# Output Feedback Reliable $H_\infty$ Control for Networked Control System

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**Abstract.** This paper investigates the static output feedback control for networked control systems with actuator failure. The failure is composed of two parts, the linear deficiency of the control gain and the nonlinearity varying with the control input. Based on the more general actuator fault model, linear matrix inequality (LMI) optimization approach is used to design the reliable $H_\infty$ output feedback control. Finally, an example is provided to demonstrate the design method.

**Keywords:** Actuator failure, Networked control system, Reliable control.

## 1 Introduction

Networked control systems (NCSs) are feedback control systems wherein the control loops are closed via real-time networks [1,2,3,4]. The past decade has witnessed continued research interest on various aspects of networked control systems, since this new type of information transmission reduces system wiring, eases maintenance and diagnosis, and increases system agility, which makes NCS a promising structure for control systems. Nevertheless, the introduction of networks also brings some new problems and challenges, such as network-induced delay, packet dropout, network scheduling and quantization problems, etc..

Most of the references concerning with NCSs assumed that the actuator/sensor operates without any flaws, i.e., the data transmit to the receiving device without any deviation, which is, unfortunately, not true in the practical systems. Due to the aging, external disturbance, etc., the actuator/sensor failure becomes a common problem [5,6]. It is worth pointing out that it plays a key role to develop a reasonable fault model in analyzing and synthesizing reliable control systems. Taking the actuator fault for example, researchers modeled the actuator fault as

$$u^F(t) = \Phi u(t) \tag{1}$$

where $u(t)$ is the true signal of the control input to be sent to the actuator device, $u^F(t)$ is the real signal of the receiver with a certain fault, $\Phi = diag\{\phi_1, \cdots, \phi_m\}$is

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a fault factor which reflects the gain missing of the actuator. In [7,8], is defined to satisfy $\phi_i \in \{0, 1\}$, if $\phi_i = 0$, it means a complete failure, otherwise, $\Phi_i = 1$ denotes the device works normally. In \cite [9,10], the authors let $\phi \in [0\ 1]$, which depicts the gain of the faulty device varies from 0 to 1. In [11,12], $\phi$ obeys a Gaussian distribution to characterize a random fault. Most of the existed references regard the faulty device as a linear gain missing. However, the transmission characteristic of the faulty device does not always present multiplicative fault, sometimes it adhered with a certain nonlinearity. To the best of our knowledge, few authors dealt with this problem, which motivates us to further investigate the problem.

In this paper, the closed-loop NCS with the actuator failure presented in Section 2, in which the failure is composed of linear and nonlinear parts. The reliable control design method is provided in Section 3. Section 4 presents the design results and simulations. Finally, the study's findings are summarized in Section 5.

## 2 Problem Formulation

In this paper, we will study the reliable control for networked control system with the actuator failures , in which the plant is given by a continuous-time linear model of the form

$$\mathcal{P}: \begin{cases} \dot{x}(t) = Ax(t) + Bu(t) + D\omega(t) \\ y(t) = C_1 x(t) \\ z(t) = C_2 x(t) \\ x(t) = \varphi(t) \quad t \in [-\eta_2, 0) \end{cases} \tag{2}$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $y(t) \in \mathbb{R}^{q_1}$ is the controlled output vector, $z(t) \in \mathbb{R}^{q_2}$ is output vector, $u(t) \in \mathbb{R}^m$ is the control input, and the process noise $\omega(t) \in \mathbb{R}^d$ including model uncertainties and external plant disturbance belongs to $l_2[0\ \infty]$. $A, B, C_1, C_2$, and $D$ are constant matrices with appropriate dimensions.

Here we do the following assumptions, which is widely adopted in NCS, that

- sensors are clock-driven; controllers and actuators are event-driven;
- the sampling data are hold by ZOH before the new event updates.
- the data are transmitted over the network by a single packet in every control period.

Under the above assumptions together with our previous work [13,14], the system (2) can be further written as

$$\dot{x}(t) = Ax(t) + Bu(i_k h) + D\omega(t) \tag{3}$$

for $t \in [i_k h + \tau_{i_k}, i_{k+1} h + \tau_{i_k+1})$, where \$h\$ is the sensor sampling period, $i_k (k = 1, 2, 3, \cdots)$ is some non-negative integer and $i_k h$ is the sensor sampling instant. It can be obviously see that $\cup_{k=1}^{\infty} [i_k h + \tau_{i_k}, i_{k+1} h + \tau_{i_k+1}) = [0\ \infty)$.

We consider the following actuator fault model

$$u^F(t) = \Xi_1 u(t) + g(u(t)) \tag{4}$$

Where $0 < \Xi_1 = diag\{e_1, e_2, \cdots, e_m\} \le I$ and the vector function $g(u(t)) = [g_1(u_1(t)), g_2(u_2(t)), \cdots, g_m(u_m(t))]^T$. For each channels of the control input, it satisfies

$$|g_i(u_i(t))| \le \alpha_i u_i(t) \tag{5}$$

where $0 \le \alpha_i$ and $e_i + \alpha_i \le 1$ for$i \in \mathcal{I} = \{i | i = 1, 2, \cdots, m\}$.
From (5), it follows

$$g^T(u(t))g(u(t)) \le u^T(t)\Xi_2 u(t) \tag{6}$$

where$\Xi_2 = diag\{\alpha_1, \alpha_2, \cdots, \alpha_m\}$.

**Remark 1.** Let $\Xi_2 \equiv 0$, it means the actuator works in a certain fixed missing gain. Specifically, if we choose $e_i = 0 (i \in \mathcal{I})$, it means that the $i\text{-th}$ channel of the control input is complete failure, and we choose $e_i = 1 (i \in \mathcal{I})$, it denotes that the actuator is intactness, otherwise, the feature of partial actuator failure can be described by defining $0 < e_i < 1$.

**Remark 2.** If $\Xi_2$ satisfies (6) and$\Xi_2 \ne 0$, then the failure model (4) characterizes the actuator failure in each channels is time-varying and coupled with different levels of line/nonlinear nature.

Consider the following memoryless output feedback controller

$$u(t) = u(i_k h) = Ky(i_k h) \tag{7}$$

For $t \in [i_k h + \tau_{i_k}, i_{k+1} h + \tau_{i_k+1})$, where $K$ is a controller gain to be designed. Combining (2), (4) and (7), we can obtain the following closed-loop system with consideration of the actuator fault for $t \in [i_k h + \tau_{i_k}, i_{k+1} h + \tau_{i_k+1})$ as

$$\dot{x}(t) = Ax(t) + B\Xi_1 KCx(i_k h) + Bg(u(i_k h)) + D\omega(t) \tag{8}$$

Define $\eta(t) = t - i_k h$ in every interval $[i_k h + \tau_{i_k}, i_{k+1} h + \tau_{i_k+1})$, it follows that

$$i_k h = t - (t - i_k h) = t - \eta(t) \tag{9}$$

Then we have

$$\eta_1 \le \tau_{i_k} \le \eta(t) \le (i_{k+1} - i_k)h + \tau_{i_{k+1}} \le \eta_2$$

So far, the closed-loop NCS with consideration of the actuator fault can be further written as

$$\begin{cases} \dot{x}(t) = \mathcal{A}\xi(t) \\ z(t) = C_2 x(t) \end{cases} \tag{10}$$

For $t \in [i_k h + \tau_{i_k}, i_{k+1} h + \tau_{i_k+1})$, where $\mathcal{A} = \begin{bmatrix} A & 0 & B\Xi_1 KC_1 & 0 & B & D \end{bmatrix}$, $\xi(t) = [x^T(t)\ x^T(t-\eta_1)\ x^T(t-\eta(t))\ x^T(t-\eta_2)\ g^T(u(t-\eta(t)))\ \omega^T(t)]^T$.

From the definition of $\eta(t)$, the nonlinear constraint condition in (6) in the time interval $t \in [i_k h + \tau_{i_k}, i_{k+1} h + \tau_{i_k+1})$ can be described as

$$\xi^T(t)[\Pi_2^T\Pi_2 - \Pi_1^T\Xi_2\Pi_1]\xi(t) \le 0 \tag{11}$$

where$\Pi_1 = \begin{bmatrix} 0 & 0 & KC_1 & 0 & 0 & 0 \end{bmatrix}, \Pi_2 = \begin{bmatrix} 0 & 0 & 0 & 0 & I & 0 \end{bmatrix}$..

## 3 Main Results

In this section, the output feedback reliable controller will be designed. Before we give the Theorems, the following lemma are first introduced, which will be used in the subsequent development.

**Lemma 1.** [15,16] Let $Y_0(\xi(t)), Y_1(\xi(t)), \cdots, Y_p(\xi(t))$ be quadratic functions of $\xi(t) \in \mathbb{R}^n$

$$Y_i(\xi(t)) = \xi(t)^T T_i \xi(t), i = 0, 1, \cdots, p \tag{12}$$

with$T_i = T_i^T$. Then, the implication

$$Y_0(\xi(t)) \le 0, \cdots, Y_p(\xi(t)) \le 0 \Longrightarrow Y_0(\xi(t)) \le 0 \tag{13}$$

holds if there exist $\kappa_1, \cdots, \kappa_p > 0$ such that

$$T_0 - \sum_{i=1}^{p} \kappa_i^{-1} T_i \le 0 \tag{14}$$

**Theorem 1.** For some given constants $\eta_1, \eta_2, \gamma, e_i, \alpha_i (i \in \mathcal{I})$ and matrix $K$, the closed-loop system (10) satisfies $H_\infty$ performance criterion, if there exist real matrices $P > 0, Q_1 > 0, Q_2 > 0$ and $R > 0$ with appropriate dimensions, such that the following inequality hold

$$\begin{bmatrix} \Theta_0 - \kappa^{-1}\Pi_2^T\Pi_2 & * & * & * \\ (\eta_2 - \eta_1)P\mathcal{A} & -PR^{-1}P & * & * \\ \mathcal{C} & 0 & -I & * \\ \Pi_1 & 0 & 0 & -\kappa\Xi_2^{-1} \end{bmatrix} \le 0 \tag{15}$$

where

$$\begin{aligned}
\Theta_0 &= \begin{bmatrix} \Gamma_0^{11} & * & * & * & * & * \\ \Gamma_0^{21} & \Gamma_0^{22} & * & * & * & * \\ 0 & R & -2R & * & * & * \\ 0 & 0 & R & -Q_2 - R & * & * \\ B^T P & 0 & 0 & 0 & 0 & 0 \\ D^T P & 0 & 0 & 0 & 0 & -\gamma^2 I \end{bmatrix}, \\
\Gamma_0^{11} &= PA + A^T P + Q_1 + Q_2, \\
\Gamma_0^{21} &= C_1^T K^T \Xi_1^T B^T P, \Gamma_0^{22} = -Q_1 - R \\
\mathcal{C} &= [C_2\ 0\ 0\ 0\ 0\ 0]
\end{aligned}$$

**Proof:** Choose a Lyapunov functional candidate for the system (10) as

$$V(x_t) = V_1(x_t) + V_2(x_t) + V_3(x_t)$$
$$V_1(x_t) = x^T(t)Px(t)$$
$$V_2(x_t) = \sum_{i=1}^{2}\int_{t-\eta_i}^{t} x^T(s)Q_i x(s)ds$$
$$V_3(x_t) = (\eta_2-\eta_1)\int_{t-\eta_2}^{t-\eta_1}\int_{s}^{t}\dot{x}^T(v)R\dot{x}(v)dvds$$

Taking the derivative of $V(x_t)$ with respect to $t$ along the trajectory of (10) yields

$$\begin{aligned}\dot{V}(x_t) &= 2x^T(t)P\mathcal{A}\xi(t)\\ &+ \sum_{i=1}^{2}x^T(t)Q_i x(t) + \sum_{i=1}^{2}x^T(t-\eta_i)Q_i x(t-\eta_i)\\ &+ (\eta_2-\eta_1)^2\dot{x}^T(t)R\dot{x}(t) - (\eta_2-\eta_1)\int_{t-\eta_2}^{t-\eta_1}\dot{x}^T(s)R\dot{x}(s)ds\end{aligned}$$

From Lemma 1 in [15], it yields

$$z^T(t)z(t) - \gamma^2\omega^T(t)\omega(t) + \dot{V}(t) \le \xi^T(t)[\Theta_0 + (\eta_2-\eta_1)^2\mathcal{A}^T R\mathcal{A} + \mathcal{C}^T\mathcal{C}]\xi(t)$$

Applying Schur complement and Lemma 1, one can see that Eq.(15) is a sufficient condition to guarantee Eq.(11) and

$$z^T(t)z(t) - \gamma^2\omega^T(t)\omega(t) + \dot{V}(t) \le 0 \tag{16}$$

Under zero initial conditions, integrating both side of Eq.(16) yields

$$V(t) \le \int_0^t [\gamma^2\omega^T(t)\omega(t) - z^T(t)z(t)]dt \tag{17}$$

Then we can conclude that $\|z(t)\|_2 \le \|\omega(t)\|_2$ for all nonzero $\omega(t) \in l_2[0,\infty)$. $H_\infty$ performance is established. The proof is completed.

With the result of Theorem 1, the reliable $H_\infty$ control for system(2) with the control law (4) is provided in the following result.

**Theorem 2.** For some given constants $\eta_1, \eta_2, \gamma, e_i, \alpha_i(i \in \mathcal{I}), \varepsilon$, the closed-loop system (10) satisfies $H_\infty$ performance criterion, if there exist real matrices $P > 0, Q_1 > 0, Q_2 > 0$ and $R > 0$ with appropriate dimensions, such that the following equalities hold

$$\begin{bmatrix}\Theta_1 - \kappa\Pi_2^T\Pi_2 & * & * & *\\ (\eta_2-\eta_1)\Pi_3 & -2\varepsilon X + \varepsilon^2\bar{R} & * & *\\ \bar{\mathcal{C}} & 0 & -I & *\\ \bar{\Pi}_1 & 0 & 0 & -\kappa\Xi_2^{-1}\end{bmatrix} \le 0 \tag{18}$$

$$C_1X = WC_1 \tag{19}$$

Moreover, the controller gain is given by $K = YW^{-1}$, where

$$
\begin{aligned}
\Theta_1 &= \begin{bmatrix} \Gamma_1^{11} & * & * & * & * & * \\ \Gamma_1^{21} & \Gamma_1^{22} & * & * & * & * \\ 0 & \bar{R} & -2\bar{R} & * & * & * \\ 0 & 0 & \bar{R} & -\bar{Q}_2 - \bar{R} & * & * \\ \kappa B^T & 0 & 0 & 0 & 0 & 0 \\ D^T & 0 & 0 & 0 & 0 & -\gamma^2 I \end{bmatrix}, \\
\Gamma_1^{11} &= AX + XA^T + \bar{Q}_1 + \bar{Q}_2, \\
\Gamma_1^{21} &= C_1^T Y^T \Xi_1^T B^T, \Gamma_1^{22} = -\bar{Q}_1 - \bar{R} \\
\bar{\mathcal{C}} &= [C_2 X \ 0\ 0\ 0\ 0\ 0], \\
\bar{\Pi}_1 &= \begin{bmatrix} 0 & 0 & KWC_1 & 0 & 0 & 0 \end{bmatrix}, \\
\Pi_3 &= \begin{bmatrix} AX & 0 & B\Xi_1 Y C_1 & 0 & \kappa B & D \end{bmatrix}
\end{aligned}
$$

**Proof:** Note that

$$(\varepsilon R - P)R^{-1}(\varepsilon R - P) \geq 0 \tag{20}$$

where $\varepsilon$ is a positive scalar. Then it is true that

$$-PR^{-1}P \leq -2\varepsilon P + \varepsilon^2 R \tag{21}$$

It follows that

$$\begin{bmatrix} \Theta_0 - \kappa \Pi_2^T \Pi_2 & * & * & * \\ (\eta_2 - \eta_1)P\mathcal{A} & -2\varepsilon P + \varepsilon^2 R & * & * \\ \mathcal{C} & 0 & -I & * \\ \Pi_1 & 0 & 0 & -\kappa \Xi_2^{-1} \end{bmatrix} \leq 0 \tag{22}$$

from Eq. (15).

Defining $X = P^{-1}, \bar{Q}_1 = XQ_1X, \bar{Q}_2 = XQ_2X, \bar{R} = XRX, Y = KW$, $J = diag\{X, X, X, X, \kappa, I, X, I, I\}$, pre- and post-multiplying (22) with J and its transposes, respectively. It can obviously see that (22) is equivalent to (18) under the condition of (19). This completes the proof.

One can see that it is difficult to find a feasible solution by Theorem 2 since Eq.(19) is not a strict inequality. Now we introduce the following algorithm to address this problem.

It is clear that Eq.(19) is equivalent to

$$\mathrm{trace}\left[(CX - WC)^T(CX - WC)\right] = 0 \tag{23}$$

which can be converted to the following optimization problem by using Schur complement

$$\begin{cases} \begin{bmatrix} -\sigma I & * \\ WC - CX & -I \end{bmatrix} < 0 \\ \sigma \to 0 \end{cases} \tag{24}$$

where the scalar $\sigma$ is a small enough positive scalar. Then the controller gain can be resolved by (18),(19) and (24).

**Remark:** To obtain a feasible solution of the SOF controller gain $K$, in some existed results, the output matrix $C_1$ is assumed to be invertible or some intelligent optimization algorithms are applied to find a sub-optimum solution. For the sake of technical simplicity, we take the above algorithm, in this paper, to tackle this problem.

## 4 A Numerical Example

An example of networked control for an unstable batch reactor [18] is used in this section to demonstrate the effectiveness of the proposed approach. The plant matrices are

$$A = \begin{bmatrix} 1.3800 & -0.2077 & 6.7150 & -5.6760 \\ 0.5814 & -4.2900 & 0 & 0.6750 \\ 1.0670 & 4.2730 & -6.6540 & 5.8930 \\ 0.0480 & 4.2730 & 1.3430 & -2.1040 \end{bmatrix},$$

$$B = \begin{bmatrix} 0 & 0 \\ 5.6790 & 0 \\ 1.1360 & -3.1460 \\ 1.1360 & 0 \end{bmatrix}, C_1 = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix},$$

$$C_2 = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}, D = \begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix}^T$$

The output $y(t)$ is communicated over the network whose parameters are assumed as $\eta_1 = 5$ ms and $\eta_2 = 50$ ms, respectively. The disturbance $\omega(t) = 4\exp^{-(2t-2)^2}$. Next, we will study the two different actuator failure scenarios, which are described as

$$\begin{cases} u_1^F(t) = 0.9u(t) + g_1(u_1(t)) \\ u_2^F(t) = 0.6u(t) + g_2(u_2(t)) \end{cases} \tag{25}$$

where $g(\cdot)$ satisfies $g_1(u_1(t)) = 0.1sat(u_1(t))$ and $g_2(u_2(t)) = 0.02sat(u_2(t))$, respectively.

Under these failure scenarios, the reliable controller $K$ can be obtained as

$$K = \begin{bmatrix} -1.3609 & -0.1032 \\ -0.6563 & 4.4042 \end{bmatrix} \tag{26}$$

by using Theorem 2 together with its algorithm.

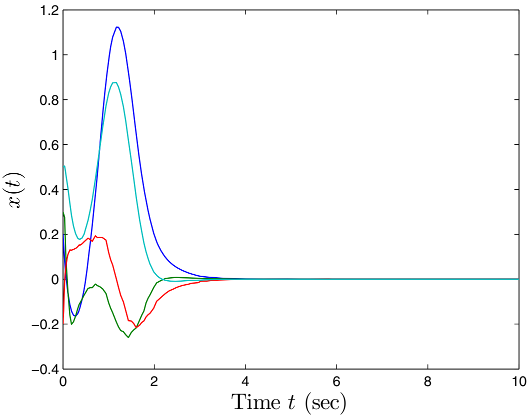


**Fig. 1.** State response x(t)

Simulation results are provided as Fig. 1 under the initial condition $\varphi(t) = [0.2\ 0.3\ \ -0.2\ 0.5]^T$. In this study, the control inputs are of $90\%$ and $60\%$ linear deficiency, respectively, meanwhile there are some different levels of saturation adhered on those two actuators. It is obviously observed from Fig. 1 that the reliable controller implemented on the system under the above failure scenarios can maintain the systems performance.

## 5 Conclusion

In this paper, the reliable output feedback control design for the NCS with actuator failure has been developed. Based on a more general actuator fault model, the reliable $H_\infty$ SOF controller is derived by using Lyapunov method, which guarantees the closed-loop system satisfies a desired $H_\infty$ disturbance attenuation constraint. An illustrative example is given to show the validity of the present control scheme.

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