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Research Article

# Event-triggered reliable $H_\infty$ filter design for networked systems with multiple sensor distortions: A probabilistic partition approach

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In this paper, the problem of event-triggered reliable $H_\infty$ filtering for networked systems with multiple sensor distortions is investigated. The interval of sensor distortion in each channel is partitioned several segments. By introducing a set of rand variables, the model of multiple sensor distortions with their information of probability distribution in each segment is established, which is more general than the one in open results. Furthermore, an event-triggered communication scheme is proposed to mitigate the utility of limited network bandwidth. Then a unified model with consideration of the event-triggered communication scheme and the sensor distortion is put forward. Based on this model, sufficient conditions of the mean square stability of the filtering error system and $H_\infty$ filter parameters are achieved. Finally, a simulation example is exploited to demonstrate the effectiveness of the presented method.


## 1. Introduction

Networked control systems (NCSs) is a kind of feedback control system using communication networks to close control loops. It has received more and more attention due to these obvious advantages, such as the higher reliability, better flexibility and lower cost. It follows that the applications of NCSs can be found in many fields such as aircrafts, automobiles, HVAC systems automation, manufacturing, robotics and industrial process control [1–5]. It is noted that the limited network bandwidth restrains the efficiency of communication, which results in a difficulty of transmitting the control information in real time. Consequently, the control performance of NCSs becomes deteriorative. In recent years, much attention has been drawn to the problem of improving the scheme of network communication to design the networked filter.

Time-triggered scheme is based on periodic sampling and releasing (see [6–10], and the references therein) in the control system, which leads to abundant data packets transmitting over the network. Some unnecessary data packets occupy the limited networked bandwidth. Moreover, frequent sampling and releasing consume much more energy which is an extremely limited resource in wireless NCSs, such as Bluetooth, wireless HART, ZigBee, etc. As an alternative communication scheme, the event-triggered communication (ETC) scheme is proposed to mitigate the need of data transmission while preserving the system performance by using a decision mechanism of data-releasing. The basic idea of this decision mechanism is that the sampled-data can be transmitted over the network only when a threshold is exceeded, and thus the amount of data-releasing is reduced significantly. The authors in [11] developed an event triggered framework of state estimation for discrete-time systems with parameter uncertainties residing in a polytope. The control design problem of event-triggered networked systems with both state and control input quantizations was discussed in [12]. In [13], the authors dealt with a problem of distributed state estimation for nonlinear discrete time-delay systems over sensor networks. In [14], the authors designed an event-triggered output feedback controller for distributed networked systems. An event-triggered fuzzy filtering was investigated for a class of nonlinear networked control systems in [15]. In [16], the authors proposed a distributed event-triggered communication scheme for $H_\infty$ filtering over sensor network. In [17], the authors investigated multiple event-triggered $H_2/H_\infty$ filtering for hybrid wired-wireless networked systems with random network-induced delays. The problem of $H_\infty$ filter design for linear and nonlinear NCSs was addressed by using discrete event-triggering scheme in [18,19]. The authors proposed an extended event-triggered scheme for data releasing of NCSs by using adaptive method in [20].

It should be pointed out that the aforementioned results are based on an assumption of the system with precise sensors. However, sensors with distortion are inevitable in real practice.

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Although it can be improved by using a better sensor, compensation approaches are often applied in the design of control systems. The authors in [21,22] addressed the filtering problem for discrete-time nonlinear systems with the probabilistic missing measurements by introducing a series of rand variables to obey a certain distribution with given expectations and variances. The filtering problem for discrete-time nonlinear system with probabilistic occurring of sensors failure by using a method of binary random variable sequence was studied in [23]. However, in some cases, it is not enough to characterize the behaviour of sensor distortion only by a certain distribution or a missing interval, for example, the missing interval of the sensor in a certain channel is $[\underline{\alpha}, \overline{\alpha}]$. We assume $\underline{\alpha} \le \alpha_1 \le \cdots \le \alpha_n \le \overline{\alpha}$. The distribution of sensor distortion in each subinterval is different from the others. Then it is difficult to model this issue by existing methods, especially for the system with multiple sensors.

In this paper, we investigate the problem of $H_\infty$ filtering for networked control systems with multiple sensor distortions based on the event-triggering scheme. The main contribution of this study is as follows: Firstly, by partitioning the interval of sensor distortion in each channel into several segments and introducing a series of rand variables to describe the distribution of each segment, the model of multiple sensor distortions is developed, which is more general than the existing results. Secondly, an overall model of the system with consideration of multiple sensor distortions and ETC scheme is established. Based on this model, a co-design method of achieving the reliable $H_\infty$ filter parameters and triggered parameters is given by using Lyapunov stability theory. Finally, an example is given to demonstrate the effectiveness and superiority of the proposed design approach.

*Notations*: Throughout this paper, the notation $Z^+$ represents the positive integer set and $\mathbb{R}^n$ represents the $n$-dimensional Euclidean space. The notation $S>0 (S<0)$ is used to denote a positive definite(negative definite) matrix. $\|\cdot\|$ denotes the spectral norms of matrices or the Euclidean norm for vectors. $\mathbb{E}\{\cdot\}$ represents mathematical expectation and the superscripts "$T$" represents the transpose of the matrix. $I$ denotes the identity matrix with appropriate dimensions. In symmetric block matrices, $*$ is used as ellipsis for terms induced by symmetry and $col\{\cdot\}$ stand for the column vector of "$\cdot$". For any square matrices $A$ and $B$, we define $\mathrm{diag}\{A, B\} = \begin{bmatrix} A & * \\ 0 & B \end{bmatrix}$.

## 2. Preliminaries

In this paper, an event-triggered filter for networked system with multiple sensor distortions is shown in Fig. 1, from which one can see that the framework is composed of sensors, samplers, event-triggered generator (ETG) and a network channel. Under ETC scheme, the network does not release the sampled data periodically. Here, we assume that the communication network works in a non-ideal condition. A network induced delay is existed when transmitting the data packet due to a limited bandwidth. Moreover, the distribution of sensor distortion in channels is known in prior. In this section, the event-triggered communication scheme and the model of multiple sensor distortion with multi-interval probabilistic distribution are put forward firstly. Then an overall network-based reliable filtering model with the consideration of event-triggered communication scheme and sensor distortion is developed.

Consider the following continuous-time linear system:

$$\begin{cases} \dot{x}(t) = Ax(t) + B\omega(t) \\ y(t) = Cx(t) \\ z(t) = Lx(t) \end{cases} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $y(t) \in \mathbb{R}^m$ is the ideal measurement output, $z(t) \in \mathbb{R}^p$ is the signal to be estimated, $\omega(t) \in \ell_2[0, \infty)$ is the disturbance input vector, $A$, $B$, $C$ and $L$ are known real matrices with appropriate dimensions.

### 2.1. The model of multiple sensor distortions with probabilistic distributions

We consider the following filter for the estimation of the output $z(t)$ of system (1):

$$\begin{cases} \dot{x}_f(t) = A_f x_f(t) + B_f \hat{y}(t) \\ z_f(t) = C_f x_f(t) \end{cases} \tag{2}$$

where $x_f(t)$ is the filter state vector, $\hat{y}(t)$ is the input of the filter, $z_f(t)$ is the estimation of $z(t)$, $A_f$, $B_f$ and $C_f$ are filter parameters to be determined.

**Remark 1.** A sensor distortion is inevitable in practice, although a better result can be achieved by using a more precise instrument, we can get a similar desiring result by using a good algorithm for the sake of cost saving. Since the distribution of the sensor distortion in each interval is different, the range of the sensor distortion and its distribution in each subinterval can be got in prior by abundant off-line experiments.

By considering sensor distortion in each channels, the measurement output $\tilde{y}(t)$ can be represented by

$$\tilde{y}(t) = \Pi(t) y(t) = \sum_{i=1}^{m} \pi_i(t) I_i y(t) \tag{3}$$

where $\Pi(t) = \mathrm{diag}\{\pi_1(t), \pi_2(t), \ldots, \pi_m(t)\}$, $I_i = \mathrm{diag}\{\underbrace{0, \ldots, 0}_{i-1}, I, \underbrace{0, \ldots, 0}_{m-i}\}$. $\pi_i(t)$ $(i \in \{1, 2, \ldots, m\})$ is a factor of sensor distortion in every channels.

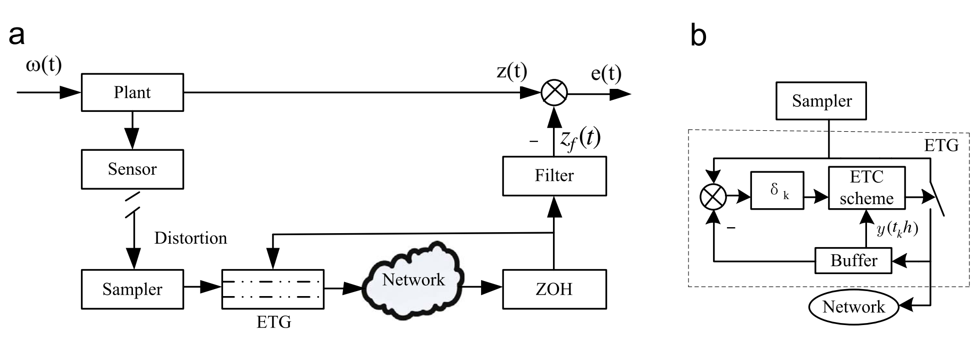


**Fig. 1.** The framework of event-triggered $H_\infty$ filtering for NCSs with multiple sensor distortions.

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**Remark 2.** If one sets $\Pi(t) \equiv 1$, then the proposed problem reduces to the case of traditional filter design under an assumption of the measuring instruments are flawless.

We introduce the following two sets:

$$F_1 = \{t : \pi_i(t) \in [\underline{\pi}_i, \pi_{i0})\},$$
$$F_2 = \{t : \pi_i(t) \in [\pi_{i0}, \overline{\pi}_i]\}$$

where $0 < \underline{\pi}_i < \pi_{i0} < \overline{\pi}_i \le 1$.

Also, the following two functions are defined:

$$\pi_{i1}(t) = \begin{cases} \pi_i(t) & t \in F_1 \\ \sigma_{i2} & t \in F_2, \end{cases} \tag{4}$$

$$\pi_{i2}(t) = \begin{cases} \pi_i(t) & t \in F_2 \\ \sigma_{i1} & t \in F_1 \end{cases} \tag{5}$$

where $\sigma_{i1} \in [\underline{\pi}_i, \pi_{i0})$ and $\sigma_{i2} \in [\pi_{i0}, \overline{\pi}_i]$.

To characterize the feature of the distribution of sensor distortion, a set of rand variables $\alpha_i(t)$ $(i = 1, 2, \dots, m)$ is defined:

$$\alpha_i(t) = \begin{cases} 1, & t \in F_1 \\ 0, & t \in F_2 \end{cases} \tag{6}$$

Assume that $\alpha_i(t)$ obeys a Bernoulli distribution with $\text{Prob}\{\alpha_i(t) = 1\} = \mathbb{E}\{\alpha_i(t)\} = \overline{\alpha}_i$, $\text{Prob}\{\alpha_i(t) = 0\} = 1 - \mathbb{E}\{\alpha_i(t)\} = 1 - \overline{\alpha}_i$, where $\mathbb{E}\{\alpha_i(t)\}$ represents the expectation of $\alpha_i(t)$.

Based on the above analysis, it yields

$$\tilde{y}(t) = \sum_{i=1}^{m} [\alpha_i(t)\pi_{i1}(t) + (1 - \alpha_i(t))\pi_{i2}(t)] I_i y(t) \tag{7}$$

where $\underline{\pi}_i \le \pi_{i1}(t) \le \pi_{i0}, \pi_{i0} \le \pi_{i2}(t) \le \overline{\pi}_i$.

For convenience to analyse, we define

$$\pi_{i1}(t) = \frac{\underline{\pi}_i + \pi_{i0}}{2} + \frac{\pi_{i0} - \underline{\pi}_i}{2} F(t) \tag{8}$$

$$\pi_{i2}(t) = \frac{\pi_{i0} + \overline{\pi}_i}{2} + \frac{\overline{\pi}_i - \pi_{i0}}{2} F(t) \tag{9}$$

where $F(t)$ satisfying $F^T(t)F(t) \le I$. Combining with (7), the real measurement can be simplified as

$$\tilde{y}(t) = \sum_{i=1}^{m} [\varepsilon_{1i}(t) + \varepsilon_{2i}(t)F(t)] I_i y(t), \tag{10}$$

where $\varepsilon_{1i}(t) = \frac{\alpha_i(t)(\underline{\pi}_i - \overline{\pi}_i) + \overline{\pi}_i + \pi_{i0}}{2}$, $\varepsilon_{2i}(t) = \frac{\alpha_i(t)(2\pi_{i0} - \underline{\pi}_i - \overline{\pi}_i) + \overline{\pi}_i - \pi_{i0}}{2}$.

Recalling the definition of $\alpha_i(t)$, one can know that

$$\mathbb{E}\{\varepsilon_{1i}(t)\} = \frac{\overline{\alpha}_i(\underline{\pi}_i - \overline{\pi}_i) + \overline{\pi}_i + \pi_{i0}}{2} \tag{11}$$

$$\mathbb{E}\{\varepsilon_{2i}(t)\} = \frac{\overline{\alpha}_i(2\pi_{i0} - \underline{\pi}_i - \overline{\pi}_i) + \overline{\pi}_i - \pi_{i0}}{2} \tag{12}$$

For presentation convenience, we denote $\varepsilon_{1i} = \mathbb{E}\{\varepsilon_{1i}(t)\}, \varepsilon_{2i} = \mathbb{E}\{\varepsilon_{2i}(t)\}$.

**Remark 3.** By introducing rand various $\alpha_i$ and diagonal matrix $I_i$ $(i = 1, 2, \dots, m)$, the model of multiple sensor distortions can be established. The distribution of sensor distortion in a subinterval is expressed by $\alpha_i(t)$. If one sets $\alpha_i(t) = 0$ $(i = 1, 2, \dots, m)$, the problem reduces to be a conventional sensor gain missing with deterministic distribution.

**Remark 4.** A better result can be achieved by using much information of the distribution of multiple sensor distortions, which needs to partition the interval of sensor distortion into more segments, then (7) will be extended to the format of (13), however, the interval in this study is decomposed into two segments for a simple presentation:

$$\tilde{y}(t) = \sum_{i=1}^{m} [\alpha_i^1(t)\pi_{i1}(t) + \alpha_i^2(t)\pi_{i2}(t) + \cdots + \alpha_i^s(t)\pi_{s2}(t)\alpha_i^{s_M}(t)\pi_{s_M 2}(t)] I_i y(t) \tag{13}$$

where $s_M \in \mathbb{N}^+$ is the number of decomposition, $\alpha_i^s(t) \in \{0, 1\}$ and it satisfies $\sum_{s=1}^{s_M} \alpha_i^s(t) = 1$ as well.

### 2.2. Event-triggered scheme

To improve the utility of limited network bandwidth, an event-triggered scheme is applied to determine whether or not the current measurement should be transmitted. In Fig. 1, the output $\tilde{y}(t)$ is sampled periodically as $\tilde{y}(t_k h + jh)$ through the sampler, where $t_k h + jh$ represents the $j$-th $(j = 1, 2, \dots, j_M)$ sampling instant after the current releasing instant. The next releasing instant $t_{k+1}h$ of the sampled data is decided by

$$t_{k+1}h = t_k h + \inf_{j \ge 1}\{jh \mid \delta_k^T(t_k h + jh - h)\Phi\delta_k(t_k h + jh - h) > \varepsilon\wp\}$$

where $\wp = \tilde{y}^T(t_k h)\Phi\tilde{y}(t_k h)$, $0 \le \varepsilon < 1$ is a given scalar parameter and $\Phi > 0$ is a positive definite weighting matrix, $\delta_k^T(t_k h + jh - h)$ represents the error between data at the current sampling instant and the latest transmitted sampling instant, that is, $\delta_k(t_k h + jh - h) = \tilde{y}(t_k h + jh - h) - \tilde{y}(t_k h)$. For the convenience of calculation, the following estimation of $\tilde{y}(t)$ is used in the event-triggering condition:

$$\overline{y}(t) = \sum_{i=1}^{m} [\overline{\alpha_i}\pi_1^i + (1 - \overline{\alpha_i})\pi_2^i] y(t)$$

where $\pi_1^i = \frac{\underline{\pi}_i + \pi_{i0}}{2}$, $\pi_2^i = \frac{\pi_{i0} + \overline{\pi}_i}{2}$.

Thus, the next releasing instant $t_{k+1}h$ turns to be

$$t_{k+1}h = t_k h + \inf_{j \ge 1}\{jh \mid \delta_k^T(t_k h + jh - h)\Phi\delta_k(t_k h + jh - h) > \varepsilon\overline{\wp}\} \tag{14}$$

where $\overline{\wp} = \overline{y}(t_k h + jh - h) - \delta_k(t_k h + jh - h)$.

**Remark 5.** From (14), one can see that not all the sampling data are transmitted to the network. The sampling data turn to be a releasing data only when it invokes the triggering condition of $\delta_k^T(t_k h + jh - h)\Phi\delta_k(t_k h + jh - h) > \varepsilon\overline{y}^T(t_k h)\Phi\overline{y}(t_k h)$, that is, the set of releasing instants is a subset of sampling instants. Thus, the networked bandwidth is saved.

### 2.3. The overall model

For analysis convenience, we define $\eta(t) = t - t_k h - jh + h$ for $t \in \chi_k^j \triangleq [t_k h + (j-1)h + \mu_{k,j}, t_{k+1}h + jh + \mu_{k+1,j})$, where

$$\mu_{k,j} = \begin{cases} \tau_k & j = 1 \\ \hat{\tau} & j = 2, \dots, j_M - 1 \\ \tau_{k+1} & j = j_M \end{cases} \tag{15}$$

where $\hat{\tau}$ is a positive constant which guarantees the left and right order of the interval $\chi_k^j$. One can clearly see that $\cup_{j=1}^{j_M}\chi_k^j = [t_k h + \tau_k, t_{k+1}h + \tau_{k+1})$. It yields,

$$\eta_m = \underline{\tau}_k \le \eta(t) \le h + \overline{\tau}_k = \eta_M \tag{16}$$

**Remark 6.** In (15), we decompose the intervals of $[t_k h + \tau_k, t_{k+1}h + \tau_{k+1})$ into $j_M$ parts. $\mu_{k,j}^j$ can be regarded as an artificial delay for $j = 2, 3, \dots, j_M - 1$.

Combining with (7) and (15), it follows that

$$\begin{aligned} \hat{y}(t) &= \tilde{y}(t_k h) = \tilde{y}(t - \eta(t)) - \delta_k(t - \eta(t)) \\ &= \sum_{i=1}^{m} [\alpha_i(t)\pi_{i1}(t) + (1 - \alpha_i(t))\pi_{i2}(t)] I_i y(t - \eta(t)) - \delta_k(t - \eta(t)) \end{aligned} \tag{17}$$

Define $\tilde{x}^T(t) = \left[x^T(t)\ x_f^T(t)\right]$, $e(t) = z(t) - z_f(t)$, the filtering error system can be written as:

$$\begin{cases} \dot{\tilde{x}}(t) = A_0\tilde{x}(t) + A_1 x(t-\eta(t)) + B_0\omega(t) + B_1\delta_k(t-\eta(t)), \\ e(t) = C_0\tilde{x}(t) \end{cases} \tag{18}$$

for $t \in \chi_k^j$, where

$$A_0 = \begin{bmatrix} A & 0 \\ 0 & A_f \end{bmatrix}, \quad A_1 = \begin{bmatrix} 0 \\ \sum_{i=1}^{m}[\varepsilon_{1i}(t)B_f I_i C + \varepsilon_{2i}(t)B_f F(t) I_i C] \end{bmatrix},$$

$$B_0 = \begin{bmatrix} B \\ 0 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 0 \\ -B_f \end{bmatrix}, \quad C_0 = \begin{bmatrix} L & -C_f \end{bmatrix}$$

## 3. Stability analysis and reliable filter design under ETC scheme

This section aims to develop an approach of the stability analysis and reliable filter design for the system (18) subject to multiple sensor distortion under ETC scheme.

Firstly we provide the following definitions and lemmas which will play a significant role in the subsequent derivation.

**Definition 1.** The filtering error system (18) is mean square stable with a $H_\infty$ norm bound $\gamma$ if the following hold:

(1) When $\omega(t) = 0$, the filtering error system (18) is mean square stable.

(2) Under zero initial condition, for a scalar $\gamma > 0$ and $\omega(t) \in \ell_2[0, \infty)$, $e(t)$ satisfies $\mathbb{E}\{\sum_{t=0}^{\infty} \| e(t) \|_2\} \le \gamma \mathbb{E}\{\sum_{t=0}^{\infty} \| \omega(t) \|_2\}$.

**Definition 2** ([24]). For a given function $V : C_{F_0}^b([-\eta_M, 0], \mathbb{R}^n) \times S$, its infinitesimal operator $\mathcal{L}$ is defined as

$$\mathcal{L}V(x_t) = \lim_{\Delta \to 0^+} \frac{1}{\Delta}[\mathbb{E}(V(x_{t+\Delta} | x_t) - V(x_t))] \tag{19}$$

**Lemma 1** ([25]). *Given matrices W, M and N with appropriate dimensions, where W is symmetric, then $W + MF(t)N + N^T F^T(t) M^T < 0$ for all matrices $F(t)$ satisfying $F^T(t)F(t) \le I$, if and only if there exists a constant $\varepsilon > 0$ such that $W + \varepsilon MM^T + \varepsilon^{-1} N^T N < 0$.*

**Lemma 2** ([26]). *For given positive integers $n, m$, a scalar $\alpha$ in the interval $(0,1)$, a given $n \times n$–matrix $R > 0$, two matrices $W_1$ and $W_2$ in $\mathbb{R}^{n \times m}$. Define, for all vector $\xi$ in $\mathbb{R}^m$, the function $\Theta(\alpha, R)$ given by $\Theta(\alpha, R) = \frac{1}{\alpha}\xi^T W_1^T R W_1 \xi + \frac{1}{1-\alpha}\xi^T W_2^T R W_2 \xi$. Then, if there exists a matrix H in $\mathbb{R}^{n \times n}$ such that $\left[\begin{smallmatrix} R & H \\ * & R \end{smallmatrix}\right] > 0$, then the following inequality holds*

$$\min_{\alpha \in (0,1)} \Theta(\alpha, R) \ge \begin{bmatrix} W_1\xi \\ W_2\xi \end{bmatrix}^T \begin{bmatrix} R & H \\ * & R \end{bmatrix} \begin{bmatrix} W_1\xi \\ W_2\xi \end{bmatrix}.$$

**Lemma 3** ([26]). *For a given $n \times n$-matrix $R > 0$, the following inequality holds for all continuously differentiable function x in $[a, b] \to \mathbb{R}^n$ : $\int_a^b \dot{x}^T(u) R \dot{x}(u) du \ge \frac{1}{b-a}(x(b) - x(a))^T R (x(b) - x(a)) + \frac{3}{b-a}\tilde{\Omega}^T R \tilde{\Omega}$, where $\tilde{\Omega} = x(b) + x(a) - \frac{2}{b-a}\int_a^b x(u) du$.*

### 3.1. Filter stability analysis

**Theorem 1.** *For some given positive constants $\varepsilon_{1i}$, $\varepsilon_{2i}$, $\overline{\alpha}_i$, $\pi_1{}^i$, $\pi_2{}^i$ $(i = 1, 2, \ldots, m)$ and $\gamma$, if there exist matrices $P = \left[\begin{smallmatrix} P_1 & P_2 \\ P_2^T & P_3 \end{smallmatrix}\right] > 0$, $S_1 > 0$, $S_2 > 0$, $R_1 > 0$, $R_2 > 0$, $\Phi > 0$, H and constants $\mu_{1i} > 0$ $(i = 1, 2, \ldots, m)$ such that*

$$\Omega_1 = \begin{bmatrix} \Omega_{11} & * & * \\ \Omega_{21} & \Omega_{22} & * \\ \Omega_{31} & 0 & \Omega_{33} \end{bmatrix} < 0, \tag{20}$$

$$\begin{bmatrix} \hat{R} & H \\ * & \hat{R} \end{bmatrix} > 0 \tag{21}$$

*where*

$$\Omega_{11} = \begin{bmatrix} \Xi_{11} & * & * & * & * & * & * & * & * \\ \Xi_{21} & \Xi_{22} & * & * & * & * & * & * & * \\ R_1 & 0 & -S_1 - R_1 & * & * & * & * & * & * \\ \Xi_{41} & \Xi_{42} & 0 & \Xi_{44} & * & * & * & * & * \\ \Xi_{51} & 0 & 0 & \Xi_{54} & \Xi_{55} & * & * & * & * \\ \frac{6}{\eta_M}R_2 & 0 & 0 & \Xi_{64} & \Xi_{65} & -\frac{12}{\eta_M}R_2 & * & * & * \\ \Xi_{71} & 0 & 0 & \Xi_{74} & \frac{6}{\eta_M}R_2 & -\frac{4}{\eta_M}H_3 & -\frac{12}{\eta_M}R_2 & * & * \\ -B_f^T P_2^T & -B_f^T P_3 & 0 & \Xi_{84} & 0 & 0 & 0 & \varepsilon\Phi - \Phi & * \\ B^T P_1 & B^T P_2 & 0 & 0 & 0 & 0 & 0 & 0 & -\gamma^2 I \end{bmatrix},$$

$$\Omega_{21} = \begin{bmatrix} \eta_M R_2 A & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \eta_M R_2 B \\ \eta_m^2 R_1 A & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \eta_m^2 R_1 B \\ L & -C_f & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\Omega_{22} = \mathrm{diag}\{-\eta_M R_2, -\eta_m^2 R_1, -I\}, \quad \Omega_{31} = \left[\Theta_{11}^T\ \ \Theta_{12}^T\ \ \cdots\ \ \Theta_{1m}^T\right]^T,$$

$$\Omega_{33} = \mathrm{diag}\{-\mu_1, -\mu_2, \ldots, -\mu_m\},$$

$$\Theta_{11} = \begin{bmatrix} \mu_{11} B_f^T P_2^T & \mu_{11} B_f^T P_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \varepsilon_{21} I_1 C & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\Theta_{1m} = \begin{bmatrix} \mu_{1m} B_f^T P_2^T & \mu_{1m} B_f^T P_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \varepsilon_{2m} I_m C & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\mu_1 = \mathrm{diag}\{\mu_{11} I, \mu_{11} I\}, \quad \mu_m = \mathrm{diag}\{\mu_{1m} I, \mu_{1m} I\}, \quad H = \begin{bmatrix} H_1 & * \\ H_2 & H_3 \end{bmatrix},$$

$$\Xi_{11} = A^T P_1 + P_1 A + S_1 + S_2 - R_1 - \frac{4}{\eta_M}R_2, \quad \Xi_{21} = A_f^T P_2^T + P_2^T A,$$

$$\Xi_{22} = A_f^T P_3 + P_3 A_f, \quad \hat{R} = \mathrm{diag}\{R_2, 3R_2\},$$

$$\Xi_{41} = \sum_{i=1}^{m} \varepsilon_{1i} C^T I_i B_f^T P_2^T - \frac{1}{\eta_M}(2R_2 + H_1^T + H_2 + H_2^T + H_3),$$

$$\Xi_{42} = \sum_{i=1}^{m} \varepsilon_{1i} C^T I_i B_f^T P_3,$$

$$\Xi_{44} = \sum_{i=1}^{m} \varepsilon[\overline{\alpha}_i \pi_1^i + (1 - \overline{\alpha}_i)\pi_2^i]^2 C^T \Phi C - \frac{1}{\eta_M}(8R_2 - 2H_1 + 2H_3),$$

$$\Xi_{51} = -\frac{1}{\eta_M}(-H_1 + H_2 - H_2^T + H_3),$$

$$\Xi_{54} = -\frac{1}{\eta_M}(2R_2 + H_1 - H_2 - H_2^T + H_3),$$

$$\Xi_{55} = -S_2 - \frac{4}{\eta_M}R_2, \quad \Xi_{64} = \frac{1}{\eta_M}(6R_2 + 2H_2 + 2H_3),$$

$$\Xi_{65} = -\frac{1}{\eta_M}(2H_2 - 2H_3),$$

$$\Xi_{71} = \frac{1}{\eta_M}(2H_2 + 2H_3), \quad \Xi_{74} = -\frac{1}{\eta_M}(2H_2 - 2H_3 - 6R_2),$$

$$\Xi_{84} = -\sum_{i=1}^{m} \varepsilon[\overline{\alpha}_i \pi_1^i + (1 - \overline{\alpha}_i)\pi_2^i]\Phi C.$$

*Then the filtering error system (18) is mean square stable with an $H_\infty$ norm bound $\gamma$.*

**Proof.** For the filtering error system (18), a Lyapunov–Krasovskii functional candidate is given by

$$V(t) = V_1(t) + V_2(t) + V_3(t)$$

where

$$V_1(t)=\tilde{x}^T(t)P\tilde{x}(t),$$

$$V_2(t)=\int_{t-\eta_M}^{t} x^T(s)S_2x(s)ds+\eta_m\int_{-\eta_m}^{0}\int_{t+s}^{t}\dot{x}^T(v)R_1\dot{x}(v)dvds,$$

$$V_3(t)=\int_{t-\eta_m}^{t} x^T(s)S_1x(s)ds+\int_{-\eta_M}^{0}\int_{t+s}^{t}\dot{x}^T(v)R_2\dot{x}(v)dvds,$$

For $t\in\chi_k^j$, the mathematical expectation of the generator $\mathcal{L}V(t)$ along the solutions of (18) is given by

$$\begin{aligned}
&\mathbb{E}\{\mathcal{L}V(t)+e^T(t)e(t)-\gamma^2\omega^T(t)\omega(t)\}\\
&\quad=\mathbb{E}\Big\{2\tilde{x}^T(t)P\dot{\tilde{x}}(t)+x^T(t)S_1x(t)-x^T(t-\eta_m)S_1x(t-\eta_m).\\
&\quad\quad+x^T(t)S_2x(t)-x^T(t-\eta_M)S_2x(t-\eta_M)+\eta_m^2\dot{x}^T(t)R_1\dot{x}(t)\\
&\quad\quad-\eta_m\int_{t-\eta_m}^{t}\dot{x}^T(s)R_1\dot{x}(s)ds+\eta_M\dot{x}^T(t)R_2\dot{x}(t)\\
&\quad\quad-\int_{t-\eta_M}^{t}\dot{x}^T(s)R_2\dot{x}(s)ds+\delta_k^T(t-\eta(t))\Phi\delta_k(t-\eta(t))\\
&\quad\quad-\delta_k^T(t-\eta(t))\Phi\delta_k(t-\eta(t))+e^T(t)e(t)-\gamma^2\omega^T(t)\omega(t)\Big\}
\end{aligned}$$

Defining $\zeta(t)=\mathrm{col}\{x(t),x_f(t),x(t-\eta_m),x(t-\eta(t)) \quad ,x(t-\eta_M),\frac{1}{\eta(t)}\int_{t-\eta(t)}^{t}x(s)ds,\ \frac{1}{\eta_M-\eta(t)}\int_{t-\eta_M}^{t-\eta(t)}x(s)ds,\ \delta_k(t-\eta(t)),\omega(t)\}$, and applying Lemma 2, it yields

$$-\int_{t-\eta_M}^{t}\dot{x}^T(s)R_2\dot{x}(s)ds\le-\zeta^T(t)\left(\frac{1}{\eta(t)}F_1^T\hat{R}F_1+\frac{1}{\eta_M-\eta(t)}F_2^T\hat{R}F_2\right)\zeta(t) \tag{22}$$

where

$$F_1=\begin{bmatrix} I & 0 & 0 & -I & 0 & 0 & 0 & 0 & 0\\ I & 0 & 0 & I & 0 & -2I & 0 & 0 & 0\end{bmatrix},$$

$$F_2=\begin{bmatrix} 0 & 0 & 0 & I & -I & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & I & I & 0 & -2I & 0 & 0\end{bmatrix}.$$

Then, applying Lemma 3, we can see that if there exists a matrix $H\in\mathbb{R}^{n\times n}$ such that $\begin{bmatrix}\hat{R} & H\\ * & \hat{R}\end{bmatrix}>0$, it yields

$$\begin{aligned}
\frac{1}{\eta(t)}F_1^T\hat{R}F_1+\frac{1}{\eta_M-\eta(t)}F_2^T\hat{R}F_2&=\frac{1}{\eta_M}\left[\frac{1}{\eta(t)/\eta_M}F_1^T\hat{R}F_1+\frac{1}{[1-\eta(t)]/\eta_M}F_2^T\hat{R}F_2\right]\\
&\ge\frac{1}{\eta_M}\begin{bmatrix}F_1\\F_2\end{bmatrix}^T\begin{bmatrix}\hat{R} & H\\ * & \hat{R}\end{bmatrix}\begin{bmatrix}F_1\\F_2\end{bmatrix}
\end{aligned} \tag{23}$$

Then it follows

$$-\int_{t-\eta_M}^{t}\dot{x}^T(s)R\dot{x}(s)ds\le-\frac{1}{\eta_M}\zeta^T(t)\begin{bmatrix}F_1\\F_2\end{bmatrix}^T\begin{bmatrix}\hat{R} & H\\ * & \hat{R}\end{bmatrix}\begin{bmatrix}F_1\\F_2\end{bmatrix}\zeta(t).$$

Then using Jensen inequality [27], it follows

$$-\eta_m\int_{t-\eta_m}^{t}\dot{x}^T(s)R_1\dot{x}(s)ds\le\begin{bmatrix}x(t)\\x(t-\eta_m)\end{bmatrix}^T\begin{bmatrix}-R_1 & *\\ R_1 & -R_1\end{bmatrix}\begin{bmatrix}x(t)\\x(t-\eta_m)\end{bmatrix}$$

For $t\in\chi_l^j$, it is clear that $\delta_k^T(t-\eta(t))\Phi\delta_k(t-\eta(t))\le\varepsilon\overline{y}^T(t_kh)\Phi\overline{y}(t_kh)$ from the triggering condition in (14). Then using Schur complement and Lemma 1, we can conclude that (20) and (21) are sufficient conditions to guarantee

$$\mathbb{E}\{\mathcal{L}V(t)+e^T(t)e(t)-\gamma^2\omega^T(t)\omega(t)\}\le 0\} \tag{24}$$

Under zero initial condition, integrating both sides of (24) from 0 to $t$ and let $t\to\infty$, we have

$$\mathbb{E}\left\{\int_{t_0}^{\infty}e^T(s)e(s)ds\right\}\le\mathbb{E}\left\{\int_{t_0}^{\infty}\gamma^2\omega^T(s)\omega(s)ds\right\} \tag{25}$$

Recalling Definition 1, the result is established. This completes the proof. □

The stable condition of filtering error system (18) with rand multi-sensor distortions is given by Theorem 1. As stated in Remark 3, if one lets $\alpha_i(t)\equiv 0$ or 1 in (7), it becomes a deterministic case. In this case, $\pi_i(t)$ belongs to interval $[\underline{\pi}_i,\overline{\pi}_i]$ in (3). Then the measurement output $\hat{y}(t)$ can be rewritten as:

$$\hat{y}(t)=\sum_{i=1}^{m}\left[\frac{\underline{\pi}_i+\overline{\pi}_i}{2}+\frac{\overline{\pi}_i-\underline{\pi}_i}{2}F(t)\right]I_iy(t-\eta(t))-\delta_k(t-\eta(t)) \tag{26}$$

The following corollary will be given for this case to guarantee the stability of the system (18) in mean square sense.

**Corollary 1.** *For some given positive parameters $\gamma$, $\underline{\pi}_i$ and $\overline{\pi}_i$, If there exist matrices $P=\begin{bmatrix}P_1 & P_2\\ P_2^T & P_3\end{bmatrix}>0$, $S_1>0$, $S_2>0$, $R_1>0$, $R_2>0$, $\Phi>0$ and $H$ such that the following LMIs are satisfied:*

$$\hat{\Omega}_1=\begin{bmatrix}\hat{\Omega}_{11} & * & *\\ \hat{\Omega}_{21} & \Omega_{22} & *\\ \hat{\Omega}_{31} & 0 & \Omega_{33}\end{bmatrix}<0, \tag{27}$$

$$\begin{bmatrix}\hat{R} & H\\ * & \hat{R}\end{bmatrix}>0, \tag{28}$$

*where*

$$\hat{\Omega}_{11}=\begin{bmatrix}
\Xi_{11} & * & * & * & * & * & * & * & *\\
\Xi_{21} & \Xi_{22} & * & * & * & * & * & * & *\\
R_1 & 0 & -S_1-R_1 & * & * & * & * & * & *\\
\hat{\Xi}_{41} & \hat{\Xi}_{42} & 0 & \Xi_{44} & * & * & * & * & *\\
\Xi_{51} & 0 & 0 & \Xi_{54} & \Xi_{55} & * & * & * & *\\
\frac{6}{\eta_M}R_2 & 0 & 0 & \Xi_{64} & \Xi_{65} & -\frac{12}{\eta_M}R_2 & * & * & *\\
\Xi_{71} & 0 & 0 & \Xi_{74} & \frac{6}{\eta_M}R_2 & -\frac{4}{\eta_M}H_3 & -\frac{12}{\eta_M}R_2 & * & *\\
-B_f^TP_2^T & -B_f^TP_3 & 0 & -\varepsilon\Phi C & 0 & 0 & 0 & \varepsilon\Phi-\Phi & *\\
B^TP_1 & B^TP_2 & 0 & 0 & 0 & 0 & 0 & 0 & -\gamma^2I
\end{bmatrix},$$

$$\hat{\Omega}_{21}=\begin{bmatrix}
\eta_MR_2A & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \eta_MR_2B\\
\eta_m^2R_1A & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \eta_m^2R_1B\\
L & -C_f & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix},$$

$$\hat{\Omega}_{31}=\begin{bmatrix}\hat{\Theta}_{11}^T & \hat{\Theta}_{12}^T & \cdots & \hat{\Theta}_{1m}^T\end{bmatrix}^T,\quad \hat{\Omega}_{32}=\begin{bmatrix}\hat{\Theta}_{21}^T & \hat{\Theta}_{22}^T & \cdots & \hat{\Theta}_{2m}^T\end{bmatrix}^T,$$

$$\hat{\Theta}_{11}=\begin{bmatrix}
\mu_{11}B_f^TP_2^T & \mu_{11}B_f^TP_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & \frac{\overline{\pi}_1-\underline{\pi}_1}{2}I_1C & 0 & 0 & 0 & 0 & 0
\end{bmatrix},$$

$$\hat{\Theta}_{1m}=\begin{bmatrix}
\mu_{1m}B_f^TP_2^T & \mu_{1\,m}B_f^TP_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & \frac{\overline{\pi}_m-\underline{\pi}_m}{2}I_mC & 0 & 0 & 0 & 0 & 0
\end{bmatrix},$$

$$\hat{\Xi}_{41}=\sum_{i=1}^{m}\frac{\underline{\pi}_i+\overline{\pi}_i}{2}C^TI_iB_f^TP_2^T-\frac{1}{\eta_M}(2R_2+H_1^T+H_2+H_2^T+H_3),$$

$$\hat{\Xi}_{42}=\sum_{i=1}^{m}\frac{\underline{\pi}_i+\overline{\pi}_i}{2}C^TI_iB_f^TP_3,\quad H=\begin{bmatrix}H_1 & *\\ H_2 & H_3\end{bmatrix}.$$

*and $\Omega_{22}$ and $\Omega_{33}$ are defined in Theorem 1, then the filtering error system (18) is mean square stable with an $H_\infty$ norm bound $\gamma$.*

The proof is similar to Theorem 1, it is omitted here for brevity.

Let $\Pi(t)\equiv 1$ in (7) as mentioned in Remark 2, the problem reduces to the filter design for the system without any sensor's flaw. The model can be expressed by

$$\dot{\tilde{x}}(t)=A_0\tilde{x}(t)+\begin{bmatrix}0\\B_fC\end{bmatrix}x(t-\eta(t))+B_0\omega(t)+\begin{bmatrix}0\\-B_f\end{bmatrix}\delta_k(t-\eta(t)),$$
$$e(t)=C_0\tilde{x}(t). \tag{29}$$

By using the similar method, we can get the following result.

**Corollary 2.** *For a given positive parameter $\gamma$, if there exist matrices $P=\left[\begin{smallmatrix}P_1 & P_2\\ P_2^T & P_3\end{smallmatrix}\right]>0$, $S_1>0$, $S_2>0$, $R_1>0$, $R_2>0$, $\Phi>0$ and $H$ such that*

$$\overline{\Omega}_1=\begin{bmatrix}\overline{\Omega}_{11} & *\\ \overline{\Omega}_{21} & \Omega_{22}\end{bmatrix}<0, \tag{30}$$

$$\begin{bmatrix}\hat{R} & H\\ * & \hat{R}\end{bmatrix}>0, \tag{31}$$

*where*

$$\overline{\Omega}_{11}=\begin{bmatrix}
\Xi_{11} & * & * & * & * & * & * & * & *\\
\Xi_{21} & \Xi_{22} & * & * & * & * & * & * & *\\
R_1 & 0 & -S_1-R_1 & * & * & * & * & * & *\\
\overline{\Xi}_{41} & C^TB_f^TP_3 & 0 & \Xi_{44} & * & * & * & * & *\\
\Xi_{51} & 0 & 0 & \Xi_{54} & \Xi_{55} & * & * & * & *\\
\frac{6}{\eta_M}R_2 & 0 & 0 & \Xi_{64} & \Xi_{65} & -\frac{12}{\eta_M}R_2 & * & * & *\\
\Xi_{71} & 0 & 0 & \Xi_{74} & \frac{6}{\eta_M}R_2 & -\frac{4}{\eta_M}H_3 & -\frac{12}{\eta_M}R_2 & * & *\\
-B_f^TP_2^T & -B_f^TP_3 & 0 & -\varepsilon\Phi C & 0 & 0 & 0 & \varepsilon\Phi-\Phi & *\\
B^TP_1 & B^TP_2 & 0 & 0 & 0 & 0 & 0 & 0 & -\gamma^2I
\end{bmatrix},$$

$$\overline{\Omega}_{21}=\begin{bmatrix}
\eta_MR_2A & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \eta_MR_2B\\
\eta_m^2R_1A & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \eta_m^2R_1B\\
L & -C_f & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix},$$

$$\overline{\Xi}_{41}=C^TB_f^TP_2^T-\frac{1}{\eta_M}(2R_2+H_1^T+H_2+H_2^T+H_3).$$

*Then the system (29) is mean square stable with an $H_\infty$ performance level $\gamma$.*

### 3.2. Filter design

With the aid of Theorem 1, we are now in a position to co-design the filter parameters $A_f, B_f, C_f, D_f$, and the event-triggered matrix $\Phi$ for the system with multiple sensor distortions.

**Theorem 2.** *For some given positive constants $\varepsilon_{1i}$, $\varepsilon_{2i}$ $(i=1,2,\ldots,m)$, $\overline{\alpha}_i$, ${\pi_1}^i$, ${\pi_2}^i$ and $\gamma$, if there exist matrices $X>0$, $Y>0$, $\Phi>0$, $S_1>0$, $S_2>0$, $R_1>0$, $R_2>0$ and constants $\mu_{1i}>0$ $(i=1,2,\ldots,m)$, matrices $H,\tilde{A}_f,\tilde{B}_f,\tilde{C}_f$ with appropriate dimensions such that*

$$\Omega_2=\begin{bmatrix}\tilde{\Omega}_{11} & * & *\\ \tilde{\Omega}_{21} & \Omega_{22} & *\\ \tilde{\Omega}_{31} & 0 & \Omega_{33}\end{bmatrix}<0, \tag{32}$$

$$\begin{bmatrix}\hat{R} & H\\ * & \hat{R}\end{bmatrix}>0, \tag{33}$$

$$X-Y>0 \tag{34}$$

*where*

$$\tilde{\Omega}_{11}=\begin{bmatrix}
\tilde{\Xi}_{11} & * & * & * & * & * & * & * & *\\
\tilde{\Xi}_{21} & \tilde{\Xi}_{22} & * & * & * & * & * & * & *\\
R_1 & 0 & -S_1-R_1 & * & * & * & * & * & *\\
\tilde{\Xi}_{41} & \tilde{\Xi}_{42} & 0 & \Xi_{44} & * & * & * & * & *\\
\Xi_{51} & 0 & 0 & \Xi_{54} & \Xi_{55} & * & * & * & *\\
\frac{6}{\eta_M}R_2 & 0 & 0 & \Xi_{64} & \Xi_{65} & -\frac{12}{\eta_M}R_2 & * & * & *\\
\Xi_{71} & 0 & 0 & \Xi_{74} & \frac{6}{\eta_M}R_2 & -\frac{4}{\eta_M}H_3 & -\frac{12}{\eta_M}R_2 & * & *\\
-\tilde{B}_f^T & -\tilde{B}_f^T & 0 & \Xi_{84} & 0 & 0 & 0 & \varepsilon\Phi-\Phi & *\\
B^TX & B^TY^T & 0 & 0 & 0 & 0 & 0 & 0 & -\gamma^2I
\end{bmatrix},$$

$$\tilde{\Omega}_{21}=\begin{bmatrix}
\eta_MR_2A & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \eta_MR_2B\\
\eta_m^2R_1A & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \eta_m^2R_1B\\
L & -\tilde{C}_f & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix},$$

$$\tilde{\Omega}_{31}=\begin{bmatrix}\tilde{\Theta}_{11}^T & \tilde{\Theta}_{12}^T & \cdots & \tilde{\Theta}_{1m}^T\end{bmatrix}^T,$$

$$\tilde{\Theta}_{11}=\begin{bmatrix}\mu_{11}\tilde{B}_f^T & \mu_{11}\tilde{B}_f^T & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & \varepsilon_{21}I_1C & 0 & 0 & 0 & 0 & 0\end{bmatrix},$$

$$\tilde{\Theta}_{1m}=\begin{bmatrix}\mu_{1m}\tilde{B}_f^T & \mu_{1m}\tilde{B}_f^T & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & \varepsilon_{2m}I_mC & 0 & 0 & 0 & 0 & 0\end{bmatrix},$$

$$\tilde{\Xi}_{11}=A^TX+XA+S_1+S_2-R_1-\frac{4}{\eta_M}R_2,$$

$$\Xi_{21}=\tilde{A}_f^T+YA,\ \tilde{\Xi}_{22}=\tilde{A}_f+\tilde{A}_f^T,$$

$$\tilde{\Xi}_{41}=\sum_{i=1}^{m}\varepsilon_{1i}C^TI_i\tilde{B}_f^T-\frac{1}{\eta_M}(2R_2+H_1^T+H_2+H_2^T+H_3),$$

$$\hat{R}=\mathrm{diag}\{R_2,3R_2\},\quad \tilde{\Xi}_{42}=\sum_{i=1}^{m}\varepsilon_{1i}C^TI_i\tilde{B}_f^T.$$

*and $\Omega_{22},\Omega_{32},\Omega_{33},\Xi_{44},\Xi_{51},\Xi_{54},\Xi_{55},\Xi_{64},\Xi_{65},\Xi_{71},\Xi_{74}$ are defined in Theorem 1, then under the event-triggered scheme, the filtering error system (18) with multiple sensor distortions is mean square stable with a $H_\infty$ norm bound $\gamma$. Moreover, the reliable filter parameters are given by*

$$A_f=Y^{-1}\tilde{A}_f,\quad B_f=Y^{-1}\tilde{B}_f,\quad C_f=\tilde{C}_f. \tag{35}$$

**Proof.** Since $P=\left[\begin{smallmatrix}P_1 & P_2\\ P_2^T & P_3\end{smallmatrix}\right]>0$, where $P_1>0$ and $P_3>0$, then define the following matrix: $J=\left[\begin{smallmatrix}I & 0\\ 0 & P_2P_3^{-1}\end{smallmatrix}\right]$. In addition, by Schur complement, $P>0$ is equivalent to

$$P_1-P_2P_3^{-1}P_2^T>0 \tag{36}$$

Then multiply (20) by $\mathrm{diag}\{J,\underbrace{I,\ldots,I}_{10+2m}\}$ from the left side and its transpose from the right side, respectively, and defining $X=P_1, Y=P_2P_3^{-1}P_2^T, \tilde{A}_f=P_2A_fP_3^{-1}P_2^T, \tilde{B}_f=P_2B_f, \tilde{C}_f=C_fP_3^{-1}P_2^T$. Then the filter parameters can be obtained as:

$$\begin{bmatrix}A_f & B_f\\ C_f & \end{bmatrix}=\begin{bmatrix}P_2^{-1}\tilde{A}_fP_2^{-T}P_3 & P_2^{-1}\tilde{B}_f\\ \tilde{C}_fP_2^TP_3 & \end{bmatrix}\Longleftrightarrow\begin{bmatrix}Y^{-1}\tilde{A}_f & Y^{-1}\tilde{B}_f\\ \tilde{C}_f & \end{bmatrix} \tag{37}$$

Finally, based on the analysis above, the matrix inequality (32) and (34) can be easily obtained from (20) and (36). Meanwhile, the suitable $H_\infty$ filter can be obtained by (35). This completes the proof. □

## 4. Examples

In this section, an example is given to demonstrate the effectiveness of the proposed design method.

**Table 1**
$H_\infty$ norm bound $\gamma_{min}$ under the distortion with/without partition.

| Case | Distribution | Criterion | $\gamma_{min}$ |
|---|---|---|---|
| Without partition | [0.3 1] | Corollary 1 | 5.70 |
| With partition | [0.3 0.8], $\alpha_i$=0.2 | Theorem 1 | 2.84 |
| | [0.8 0.1], $1-\overline{\alpha}_i=0.8$ | | |

**Table 2**
The parameters of multiple sensor distortions.

| Sensor | Subinterval 1 | Subinterval 2 |
|---|---|---|
| Sensor 1 | [0.3 0.8], $\overline{\alpha}_1 = 0.2$ | [0.3 0.8], $1-\overline{\alpha}_1 = 0.8$ |
| Sensor 2 | [0.4 0.9], $\overline{\alpha}_2 = 0.1$ | [0.9 1.0], $1-\overline{\alpha}_2 = 0.9$ |

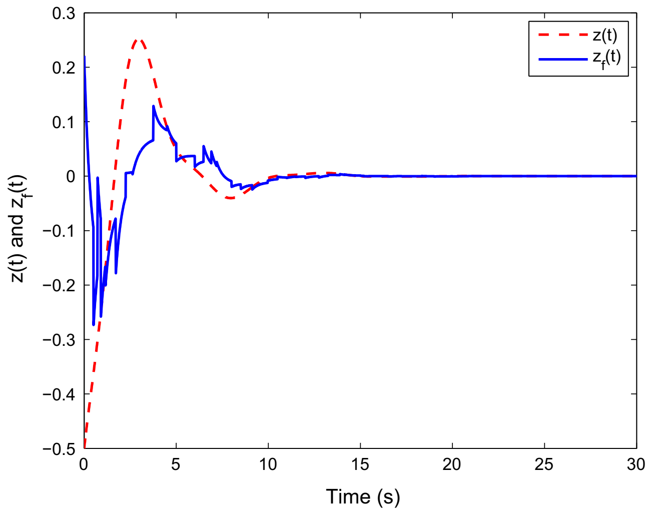


**Fig. 2.** Responses of $z(t)$ and estimation $z_f(t)$.

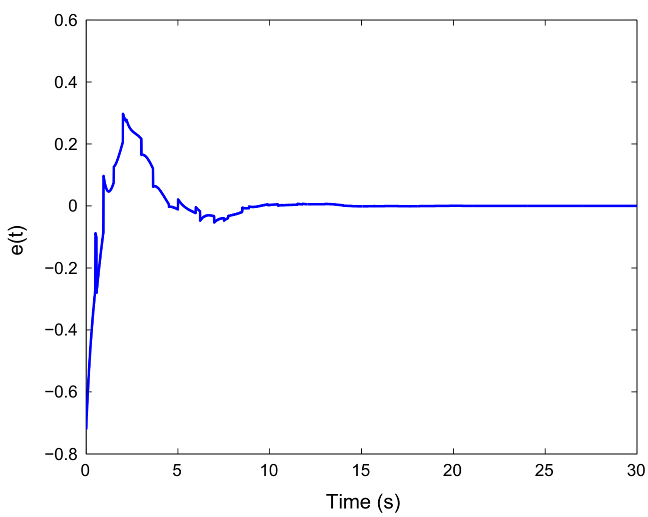


**Fig. 3.** The estimation error $e(t)$.

**Example 1.** Consider the following linear continuous-time system [19] with the matrices:

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -2 & 1 & -0.5 & 0 \\ 2 & -2 & 0 & -1 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix},$$

$$L = [1 \;\; 1 \;\; 0 \;\; 0]$$

Assume $\eta_m = 0.1$, $\eta_M = 0.61$ in (16), and the threshold of ETC scheme $\varepsilon = 0.1313$ in (14). The intervals of two sensor distortions are [0.3 1], that is, $\underline{\pi}_i = 0.3, \overline{\pi}_i = 1$ $(i = 1, 2)$. To illustrate the effectiveness of the proposed method, the following two cases are given: the one is the interval of sensor distortion without partition, the other is the interval of sensor distortion with partition, which is shown in Table 1. The expectation of the distortion in the subinterval [0.3 0.8) and [0.8 1.0] are 0.2 and 0.8, respectively. By using Corollary 1 and Theorem 1, we can obtain the $H_\infty$ norm bound $\gamma_{\min}$ are 5.70 and 2.84, respectively, from which we can see the proposed method by partitioning the interval of sensor distortion leading to a less conservativeness.

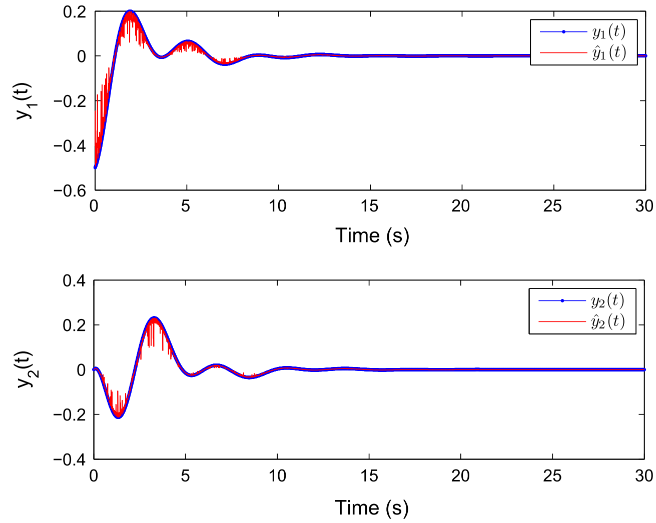


**Fig. 4.** Responses of $y(t)$ and $\hat{y}(t)$. (For interpretation of the references to colour in this figure, the reader is referred to the web version of this paper.)

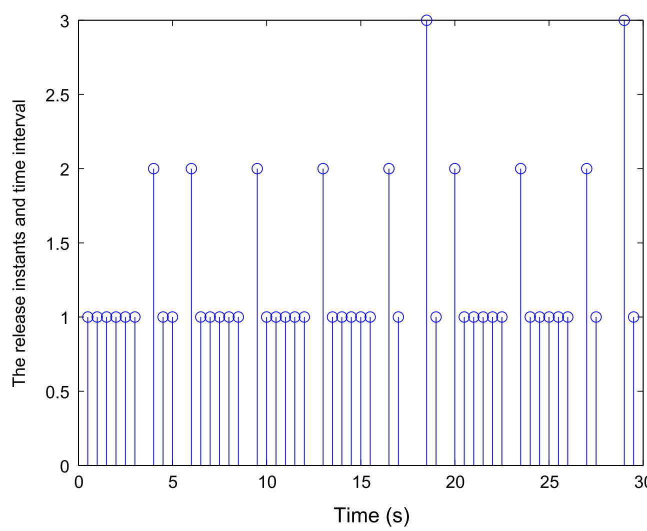


**Fig. 5.** Release instants and the number of active packet loss.

Next, we discuss the problem of co-designing between reliable filter parameters in (35) and the parameter of ETC scheme in (14). The parameters of the sensor distortion and the distribution of their partition are shown in Table 2. We can obtain the following parameters by using Theorem 2 with $\gamma = 7$, $\eta_m = 0.1$, $\varepsilon = 0.1313$:

$$\Phi = \begin{bmatrix} 20.7285 & -0.0315 \\ -0.0315 & 9.5605 \end{bmatrix},$$

$$A_f = \begin{bmatrix} -2.5949 & -0.3170 & 0.1880 & -0.0006 \\ -0.3170 & -1.3290 & -0.2837 & -0.0018 \\ 0.1880 & -0.2837 & -0.5646 & 0.2553 \\ -0.0006 & -0.0018 & 0.2553 & -0.5212 \end{bmatrix},$$

$$B_f = \begin{bmatrix} -1.2918 & -0.0110 \\ -0.3834 & -0.3082 \\ 0.1426 & -0.0971 \\ -0.1564 & 0.1247 \end{bmatrix},$$

$C_f = [-1.8538 \quad -1.0027 \quad -0.1141 \quad -0.1504]$.

Under the above parameters, we can obtain the following responses shown in Figs. 2–5 for the filtering system with initial condition $x_0 = [-0.5\ 0\ 0.2\ 0.1]^T$ and external disturbance $\omega(t) = e^{-0.2t}$. The output of $z(t)$ and its estimation $z_f(t)$ are shown in Fig. 2. The response of filter error $e(t)$ is depicted in Fig. 3. In Fig. 4, the blue line represents the response of measurement output $y(t)$ and the red line represents $\hat{y}(t)$, which indicates that the sensors in two channels are all with distortion. From Fig. 5, one can see that much sampled data are dropped actively due to ETC scheme. Under these situations, one can clearly observe that the designed event-triggered $H_\infty$ filter can still get a good estimation (see Figs. 2 and 3). From these results, we can draw a conclusion that (1) the design method can improve the resource utilization by using ETC scheme; and (2) reliable filter design can better compensate the sensor distortion by utilizing more distribution information on sensor distortions.

## 5. Conclusion

In this paper, a co-design of reliable filtering and ETC scheme for a class of NCSs with multiple sensor distortion has been investigated. By using a method of interval partition to sensor distortions and ETC scheme, we develop a new overall model. Based on this model, sufficient conditions have been developed to guarantee the filtering error system to be stable in mean square sense with the desired $H_\infty$ performance. Under the proposed model and the designed scheme, we can get a less conservativeness result, moreover, the data communication frequency and network bandwidth usages can be reduced as well.

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## References

[1] Yue D, Tian E, Han Q-L. A delay system method for designing event-triggered controllers of networked control systems. IEEE Trans Autom Control 2013;58 (2):475–81.
[2] Tian E, Yue D, Yang TC, Gu Z, Lu G. T–S fuzzy model-based robust stabilization for networked control systems with probabilistic sensor and actuator failure. IEEE Trans Fuzzy Syst 2011;19(3):553–61.
[3] Peng C, Yang TC. Event-triggered communication and $H_\infty$ control co-design for networked control systems. Automatica 2013;49(5):1326–32.
[4] Lu R, Xu Y, Xue A, Zheng J. Networked control with state reset and quantized measurements: observer-based case. IEEE Trans Ind Electron 2013;60 (11):5206–13.
[5] Qu F-L, Hu B, Guan Z-H, Wu Y-H, He D-X, Zheng D-F. Quantized stabilization of wireless networked control systems with packet losses. ISA Trans 2016;64:92-7.
[6] Sahebsara M, Chen T, Shah SL. Optimal $H_\infty$ filtering in networked control systems with multiple packet dropouts. Syst Control Lett 2008;57(9):696–702.
[7] He X, Wang Z, Zhou D. Robust $H_\infty$ filtering for networked systems with multiple state delays. Int J Control 2007;80(8):1217–32.
[8] Song H, Yu L, Zhang W-A. $H_\infty$ filtering of network-based systems with random delay. Signal Process 2009;89(4):615–22.
[9] Zhang H, Shi Y, Mehr AS. Robust weighted $H_\infty$ filtering for networked systems with intermittent measurements of multiple sensors. Int J Adapt Control Signal Process 2011;25(4):313–30.
[10] Jiang S, Fang H. $H_\infty$ static output feedback control for nonlinear networked control systems with time delays and packet dropouts. ISA Trans 2013;52 (2):215–22.
[11] Meng X, Chen T. Event triggered robust filter design for discrete-time systems. IET Control Theory Appl 2014;8(January (2)):104–13.
[12] Hu S, Yue D. Event-triggered control design of linear networked systems with quantizations. ISA Trans 2012;51(1):153–62.
[13] Yan L, Zhang X, Zhang Z, Yang Y. Distributed state estimation in sensor networks with event-triggered communication. Nonlinear Dyn 2014;76(1):169–81.
[14] Mahmoud MS, Sabih M, Elshafei M. Event-triggered output feedback control for distributed networked systems. ISA Trans 2016;60:294-302.
[15] Wang H, Shi P, Zhang J. Event-triggered fuzzy filtering for a class of nonlinear networked control systems. Signal Process 2015;113:159–68.
[16] Ge X, Han Q-L. Distributed event-triggered $H_\infty$ filtering over sensor networks with communication delays. Inf Sci 2015;291:128–42.
[17] Du D, Qi B, Fei M, Peng C. Multiple event-triggered $H_2/H_\infty$ filtering for hybrid wired-wireless networked systems with random network-induced delays. Inf Sci 2015;325:393–408.
[18] Hu S, Yue D. Event-based $H_\infty$ filtering for networked system with communication delay. Signal Process 2012;92(9):2029–39.
[19] Zhang X-M, Han Q-L. Event-based $H_\infty$ filtering for sampled-data systems. Automatica 2015;51:55–69.
[20] Zhang J, Peng C, Du D, Zheng M. Adaptive event-triggered communication scheme for networked control systems with randomly occurring nonlinearities and uncertainties. Neurocomputing 2016;174:475–82.
[21] Wei G, Wang Z, Shu H. Robust filtering with stochastic nonlinearities and multiple missing measurements. Automatica 2009;45(3):836–41.
[22] Wang Z, Shen B, Liu X. $H_\infty$ filtering with randomly occurring sensor saturations and missing measurements. Automatica 2012;48(3):556–62.
[23] Shen B, Wang Z, Shu H, Wei G. On nonlinear filtering for discrete-time stochastic systems with missing measurements. IEEE Trans Autom Control 2008;53(9):2170–80.
[24] Mao X. Exponential stability of stochastic delay interval systems with markovian switching. IEEE Trans Autom Control 2002;47(10):1604–12.
[25] Tian E, Zhao X. Robust $H_\infty$ control for uncertain networked systems with communication constraints. J Frankl Inst 2013;350:1926–43.
[26] Seuret A, Gouaisbaut F. Wirtinger-based integral inequality: application to time-delay systems. Automatica 2013;49(9):2860–6.
[27] Briat C. Convergence and equivalence results for the Jensen's inequality application to time-delay and sampled-data systems. IEEE Trans Autom Control 2011;56(7):1660–5.

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