

(c) Table of communities

Figure 4.11: Indices of nodes/electrodes belonging to same communities

5. Conclusion and future works

This research work aimed to incorporate the multiplex network to untangle the relationship in electroencephalogram (EEG) signal reflecting brain activity during a motor move. Precisely, the central idea here was to understand the nature of the cerebral signals translating the movement of opening and closing either the left or right fist.

In view of the results recorded, it can be confirmed that the use of the multiplex network was decisive in revealing the nature and interaction of the cerebral signals reflecting the cerebral activity of these individuals in question. It allowed us to show that the Delta [0-4 Hz] and Theta [4-8 Hz] brain waves mainly characterize brain connectivity when a healthy individual performs the mentioned motor task. It has also revealed that these two frequency bands play a very crucial role in the interaction and communication between the different brain regions during the execution of this movement in question.

Moreover, the multiplex network allowed us to reveal the very weak cerebral communication via the high frequencies and the very strong communication via the weak Delta, Theta, and Alpha frequency bands, of course during the execution of this movement. These strong cerebral communications via these low-frequency bands have revealed the existence of 2 large cerebral communities synchronized locally via these low frequencies and each of which plays a major role during the execution of this movement.

The findings of this study might serve as a basis to conduct further and more detailed investigations, particularly in the mentioned two frequency bands. This may lead to more valuable insights that might be used as a baseline for the connectivity structure of brains suffering from diseases to design novel treatment approaches.

Moreover, given the richness of the datasets in question and the excellent results obtained by this method, we intend to use the same method in the remainder of this study in the case where the individuals imagined only making the same movement in question, as this may reveal additional information that can be very useful in the brain-computer-interface.

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