Decentralized Adaptive Event-Triggered H_∞ Filtering for a Class of Networked Nonlinear Interconnected Systems

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***Abstract*—This paper focuses on the issue of designing an adaptive event-triggered scheme to the decentralized filtering for a class of networked nonlinear interconnected system. A novel adaptive event-triggered condition is proposed by constructing an adaptive law for the threshold. This new type of threshold mainly depends on the error between the states at the current sampling instant and the latest releasing instant, by which the data release rate is adapted to the variation of the system. The limitation of network bandwidth is alleviated on account of a large amount of "unnecessary" packets being dropped out before accessing the network. Sufficient conditions are derived such that the overall filtering error system under the proposed adaptive data-transmitting scheme is asymptotically stable with a prescribed disturbance attenuation level. An example is given to show the effectiveness of the proposed scheme.**

***Index Terms*—Adaptive event-triggered scheme, filtering, nonlinear networked interconnected system, Takagi–Sugeno (T–S) fuzzy model.**

I. Introduction

LARGE-SCALE system has gained a growing attention since the 1970s [1], [2]. It has many practical applications, such as transportation systems [3], power systems [4], multiagent systems [5], aerospace vehicles [6], network of Chua's chaotic circuits [7], and ecosystems [8]. Such a system is composed of several connected low-order subsystems, which is also called as an interconnected system. A common feature of such a system is that the subsystems or components are usually widely located in space [9]–[12]. Decentralized strategy allows the control implementation to be more feasible and flexible. But signal transmission becomes more complicated when using point-to-point wired connection under this control strategy. Wireless/wired communication network is an alternative to transform the control system into a networked control system (NCS). The information is exchanged over the network among subsystems and control components. However, the limitation of network bandwidth or some other factors restraining the network, such as, the consumption of power while using wireless network, will lead to increasing difficulties and challenges of analysis and synthesis [10], [13].

Nonlinearity is an inherent nature of many practical systems that should not be neglected in modeling system. In the past decades, research on control/estimation problems for nonlinear systems has received considerable attention [14]–[17]. Takagi–Sugeno (T–S) fuzzy model technique has been proved to be a successful approach in dealing with the problem of modeling and analyzing nonlinear dynamic control systems [18]–[20]. Based on T–S fuzzy mode, an H_∞ filter design for a class of continuous-time/NCSs with multiple state-delays was investigated in [14]. The objective of H_∞ filtering is to design an estimator for a given system such that the $\mathcal{L}_2$ gain from the exogenous disturbance to the estimation error is less than a given level [21]. Xu *et al.* [22] investigated the H_∞ filtering for discrete-time nonlinear systems using T–S fuzzy model with consideration of multiple sensor faults. The sensor saturation and missing measurements were considered in the filtering of T–S fuzzy NCSs in [23] and [24]. For NCSs, there are three ways to improve the performance of the control system.

1) By improving the method of control design to adapt network inherent defectives, such as packet drop-out, network-induced delay, etc.
2) By enhancing the network quality of service (QoS), such as the method of quantization, event-triggered mechanism, etc.
3) By co-designing method.

Zhang *et al.* [25] studied the problem of quantized H_∞ filtering for T–S fuzzy systems. However, quantization method is not

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a efficient way to mitigate the burden of network bandwidth, especially for large-scale systems with a large amount of information to be transmitted, since the number of packets released into the network cannot be decreased. Recently, the issue of event-triggered scheme (ETS) to NCSs has so far attracted much attention. Under this scheme, the packet is transmitted over the network only when a specified threshold is exceeded. Consequently, it results in a great reduction of the amount of releasing data into the network. Meanwhile, the energy consumption is reduced as well owing to the event-triggered releasing mechanism with a lower releasing frequency. It is a crucial problem for a wireless sensor network. The input-to-state stability of a nonlinear system was investigated in [26] based on ETS. A self-triggered feedback control strategy and its extended result under ETS for decentralized NCSs were proposed in [27] and [28], respectively. The authors aimed to co-design the controller and ETS for NCSs by using a discrete event-triggered communication scheme (DETCS) in [16], [29], and [30]. In comparison with the problem of stabilization, the filter design is much more complicated, especially for large scaled systems. From the published literature, few results are in reference to the problem of filtering of nonlinear interconnected systems based on the ETS, which gives rise to the motivation for our current investigation.

It is known that the threshold of ETS/DETCS plays a significant role in deciding whether or not to release the data into the network. For example, the threshold with a small positive constant yields to the case of time-triggered scheme as in [29]. Even worse, Zeno behavior may occur using the method in [28] with an unsuitable threshold, which is unacceptable for hardware. Obviously, it would be better for the threshold regulating with the disturbance of the system, network QoS, etc., to improve the performance of the system adaptively. Note that the threshold in the existing literature was a prior predetermined constant. To the best of authors' knowledge, there is no related research on the issue of adaptive ETS for the filtering of interconnected systems, which is another motivation of this paper.

This paper is devoted to investigating an adaptive ETS for the filtering of networked nonlinear interconnected systems to save the limited network resource while maintaining the desired filter performance. Under the proposed adaptive ETS, a co-design method to decentralized H_∞ filtering of T–S fuzzy model-based interconnected systems is put forward. The main contributions of this paper are summarized as follows.

1) A framework of decentralized H_∞ filter is developed by introducing a distributed adaptive event-triggered generator (AETG).
2) A novel adaptive ETS is proposed by constructing a new adaptive law of threshold, which is accommodated with to the variation of the system.
3) A co-design method is developed with the aid of a new Lyapunov function to achieve the parameters of both filter and adaptive ETS. Furthermore, simulation results illustrate the proposed adaptive ETS has a good effect on a tradeoff between the occupied network resources and the desired filter performance.

II. Problem Statement

In this paper, we are in a position to investigate a novel adaptive ETS for the filtering of networked nonlinear interconnected systems. As shown in Fig. 1, the signals are transferred via a communication network; zero-order hold (ZOH) holds the value received from the network till the next packet arrives. The filter estimates the output of the system in the light of the signals transmitted over the network. To mitigate the burden of network-bandwidth, an AETG is introduced in this paper to decide whether or not the sampling data is necessary for the filter. For this purpose, the threshold of ETS is improved to make it regulate with the error of sampling data adaptively such that a good performance of the filter can be preserved.

A. Physical Plant

Consider a nonlinear interconnected system which is composed of J subsystems. The subsystem S_i ($i \in \mathcal{J} := \{1, 2, \ldots, J\}$) is modeled by IF-THEN fuzzy rules as follows.

Plant Rule p: IF $\theta_{i1}(t)$ is $W_{i1}^p, \ldots$ and $\theta_{ig_i}(t)$ is $W_{ig_i}^p$ THEN

$$\begin{cases} \dot{x}_i(t) = A_{ip}x_i(t) + \sum_{j=1,j\neq i}^{J} B_{ijp}x_j(t) + D_{ip}v_i(t) \\ y_i(t) = C_{ip}x_i(t) \\ z_i(t) = E_{ip}x_i(t) \end{cases} \tag{1}$$

where $p \in \Psi := \{1, \ldots, r_i\}$ denotes the pth fuzzy inference rule; r_i is the number of inference rules of subsystem S_i; W_{is}^p ($s = 1, 2, \ldots, g_i$) is the fuzzy set; $x_i(t) \in \mathbb{R}^{n_{xi}}$ is the state vector of subsystem S_i; $y_i(t) \in \mathbb{R}^{n_{yi}}$ is the measurement output of subsystem S_i; $z_i(t) \in \mathbb{R}^{n_{zi}}$ is the signal to be estimated and $v_i(t) \in \mathbb{R}^{n_{vi}}$ is the disturbance input which belongs to $l_2[0, \infty)$; and $A_{ip}, B_{ijp}, C_{ip}, D_{ip}$, and E_{ip} are known matrices with appropriate dimensions, where B_{ijp} represents the interconnection between the ith and the jth subsystem.

Denote $\theta_i(t) = [\theta_{i1}(t), \theta_{i2}(t), \ldots, \theta_{ig_i}(t)]^T$. The overall dynamic fuzzy model of subsystem S_i can be inferred by using the center-average defuzzifier, product inference and singleton fuzzifier as follows:

$$\begin{cases} \dot{x}_i(t) = \sum_{p=1}^{r_i} h_{ip}(\theta_i(t))\Big[A_{ip}x_i(t) + \sum_{j=1,j\neq i}^{J} B_{ijp}x_j(t) + D_{ip}v_i(t)\Big] \\ y_i(t) = \sum_{p=1}^{r_i} h_{ip}(\theta_i(t))C_{ip}x_i(t) \\ z_i(t) = \sum_{p=1}^{r_i} h_{ip}(\theta_i(t))E_{ip}x_i(t) \end{cases} \tag{2}$$

with

$$h_{ip}(\theta_i(t)) = \frac{\mu_{ip}(\theta_i(t))}{\sum_{p=1}^{r_i} \mu_{ip}(\theta_i(t))}, \quad \mu_{ip}(\theta_i(t)) = \prod_{s=1}^{g_i} W_{is}^p(\theta_{is}(t))$$

where $W_{is}^p(\theta_{is}(t))$ being the grade membership value of $\theta_{is}(t)$ in W_{is}^p, and $h_{ip}(\theta_i(t))$ satisfies

$$h_{ip}(\theta_i(t)) \geq 0, \quad \sum_{p=1}^{r_i} h_{ip}(\theta_i(t)) = 1. \tag{3}$$

Here, we assume the system (2) is well controlled. Our task is to design a filter to estimate the signal $z_i(t)$ of each

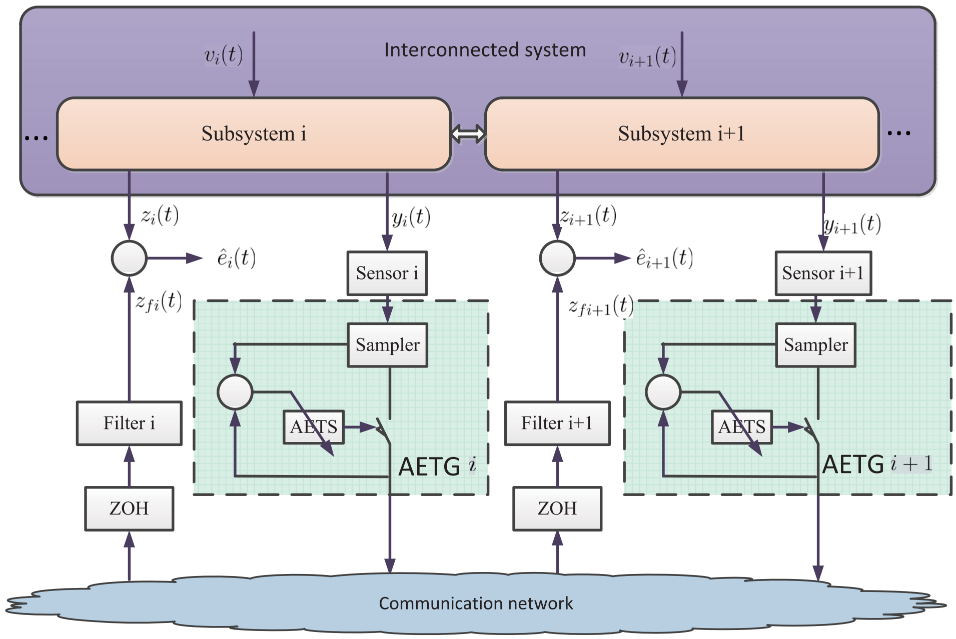

Fig. 1. Framework of the adaptive ETS-based filtering of interconnected systems.

subsystem with some external disturbances precisely. The full-order decentralized fuzzy filter is considered as follows:

$$\begin{cases} \dot{x}_{fi}(t) = \sum_{q=1}^{r_i} h_{iq}(\theta_i(t))\big[A_{fiq}x_{fi}(t) + B_{fiq}\hat{y}_i(t)\big] \\ z_{fi}(t) = \sum_{q=1}^{r_i} h_{iq}(\theta_i(t))L_{fiq}x_{fi}(t) \end{cases} \tag{4}$$

where $x_{fi}(t)$ is the state vector of the filter; $z_{fi}(t)$ is the estimation of $z_i(t)$; the input of the filter $\hat{y}_i(t)$ is the measurement of the subsystem $y_i(t)$ transmitted over the network; and A_{fiq}, B_{fiq}, and L_{fiq} are filter parameters to be determined.

For notational simplicity, h_{ip} and h_{iq} are used to represent $h_{ip}(\theta_i(t))$ and $h_{iq}(\theta_i(t))$, respectively, in the subsequent description.

B. Adaptive Data Releasing Scheme

For clear elaboration of the scheme of adaptive ETS, an example of time sequence of data packet is given, which is shown in Fig. 2. "★," "•," and "∅," in this figure, represent the releasing instant, arriving instant and the instant of packet-dropping, respectively. The set $\{s_k\}_{k=0}^{\infty} = \{0, 1, 2, 3, 4, 5, 6, 7, \ldots\}$ denotes the sequence of sampling instants, and $\{r_k^i\}_{k=0}^{\infty} = \{0, 2, 3, 7, \ldots\}$ is the set of the sequence of releasing instants of the subsystem S_i. Obviously, the set $\{r_k^i\}_{k=0}^{\infty}$ is a subset of $\{s_k\}_{k=0}^{\infty}$. The input of the filter keeps the value of the latest released packet till a new packet updates the old one thanks to ZOH, therefore, for $t \in [r_k^i h + \tau_{r_k^i}, r_k^i h + \tau_{r_{k+1}^i}) \triangleq \mathscr{L}_{r_k^i}$, we have

$$\hat{y}_i(t) = y_i\big(r_k^i h\big) \tag{5}$$

where h is the sampling period. $\tau_{r_k^i}$ with $\underline{\tau}_i \le \tau_{r_k^i} \le \bar{\tau}_i$ is the network-induced delay at the instant $r_k h$ of the subsystem S_i. To release the packet from the sampler into the network or not depends on whether the current sampled packet at instant $r_k^i h + lh$ $(l = 0, 1, 2, \ldots)$ invokes the following condition:

$$\begin{aligned} &\big(y_i\big(r_k^i h\big) - y_i\big(r_k^i h + lh\big)\big)^T \Phi_i\big(y_i\big(r_k^i h\big) - y_i\big(r_k^i h + lh\big)\big) \\ &\quad - \varsigma_i(t)y_i^T\big(r_k^i h + lh\big)\Phi_i y_i(r_k^i h + lh) < 0 \end{aligned} \tag{6}$$

where $\Phi_i > 0$, $0 < \varsigma_i(0) < 1$, and $\varsigma_i(t)$ is an error-dependent threshold function.

Remark 1: An inspection should be taken to each sampled data by AETG before entering the network to decide whether it is a necessary packet for the filter. The packets at instant $1h, 4h, 5h, 6h, \ldots$, in Fig. 2, are discarded due to the event-triggering condition in (6) not being invoked.

Assume the lth data packet is the last packet satisfying the condition (6) after the latest releasing instant $r_k^i h$, then the next packet to be released into the network is triggered at

$$r_{k+1}^i h = r_k^i h + (\bar{l} + 1)h. \tag{7}$$

Taking the networked induced delay into account, we partition the interval $\mathscr{L}_{r_k^i}$ into $\bar{l} + 1$ segments as $\mathscr{L}_{r_k^i}^l$ $(l = 0, 1, \ldots, \bar{l})$, where $\mathscr{L}_{r_k^i}^l \triangleq [r_k^i h + lh + \tau_{r_k^i}^l, r_k^i h + lh + h + \tau_{r_k^i}^{l+1})$ with $\tau_{r_k^i}^0 = \tau_{r_k^i}$, $\tau_{r_k^i}^{\bar{l}+1} = \tau_{r_{k+1}^i}$ and $\tau_{r_k^i}^l = \hat{\tau}_{r_k^i}$ for $l = 1, \ldots, \bar{l}$. It yields $\mathscr{L}_{r_k^i} = \cup_{l=0}^{\bar{l}} \mathscr{L}_{r_k^i}^l$.

Remark 2: $\tau_{r_k^i}^l$ for $l = 0$ or $\bar{l} + 1$ is a real communication delay at instant $r_k^i h$ or $r_{k+1}^i h$, while $\hat{\tau}_{r_k^i}$ is an artificial delay which is defined for the requirement of analysis.

The threshold of event-triggering condition in this paper is an error-dependent function, which is decided by

$$\dot{\varsigma}_i(t) = \frac{1}{\varsigma_i(t)}\left(\frac{1}{\varsigma_i(t)} - \phi_i\right)e_i^T(t)\Phi_i e_i(t) \tag{8}$$

with $0 < \varsigma(0) \le 1$ and a given constant $\phi_i(1 < \phi_i)$ for $t \in \mathscr{L}_{r_k^i}^l$, where $e_i(t) = y_i(r_k^i h) - y_i(r_k^i h + lh)$ denotes the absolute

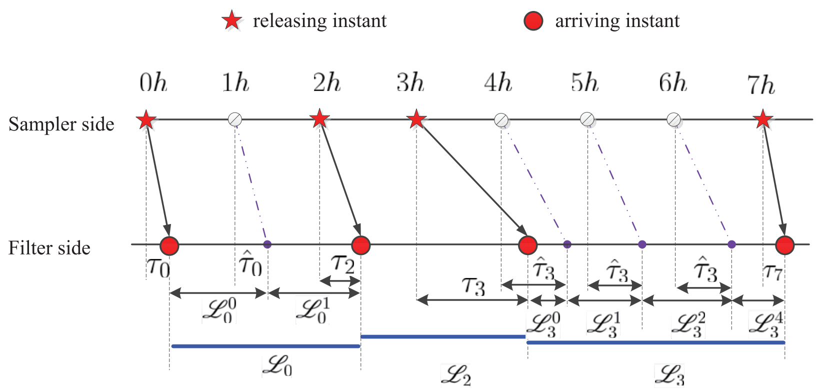

Fig. 2. Example of time sequence of adaptive ETS.

error between the states at the current sampling instant and the latest releasing instant.

Remark 3: From (6), one can see how important it is to take a proper value of the threshold for the conventional ETS. For example, if one chooses $\varsigma_i(t) \to 0$, the condition will be invoked at each sampling instant due to $\bar{l} = 0$, which gives rise to $r^i_{k+1} = r^i_k + 1$. That means the case reduces to a time-triggered scheme. The condition (6) will be more easily satisfied if $\varsigma_i(t)$ takes a bigger value. It results in a lower releasing rate. A poor filtering performance will be achieved for lack of input information under this situation. Therefore, it is reasonable for the threshold being a variable while not a predetermined constant as in the existing literature.

Remark 4: From the adaptive law in (8), one can see that the threshold is depended on the dynamic error of the measurements between the latest released packet and the current sampled packet. When the error tends to zero, for example, the system tends to be stable at the equilibrium, the threshold then keeps a constant, that is to say, the threshold is regulated with the dynamic error. If the error is disappeared, the regulation is ended.

Remark 5: In [29] and [31], the threshold ς_i is a predetermined constant satisfying $0 < \varsigma_i < 1$. If one sets $\phi_i = (1/\varsigma_i(0))$ in (8) yields $\varsigma_i(t) \equiv \varsigma_i(0)$, then it reduces to the case of DETCS as in [29] and [31]. It should be pointed out that $\varsigma_i(t)$, in this paper, is a result of adaptive regulation. If $\varsigma_i(t) < 0$ means that the packet at this instant should be released, for the triggering condition in (6) is not satisfied under this case.

C. Adaptive ETS-Based Filtering Error System

For $t \in \mathscr{L}^l_{r^i_k}$, we define $\eta_i(t) = t - (r^i_k h + lh)$, then it is true that

$$0 \le \eta_{i1} \le \eta_i(t) \le \bar{\tau}_i + h = \eta_{i2}. \tag{9}$$

Therefore, the dynamic of filter in (4) can be rewritten as

$$\dot{x}_{fi}(t) = \sum_{p=1}^{r_i}\sum_{q=1}^{r_i} h_{ip}h_{iq}\big[A_{fiq}x_{fi}(t) + B_{fiq}e_i(t) + B_{fiq}C_{ip}x_i(t-\eta_i(t))\big]. \tag{10}$$

Defining new state vectors $\xi(t) = [x_i(t)\ x_{fi}(t)]^T$ and $\hat{e}_i(t) = z_i(t) - z_{fi}(t)$, one can get the filtering error subsystem for $t \in \mathscr{L}^l_{r^i_k}$ as follows:

$$\begin{cases} \dot{\xi}_i(t) = \sum_{p=1}^{r_i}\sum_{q=1}^{r_i} h_{ip}h_{iq}\Big[\mathcal{A}_{ipq}\xi_i(t) + \mathcal{B}^1_{iq}e_i(t) \\ \qquad + \mathcal{B}^2_{ipq}x_i(t-\eta_i(t)) \\ \qquad + \sum_{j=1,j\ne i}^{J}\mathcal{B}^3_{ijp}x_j(t) + \mathcal{D}_{ip}v_i(t)\Big] \\ \hat{e}_i(t) = \sum_{p=1}^{r_i}\sum_{q=1}^{r_i} h_{ip}h_{iq}\mathcal{E}_{ipq}\xi_i(t) \end{cases} \tag{11}$$

where

$$\mathcal{A}_{ipq} = \begin{bmatrix} A_{ip} & 0 \\ 0 & A_{fiq}\end{bmatrix}, \mathcal{B}^1_{iq} = \begin{bmatrix} 0 \\ B_{fiq}\end{bmatrix}, \mathcal{B}^2_{ipq} = \begin{bmatrix} 0 \\ B_{fiq}C_{ip}\end{bmatrix}$$
$$\mathcal{B}^3_{ijp} = \begin{bmatrix} B_{ijp} \\ 0\end{bmatrix}, \mathcal{D}_{ip} = \begin{bmatrix} D_{ip} \\ 0\end{bmatrix}, \mathcal{E}_{ipq} = \begin{bmatrix} E_{ip} & -L_{fiq}\end{bmatrix}.$$

$\hat{e}_i(t)$ in (11) tending to zero means that the output of the filter can estimate the one of the system perfectively. Therefore, the objective of this paper is to co-calculate the parameters of both filter in (4) and adaptive ETS in (6) such that the overall filtering error system (11) with the proposed adaptive ETS is asymptotically stable. The overall H_∞ performance satisfies

$$\left\|\hat{e}(t)\right\|_2 < \gamma\|v\|_2 \tag{12}$$

for all nonzero $v(t) \in l_2[0,\infty)$ under zero initial condition, where $\gamma > 0$ is a prescribed scalar, $\hat{e}(t) = [\hat{e}^T_1(t)\ \hat{e}^T_2(t),\dots,\hat{e}^T_J(t)]^T$, and $v(t) = [v^T_1(t)\ v^T_2(t),\dots,v^T_J(t)]^T$.

III. Adaptive ETS-Based H_∞ Filtering Performance Analysis

In this section, we aim to derive a stability criterion for the augmented filtering error system based on the adaptive ETS. First, we present the following lemma which will be used in the proof of the theorem below.

Lemma 1 [32]: For any symmetric positive-definite matrix R, and vector function $\dot{\varphi} : [a,b] \to \mathbb{R}^n$ such that $l_R(\varphi)$ is well defined, then the inequality

$$l_R(\dot{\varphi}) \ge \frac{1}{b-a}\Big[(\varphi(b)-\varphi(a))^T R(\varphi(b)-\varphi(a)) + 3\tilde{\Omega}^T R\tilde{\Omega}\Big] \tag{13}$$

holds, where $l_R(\varphi) = \int_a^b \varphi^T(u)R\varphi(u)du$ and $\tilde{\Omega} = \varphi(b) + \varphi(a) - (2/[b-a])\int_a^b \varphi(u)du$.

Lemma 2 [33]: For a scalar $\vartheta \in (0,1)$, matrix $S > 0$, and matrices T_1 and T_2 with appropriate dimensions. If there exists a matrix U such that $\begin{bmatrix} S & * \\ U & S \end{bmatrix} > 0$, then it holds that

$$\min_{\vartheta\in(0,1)} \Theta(\vartheta, S) \ge \begin{bmatrix} T_1\xi \\ T_2\xi \end{bmatrix}^T \begin{bmatrix} S & * \\ U & S \end{bmatrix} \begin{bmatrix} T_1\xi \\ T_2\xi \end{bmatrix}$$

for all vector ξ, where the function $\Theta(\vartheta, S) = (1/\vartheta)\xi^T T_1^T S T_1 \xi + (1/[1-\vartheta])\xi^T T_2^T S T_2 \xi$.

For convenience of description, let we denote $\mathscr{I}_c = 8n_{xi} + n_{yi} + n_{vi})$ and

$$\begin{aligned}
\mathscr{I}_1 &= \begin{bmatrix} I_{n_{xi}\times n_{xi}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}_{n_{xi}\times \mathscr{I}_c} \\
\mathscr{I}_2 &= \begin{bmatrix} 0 & I_{n_{xi}\times n_{xi}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}_{n_{xi}\times \mathscr{I}_c} \\
\mathscr{I}_3 &= \begin{bmatrix} 0 & 0 & I_{n_{xi}\times n_{xi}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}_{n_{xi}\times \mathscr{I}_c} \\
\mathscr{I}_4 &= \begin{bmatrix} 0 & 0 & 0 & I_{n_{xi}\times n_{xi}} & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}_{n_{xi}\times \mathscr{I}_c} \\
\mathscr{I}_5 &= \begin{bmatrix} 0 & 0 & 0 & 0 & I_{n_{xi}\times n_{xi}} & 0 & 0 & 0 & 0 & 0 \end{bmatrix}_{n_{xi}\times \mathscr{I}_c} \\
\mathscr{I}_6 &= \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & I_{n_{yi}\times n_{yi}} & 0 & 0 & 0 & 0 \end{bmatrix}_{n_{yi}\times \mathscr{I}_c} \\
\mathscr{I}_7 &= \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & I_{n_{vi}\times n_{vi}} & 0 & 0 & 0 \end{bmatrix}_{n_{vi}\times \mathscr{I}_c} \\
\mathscr{I}_8 &= \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & I_{n_{xi}\times n_{xi}} & 0 & 0 \end{bmatrix}_{n_{xi}\times \mathscr{I}_c} \\
\mathscr{I}_9 &= \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & I_{n_{xi}\times n_{xi}} & 0 \end{bmatrix}_{n_{xi}\times \mathscr{I}_c} \\
\mathscr{I}_{10} &= \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & I_{n_{xi}\times n_{xi}} \end{bmatrix}_{n_{xi}\times \mathscr{I}_c}.
\end{aligned}$$

Theorem 1: For given scalars $\gamma_i, \phi_i, \kappa_{i1}, \kappa_{i2}$, and κ_{i3} and matrices A_{fiq}, B_{fiq}, and L_{fiq}, the filtering error system in the form of (11) under the adaptive ETS in (6) is asymptotically stable if there exist matrices $P_i > 0, Q_{1i} > 0, Q_{2i} > 0, R_{i1} > 0, R_{i2} > 0, \Phi_i > 0$ and matrix U_i $(i \in \mathcal{J}; p, q \in \Psi)$ with appropriate dimensions such that

$$\Pi_{ipq} + \Pi_{iqp} < 0, \quad p \le q \tag{14}$$

$$\begin{bmatrix} \bar{R}_{i2} & * \\ U_i & \bar{R}_{i2} \end{bmatrix} > 0 \tag{15}$$

where

$$\Pi_{ipq} = \begin{bmatrix} \Pi_{ipq}^{11} & * & * & * & * \\ \Pi_{ipq}^{21} & -\Pi_i^{22} & * & * & * \\ \Pi_{ipq}^{31} & 0 & -\Pi_{ipq}^{33} & * & * \\ \Pi_{ipq}^{41} & 0 & 0 & -\Pi_{ipq}^{44} & * \\ \mathscr{E}_{ipq}H & 0 & 0 & 0 & -I \end{bmatrix}$$

$$\begin{aligned}
H &= \begin{bmatrix} \mathscr{I}_1^T & \mathscr{I}_2^T \end{bmatrix}^T, \bar{R}_{i2} = \mathrm{diag}\{R_{i2}, 3R_{i2}\} \\
\mathscr{R}_i &= \eta_{i1}^2 R_{i1} + (\eta_{i2} - \eta_{i1})^2 R_{i2} \\
\Pi_{ipq}^{11} &= H^T P_i \mathscr{A}_{ipq} + \mathscr{A}_{ipq}^T P_i H + \kappa_{i1}^{-1} H^T P_i H \\
&\quad + \mathscr{I}_1^T (Q_{i1} + Q_{i2})\mathscr{I}_1 - \mathscr{I}_3^T Q_{i1}\mathscr{I}_3 \\
&\quad - \mathscr{I}_5^T Q_{i2}\mathscr{I}_5 - (\mathscr{I}_1 - \mathscr{I}_3)^T R_{i1}(\mathscr{I}_1 - \mathscr{I}_3) \\
&\quad - 3(\mathscr{I}_1 + \mathscr{I}_3 - 2\mathscr{I}_8)^T R_{i1}(\mathscr{I}_1 + \mathscr{I}_3 - 2\mathscr{I}_8) \\
&\quad - \begin{bmatrix} T_1 \\ T_2 \end{bmatrix}^T \begin{bmatrix} \bar{R}_{i2} & * \\ U_i & \bar{R}_{i2} \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \end{bmatrix} \\
&\quad + \mathscr{I}_4^T C_{ip}^T \Phi_i C_{ip}\mathscr{I}_4 - \mathscr{I}_6^T \phi_i \Phi_i \mathscr{I}_6 - \mathscr{I}_7^T \gamma_i^2 I \mathscr{I}_7 \\
T_1 &= \begin{bmatrix} \mathscr{I}_4 - \mathscr{I}_5 \\ \mathscr{I}_4 + \mathscr{I}_5 - 2\mathscr{I}_9 \end{bmatrix}, T_2 = \begin{bmatrix} \mathscr{I}_3 - \mathscr{I}_4 \\ \mathscr{I}_3 + \mathscr{I}_4 - 2\mathscr{I}_{10} \end{bmatrix}
\end{aligned}$$

$$\begin{aligned}
\Pi_{ipq}^{21} &= \begin{bmatrix} \mathscr{R}_i \bar{\mathscr{A}}_{ip} \\ \mathscr{R}_i A_{ip}\mathscr{I}_1 \\ \mathscr{R}_i D_{ip}\mathscr{I}_7 \end{bmatrix}, \Pi_i^{22} = \mathrm{diag}\{\mathscr{R}_i, \kappa_{i2}\mathscr{R}_i, \kappa_{i3}\mathscr{R}_i\} \\
\bar{\mathscr{A}}_{ip} &= A_{ip}\mathscr{I}_1 + D_{ip}\mathscr{I}_7 \\
\mathscr{A}_{ipq} &= \mathcal{A}_{ipq}H + \mathcal{B}_{iq}^1\mathscr{I}_6 + \mathcal{B}_{ipq}^2\mathscr{I}_4 + \mathcal{D}_{ip}\mathscr{I}_7 \\
\Pi_{ipq}^{31} &= \begin{bmatrix} \mathscr{I}_1^T B_{1ip}^T \mathscr{R}_1 \cdots \mathscr{I}_1^T B_{jip}^T \mathscr{R}_{j,j\neq i} \cdots \mathscr{I}_1^T B_{Jip}^T \mathscr{R}_J \end{bmatrix}^T \\
\Pi_{ipq}^{33} &= (J-1)^{-1}\mathrm{diag}\Big\{(1+\kappa_{12}+\kappa_{13})^{-1}\mathscr{R}_1, \ldots \\
&\qquad \left(1+\kappa_{j2}+\kappa_{j3}\right)_{j\neq i}^{-1}\mathscr{R}_{j,j\neq i}, \ldots \\
&\qquad (1+\kappa_{J2}+\kappa_{J3})^{-1}\mathscr{R}_J\Big\} \\
\Pi_{ipq}^{41} &= \begin{bmatrix} \mathscr{I}_1^T \mathcal{B}_{1ip}^{3T} P_i \cdots \mathscr{I}_1^T \mathcal{B}_{jip}^{3T} P_{j,j\neq i} \cdots \mathscr{I}_1^T \mathcal{B}_{Jip}^{3T} P_J \end{bmatrix}^T \\
\Pi_{ipq}^{44} &= (J-1)^{-1}\mathrm{diag}\Big\{\kappa_{11}^{-1}P_1, \ldots, \kappa_{j1}^{-1}P_{j,j\neq i}, \ldots, \kappa_{J1}^{-1}P_J\Big\}.
\end{aligned}$$

Proof: Construct a Lyapunov function for the filtering error system (11) as

$$V(t) = \sum_{i=1}^{J}(V_{1i}(t) + V_{2i}(t) + V_{3i}(t) + V_{4i}(t)) \tag{16}$$

where

$$\begin{aligned}
V_{1i}(t) &= \xi_i^T(t)P_i\xi_i(t) \\
V_{2i}(t) &= \int_{t-\eta_{i1}}^{t} x_i^T(s)Q_{i1}x_i(s)ds + \int_{t-\eta_{i2}}^{t} x_i^T(s)Q_{i2}x_i(s)ds \\
V_{3i}(t) &= \eta_{i1}\int_{-\eta_{i1}}^{0}\int_{t-s}^{t} \dot{x}_i^T(v)R_{i1}\dot{x}_i(v)dvds \\
&\quad + (\eta_{i2} - \eta_{i1})\int_{-\eta_{i2}}^{-\eta_{i1}}\int_{t-s}^{t} \dot{x}_i^T(v)R_{i2}\dot{x}_i(v)dvds \\
V_{4i}(t) &= \frac{1}{2}\varsigma_i^2(t).
\end{aligned}$$

Along the trajectories of (11), the corresponding time derivative of $V_{ki}(t)$ $(k = 1, 2, 3)$ is given by

$$\begin{aligned}
\dot{V}_{1i}(t) &= \sum_{p=1}^{r_i}\sum_{q=1}^{r_i} h_{ip}h_{iq}2\xi_i^T(t)P_i\Bigg[\mathcal{A}_{ipq}\xi_i(t) + \mathcal{B}_{iq}^1 e_i(t) \\
&\qquad + \mathcal{B}_{ipq}^2 x_i(t-\eta_i(t)) \\
&\qquad + \sum_{j=1, j\neq i}^{J}\mathcal{B}_{ijp}^3 x_j(t) + \mathcal{D}_{ip}v_i(t)\Bigg] \\
\dot{V}_{2i}(t) &= x_i^T(t)(Q_{i1} + Q_{i2})x_i(t) \\
&\quad - \sum_{k=1}^{2} x_i^T(t-\eta_{ik})Q_{ik}x_i(t-\eta_{ik}) \\
\dot{V}_{3i}(t) &= \dot{x}_i^T(t)\mathscr{R}_i\dot{x}_i(t) - \eta_{i1}\int_{t-\eta_{i1}}^{t}\dot{x}_i^T(s)R_{i1}\dot{x}_i(s)ds \\
&\quad - (\eta_{i2} - \eta_{i1})\int_{t-\eta_{i2}}^{t-\eta_{i1}}\dot{x}_i^T(s)R_{i2}\dot{x}_i(s)ds.
\end{aligned}$$

Define $\zeta_{i1}(t) = [x_i^T(t)\ x_{fi}^T(t)\ x_i^T(t-\eta_{i1})\ x_i^T(t-\eta_i(t))x_i^T(t-\eta_{i2})\ e_i^T(t)\ v_i^T(t)]^T$, $\zeta_{i2}(t) = [(1/\eta_{i1})\int_{t-\eta_{i1}}^{t} x^T(s)ds(1/[\eta_{i2} - \eta_i(t)])\int_{t-\eta_{i2}}^{t-\eta_i(t)} x^T(s)ds(1/[\eta_i(t) - \eta_{i1}])\int_{t-\eta_i(t)}^{t-\eta_{i1}} x^T(s)ds]^T$, and $\zeta_i(t) = \begin{bmatrix}\zeta_{i1}^T(t) & \zeta_{i2}^T(t)\end{bmatrix}^T$.

From Lemma 1, we have

$$-\eta_{i1}\int_{t-\eta_{i1}}^{t}\dot{x}_i^T(s)R_{i1}\dot{x}_i(s)ds \le -\zeta_i^T(t)\big[(\mathscr{I}_1-\mathscr{I}_3)^TR_{i1}(\mathscr{I}_1-\mathscr{I}_3) - 3(\mathscr{I}_1+\mathscr{I}_3-2\mathscr{I}_8)^T \times R_{i1}(\mathscr{I}_1+\mathscr{I}_3-2\mathscr{I}_8)\big]\zeta_i(t).$$

Defining $\vartheta(t)=([\eta_i(t)-\eta_{i1}]/[\eta_{i2}-\eta_{i1}])$ yields $0<\vartheta(t)<1$ due to $\eta_{i1}<\eta_i(t)<\eta_{i2}$. Using Lemma 1, we can obtain

$$\begin{aligned}-&(\eta_{i2}-\eta_{i1})\int_{t-\eta_{i2}}^{t-\eta_{i1}}\dot{x}_i^T(s)R_{i2}\dot{x}_i(s)ds\\ &=-(\eta_{i2}-\eta_{i1})\left[\int_{t-\eta_{i2}}^{t-\eta_i(t)}\dot{x}_i^T(s)R_{i2}\dot{x}_i(s)ds + \int_{t-\eta_i(t)}^{t-\eta_{i1}}\dot{x}_i^T(s)R_{i2}\dot{x}_i(s)ds\right]\\ &\le \zeta_i^T(t)\Big[-\frac{1}{\vartheta(t)}(\mathscr{I}_4-\mathscr{I}_5)^TR_{i2}(\mathscr{I}_4-\mathscr{I}_5)\\ &\quad-\frac{3}{\vartheta(t)}(\mathscr{I}_4+\mathscr{I}_5-2\mathscr{I}_9)^TR_{i2}(\mathscr{I}_4+\mathscr{I}_5-2\mathscr{I}_9)\\ &\quad-\frac{1}{1-\vartheta(t)}(\mathscr{I}_3-\mathscr{I}_4)^TR_{i2}(\mathscr{I}_3-\mathscr{I}_4)\\ &\quad-\frac{3}{1-\vartheta(t)}(\mathscr{I}_3+\mathscr{I}_4-2\mathscr{I}_{10})^T\\ &\quad\times R_{i2}(\mathscr{I}_3+\mathscr{I}_4-2\mathscr{I}_{10})\Big]\zeta_i(t)\\ &=-\frac{1}{\vartheta(t)}\zeta_i^T(t)T_1^T\bar{R}_{i2}T_1\zeta_i(t)-\frac{1}{1-\vartheta(t)}\zeta_i^T(t)T_2^T\bar{R}_{i2}T_2\zeta_i(t).\end{aligned}$$

Using Lemma 2 follows:

$$-(\eta_{i2}-\eta_{i1})\int_{t-\eta_{i2}}^{t-\eta_{i1}}\dot{x}_i^T(s)R_{i2}\dot{x}_i(s)ds \le -\zeta_i^T(t)\begin{bmatrix}T_1\\T_2\end{bmatrix}^T\begin{bmatrix}\bar{R}_{i2} & *\\ U & \bar{R}_{i2}\end{bmatrix}\begin{bmatrix}T_1\\T_2\end{bmatrix}\zeta_i(t). \tag{17}$$

Note that

$$\sum_{i=1}^{J}\left(\sum_{j=1,j\neq i}^{J}x_j^T(t)B_{ijp}^T\mathscr{R}_i\sum_{j=1,j\neq i}^{J}B_{ijp}x_j(t)\right) \le \sum_{i=1}^{J}\left((J-1)\sum_{j=1,j\neq i}^{J}x_i^T(t)B_{jip}^T\mathscr{R}_jB_{jip}x_i(t)\right). \tag{18}$$

The following inequalities can be got by using the similar method:

$$\begin{aligned}\sum_{i=1}^{J}&2\xi_i^T(t)P_i\sum_{j=1,j\neq i}^{J}\mathcal{B}_{ijp}^3x_j(t)\\ &\le\sum_{i=1}^{J}\left(\kappa_{i1}\sum_{j=1,j\neq i}^{J}x_j^T(t)\mathcal{B}_{ijp}^{3T}P_i\sum_{j=1,j\neq i}^{J}\mathcal{B}_{ijp}x_j(t) + \kappa_{i1}^{-1}\xi_i^T(t)P_i\xi_i(t)\right)\\ &\le\sum_{i=1}^{J}\left((J-1)\sum_{j=1,j\neq i}^{J}\kappa_{j1}x_i^T(t)\mathcal{B}_{jip}^{3T}P_j\mathcal{B}_{jip}x_i(t) + \kappa_{i1}^{-1}\xi_i^T(t)P_i\xi_i(t)\right)\end{aligned} \tag{19}$$

$$\begin{aligned}\sum_{i=1}^{J}&\left(2x_i^T(t)A_{ip}^T\mathscr{R}_i\sum_{j=1,j\neq i}^{J}B_{ijp}x_j(t)\right)\\ &\le\sum_{i=1}^{J}\left((J-1)\sum_{j=1,j\neq i}^{J}\kappa_{j2}x_i^T(t)B_{jip}^T\mathscr{R}_jB_{jip}x_i(t) + \kappa_{i2}^{-1}x_i^TA_{ip}^T\mathscr{R}_iA_{ip}x_i(t)\right)\end{aligned} \tag{20}$$

$$\begin{aligned}\sum_{i=1}^{J}&\left(2v_i^T(t)D_{ip}^{1T}\mathscr{R}_i\sum_{j=1,j\neq i}^{J}B_{ijp}x_j(t)\right)\\ &\le\sum_{i=1}^{J}\left((J-1)\sum_{j=1,j\neq i}^{J}\kappa_{j3}x_i^T(t)B_{jip}^T\mathscr{R}_jB_{jip}x_i(t) + \kappa_{i3}^{-1}v_i^TD_{ip}^T\mathscr{R}_iD_{ip}v_i(t)\right).\end{aligned} \tag{21}$$

From the adaptive law of threshold in (8), it follows that:

$$\begin{aligned}\dot{V}_{4i}(t)&=\varsigma_i(t)\dot{\varsigma}_i(t)\\ &=\frac{1}{\varsigma_i(t)}e_i^T(t)\Phi_ie_i(t)-\phi_ie_i^T(t)\Phi_ie_i(t).\end{aligned}$$

Recalling the event-triggering condition in (6) together with the definition of $\eta_i(t)$, we have

$$\begin{aligned}\dot{V}_{4i}(t)\le\;& x_i^T(t-\eta_i(t))C_{ip}^T\Phi_iC_{ip}x_i(t-\eta_i(t))\\ &-\phi_ie_i^T(t)\Phi_ie_i(t).\end{aligned} \tag{22}$$

Combining (17)–(22), we can obtain

$$\begin{aligned}\dot{V}&(t)+\hat{e}^T(t)\hat{e}(t)-\gamma^2v^T(t)v(t)\\ &\le\sum_{i=1}^{J}\Bigg\{\sum_{p=1}^{r_i}\sum_{q=1}^{r_i}h_{ip}h_{iq}\zeta_i^T(t)\\ &\quad\times\Big(H^TP_i\mathscr{A}_{ipq}+\mathscr{A}_{ipq}^TP_iH+\kappa_{i1}^{-1}H^TP_iH\\ &\quad+\mathscr{I}_1^T(Q_{i1}+Q_{i2})\mathscr{I}_1-\mathscr{I}_3^TQ_{i1}\mathscr{I}_3\\ &\quad-\mathscr{I}_5^TQ_{i2}\mathscr{I}_5-(\mathscr{I}_1-\mathscr{I}_3)^TR_{i1}(\mathscr{I}_1-\mathscr{I}_3)\\ &\quad-3(\mathscr{I}_1+\mathscr{I}_3-2\mathscr{I}_8)^TR_{i1}(\mathscr{I}_1+\mathscr{I}_3-2\mathscr{I}_8)\\ &\quad-\begin{bmatrix}T_1\\T_2\end{bmatrix}^T\begin{bmatrix}\bar{R}_{i2} & *\\ U_i & \bar{R}_{i2}\end{bmatrix}\begin{bmatrix}T_1\\T_2\end{bmatrix}\\ &\quad+\mathscr{I}_4^TC_{ip}^T\Phi_iC_{ip}\mathscr{I}_4-\mathscr{I}_6^T\phi_i\Phi_i\mathscr{I}_6\\ &\quad-\mathscr{I}_7^T\gamma_i^2I\mathscr{I}_7+\Pi_{ipq}^{21\,T}(\Pi_i^{22})^{-1}\Pi_{ipq}^{21}\\ &\quad+\Pi_{ipq}^{31\,T}\left(\Pi_{ipq}^{33}\right)^{-1}\Pi_{ipq}^{31}\\ &\quad+\Pi_{ipq}^{41\,T}\left(\Pi_{ipq}^{44}\right)^{-1}\Pi_{ipq}^{41}+H^T\mathscr{E}_{iqp}^T\mathscr{E}_{iqp}H\Big)\zeta_i(t)\Bigg\}.\end{aligned}$$

By Schur complement, it can be known that if (14) and (15) hold, then

$$\dot{V}(t) \leq -\hat{e}^T(t)\hat{e}(t) + \gamma^2 v^T(t)v(t). \tag{23}$$

Take the integral of (23) from 0 to ∞ with respect to t, it follows:

$$V(\infty) - V(0) \leq \lim_{t\to\infty}\int_0^t \hat{e}^T(t)\hat{e}(t) - \lim_{t\to\infty}\int_0^t \gamma^2 v^T(t)v(t). \tag{24}$$

Then it is true that $\|\hat{e}(t)\|_2 < \gamma^2\|v\|_2$ under zero initial condition. With the condition of $v(t) = 0$, we can conclude that $\dot{V}(t) < 0$ from (23). Thus the proof is completed. ■

IV. Adaptive ETS-Based H_∞ Filter Design

With the aid of Theorem 1, we are now ready to develop an approach of filter design for networked interconnected fuzzy systems with the proposed adaptive ETS.

Theorem 2: For given scalars $\gamma_i, \phi_i, \kappa_{i1}, \kappa_{i2}$, and κ_{i3}, a full-order H_∞ filter in the form of (4) for the networked interconnected system (2) under the adaptive ETS in (6) exists, if there exist matrices $P_{1i} > 0, \bar{P}_{2i} > 0, Q_{1i} > 0, Q_{2i} > 0, R_{i1} > 0, R_{i2} > 0$, and $\Phi_i > 0$ and matrices $U_i, \bar{A}_{fiq}, \bar{B}_{fiq}$, and $\bar{L}_{fiq}$ $(i \in \mathcal{J}; p, q \in \Psi)$ with appropriate dimensions such that

$$\tilde{\Pi}_{ipq} + \tilde{\Pi}_{iqp} < 0, \quad p \leq q \tag{25}$$

$$\begin{bmatrix} \bar{R}_{i2} & * \\ U_i & \bar{R}_{i2} \end{bmatrix} > 0 \tag{26}$$

$$P_{1i} - \bar{P}_{2i} > 0. \tag{27}$$

Moreover, the filter gains are given by

$$A_{fiq} = \bar{P}_{2i}^{-1}\bar{A}_{fiq}, B_{fiq} = \bar{P}_{2i}^{-1}\bar{B}_{fiq}, L_{fiq} = \bar{L}_{fiq} \tag{28}$$

where

$$\tilde{\Pi}_{ipq} = \begin{bmatrix} \tilde{\Pi}_{ipq}^{11} & * & * & * & * \\ \Pi_{ipq}^{21} & -\Pi_i^{22} & * & * & * \\ \Pi_{ipq}^{31} & 0 & -\Pi_{ipq}^{33} & * & * \\ \tilde{\Pi}_{ipq}^{41} & 0 & 0 & -\tilde{\Pi}_{ipq}^{44} & * \\ \mathfrak{L}_{fipq}H & 0 & 0 & 0 & -I \end{bmatrix}$$

$$\begin{aligned}
\tilde{\Pi}_{ipq}^{11} &= H^T\mathfrak{A}_{ipq} + \mathfrak{A}_{ipq}^T H + \kappa_{i1}^{-1}H^T\mathfrak{P}_i H \\
&\quad + \mathscr{I}_1^T(Q_{i1} + Q_{i2})\mathscr{I}_1 - \mathscr{I}_3^T Q_{i1}\mathscr{I}_3 - \mathscr{I}_5^T Q_{i2}\mathscr{I}_5 \\
&\quad - (\mathscr{I}_1 - \mathscr{I}_3)^T R_{i1}(\mathscr{I}_1 - \mathscr{I}_3) \\
&\quad - 3(\mathscr{I}_1 + \mathscr{I}_3 - 2\mathscr{I}_8)^T R_{i1}(\mathscr{I}_1 + \mathscr{I}_3 - 2\mathscr{I}_8) \\
&\quad - \begin{bmatrix} T_1 \\ T_2 \end{bmatrix}^T \begin{bmatrix} \bar{R}_{i2} & * \\ U_i & \bar{R}_{i2} \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \end{bmatrix} \\
&\quad + \mathscr{I}_4^T C_{ip}^T \Phi_i C_{ip}\mathscr{I}_4 - \mathscr{I}_6^T \phi_i \Phi_i \mathscr{I}_6 - \mathscr{I}_7^T \gamma_i^2 I \mathscr{I}_7 \\
\mathfrak{A}_{ipq} &= \mathfrak{A}_{ipq}^1 H + \mathfrak{B}_{iq}^1\mathscr{I}_6 + \mathfrak{B}_{ipq}^2\mathscr{I}_4 + \mathfrak{D}_{ip}\mathscr{I}_7 \\
\mathfrak{A}_{ipq}^1 &= \begin{bmatrix} P_{1i}A_{ip} & \bar{A}_{fiq} \\ \bar{P}_{2i}A_{ip} & \bar{A}_{fiq} \end{bmatrix}, \mathfrak{B}_{iq}^1 = \begin{bmatrix} \bar{B}_{fiq} \\ \bar{B}_{fiq} \end{bmatrix} \\
\mathfrak{B}_{ipq}^2 &= \begin{bmatrix} \bar{B}_{fiq}C_{ip} \\ \bar{B}_{fiq}C_{ip} \end{bmatrix}, \mathfrak{B}_{jip}^3 = \begin{bmatrix} P_{1i}B_{jip} \\ \bar{P}_{2i}B_{jip} \end{bmatrix} \\
\mathfrak{D}_{ip} &= \begin{bmatrix} P_{1i}D_{ip} \\ \bar{P}_{2i}D_{ip} \end{bmatrix}, \mathfrak{P}_i = \begin{bmatrix} P_{1i} & \bar{P}_{2i} \\ \bar{P}_{2i} & \bar{P}_{2i} \end{bmatrix} \\
\mathfrak{L}_{fipq} &= \begin{bmatrix} E_{ip} & -\bar{L}_{fiq} \end{bmatrix}
\end{aligned}$$

$$\tilde{\Pi}_{ipq}^{41} = \begin{bmatrix} \mathscr{I}_1^T\mathfrak{B}_{1ip}^{3T} & \cdots & \mathscr{I}_1^T\mathfrak{B}_{jip,j\neq i}^{3T} & \cdots & \mathscr{I}_1^T\mathfrak{B}_{Jip}^{3T} \end{bmatrix}^T$$

$$\tilde{\Pi}_{ipq}^{44} = (J-1)^{-1}\text{diag}\left\{\kappa_{11}^{-1}\mathfrak{P}_1, \ldots, \kappa_{j1}^{-1}\mathfrak{P}_{j,j\neq i}, \ldots, \kappa_{J1}^{-1}\mathfrak{P}_J\right\}.$$

Proof: Defining

$$P_i = \begin{bmatrix} P_{1i} & P_{2i} \\ P_{2i}^T & P_{3i} \end{bmatrix} > 0, Y_i = \begin{bmatrix} I & 0 \\ 0 & P_{2i}P_{3i}^{-1} \end{bmatrix}$$

and $\bar{A}_{fiq} = P_{2i}A_{fiq}P_{3i}^{-1}P_{2i}^T, \bar{B}_{fiq} = P_{2i}B_{fiq}, \bar{P}_{2i} = P_{2i}P_{3i}^{-1}P_{2i}^T$, and $\bar{L}_{fiq} = L_{fiq}P_{3i}^{-1}P_{2i}^T$.

Using Schur complement, one can know that $P_{1i} > 0$ and $P_{1i} - \bar{P}_{2i} > 0$ if $P_i > 0$.

Define $Y_{1i} = \text{diag}\{Y_i, \underbrace{I, \ldots, I}_{8}\}$, $Y_{2i} = \text{diag}\{\underbrace{I, \ldots, I}_{J-1}\}$, $Y_{3i} = \text{diag}\{\underbrace{Y_i, \ldots, Y_i}_{J-1}\}$. Multiplying both side of (14) with $\text{diag}\{Y_{1i}, I, I, I, Y_{2i}, Y_{3i}, I\}$ and its transpose, one can be know that (25) is equivalent to (14). Using an equivalent transformation for the transfer function of the system (4) from $y_i(t)$ to $z_{fi}(t)$ yields (28). This completes the proof. ■

V. Numerical Example

Consider a double-inverted pendulums system connected by a spring formulated in [34] and [35]. The two rules fuzzy model of the nonlinear plant (2) with the parameters

$$\begin{aligned}
A_{11} &= \begin{bmatrix} 0 & 1 \\ -44.75 & 20 \end{bmatrix}, A_{12} = \begin{bmatrix} 0 & 1 \\ -44.19 & 20 \end{bmatrix} \\
A_{21} &= \begin{bmatrix} 0 & 1 \\ -42.55 & 20 \end{bmatrix}, A_{22} = \begin{bmatrix} 0 & 1 \\ -38.99 & 20 \end{bmatrix} \\
B_{121} &= \begin{bmatrix} 0 & 0 \\ 0.8 & 0 \end{bmatrix}, B_{122} = \begin{bmatrix} 0 & 0 \\ 0.8 & 0 \end{bmatrix}, B_{211} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} \\
B_{212} &= \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, D_{11} = \begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix}, D_{12} = \begin{bmatrix} 0 \\ 0.5 \end{bmatrix} \\
D_{21} &= \begin{bmatrix} 0 \\ 0.4 \end{bmatrix}, D_{22} = \begin{bmatrix} 0 \\ 0.4 \end{bmatrix}, E_{11} = \begin{bmatrix} 1 & 0 \end{bmatrix}, E_{12} = \begin{bmatrix} 1 & 0 \end{bmatrix} \\
E_{21} &= \begin{bmatrix} 1 & 0 \end{bmatrix}, E_{22} = \begin{bmatrix} 1 & 0 \end{bmatrix}.
\end{aligned}$$

In (2), $h_{i1} = |0.637x_{i1}|, h_{i2} = 1 - |0.637x_{i1}|$, and $x_{i1} \in [-\pi/2, \pi/2]$ $(i = 1, 2)$. The disturbances are chosen by $v_1(t) = 4\cos(0.001(t-4))e^{-0.001(t-4)}$, and $v_2(k) = 1.6\sin(0.001(t-4))e^{-0.001(t-4)}$.

By using Theorem 2 with $\bar{\tau}_1 = \bar{\tau}_2 = 0.03$, $\phi_1 = 0.9$, $\phi_2 = 0.8$, $\gamma_1 = 1$, $\gamma_2 = 0.9$, and $\kappa_{ij} = 5$, $(i = 1, 2; j = 1, 2, 3)$, we can get both triggering parameters and filter parameters as

$$\begin{aligned}
&\Phi_1 = 1.1675, \Phi_2 = 5.0127 \\
&A_{f11} = \begin{bmatrix} -5.3378 & 1.9853 \\ -44.5718 & -10.1088 \end{bmatrix} \\
&A_{f12} = \begin{bmatrix} -5.6016 & 0.5890 \\ -40.9074 & -17.9413 \end{bmatrix} \\
&A_{f21} = \begin{bmatrix} -9.3482 & 0.6553 \\ -39.8035 & -6.5414 \end{bmatrix} \\
&A_{f22} = \begin{bmatrix} -8.6514 & 0.6351 \\ -21.2546 & -15.0654 \end{bmatrix} \\
&B_{f11} = \begin{bmatrix} -0.8736 \\ 4.1808 \end{bmatrix}, B_{f12} = \begin{bmatrix} 4.6240 \\ 31.6200 \end{bmatrix}
\end{aligned}$$

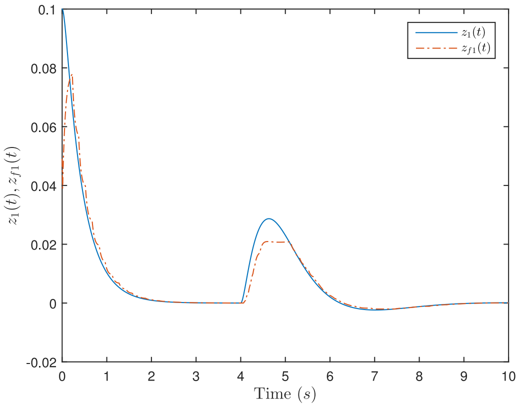

Fig. 3. Trajectories of $z_1(t)$ and its estimation $z_{1f}(t)$.

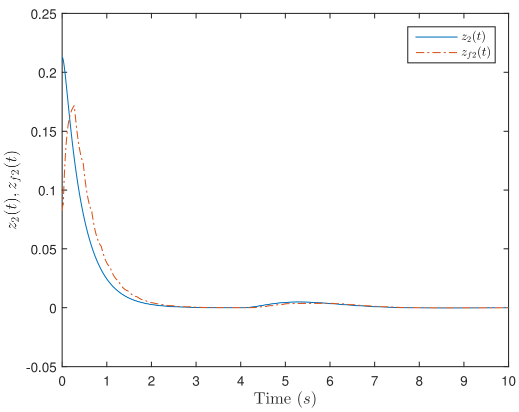

Fig. 4. Trajectories of $z_2(t)$ and its estimation $z_{2f}(t)$.

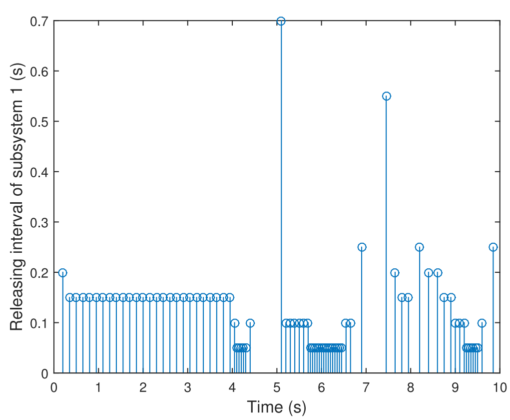

Fig. 5. Releasing instant and its interval of subsystem 1.

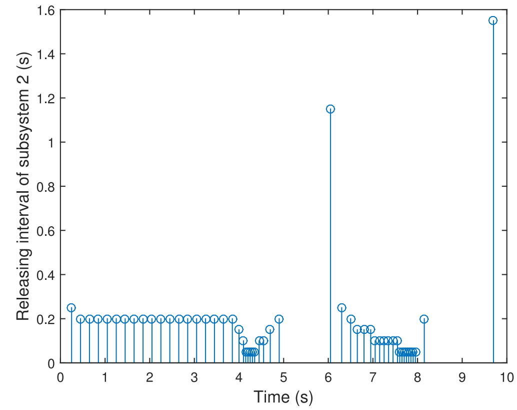

Fig. 6. Releasing instant and its interval of subsystem 2.

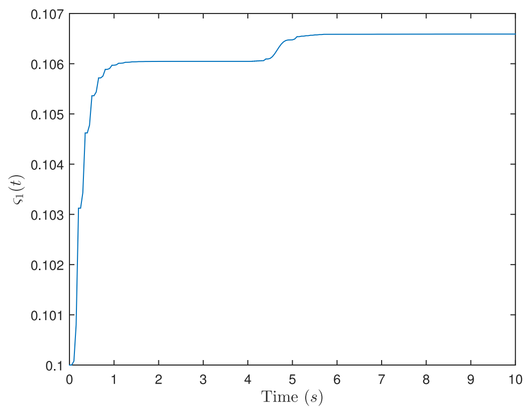

Fig. 7. Trajectory of threshold of subsystem 1.

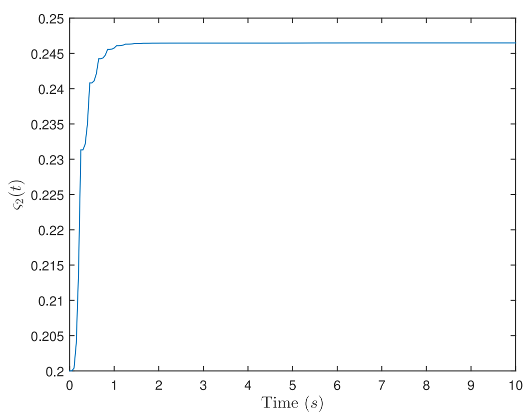

Fig. 8. Trajectory of threshold of subsystem 2.

$$B_{f21} = \begin{bmatrix} 2.9800 \\ 24.1082 \end{bmatrix}, B_{f22} = \begin{bmatrix} -5.1000 \\ 1.0880 \end{bmatrix}$$
$$L_{f11} = \begin{bmatrix} -0.9008 & 0.4700 \end{bmatrix}, L_{f12} = \begin{bmatrix} 1.1100 & 0.3468 \end{bmatrix}$$
$$L_{f21} = \begin{bmatrix} 0.9610 & 0.4405 \end{bmatrix}, L_{f22} = \begin{bmatrix} -1.0132 & 0.4572 \end{bmatrix}.$$

The initial conditions, in this simulation, are given by $x_1(t) = [0.5, 0]^T$, $x_2(t) = [0.2, 0]^T$, $x_{f1}(t) = [-0.05, 0]^T$, and $x_{f2}(t) = [-0.1, 0]^T$. Assume the sampling period $h = 0.5$ s together with the solutions above, we can get the results as shown in Figs. 3–8. From Figs. 3 and 4, one can clearly see that the filter can well estimate the outputs of the plant although the release rates of the sampled data of the subsystems 1 and 2 are only 39.5% and 26.5%, respectively. It illustrates that some sampling data have litter contribution to the filtering system. The releasing instants and the releasing periods of the two subsystems are shown in Figs. 5 and 6, from which one can see the period of data releasing is not a fixed clock period. The event of data-releasing is generated only when the triggering condition is invoked. Moreover, the average releasing periods are $\bar{h}_1 = 0.13$ s and $\bar{h}_2 = 0.19$ s, respectively, which is much longer than the sampling period $h = 0.05$ s. Obviously, during the releasing period, lots of "unnecessary" sampled data are discarded, and thus the occupation of the communication resource can be alleviated. Figs. 7 and 8 depict the responses of the threshold of triggering condition in (6). The threshold is depended on the dynamic error between the latest releasing data and the current sampling data of each subsystems. It is not a preset constant as presented in the existing literature but a result of on-line optimization. From Figs. 7 and 8 together with the adaptive law in (8), one can see that the threshold keeps a certain value when the system is stable due to $e_i(t) \to 0$, that is to say, the threshold is regulated with the dynamic of the plant.

VI. Conclusion

In this paper, a new adaptive ETS is proposed for networked interconnected fuzzy systems. Threshold with dynamic regulation is designed to adapt the variation of the system and the requirement of filter performance. The decentralized filter is designed based on the proposed adaptive ETS by which a plenty of unnecessary sampling data are dropped out. These communication resource can be allocated to some other useful task. Furthermore, a lower releasing frequency saves the energy consumption by using adaptive ETS, which is important for the device of wireless network with battery. The parameters of the proposed adaptive ETS and the decentralized filter are co-designed by using Lyapunov function method. The application to a double-inverted pendulums system connected by a spring has shown the effectiveness of the proposed filter design method.

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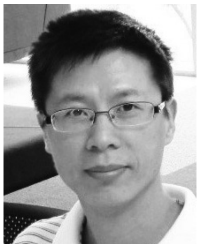

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