

Sampling period scheduling of networked control systems with multiple-control loops

Chen Peng^a, Dong Yue^{a,d,*}, Zhou Gu^b, Feng Xia^c

^a School of Electrical and Automation Engineering, Nanjing Normal University, Nanjing, Jiangsu 210042, China

^b College of Power Engineering, Nanjing Normal University, Nanjing, Jiangsu 210042, China

^c College of Computer Science and Technology, Zhejiang University, Hangzhou 310027, China

^d Department of Control Science and Engineering, Huazhong University of Science and Technology, Wuhan, Hubei 430074, China

Received 23 April 2007; received in revised form 29 February 2008; accepted 21 June 2008

Available online 5 July 2008

Abstract

This paper addresses the sampling period scheduling of Networked Control Systems (NCSs) with multiple control loops. The generalized exponential function is employed to describe Integral Absolute Error (IAE) performance versus sampling period by Truetime toolbox under Matlab environment, and the sampling periods are scheduled to obtain the optimal integrated performance based on Kuhn–Tucker Theorem, which are subject to the stability of every control loop and the bandwidth on available network resource. Numerical examples are given to show the effectiveness of our method.

© 2008 IMACS. Published by Elsevier B.V. All rights reserved.

Keywords: Sampling period scheduling; Kuhn–Tucker theorem; Networked control systems (NCSs); Optimization

1. Introduction

In distributed control systems, feedback control loops are often closed through a communication network medium. The performance of the control loops depends not only on the designed control algorithm but also on the scheduling of the shared network resource [13]. Although the higher sampling rates improve system performance in traditional computer control systems, they also induce the higher traffic loads on the communication medium in Networked control systems (NCSs) [13,15,14,9,10]. In limited-bandwidth network, high traffic loads increase the network-induced delay and can degrade control performance. Therefore, finding an appropriate sampling period that can both tolerates the network-induced delay and achieves desired integrated performance is an important topic in the analysis and design of NCSs [9,8,12].

There have been some reports on task scheduling in real-time control systems [7,11]. For example, Layland et al. developed an optimal static and dynamic, priority-based, pre-emptive scheduling algorithm for a set of independent and periodic real-time task set [7], and assumed that characteristics of the task set were fixed and known. Sha and Shin et al. proposed an algorithm to optimize task frequency with the limited available resource [11]. It is noted that task scheduling and NCSs scheduling have many similar characters [15]. There have been some studies on NCSs scheduling [13,15,16,1,3]. For example, an exponential cost was used as the performance measure and Rate Monotonic

* Corresponding author. Tel.: +86 25 85481197; fax: +86 25 85481197.

E-mail addresses: 63050@njnu.edu.cn (C. Peng), medongy@vip.163.com (D. Yue).

(RM) algorithm was applied to scheduling optimization problem of NCS based on `Mfincon.m`/Matlab in [1], but only the suboptimal solution can be obtained. A Maximum Error First with Try-Once-Discard (MEF-TOD) scheduling algorithm was presented in [13], which based on the assumption that if data packet failed to win the competition for network access, it would be discarded and new data would be used at the next time. The authors of [3] presented a scheduling algorithm which could allocate the bandwidth and determine the sampling periods of sensors under constraint that the network-induced delays were less than the sampling periods. Concerned with different types of data, a scheduling method for NCSs was proposed in [8] under the assumption that the other control loops' sampling period was the integral multiples of the basic sampling period. In addition, a calculational method to show the relationship between network-induced delay and the sampling period was proposed in [14], but how to optimize the sampling period is not considered. In aforementioned papers, the assumption that the sampling period is less than Maximum Allowable Delay Bounds (MADB) is all needed and the relationship between sampling period and integrated performance is not directly revealed. This motivates the research of this work.

As is known, if the data generated by the sensors is more than the data that network can transmit, then the network will be saturated and data will be queued at the buffer (unless it is discarded). So, both the effective bandwidth and the sampling period should be simultaneously considered in the design of NCSs. If the original system parameters are schedulable, one can change the sampling period of control loops in the given set under the constraints of Quality of Performance (QoP) and Quality of Services (QoSs) to obtain the optimal integrated performance. In this paper, as far as the overall system performance be concerned, the interaction between the control performance the sampling period will be investigated, and based on dynamic and static scheduling algorithm, an optimal scheduling algorithm for multiple control loops will be presented under constraints that the requirement of control performance and the limitation on available network resource.

This paper is organized as follows. Section 2 proposes the relationship between the performance index and the sampling period based on Truetime toolbox. In Section 3, an optimal scheduling of sampling period is presented based on Kuhn–Tucker Theorem. Numerical examples are given in Section 4 to demonstrate the effectiveness of the proposed method. Section 5 concludes the paper.

2. Relationship between IAE performance and sampling period

To design an NCS, a better approach is to optimize the integrated performance considering both QoP and QoS. To illustrate the effect of sampling period on system performance, an actual Direct Current (DC) motor control system was chosen to construct a networked control system.

In this paper, IAE (Integral Absolute Error) is chosen as the performance index, which means $J(kh) = \sum_{i=1}^k |e(ih)|$, where k is the simulation step. The transfer function of DC motor is $(1000/(s^2 + 2s + 2))$ and the controller is PI (Proportional Integrated controller). Truetime toolbox [2] is chosen as the network simulation tool, CAN and Ethernet are chosen as the communication medium, typical data rate are 0.5 Mb/s and 10 Mb/s, the data packet size is 72 bytes and 8 bytes, respectively. The simulation time is 0.6 s. The Simulink structural graph under Simulink/Matlab environment is shown in Fig. 1. The sampling period is chosen from 0.001 s to 0.016 s. According to the different sampling period and network topology, the simulation results are listed in Table 1.

Based on Table 1, we use the polynomial function `Polyfit.m` to obtain the coefficients of a polynomial $P(X)$ of degree N in a least-squares sense, $P(X(I)) \approx Y(I)$, where N is set to 2. The polynomial fit results are shown in (1) and the sampling period versus IAE performance is shown in Fig. 2.

$$\begin{cases} p1(t) = (4.8432t^2 - 0.0826t + 0.0004) \times 1000 \\ p2(t) = (4.1315t^2 - 0.0724t + 0.0004) \times 1000 \end{cases} \quad (1)$$

Table 1
IAE (10^{-2}) versus sampling period (10^{-3} s)

| | Sampling period | | | | | | | | | | | | | | | |
|----------------|-----------------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
| IAE (CAN) | 3.18 | 2.67 | 2.29 | 1.01 | 1.63 | 1.47 | 1.32 | 1.09 | 1.11 | 0.97 | 1.21 | 1.27 | 1.37 | 1.57 | 1.83 | 2.79 |
| IAE (Ethernet) | 3.13 | 2.63 | 2.25 | 1.08 | 1.61 | 1.45 | 1.31 | 1.08 | 1.10 | 0.96 | 1.21 | 1.26 | 1.34 | 1.54 | 1.75 | 2.45 |

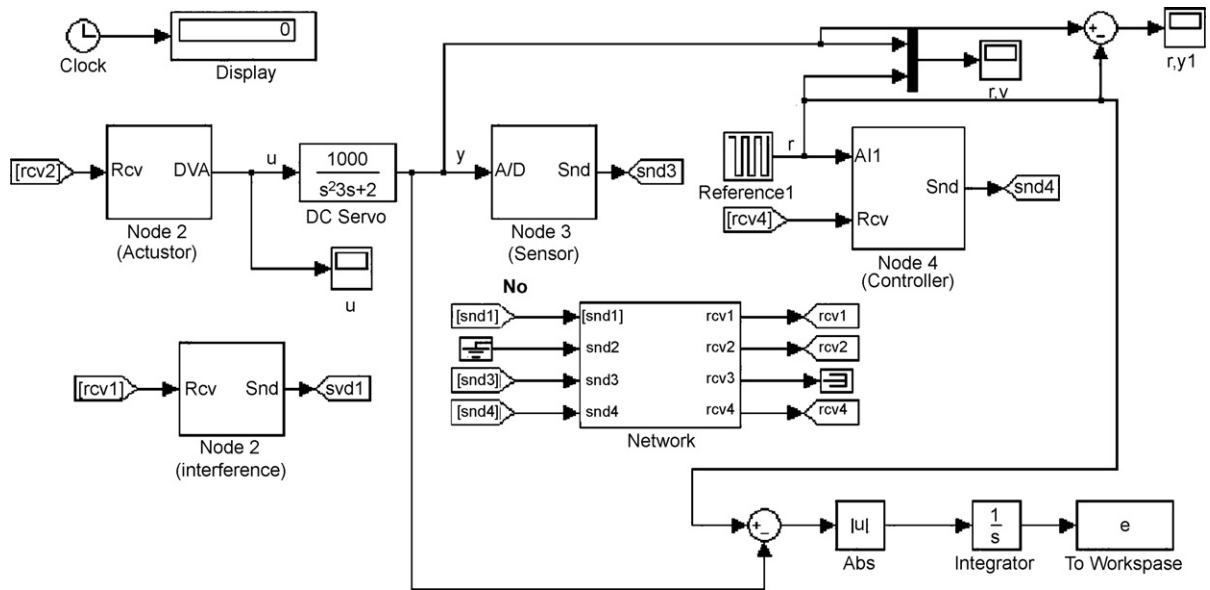


Fig. 1. Simulation structure of NCS based on true-time toolbox.

It can be seen from (1) that the relationship between IAE and sampling period is not monotone, but a concave conic. At first, the sampling period becomes smaller along with IAE becomes smaller, but to some degree, the sampling period becomes smaller along with IAE becomes larger. This shows that there exist the upper and lower bound of sampling period in NCS to satisfying QoP. For example, T_{\min} and T_{\max} in Fig. 2. It is clear that the smaller sampling period, the more data packets need to be transmitted, which heavies the network traffic load, increases the possibility of data loss and network-induced delay in bandwidth-limited network, and degrades the system performance [5,6]. Otherwise, from Table 1 and Fig. 2, it can be seen that the minimum of IAE(CAN) and IAE(Ethernet) occur when the sampling period is taken to be 0.01 s and 0.009 s respectively, there are table-to-figure matching due to Fig. 2 is drawn based on polynomial fit results (1) of Table 1.

Taking both the sampling period of multiple control loops and network utilization into considered, we can enhance integrated performance of NCS by appropriate optimization scheme.

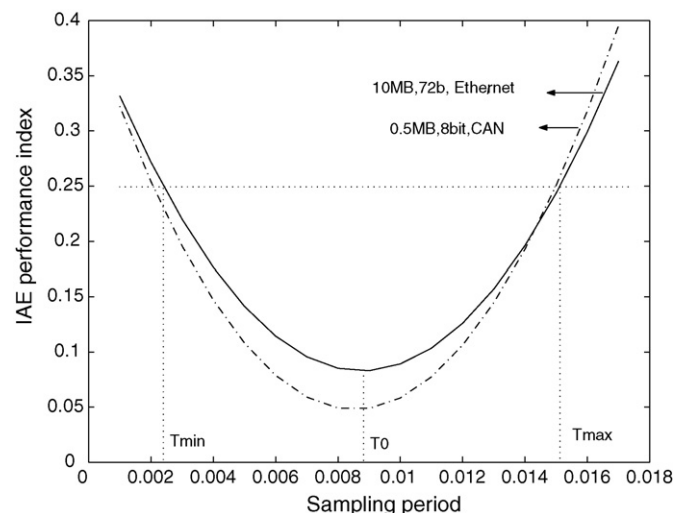


Fig. 2. Sampling period versus IAE performance.

Remark 1. The limitation on shared network resource among multiple control loops imposes a lower bound T_{\min} on the sampling period for each control loop. The control performance, such as stability, imposes an upper bound T_{\max} on the sampling period for each periodic transmitted task. Therefore, there have the upper and lower bound on the sampling period of NCSs.

3. Optimal scheduling of sampling period

In this section, the scheduling method of sampling period for multiple-control loops is proposed such that the integrated performance is optimized. To schedule the sampling period of multiple control loops, both dynamic (EDF, Earliest Deadline First) and static (RM, Rate Monotonic) priority assignment schemes will be considered. It shows in Fig. 2 that large sampling period lead to the same performance as that by small sampling period. However, more network bandwidth is needed when using smaller sampling period. Therefore, only the lowest point to allowable performance of right part of Fig. 2 is chosen as the region of sampling period scheduling. And Fig. 2 can be simplified as monotone curve and as a convex function. To facilitate development, in the following, we apply sampling frequency instead of sampling period. Moreover, the generalized exponential distribution is proposed to describe the relationship between IAE performance and the sampling frequency:

$$\Delta J_i = \omega_i \alpha_i e^{-\alpha_i(f_i - \beta_i)} \quad (2)$$

where $\omega_i (i = 1, \dots, n)$ are the weights, n is the number of control loop, f_i is sampling frequency of each control loop, α_i and β_i are correlative performance parameters of each control loop.

Remark 2. Although (2) is presented based on effective utilization of network bandwidth and scheduling of sampling frequency, it describes an general expression of performance versus sampling period. Similar results can be seen in [5,6,16].

To sampling period scheduling of multiple control loops, we will adjust the sampling frequency to optimize the overall system control performance $\sum_{i=1}^n \Delta J_i$, which are subject to two constraints:

(1) The performance of each control loops (the lower and upper bounds on sampling frequency):

$$f_{mi} \leq f_i \leq f_{ni} \quad (3)$$

(2) The underlying scheduling algorithm and the limitation on available network resource:

$$\sum_{i=1}^n c_i f_i \leq A, 0 < A \leq 1 \quad (4)$$

where $c_i (i = 1, \dots, n)$ is the data packet transmission time, A is the network utilization bound, which relates with chosen scheduling algorithm. The sampling frequency f_i should be chosen as near as possible to the upper bound f_{ni} for each closed control loop L_i . For guaranteeing the initial scheduling of sampling frequency of NCSs, the following assumption is necessary.

Assumption 1. When $f_i = f_{mi}$, $\sum_{i=1}^n c_i f_i \leq A$, and when $f_i = f_{ni}$, $\sum_{i=1}^n c_i f_i > A$, where f_{mi} and f_{ni} are the lower and upper bound of sampling frequency, respectively.

The following Lemma is crucial to the development of our main result.

Lemma 1. ([4]) A set of N independent, pre-emptive, periodic tasks can be feasibly scheduled by RM algorithm if its total utilization U satisfies

$$U = \sum_{i=1}^N c_i f_i \leq N(2^{1/N} - 1) \quad (5)$$

where f_i is the task frequency and c_i is the transmission time for each of the i th process.

Remark 3. In Lemma 1, independent means the initiation and completion of different tasks do not depend on each other, pre-emptive means the currently executing task is preempted by a newly arrived task with shorter period. Otherwise, Lemma 1 can be used to test the schedulability of NCSs connected by the same network medium.

It is noted that the optimization under constraints of (3) and (4) is a nonlinear programming problem. More precisely, this problem can be described as

$$\min_{(f_1, \dots, f_n)} \Delta J = \sum_{i=1}^n \omega_i \alpha_i e^{-\alpha_i(f_i - \beta_i)} \quad (6)$$

$$\text{subject to : } \sum_{i=1}^n c_i f_i \leq A, \quad 0 < A \leq 1 \quad (7)$$

$$f_{mi} \leq f_i \leq f_{ni} \quad (8)$$

The solutions of f_i^* based on (6)–(8) is generally referred to as constrained nonlinear optimization or nonlinear programming. It can be seen that ΔJ in (6) is a convex function and the constraint function (7) and (8) are linear, so Kuhn–Tucker theorem can be applied to obtain the optimal solutions of f_i^* . Based on above analysis, we can draw the following conclusion.

Theorem 1. Given the object performance function (6), there exist the optimal solutions given by

$$f_i = f_{mi}, \quad i = 1, \dots, p \quad (9)$$

$$f_j = \frac{\ln \Gamma_j - \Phi}{\alpha_j} + \beta_j, \quad j = p+1, \dots, n \quad (10)$$

where f_{mk} are ordered by

$$\Gamma_1 e^{-\alpha_1(f_{m1} - \beta_1)} \leq \Gamma_2 e^{-\alpha_2(f_{m2} - \beta_2)} \leq \dots \leq \Gamma_n e^{-\alpha_n(f_{mn} - \beta_n)}$$

$p \in [1, \dots, n]$ is the smallest integer such that

$$\sum_{i=1}^p c_i f_i + \sum_{j=p+1}^n c_j \left(\frac{\ln \Gamma_j - \ln \Gamma_p + \alpha_p(f_{mp} - \beta_p)}{\alpha_j} + \beta_j \right) \geq A \quad (11)$$

and

$$\begin{aligned} \Gamma_i &= \omega_i \alpha_i^2 / c_i \\ \Phi &= \frac{1}{\sum_{j=p+1}^n \frac{c_j}{\alpha_j}} \left(\sum_{i=1}^p c_i f_i + \sum_{j=p+1}^n c_j \left(\frac{\ln \Gamma_j}{\alpha_j} + \beta_j \right) - A \right) \end{aligned} \quad (12)$$

to satisfy the constraints of (7) and (8).

Proof. First, by introducing Lagrange Multipliers λ , λ_{i1} and λ_{i2} , the Kuhn–Tucker conditions can be written as

$$-\frac{\omega_i \alpha_i^2}{c_i} e^{-\alpha_i(f_i - \beta_i)} + \lambda - \frac{\lambda_{i1}}{c_i} + \frac{\lambda_{i2}}{c_i} = 0 \quad (13)$$

$$\lambda(A - \sum_{i=1}^n c_i f_i) = 0 \quad (14)$$

$$\lambda_{i1}(f_i - f_{mi}) = 0, \quad \lambda_{i2}(f_{ni} - f_i) = 0, \quad \lambda, \quad \lambda_{i1}, \lambda_{i2} \geq 0 \quad (15)$$

where $i = 1, \dots, n$. Based on Assumption 1, when $f_i = f_{mi}$ ($i = 1, \dots, n$), we have

$$\sum_{i=1}^n c_i f_i \leq A \quad (16)$$

By (15), when increasing the sampling frequency from f_{mp} for each $f_{ni} > f_i > f_{mi}$, we have $\lambda_{i1} = \lambda_{i2} = 0$, based on (13), defining $\Gamma_i = \omega_i \alpha_i^2 / c_i$, we obtain

$$\lambda = \Gamma_i e^{-\alpha_i(f_i - \beta_i)}, \quad i = p, \dots, n \quad (17)$$

The smallest integer p should be found to make

$$\sum_{i=1}^p c_i f_{mi} + \sum_{j=p+1}^n c_j f_j \geq A \quad (18)$$

According to (17), when $f_{ni} > f_i > f_{mi}$, $\lambda = \Gamma_i e^{-\alpha_i(f_i - \beta_i)}$, $i = p, \dots, n$

$$f_i = \frac{\ln \Gamma_i - \ln \lambda}{\alpha_i} + \beta_i, \quad i = p, \dots, n \quad (19)$$

Substituting f_i of (19) into (18), we have

$$\sum_{i=1}^p c_i f_i + \sum_{j=p+1}^n c_j \left(\frac{\ln \Gamma_j - \ln \lambda}{\alpha_j} + \beta_j \right) \geq A \quad (20)$$

According to (17), when $i = p$, $\lambda = \Gamma_p e^{-\alpha_p(f_p - \beta_p)}$ (20) is equivalent to

$$\sum_{i=1}^p c_i f_i + \sum_{j=p+1}^n c_j \left(\frac{\ln \Gamma_j - \ln \Gamma_p + \alpha_p(f_p - \beta_p)}{\alpha_j} + \beta_j \right) \geq A, \quad (21)$$

so (11) is obtained. After the integer p has been identified, the frequencies f_i must be chosen as that described in (9) are obtained, i.e., $f_i = f_{mi}$ ($i = 1, \dots, p$).

Since $\lambda_{i1,2} \geq 0$, $\Gamma_j > 0$, $\forall i$, from (14) we obtain

$$\text{when } \sum_{i=1}^n c_i f_i - A = 0, \quad \lambda > 0 \quad (22)$$

From (20) and (22), we have

$$\begin{cases} \sum_{i=1}^p c_i f_{mi} + \sum_{j=p+1}^n c_j \left(\frac{\ln \Gamma_j - \ln \lambda}{\alpha_j} + \beta_j \right) = A \\ \frac{1}{\sum_{j=p+1}^n \frac{c_j}{\alpha_j}} \left(\sum_{i=1}^p c_i f_{mi} + \sum_{j=p+1}^n c_j \left(\frac{\ln \Gamma_j}{\alpha_j} + \beta_j \right) - A \right) = \ln \lambda \end{cases} \quad (23)$$

Define $\Phi = \ln \lambda$ and according to (19), when $f_{ni} > f_i > f_{mi}$,

$$f_i = \frac{\ln \Gamma_i - \Phi}{\alpha_i} + \beta_i, \quad i = p+1, \dots, n \quad (24)$$

as given in (10), This completes the proof. \square \square

Remark 4. In this paper, we only consider the performance index of NCS characterized by (17) and samplers have variable sampling frequency. If part of samplers have invariable sampling frequency, we only need to modify A in (4) by subtracting $\sum_{i \in J} c_i f_i$ from the scheduling network utilization, where J is the index of sampler with the sampling frequency being fixed. When the computing results in (24) satisfying $f_i \geq f_{ni}$, we set $f_i = f_{ni}$ and modify A in (4) by subtracting $\sum_{i \in L} c_i f_{ni}$ from the scheduling network utilization, where L is the indices of the sampler with the sampling frequency satisfying $f_i \geq f_{ni}$.

Remark 5. In Theorem 1, A is related with the chosen scheduling algorithm of transmitted data packet in multiple control loops. When dynamic EDF scheduling is chosen [11], A is set to 1, when static RM scheduling is chosen, A is a function of N , c_i and f_i are defined in Lemma 1.

Theorem 1 provides a method to optimizing the sampling frequency of multiple control loops of NCSs. Based on **Theorem 1**, we obtain the following optimal algorithm of the sampling frequency.

Algorithm. Optimal algorithm of the sampling frequency

- (1) Based on priority scheduling algorithm to decide A , when EDF is chosen, $A = 1$; when RA is chosen, A is decided by (5).
- (2) Use (7) and (8) to compute U , if $\sum_{i=1}^N c_i f_{mi} < A < \sum_{i=1}^N c_i f_{ni}$, it means that f_i is schedulable, goto step 4. Otherwise, f_i is unschedulable, goto step 3.
- (3) Reduce the number of closed-loop L_i until $\sum_{i=1}^N c_i f_{mi} < A$.
- (4) In terms of **Theorem 1**, compute and output f_i .

Compared with the sampling period scheduling algorithm presented in the existing work, neither the assumption that the sampling periods are less than the Maximum Allowable Deadline Bounds (MADB) [3] nor the assumption that the other control loop's sampling periods are the integral multiples of the basis sampling period [8] are necessary in this paper. Otherwise, compared with the non-optimal sampling period scheduling algorithm introduced in [8,3], it is clear that Kuhn–Tucker Theorem based scheduling algorithm is an optimal sampling period algorithm.

4. Scheduling example

Consider networked DC motor control system, suppose five such subsystems with shared communication medium, different distance with the corresponding PI controller. $\Delta J_i = \omega_i \alpha_i e^{-\alpha_i(f_i - \beta_i)} (i = 1, \dots, 5)$, which means the relationship between IAE performance and the sampling frequency. For each control system, let c_i be the data packet transmission time of each sampling period of sensor. f_{mi} and f_{ni} are the lower and upper bound on sampling frequency, respectively. ω_i , α_i and β_i are the parameters assigned to system L_i . The parameters listed in Table 2 are given for the sampling frequency scheduling and where the optimal sampling frequencies $f_i (i = 1, \dots, 5)$ must to be determined.

For the scheduling of sampling frequency, we consider both EDF and RM approaches. To EDF, A is set to 1, to RM, A is set based on Lemma 1.

According to the sampling period scheduling algorithm, we first check whether the sampling period is schedulable under EDF. When all sampling frequencies are set to their lower bound f_{mi} , $\sum_{i=1}^5 C_i f_{mi} = 0.964 < 1$, which indicates that all sampling frequencies are schedulable. But when all sampling frequencies are set to their upper bound f_{ni} , $\sum_{i=1}^5 C_i f_{ni} = 2.074 > 1$, which indicates that all sampling period do not able to achieve their upper bound. In terms of **Theorem 1**, we obtain the optimal frequency for each closed-loop as follows. Let

$$F(p) = \sum_{i=1}^p c_i f_i + \sum_{j=p+1}^n c_j \left(\frac{\ln \Gamma_j - \ln \Gamma_p + \alpha_p(f_{mp} + \beta_p)}{\alpha_j} + \beta_j \right) \geq A, \quad (25)$$

by calculating we obtain $F(5) = 0.964 < 1$, $F(4) = 0.9756 < 1$, $F(3) = 0.981 < 1$, $F(2) = 1.361 > 1$, where $p = 2$, $\Phi = 4.0838$.

Therefore, according to (9), when f_i is assigned as follows

$$f_i = f_{mi}, \quad i = 1, \dots, p, \quad f_1 = 18, \quad f_2 = 14, \quad (26)$$

Table 2

Parameters of different control loops Applied in scheduling example

| Number of DC | ω_i | α_i | β_i | c_i | f_{mi} | f_{ni} |
|--------------|------------|------------|-----------|-------|----------|----------|
| DC1 | 9 | 0.32 | 13 | 18 | 18 | 28 |
| DC2 | 7 | 0.33 | 10 | 19 | 14 | 24 |
| DC3 | 8 | 0.34 | 9 | 20 | 8 | 18 |
| DC4 | 5 | 0.35 | 8 | 26 | 5 | 15 |
| DC5 | 4 | 0.36 | 7 | 28 | 4 | 14 |

$f_{3,4,5}$ could be computed by

$$\begin{cases} f_3 = \frac{\ln \Gamma_3 - \Phi}{\alpha_3} + \beta_3 = 8.2648 \\ f_4 = \frac{\ln \Gamma_4 - \Phi}{\alpha_4} + \beta_4 = 5.359 \\ f_5 = \frac{\ln \Gamma_5 - \Phi}{\alpha_5} + \beta_5 = 3.7632 \\ \Delta J = \sum_{i=1}^5 \omega_i \alpha_i e^{-\alpha_i(f_i - \beta_i)} = 13.719 \end{cases} \quad (27)$$

The optimal results yield the total 100% of network utilization, and optimal IAE performance is 13.719. it is obvious that the final set of transmitted task are schedulable.

When static scheduling algorithm *RM* is chosen, according to Theorem 1, we obtain

$$\begin{cases} N = 5, U = \sum_{i=1}^N c_i f_i = 0.964 > 5(2^{1/5} - 1) = 0.7435 \\ N = 4, U = \sum_{i=1}^N c_i f_i = 0.88 > 4(2^{1/4} - 1) = 0.7568 \\ N = 3, U = \sum_{i=1}^N c_i f_i = 0.75 < 4(2^{1/4} - 1) = 0.7798 \end{cases} \quad (28)$$

It is obvious that when units 4 and 5 are included, the multiple control loops are unschedulable. Assume that units 4 and 5 are removed, then scheduling the units 1, 2 and 3, we obtain $F(3) = 0.75 < A = 0.7798$, $F(2) = 0.856 > A = 0.7568$, $p = 2$, $\Phi = 3.6679$.

Therefore, according to (9), when f_i is assigned as

$$f_i = f_{mi}, \quad i = 1, 2, \quad f_1 = 18, \quad f_2 = 14, \quad (29)$$

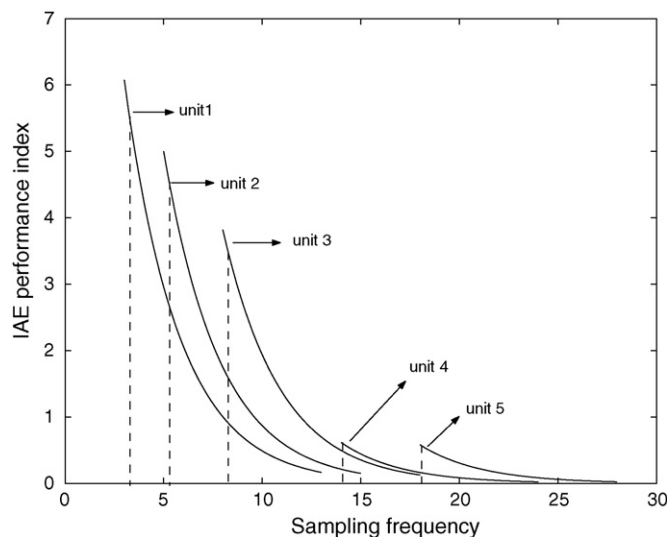


Fig. 3. Optimal result of sampling frequency.

f_3 could be computed by

$$f_3 = \frac{\ln \Gamma_3 - \Phi}{\alpha_3} + \beta_3 = 9.4882 \quad (30)$$

Then, $\Delta J = \sum_{i=1}^3 \omega_i \alpha_i e^{-\alpha_i(f_i - \beta_i)} = 3.5026$.

The total network utilization is 77.98% based on this optimal results and optimal IAE performance is 3.5026.

Fig. 3 shows the curve of the variation of IAE performance with the different sampling frequencies, the optimal result of sampling frequencies are shown in vertical line. From it we can see that $f_{ni} > f_i \geq f_{mi}$ and obtain the minimum IAE and achieve maximum network utilization.

5. Conclusion

This paper is concerned with the multiple-control loops' sampling period scheduling of networked control systems. Although the relationship between IAE and sampling period is not monotone, we have found that the performance indices can be translated into monotone decrease and the sampling frequency has the upper and lower bound. An exponential function to describe IAE performance versus sampling period is presented based on Truetime simulation, and an optimal sampling period scheduling algorithm is proposed based on Kuhn–Tucker Theorem. If the chosen initial lower bounds of sampling frequency are all schedulable, the presented algorithm guarantees the system performance will be optimized subject to the limitation of control performance and available network resource. If the chosen initial lower bounds of sampling frequency are not all schedulable, the presented algorithm guarantees the priority control loops performance. The controller and scheduling codesign to optimize the overall performance index are left for our future studies.

Acknowledgments

The authors would like to acknowledge Natural Science Foundation of China (NSFC) for its support under grant numbers 60704024 and 60774060 and the High Technology Project of Jiangsu Province (BG2006042). Peng is also grateful to Natural Science Foundation of Jiangsu Province of China for its support under grant number BK2006573. Special thanks also go to the anonymous reviewers for their constructive comments and suggestions for improvement of the quality and readability of the paper.

References

- [1] M.S. Branicky, S.M. Phillips, W. Zhang, Scheduling and feedback co-design for networked control systems, in: Proceedings of 41st IEEE conference on decision and control, Las Vegas, Nevada USA, December, 2002, pp. 670–675.
- [2] A. Cervin, D. Henriksson, B. Lincoln, J. Eker, K. Arzen, How does control timing affect performance? Analysis and simulation of timing using Jitterbug and TrueTime, *Control Systems Magazine, IEEE* 23 (3) (2003) 16–30.
- [3] S.H. Hong, Scheduling algorithm of data sampling times in the integrated communication and control systems, *IEEE Transactions on Control System Technology* 3 (1995) 225–231.
- [4] W. Jane, S. Liu, *Real-Time Systems*, Prentice Hall, Upper Saddle River, NJ, 2000.
- [5] F.L. Lian, J.R. Moyne, D.M. Tilbury, Performance evaluation of control networks: Ethernet, controlnet, and devicenet, *Control Systems Magazine, IEEE* 21 (2001) 66–83.
- [6] F.L. Lian, J.R. Moyne, D.M. Tilbury, Network design consideration for distributed control systems, *IEEE Transactions on Control Systems Technology* 10 (2) (2002) 297–307.
- [7] C.L. Liu, J.W. Layland, Scheduling algorithms for multiprogramming in a hard real time environment, *Journal of Association for Computing Machinery* 20 (1973) 46–61.
- [8] H.S. Park, Y.H. Kim, D.S. Kim, W.H. Kwon, A scheduling method for network based control systems, *IEEE Transactions on Control System Technology* 10 (2002) 318–330.
- [9] C. Peng, Y.-C. Tian, Networked H_∞ control of linear systems with state quantization, *Information Sciences* 177 (2007) 5763–5764.
- [10] C. Peng, Y.-C. Tian, M.O. Tade, State feedback controller design of networked control systems with interval time-varying delay and nonlinearity, *International Journal of Robust and Nonlinear Control* 18 (2008) 1285–1301.
- [11] D. Seto, J. Lehoczky, L. Sha, K. Shin, On task schedulability in real-time control systems, in: *Real-Time Systems Symposium*, 1996., 17th IEEE, 1996, pp. 13–21.

- [12] Y.-C. Tian, Z.-G. Yu, C. Fidge, Multifractal nature of network induced time delay in networked control systems, *Physics Letters A* 361 (2007) 103–107.
- [13] G.C. Walsh, Y. Hong, Scheduling of networked control systems, *IEEE Control Systems Magazine* 21 (2001) 57–65.
- [14] D. Yue, Q.L. Han, C. Peng, State feedback controller design of networked control systems, *Circuits and Systems II: Express Briefs, IEEE Transactions* 51 (2004) 640–644.
- [15] W. Zhang, Stability analysis of networked control systems, Ph.D. thesis, Ph. D Dissertation, Department of Electrical Engineering and Computer Science, Case Western Reserve University (2001).
- [16] C. Peng, D. Yue, Z. Gu, Multiple sampling periods scheduling of networked control systems, in: *Proceedings of the 27th chinese control conference*, Kunming, Yunan, China, July, 2008, pp. 447–451.