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Level 1 Mathematics and Statistics, 2017

91031 Apply geometric reasoning in solving problems

9.30 a.m. Monday 20 November 2017

Credits: Four

Achievement	Achievement with Merit	Achievement with Excellence
Apply geometric reasoning in solving problems.	Apply geometric reasoning, using relational thinking, in solving problems.	Apply geometric reasoning, using extended abstract thinking, in solving problems.

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

You should attempt ALL the questions in this booklet.

Show ALL working.

If you need more space for any answer, use the page(s) provided at the back of this booklet and clearly number the question.

Check that this booklet has pages 2–15 in the correct order and that none of these pages is blank.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

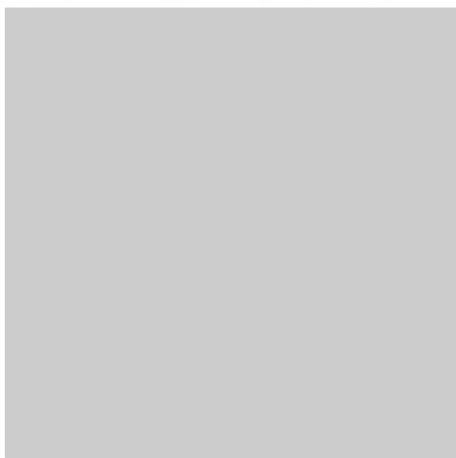
Merit

TOTAL

18

ASSESSOR'S USE ONLY

MOSAICS AND TESSELLATIONS

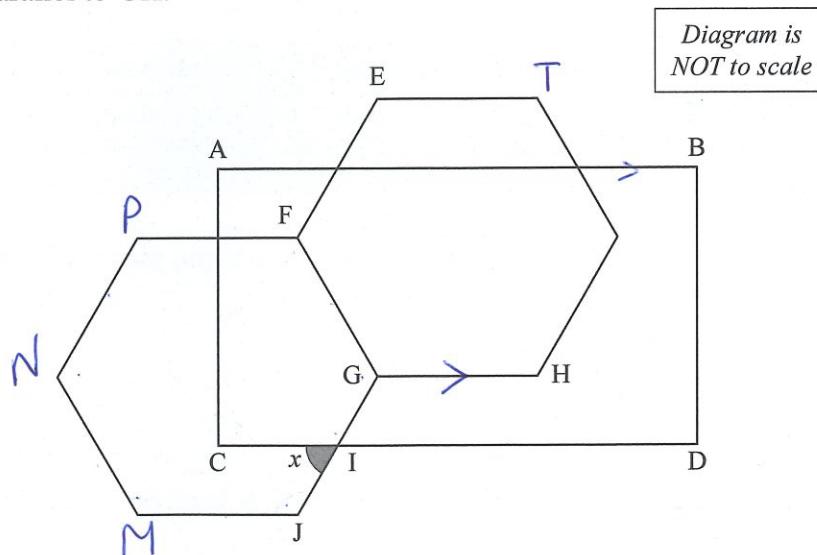


<http://mosaicsbypost.com/banded-stars/banded-stars-black-195m2>

A tessellation is a pattern of repeating shapes fitting together and leaving no gaps. These patterns can be made into mosaic pictures as shown above.

QUESTION ONE

- (a) The pattern below is made up of two regular hexagons with a rectangle overlaying them.
AB is parallel to GH.



Calculate the size, x , of angle CIJ.

Justify your answer with clear geometric reasoning.

$$\begin{aligned} \text{+) } \angle PFG &= \angle FGJ = \angle GJM = \angle JMN = \angle MNP = \angle NPF \quad \checkmark \\ &= \frac{(6-2) \times 180^\circ}{6} = \frac{720^\circ}{6} = 120^\circ. \end{aligned}$$

+) AB is \parallel to GH \Rightarrow AB is \parallel MJ \Rightarrow MJ is \parallel CI.

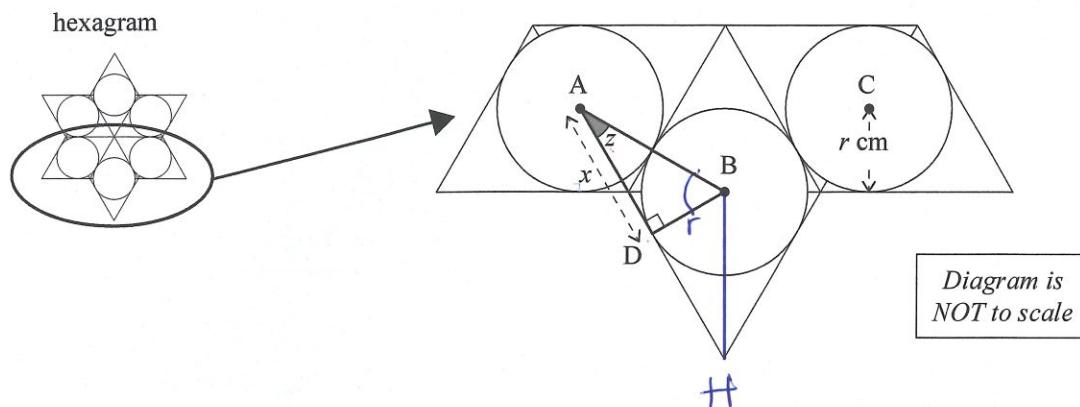
$$\begin{aligned} &\Rightarrow x + \angle MJG = 180^\circ \quad (\text{co-int L's add up to } 180^\circ \text{ // lines}) \\ &x + 120^\circ = 180^\circ \Rightarrow x = 60^\circ \quad // \end{aligned}$$

- (b) Circles can be drawn inside **half** a hexagram (which is a regular six-pointed star) as shown in the pattern below.

Points A, B, and C are the centres of the three circles.

The radius of all the circles is r cm.

A triangle ABD is drawn across two of the circles.



- (i) Prove that the angle z is equal to 30° .

Show your working clearly.

- (ii) Calculate the length, x , of the line AD in terms of r .

Show your working clearly.

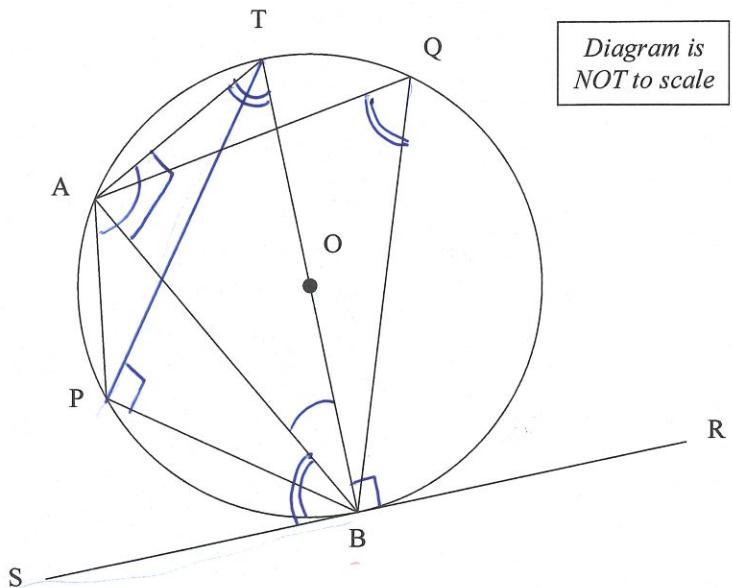
$$\tan Z = \frac{BD}{AD}$$

$$\Rightarrow \tan z = \frac{r}{x} \Rightarrow \tan z_{xx} = \frac{r}{x}$$

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(c) In the diagram below, the line SR is a tangent to the circle.

The line BT passes through O, the centre of the circle.



Prove that angle ABS equals angle AQB.

Justify your answer with clear geometric reasoning.

$$\Rightarrow \angle TAB = \angle AQB \text{ (L's = same arc)} \quad (1)$$

$$\Rightarrow \angle TAB = 90^\circ \text{ (L at semi circle).}$$

$$\Rightarrow \angle TBA + \angle ABS = \angle TBS = 90^\circ \quad (2)$$

$$\therefore \triangle TAB : \angle TAB + \angle TBA + \angle TAB = 180^\circ \text{ (sum of angles in a triangle)}$$

$$\Rightarrow 90^\circ + \angle TBA + \cancel{\angle TAB} = 180^\circ \quad \triangle = 180^\circ.$$

$$\Rightarrow \angle TBA + \cancel{\angle TAB} = 90^\circ. \quad ?$$

$$\text{which} : \angle ABS + \angle TBA = 90^\circ \quad (2)$$

~~$$\Rightarrow \angle TAB = \angle ABS.$$~~

~~$$\Rightarrow \angle ABS = \angle TAB. \quad ?$$~~

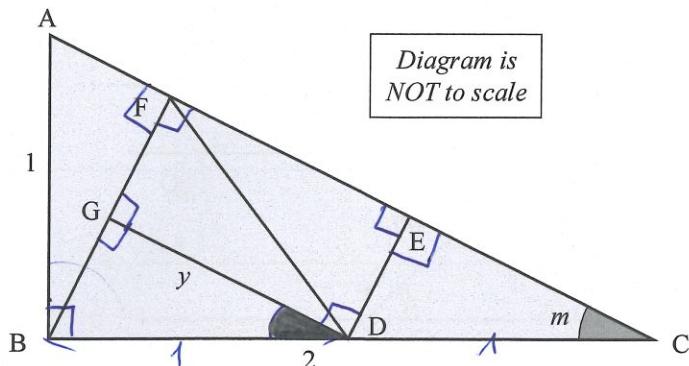
~~$$\text{which} : \angle TAB = \angle AQB \quad (1).$$~~

~~$$\Rightarrow \boxed{\angle ABS = \angle AQB}$$~~

QUESTION TWO

- (a) The shape below is an example of Pinwheel Tiling, where identical right-angled triangles are used to create a tessellation within a larger right-angled triangle.

AB is 1 unit long and BC is 2 units long.



- (i) Calculate the size, m , of angle ACB.

Show your working clearly.

$$\tan \angle ACB = \frac{AB}{BC} = \frac{1}{2}$$

$$\Rightarrow \angle ACB = 26.5651^\circ \\ \approx 26.57^\circ$$

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- (ii) Calculate the length, y , of the line GD.

Show your working clearly.

$$\therefore \angle GDB = \angle ECD = 26.57^\circ$$

$$\therefore BD = AB = 1 \text{ unit long.}$$

$$\therefore \triangle GBD : \cos \angle 26.57^\circ = \frac{GD}{BD}$$

$$\Rightarrow \cos 26.57^\circ = \frac{y}{1}$$

$$\Rightarrow y = \cos 26.57^\circ$$

$$\Rightarrow y = 0.8944 \text{ unit long.}$$

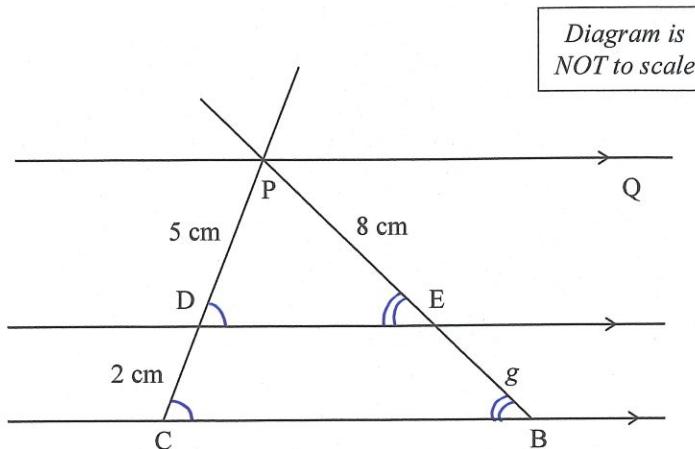
~~your answer~~

- (b) In the diagram below, PQ is parallel to the lines DE and CB.

PE is 8 cm long.

PD is 5 cm long.

DC is 2 cm long.



- (i) Calculate the length, g , of the line segment BE.

Show your working clearly.

$\Rightarrow \triangle PDE \text{ and } \triangle PCB : \angle BDE = \angle PCB \text{ (corresp L's =, || lines)}$

$\angle PED = \angle PBC \text{ (corresp L's =, || lines)}$

$\angle CPB$.

$\Rightarrow \triangle PDE \text{ and } \triangle PCB \text{ is similar triangle.}$

$$\Rightarrow \frac{PD}{PC} = \frac{PE}{PB} = \frac{DE}{CB}$$

$$\Rightarrow \frac{PD}{PC} = \frac{PE}{PB} \Rightarrow \frac{5}{5+2} = \frac{8}{PB}$$

$$\Rightarrow 5PB = 56$$

$$\Rightarrow PB = 11.2 \text{ cm.}$$

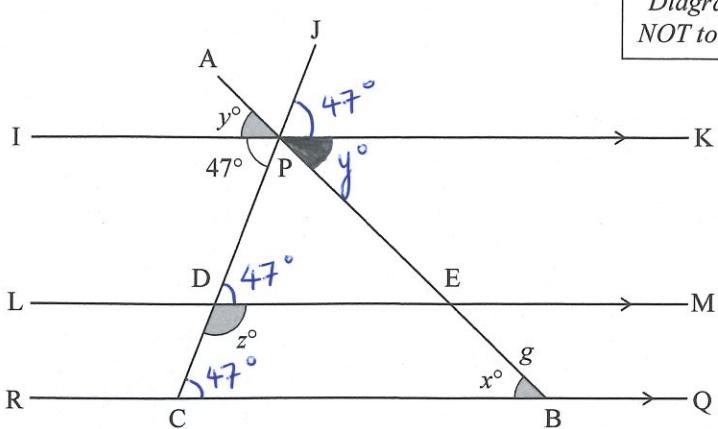
$$\Rightarrow PE + EB = PB$$

$$8 + g = 11.2$$

$$\Rightarrow g = 3.2 \text{ cm}$$

✓

(ii) Angle API is y° , angle CDE is z° and angle IPC is 47° .



*Diagram is
NOT to scale*

Express angle x in terms of y and z .

Justify your answer with clear geometric reasoning. $\Rightarrow \angle DPE = \angle APJ$ (vert opp \angle 's =).

$\Rightarrow \angle IPC = \angle JPK = 47^\circ$ (vert opp L's =).

$$+) \angle API = \angle kPE = y^\circ \text{ (vert opp L's =).}$$

$$\Rightarrow \angle API + \angle IPD + \angle DPE + \angle EPK + \angle KPJ + \angle JPA = 360^\circ \text{ (L's at a point add up to } 360^\circ\text{).}$$

$$\Rightarrow 2 \angle DPE + 2y^\circ = 360^\circ.$$

$$2 \text{ LDPE} + 2y^\circ = 266^\circ$$

$$2 \angle DPE = 266^\circ - 24^\circ$$

$$\Rightarrow LDPE = \frac{266^\circ - 24^\circ}{2}$$

$\Rightarrow \angle PDM = \angle PCB = 47^\circ$ (corresp \angle 's
 $\Rightarrow \parallel$ lines)

$\Rightarrow \triangle PCB: \angle CPB + \angle PCB + \angle PBC = 180^\circ$ (Lin D)

$$\Rightarrow \frac{266^\circ - 24^\circ}{2} + 47^\circ + \cancel{\text{LRB}} x^\circ = 180^\circ \text{ add to } 180^\circ$$

$$\rightarrow x^\circ + \frac{266^\circ - 24^\circ}{2} = 133^\circ.$$

$$x = \frac{133^\circ - 266^\circ - 24^\circ}{2}$$

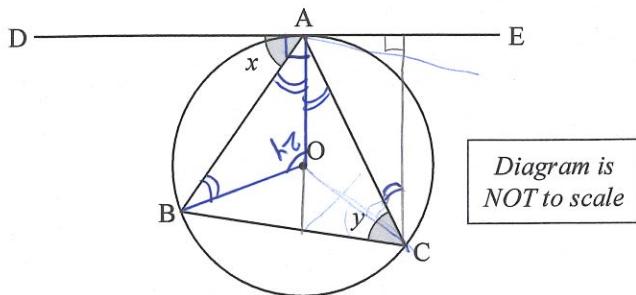
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<continue>
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- (c) A triangle ABC is drawn inside a circle.

O is the centre of the circle.

DE is a tangent to the circle. Point A is where DE touches the circle.



Prove that angle x equals angle y.

Justify your answer with clear geometric reasoning.

$$\text{+) } \angle AOB = 2 \angle ACB \text{ (Angle at center} = 2 \times \text{Angle at circumference)}$$

$$\rightarrow \angle AOB = 2y.$$

$$\text{+) } \triangle AOB \text{ is an isosceles triangle } (OB = OA \text{ radii})$$

$$\Rightarrow \angle OAB = \angle OBA = \frac{180^\circ - 2y}{2} = 90^\circ - \frac{2y}{2}$$

$$= 90^\circ - \frac{y}{2}$$

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M6

This page has been deliberately left blank.
The examination continues on the following page.

QUESTION THREE

- (a) In the diagram alongside:

BC and BE are radii of the circle centre B.

GI is a tangent to the circle.

Angle CEG is 38°

- (i) Calculate the size, x , of angle BCE.

Justify your answer with clear geometric reasoning.

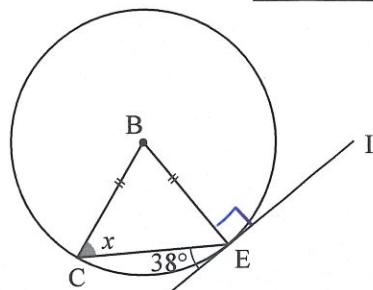
$$\therefore \angle GEC + \angle CEB + \angle BEI = 180^\circ$$

$$\Rightarrow \angle CEB = 180^\circ - 38^\circ - 90^\circ = 52^\circ \quad \textcircled{1}$$

$\therefore \triangle BCE$ is an isosceles triangle ($BC = BE$).

$$\Rightarrow \angle BCE = \angle CEB \quad \textcircled{2}$$

$$\text{From } \textcircled{1} \text{ and } \textcircled{2} \Rightarrow \angle BCE = x = 52^\circ$$



Two overlapping circles, with centres A and B and the same radii, are drawn in a quadrilateral which is symmetrical through HG.

The distance between A and B is equal to the radius of the circles.

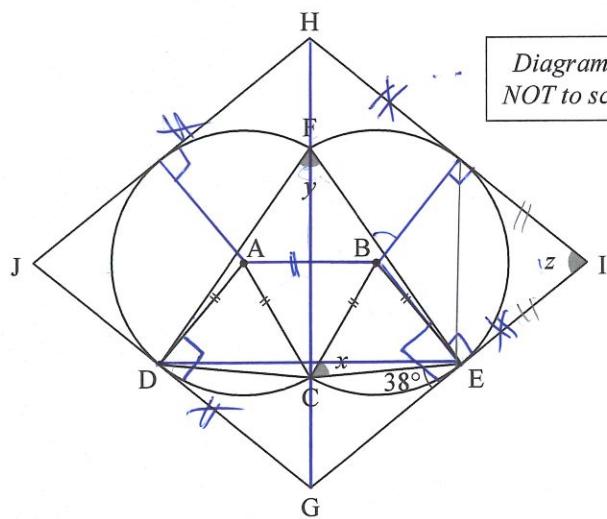
HIGJ is a rhombus.

HI, IG, GJ, and HJ are tangents to the circles.

Angle CEG is 38° .

AD is perpendicular to JG and BE is perpendicular to GI.

*Diagram is
NOT to scale*



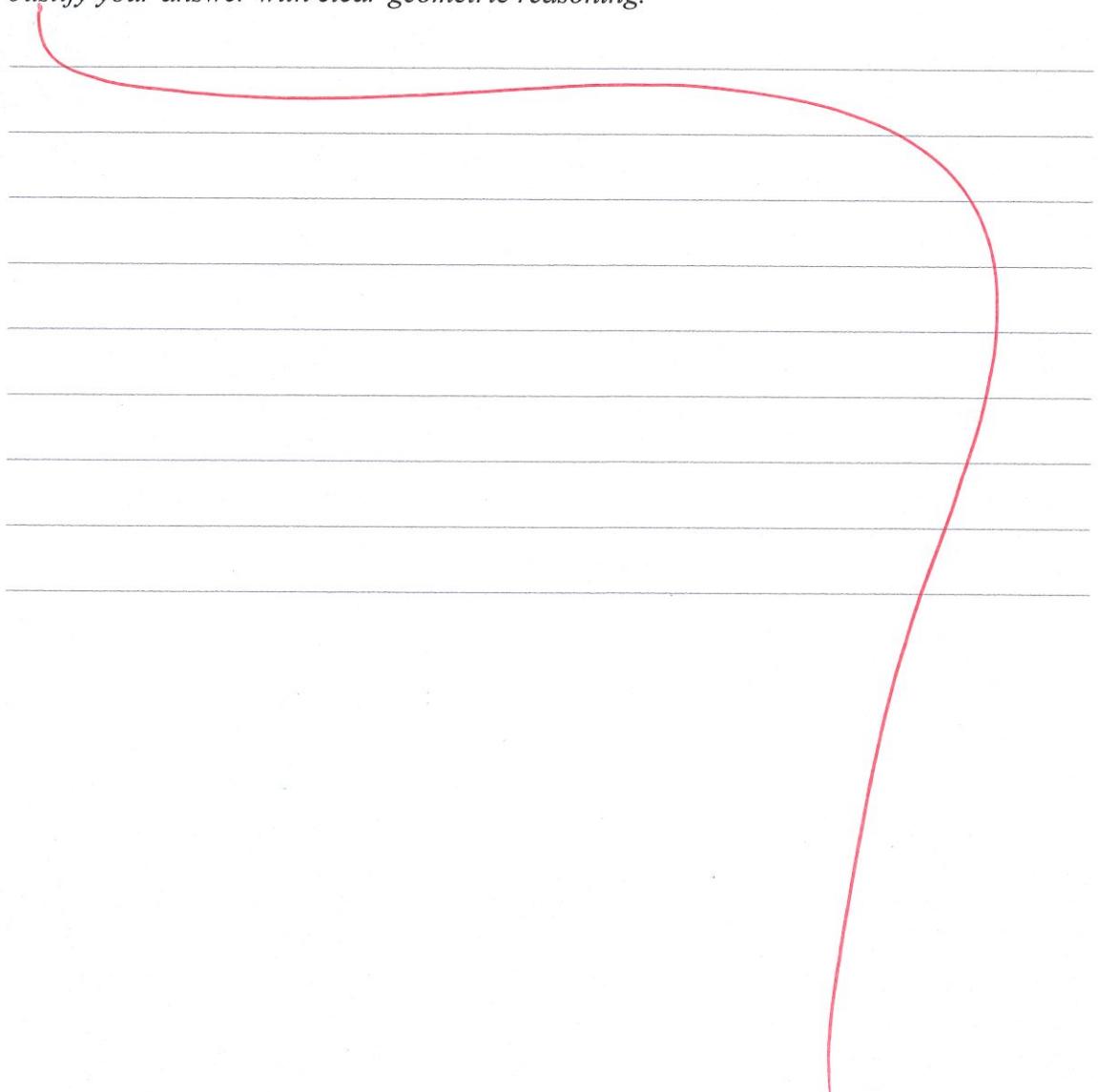
- (ii) Calculate the size, y , of angle DFE.

Justify your answer with clear geometric reasoning.



- (iii) Calculate the size, z , of angle HIG.

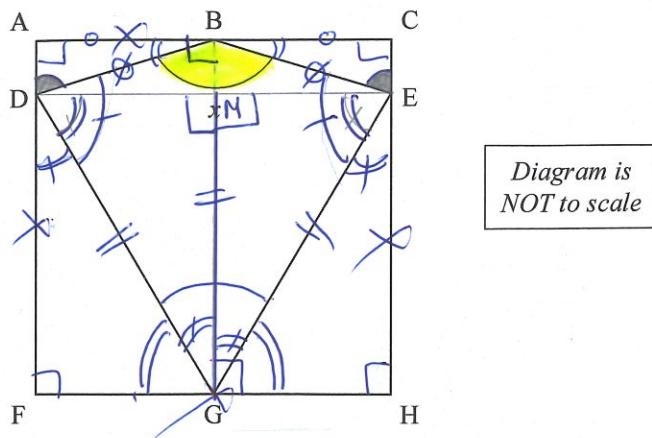
Justify your answer with clear geometric reasoning.



**Question Three continues
on the following page.**

- (b) The kite GDBE is placed in the square ACHF.

$$DG = GB = EG$$



Calculate the size, x , of angle DBE.

Justify your answer with clear geometric reasoning.

$$\rightarrow \angle FGD + \angle DGE + \angle EGH = 180^\circ \quad (\text{L's on a line add to } 180^\circ) \quad \textcircled{1} \quad \text{NS}$$

$$\begin{aligned} \rightarrow \triangle DGE &\text{ is isosceles triangle } (DG = GE) \\ \rightarrow \angle DEG &= \angle EDG. \end{aligned}$$

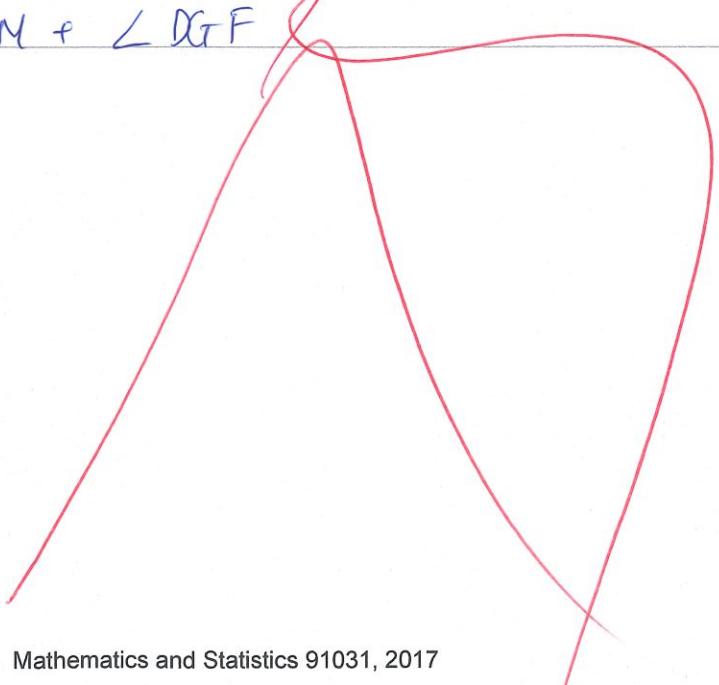
$$\begin{aligned} \rightarrow \triangle DGE &\text{ is isosceles : } GM \perp DE \\ \rightarrow \angle DGM &= \angle EGN \quad \textcircled{2} \end{aligned}$$

$$\rightarrow \angle FGD = \angle EGH. \quad \textcircled{3}$$

From $\textcircled{1}$, $\textcircled{2}$, $\textcircled{3} \Rightarrow$

$$\begin{aligned} \rightarrow 2\angle DGM + 2\angle DGF &= 180^\circ. \\ \rightarrow \angle DGM + \angle DGF &= 90^\circ. \end{aligned}$$

$$\rightarrow \angle DGM + \angle DGF$$



Extra paper if required.
Write the question number(s) if applicable.

2 (b ii)

$\Rightarrow LN \text{ is } \parallel RQ \Rightarrow z + \angle PCB = 180^\circ$
(\angle int L's add up to 180°).

$\Rightarrow \triangle PCB \cong \triangle PBC$

$$\angle PBC + z + \angle PCB + \frac{266^\circ - 2y}{2} = 180^\circ$$

$$\Rightarrow \angle PBC = 180^\circ - z - \angle PCB - \frac{266^\circ - 2y}{2}$$

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Subject:		Mathematics and Statistics	Standard:	91031	Total score:	18
Q	Grade score	Annotation				
1	E7	This is E7 because clear geometric reasoning has been used to "Prove that angle ABS equals angle AQB". The reasoning is easy to follow because: <ul style="list-style-type: none"> • one line is used for each statement; • "=" signs are used appropriately; • most statements are made <i>together with</i> the appropriate reason, or clearly referenced back to an earlier statement. It is not E8, because in (b)ii, the value of z was not used to evaluate the value of x as far as possible in terms of r.				
2	M6	This is M6 because clear mathematical statements, trigonometry and geometric reasoning have all been used to find the sizes of angles and the lengths of lines. The reasoning is easy to follow because: <ul style="list-style-type: none"> • one line is used for each statement; • "therefore" and "because" are used to link statements; • "=" signs are used appropriately; each statement is made <i>together with</i> the appropriate reason. To reach E7, the proof in (c) would need to have been completed or, an expression for x in terms of y and z in (b)ii would need to have been formed.				
3	M5	This is M5 because clear mathematical statements and geometric reasoning were used to find the size of angle x in (a)i. To reach M6 another chain of correct geometric reasoning needed to be used to find the size of at another angle in another part of the question.				