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91028



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# Level 1 Mathematics and Statistics, 2017

## 91028 Investigate relationships between tables, equations and graphs

9.30 a.m. Monday 20 November 2017

Credits: Four

Achievement	Achievement with Merit	Achievement with Excellence
Investigate relationships between tables, equations and graphs.	Investigate relationships between tables, equations and graphs, using relational thinking.	Investigate relationships between tables, equations and graphs, using extended abstract thinking.

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

**You should attempt ALL the questions in this booklet.**

Show ALL working.

Grids are provided on some pages. This is working space for the drawing of a graph or a diagram, constructing a table, writing an equation, or writing your answer.

If you need more space for any answer, use the page(s) provided at the back of this booklet and clearly number the question.

Check that this booklet has pages 2–15 in the correct order and that none of these pages is blank.

**YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.**

Merit

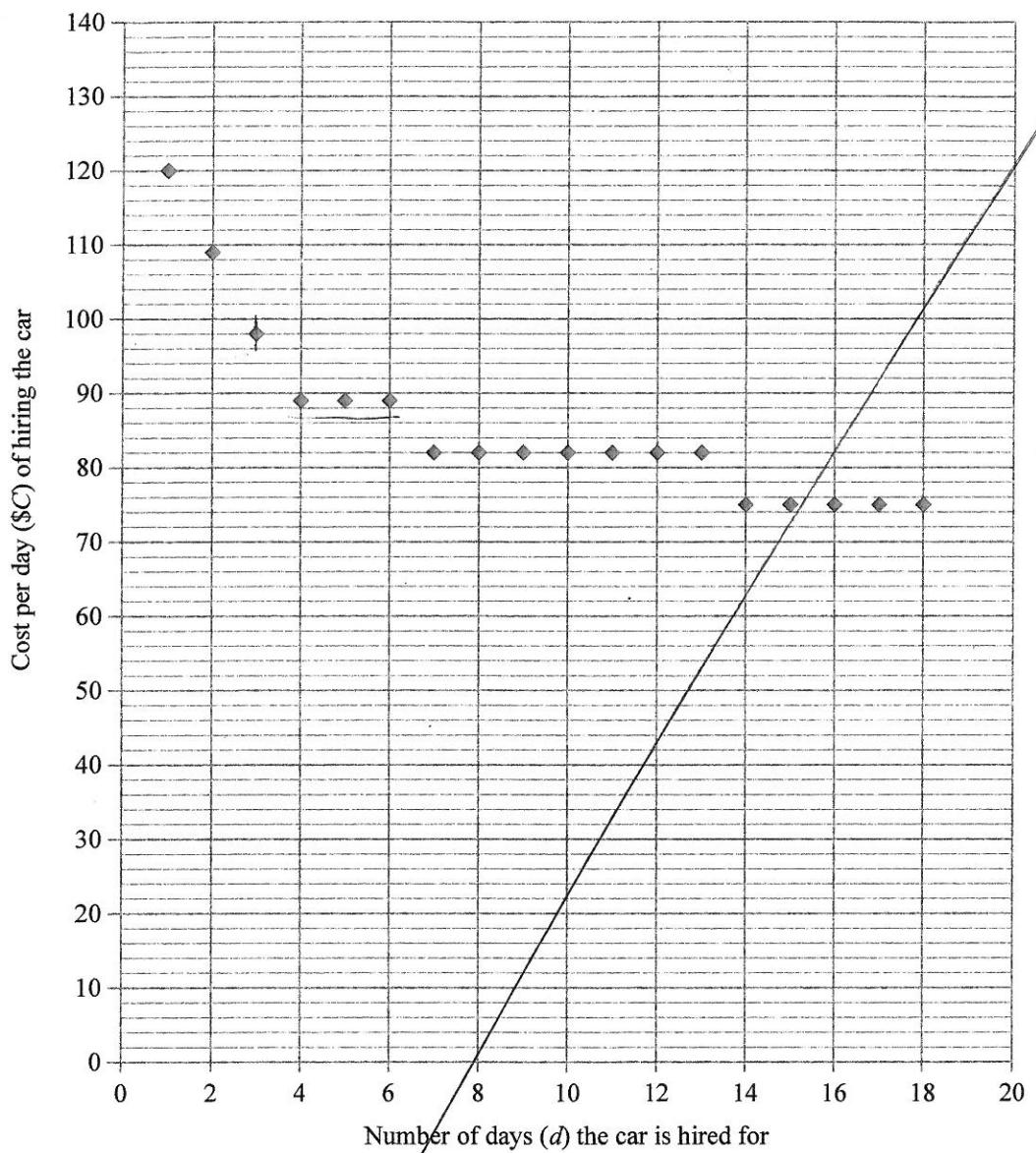
TOTAL

16

ASSESSOR'S USE ONLY

**QUESTION ONE**

- (a) *Rent A Car* is a car rental company. The graph below shows the cost per day ( $\$C$ ), of hiring one of their standard-sized cars, as the number of days the car is hired for ( $d$ ) increases.



~~98 for 3      120 for 4~~

3

- (i) How much cheaper per day is it to hire the car for 3 days than 1 day?

$$\underline{32.67 \text{ per day}} \quad \underline{120 \quad 32.67} \quad \underline{120 - 98 = 22}$$

it is \$22.00 cheaper per day

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- (ii) Give the equation for the cost per day of hiring the car:

- (1) for 4 to 6 days



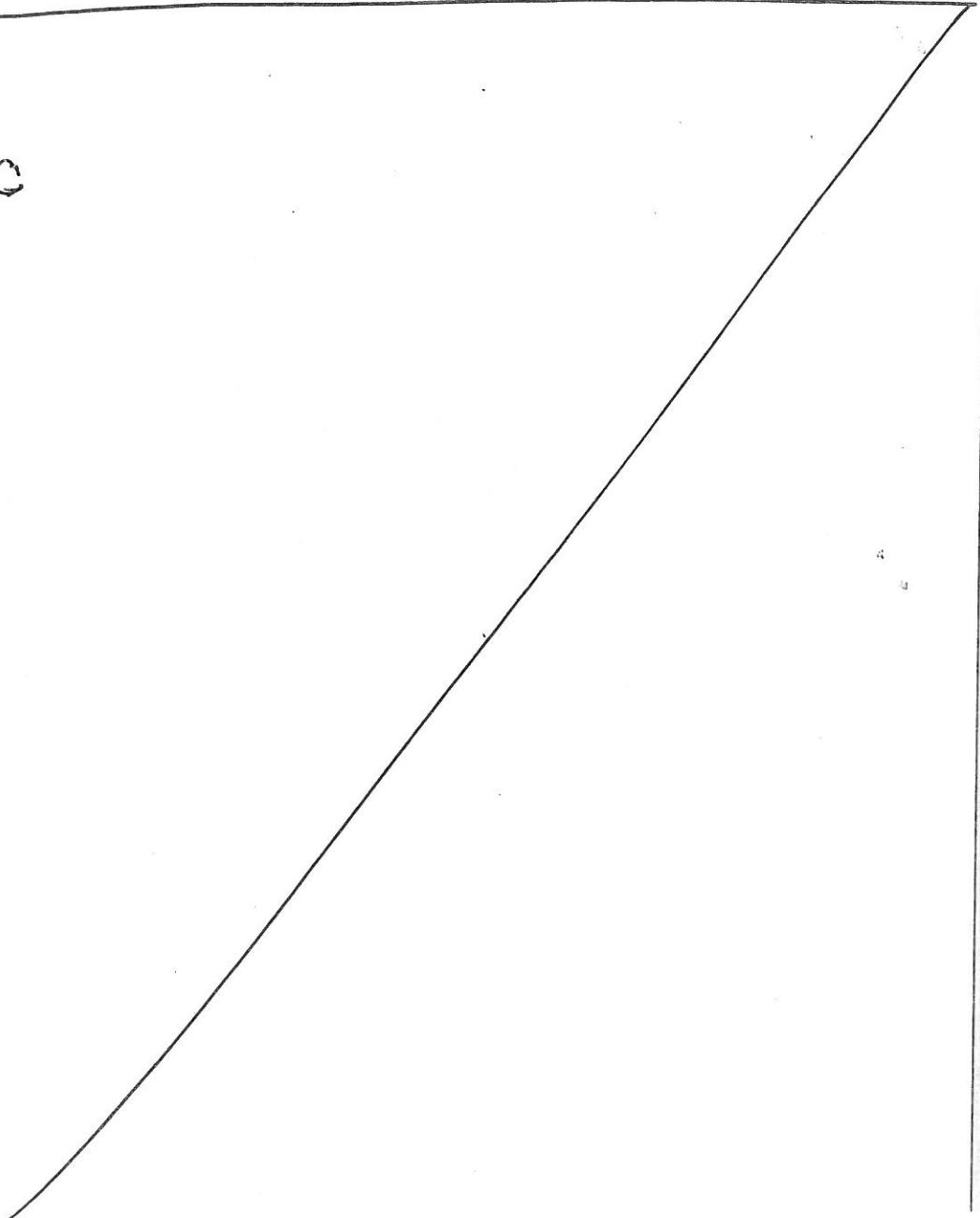
- (2) for the first 3 days.

~~your diagram~~

$$y = 0.090x + 11.90$$

N

$$y = mx + c$$



- (b) Rent A Car decides to introduce a special deal, and produces a sign as shown on the right.

Mere is trying to find the cheapest option for renting a car. She asks what this 'SPECIAL DEAL' actually means.

The company gives Mere the formula they use to work out the daily rate.

$$C = 140 \times 0.9^{d-1}$$

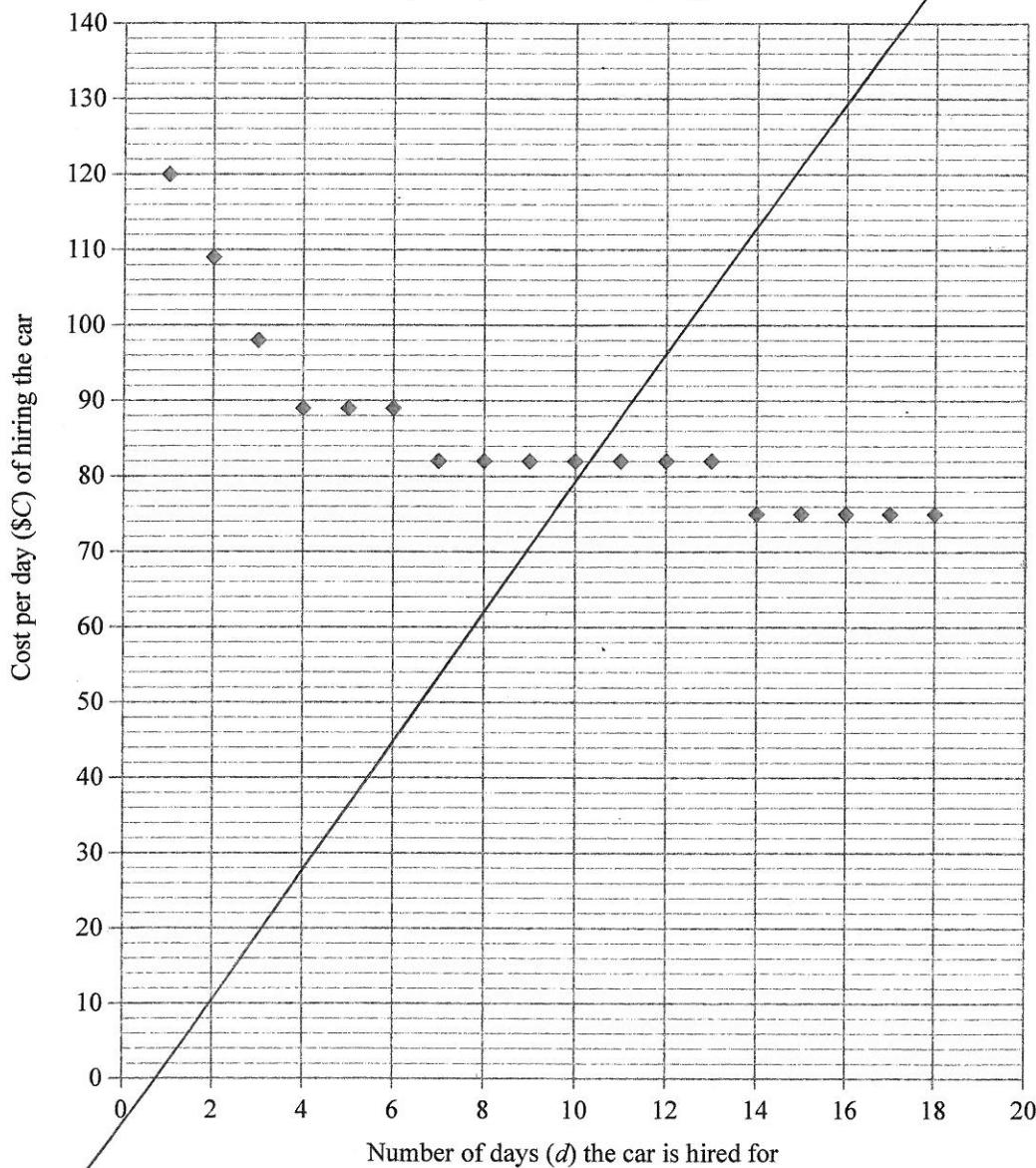
where  $C$  is the daily cost and  $d$  is the number of days for which the car is hired.

Investigate, using an equation, table, or graph, whether Mere is any better off with this 'special deal' offer compared to the original price, as shown on the graph from page 2 (reproduced below).

*Justify your answer.*



Graph repeated from Page 2



$$C = 140 \times 0.9^{t-1} = 125$$

to hire the car for one day using the special deal equation it cost \$125 whereas the normal price is only \$120

Table comparing normal price to special deal price

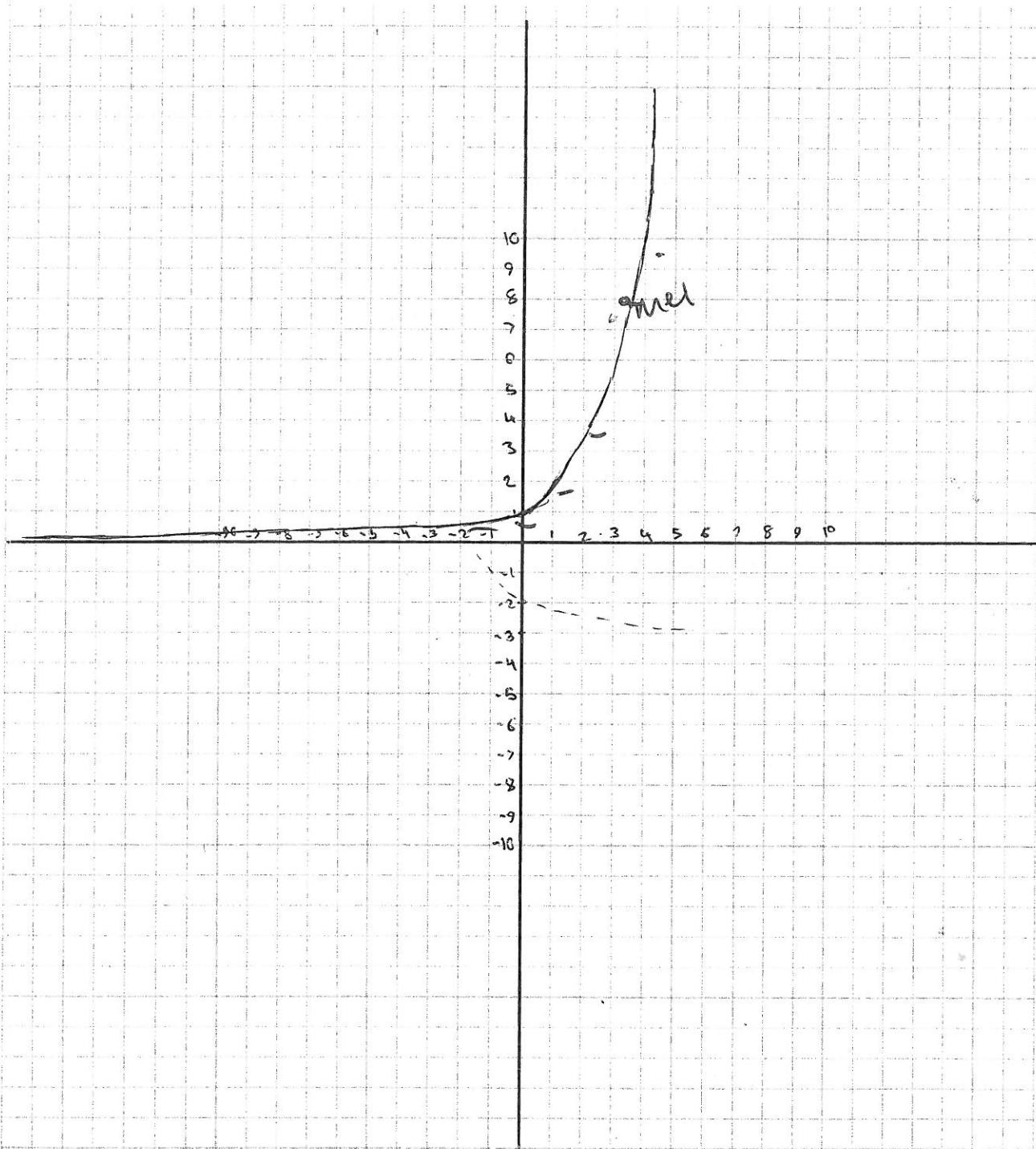
amount of days	special deal price	original price	per day
1	\$125	\$120	
2	\$112.4	\$109	
3	\$101.06	\$98	
4	\$90.85	\$89	
5	\$81.67	\$89	
6	\$73.40	\$89	
7	\$65.96	\$82	
8	\$59.27	\$82	
9	\$53.23	\$82	
10	\$47.81	\$82	

through the table we see that if mere  
is only thinking of renting a car  
for 1-4 days she is better off  
with the ~~normal~~ prices original  
prices but if she wants to rent  
a car for a longer period  
of time (4 or more days) she  
is better off with the special  
deal prices as it becomes cheaper  
over a longer period of time, which  
is seen in the graph //

M5

## QUESTION TWO

- (a) (i) Sketch the graph of  $y = 2^x$ .



- (ii) Give the equation of this graph if it is translated down by 3 units, and then reflected in the  $y$ -axis.

$$y = -2^{-x} - 3 //$$

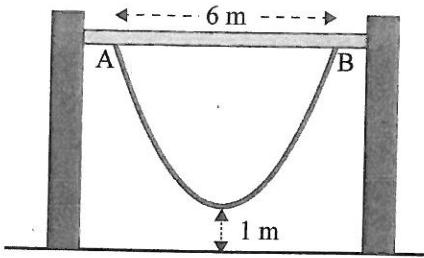
- (b) In a children's playground there is a rope hanging from two points, A and B, on a horizontal beam.

A and B are 6 metres apart.

The lowest point of the rope is 1 m above the ground.

The shape of the rope can be modelled by

$$y = \frac{x}{3}(x - p) + 4$$



where  $y$  is the height above the ground, and  $x$  is the distance from A.

- (i) How high above the ground is A?

$$y = \frac{0}{3}(0 - p) + 4$$

A is 4m above the ground

- (ii) Give the value of  $p$ .

$$4 = \frac{0}{3}(0 - p) + 4$$

$$4 - 4 = \frac{0}{3}(0 - p)$$

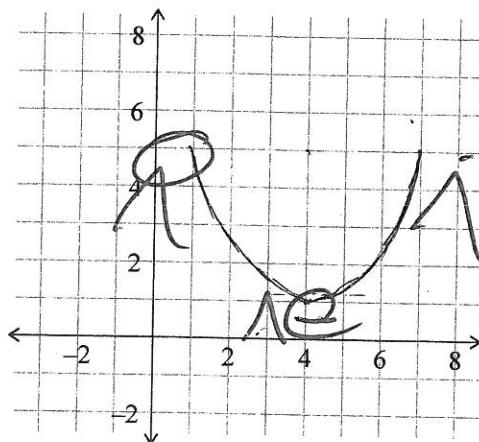
$$0 - p = 4 - 4 + \frac{0}{3}$$

$$0 = 4 - 4 + \frac{0}{3} + 0$$

$$p =$$

$$\textcircled{P} = \frac{0}{3}$$

- (iii) On the grid below sketch the graph that models the shape of the rope.

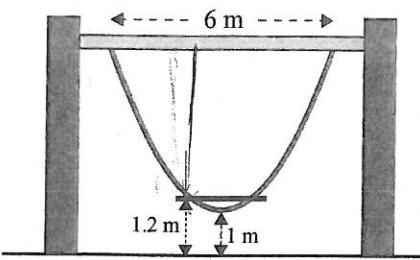


*Wet Shelly*

- (iv) Holes are drilled through a 2 m long horizontal board.

The rope passes through the holes to make the seat of a swing.

The height of the seat is 1.2 metres above the ground.



How far apart would the holes in the board need to be if the shape of the rope above the seat stays the same?

Give your answer to 2 dp.

$$y = \frac{x}{3}(x - p) + 4$$

$$1.2 = \frac{x}{3}(x - p) + 4$$

$$y = \frac{x}{3}(x - p) + 4$$

$$y = \frac{1.2}{3}(1.2 - p) + 4 = \cancel{\text{_____}}$$

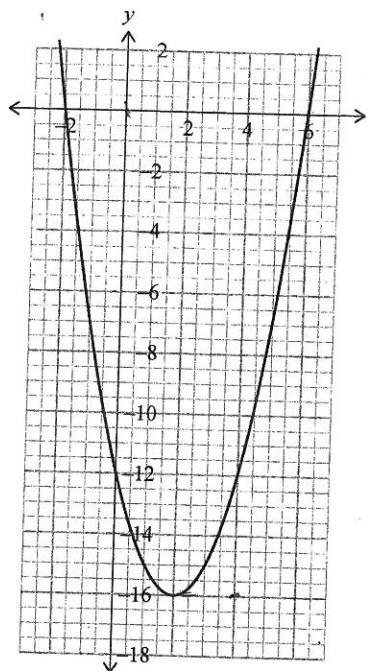
$$1.2 = \frac{x}{3}(1.2 - p) + 4$$

N

M5

## QUESTION THREE

- (a) (i) Give the equation of the parabola shown below.



$$0 = K(8)$$

$$0 = K(8+2)(8-6)$$

$$0 = K(-8)$$

$$0 = K(-8)$$

$$y = K(x+2)(x-6)$$

$$0 = K(-2+2)(-2-6)$$

equation =  $y = (x+2)(x-6)$

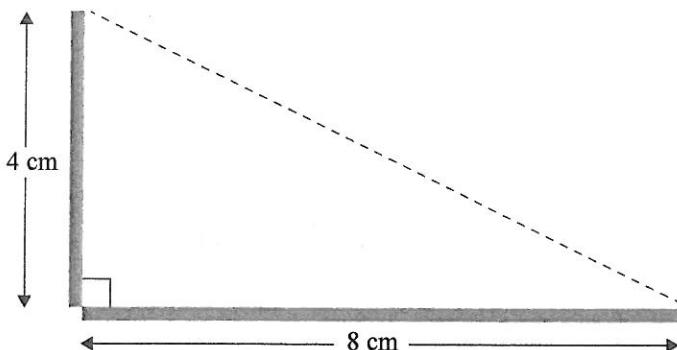
- (ii) Give the equation of the above graph if it is translated by 2 units to the right.

$y = x(x-8)$

- (b) Jono has some strips of plastic that are each 12 cm long.

He cuts one of these strips into two pieces and uses them as the two shorter sides of a right-angled triangle.

He starts by cutting a piece 4 cm long from a 12 cm strip, and uses this as one side of a right-angled triangle. He places the remaining 8 cm piece at right angles as the second side, as shown below.



He then calculates the area of the triangle that would be formed by joining the two end points.

- (i) Use a table, equation, or graph to investigate the relationship between the area of the triangle, and the different lengths of the piece of plastic that can be cut from the 12 cm strip.

table showing different combinations of plastic lengths

length one	length two	area
1 cm	11 cm	<del>5.5 cm<sup>2</sup></del>
2 cm	10 cm	<del>10 cm<sup>2</sup></del>
3 cm	9 cm	<del>13.5 cm<sup>2</sup></del>
4 cm	8 cm	<del>16 cm<sup>2</sup></del>
5 cm	7 cm	<del>17.5 cm<sup>2</sup></del>
6 cm	6 cm	<del>18 cm<sup>2</sup></del>

~~equation for the relationship between the length of the plastic and the area~~

~~area of the triangle is 18 cm<sup>2</sup> and the smallest is 5.5 cm<sup>2</sup>~~

as seen in the table the largest area that can be made is 18 cm<sup>2</sup> and the smallest is 5.5 cm<sup>2</sup>

U

State the equation that best represents the relationship between the area of the triangle and the length of plastic cut from the 12 cm strip.

let the short side of the triangle be  $B$  and the long side be  $H$ .  $A = \frac{1}{2} B \times H$

- (ii) What features can be noticed about the area when Jono increases the length of the strip of plastic that he cuts from the 12 cm strip?

When Jono cuts a longer piece of plastic off course the length of the piece left remaining decreases.

as the piece of plastic cut increases from 1-6cm so does the area of the triangle formed

the maximum area that can be formed is  $18\text{cm}^2$ , this is formed when a 6cm piece of plastic is cut therefore also leaving another 6cm piece remaining.

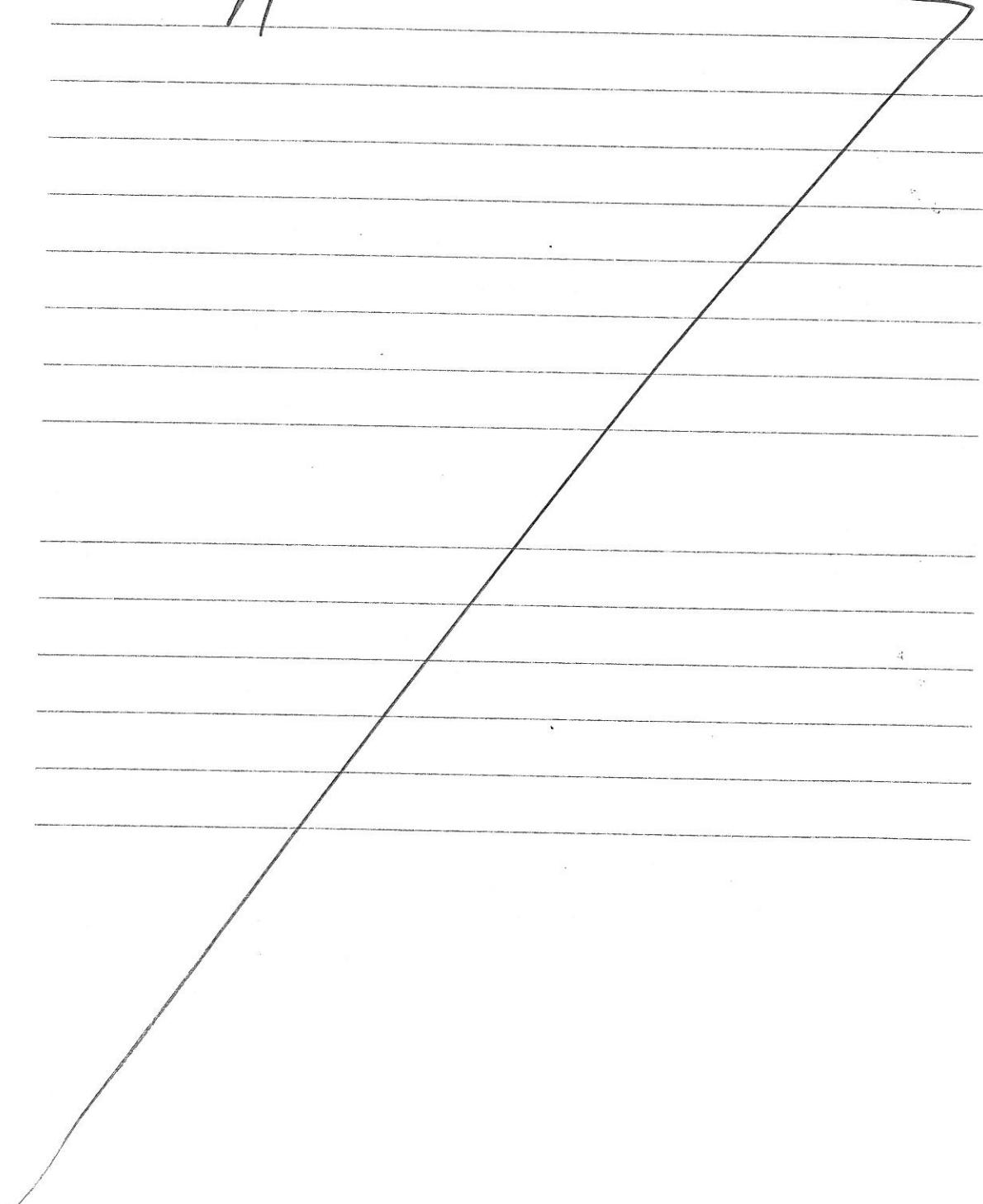
Overall as the length of strip increases up to 6cm the area of the triangle also increases, but when he cuts anything longer than 6cm the area then begins to decrease again

- (iii) Clearly describe how the features of the graph of the relationship would change if the total length of the strip of plastic was  $n$  cm longer.

Include the co-ordinates of the vertex of the parabola.

*NOTE: You do not need to draw the graph.*

~~the parabola would now be  
thinner than the original  
Graph.~~



N

M6

Subject:		Level 1 Mathematics	Standard:	91028	Total score:	16
Q	Grade score	Annotation				
1	M5	Candidate was able to relate their findings to the context consistently. Their communication was appropriate for the question. To gain an M6 the candidate needed to relate and link the equations to the graph more fully and display more understanding of the graphs in part a).				
2	M5	Candidate was able to link the equation and the graph in a) i. and recognise a vertical shift. In part b) the candidate displayed a limited understanding of the equation and context. For a M6 in this question the candidate would have needed to find p successfully in part b) and then display on a graph.				
3	M6	Candidate was able to link the equations to the graph and a horizontal shift in part a. The candidate devised a strategy to start the investigation but did not form the equation to fully connect the concepts. In communicating their thinking the candidate found the maximum area and was able to communicate how the area would increase up to 6cm and then start to decrease. For an E7 in this questions the candidate would have needed to develop a strategy to find the equation of the parabola formed during the investigation in part b).				