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1

91028



910280



NEW ZEALAND QUALIFICATIONS AUTHORITY
MANA TOHU MĀTAURANGA O AOTEAROA

QUALIFY FOR THE FUTURE WORLD
KIA NOHO TAKATŪ KI TŌ ĀMUA AO!

SUPERVISOR'S USE ONLY

Level 1 Mathematics and Statistics, 2015

91028 Investigate relationships between tables, equations and graphs

9.30 a.m. Monday 9 November 2015

Credits: Four

Achievement	Achievement with Merit	Achievement with Excellence
Investigate relationships between tables, equations and graphs.	Investigate relationships between tables, equations and graphs, using relational thinking.	Investigate relationships between tables, equations and graphs, using extended abstract thinking.

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

You should attempt ALL the questions in this booklet.

Show ALL working.

If you need more space for any answer, use the page(s) provided at the back of this booklet and clearly number the question.

Check that this booklet has pages 2–16 in the correct order and that none of these pages is blank.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

Merit

TOTAL

16

ASSESSOR'S USE ONLY

QUESTION ONE

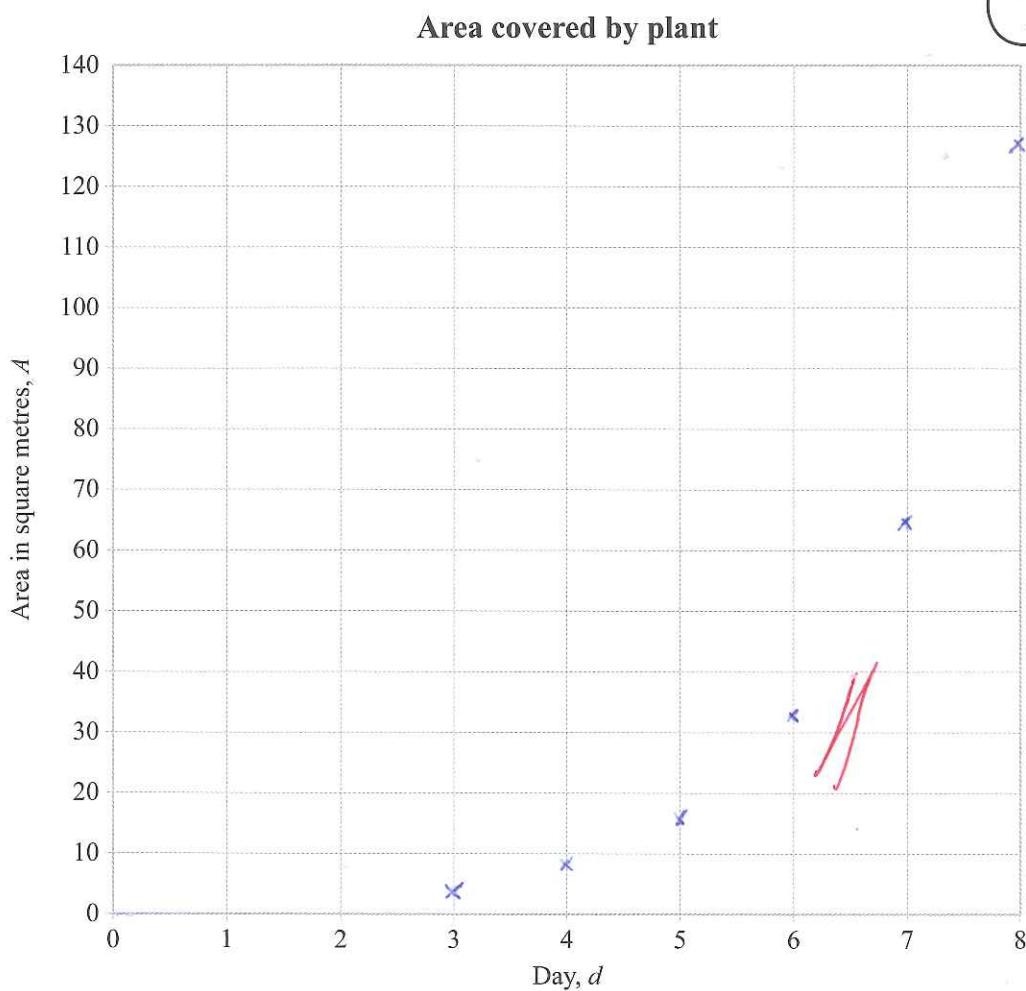
A plant is growing on the surface of a pond. Hank noticed the plant on Day 1. Two days later Hank was worried about the plant and started measuring the area that the plant covered.

- (a) Each day (at 5 pm) Hank measures the area of water (in square metres) covered by the plant. He records his measurements in the table below.

Day, d	Area covered by plant, A
1	1
2	2
3	4
4	8
5	16
6	32
7	64
8	128

d is the number of days since Hank first noticed the plant.

- (i) Show how the area of the pond covered by the plant changes with time.



If you
need to
redraw this
graph, use
the grid on
page 14.

- (ii) The plant followed the same pattern of growth from the time when it was first noticed.

What area of the pond was covered by the plant when it was first noticed?

Explain your answer.

$1m^2$ because every day the area of the pond that is covered by plant increases by $\frac{1}{2}$ each from the last day.

- (iii) Give the equation that describes the area of the plant covering the pond after d days.

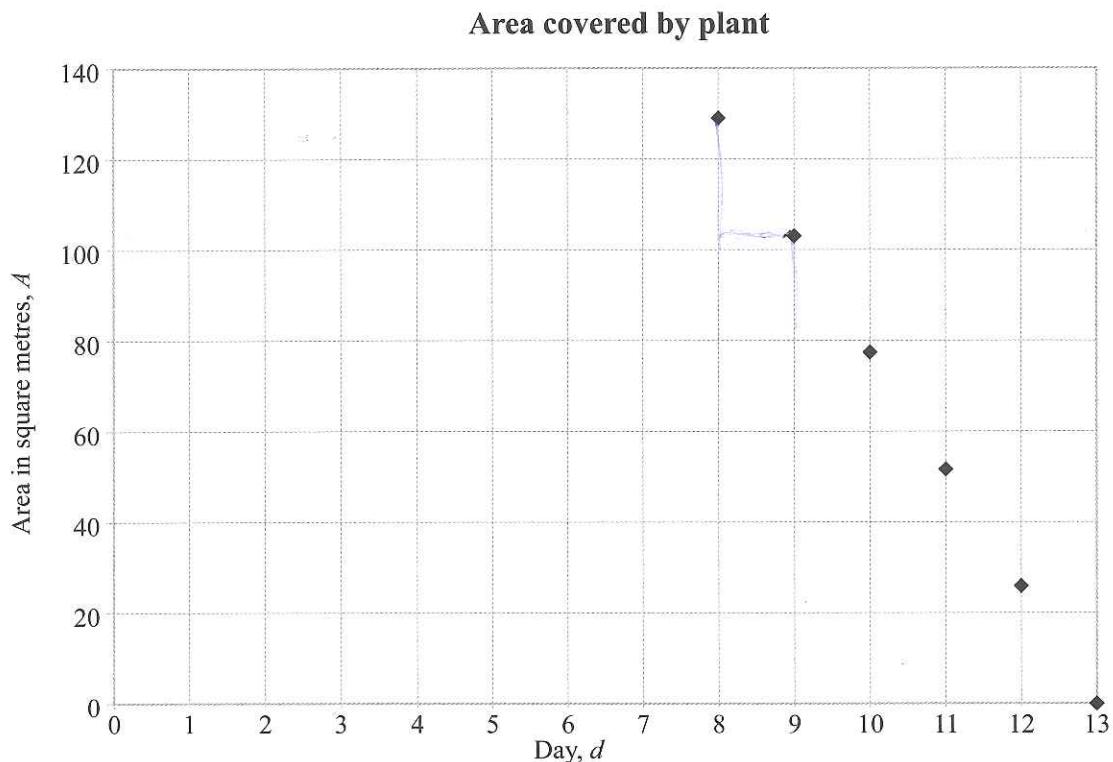
$$A = \frac{1}{2}d$$

- (iv) If no intervention takes place, on which day will Hank first measure the area of the plant to be more than 500 square metres?

$$500 = \frac{1}{2}d$$

- (b) Hank and some friends start removing the plant on Day 9.

The graph of the area covered by the plant from **Day 8** (when it covers 128 square metres), below, shows what Hank hopes will happen to the area of pond covered by the plant.



- (i) What is the equation for the area covered by the plant as shown in this graph?

$$A = \frac{1}{2}x^2 - 23x + 312$$

N

- (ii) What is unrealistic about this graph?

Write at least TWO comments with justification.

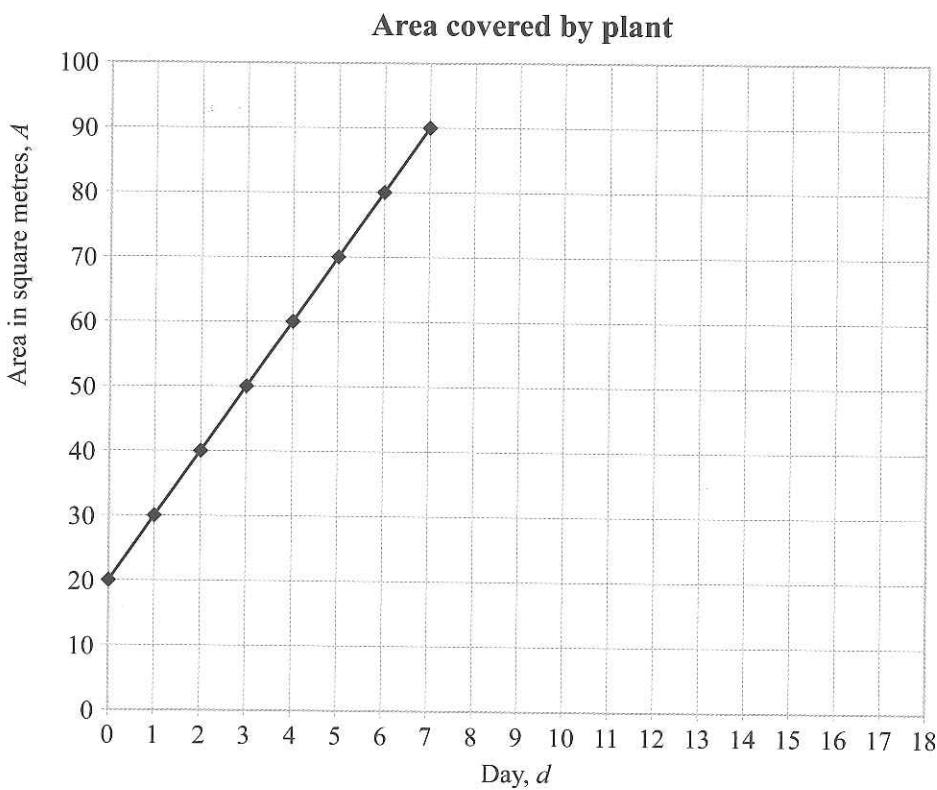
everyday they aren't going to get rid of the same amount of plant, some days they will take more breaks or ~~need to stay at home have~~ less time to complete the work which will result in less work getting done. another unrealistic thing is that they have considered the weather. Some days it may be raining, since this is outdoors work they will not be able to do it during that day. //

11

M5

QUESTION TWO

The next year, when the plant begins to grow back, Hank tries to stop it from spreading across the pond so quickly. As soon as he notices the plant, he begins removing it. The graph of the area of pond covered by the plant in this year is shown below:



- (a) How much more area is the plant covering each day?

$$10 \text{ m}^2/\text{day}$$

- (b) What day will it be when the plant covers 200 square metres if the conditions remain the same?

Show your working.

$$\begin{aligned} & 90 \text{ m}^2 \text{ in } 7 \text{ days} \\ & 200 - 90 = 110 \text{ m}^2 \text{ till } 200 \\ & 110 \div 10 \text{ m}^2 \text{ per day} \end{aligned}$$

$$= 11 \text{ days}$$

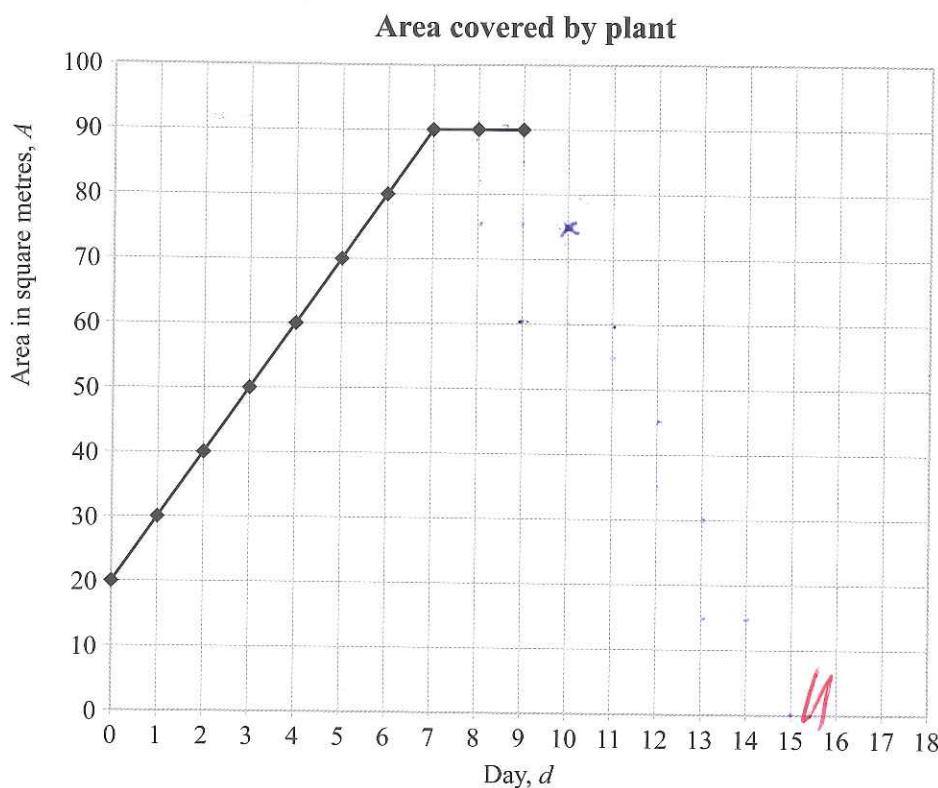
$$7 + 11 = 18 \text{ days}$$

day 18 will be 200 m^2

After 7 days removing some of the plant by himself, Hank decides to get help.

- (c) One friend helps on Day 8 and Day 9.

The area covered by the plant stays the same for Day 8 and Day 9.



If you
need to
redraw this
graph, use
the grid on
page 14.

- (i) What is the equation of this new section of the graph on Day 8 and Day 9?

$$A = 90 \text{ II}$$

- (ii) What does this section of the graph mean?

this section of the graph means that
with the help of his friends he
managed to keep the plant down to 90ms
during those days.

- (d) Two more friends come to help. Now the area covered by the plant decreases by 15 square metres each day until the plant is completely removed.

- (i) Draw a graph on the grid above to show the area of pond covered by the plant from Day 10.

- (ii) On what day will there be no plant left?

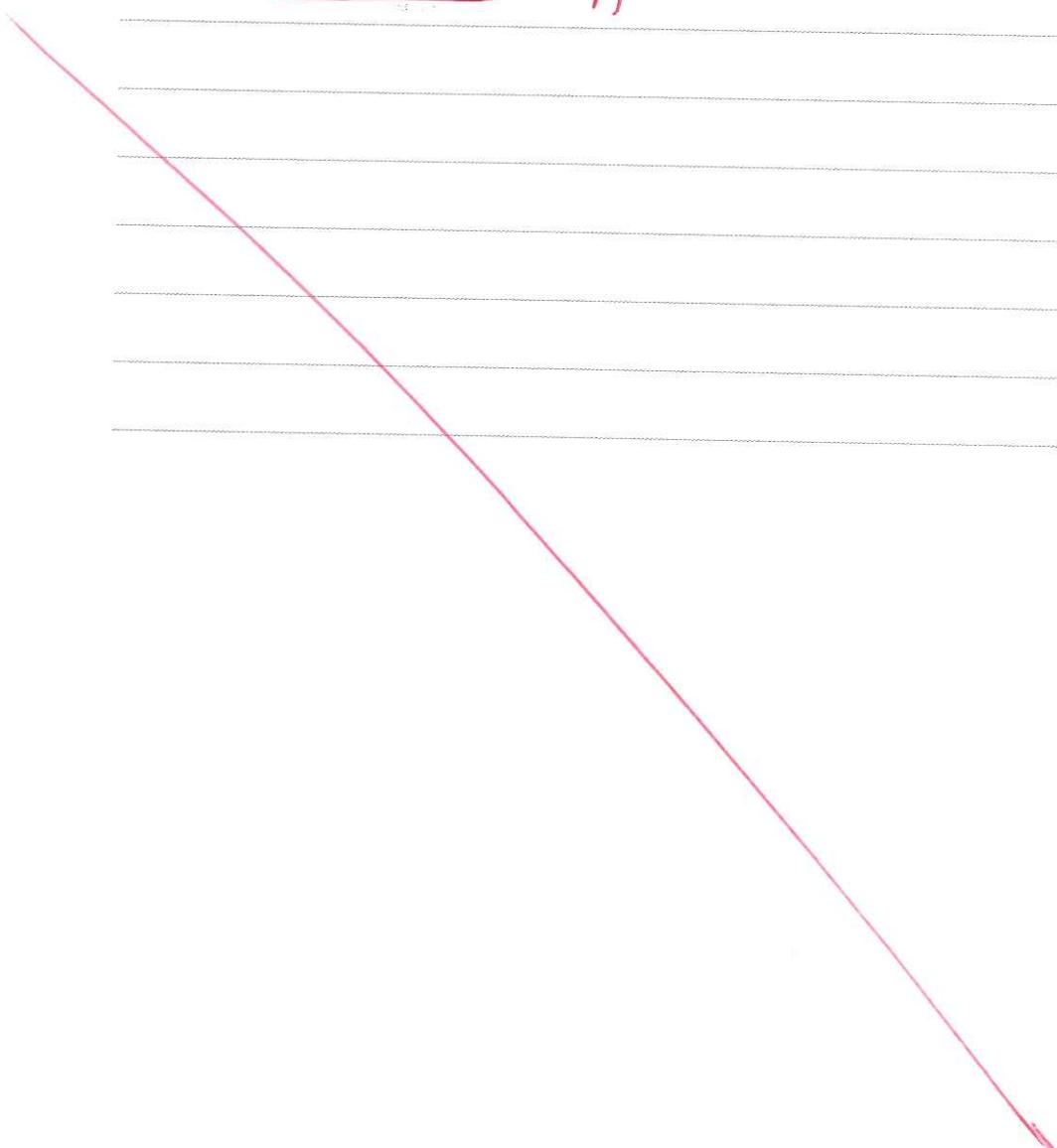
if this is continued the plant would
be removed on day 15.

- (e) The equation of the line for Day 9 onwards is $A = 225 - 15d$.

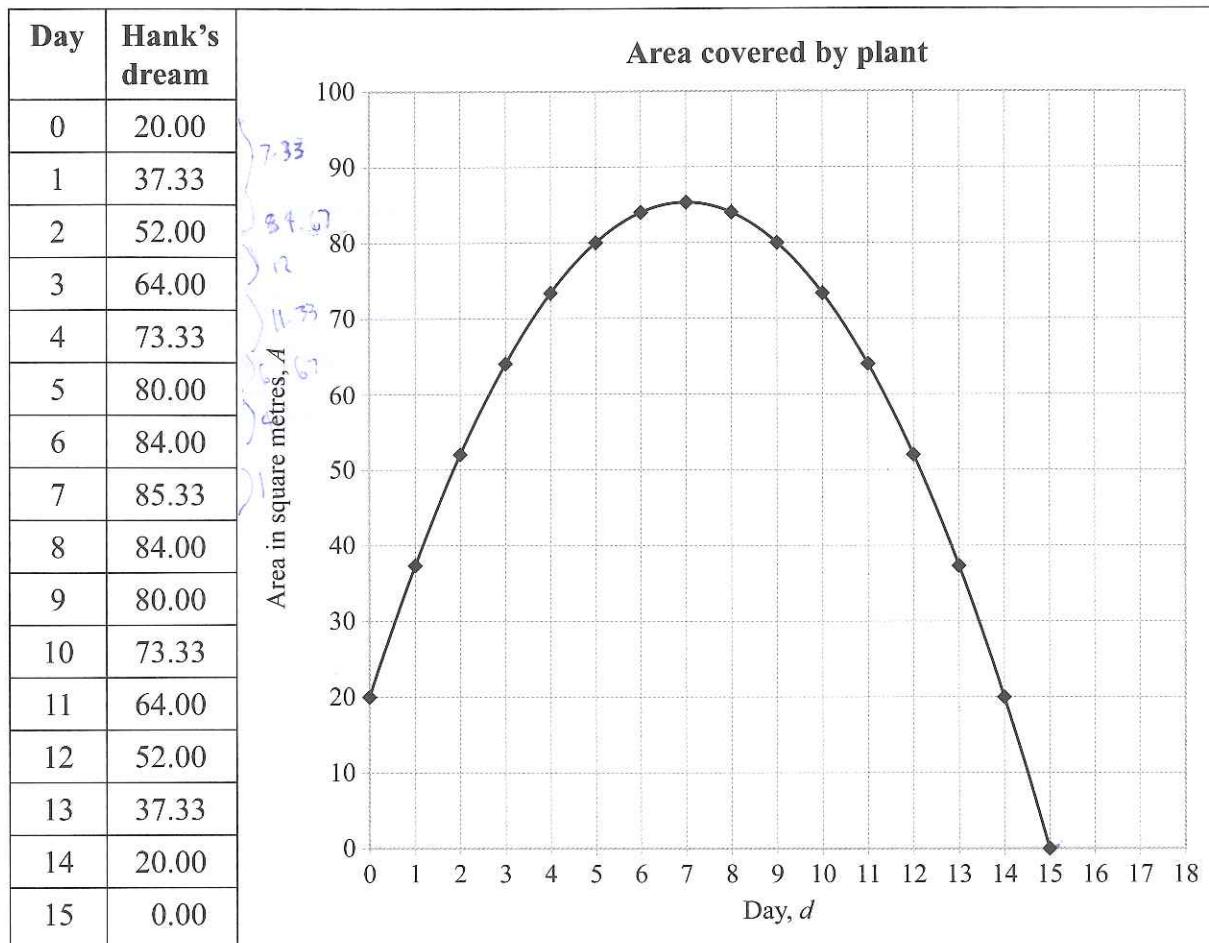
If Hank's 2 friends had come on Day 8, what would the equation of this line have been?
Explain your reasoning.

$$\underline{A = 195 - 15d}$$

$$\cancel{225 - 15d} \cancel{- 30}$$



- (f) Hank had a dream that he and his friends made the area of the pond covered by the plant follow the parabola given below:



What is the equation of this graph?

$$A = -$$

21 37 52 64 73 80 85
 17 20 27 33 40 47 54
 14 17 24 31 38 45 52
 11 14 21 28 35 42 49
 8 11 18 25 32 39 46
 5 8 15 22 29 36 43
 2 5 12 19 26 33 40
 0 3 10 17 24 31 38

5

M6

- (a) Jodie sets her friends a mathematical problem. She says:

I think of an integer

When I add 1 to my number, I get A

But if I take 4 off my number, I get B

When A is multiplied by B, I get an answer of 6.

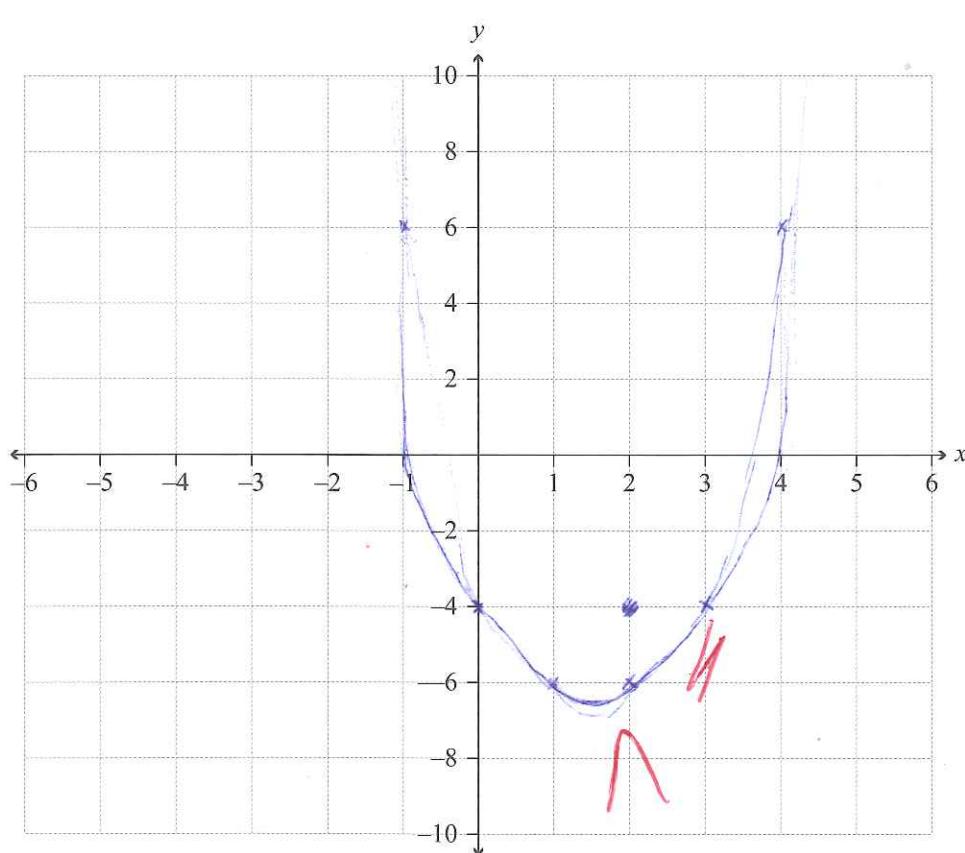
What's my number?

Her friends start by writing a table:

x : Jodie's number	$A = x + 1$	$B = x - 4$	$y = AB$
0	1	-4	-4
1	2	-3	-6
2	3	-2	-6
3	4	-1	-4
4	5	0	0
5	6	1	6

- (i) Draw the graph of y against x .

Use the set of axes below.



If you
need to
redraw this
graph, use
the grid on
page 15

- (ii) What is the equation of the graph that matches the table above, in terms of x ?

$$y = (x+1)(x-4)$$

$$y =$$

- (iii) Explain how Jodie's number can be found from the graph if the answer is 6.

$$6 = (x+1)(x-4)$$

~~you must find a number that adds with +1 and -4 to make~~

$$\begin{matrix} 3, 2 \\ \text{or } 6, 1 \end{matrix}$$

$$x = 5$$

$$5+1 = 6 \quad 5-4 = 1$$

$$6 + 1 = 6$$

- (iv) Suppose Jodie had said "A multiplied by B gives me -10".

What does your graph tell you about the solutions to this new problem?

H

H

- (b) Tom thinks of a puzzle to challenge Jodie.

He starts by saying:

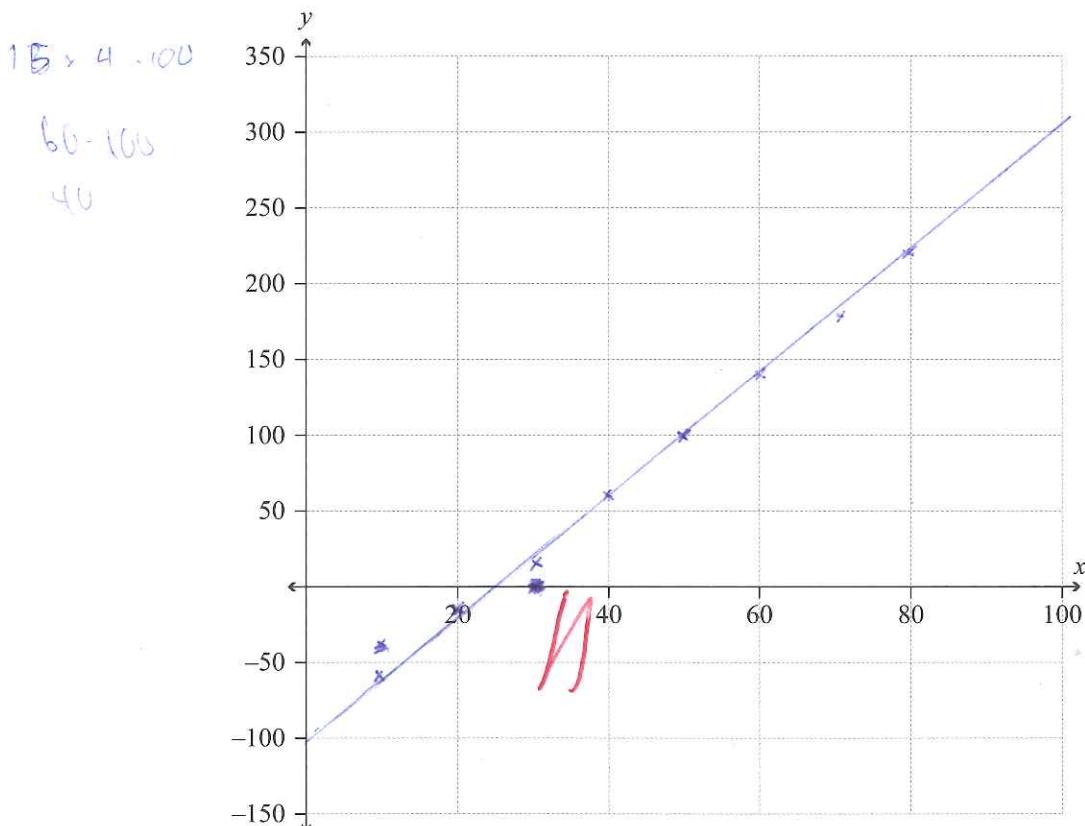
I think of a two-digit number.

I multiply it by 4 and take away 100 ...

- (i) ~~What equation would you use to describe this relationship?~~

$$\cancel{x} = 4 \cancel{x} - 100$$

- (ii) ~~Draw the graph of this relationship on the axes below.~~



If you
need to
redraw this
graph, use
the grid on
page 15

- (iii) Tom's whole puzzle is:

Guess my 2-digit number:

If I multiply it by 4 and take away 100 ...

I get the same as when I add 47 to it and then multiply the result by 1.12

Explain how the solution to Tom's question can be found, and give the solution as accurately as possible.

$$\cancel{4x - 100} = (\cancel{x} + 47) \times 1.12$$

$$4x - 100 = 1.12x + 52.64$$

$$4x - \cancel{47.36} = 1.12x - \cancel{47.36}$$

$$\cancel{3.78x - 47.36} = \cancel{0} + 47.36$$

$$x = 12.33 \text{ (2dp)}$$

u

M5

Annotated Exemplar Template

Merit exemplar for 91028 2015			Total score	16
Q	Grade score	Annotation		
1	M5	<p>a(i). Points correctly plotted.</p> <p>a(ii) Correct answer and recognition of doubling for r.</p> <p>a(ii) and (iii). Incorrect.</p> <p>b(i). Incorrect.</p> <p>b(ii). Candidate has discussed context but has not related to graph.</p>		
.2	M6	<p>a) Correct.</p> <p>b) Correct and some indication of working for r.</p> <p>c) Equation and explanation correct.</p> <p>d) Both parts correct.</p> <p>e) Correct equation for r but no explanation.</p> <p>f) Incorrect.</p>		
3	M5	<p>a(i). Graph is not pretty but OK for u. Should be discrete points for r.</p> <p>a(ii). Correct.</p> <p>a(iii). No obvious explanation.</p> <p>a(iv). No attempt.</p> <p>b(i) Equation incorrect.</p> <p>b(ii). Graph correct.</p> <p>b(iii). Candidate has correct equation to solve for u but has failed to solve correctly.</p>		

1

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Show ALL working.

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Merit

TOTAL

18

ASSESSOR'S USE ONLY

QUESTION ONE

A plant is growing on the surface of a pond. Hank noticed the plant on Day 1. Two days later Hank was worried about the plant and started measuring the area that the plant covered.

- (a) Each day (at 5 pm) Hank measures the area of water (in square metres) covered by the plant. He records his measurements in the table below.

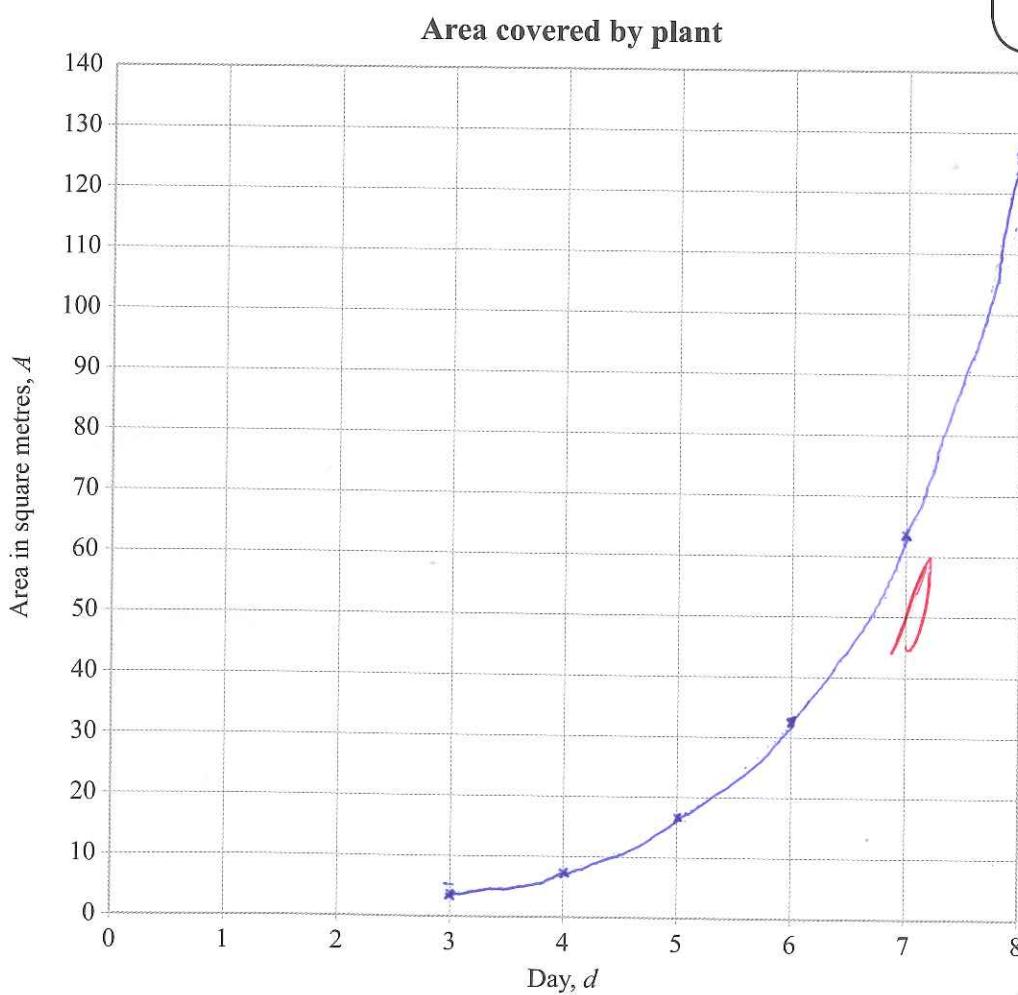
1	1	2
10	10	512

Day, d	Area covered by plant, A
1	1
2	2
3	4
4	8
5	16
6	32
7	64
8	128

d is the number of days since Hank first noticed the plant.

- (i) Show how the area of the pond covered by the plant changes with time.

If you
need to
redraw this
graph, use
the grid on
page 14.



- (ii) The plant followed the same pattern of growth from the time when it was first noticed.

What area of the pond was covered by the plant when it was first noticed?

Explain your answer.

1 square meter as the plant doubles in size each day. day 1 would be half of day 2 and day 2 would be half of day 3.

- (iii) Give the equation that describes the area of the plant covering the pond after d days.

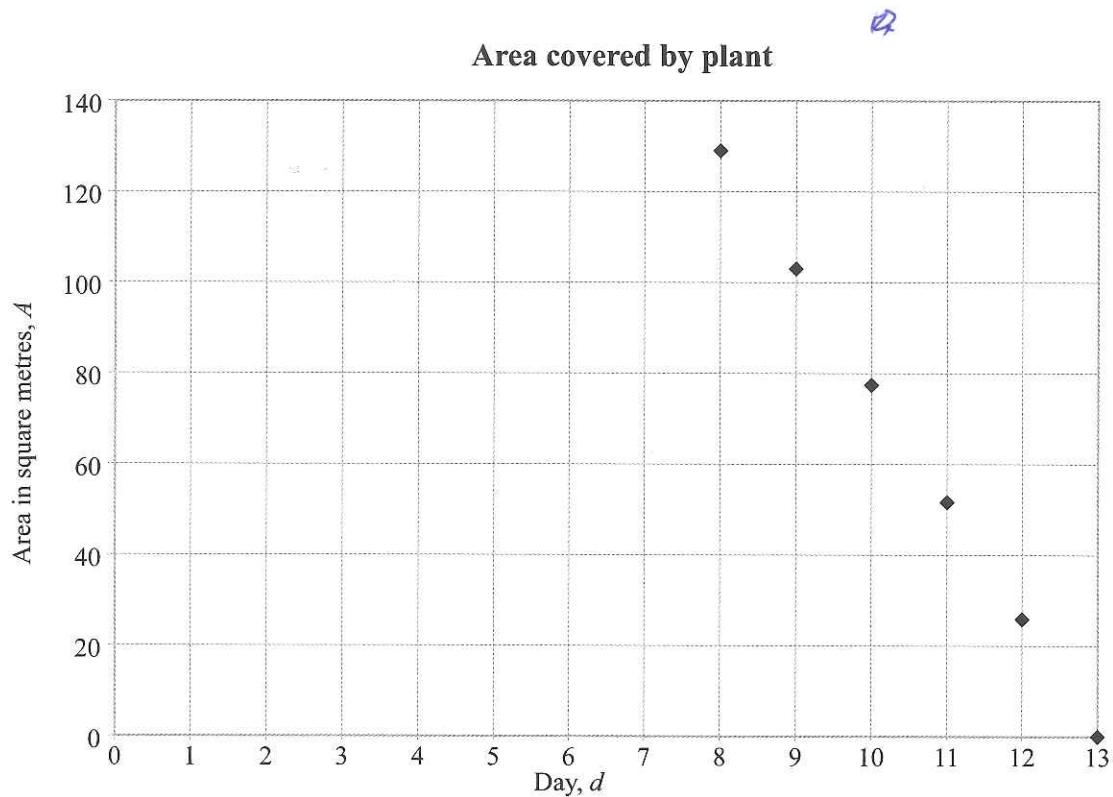
$$d \text{ days } A = 1$$

- (iv) If no intervention takes place, on which day will Hank first measure the area of the plant to be more than 500 square metres?

day 10

- (b) Hank and some friends start removing the plant on Day 9.

The graph of the area covered by the plant from Day 8 (when it covers 128 square metres), below, shows what Hank hopes will happen to the area of pond covered by the plant.



- (i) What is the equation for the area covered by the plant as shown in this graph?

$$y = mx + c$$

$$y = -25\frac{3}{5}x + 332.8$$

$$y = -25\frac{3}{5}x + 332.8$$

$$y = -\frac{128}{5}x + 128$$

$$y = -25.6x + 332.8$$

$$y = -25\frac{3}{5}x + 332.8 \quad \boxed{1}$$

128 0

- (ii) What is unrealistic about this graph?

Write at least TWO comments with justification.

The graph

~~The plant 2 should be doubling each day. So when the 25.6 square meters is removed and it is $128 - 25.6 = 102.4\text{m}$. The day after should the plant now been 204.8 square meters now?~~

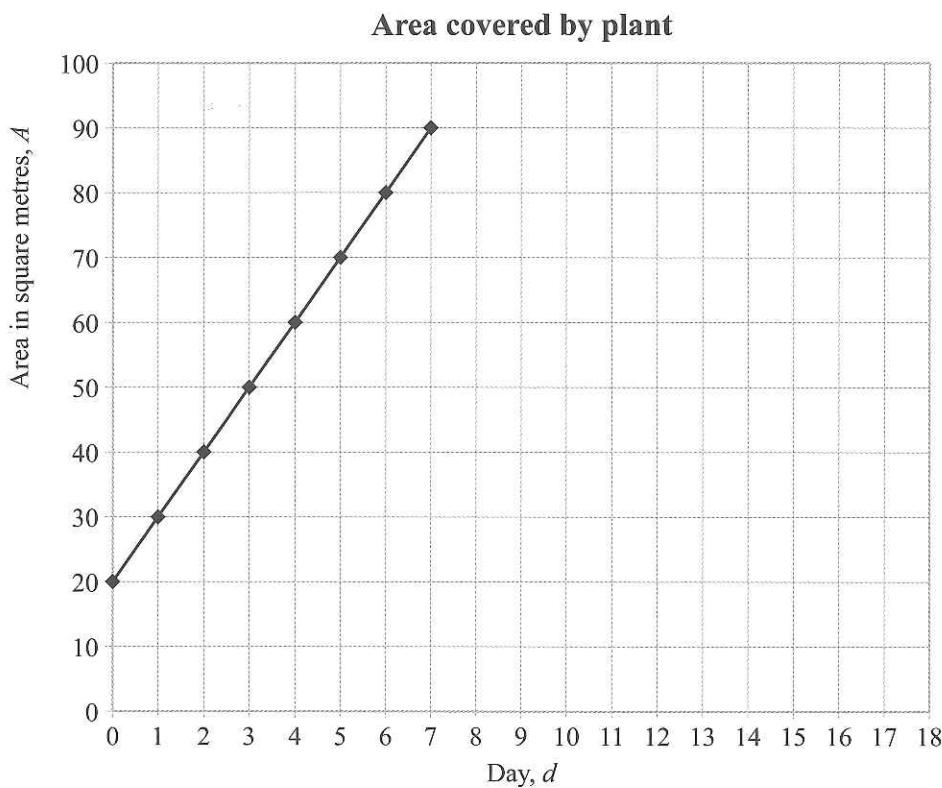
H

I don't know

M6

QUESTION TWO

The next year, when the plant begins to grow back, Hank tries to stop it from spreading across the pond so quickly. As soon as he notices the plant, he begins removing it. The graph of the area of pond covered by the plant in this year is shown below:



- (a) How much more area is the plant covering each day?

10 Square meters $\boxed{1}$

- (b) What day will it be when the plant covers 200 square metres if the conditions remain the same?

Show your working

$$Y = mx + c \quad Y = 10x + 20$$

$$A = 10d + 20$$

$$d = A$$

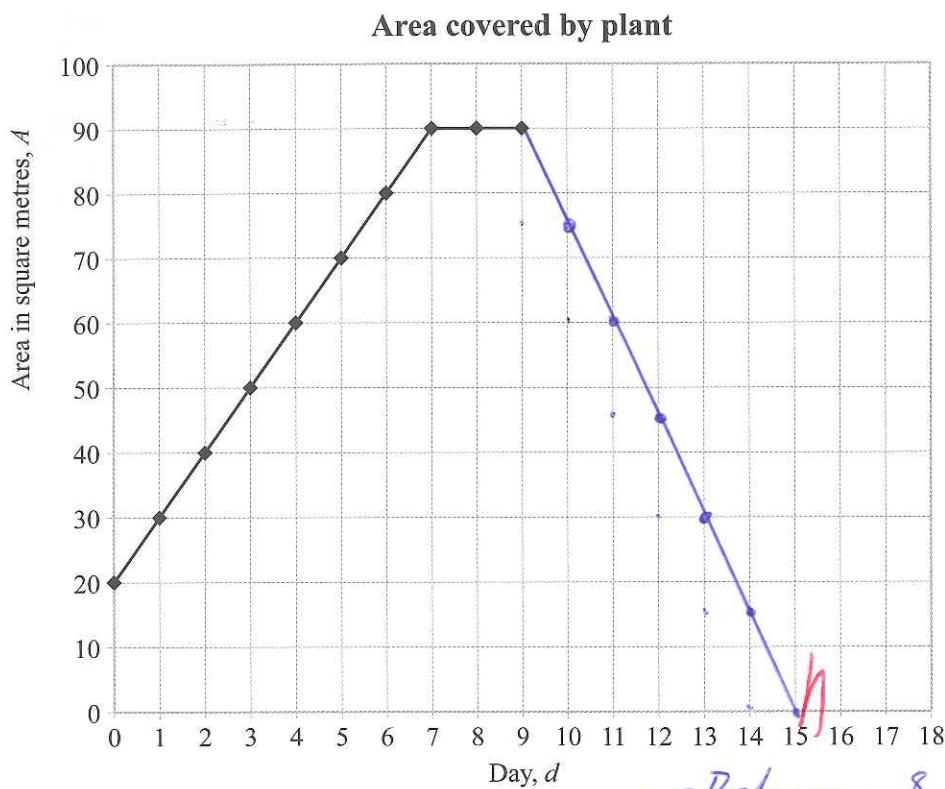
$$200 \text{ square metres} = 18 \text{ days}$$

Answered $\boxed{1}$

After 7 days removing some of the plant by himself, Hank decides to get help.

- (c) One friend helps on Day 8 and Day 9.

The area covered by the plant stays the same for Day 8 and Day 9.



If you
need to
redraw this
graph, use
the grid on
page 14.

- (i) What is the equation of this new section of the graph on Day 8 and Day 9?

$$y = 90 \text{ } h$$

- (ii) What does this section of the graph mean?

No increase of plant growth
between the start of day 8 and the
start of day 9

- (d) Two more friends come to help. Now the area covered by the plant decreases by 15 square metres each day until the plant is completely removed.

- (i) Draw a graph on the grid above to show the area of pond covered by the plant from Day 10.

~~crosses~~

- (ii) On what day will there be no plant left?

Day 15

- (e) The equation of the line for Day 9 onwards is $A = 225 - 15d$.

If Hank's 2 friends had come on Day 8, what would the equation of this line have been?

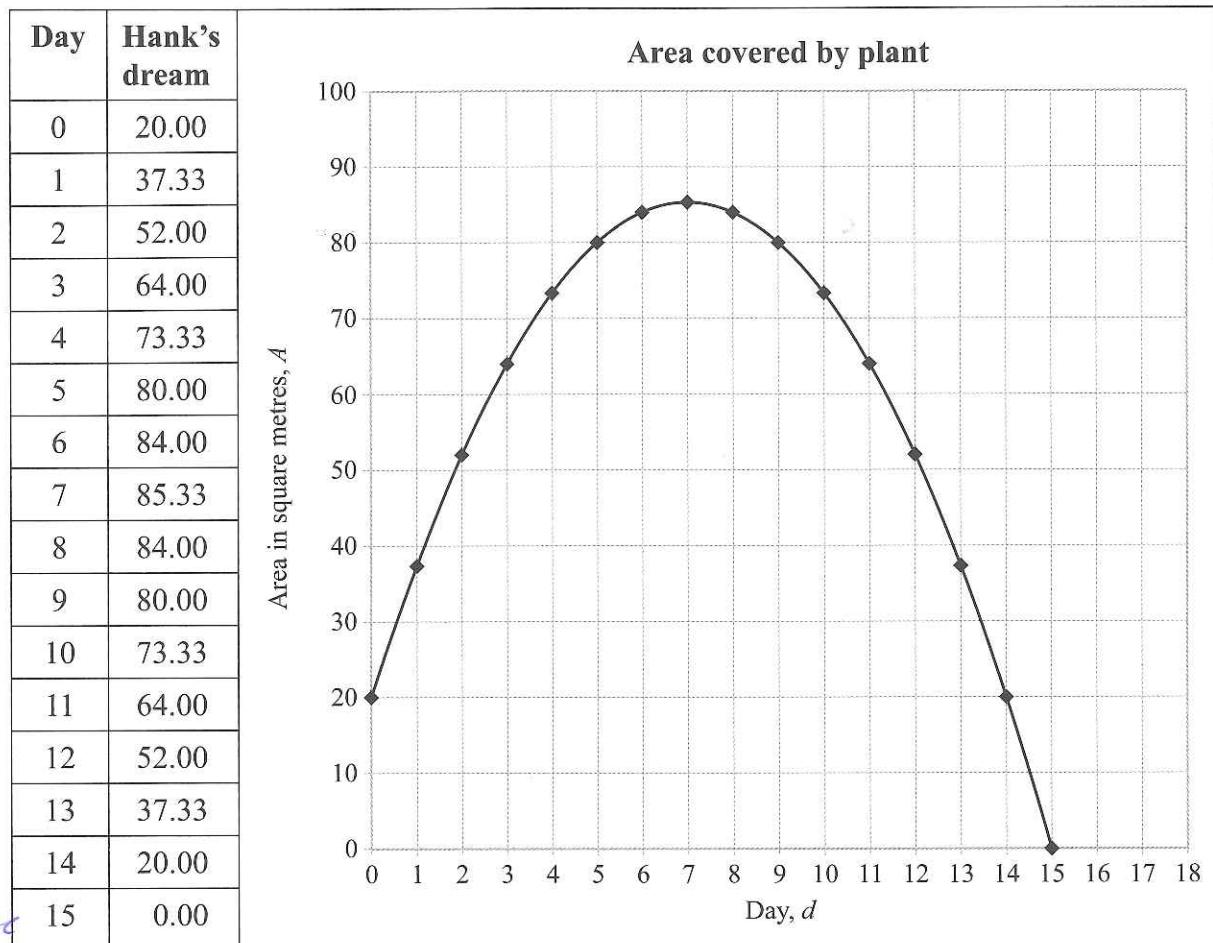
Explain your reasoning.

x



n

- (f) Hank had a dream that he and his friends made the area of the pond covered by the plant follow the parabola given below:



What is the equation of this graph?

$$Y = ax^2 + bx + c$$

A

$$Y = -1.3333248x^2 + 18.6665598x + 19.9990808 //$$

E7

QUESTION THREE

- (a) Jodie sets her friends a mathematical problem. She says:

I think of an integer

When I add 1 to my number, I get A

But if I take 4 off my number, I get B

When A is multiplied by B, I get an answer of 6.

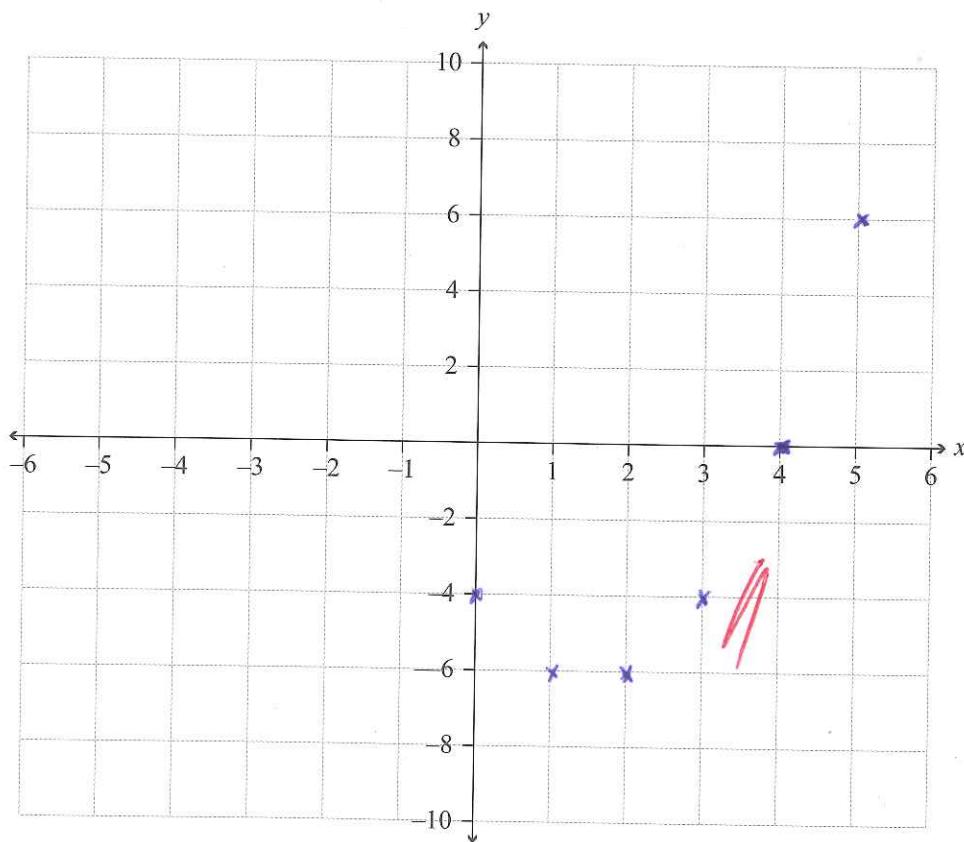
What's my number? 

Her friends start by writing a table:

x : Jodie's number	$A = x + 1$	$B = x - 4$	$y = AB$
0	1	-4	-4
1	2	-3	-6
2	3	-2	-6
3	4	-1	-4
4	5	0	0
5	6	1	6
6	7	2	14

- (i) Draw the graph of y against x .

Use the set of axes below.



- (ii) What is the equation of the graph that matches the table above, in terms of x ?

$$y = 1x^2 + -3 + -4$$

$$y = 1x^2 + -3 + -4$$

n

- (iii) Explain how Jodie's number can be found from the graph if the answer is 6.

Her number is x
 AB is 6 (Y) so If we
 look at 6 on Y another
 coordinate on 6Y is 5x
 then Ans Her number is 5



n

- (iv) Suppose Jodie had said "A multiplied by B gives me -10".

What does your graph tell you about the solutions to this new problem?

nothing really, graphs don't speak

n

- (b) Tom thinks of a puzzle to challenge Jodie.

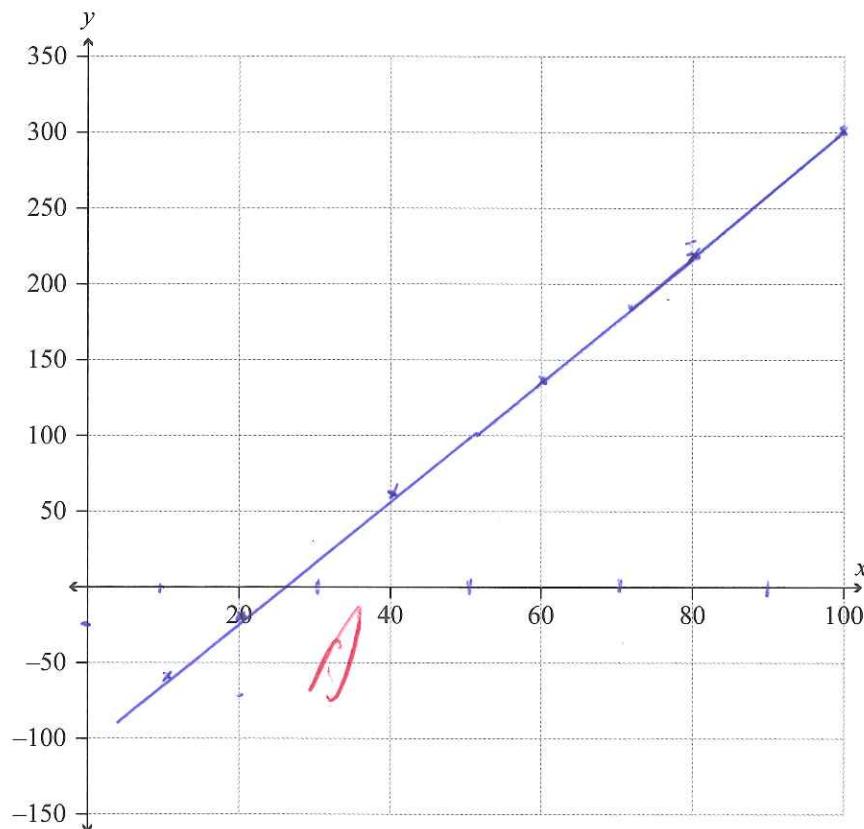
He starts by saying:

*I think of a two-digit number.
I multiply it by 4 and take away 100 ...*

- (i) What equation would you use to describe this relationship?

Linear

- (ii) Draw the graph of this relationship on the axes below.



If you
need to
redraw this
graph, use
the grid on
page 15

$$x = 2 \quad \cancel{x+4}$$

$$\text{or} \quad (x+4) \times 1.12$$

(iii) Tom's whole puzzle is:

Guess my 2-digit number:

If I multiply it by 4 and take away 100 ...

I get the same as when I add 47 to it and then multiply the result by 1.12

Explain how the solution to Tom's question can be found, and give the solution as accurately as possible.

ASSESSOR'S
USE ONLY

h

M5

Annotated Exemplar Template

Merit exemplar for 91028 2015			Total score	18
Q	Grade score	Annotation		
1	M6	<p>a(i). Graph correct.</p> <p>a(ii). Answer and explanation correct.</p> <p>a(iii). No equation.</p> <p>a(iv). Correct answer.</p> <p>b(i). Correct answer but candidate has missed “x” third equation. MEI (minor error ignored) because candidate understands what is required despite errors.</p> <p>b(ii). Candidate has been unable to link context and graph.</p>		
.2	E7	<p>a) Correct answer.</p> <p>b) Correct answer with evidence although minimal.</p> <p>c) Equation correct but explanation incorrect. Should refer to area rather than growth.</p> <p>d) Both parts correct.</p> <p>e) No attempt.</p> <p>f) Correct answer with very bad rounding from graphics calculator. This is because the numbers in the table are rounded.</p>		
3	M5	<p>a(i). Correct points and discrete for r.</p> <p>a(ii). Missing “x” beside -3 so n.</p> <p>a(iii). Some understanding of link between graph and solution but has missed second solution.</p> <p>a(iv). Incorrect.</p> <p>b(i). Incorrect.</p> <p>b(ii). Graph correct.</p> <p>b(iii). No attempt.</p>		