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91031



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# Level 1 Mathematics and Statistics, 2017

## 91031 Apply geometric reasoning in solving problems

9.30 a.m. Monday 20 November 2017

Credits: Four

Achievement	Achievement with Merit	Achievement with Excellence
Apply geometric reasoning in solving problems.	Apply geometric reasoning, using relational thinking, in solving problems.	Apply geometric reasoning, using extended abstract thinking, in solving problems.

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

**You should attempt ALL the questions in this booklet.**

Show ALL working.

If you need more space for any answer, use the page(s) provided at the back of this booklet and clearly number the question.

Check that this booklet has pages 2–15 in the correct order and that none of these pages is blank.

**YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.**

**Achievement**

**TOTAL**

**09**

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## MOSAICS AND TESSELLATIONS



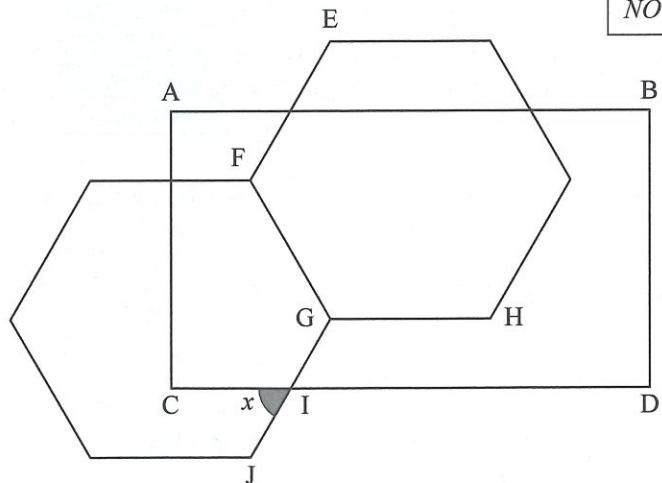
<http://mosaicsbypost.com/banded-stars/banded-stars-black-195m2>

A tessellation is a pattern of repeating shapes fitting together and leaving no gaps. These patterns can be made into mosaic pictures as shown above.

## QUESTION ONE

- (a) The pattern below is made up of two regular hexagons with a rectangle overlaying them.  
AB is parallel to GH.

Diagram is  
NOT to scale



Calculate the size,  $x$ , of angle CIJ.

Justify your answer with clear geometric reasoning.

Each angle of hexagon =  $180 \times (6 - 2)$

$$= 720 \div 6 \quad \text{Sum of } L's \text{ in polygon}$$

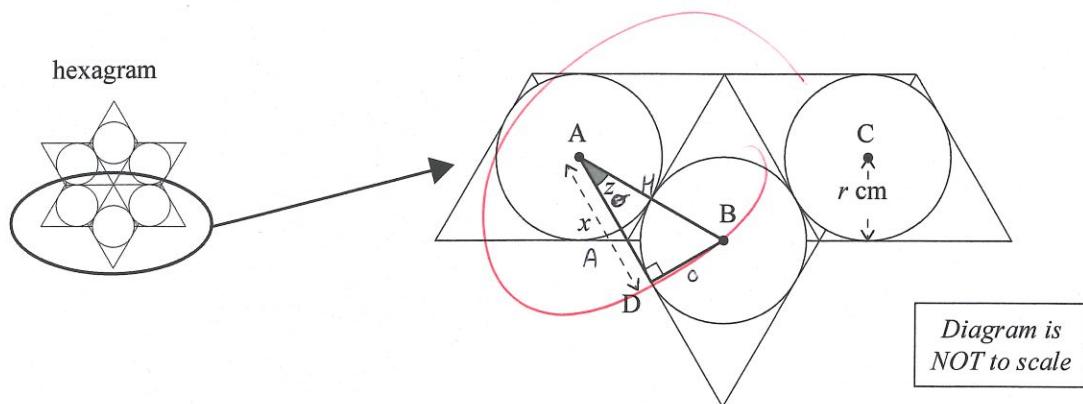
$= 120$

- (b) Circles can be drawn inside **half** a hexagram (which is a regular six-pointed star) as shown in the pattern below.

Points A, B, and C are the centres of the three circles.

The radius of all the circles is  $r$  cm.

A triangle ABD is drawn across two of the circles.



- (i) Prove that the angle  $z$  is equal to  $30^\circ$ .

Show your working clearly.

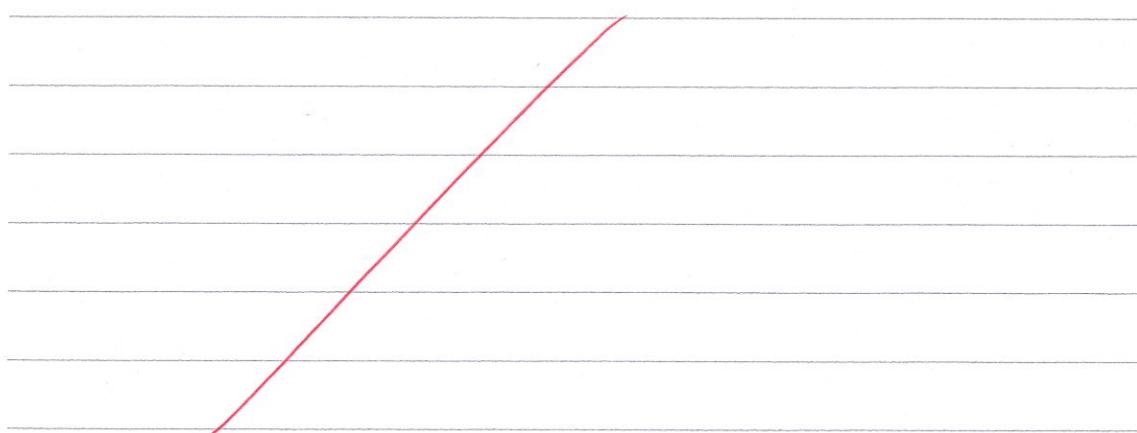
SOH CAH TOA

$$\tan z = \frac{r}{x}$$

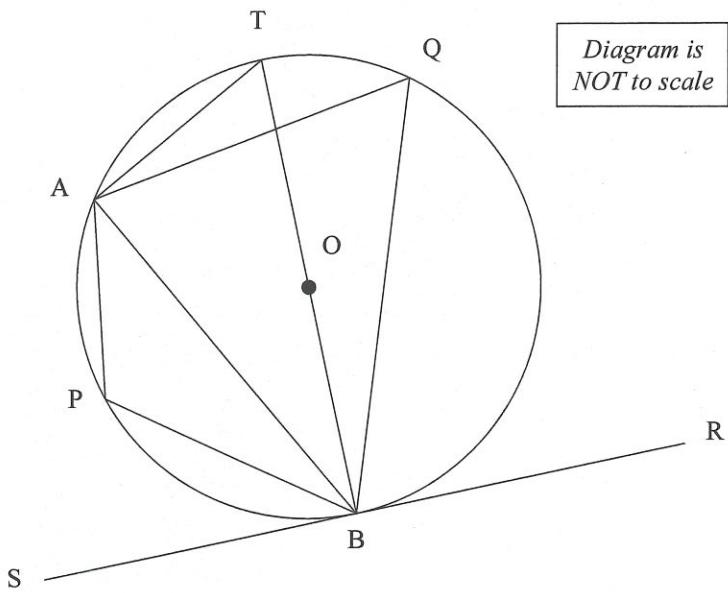
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- (ii) Calculate the length,  $x$ , of the line AD in terms of  $r$ .

Show your working clearly.



- (c) In the diagram below, the line SR is a tangent to the circle.  
The line BT passes through O, the centre of the circle.



Prove that angle ABS equals angle AQB.

*Justify your answer with clear geometric reasoning.*

||  $TBS = 90^\circ$  tgt  $\perp$  radius  
 $SBR = 180^\circ$   $\angle$ 's line add to  $180^\circ$  //

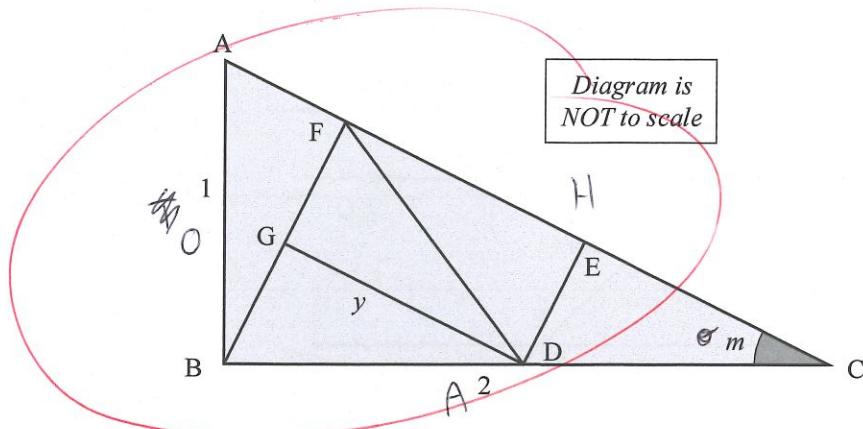
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A3

## QUESTION TWO

- (a) The shape below is an example of Pinwheel Tiling, where identical right-angled triangles are used to create a tessellation within a larger right-angled triangle.

AB is 1 unit long and BC is 2 units long.



- (i) Calculate the size,  $m$ , of angle ACB.

Show your working clearly.

SOH CAH TOA

$$\tan m^{-1} = \frac{1}{2}$$

$$= 26.6^\circ$$

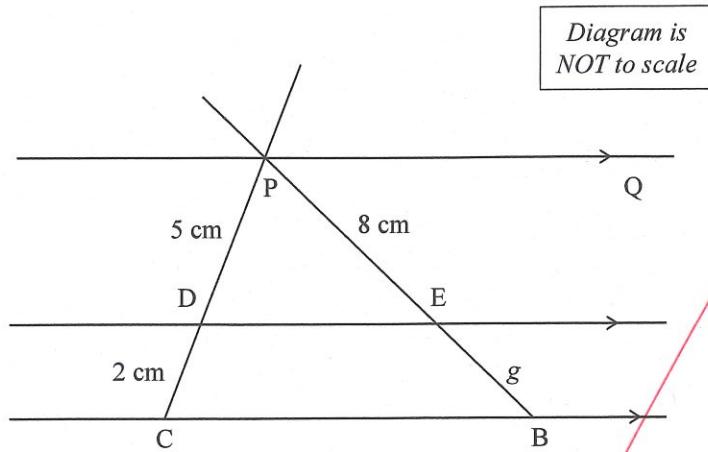
- (ii) Calculate the length,  $y$ , of the line GD.

Show your working clearly.

~~$$y = FD^2 + FG^2$$~~

~~$$y = \sqrt{FD^2 - FG^2}$$~~

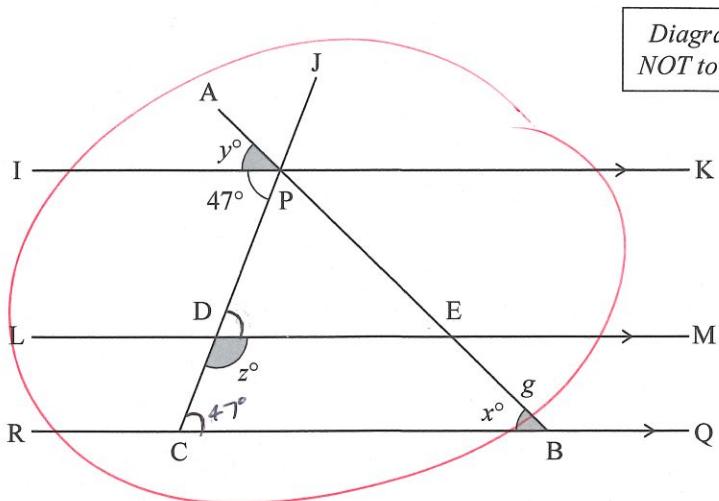
- (b) In the diagram below, PQ is parallel to the lines DE and CB.  
PE is 8 cm long.  
PD is 5 cm long.  
DC is 2 cm long.



- (i) Calculate the length,  $g$ , of the line segment BE.

*Show your working clearly.*

- (ii) Angle API is  $y^\circ$ , angle CDE is  $z^\circ$  and angle IPC is  $47^\circ$ .



Express angle  $x$  in terms of  $y$  and  $z$ .

Justify your answer with clear geometric reasoning.

$$\angle PDE = 47^\circ \quad \text{Alt } \angle's, \parallel \text{ lines}$$

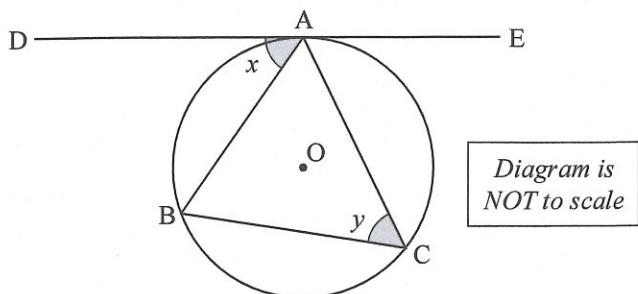
$$\angle DCB = 47^\circ \quad \text{Corresponding } \angle's, \parallel \text{ lines}$$

$$\angle x = y \quad \text{Corresponding } \angle's, \parallel \text{ lines} \quad //$$

- (c) A triangle ABC is drawn inside a circle.

O is the centre of the circle.

DE is a tangent to the circle. Point A is where DE touches the circle.



Prove that angle  $x$  equals angle  $y$ .

*Justify your answer with clear geometric reasoning.*

$\angle C = \angle y$        $\angle x = \angle$  equal to  $\angle$   
 tangent angle opposite segment

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**The examination continues on the following page.**

**QUESTION THREE**

*Diagram is  
NOT to scale*

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- (a) In the diagram alongside:

BC and BE are radii of the circle centre B.

GI is a tangent to the circle.

Angle CEG is  $38^\circ$

- (i) Calculate the size,  $x$ , of angle BCE.

*Justify your answer with clear geometric reasoning.*

$$\angle CBE = 38^\circ$$

~~tgt  $\angle \Delta$  equal  $\angle$  opposite segment~~

tgt  $\angle \Delta$  equal  $\angle$  opposite segment

$$\angle BCE = 180 - 38$$

$$= 142 \div 2$$

$$= 71^\circ$$

base  $\angle$ 's isos  $\Delta$

Two overlapping circles, with centres A and B and the same radii, are drawn in a quadrilateral which is symmetrical through HG.

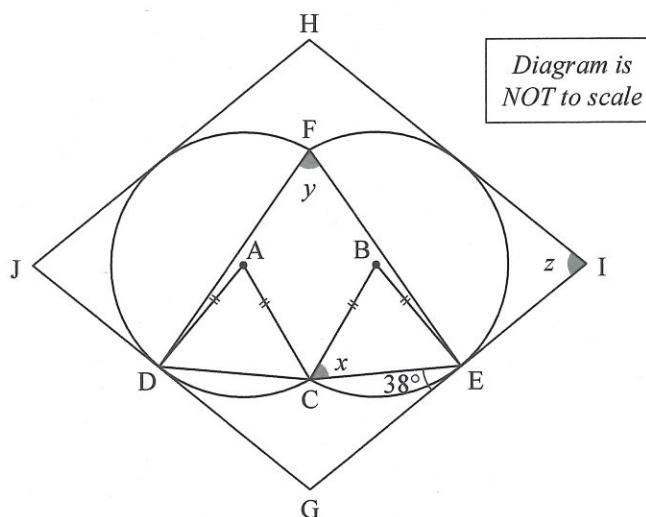
The distance between A and B is equal to the radius of the circles.

HIGJ is a rhombus.

HI, IG, GJ, and HJ are tangents to the circles.

Angle CEG is  $38^\circ$ .

AD is perpendicular to JG and BE is perpendicular to GI.



- (ii) Calculate the size,  $y$ , of angle DFE.

*Justify your answer with clear geometric reasoning.*

$$y = 38^\circ \quad \text{tgc } \angle \Delta \text{ equal to } \cancel{\text{opposite}} \\ \angle \text{ opposite segment}$$

n.

- (iii) Calculate the size,  $z$ , of angle HIG.

*Justify your answer with clear geometric reasoning.*

~~Sum of  $\angle$ 's in rhombus~~

$$= 180 \times (4 - 2)$$

$$= 360 \quad \text{Sum of } \angle \text{'s in a polygon}$$

$$z = 360 \div 4$$

$$= 90^\circ$$

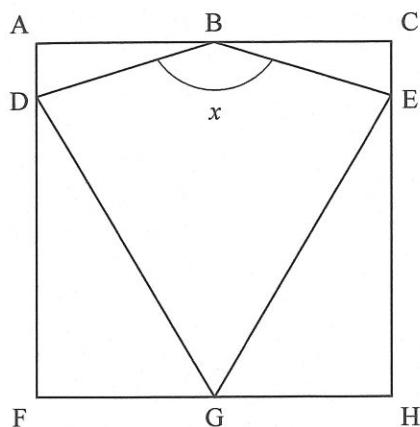
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n.

Question Three continues  
on the following page.

- (b) The kite GDBE is placed in the square ACHF.

$$\mathbf{DG} = \mathbf{GB} = \mathbf{EG}$$



*Diagram is  
NOT to scale*

Calculate the size,  $x$ , of angle DBE.

*Justify your answer with clear geometric reasoning.*

N2

Subject:		Mathematics and Statistics	Standard:	91031	Total score:	09
Q	Grade score	Annotation				
1	A3	<p>Three of the four parts of this question were attempted. This is A3 because in two of those parts, in attempting to solve the problem, one correct step with supporting reason was evident in each case.</p> <p>To reach M5 or better, the correct value of <math>x</math> in (a) would need to have been found or another correct step in the proof for (c) with supporting reason would have been needed.</p>				
2	A4	<p>Four of the five parts of this question were attempted. This is A4 because in three of those parts, in attempting to solve the problem, one correct step with supporting reason was evident in each case.</p> <p>To reach M5 or better, the actual value of <math>y</math> in (a)ii would need to have been found or an expression for <math>x</math> in terms of <math>y</math> and <math>z</math> found in (b)ii or other correct step in the proof for (c) with supporting reason would have been needed.</p>				
3	A3	<p>Three of the four parts of this question were also attempted. This is N2 because in one of those parts, (a)i, the first step was an incorrect application of the alternate segment theorem, but the second step was correctly applied with supporting reason.</p> <p>To reach M5, even though the value of <math>x</math> was incorrect, correct geometric reasoning could have been shown thereafter in (a)ii and (a)iii.</p>				