H_∞ Network-Servo Tracking Control

GU Zhou[1,2]
1. College of Automation
Nanjing University of Aeronautics Astronautics
Nanjing, China
guzhouok@yahoo.com.cn

WANG Daobo[1]
2.School of Power Engineering
Nanjing Normal University
Nanjing, China

***Abstract*—This paper studies the problems of H_∞ tracking performance analysis and controller design for servo control systems using data networks. An augmented model of network-servo system is established in consideration of network induced delay and data dropped conditions, and a linear matrix inequality(LMI)-based procedure is proposed for designing state-feedback controllers, which guarantee that the position signals tracks the output of a given reference model well in the H_∞ sense. Numerical examples show the effectiveness of the proposed method.**

Keywords-tracking control; LMI; H_∞

I. INTRODUCTION

With the development of networked technology, more and more networkes (e.g.Internet) have been applied to distributed control system, whose feedback loop is closed through a real-time network, which is termed networked control system (NCS). NCSs are applied to many fields ranging from DC motors, advanced aircraft, and spacecraft automotive due to their safety, low cost, reduced weight and power requirement, simple installation and maintenance [1]. However, integration of communication networks into feedback control loops inevitably leads to non-ideal network Quality of Services (QoS), e.g. network-induced communication delays, data packet dropout, and out of order packet sequences. Those makes the analysis and design of NCSs more complex than those for traditional control system. Therefore ,increasing attention focus on NCSs control with non-idea QoS in recent years[2][3].

Servo tracking control has wide applications in dynamic processes industry, It has been well recognized that tracking control design is more general and more difficult than stabilization[4]. Servo tracking control without consideration of network has been investigated in [5],[6] $et\psi\ al$. In fact, the components of control system (such as sensor,actuator, controller) are often difficult to be located at the same place, and thus network is required to transmit signals. because of negative effects of non-ideal network, the performance of servo tracking control will be decreased, It even leads to instability. Therefore, the studies of network-tracking control are quite significance.

This paper will address the problem of designing the H_∞ controller for network-servo system. Under consideration of non-ideal network QoS, a new augmented network-servo traking model is established. Suppose the sensor is clock driven, the controller and actuator is event driven and the data is single packet.

The organization of the paper is as follows. In section 2, the problem of network-servo control is precisely formulated. Section 3 and 4 deal with the H_∞ stability analysis and controller design, respectively. The proposed approach is illustrated in Section 5, and Section 6 concludes the work.

II. MODELING OF NETWORK-SERVO CONTROL SYSTEMS

Suppose the model of DC servo system is described as

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) + E\omega(t) \\ y(t) &= Cx(t) \end{aligned} \tag{1}$$

where $x(t) \in \mathbb{R}^n, u(t) \in \mathbb{R}^m, y(t) \in \mathbb{R}^q, \omega(t) \in L_2[t_0, \infty)$ are the state vector, control input vector, output vector and disturbance input vector. A, B, E, C are systems constant matrix with appropriate dimensions.

As shown in Fig. 1, to facilitate design controller, We introduce a reference model, so that position signals of the servo system track the reference model output.

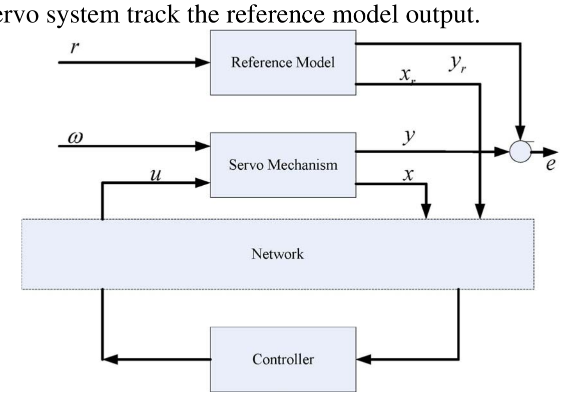

Figure 1. Network-servo control system

Define reference model as

$$\begin{aligned} \dot{x}_r(t) &= Gx_r(t) + r(t) \\ y_r(t) &= Hx_r(t) \end{aligned} \tag{2}$$

978-0-7695-3498-5/08 $25.00 © 2008 IEEE
DOI 10.1109/ISCSCT.2008.39

where, $r(t)\in\mathbb{R}^r, y_r(t)\in\mathbb{R}^q, x_r(t)\in\mathbb{R}^r$ are bounded reference input, reference model output, and reference state vector, respectively. G,H are constant matrix with appropriate dimensions. At the same time, we assume reference model is stable, and reference state vector is measurable.

As supposition in Section 1, the real control system can be modeled as

$$\dot{\zeta}(t)=\bar{A}\zeta(t)+\bar{B}\bar{K}u(t)+\bar{E}v(t) \tag{3}$$

$$e(t)=\bar{C}\zeta(t) \qquad t\in[i_kh+\tau_k, i_{k+1}h+\tau_{k+1})$$

$$u(t^+)=K_1x(t-\tau_k)+K_2x_r(t-\tau_k)t\in\{i_kh+\tau_k, k=1,2,\cdots\} \tag{4}$$

where,

$$\zeta^T(t)=[x^T(t)\ x_r^T(t)], v^T(t)=[w^T(t)\ r^T(t)], e(t)=y(t)-y_r(t),$$

$$\bar{K}=[K_1\ K_2], \bar{C}=[C-H], \bar{A}=\begin{bmatrix}A & 0\\ 0 & G\end{bmatrix}, \bar{B}=\begin{bmatrix}B\\ 0\end{bmatrix},$$

$\bar{E}=\begin{bmatrix}E & 0\\ 0 & I\end{bmatrix}$, and K_1, K_2 is the controller gain, $h\psi$is the sampling period, $i_k, k=1,2,\cdots$ are some integers and $i_kh\ \psi$ are time delay which denotes the time from the instant $i_kh\psi$ when sensor nodes sample signals of servo mechanism to the instant when control input signal are accepted by servo mechanism. Obviously $\bigcup_{k=1}^{\infty}[i_kh+\tau_k, i_{k+1}h+\tau_{k+1})=[t_0,\infty)$.

Then the close-loop system (3) can be rewritten as the following equivalent form:

$$\begin{aligned}&\dot{\zeta}(t)=\bar{A}\zeta(t)+\bar{B}\bar{K}\zeta(i_kh)+\bar{E}v(t)\\ &e(t)=\bar{C}\zeta(t)\end{aligned} \tag{5}$$

Definition 1: Let the constant $\gamma>0$ be given, close-loop system (5)is said to be stable with H_∞ norm bound γ if system (5) satisfy the following properties:

1) System (5) with $v(t)\equiv 0$ is asymptotically stable;

2) For the zero initial condition of $\zeta(t)$ and non-zero $v(t)$, the following condition holds

$$\|e(t)\|_2\le\gamma\|v(t)\|_2$$

Obviously, our main purpose is to find conditions to satisfy the Definition 1, so that it can meet servo control performance.

III. PERFORMANCE ANALYSIS OF NETWORK-SERVO SYSTEM

In this section, a stability criterion for the argumented system (5) will be given first.

Theorem 1: For given a scalar η, and matrices $\bar{A},\bar{B}$,$\bar{C}$,$\bar{E},G,H,K$,where $K\psi$is controller gain, if there exist matrices $P=P^T>0, Q=Q^T>0, Z=Z^T>0$ and $M\psi$ of appropriate dimensions such that, (6)↖←(8) are true, then the argumented system (5) is asymptotically stable.

$$\begin{bmatrix}\Phi_{11} & \Phi_{12} & -M_1 & P\bar{E} & \eta\bar{A}^TZ & C^T\\ * & \Phi_{22} & -M_2 & 0 & \eta\bar{K}^T\bar{B}^TZ & 0\\ * & * & -Q & 0 & 0 & 0\\ * & * & * & -\gamma^2I & \eta\bar{E}^TZ & 0\\ * & * & * & * & -\eta Z & 0\\ * & * & * & * & * & -I\end{bmatrix}<0 \tag{6}$$

$$\Theta_1=\begin{bmatrix}X & M\\ * & Z\end{bmatrix}\ge 0 \tag{7}$$

$$\Theta_2=\begin{bmatrix}X & N\\ * & Z\end{bmatrix}\ge 0 \tag{8}$$

Where,

$$\Phi_{11}=P\bar{A}+\bar{A}^TP+Q+N_1+N_1^T+\eta X_{11}$$

$$\Phi_{12}=P\bar{B}\bar{K}-N_1+M_1+N_2^T+\eta X_{12}$$

$$\Phi_{22}=-N_2-N_2^T+M_2+M_2^T+\eta X_{22}$$

Proof: Contruct a Lyapunov-Krasovskii functional as

$$\begin{aligned}V(t)&=\zeta^T(t)P\zeta(t)+\int_{t-\eta}^{t}\zeta^T(s)Q\zeta(s)ds\\ &+\int_{-\eta}^{0}\int_{t+\theta}^{t}\dot{\zeta}^T(s)Z\dot{\zeta}(s)dsd\theta\end{aligned} \tag{9}$$

where, $P=P^T>0, Q=Q^T>0, Z=Z^T>0$.

Taking the time derivative of $V(t)$ for $t\in[i_kh+\tau_k$, $i_{k+1}h+\tau_{k+1})$ yields

$$\begin{aligned}\dot{V}(t)&=2\zeta^T(t)P[\bar{A}\zeta(t)+\bar{B}\bar{K}\zeta(i_kh)+\bar{E}v(t)]\\ &+\zeta^T(t)Q\zeta(t)-\zeta^T(t-\eta)Q\zeta(t-\eta)\\ &+\eta\dot{\zeta}^T(t)Z\dot{\zeta}(t)-\int_{i_kh}^{t}\dot{\zeta}^T(s)Z\dot{\zeta}(s)ds-\int_{t-\eta}^{i_kh}\dot{\zeta}^T(s)Z\dot{\zeta}(s)ds\\ &+2\xi^T(t)N[\zeta(t)-\zeta(i_kh)-\int_{i_kh}^{t}\dot{\zeta}(s)ds]\\ &+2\xi^T(t)M[\zeta(i_kh)-\zeta(t-\eta)-\int_{t-\eta}^{i_kh}\dot{\zeta}(s)ds]\\ &+\eta\xi^T(t)X\xi(t)-\int_{i_kh}^{t}\xi^T(t)X\xi(t)ds-\int_{t-\eta}^{i_kh}\xi^T(t)X\xi(t)ds\end{aligned}$$

where, $\xi^T(t)=[\zeta^T(t)\quad \zeta^T(i_kh)]$, $M^T=[M_1^T\quad N_2^T]^T$, $N^T=[N_1\quad N_2^T]^T$, then we can show that, for $t\in[i_kh+\tau_k, i_{k+1}h+\tau_{k+1})$

$$\begin{aligned}\dot{V}(t)&=\xi_1^T(t)\Xi\xi_1(t)-\int_{t-\eta}^{i_kh}\dot{\xi}_2^T(s)\Theta_1\dot{\xi}_2(s)ds-\int_{i_kh}^{t}\dot{\xi}_2^T(s)\Theta_2\dot{\xi}_2(s)ds\\ &-e^T(t)e(t)+\gamma^2v^T(t)v(t)\end{aligned} \tag{10}$$

$$\Xi = \begin{bmatrix} \Phi_{11} + \eta\bar{A}^T Z\bar{A} + C^T C & \Phi_{12} + \eta\bar{A}^T Z\bar{B}\bar{K} & -M_1 & P\bar{E} + \eta\bar{A}^T Z\bar{E} \\ * & \Phi_{22} + \eta\bar{K}^T\bar{B}^T Z\bar{B}\bar{K} & -M_2 & \eta\bar{K}^T\bar{B}^T Z\bar{E} \\ * & * & -Q & 0 \\ * & * & * & -\gamma^2 + \eta\bar{E}^T\bar{E} \end{bmatrix}$$

where,

$\xi_1^T(t) = [\zeta^T(t)\ \ \zeta^T(i_k h)\ \ \zeta^T(t-\eta)\ \ v^T(t)]$, $\xi_2^T(t) = [\zeta^T(t)\ \ \zeta^T(i_k h)\ \ \dot{\zeta}^T(s)]$.

Thus, if $\Theta_1 \geq 0, \Theta_2 \geq 0,$ *and* $\Xi < 0$ which is equivalent to (6) by using Schur complement , combining (6)-(8) and (10) we can obtain

$$\dot{V}(t) \leq -e^T(t)e(t) + \gamma^2 v^T(t)v(t) \qquad (11)$$

Integrating both sides of (11) in intervals of t, we have

$$V(t) - V(i_k h + \tau_k) \ \leq \ -\int_{i_k h+\tau_k}^{t} e^T(s)e(s)ds + \int_{i_k h+\tau_k}^{t} \gamma^2 v^T(s)v(s)ds \qquad (12)$$

Since $\bigcup_{k=1}^{\infty}[i_k h + \tau_k, i_{k+1}h + \tau_{k+1}) = [t_0, \infty)$, $t_0 > 0$ and $V\psi(t)$ is continuous in t, therefore from 12, we can see that

$$V(t) - V(t_0) \leq -\int_{t_0}^{t} e^T(s)e(s)ds + \int_{t_0}^{t} \gamma^2 v^T(s)v(s)ds$$

Then, $t \to \infty$ and under zero initial condition, we can see that

$$\int_{t_0}^{t} e^T(s)e(s)ds \leq \int_{t_0}^{t} \gamma^2 v^T(s)v(s)ds \qquad (13)$$

that is $\| e(t)\|_2 \leq \gamma \| v(t)\|_2$, then, by the Definition 1,we can complete the proof.

IV. DESIGN OF H_∞ NETWORK-SERVO CONTROLLER

In this section, our purpose is to determine the controller gain of the argumented system (5) based on Theorem 1.

Theorem 2 For given a scalar $\eta, \gamma,$ ψif there exist matrices $L = L^T > 0,$ $W = W^T > 0,$ $R = R^T > 0$ $Y = \begin{bmatrix} Y_{11} & Y_{12} \\ * & Y_{22} \end{bmatrix} > 0, S = \begin{bmatrix} S_1 \\ S_2 \end{bmatrix}, T = \begin{bmatrix} T_1 \\ T_2 \end{bmatrix}$ and $V\psi$, such that(14)–(15) are satisfied, then, the argumented system (5)is asymptotically stable with controller gain $K = VL^{-1}$.

$$\begin{bmatrix} \Omega_{11} & \Omega_{12} & -T_1 & \bar{E} & \eta L\bar{A}^T & LC^T \\ * & \Omega_{22} & -T2 & 0 & \eta V^T\bar{B}^T & 0 \\ * & * & -W & 0 & 0 & 0 \\ * & * & * & -\gamma^2 I & \eta\bar{E}^T & 0 \\ * & * & * & * & -\eta R & 0 \\ * & * & * & * & * & -I \end{bmatrix} < 0 \qquad (14)$$

$$\begin{bmatrix} Y & S \\ * & LR^{-1}L \end{bmatrix} \geq 0 \quad , \begin{bmatrix} Y & T \\ * & LR^{-1}L \end{bmatrix} \geq 0 \qquad (15)$$

where,

$\Omega_{11} = \bar{A}L + L\bar{A}^T + W + S_1 + S_1^T + \eta Y_{11}$

$\Omega_{12} = \bar{B}V - S_1 + T_1 + \eta Y_{12} + S_2^T$

$\Omega_{22} = -S_2 - S_2^T + T_2 + T_2^T + \eta Y_{22}$

Proof:

Define $L = P^{-1}, R = Z^{-1}, S_i = LN_i L, T_i = LM_i L, W_i = LQ_i L,$ $Y = diag\{P^{-1}\ \ P^{-1}\}X\{P^{-1}\ \ P^{-1}\}, V = \bar{K}L$, then pre and post-multiplying the left-hand side of inequalities (6) with $diag\{P^{-1}\ P^{-1}\ P^{-1}\ I\ Z^{-1}\ I\}$ and its transpose, respectively, and then, pre- and post-multiplying the left-hand side of inequalities (7)–(8) with $diag\{P^{-1}\ P^{-1}\ P^{-1}\}$ and its transpose, we can obtain (14)-(15).

At this stage, we can say that there exsits a feedback gain $K = VL^{-1}$, which guarantee the asymptotical stability of the argumented system (5) if the above problem of (14)-(15) have solution. This complete the proof.

It is worth mentioning that the obtained conditions in Therom 2 are not strict LMI conditions due to the term $LR^{-1}L$ in (15). However, we chan solve this non-convex feasibility problem by formulating it into a sequential optimization problem subject to LMI constraints[7].

Define a new variable $U\psi$such that

$$U \leq LR^{-1}L \qquad (16)$$

Therefore inequalities (15) can be approximately translated into

$$\begin{bmatrix} Y & S \\ * & U \end{bmatrix} > 0 \quad \begin{bmatrix} Y & T \\ * & U \end{bmatrix} > 0 \qquad (17)$$

Inequality (16) is equivalent to (18) by Schur complement.

$$\begin{bmatrix} U^{-1} & L^{-1} \\ * & R^{-1} \end{bmatrix} \geq 0 \qquad (18)$$

Define $P = L^{-1}, H = U^{-1}, Z = R^{-1}$, by introducing new variables P, H, Z then it has

$$\begin{bmatrix} H & P \\ * & Z \end{bmatrix} > 0 \qquad (19)$$

Using the cone complementarity approach, we formulate the following nonlinear minimization problem with consideration of LMI conditions instead of the original nonconvex feasibility problem.

$$\begin{bmatrix} L & I \\ * & P \end{bmatrix} > 0, \begin{bmatrix} U & I \\ * & H \end{bmatrix} > 0, \begin{bmatrix} R & I \\ * & Z \end{bmatrix} > 0 \qquad (20)$$

$$\begin{cases} min\ tr\{LP + UH + RZ\} & s.t.(14)(17)(19)(20) \\ and \quad H > 0, P > 0, Z > 0 \end{cases} \qquad (21)$$

V. ILLUSTRATIVE EXAMPLE

The servo mechanism is composed of a permanent magnet DC motor coupled to the output shaft by means of a reduction gear. Suppose no various nonlinear effects introduced by friction (stiction, Stribeck effect, hysteresis, etc.) and backlash. using the following classical DC model [8].

$$\frac{\theta(s)}{u_{in}(s)} = \frac{\hat{K}_m}{s(\hat{T}_m s+1)} \tag{22}$$

Here we choose $\hat{K}_m$ ψand $\hat{T}_m$ ψas 32 and 0.05, respectively. Choose θ ψand ω ψas state variable, angular position as system's output. then the form of the state-space equation of servo mechanism is depicted by (1), and the matrices described in Section 2 are $A=\begin{bmatrix}0 & 1\\ 0 & -20\end{bmatrix}, B=\begin{bmatrix}0\\ 640\end{bmatrix}$, $C=[1 \quad 0]$, $E=[0.2 \quad 0.1]^T$, $G=-1$, and $H=0.5$ It is assumed that $\omega(t)=0.1\sin 3t$, $r(t)=0.8\sin t$, and the initial state of servo mechanism and reference model is $x_0=[0.5 \quad 0]^T, x_{r0}=0.1$, respectively, the sampling period $h=10ms$, $\gamma=3$, networked induced delay upper-bound $\eta=50$. Based on Theorem 2 and Algorithm, utilizing the Matlab LMI Toolbox, we can obtain the controller gain in (4) $K_1=[-0.0243 \quad -0.0039], K_2=0.0053$.

To demonstrate the effectiveness of the controller, we choose 2 methods to simulate, one is H_∞ tracking servo control proposed in this paper ,which is depicted in Fig. 1, the other is to use conventional PI controller whose parameter are KPψ=-5,KIψ=-0.05, we can find that using the proposed controller tracks reference output well, even the network induced delay is up to 50ms , while using PI controller , the system is unstable when network induced delay comes to 6.8ms, depicted in Fig.2.

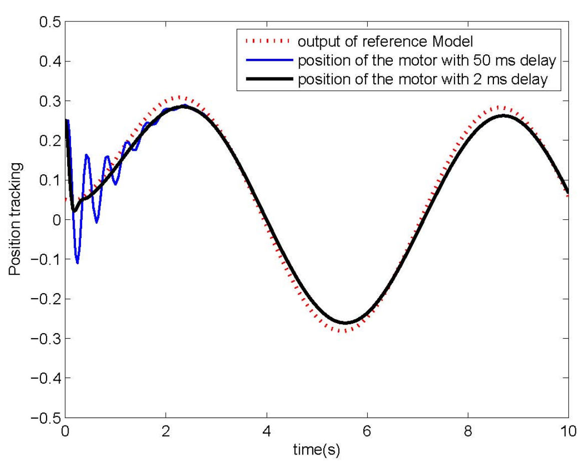

Figure 2. Output of networked tracking

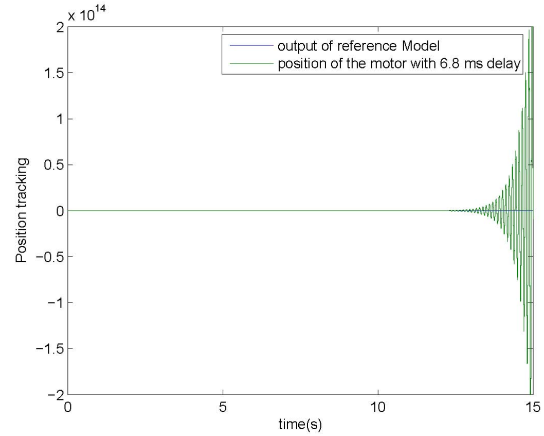

Figure 3. Output of networked tracking with 6.8ms delay

VI. CONCLUSIONS

In this paper, we have investigated the stabilization and H_∞ feedback controller design of networked servo control system. An new augmented model has been presented for networked servo control with non-ideal network conditions. The feedback gain of a memoryless controller can be derived by solving a set of LMIs based on the Lyapunov functional method, which guarantees the angular position tracks the reference position well in H_∞. An illustrative example is given to show the usefulness and effectiveness of our proposed method. It is worth mentioning that our purposed controller is illustrated by numerical simulation, the experimental illustration will be further researched.

REFERENCES

[1] D. Yue, Q. Han, and C. Peng, "State feedback controller design of networked control systems," *Control Applications,2004. Proceedings of the 2004 IEEE Internationa Conference on*, vol. 1, 2004.

[2] C. PENG, Y. TIAN, and Y. TIPSUWAN, "Robust HŁ control of networked control systems with parameter uncertainty and state-delay," *European journal of control*, vol. 12, no. 5, pp. 471–482, 2006.

[3] G. Walsh, H. Ye, L. Bushnell, C. Technol, and C. Oakland, "Stability analysis of networked control systems," *Control Systems Technology, IEEE Transactions on*, vol. 10, no. 3, pp. 438–446, 2002.

[4] H. Gao and T. Chen, "Network-Based *H1* Output Tracking Control," *Automatic Control, IEEE Transactions on*, vol. 53, no. 3, pp. 655–667, 2008.

[5] H. Schulte and H. Hahn, "Fuzzy state feedback gain scheduling control of servo-pneumatic actuators," *Control Engineering Practice*, vol. 12, no. 5, pp. 639–650, 2004.

[6] K. Tang, S. Huang, K. Tan, and T. Lee, "Combined PID and adaptive nonlinear control for servo mechanical systems," *Mechatronics*, vol. 14, no. 6, pp. 701–714, 2004.

[7] L. El Ghaoui, F. Oustry, and M. AitRami, "A cone complementarity linearization algorithm for staticoutputfeedback and related problems," *Automatic Control, IEEE Transactions on*, vol. 42, no. 8, pp. 1171–1176, 1997.

[8] B. Kuo, Automatic Control Systems. Prentice Hall, 1995.