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SUPERVISOR'S USE ONLY

**91028**

910280

NEW ZEALAND QUALIFICATIONS AUTHORITY
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# Level 1 Mathematics and Statistics, 2017

## 91028 Investigate relationships between tables, equations and graphs

9.30 a.m. Monday 20 November 2017
Credits: Four

| Achievement | Achievement with Merit | Achievement with Excellence |
|---|---|---|
| Investigate relationships between tables, equations and graphs. | Investigate relationships between tables, equations and graphs, using relational thinking. | Investigate relationships between tables, equations and graphs, using extended abstract thinking. |

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

**You should attempt ALL the questions in this booklet.**

Show ALL working.

Grids are provided on some pages. This is working space for the drawing of a graph or a diagram, constructing a table, writing an equation, or writing your answer.

If you need more space for any answer, use the page(s) provided at the back of this booklet and clearly number the question.

Check that this booklet has pages 2–15 in the correct order and that none of these pages is blank.

**YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.**

Excellence

**TOTAL** 22

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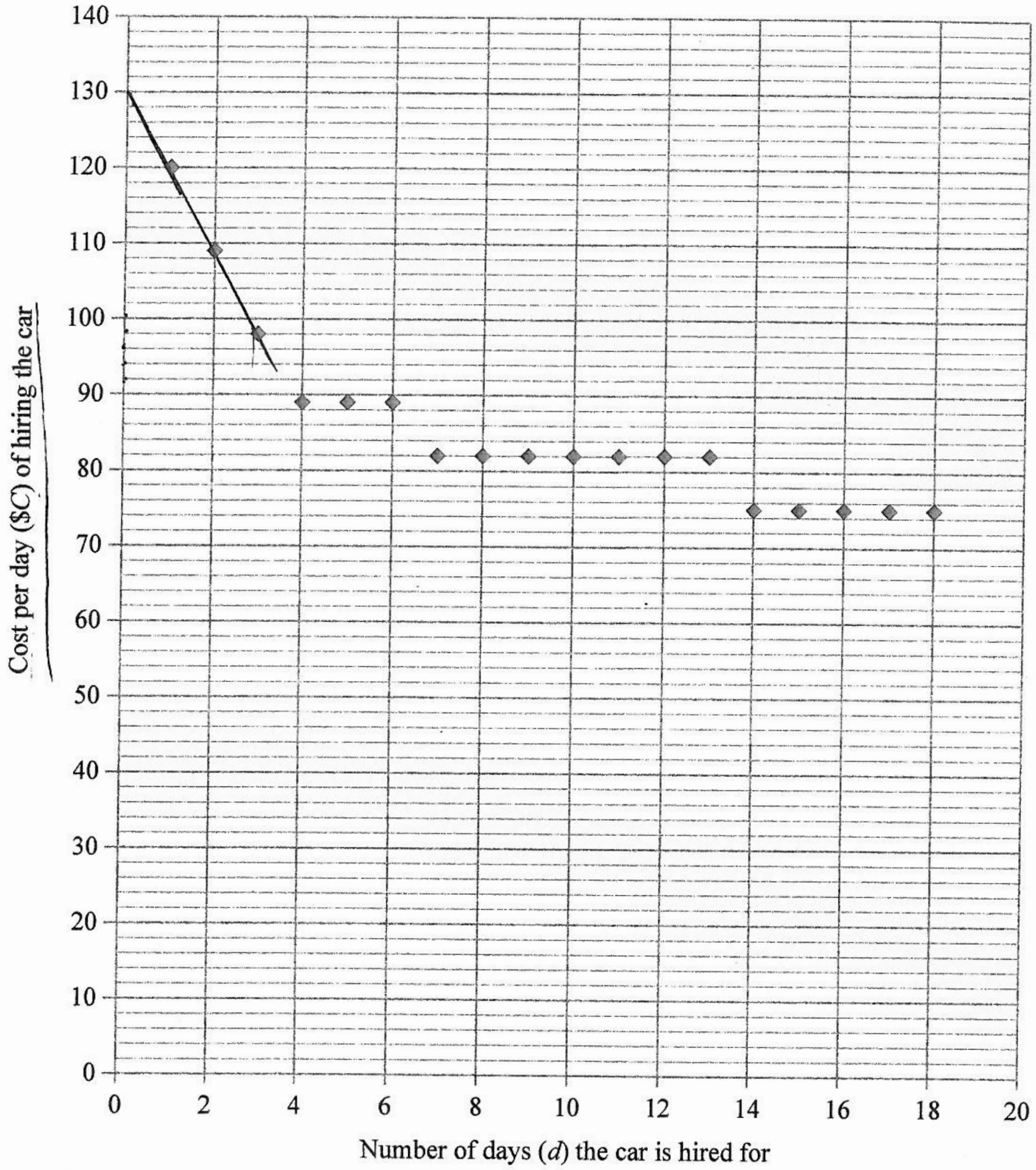
## QUESTION ONE

(a) *Rent A Car* is a car rental company. The graph below shows the **cost per day ($C)** of hiring one of their standard-sized cars, as the **number of days the car is hired for (*d*)** increases.



(i) How much cheaper per day is it to hire the car for 3 days than 1 day?

3days = \$98 per day, 1 day = \$120

$120 - 98 =$ \$22 cheaper

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(ii) Give the equation for the **cost per day** of hiring the car:

(1) for 4 to 6 days

$\$c = 89$

u

(2) for the first 3 days.

~~$y = ax + y$. y intercept $= 130$.~~

~~(d, \$c)~~

~~$y = ax + 130$, Substitute one point $(1, 120)$~~

~~$120 = a + 130$, $a = -10$.~~

~~$\therefore y = -10x + 130$.~~

(d, \$c)

Points $(1, 120)$, $(2, 109)$, $(3, 98)$.

gradient of $-11$.

$y = -11x + y$.

Substitute one value

$109 = -22 + y$. $y = 131$

$\therefore y = -11x + 131$

$\downarrow$

$\$c = -11d + 131$

r

(b) *Rent A Car* decides to introduce a special deal, and produces a sign as shown on the right.

Mere is trying to find the cheapest option for renting a car. She asks what this 'SPECIAL DEAL' actually means.

The company gives Mere the formula they use to work out the daily rate.

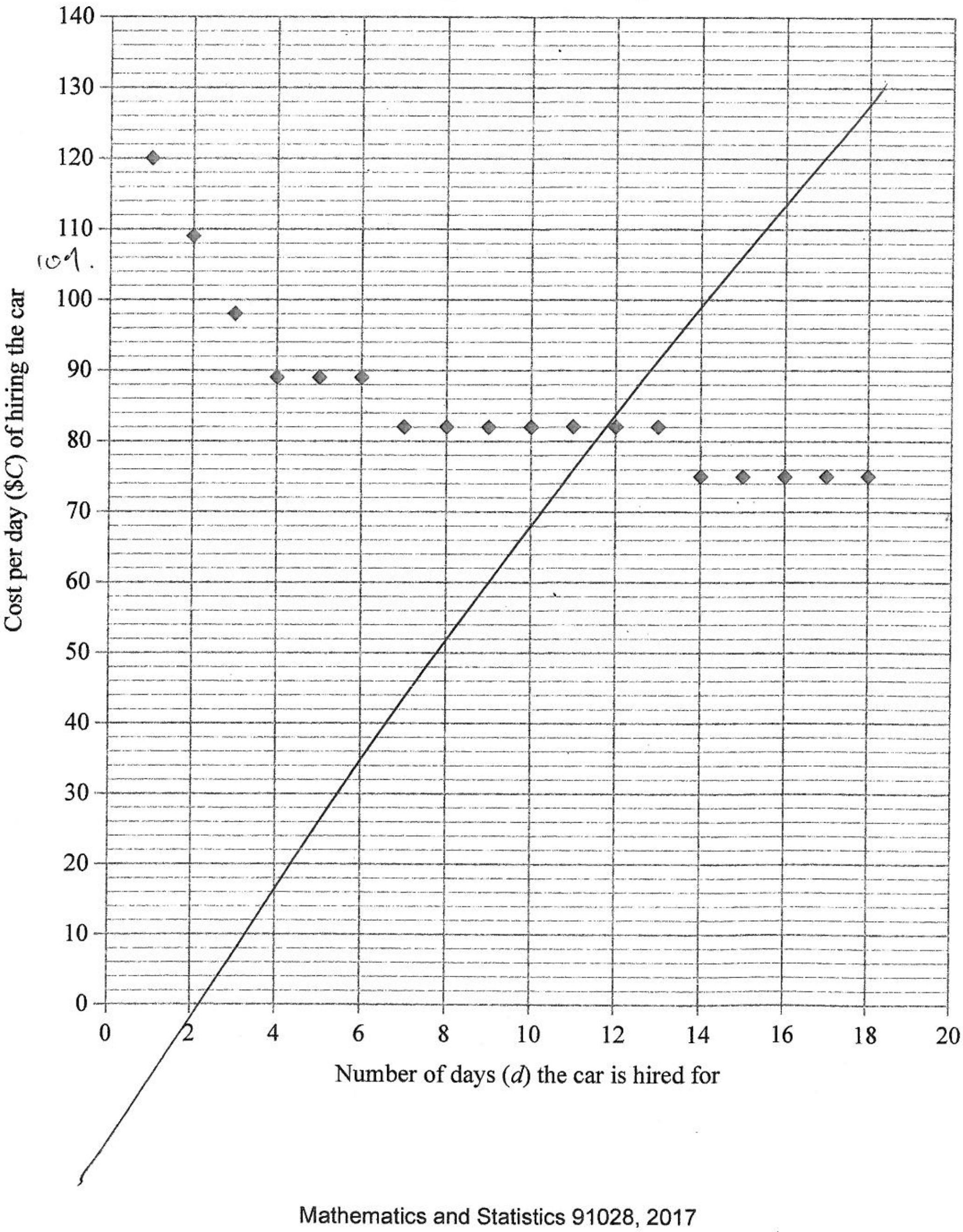
$$C = 140 \times 0.9^{d-1}$$

where $C$ ~~is the daily~~ cost and $d$ is the number of days for which the car is hired.


Investigate, using an equation, table, or graph, whether Mere is any better off with this 'special deal' offer compared to the original price, as shown on the graph from page 2 (reproduced below).

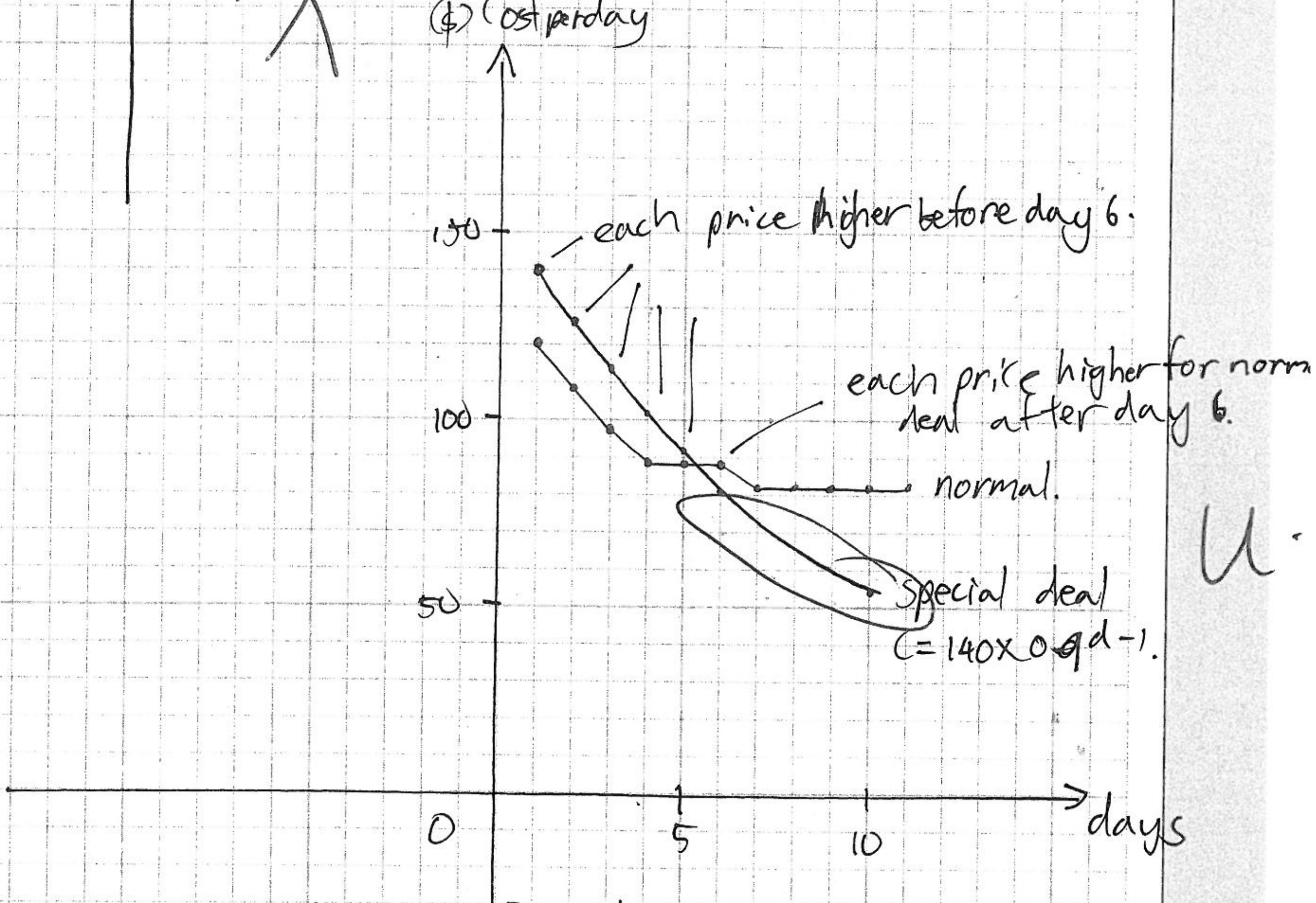
*Justify your answer.*

**Graph repeated from Page 2**



ASSESSOR'S USE ONLY

| days | Cost perday |
|---|---|
| 1 | 140 |
| 2 | 126 |
| 3 | 113.4 |
| 4 | 102.06 |
| 5 | 91.854 |
| ⋮ | ⋮ |




From the graph above I was able to investigate and make the call that Mere is better off with this deal if she is hiring the car for more than 5 days ($d > 5$) If she is hiring for less than 5 days, it is better to go with the normal deal, as the daily cost for normal deal is less ~~than~~ until day 5.

U

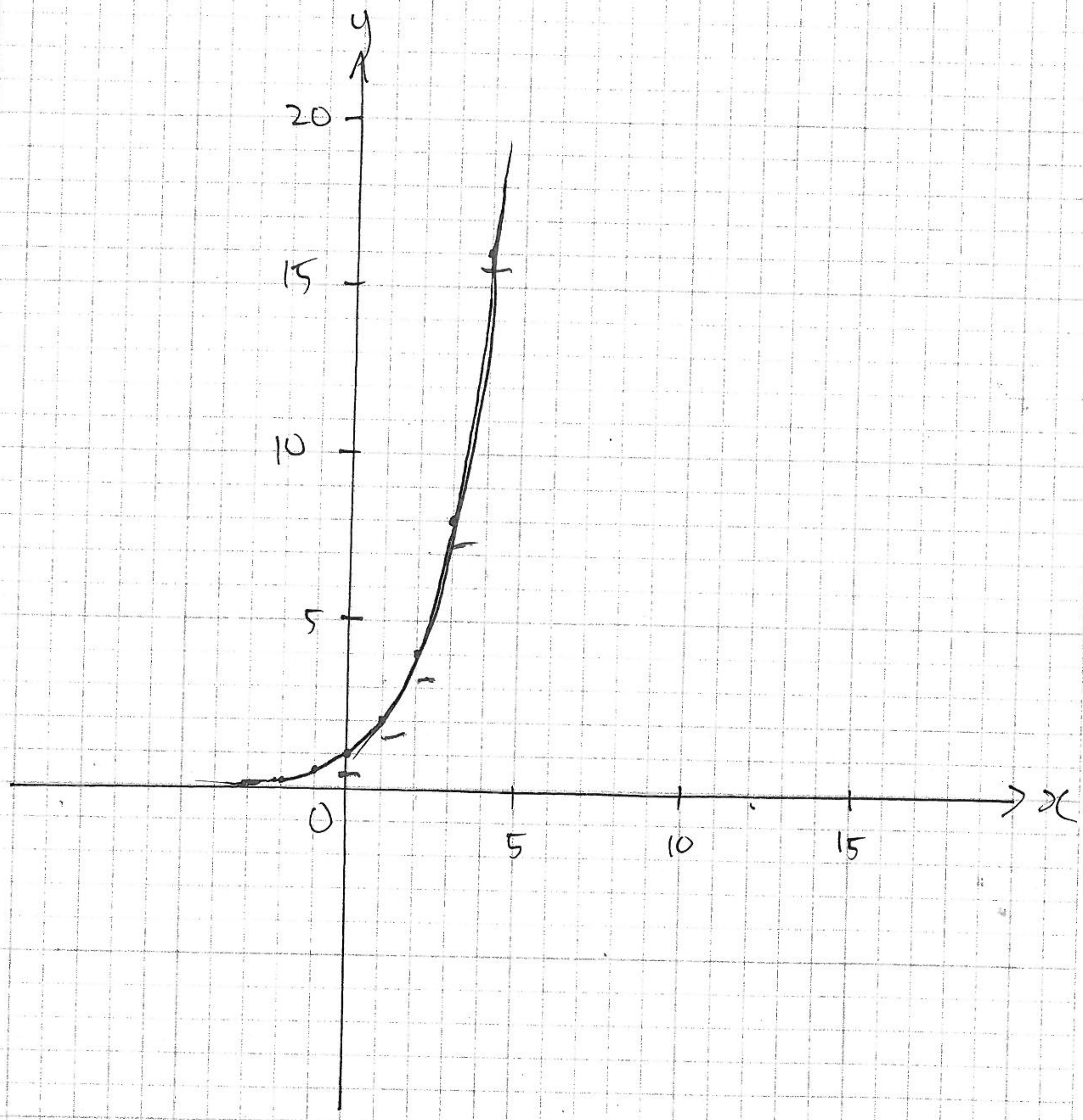
r c

M6

## QUESTION TWO

ASSESSOR'S USE ONLY

(a) (i) Sketch the graph of $y = 2^x$.

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(ii) Give the equation of this graph if it is translated down by 3 units, and then reflected in the $y$-axis.

$y = 2^{-x} - 3.$

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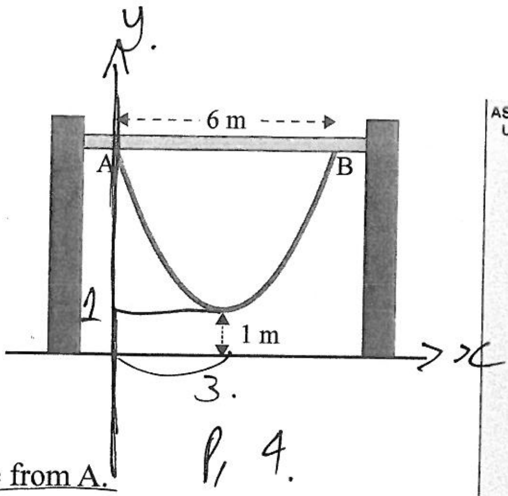



(b) In a children's playground there is a rope hanging from two points, A and B, on a horizontal beam.

A and B are 6 metres apart.

The lowest point of the rope is 1 m above the ground.

The shape of the rope can be modelled by

$$y = \frac{x}{3}(x - p) + 4$$

where $y$ is the height above the ground, and $x$ is the distance from A.

ASSESSOR'S USE ONLY

(i) How high above the ground is A?

$y = \frac{x^2}{3} - \frac{x}{3}p + 4$. $y = \frac{1}{3}x^2 - \frac{1}{3}xp + 4$. 4m

height of A is value of $y$ when $x = 0$.

u

(ii) Give the value of $p$.

The vertex of the rope has co-ordinates (3, 1)

so substitute that in to. $y = \frac{x}{3}(x-p) + 4$.

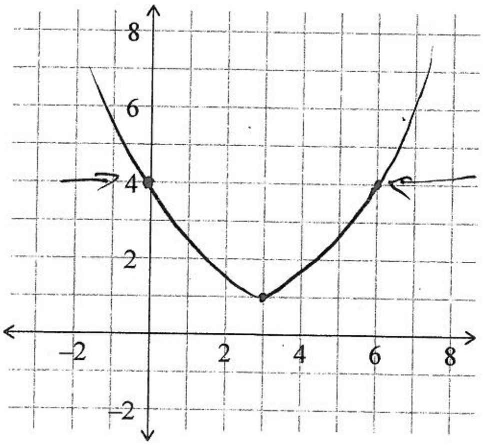
$1 = 3 - p + 4$. $y = \frac{1}{3}x^2 - 2x + 4$.

$p = 6$.

r

(iii) On the grid below sketch the graph that models the shape of the rope. $y = \frac{1}{3}(x^2 - 6x + 9 - 9) + 4$

$y = \frac{1}{3}(x-3)^2 + 1$.

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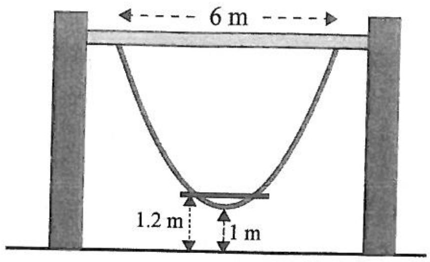
ASSESSOR'S USE ONLY

(iv) Holes are drilled through a 2 m long horizontal board.

The rope passes through the holes to make the seat of a swing.

The height of the seat is 1.2 metres above the ground.




How far apart would the holes in the board need to be if the shape of the rope above the seat stays the same?

Give your answer to 2 dp.

~~The co-ordinates of both holes can be found~~

The co-ordinates of the holes can be found by finding the intercepts of the two graphs $y=\frac{1}{3}(x-3)^2+1$ (rope equation) and $y=1.2$ (seat equati

To find the intercepts, substitute 1.2 into the equation for the rope;

$1.2=\frac{1}{3}(x-3)^2+1.$

$0.2=\frac{1}{3}(x-3)^2 \longrightarrow \quad 0.6=(x-3)^2.$

$\Downarrow$

$\sqrt{0.6}=\pm(x-3)$

$\Downarrow$

$0.77(2\,dp)=\pm(x-3).$

hence the $x$ co-ordinate of the seats are 3.77 and 2.23, so the distance between them is 1.54m

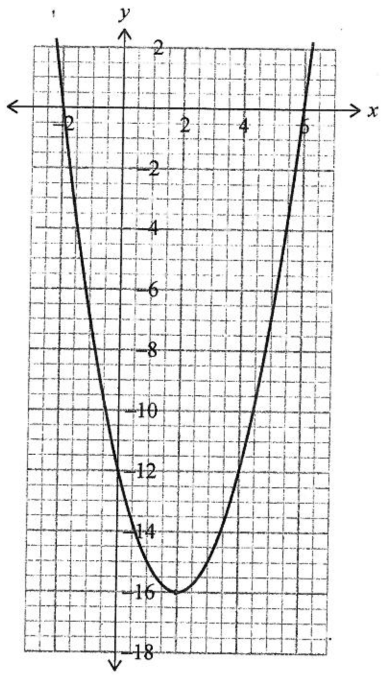
E

E8

## QUESTION THREE

ASSESSOR'S USE ONLY

(a) (i) Give the equation of the parabola shown below.




vertex $= (2, -16)$. $\rightarrow$ $y = a(x-2)^2 - 16$.

y intercept $= (0, -12)$. substitute $-12 = 4a - 16$.

$y = (x-2)^2 - 16$. $a = 1$. u.

(ii) Give the equation of the above graph if it is translated by 2 units to the right.

~~Right is positive~~ the new vertex is $(4, -16)$

$y = (x-4)^2 - 16$. r.

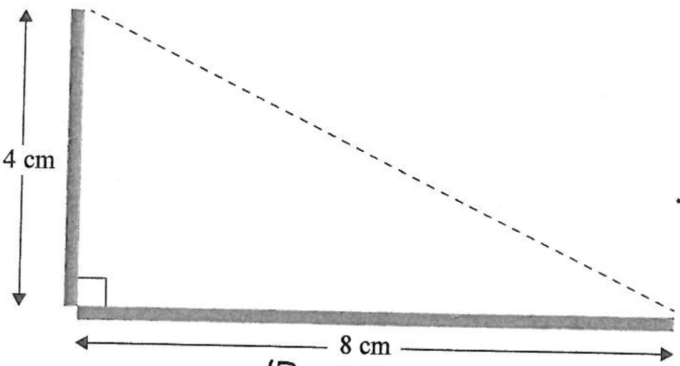
(b) Jono has some strips of plastic that are each 12 cm long.

~~He cuts one of these strips into two pieces and uses them as the two shorter sides of a right-angled triangle.~~

He starts by cutting a piece 4 cm long from a 12 cm strip, and uses this as one side of a right-angled triangle. He places the remaining 8 cm piece at right angles as the second side, as shown below.

ASSESSOR'S USE ONLY

$x$




$12-x.$

$$\frac{-x^2+12x}{2}.$$

$$-\frac{1}{2}x^2+6x.$$

$$-\frac{1}{2}(x^2-12+36-36)$$

$$-\frac{1}{2}(x-6)^2+18.$$

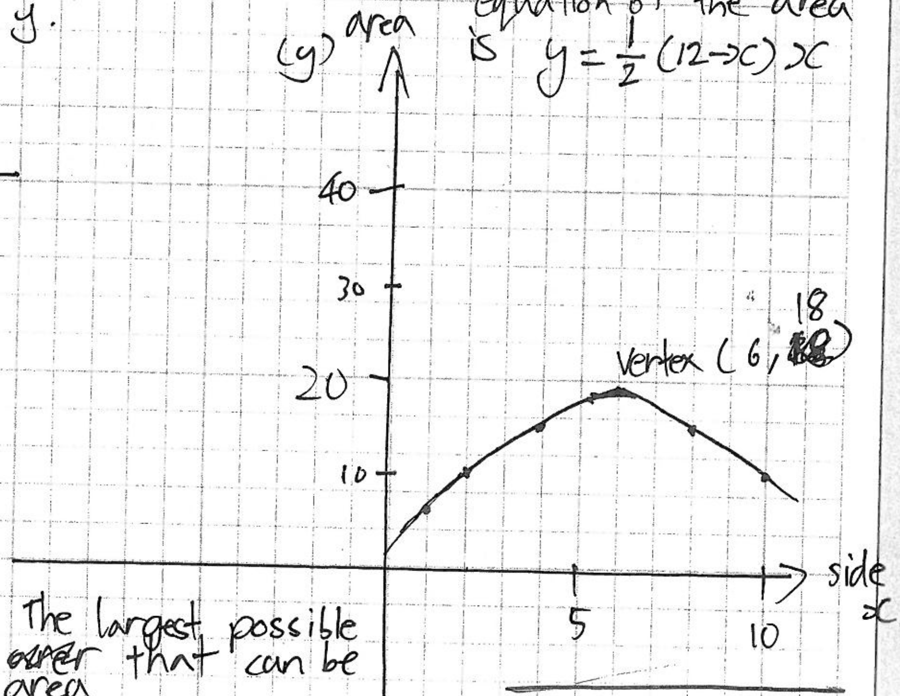
He then calculates the area of the triangle that would be formed by joining the two end points.

(i) Use a table, equation, or graph to investigate the relationship between the area of the triangle, and the different lengths of the piece of plastic that can be cut from the 12 cm strip.

let the perpendicular ~~size~~ side be $x$, and the other $12-x$. Let area be $y$.

equation of the area is $y=\frac{1}{2}(12-x)x$

| (cm) one side | (cm) other side | area. |
|---|---|---|
| 1 | 11 | $\frac{11}{2}$ |
| 2 | 10 | 10 |
| 3 | 9 | $\frac{27}{2}$ |
| 4 | 8 | 16 |
| 5 | 7 | $\frac{35}{2}$ |
| 6 | 6 | 18 |
| 7 | 5 | $\frac{35}{2}$ |
| ⋮ | ⋮ | |




The largest possible ~~arer~~ area that can be created is 18 cm², when both sides are 6 cm. long.

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State the equation that best represents the relationship between the area of the triangle and the length of plastic cut from the 12 cm strip.

$y=$ ~~...~~ $-\frac{1}{2}x(x-12)$ where $x$ is lenth cut

$y$ is area.

(ii) What features can be noticed about the area when Jono increases the length of the strip of plastic that he cuts from the 12 cm strip?

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Starting from the length of the strip cut being 1 cm, the area of the triangle increases in a quadratic pattern until the strip cut off is increased to 6 cm. At 6 cm, the triangle's area is at its max with the area of $18 cm^2$ from $\frac{1}{2} \times 6 \times 6$. This point (6, 18) represents the vertex of my graph, of the equation $y = -\frac{1}{2}x(x-12)$ or $-\frac{1}{2}(x-6)^2+18$. When the strip cut off starts increasing again from 6 cm onwards, the area ~~is~~ starts to decrease. From 6 cm onwards, the strip cut off is basically the strip remaining before the 6 cm mark. E.g. ~~At 8~~ the area you get with the strip that is 3 cm (cut off) and 9 cm (remaining) is the same as 9 cm (cut off) and ~~a~~ 3 cm (remaining.). Hence the graph is symmetrical ~~through~~ to a vertical line going through the vertex//

(iii) Clearly describe how the features of the graph of the relationship would change if the total length of the strip of plastic was *n* cm longer.

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Include the co-ordinates of the vertex of the parabola.

*NOTE: You do not need to draw the graph.*

the new strip would be ~~12cm~~ 12+n cm.
if the strip cut off is represented with $x$ again, the remaining strip would be $12+n-x$.
the area would be represented by the equation $y=\frac{1}{2}x(12+n-x)$.
which can be changed to $y=-\frac{1}{2}(x-\frac{12+n}{2})^2+\frac{(12+n)^2}{8}$.

The vertex of the new parabola would be

$\left(\frac{12+n}{2}, \frac{(12+n)^2}{8}\right)$

which means that the area of the triangle will be at max when both the side lengths of the triangle are equal to half of the total strip's lenght (half of 12+n) which at that point the area would be $\frac{1}{2}\times\left(\frac{12+n}{2}\right)^2 \longrightarrow \frac{(12+n)^2}{8}$.

E

E8

QUESTION NUMBER working

**Extra paper if required.**
**Write the question number(s) if applicable.**

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$y = \frac{1}{2}(-x^2 + 12x + nx)$

$-\frac{1}{2}(x^2 + (6 + \frac{1}{2}n)x)$

$y = -\frac{1}{2}x^2 + 6x + \frac{1}{2}nx$

$-\frac{1}{2}(x^2 + 6 + \frac{1}{2}n$

$y = -\frac{1}{2}(x^2 - 12x + 36 - 36) + \frac{1}{2}nx$

$y = -\frac{1}{2}(x-6)^2 + 18 + \frac{1}{2}nx$

$-\frac{1}{2}(x^2 - \frac{12+n}{2}x +$

$y = -\frac{1}{2}x^2 + \frac{12+n}{2}x$

$\frac{12+n}{2}$

$y = -\frac{1}{2}(x^2 - (12+n)x + \frac{(12+n)^2}{2} - (\frac{12+n}{2})^2)$

$y = -\frac{1}{2}(x - \frac{12+n}{2})^2 + \frac{(12+n)^2}{4}$

| Subject: | Level 1 Mathematics | Standard: | 91028 | Total score: | 22 |
|---|---|---|---|---|---|

| Q | Grade score | Annotation |
|---|---|---|
| 1 | M6 | Candidate has related the equations to the graph and the context in part a. In part b the candidate has used the formula to find points and graphed them correctly. However, the candidate has not worked with the minimum value of $80. To gain an E7 or E8 a fuller understanding of the context was needed as well as understanding how the minimum price affects the solution. |
| 2 | E8 | Candidate has linked the equations to the graphs in both parts a and b. The candidate could use the algebra and the equations skilfully. Candidate has good communication and strategies are well explained. The candidate has displayed a good chain of logical reasoning in part b) to solve the problem. |
| 3 | E8 | Candidate is able to link graphs and equations in part a). In part b) the candidate has devised a successful strategy to investigate the area of the triangles. Candidate shows insight and communicates this effectively. In b) iii the candidate formed a correct generalisation. |