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91267



NEW ZEALAND QUALIFICATIONS AUTHORITY
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SUPERVISOR'S USE ONLY

Level 2 Mathematics and Statistics, 2015

91267 Apply probability methods in solving problems

2.00 p.m. Tuesday 10 November 2015

Credits: Four

Achievement	Achievement with Merit	Achievement with Excellence
Apply probability methods in solving problems.	Apply probability methods, using relational thinking, in solving problems.	Apply probability methods, using extended abstract thinking, in solving problems.

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

You should attempt ALL the questions in this booklet.

Make sure that you have Resource Sheet L2–MATHF.

Show ALL working.

If you need more space for any answer, use the page(s) provided at the back of this booklet and clearly number the question.

Check that this booklet has pages 2–16 in the correct order and that none of these pages is blank.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

Merit

TOTAL

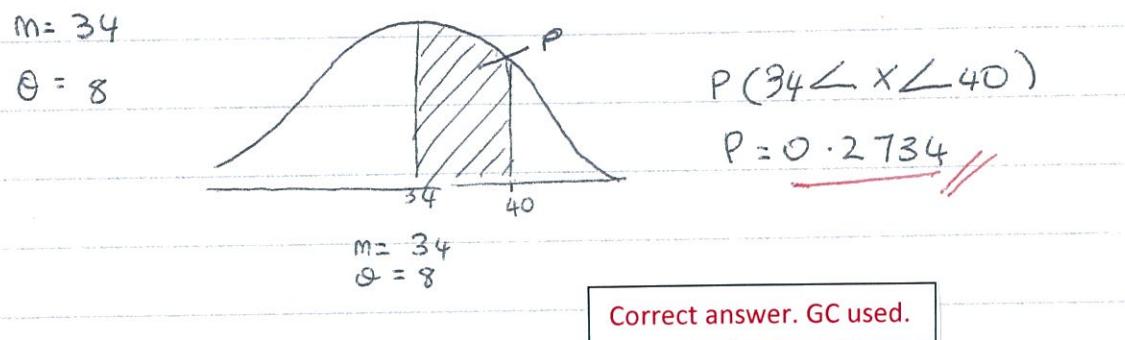
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ASSESSOR'S USE ONLY

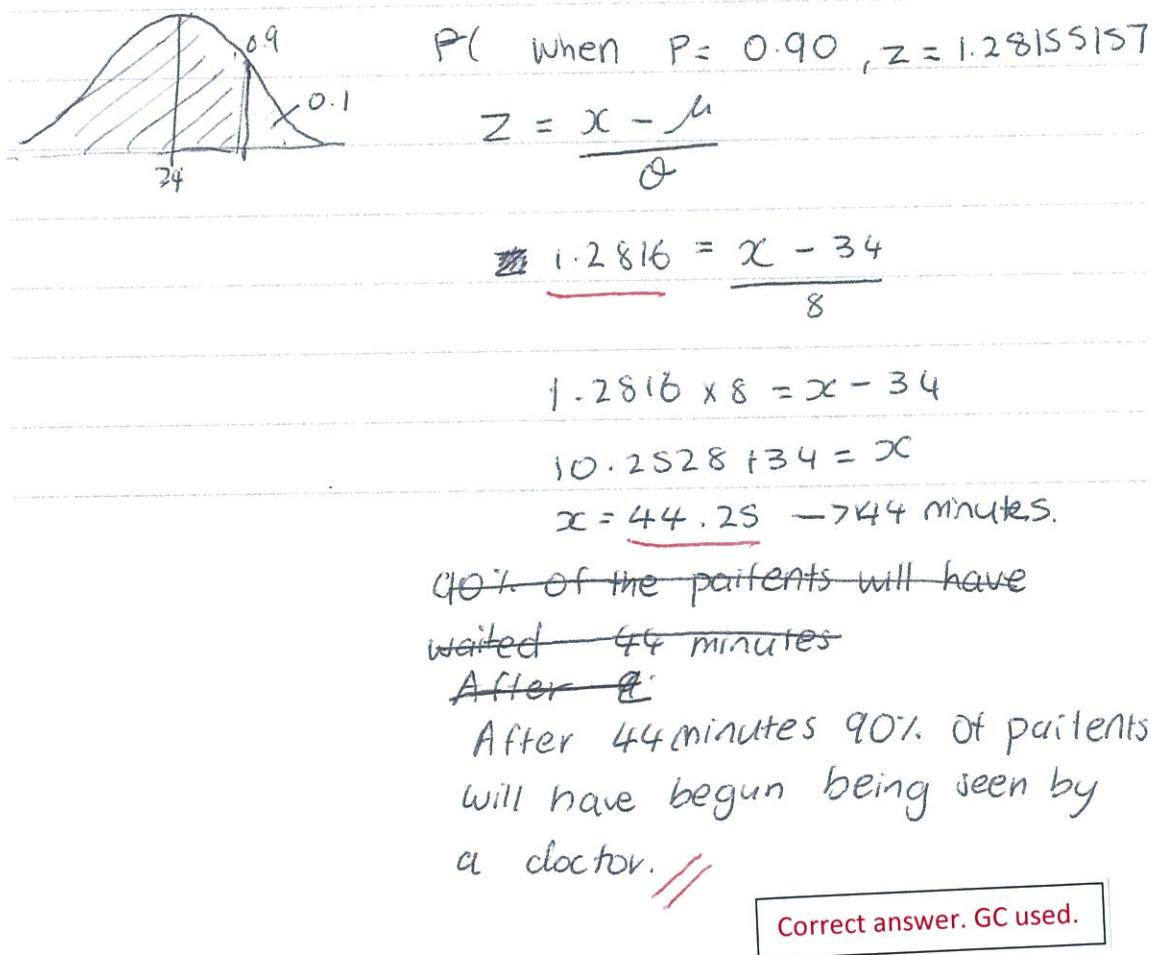
QUESTION ONE

- (a) The waiting time for a patient attending a medical centre before seeing a doctor is approximately normally distributed, with a mean of 34 minutes and a standard deviation of 8 minutes.

- (i) Find the probability that a patient will wait between 34 and 40 minutes.



- (ii) After how many minutes will 90% of patients have begun being seen by a doctor?

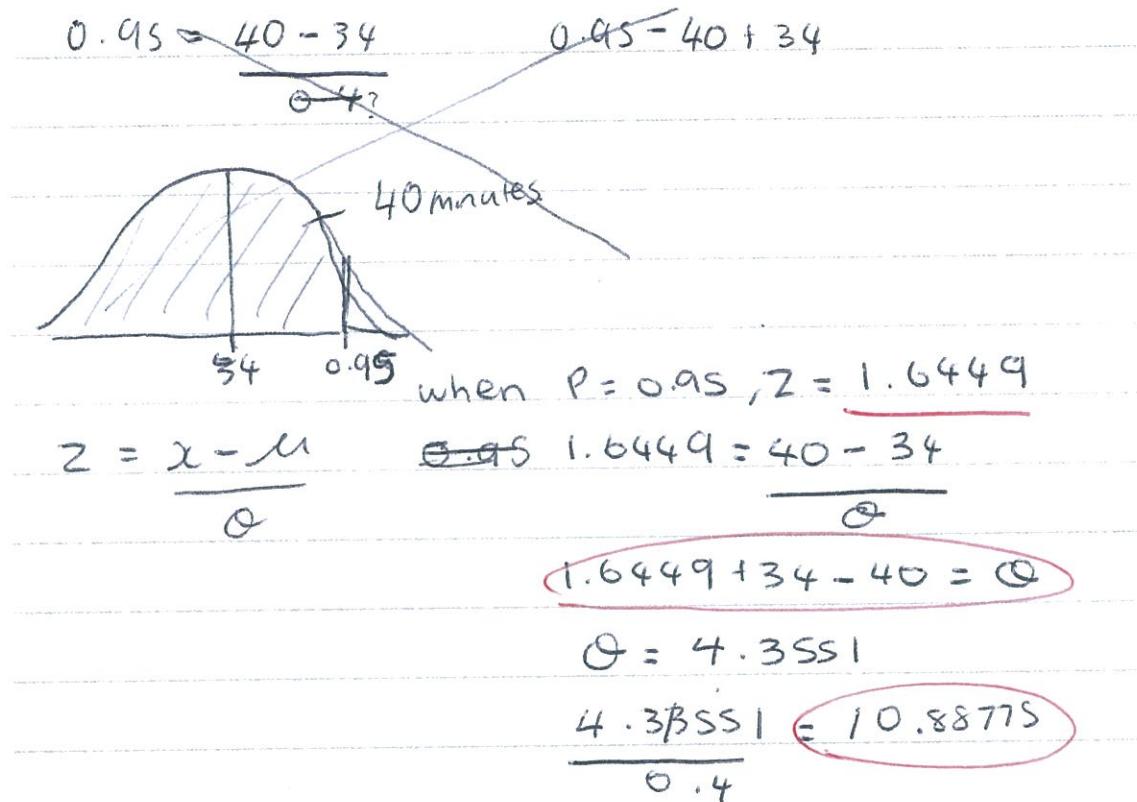


- (iii) It is decided that waiting times must be changed so that at least 95% of patients will be seen by a doctor within 40 minutes. $\sim \mu$ - 34

Because of the administration required, the mean time cannot change, but it is known that for each doctor added to the duty teams, the standard deviation will reduce by 0.4 minutes.

How many doctors must be added to meet the new requirement?

i) find



So therefore 10 new doctors must be added to meet the new requirements.

Correct z-value identified.

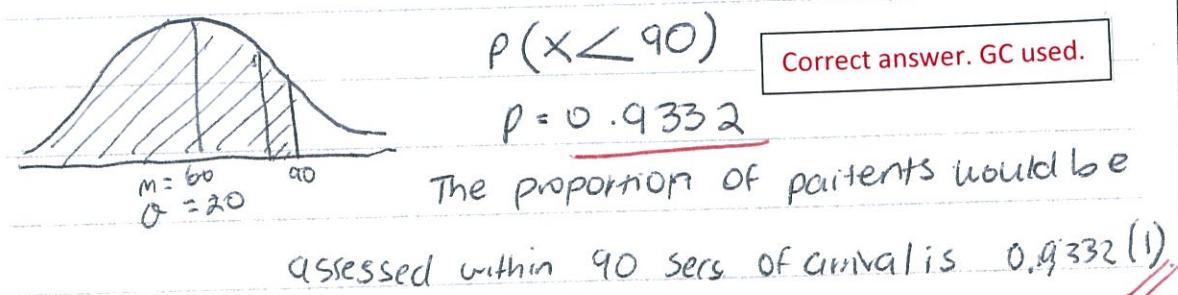
Incorrect standard deviation found

and incorrect working.

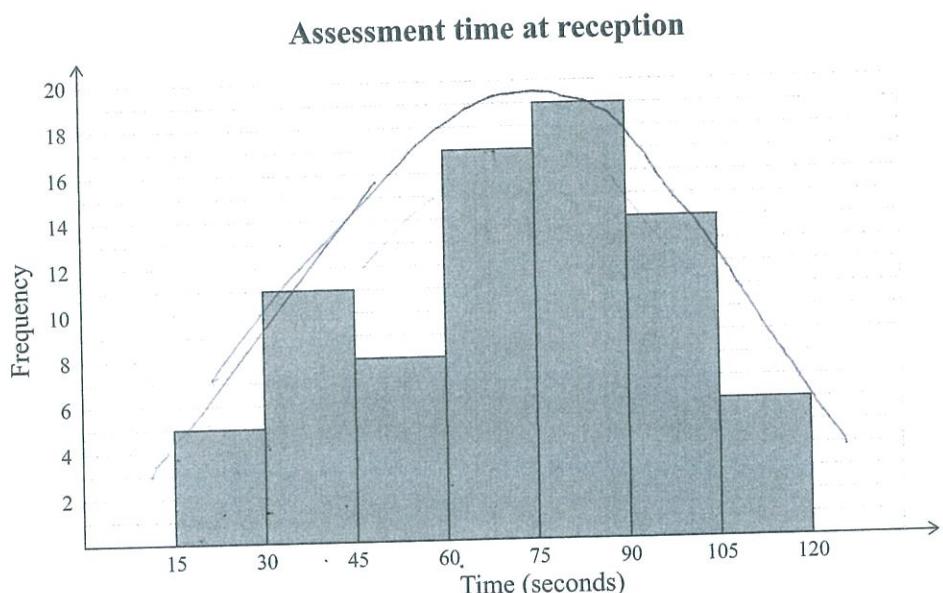
- (b) At reception, patients are assessed on the urgency of their condition. This is done within two minutes of arrival.

It is thought that the waiting time before an assessment is done is approximately normally distributed with a mean of 60 seconds and standard deviation of 20 seconds.

- (i) What proportion of patients would be assessed at reception within 90 seconds of arrival?



- (ii) A survey is carried out on 80 patients who arrive at reception. Patients are selected at random on a particular day. The results are shown in the frequency histogram below.

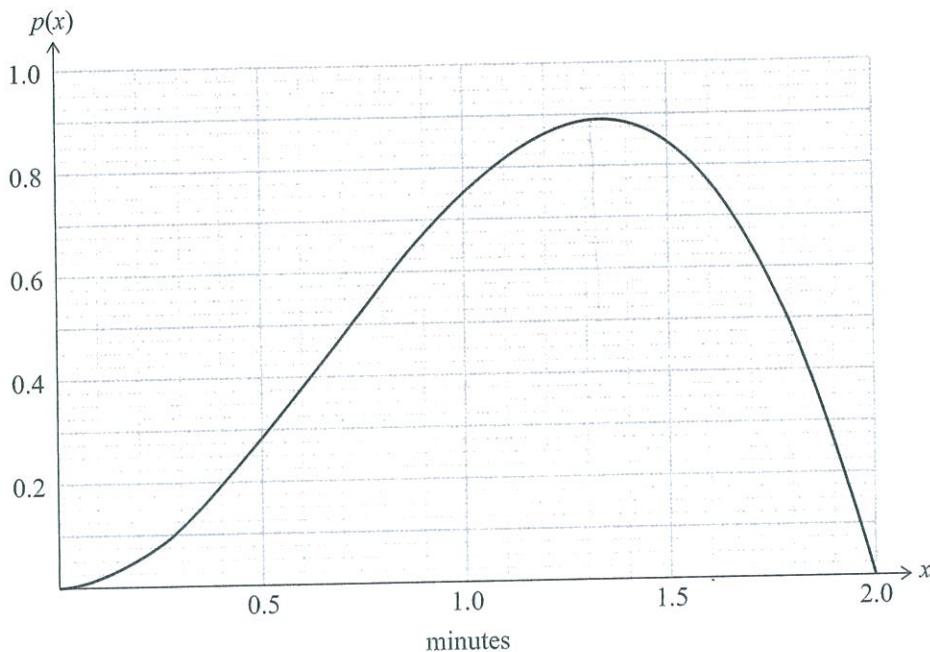


What proportion of patients in the survey were assessed at reception within 90 seconds of arrival?

$$5+11+8+17+19 = 60 / 80 = 0.75$$

Correct answer.

- (iii) A statistician states that the assessment times are not normally distributed, but are more likely to approximate the distribution $p(x)$ below.



The associated probabilities (with minutes converted to seconds) are given in the following table:

Assessment Time (seconds)	0 –	15 –	30 –	45 –	60 –	75 –	90 –	105 – 120
Probability	0.01	0.05	0.10	0.16	0.21	0.22	0.17	0.08

Compare the frequency histogram for the survey of 80 patients with the distribution curve $p(x)$.

You should comment on the comparative shape, centre, and spread of the two distributions.

It is important to give numerical values to support your statements where possible.

Step The shape of the histogram for the 80 patients is not normally distributed as it has a few peaks from 60 seconds – 90 seconds, compared to the distribution of the curve which is slightly skewed to the left but is ^{a little} quite symmetrical compared to the histogram. The centre of the histogram is lies between 60 and 90 seconds compared to the distribution of the curve which has a mean of 45 – 60 seconds. The average of the histogram is quite ~~at the~~ around 75 – 90 seconds ~~and~~ although which has

There is more space for your answer on the following page.

the same average for the distributed curve of 75-90 seconds. The spread of the histogram is quite evenly spread out with not many tails compared to the distributed curve which is skewed to the left with a tail. This shows that the stats from the distributed curve say that it's faster than the histogram as the probability of someone getting served at 0-15 seconds is 1 and the patients only start getting assessed after 15 seconds which show the distributed curve is faster.

Statement about shape but a lot of incorrect statements about other aspects.

u

M5

QUESTION TWO

A study is conducted of 1500 randomly selected candidates for an international examination to investigate whether Year 12 candidates were as successful as those from Year 13.

The results are summarised in the table below:

	Year 12	Year 13	Total
Passed	347	853	1200
Failed	33	267	300
Total	380	1120	1500

- (a) (i) What proportion of candidates in the study passed the examination?

$$\frac{1200}{1500} = 0.8$$

Correct answer.

- (ii) What proportion of candidates who failed the examination were from Year 12?

$$\frac{33}{300} = 0.11$$

Correct answer.

- (iii) There were about 52 500 candidates from Year 12 and Year 13 who attempted the examination.

Using the results of this study, how many candidates would be expected to be from Year 13, and pass the examination?

$$\frac{853}{1120} = 0.7616$$

$$\text{expected number} = 0.7616 \times 52500$$

$$= 39984.375$$

$= 39984$ students from year 13 would pass the examination out of 52 500 candidates from yr 12 and yr 13 who attempted the examination.

Incorrect answer.

1120 used instead of 1500 – a common error.

- (iv) It is claimed that Year 13 candidates are four times more likely to fail the examination than Year 12 candidates.

State whether or not you agree with this claim, showing full calculations to support your view.

$$\text{Year 13} = \left(\frac{110}{4} / 110 \right) \text{ mei} \\ = 0.2419 \text{ (4.dp)}$$

$$\text{Year 12} = \left(\frac{33}{380} \right) \\ = 0.0868 \text{ (4.dp)}$$

$$\text{relative risk} = \frac{0.2419}{0.0868} = 2.786866359 \\ = 3 \text{ times more likely (rounded)}$$

The probability of Yr 13 candidates failing the examination is 0.2419 compared to the probability of the Yr 12 candidates failing the examination which is 0.0868. I do not agree with this claim as relative risk showed that Yr 13 candidates were 3 times more likely to fail their examination compared to Yr 12 candidates and the claim stated 4 times which is far off from three, therefore the claim should rather say Yr 13 candidates are 3 times more likely to fail their examination compared to Yr 12 candidates. //

Correct absolute risks and relative risk found.

Correct supporting statement.

- (b) The same study also considered the number of subjects the candidates were taking in their normal academic courses. It found that of the same sample of 1500 candidates, 682 were taking six subjects, while the rest were taking five subjects. Of the candidates who were taking five subjects, 192 failed the examination.

The table from page 7 is repeated here to help you answer the questions that follow.

	Year 12	Year 13	Total
Passed	347	853	1200
Failed	33	267	300
Total	380	1120	1500

	5 Subjects	Yes 6 Subjects	
Passed	626	574	1200
Failed	192	108	300
Total	818	682	1500

- (i) What proportion of candidates in the study took six subjects and passed?

$$\frac{574}{1500} = 0.383$$

Correct answer.

- (ii) On the evidence of this study, would you recommend that candidates take six subjects?

Support your answer with numerical calculations that consider the absolute and relative risks. You may also wish to comment on the sensibility of drawing any conclusions on this evidence.

\ accurate

$6 \text{ subjects failed} = 108 / 682$	$6 / p = 574 / 682$
$= 0.159$	$= 0.842$
$5 \text{ Subjects failed} = 198 / 818$	$S / P = 626 / 818$
$= 0.242$	$= 0.765$

$$\cancel{RR = \frac{0.159}{0.242}} = \cancel{0.607}$$

Correct absolute risks. Has not found the relative risk to acceptable accuracy.

$$RR = \frac{0.159}{0.242} = 0.607$$

$$RR = \frac{0.842}{0.765} = 1.1$$

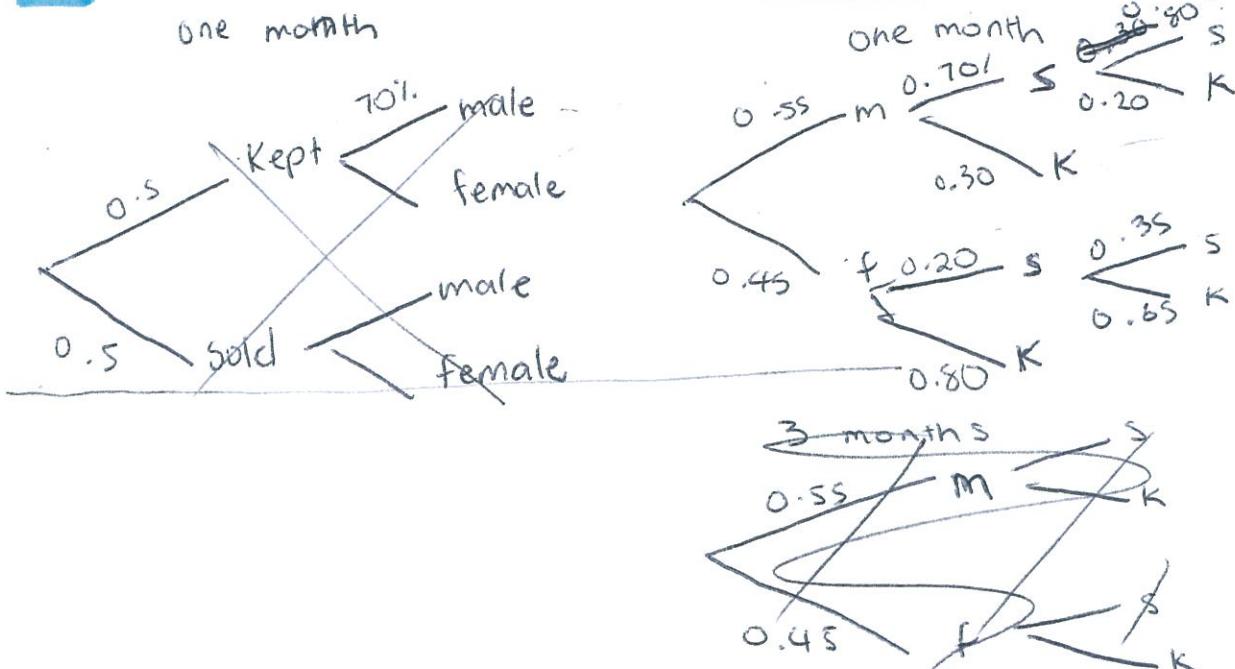
The probability that a candidate took 6 subjects and passed is 0.842 compared to the probability of a candidate that took 5 subjects and passed is 0.765, this shows that it is one times more likely of candidates taking 6 subjects and passing compared than taking 5 subjects and passing, although the probability of a candidate taking 6 subjects and failing is 0.159 compared to a candidate taking 5 subjects and failing, this shows that $\frac{14}{13}$ times more likely if you will fail if you take 6 subjects instead of 5. In conclusion I believe that the numbers from this study is not very accurate as you may need to take a sample from a wider range of candidates.

E7

QUESTION THREE

- (a) When calves are born into a pedigree beef herd, decisions are made after they are one month old, and again when they are three months old, as to whether they will be kept in the herd or sold.

55% of calves born are male. At age one month, 70% of male and 20% of female calves are sold. Of the remainder, at age three months, 80% of males and 35% of females are also sold.



- (i) Find the probability that a randomly chosen calf born into the herd will be male and sold at age one month.

$$(0.55 \times 0.70) = 0.39$$

Correct answer.

- (ii) Find the probability that a randomly chosen calf born into the herd will be female and sold at age three months.

$$(0.45 \times 0.20 \times 0.35) = 0.13$$

Correct answer.

- (iii) What percentage of calves will eventually be kept in the pedigree herd?

$$m (0.55 \times 0.30 \times 0.20) = 0.033$$

$$f(0.45 \times 0.80 \times 0.65) = 0.234$$

$$0.033 + 0.234 = 0.267 \times 100$$

$$= 26.7 \rightarrow 27\%$$

Correct answer.

- (iv) In a particular year 550 calves were born.

How many male calves can be expected to be kept in the pedigree herd?

$$m (0.55 \times 0.30 \times 0.20)$$

$$= 0.033$$

$$\text{expected number} = 550 \times 0.033$$

≈ 18 calves are expected to be
kept //

Correct answer. u

- (v) The ratio of male to female calves being kept in the herd after three months is about one male to every seven females. This is to be changed to one male to every ten females.

If the number of male calves remains the same, what proportion of females would have been sold?

One male to every seven females

$$= m (\cancel{3}) (3 \text{ calves})$$

$$= f (23 : \text{calves})$$

3 males to 30 females kept in the herd ~~each~~ after three months.

$$p(0.45 \times 0.80 \times 0.35)$$

$$P = 0.18$$

So the proportion of female calves that would have been sold is $0.18 = 0.2$. //

Incorrect answer. n

- (b) New Zealand fantails are birds which are either pied or black.



Pied fantail

Black fantail

Cherry Mariner, www.nzbirdsonline.org.nz/species/new-zealand-fantail

They interbreed, and pairs with successful nests are found in the following proportions:

Pair	Two pied fantails	One pied and one black fantail	Two black fantails
Proportion	0.75	0.2	0.05

Successful nests have between one and four eggs. The proportions of eggs are given in the table below.

Pair	Two pied fantails	One pied and one black fantail	Two black fantails
One egg	0.15	0.2	0.3
Two eggs	0.3	0.35	0.5
Three eggs	0.4	0.35	0.15
Four eggs	0.15	0.1	0.05
	0.100	0.100	0.53

- (i) What proportion of pairs with two pied fantails will have a successful nest with more than one egg?

$$\cancel{3/4} - \cancel{0.42} = 0.85/100$$

$$0.85/100$$

Incorrect answer.

- (ii) A researcher claims that only one out of every 50 nests found with three eggs is likely to be from a pair of two black fantails.

Use calculations to show that the researcher's claim is justified.

$$\frac{0.15}{64} =$$

$$0.4 / 0.15 = 2.66 = 3 \text{ nests.}$$

Therefore 3 in every 50 nests are found with eggs from a pair of two black fantails. This means that the researchers claim is incorrect as the stated one in every 50 nests are found with 3 eggs likely to be from a pair of two black fantails; therefore it should say ~~is only~~ ^{out of} 3 ~~of~~ 50 every 50 nests found have three eggs which are likely to be from a pair of two black fantails.

Incorrect answer.

M5

2

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Merit

TOTAL

18

ASSESSOR'S USE ONLY

QUESTION ONE

- (a) The waiting time for a patient attending a medical centre before seeing a doctor is approximately normally distributed, with a mean of 34 minutes and a standard deviation of 8 minutes.

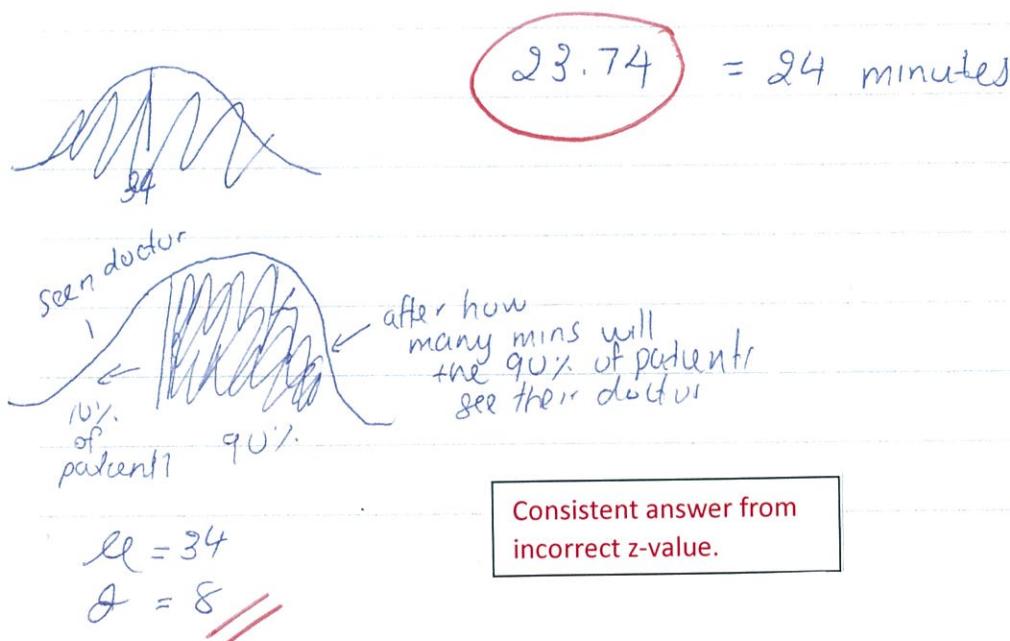
- (i) Find the probability that a patient will wait between 34 and 40 minutes.

0.2733 //

Correct answer. GC used.

u

- (ii) After how many minutes will 90% of patients have begun being seen by a doctor?



u

- (iii) It is decided that waiting times must be changed so that at least 95% of patients will be seen by a doctor within 40 minutes.

Because of the administration required, the mean time cannot change, but it is known that for each doctor added to the duty teams, the standard deviation will reduce by 0.4 minutes.

$$\mu = 34 \quad \sigma = 4 \quad \sigma = 0.4$$

How many doctors must be added to meet the new requirement?

~~22.66~~

0.2266

$$0.2266 \times 100 = 22.66 \approx$$

= 23 doctors that must be added to meet the medical centre's new requirements

Incorrect answer.

n

- (b) At reception, patients are assessed on the urgency of their condition. This is done within two minutes of arrival.

It is thought that the waiting time before an assessment is done is approximately normally distributed with a mean of 60 seconds and standard deviation of 20 seconds.

- (i) What proportion of patients would be assessed at reception within 90 seconds of arrival?

$$0.9318$$

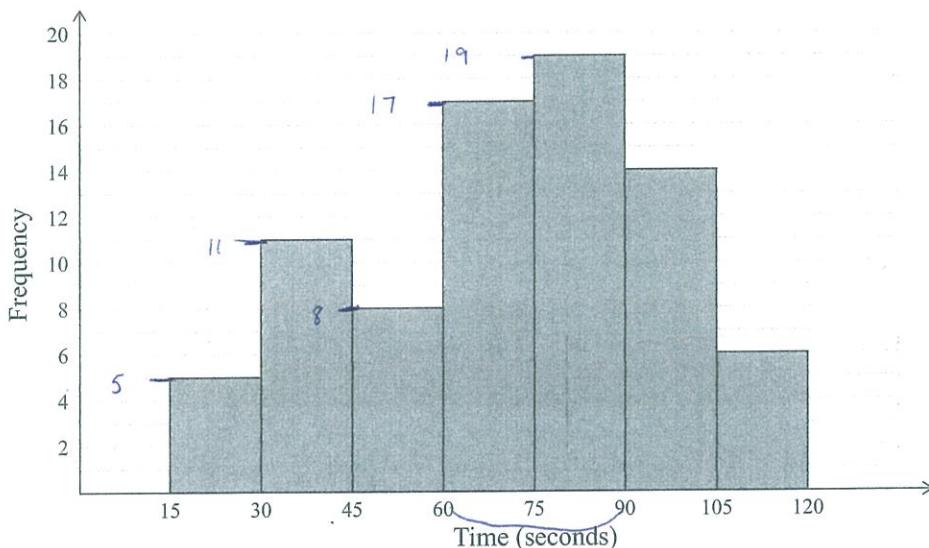
$$0.9318 - 0.5$$

$$= 0.4318 //$$

Incorrect subtraction
from 0.5

- (ii) A survey is carried out on 80 patients who arrive at reception. Patients are selected at random on a particular day. The results are shown in the frequency histogram below.

Assessment time at reception

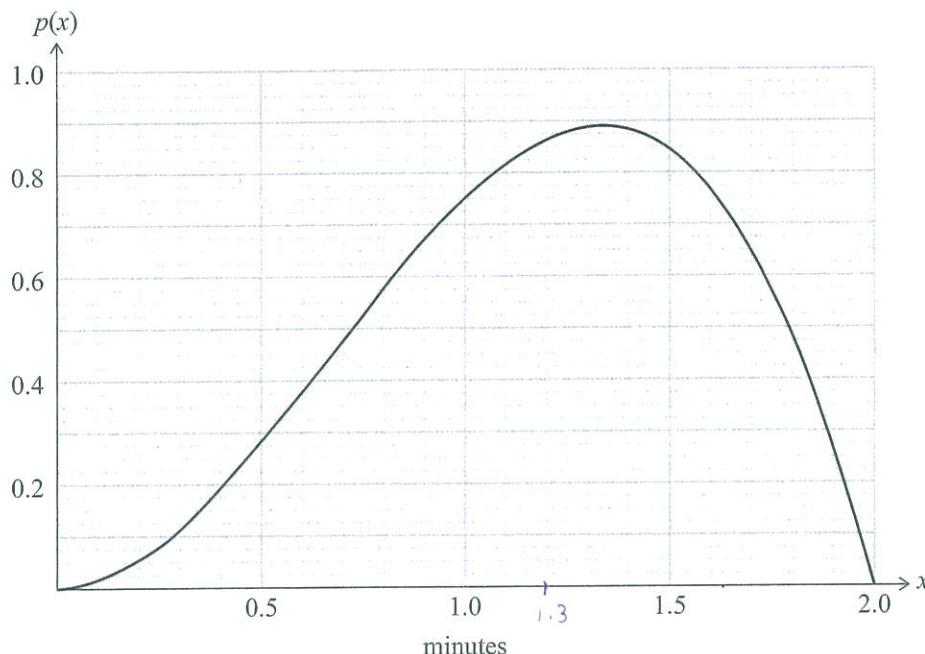


What proportion of patients in the survey were assessed at reception within 90 seconds of arrival?

$$5 + 11 + 17 + 19 = 52/80 = 13/20 //$$

Incorrect answer.

- (iii) A statistician states that the assessment times are not normally distributed, but are more likely to approximate the distribution $p(x)$ below.



The associated probabilities (with minutes converted to seconds) are given in the following table:

Assessment Time (seconds)	0 –	15 –	30 –	45 –	60 –	75 –	90 –	105 – 120
Probability	0.01	0.05	0.10	0.16	0.21	0.22	0.17	0.08

Compare the frequency histogram for the survey of 80 patients with the distribution curve $p(x)$.

You should comment on the comparative shape, centre, and spread of the two distributions.

It is important to give numerical values to support your statements where possible.

The frequency ~~but~~ of the histogram

The frequency histogram is not normally distributed & is unsymmetrical, with the highest values ^{& a sudden drop of 45-60 seconds} lying between 60 & 90 seconds¹. The distribution curve, however, is slightly more symmetrical, the highest values lying between 1.3 & 1.5 minutes.

The units on each graph is ^{the x-axis of} different, the histogram having

There is more space for your answer on the following page.

seconds & the distribution graphs being in minutes.

The centre of the frequency histogram has a value of 60 - 75 seconds & has a much lower frequency than which is between the highest values in the graph, the centre histogram bar has a smaller frequency of 17, compared to 19 making it look unsymmetrical as it is next to the highest bar in the graph, with frequency of 19. The centre of the distribution curve looks like it's shifted to the right.

The spread of the distribution curve is more spread towards the left, making the curve look uneven. The higher frequencies of the histogram are more shifted to the right, same with the distribution curve showing a trend, the higher the frequency & the longer the both have high values shifted to the right & then it drops down. The histogram shows us its sample size of 80 people whereas the distribution curve doesn't tell us the sample size //

Two correct comparative statements about two aspects, including numerical evidence.

MS

QUESTION TWO

A study is conducted of 1500 randomly selected candidates for an international examination to investigate whether Year 12 candidates were as successful as those from Year 13.

The results are summarised in the table below:

	Year 12	Year 13	Total
Passed	347	853	1200
Failed	33	267	300
Total	380	1120	1500

- (a) (i) What proportion of candidates in the study passed the examination?

$$\frac{1200}{1500} = \frac{4}{5}$$

Correct answer.

- (ii) What proportion of candidates who failed the examination were from Year 12?

$$\frac{33}{300} = \frac{11}{100}$$

Correct answer.

- (iii) There were about 52 500 candidates from Year 12 and Year 13 who attempted the examination.

Using the results of this study, how many candidates would be expected to be from Year 13, and pass the examination?

$$P(\text{year 13 \& failed}) = \frac{267}{1120} = 0.23$$

$$P(\text{year 13 \& passed}) = \frac{853}{1120} = 0.76$$

$$P(\text{year 12 \& passed}) = \frac{347}{380} = 0.91$$

52,500 people from year 12 & 13 attempted the paper.

$$0.76 \times 52,500 = 39,900$$

$$52,500 \text{ candidates did the paper}$$

$$\text{proportion yr 13 did exam \& passed} = 0.76 \times 52,500$$

$$= 39,900$$

Incorrect answer.

1120 used instead of 1500 – a common error.

- (iv) It is claimed that Year 13 candidates are four times more likely to fail the examination than Year 12 candidates.

State whether or not you agree with this claim, showing full calculations to support your view.

$$\begin{aligned} P(\text{year } 13 \text{ & failed exam}) &= \frac{267}{1120} = \underline{\underline{0.23}} \\ P(\text{year } 12 \text{ & failed exam}) &= \frac{33}{380} = \underline{\underline{0.08}} \end{aligned}$$

$$\frac{0.23}{0.08} = \underline{\underline{2.88}} = 3$$

Yr 13 Candidates are 4 times more likely to fail exam than year 12.

I do not agree with the claim that year 13 candidates are 4 times more likely to fail than year 12 candidates. However as shown by the relative risk, ~~the~~ Year 13 candidates are three times more likely to fail than year 12 candidates which is close to 4.

^{Yr 13}
Therefore though the students aren't 4 times times likely to fail, they have a higher chance of failing compared to the year 12 students, who have a much lower probability ~~of~~ (0.08) of failing.

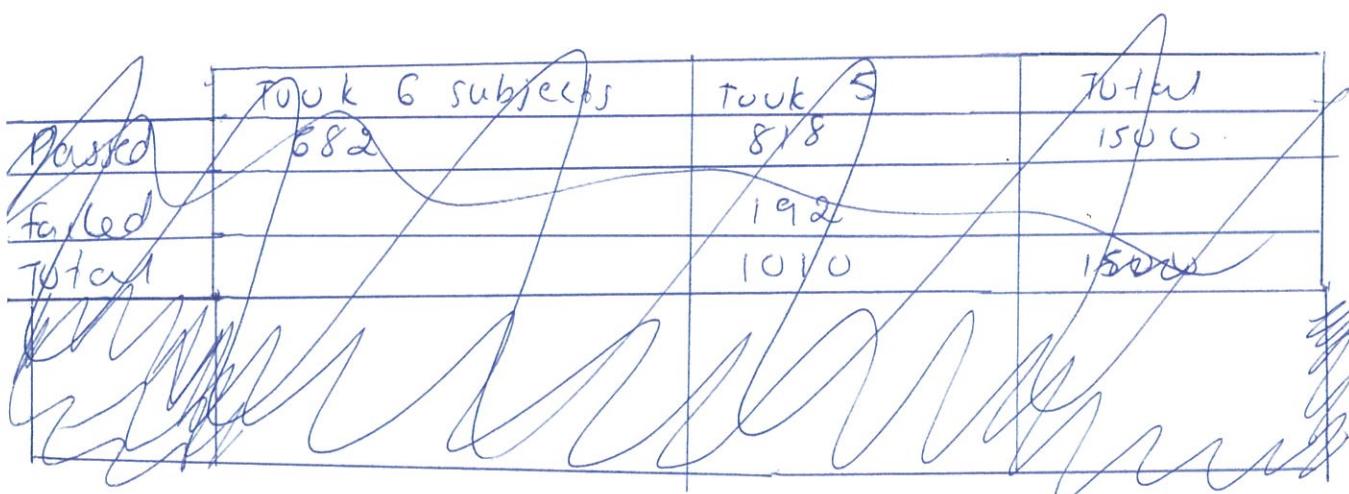
Correct absolute risks and relative risk found.

Correct supporting statement.

- (b) The same study also considered the number of subjects the candidates were taking in their normal academic courses. It found that of the same sample of 1500 candidates, 682 were taking six subjects, while the rest were taking five subjects. Of the candidates who were taking five subjects, 192 failed the examination.

The table from page 7 is repeated here to help you answer the questions that follow.

	Year 12	Year 13	Total
Passed	347	853	1200
Failed	33	267	300
Total	380	1120	1500



- (i) What proportion of candidates in the study took six subjects and passed?

$$\frac{574}{682}$$

$$= \frac{287}{341}$$

Incorrect answer.

682 used instead of 1500 – a common error.

n

	Took 6	Took 5	Total
Passed	574	626	1200
Failed	108	192	300
Total	682	818	1500

- (ii) On the evidence of this study, would you recommend that candidates take six subjects? Support your answer with numerical calculations that consider the absolute and relative risks. You may also wish to comment on the sensibility of drawing any conclusions on this evidence.

$$P(\text{take 6 subjects} \& \text{pass}) = \underline{\underline{0.84}}$$

$$P(\text{take 5 subjects} \& \text{pass}) = \underline{\underline{0.76}}$$

$$\text{relative risk}^1 = \frac{0.84}{0.76} = \underline{\underline{1.10}}$$

$$\text{relative risk (5 subjects)} = \frac{0.76}{0.84} = \underline{\underline{0.90}}$$

$$P(6 \text{ sub} \& \text{fail}) = 108/682 = 0.15 \quad \frac{0.15}{0.23} = 0.65$$

$$P(5 \text{ sub} \& \text{fail}) = 192/818 = 0.23 \quad \frac{0.23}{0.15} = 1.53$$

Based on my calculations, I ~~recommend~~ recommend that ~~candidates~~ candidates take 6 subjects rather than 5 subjects. As you can see from my calculations, if you take 6 subjects, ~~you are more~~ you have a higher chance of ~~passing~~ (1.10%) of passing & you have a much lower chance (0.65%) of failing compared to taking 5 subjects who have a ^{lower} (0.90%) chance of passing and have a much higher chance of (1.53%) of failing.

Correct absolute risks and relative risk found.

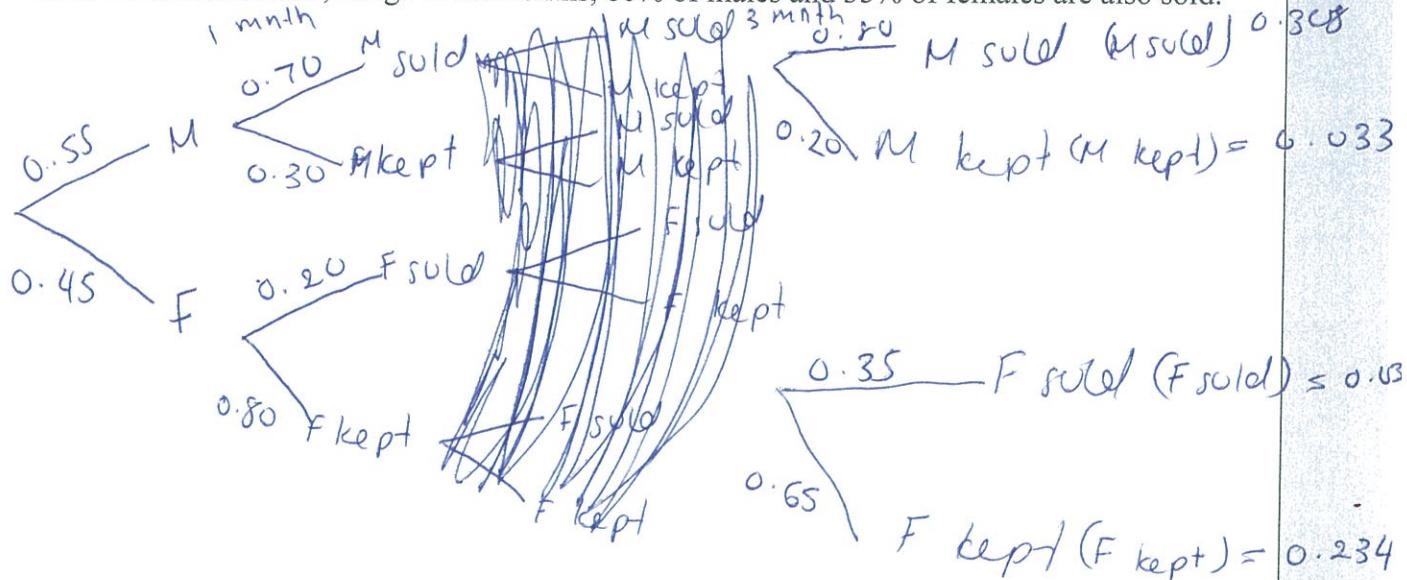
Correct supporting statement.

E8

QUESTION THREE

- (a) When calves are born into a pedigree beef herd, decisions are made after they are one month old, and again when they are three months old, as to whether they will be kept in the herd or sold.

55% of calves born are male. At age one month, 70% of male and 20% of female calves are sold. Of the remainder, at age three months, 80% of males and 35% of females are also sold.



- (i) Find the probability that a randomly chosen calf born into the herd will be male and sold at age one month.

$$\begin{aligned} P(\text{Male \& sold at 1 month}) &= 0.55 \times 0.70 \\ &= 0.385 \end{aligned}$$

Correct answer.

u

- (ii) Find the probability that a randomly chosen calf born into the herd will be female and sold at age three months.

$$\begin{aligned} P(\text{Female \& sold at 3 months}) &= 0.45 \times 0.35 \\ &= 0.1575 \end{aligned}$$

Incorrect answer.

n

- (iii) What percentage of calves will eventually be kept in the pedigree herd?

$$(M \text{ kept}) = 0.033 = 3.3\%$$

$$(F \text{ kept}) = 0.234 = 23.4\%$$

$$0.033 \times 0.234 \approx 0.0077 = 26.7\%$$

$$= 27\%$$

Correct answer.

r

- (iv) In a particular year 550 calves were born.

How many male calves can be expected to be kept in the pedigree herd?

$$(M_{\text{kept}}) = 0.033$$

$$0.033 \times 550 = 18.15 = 19 \text{ calves}$$

u
Correct answer.

- (v) The ratio of male to female calves being kept in the herd after three months is about one male to every seven females. This is to be changed to one male to every ten females.

If the number of male calves remains the same, what proportion of females would have been sold?

n
No response.

- (b) New Zealand fantails are birds which are either pied or black.



Pied fantail

Black fantail

Cherryl Mariner, www.nzbirdsonline.org.nz/species/new-zealand-fantail

They interbreed, and pairs with successful nests are found in the following proportions:

Pair	Two pied fantails	One pied and one black fantail	Two black fantails
Proportion	0.75	0.2	0.05

Successful nests have between one and four eggs. The proportions of eggs are given in the table below.

Pair	Two pied fantails	One pied and one black fantail	Two black fantails
One egg	0.15	0.2	0.3
Two eggs	0.3	0.35	0.5
Three eggs	0.4	0.35	0.15
Four eggs	0.15	0.1	0.05

- (i) What proportion of pairs with two pied fantails will have a successful nest with more than one egg?

$$0.75 + 0.3 + 0.4 + 0.15 = \textcircled{1.6}$$

Incorrect answer.

- (ii) A researcher claims that only one out of every 50 nests found with three eggs is likely to be from a pair of two black fantails.

Use calculations to show that the researcher's claim is justified.

$$1/50 = 0.02$$

Proportion of 2 black fantails = 0.05
 3 eggs from 2 black fantails = 0.15

$$\frac{0.05}{0.15} = 0.33$$

Incorrect answer.

The researchers claim is not justified as the relative risk that I got based on my calculations is 0.33 which is not even close to 1/50 which is 0.02. There is a difference of 0.31.

MS