Event-Triggered Synchronization of Chaotic Lur'e Systems via Memory-Based Triggering Approach

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***Abstract*—This brief focus on the event-triggered synchronization of networked chaotic Lur'e systems. To relieve the burden of communication network with limited bandwidth, a novel memory-based event-triggered mechanism with the past information of the system is presented. The synchronization error system under the memory-based event-triggered mechanism is modeled by a distributed delay system. Applying an integral inequality based on Legendre polynomials, a novel synchronization condition is derived via linear matrix inequalities. Finally, a Chua's circuit is adopted to illustrate the advantage of the proposed control strategy.**

***Index Terms*—Memory-based event-triggered mechanism, chaotic Lur'e systems, Chua's circuit.**

I. Introduction

As a classical kind of nonlinear system, chaotic Lur'e systems (CLSs) can be used to describe some practical systems, such as Chua's circuits [1], [2], artificial neural networks [3], [4], and so on. The synchronization problem of CLSs is a meaningful topic, which has been widely applied in secure communication [5] and image processing [6]. The existing results mainly focus on designing effective synchronization control methods, such as pinning control [7], [8], finite-time cluster synchronization [9], and the references therein. It is noticed that most of the outcomes are based on the continuous measurement outputs. With the integration of a communication network, such continuous or time-triggered communication scheme does lead to redundant transmissions and consume limited network bandwidth.

To release the burden of the network, an aperiodic manner called event-triggered mechanism (ETM) is utilized to transmit the control signal only when some pre-defined events happen [11], [12]. In recent years, the event-triggered synchronization problem of CLSs has attracted extensive attention from researchers [13], [14]. Specifically, the master-slave event-triggered synchronization issue of CLSs is addressed in [13], where the quantized measurements and constant time delay are considered for control signals. In [14], the exponential synchronization of CLSs with time-varying transmission delays with an ETM is developed. Compared with some existing results requiring the positivity of the LyapunovKrasovskii (LKF) functional over the sampling intervals, the LKF is positive definite only at sampling instants by using a novel inequality technique. Thus, this scheme is more practical than the conventional ETM with a time-invariant triggering threshold. For CLSs with sampled data, [15] studies the event-triggered synchronization issue with an adaptive triggering threshold parameter, which can be adjusted along with system dynamics. Different from [15] with periodic sampling, [16] investigates the synchronization issue of CLSs with ETM and variable sampling. It is noted that the systems are usually disturbed by external noises or abrupt disturbances, which could result in false triggers under the previous ETMs are based on instantaneous system information. Then, such false triggers may lead to the waste of network resources and stability deterioration. To avoid this problem, a memory ETM using the past system information is addressed for event-triggered H_∞ filtering of unmanned surface vehicles in [17]. However, it is known that the historical information matched with different weights will be more practical and challenging.

In this brief, the event-triggered synchronization problem for CLSs is investigated via a memory-based triggering method. The Chua's circuit is implemented to indicate the validity of the developed strategy. The main contributions are summarized as follows:

i) A novel memory-based event-triggered mechanism (METM) is presented by introducing the past measurement outputs with different weights. The closer the past information is to the current time, the greater the weight is set. Under such a METM, unexpected triggering events caused by random fluctuation of the system can be effectively decreased.

ii) The synchronization error system under the METM is converted to a distributed delay system. With the aid of the integral inequality based on Legendre polynomials, a new synchronization condition is achieved to ensure the asymptotic stability of the CLSs.

Notations: $\mho\{N_1, N_2\}$ stands for $N_2^T N_1 N_2$. $He\{N\}$ means the sum of matrix N and its transpose N^T.

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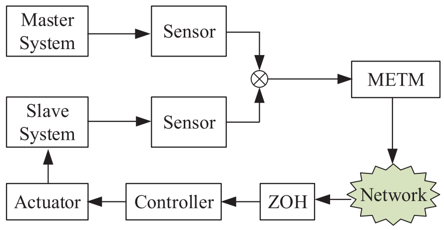

Fig. 1. The block of memory-based event-triggered synchronization for CLSs.

II. Problem Formulation

The following CLS with the master-slave form is considered as:

$$\mathbb{M}:\begin{cases}\dot{w}(t)=Aw(t)+B\theta(Cw(t))\\ y(t)=Dw(t),\end{cases}$$
$$\mathbb{S}:\begin{cases}\dot{\hat{w}}(t)=A\hat{w}(t)+B\theta(C\hat{w}(t))+u(t)\\ \hat{y}(t)=D\hat{w}(t),\end{cases}\quad(1)$$

where $w(t)\in\mathbb{R}^n$ and $\hat{w}(t)\in\mathbb{R}^n$ are the states of system $\mathbb{M}$ and system $\mathbb{S}$; $\theta(\cdot)\in\mathbb{R}^m$ is a nonlinear vector with $\theta_i(\cdot)\in[0,\varpi_i]$ for $i=1,2,\ldots,m$; $u(t)$ is the control input; $y(t)\in\mathbb{R}^p$ and $\hat{y}(t)\in\mathbb{R}^p$ are system outputs; A,B,C and D are known matrices with appropriate dimensions.

The synchronization controller is designed as

$$u(t)=K(y(t)-\hat{y}(t)).\quad(2)$$

Letting $x(t)=w(t)-\hat{w}(t)$, and substituting (2) to (1), we have the synchronization error system as:

$$\begin{cases}\dot{x}(t)=Ax(t)+B\phi(x(t))-KDx(t)\\ z(t)=y(t)-\hat{y}(t),\end{cases}\quad(3)$$

where $\phi(x(t))=\theta(Cx(t)+C\hat{w}(t))-\theta(C\hat{w}(t))$ satisfies

$$0\le\frac{\theta_i(c_i^Tx+c_i^T\hat{w})-\theta_i(c_i^T\hat{w})}{c_i^Tx}=\frac{\phi_i(x)}{c_i^Tx}\le\vartheta_i,\quad(4)$$

for $i=1,2,\ldots,m$, where c_i^T means the ith row of C.

From (4), one has

$$\phi_i^T(x)[\phi_i(x)-\vartheta_ic_i^Tx]\le0.\quad(5)$$

To release the load of the communication network, a novel METM using past system measurements is proposed to update the control signals, which is drawn in Fig. 1 and designed as:

$$t_{k+1}=\min_t\{t\ge t_k|\psi(t)\ge0\},\quad t\in[t_k,\ t_{k+1})\quad(6)$$

where

$$\psi(t)=\varepsilon^T(t)M\varepsilon(t)-\varpi z^T(t_k)Mz(t_k),$$
$$\varepsilon(t)=D\sum_{j=1}^{N}\frac{\lambda_j}{s}\epsilon_j(t)-z(t_k),\quad\sum_{j=1}^{N}\lambda_j=1/N,$$
$$\epsilon_j(t)\triangleq\int_{t-js}^{t-(j-1)s}x(v)dv,$$

and $\varpi\in[0,1)$, $M>0$ is the triggering matrix, $h=sN$ means the overall interval length of past system outputs and is divided into N intervals, s denotes the length of each divided interval, λ_j $(j=1,\ldots,N)$ means the weight of the past system outputs over the interval $(t-js,t-(j-1)s]$.

Remark 1: When the weight of each interval is set as the same value $\lambda_j=1/N^2$, the METM in (6) is expressed as:

$$t_{k+1}=\sup_t\Big\{t\ge t_k|\hat{\psi}(t)\ge0\Big\},\quad(7)$$

which covers the existing METM [17] as a special case with $\hat{\psi}(t)=\hat{\varepsilon}^T(t)M\hat{\varepsilon}(t)-\varpi z^T(t_k)Mz(t_k)$, $\hat{\varepsilon}(t)=\frac{D}{h}\int_{t-h}^{t}x(v)dv$.

In addition, (6) can be written as

$$t_{k+1}=\sup_t\Big\{t\ge t_k|\tilde{\varepsilon}^T(t)M\tilde{\varepsilon}(t)\le\varpi z^T(t_k)Mz(t_k)\Big\}\quad(8)$$

with $\tilde{\varepsilon}(t)=D(x(t)-x(t_k))$, which reduces to [10] with $D=I$. Namely, the proposed METM is general than some existing ETMs.

Remark 2: Different from the existing ETMs, such as in [15], the proposed METM in (6) utilizes the historical information with different weight rather than the current information, by which some unexpected triggering events can be reduced, thereby saving the limited network-bandwidth.

Based on the constructed METM, the synchronization controller is rewritten as:

$$u(t)=Kz(t_k)=KD\textstyle\sum_{j=1}^{N}\frac{\lambda_j}{s}\epsilon_j(t)-K\varepsilon(t).\quad(9)$$

Then, the system (3) is expressed as:

$$x(t)=Ax(t)+B\phi(x(t))+KD\sum_{j=1}^{N}\frac{\lambda_j}{s}\epsilon_j(t)-K\varepsilon(t).\quad(10)$$

Before ending this section, a useful lemma is provided as follows to achieve the main results.

Lemma 1 [19]: For a vector function $x:[a_1,a_2]\to\mathbb{R}^n$, and a symmetric matrix $Q>0$, one has

$$\int_{a_1}^{a_2}\mho\{Q,x(v)\}dv\ge\frac{1}{a_2-a_1}\mho\{\mathcal{Q},\mathscr{X}(t)\},\quad(11)$$

where $\mathcal{Q}=\mathscr{P}\otimes Q$, $\mathscr{P}=diag\{1,3,\ldots,2r-1\}$, $\mathscr{X}(t)=\int_{a_1}^{a_2}F(v)x^T(t+v)dv$, $F(v)=[f_0(v)\cdots f_i(v)\cdots f_r(v)]\otimes I_n$, and $f_i(v)$ is the Legendre polynomials defined in [19].

The main objective of this brief is to design the synchronization controller in (9) and the METM in (6).

III. Controller and METM Design

In this section, some novel conditions for designing a synchronization controller of CLSs are derived by Lemma 1 and presented in the next theorem.

Theorem 1: For given scalars ϖ,s,α and λ_i $(i=1,\ldots,N)$, the synchronization error system (10) under the METS (6) is asymptotically stable, if there exist symmetric matrices R, $T_j>0$, $Q_j>0$ $(j=1,2,\ldots,N)$, $M>0$, and matrices G, Y such that

$$\mathcal{R}>0,\quad(12)$$
$$\Phi+He\big(\widehat{\mathscr{G}\mathscr{M}}\big),\quad(13)$$

where

$$\mathcal{R}=R+diag\Big\{0,\frac{1}{s}\mathcal{T}_1,\ldots,\frac{1}{s}\mathcal{T}_N\Big\},$$
$$\mathcal{T}_j=\mathscr{P}\otimes T_j,\quad\mathcal{Q}_j=\mathscr{P}\otimes Q_j,$$
$$\Phi=He(\mathscr{U}^TR\mathscr{V}+\mathbb{J}_2^TC^T\Upsilon U\mathbb{J}_\phi-\mathbb{J}_\phi^TU\mathbb{J}_\phi)$$

$$
\begin{aligned}
&+\sum_{j=1}^{N}\left[\mho\left\{T_j,\mathbb{J}_{j+1}\right\}-\mho\left\{T_j,\mathbb{J}_{j+2}\right\}\right]\\
&+\sum_{j=1}^{N}\left[\mho\left\{sQ_j,\mathbb{J}_{j+1}\right\}-\frac{1}{s}\mho\left\{\mathcal{Q}_j,\mathbb{J}_N^j\right\}\right]\\
&+\varpi\mho\left\{M,\left(D\sum_{j=1}^{N}\frac{\lambda_j}{s}\mathbb{H}_0\mathbb{J}_N^j-\mathbb{J}_\varepsilon\right)\right\}-\mho\{M,\mathbb{J}_\varepsilon\},
\end{aligned}
$$

$$
\mathscr{U}=\begin{bmatrix}\mathbb{J}_2\\ \mathbb{J}_N^1\\ \vdots\\ \mathbb{J}_N^j\\ \vdots\\ \mathbb{J}_N^N\end{bmatrix},\quad \mathscr{V}=\begin{bmatrix}\mathbb{J}_1\\ \mathbf{I}\mathbb{J}_2-\hat{\mathbf{I}}\mathbb{J}_3-\frac{\Lambda}{s}\mathbb{J}_N^1\\ \vdots\\ \mathbf{I}\mathbb{J}_{j+1}-\hat{\mathbf{I}}\mathbb{J}_{j+2}-\frac{\Lambda}{s}\mathbb{J}_N^j\\ \vdots\\ \mathbf{I}\mathbb{J}_{N+1}-\hat{\mathbf{I}}\mathbb{J}_{N+2}-\frac{\Lambda}{s}\mathbb{J}_N^N\end{bmatrix},
$$

$$
\mathbf{I}=\begin{bmatrix}I_n\\ \vdots\\ I_n\end{bmatrix}_{n(r+1)\times n},\quad \hat{\mathbf{I}}=\begin{bmatrix}I_n\\ \vdots\\ (-1)^r I_n\end{bmatrix}_{n(r+1)\times n},
$$

$$
\widehat{\mathscr{M}}=-G\mathbb{J}_1+GA\mathbb{J}_2+GB\mathbb{J}_\phi+YD\sum_{j=1}^{N}\frac{\lambda_j}{s}\mathbb{H}_0\mathbb{J}_N^j-Y\mathbb{J}_\varepsilon,
$$

$$
\widehat{\mathscr{G}}=\mathbb{J}_1^T+\alpha\mathbb{J}_2^T,\quad \mathbb{H}_0=\begin{bmatrix}I_n & 0_{n,(r-1)n}\end{bmatrix}.
$$

In addition, the controller gain is obtained by $K=G^{-1}Y$.

Proof: Define

$$
\begin{aligned}
\eta(t)&=\left[x^T(t)\ \ \mathscr{X}_0(t)\ \cdots\ \mathscr{X}_j(t)\ \cdots\ \mathscr{X}_N(t)\right]^T,\\
\mathscr{X}_j(t)&=\int_{-js}^{-(j-1)s}F(v)x^T(t+v)dv.
\end{aligned}
$$

We construct the following LKF as

$$
V(t)=V_0(t)+\sum_{j=1}^{N}V_j(t), \tag{14}
$$

where

$$
\begin{aligned}
V_0(t)&=\mho\{R,\eta(t)\},\\
V_j(t)&=\int_{-js}^{-(j-1)s}\mho\left\{T_j+(v+s)Q_j,x(t+v)\right\}dv.
\end{aligned}
$$

From Lemma 1, one can know that

$$
\int_{-js}^{-(j-1)s}\mho\left\{T_j,x(t+v)\right\}dv\geq\frac{1}{s}\mho\left\{\mathcal{T}_j,\mathscr{X}_j(t)\right\}. \tag{15}
$$

Due to $\mathcal{R}\geq 0$, $Q_j\geq 0$, we have

$$
V(t)\geq\mho\{\mathcal{R},\eta(t)\}+\sum_{j=1}^{N}\int_{-js}^{-(j-1)s}\mho\left\{(v+s)Q_j,x(t+v)\right\}dv>0, \tag{16}
$$

where $\mathcal{R}=R+diag\{0,\mathcal{T}_1,\ldots,\mathcal{T}_N\}$ and $\mathcal{T}_j=\mathscr{P}\otimes T_j$.

Then, the time derivative of $V(t)$ is calculated as

$$
\begin{aligned}
\dot{V}(t)&=2\eta^T(t)R\dot{\eta}(t)\\
&\quad+\sum_{j=1}^{N}\left[\mho\left\{T_j,x(t-(j-1)v)\right\}-\mho\left\{T_j,x(t-jv)\right\}\right]\\
&\quad+\sum_{j=1}^{N}\left[\mho\left\{sQ_j,x(t-(j-1)v)\right\}\right.\\
&\qquad\left.-\int_{-js}^{-(j-1)s}\mho\left\{Q_j,x(t+v)\right\}dv\right].
\end{aligned} \tag{17}
$$

The derivative of $\mathscr{X}_j(t)$ is computed as

$$
\begin{aligned}
\dot{\mathscr{X}}_j(t)&=\mathbf{I}x(t-(j-1)s)-\hat{\mathbf{I}}x(t-js)-\frac{\Lambda}{s}\mathscr{X}_j(t)\\
&\triangleq\mathscr{Y}(t).
\end{aligned} \tag{18}
$$

Similar to (15), one can obtain

$$
-\int_{-js}^{-(j-1)s}\mho\left\{Q_j,x(t+v)\right\}dv\leq-\frac{1}{s}\mho\left\{\mathcal{Q}_j,\mathscr{X}_j(t)\right\}. \tag{19}
$$

By defining $\varsigma(t)=[\dot{x}^T(t)\ \ x^T(t)\ \ x^T(t-s)\ \ \cdots\ \ x^T(t-Ns)\ \ \mathscr{X}_1^T(t)\ \ \cdots\ \ \mathscr{X}_N^T(t)\ \ \varepsilon^T(t)\ \ \phi^T(x(t))]^T$, and

$$
\begin{aligned}
\mathbb{J}_1&=[I_n\ \ 0_n\ \ 0_{n\times nN}\ \ 0_{n\times nrN}\ \ 0_{n\times p}\ \ 0_{n\times m}],\\
\mathbb{J}_2&=[0_n\ \ I_n\ \ 0_{n\times nN}\ \ 0_{n\times nrN}\ \ 0_{n\times p}\ \ 0_{n\times m}],\\
\mathbb{J}_{j+2}&=[0_{n\times 2n}\ \ 0_{n\times n(j-1)}\ \ I_n\ \ 0_{n\times n(N-j)}\ \ 0_{n\times(nrN+p+m)}],\\
\mathbb{J}_N^j&=[0_{rn\times(2n+nN)}\ 0_{rn\times rn(j-1)}\ I_{rn}\ 0_{rn\times rn(N-j)}\ 0_{rn\times(p+m)}],\\
\mathbb{J}_\varepsilon&=[0_{p\times n}\ \ 0_{p\times n}\ \ 0_{p\times nN}\ \ 0_{p\times nrN}\ \ I_p\ \ 0_{p\times m}],\\
\mathbb{J}_\phi&=[0_{m\times n}\ \ 0_{m\times n}\ \ 0_{m\times nN}\ \ 0_{n\times nrN}\ \ 0_{m\times p}\ \ I_m],
\end{aligned}
$$

it leads to

$$
\eta(t)=\mathscr{U}\varsigma(t),\quad \dot{\eta}(t)=\mathscr{V}\varsigma(t). \tag{20}
$$

From the event-triggering condition (6), one can know that $\psi(t)<0$ equals to

$$
\varpi\mho\left\{M,\left[D\sum_{j=1}^{N}\frac{\lambda_j}{s}\epsilon_j(t)-\varepsilon(t)\right]\right\}-\mho\{M,\varepsilon(t)\}>0, \tag{21}
$$

which is further written as

$$
\varpi\mho\left\{M,\left(D\sum_{j=1}^{N}\frac{\lambda_j}{s}\mathbb{H}_0\mathbb{J}_N^j-\mathbb{J}_\varepsilon\right)\varsigma(t)\right\}-\mho\{M,\mathbb{J}_\varepsilon\varsigma(t)\}>0. \tag{22}
$$

Adopting $U=diag\{u_{l1},\ldots,u_{lm}\}$ to (5), one can get

$$
-2\sum_{j=1}^{m}u_{li}\phi_i^T(x(t))(\phi_i(x(t))-\vartheta_i c_i^T x(t))\geq 0, \tag{23}
$$

which can be expressed as

$$
\begin{aligned}
&He\big(x^T(t)C^T\Upsilon U\phi(x(t))-\phi^T(x(t))U\phi^T(x(t))\big)\\
&=He\Big(\varsigma^T(t)\Big(\mathbb{J}_2^TC^T\Upsilon U\mathbb{J}_\phi-\mathbb{J}_\phi^TU\mathbb{J}_\phi\Big)\varsigma(t)\Big)\geq 0,
\end{aligned} \tag{24}
$$

where $\Upsilon=diag\{\vartheta_1,\ldots,\vartheta_m\}$.

Then, it gives

$$
\begin{aligned}
\dot{V}(t)&<\varsigma^T(t)He(\mathscr{U}^TR\mathscr{V}+\mathbb{J}_2^TC^T\Upsilon U\mathbb{J}_\phi-\mathbb{J}_\phi^TU\mathbb{J}_\phi)\varsigma(t)\\
&\quad+\sum_{j=1}^{N}\left[\mho\left\{T_j,\mathbb{J}_{j+1}\varsigma(t)\right\}-\mho\left\{T_j,\mathbb{J}_{j+2}\varsigma(t)\right\}\right]\\
&\quad+\sum_{j=1}^{N}\left[s\mho\left\{Q_j,\mathbb{J}_{j+1}\varsigma(t)\right\}-\frac{1}{s}\mho\left\{\mathcal{Q}_j,\mathbb{J}_N^j\varsigma(t)\right\}\right]
\end{aligned}
$$

$$+ \varpi \mho\left\{M, \left(D\sum_{j=1}^{N}\frac{\lambda_j}{s}\mathbb{H}_0\mathbb{J}_N^j - \mathbb{J}_\varepsilon\right)\varsigma(t)\right\} - \mho\{M, \mathbb{J}_\varepsilon\varsigma(t)\}. \tag{25}$$

To ensure the asymptotic stability of system (10), it requires $\dot{V}(t) < 0$, which can be guaranteed by

$$\dot{V}(t) < \varsigma^T(t)\Phi\varsigma(t) < 0, \tag{26}$$

where

$$\begin{aligned}\Phi = {} & He(\mathscr{U}^T R\mathscr{V} + \mathbb{J}_2^T C^T \Upsilon U\mathbb{J}_\phi - \mathbb{J}_\phi^T U\mathbb{J}_\phi) \\ & + \sum_{j=1}^{N}\left[\mho\{T_j, \mathbb{J}_{j+1}\} - \mho\{T_j, \mathbb{J}_{j+2}\}\right] \\ & + \sum_{j=1}^{N}\left[\mho\{sQ_j, \mathbb{J}_{j+1}\} - \frac{1}{s}\mho\left\{Q_j, \mathbb{J}_N^j\right\}\right] \\ & + \varpi\mho\left\{M, \left(D\sum_{j=1}^{N}\frac{\lambda_j}{s}\mathbb{H}_0\mathbb{J}_N^j - \mathbb{J}_\varepsilon\right)\right\} - \mho\{M, \mathbb{J}_\varepsilon\}.\end{aligned}$$

Revisiting the description of system (10) yields

$$\mathscr{M}\varsigma(t) = 0, \tag{27}$$

where $\mathscr{M} = -\mathbb{J}_1 + A\mathbb{J}_2 + B\mathbb{J}_\phi + KD\sum_{j=1}^{N}\frac{\lambda_j}{s}\mathbb{H}_0\mathbb{J}_N^j - K\mathbb{J}_\varepsilon$.

By constructing $\mathscr{G} = \mathbb{J}_1^T G + \alpha\mathbb{J}_2^T G$ such that $\mathscr{G}\mathscr{M}\varsigma(t) = 0$, it yields

$$\Phi + He(\mathscr{G}\mathscr{M}) < 0 \tag{28}$$

Defining a new variable $Y = GK$, we can get the condition (13) holds, which completes the proof. ■

Remark 3: Theorem 1 is derived under the assumption (4) for nonlinear item $\theta_i(\cdot)$, under which one can conveniently get the parameters of both controller and METM off-line by the LMI toolbox in MATLAB. However, (4) is a strong constrain condition, some meta heuristic algorithms used in [20]–[22] can be referred in the future study to find feasible solution of the synchronization controller parameters.

IV. Simulation Example

In this section, we will use a well-known nonlinear electronic model of Chua's circuit which is a typical application of CLSs to demonstrate the effectiveness of the proposed method. The single Chua's circuit depicted in Fig. 2 is described by the following equations.

$$\begin{cases}\dot{w}_1(t) = a(w_2(t) - \chi_1 w_1(t) + \chi(w_1(t))) \\ \dot{w}_2(t) = w_1(t) - w_2(t) + w_3(t) \\ \dot{w}_3(t) = -bw_2(t)\end{cases} \tag{29}$$

with nonlinear function

$$\chi(w_1(t)) = 0.5(\chi_1 - \chi_0)(|w_1(t) + 1| - |w_1(t) - 1|) \in [0,\ 1],$$

where $\chi_0 = -1/7$, $\chi_1 = 2/7$, $a = 9$ and $b = 14.28$.

Then, system (29) is formulated as the Lur'e form with

$$A = \begin{bmatrix}-a\chi_1 & a & 0 \\ 1 & -1 & 1 \\ 0 & -b & 0\end{bmatrix}, B = \begin{bmatrix}\frac{3}{7}a \\ 0 \\ 0\end{bmatrix}, C = D = \begin{bmatrix}1 \\ 0 \\ 0\end{bmatrix}^T.$$

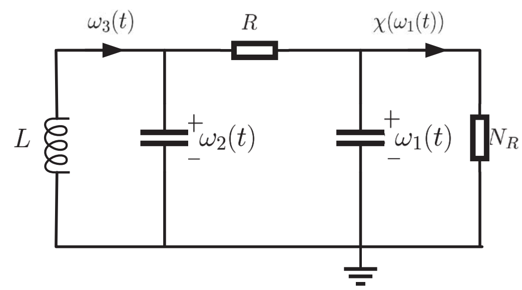

Fig. 2. The single Chua's circuit (for details, please refer to [23]).

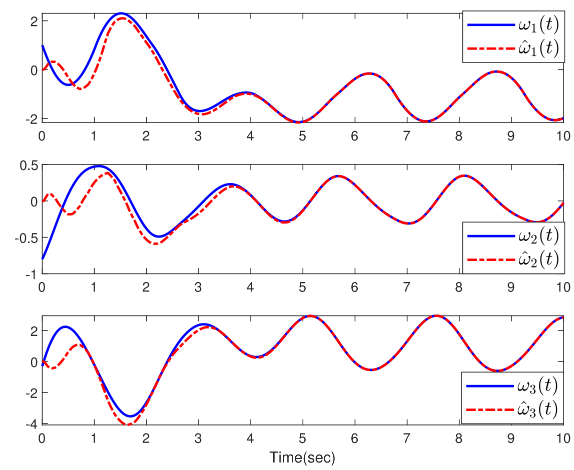

Fig. 3. Responses of the CLSs based on the memory-based event-triggered synchronization.

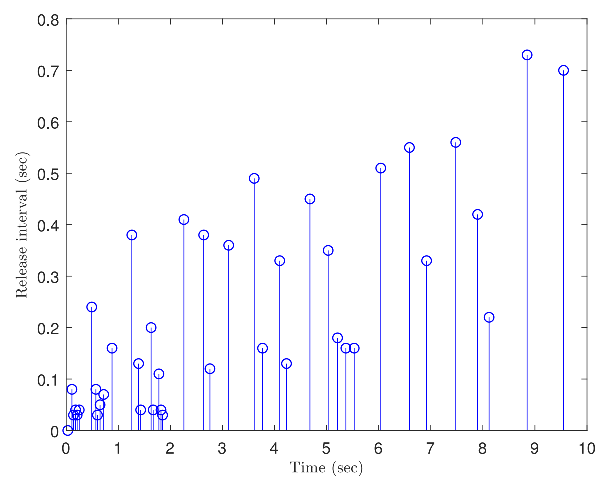

Fig. 4. Releasing time intervals of the CLSs based on the proposed METM.

Then, we select $N = 3$, $\lambda_1 = 0.166$, $\lambda_2 = 0.100$, $\lambda_3 = 0.067$, $s = 0.02s$, $\alpha = 2$, $\varpi = 0.1$. By solving the conditions in Theorem 1, the synchronization controller and triggering matrix are derived as

$$K = \begin{bmatrix}-3.0697 & -0.8913 & 1.5456\end{bmatrix}^T, \quad M = 6.046.$$

In the simulation, under the initial conditions $w(0) = [1\ \ -0.8\ \ -0.3]^T$ and $\hat{w}(0) = [0\ 0\ 0]^T$, the responses of the closed-loop master-slave CLS and triggering intervals are drawn in Fig. 3 and Fig. 4, respectively, from which one can see that the signal $\hat{w}_i$ can well synchronize with the signal w_i

TABLE I
AVERAGE TRIGGERING TIME INTERVAL

	METM	ETM
$\varpi = 0.05$	0.2041	0.1369
$\varpi = 0.10$	0.2439	0.1754
$\varpi = 0.15$	0.3125	0.1851

(i = 1, 2, 3) under the proposed METM. The comparison results of the average triggering time interval between our proposed METM and the conventional ETM under the same condition are given in Table I. From these figures, one can observe that the designed memory-based event-triggered controller is effective for achieving the synchronization of the master and slave systems. Moreover, Table I denotes that the average triggering time interval generated by METM is increased by 49.1%, 39.1% and 68.8% compared with the traditional ETM. This indicates that a lower triggering frequency is obtained by our developed METM than the conventional ETM.

V. CONCLUSION

This brief has investigated the memory-based synchronization problem of CLSs. The historical measurement outputs are introduced to construct the novel METM, where the past information is matched with different weights. The closed-loop system is then established as a distributed delay system. A co-design approach for both the METM and the synchronization controller is addressed to ensure the asymptotic stability of CLSs. Lastly, the Chua's circuit is executed to demonstrate the benefit of the developed method. In the future study, we will extend our proposed communication mechanism to the issue of remote robot control by using some intelligent method as in [24], [25].

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