Proceedings of the 7th Asian Control Conference,
Hong Kong, China, August 27-29, 2009

Reliable Observer-based Control for Networked Control System with State Time Delay

Zhou Gu , Dong Yue and Daobo Wang

Abstract— **This paper aims to design an innovative observer-based reliable controller for networked control system(NCS). The state and sensor failures are estimated as the controller's input. The gains of Observer and observer-based feedback controller are solved via the linear matrix inequality (LMI) technology. An illustrative example is given to show the reliable output can works well even if the sensor failures are unbounded.**

I. INTRODUCTION

Networked control systems (NCSs), due to their advantages, are applicable to many fields ranging from spacecraft automotive, remote robots, and manufacturing process. However, integration of communication networks into feedback control loops inevitably leads to non-ideal network quality of services(QoS), especially network-induced delay, which is usually the major cause of the deterioration of system dynamic performance and potential system instability. Furthermore, network-induced time delay in NCS is different from conventional time delay systems. Therefore, the problems of stability analysis and controller synthesis for NCSs have received considerable attention[1], [2], [3], [4].

Reliable control is introduced to tolerate the failures of sensor and actuator and maintain the system stability and performance. It is well known that to maintain high reliability for practical control systems against possible failures in systems devices is an essential requirement, which may lead to serious and even disastrous situations[5]. Many efforts have been made to design reliable control[5], [6], [7], [8], [9], [10], [11], [12]. In [6], [7], [8], an algebraic Riccati-equation approach is presented to guarantee closed-loop systems stability in some admissible component failures. However, Riccati-equation approaches have difficulties in finding feasible solutions. [9] deals with the problem of reliable H_∞ fuzzy control for a class of discrete-time non-linear systems with actuator faults by using multiple fuzzy Lyapunov functions. [11] handles the problem of reliable observer-based control for fixed time delay systems. In [12], the robust reliable H_∞ control problem for discrete-time Markovian jump systems with actuator failures is studied.

This work was supported by the Natural Science Foundation of China (60704024, 60774060), and the Key Natural Science Foundation of China (60834002)

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To the best of our knowledge, there is very little work on reliable observer-based control for NCSs with time-varying delay in state, which has motivated the work of this paper. In this paper, a new observer model is constructed to estimate the state of augmented descriptor system. Then, the augmented error systems can be formulated as a general time delay systems. Finally, the reliable controller is designed by utilizing aforementioned reliable observer technique .

The paper is organized as follows. In section 2, An augmented model with state and unexpected sensor fault signals is established. Section 3 the gain of observer is designed by using a method of Lyapunov functions. An observer-based controller are designed by using estimates information of the observer in section 4. Section 5 gives numerical example to demonstrate the effectiveness of the proposed method. Finally, Section 6 concludes the paper.

Notation: $\mathbb{R}^n$ denotes the n-dimensional Euclidean space, $\mathbb{R}^{n\times m}$ is the set of real $n\times m$ matrices, I is the identity matrix of appropriate dimensions, $\|\cdot\|$ stands for the Euclidean vector norm or spectral norm as appropriate. The notation $X>0$ (respectively, $X<0$), for $X\in\mathbb{R}^{n\times n}$ means that the matrix X is a real symmetric positive definite (respectively, negative definite). The asterisk $*$ in a matrix is used to denote term that is induced by symmetry.

II. PROBLEM FORMULATION

Similar to [10], The networked control systems discussed in this paper are depicted in Fig.1. The net only exists between sensors and controllers, which causes the controller to obtain the delayed information from the system.

Assumption 1: The sensor is clock-driven, the data are transmitted with a singer-packet and the full state variables are available for measurements.

Assumption 2: The real input realized through a zero-order holding in is a piecewise constant function.

Assumption 3: To simplify analysis, actuators are assumed to be reliable.

Consider the following state-delayed dynamic system:

$$\begin{aligned}\Sigma:\quad \dot{x}(t) &= Ax(t)+A_d x(t-\tau_1(t))+Bu(t) \qquad (1)\\ & \qquad t\in[i_k h+\tau_{i_k}, i_{k+1}h+\tau_{i_{k+1}})\\ y(t) &= Cx(i_k h)+\omega(t) \qquad (2)\\ x(t) &= \phi(t) \quad t\in[-\tau_1 \quad t_0] \qquad (3)\end{aligned}$$

where $x(t)\in\mathrm{R}^{\mathrm{n}}$ is the state vector, $u(t)\in\mathrm{R}^{\mathrm{m}}$ is the control input vector, $y(t)\in\mathrm{R}^{\mathrm{q}}$ is the measurable output vector over network, $\omega(t)\in\mathrm{R}^{\mathrm{p}}$ is the unexpected sensor fault signal, A,B,C are known const matrices with appropriate

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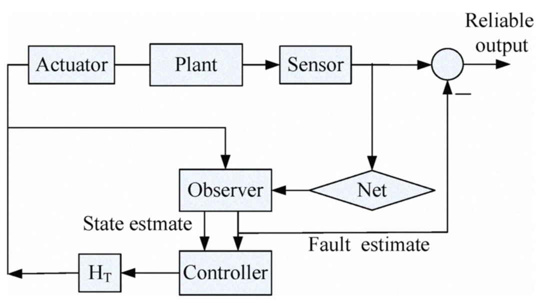

Fig. 1. The block of the NCS

dimensions. h denotes the sampling period. The time-varying delay is a continuous function which satisfies

$$0 \le \tau_1(t) \le \tau_{1M}, \quad \dot{\tau}_1(t) \le \mu \tag{4}$$

Remark 1: In traditional methods, sensor fault model are described by a scaling factor, such as, $\beta_l \in \Omega \triangleq \{\beta_l = diag[\beta_{l_1} \quad \beta_{l_2}, \cdots, \beta_{l_q}] \beta_{l_i} = 0 \; or \; 1, i = 1,2,\cdots,q\}$[13]. In this paper, fault singal $\omega(t)$ can be any forms vector. Therefore it can describe the sensor failure more generally than the traditional methods, and it corresponds to practical situation.

Remark 2: In (1), $i_k(k = 1,2,\cdots)$ are some integers such that $\{i_1, i_2, \cdots\} \subset \{0,1,2,\cdots\}$.$\tau_{i_k}$ is the time from the instant $i_k h$ when sensors sample from the plant to the instant when actuators send control actions to the plant. Obviously, $\bigcup_{k=1}^{\infty}[i_k h + \tau_{i_k}, i_{k+1}h + \tau_{i_{k+1}}) = [t_0, \infty), t \ge 0$. Since $x(i_k h) = x[t-(t-i_k h)]$, define $\tau_2(t) = t - i_k h, t \in \{i_k h + \tau_{i_k}, k = 1,2,\ldots\}$, then (2) becomes

$$y(t) = Cx(t - \tau_2(t)) + \omega(t) \tag{5}$$

As Shown in Fig.1, data networks as the media to interconnect the plant and the controller, therefore, the main purpose is to construct an observer to estimate the original state vector and fault vector, which are introduced to the control feedback loop to compensate sensor signals, then a reliable output can be established.

III. THE DESIGN OF OBSERVER

In this section, we will establish an observer to estimate the original state and unexpected sensor fault signals.

From (1) and (5), we can rewritten those equations as:

$$\tilde{E}\dot{\tilde{x}}(t) = \tilde{A}\tilde{x}(t) + \tilde{A}_d\tilde{x}(t-\tau_1(t)) + \tilde{B}u(t) \tag{6}$$
$$y(t) = \tilde{C}\tilde{x}(t-\tau_2(t)) + \tilde{D}\tilde{x}(t) \tag{7}$$

where,

$$\tilde{x}(t) = \begin{bmatrix} x(t) \\ \omega(t) \end{bmatrix} \quad \tilde{A} = \begin{bmatrix} A & 0 \\ 0 & 0 \end{bmatrix} \quad \tilde{B} = \begin{bmatrix} B \\ 0 \end{bmatrix} \quad \tilde{A}_d = \begin{bmatrix} A_d & 0 \\ 0 & 0 \end{bmatrix}$$
$$\tilde{C} = \begin{bmatrix} C & 0 \end{bmatrix} \quad \tilde{D} = \begin{bmatrix} 0 & I \end{bmatrix} \quad \tilde{E} = \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix} \tag{8}$$

To estimate the state and sensor fault signals, we can construct the observer as follows:

$$\begin{cases} \tilde{S}\dot{\xi}(t) = (\tilde{A} - \tilde{F}\tilde{D})\xi(t) + \tilde{A}_d\xi(t-\tau_1(t)) + \tilde{B}u(t) \\ \hat{\tilde{x}}(t) = \xi(t) + \tilde{S}^{-1}\tilde{L}(y(t) - Cx(t-\tau_2(t))) \end{cases} \tag{9}$$

where $\tilde{S} = \tilde{E} + \tilde{L}\tilde{D}$, Specially we select

$$\tilde{L} = \begin{bmatrix} 0_{n\times p} \\ I_p \end{bmatrix}$$

then $\tilde{S} = \begin{bmatrix} I_n & 0 \\ C & I_p \end{bmatrix}$, and we can also compute the following: $\tilde{D}\tilde{S}^{-1}\tilde{L} = I_p, \tilde{A}\tilde{S}^{-1}\tilde{L} = 0_{(n+p)\times p}, \tilde{A}_d\tilde{S}^{-1}\tilde{L} = 0_{(n+p)\times p}$

According (9), it can be obtained that

$$\begin{aligned} \tilde{S}\dot{\hat{\tilde{x}}}(t) &= (\tilde{A} - \tilde{F}\tilde{D})(\hat{\tilde{x}}(t) - \tilde{S}^{-1}\tilde{L}(y(t) - \tilde{C}\tilde{x}(t-\tau_2(t)))) \\ &+ \tilde{B}u(t) + \tilde{A}_d(\hat{\tilde{x}}(t-\tau_1(t)) - \tilde{S}^{-1}\tilde{L}(y(t-\tau_1(t)) \\ &- \tilde{C}\tilde{x}(t-\tau_1(t)-\tau_2(t)))) + L(\dot{y}(t) - \tilde{C}\dot{x}(t-\tau_2(t))) \\ &= (\tilde{A} - \tilde{F}\tilde{D})\hat{\tilde{x}}(t) + \tilde{F}[y(t) - \tilde{C}\tilde{x}(t-\tau_2(t))] \\ &+ \tilde{A}_d\hat{\tilde{x}}(t-\tau_1(t)) + \tilde{B}u(t) + \tilde{L}[\dot{y}(t) - \tilde{C}\dot{\tilde{x}}(t-\tau_2(t))] \\ &= (\tilde{A} - \tilde{F}\tilde{D})\hat{\tilde{x}}(t) + \tilde{F}\tilde{D}\tilde{x}(t) + \tilde{A}_d\hat{\tilde{x}}(t-\tau_1(t)) \\ &+ \tilde{B}u(t) + \tilde{L}\tilde{D}\dot{\tilde{x}}(t) \end{aligned} \tag{10}$$

Adding $\tilde{L}\tilde{D}\dot{\tilde{x}}(t)$ to both side of (6), have

$$\begin{aligned} \tilde{S}\dot{\tilde{x}}(t) &= (\tilde{A} - \tilde{F}\tilde{D})\tilde{x}(t) + \tilde{F}\tilde{D}\tilde{x}(t) + \tilde{A}_d\tilde{x}(t-\tau_1(t)) \\ &\quad + \tilde{B}u(t) + \tilde{L}\tilde{D}\dot{\tilde{x}}(t) \end{aligned} \tag{11}$$

subtract (11) from (10) and define

$$e(t) = \tilde{x}(t) - \hat{\tilde{x}}(t) \tag{12}$$

We have

$$\tilde{S}\dot{e}(t) = (\tilde{A} - \tilde{F}\tilde{D})e(t) + \tilde{A}_d e(t-\tau_1(t)) \tag{13}$$

It is equivalent that

$$\dot{e}(t) = \tilde{S}^{-1}(\tilde{A} - \tilde{F}\tilde{D})e(t) + \tilde{S}^{-1}\tilde{A}_d e(t-\tau_1(t)) \tag{14}$$

Defining $\bar{A} = \tilde{S}^{-1}(\tilde{A} - \tilde{F}\tilde{D})$, and $\bar{A}_d = \tilde{S}^{-1}\tilde{A}_d$, then (14) can be rewritten as

$$\dot{e}(t) = \bar{A}e(t) + \bar{A}_d e(t-\tau_1(t)) \tag{15}$$

Theorem 1: Given scalars $\tau_{1M} > 0$ and $\mu < 1$, the system (15) is asymptotically stable if there exist symmetric positive-define matrices $P > 0, X = \begin{bmatrix} X_{11} & X_{12} \\ (*) & X_{22} \end{bmatrix} > 0$, and any appropriately dimensioned matrices Y, M, N such that the LMIs (16) and (17) are true.

$$\begin{bmatrix} \hat{\Upsilon}_{11} & \hat{\Upsilon}_{12} & \tau_{1M}\bar{A}^T Z \\ (*) & \hat{\Upsilon}_{22} & \tau_{1M}\bar{A}_d^T Z \\ (*) & (*) & -\tau_{1M}Z \end{bmatrix} < 0 \tag{16}$$
$$\Theta_2 > 0 \tag{17}$$

where,

$$\begin{aligned} \hat{\Upsilon}_{11} &= P\bar{A} + \bar{A}^T P + M + M^T + Q + \tau_{1M}X_{11} \\ \hat{\Upsilon}_{12} &= P\bar{A}_d - M + N^T + \tau_{1M}X_{12} \\ \hat{\Upsilon}_{22} &= -N - N^T - (1-\mu)Q + \tau_{1M}X_{22} \end{aligned}$$

Proof: Construct a Lyapunov function for system (15) as

$$\begin{aligned} V(e_t) &= e^T(t)Pe(t) + \int_{t-\tau_1(t)}^{t} e^T(s)Qe(s)ds \\ &+ \int_{-\tau_{1M}}^{0}\int_{t+\theta}^{t} \dot{e}^T(s)Z\dot{e}(s)dsd\theta \end{aligned} \tag{18}$$

where, $P = P^T > 0, Q = Q^T > 0$, and $Z = Z^T > 0$ are to be determined.

It is noted that, for any semi-defined matrices $X \geq 0$, the following holds

$$\tau_{1M}\zeta^T(t)X\zeta(t) - \int_{t-\tau_1(t)}^{t} \zeta^T(t)X\zeta(t)ds > 0 \tag{19}$$

where $\zeta(t) = [e^T(t) \quad e^T(t-\tau_1(t))]^T$. Then, calculating the derivative of $V(e_t)$ yields

$$\begin{aligned}
\dot{V}(e_t) &= e^T(t)(P\bar{A}+\bar{A}^TP)e(t)+2e^T(t)P\bar{A}_d e(t-\tau_1(t)) \\
&\quad +e^T(t)Qe(t)-(1-\dot{\tau}_1(t))e(t-\tau_1(t))Qe(t-\tau_1(t)) \\
&\quad +\tau_{1M}\dot{e}^T(t)Z\dot{e}(t)-\int_{t-\tau_{1M}}^{t} e^T(s)Z\dot{e}(s)ds \\
&\leq e^T(t)(P\bar{A}+\bar{A}^TP)e(t)+2e^T(t)P\bar{A}_d e(t-\tau_1(t)) \\
&\quad +e^T(t)Qe(t)-(1-\mu)e(t-\tau_1(t))Qe(t-\tau_1(t)) \\
&\quad +\tau_{1M}\dot{e}^T(t)Ze(t)-\int_{t-\tau_1(t)}^{t} e^T(s)Z\dot{e}(s)ds \\
&\quad +2\left[e^T(t)M+e^T(t-\tau_1(t))N\right] \\
&\quad \times\left[e(t)-e(t-\tau_1(t))-\int_{t-\tau_1(t)}^{t}\dot{e}(t)ds\right] \\
&\quad +\tau_{1M}\zeta^T(t)X\zeta(t)-\int_{t-\tau_1(t)}^{t}\zeta^T(t)X\zeta(t)ds \\
&= \zeta^T(t)\Theta_1\zeta(t)-\int_{t-\tau_1(t)}^{t}\zeta^T(t,s)\Theta_2\zeta(t,s)ds
\end{aligned}$$

where

$$\zeta(t,s) = \begin{bmatrix} e^T(t) & e^T(t-\tau_1(t) & \dot{e}^T(s)\end{bmatrix}^T \tag{20}$$

$$\Theta_1 = \begin{bmatrix} \hat{\Upsilon}_{11}+\tau_{1M}\bar{A}^TZ\bar{A} & \hat{\Upsilon}_{12}+\tau_{1M}\bar{A}^TZ\bar{A}_d \\ (*) & \hat{\Upsilon}_{22}+\tau_{1M}A_d^TZA_d \end{bmatrix} \tag{21}$$

$$\Theta_2 = \begin{bmatrix} X_{11} & X_{12} & M \\ (*) & X_{22} & N \\ (*) & (*) & Z \end{bmatrix} \tag{22}$$

$\hat{\Upsilon}_{ij}(i,j=1,2)$ are defined in (16). If are $\Theta_1 < 0$, and $\Theta_2 > 0$, then $\dot{V}(e_t) < 0$. Applying Schur Complement, (16) implies $\Theta_1 < 0$. So the system (15) is asymptotically is LMIs (16) and (17) are hold. This completes the proof. ∎

Next, we will consider the gain of the observer design. By Theorem 1, we have the following theoretical result.

Theorem 2: For given scalars $\tau_{1M} > 0, \rho > 0$ and $\mu < 1$, the system (14) is asymptotically stable if there exist symmetric positive-define matrices $P > 0, X = \begin{bmatrix} X_{11} & X_{12} \\ (*) & X_{22}\end{bmatrix} > 0$, and any appropriately dimensioned matrices Y, M, N such that the LMIs (23) and (24) are true. Furthermore, the gain of observer (9) is given by $\tilde{F} = \tilde{S}P^{-1}Y$.

$$\begin{bmatrix} \Upsilon_{11} & \Upsilon_{12} & \tau_{1M}\rho(\tilde{A}^T\tilde{S}^{-T}P-\tilde{D}^TY^T) \\ (*) & \Upsilon_{22} & \tau_{1M}\rho\tilde{S}^{-1}\tilde{A}_d^TP \\ (*) & (*) & -\tau_{1M}\rho \end{bmatrix} < 0 \tag{23}$$

$$\Theta_2 > 0 \tag{24}$$

$$\begin{aligned}
\Upsilon_{11} &= P\tilde{S}^{-1}\tilde{A}+Y\tilde{D}+\tilde{A}^T\tilde{S}^{-T}P-\tilde{D}^TY^T \\
&\quad +M+M^T+Q+\tau_{1M}X_{11} \\
\Upsilon_{12} &= P\tilde{S}^{-1}\tilde{A}_d-M+N^T+\tau_{1M}X_{12} \\
\Upsilon_{22} &= -N-N^T-(1-\mu)Q+\tau_{1M}X_{22}
\end{aligned}$$

Proof: Substitute $\bar{A} = \tilde{S}^{-1}(\tilde{A}-\tilde{F}\tilde{D})$, and $\bar{A}_d = \tilde{S}^{-1}\tilde{A}_d$ in (16), and define $Z = \rho P, Y = P\tilde{S}^{-1}\tilde{F}$, we have (23) and (24). This complete the proof. ∎

Remark 3: From the definition (8) we have

$$\hat{x}(t) = [I_n \quad 0_{n\times p}]\hat{\bar{x}}(t) \tag{25}$$

$$\hat{\omega}(t) = [0_{p\times n} \quad I_p]\hat{\bar{x}}(t) \tag{26}$$

Therefore, Theorem 2 gives the gain of the observer (9), such that the estimated state and fault signals are closed to original signals.

IV. THE DESIGN OF RELIABLE CONTROLLER AGAINST SENSOR FAILURES

In this section, we will develop an observer-based controller for NCS with unbounded sensor failure, which uses estimated state as feedback information, rather than original measured data over network. By this method, one can escape from considering network-induced time delay.

The observer-based controller is represented by

$$u(t) = \hat{K}\hat{\bar{x}}(t) \tag{27}$$

where $\hat{K} = [K \quad 0_p]$. From definition (8) and (12), we have

$$\begin{aligned}
u(t) &= \hat{K}\hat{\bar{x}}(t) \\
&= [K \quad 0]\bar{x}(t)-\hat{K}e(t) \\
&= Kx(t)-\hat{K}e(t)
\end{aligned} \tag{28}$$

By substituting (28) and (26) into (1) and (2), respectively, the integrated form of the corresponding closed-loop system and reliable output can be formulated as

$$\begin{cases} \dot{x}(t) = (A+BK)x(t)+A_dx(t-\tau_1(t))-B\hat{K}e(t) \\ y_r(t) = y(t)-\hat{\omega}(t) = Cx(t-\tau_2(t))+[0 \quad I_p]e(t) \end{cases} \tag{29}$$

Then , we have the following results:

Theorem 3: For given scalars $0 < \mu < 1$, the system (29) is asymptotically stable, if there exist symmetric matrices $X > 0, Q > 0$, and a matrix Y with appropriate dimensions, such that Theorem 2 and (30) are hold, then system is asymptotically stable. Furthermore, $K = YX^{-1}$.

$$\begin{bmatrix} XA^T+AX+(1-\mu)^{-1}A_dQA_d^T+BY+Y^TB^T & X \\ (*) & -Q \end{bmatrix} < 0 \tag{30}$$

Proof: Defining a Lyapunov function as

$$V_1(x_t) = x^T(t)Px(t)+\int_{t-\tau_1(t)}^{t} x^T(s)Hx(s)ds \tag{31}$$

where $P > 0, T > 0$, Taking the time derivate of $V_1(x_t)$ with respect to t along the trajectory of (29)yields

$$\begin{aligned}
\dot{V}_1(x_t) &= x^T(t)[(A+BK)^TP+P(A+BK)+H]x(t) \\
&\quad +2x^T(t)PA_dx(t-\tau_1(t))-2x^T(t)PBKe(t) \\
&\quad -(1-\dot{\tau}_1(t))x(t-\tau_1(t))^THx(t-\tau_1(t)) \\
&\leq \xi^T(t)\Omega\xi(t)-2x^T(t)PBKe(t)
\end{aligned} \tag{32}$$

where $\xi(t) = \begin{bmatrix} x^T(t) & x^T(t-\tau_1(t)) \end{bmatrix}^T$,

$$\Omega = \begin{bmatrix} (A+BK)^T P + P(A+BK) + H & PA_d \\ (*) & -(1-\mu)H \end{bmatrix} < 0 \quad (33)$$

Redefining a Lyapunov function as

$$V(x_t, e_t) = \varepsilon V(e_t) + V_1(x_t) \quad (34)$$

where $V(e_t)$ is defined as (18), ε satisfies $\varepsilon \geq \frac{\varepsilon}{\sqrt{\lambda_{\Theta_1}\lambda_\Omega}}$, and β are positive number. Define $\eta(t) = [e^T(t) \quad e^T(t-\tau_1(t)) \quad e^T(t-\tau_2(t))]^T$

$$\begin{aligned} \dot{V}(x_t, e_t) &\leq \varepsilon \zeta^T(t)\Theta_1\zeta(t) + \xi^T(t)\Omega\xi(t) \\ &\quad -2x^T(t)PBKe(t-\tau_2(t)) \\ &\quad -\varepsilon \int_{t-\tau_1(t)}^{t} \zeta^T(t,s)\Theta_2\zeta(t,s)ds \\ &\leq -\varepsilon\lambda_{\Theta_1}\|\zeta(t)\|^2 - \lambda_\Omega\|\xi\|^2 + \varepsilon\|\zeta(t)\|\|\xi(t)\| \\ &\leq -\frac{(\varepsilon\lambda_{\Theta_1} + \lambda_\Omega)}{2}(\|\zeta(t)\| + \|\xi(t)\|)^2 < 0 \quad (35) \end{aligned}$$

where, Θ_1 and Θ_2 are defined in (21) and (22), respectively, and $\lambda_{\Theta_1} = \lambda_{min}(-\Theta_1), \lambda_\Omega = \lambda_{min}(-\Omega), \varepsilon = 2\|PBK\|$.

Remark 4: From (35), we can conclude that $x \to 0$ and $e \to 0$ as $t \to \infty$, which means the system can work well even if sensor fault exists.

Applying Schur Complement two times for (33), has

$$\begin{bmatrix} \Xi_{11} & I \\ (*) & -H \end{bmatrix} < 0 \quad (36)$$

where, $\Xi_{11} = A^T P + PA^T + (1-\mu)^{-1} PA_d H^{-1} A_d^T P + PBK + K^T B^T P$. By defining $X := P^{-1}, Y := KX, Q := H^{-1}$, and pre and post multiplying $diag\{X^{-1}, I, I\}$ on both sides of (30), we can know that (30) is equivalently expressed as (36). This completes the proof. ■

V. NUMERICAL EXAMPLES

In this section, an example is used to demonstrate that the method presented in this paper is effective.

Consider the system Σ with the following parameters:

$$A = \begin{bmatrix} -2 & 1 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, B = \begin{bmatrix} 2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0.6 \end{bmatrix}, A_d = \begin{bmatrix} 0.5 & 0 & 0 \\ 0.2 & 0 & 0 \\ 0 & 0 & -5 \end{bmatrix}, C = \begin{bmatrix} -1 & 1 & 0 \end{bmatrix}$$

For explaining the effectiveness of the unexpected sensor signal on the reliable output, we choose $\omega(t)$ as:

$$\omega(t) = \begin{cases} 1.5e^{-2t} & 0 < t \leq 6 \\ e^{0.01t} sin15t & 6 < t \leq 15 \\ 0.5e^{0.05t} & 15 < t \leq 20 \\ 0 & 20 < t \end{cases} \quad (37)$$

In addition, the initial condition is assumed as $x(t) = [-0.1 \quad -e^t \quad 0.2]^T, t \in [-\tau_{1M} \quad 0]$.

By solving the LMIs in Theorem 2 and Theorem 3, utilizing the Matlab LMI Toolbox, we obtain the following gain matrices for the observer and observer-based feedback controller in (9) and (27) :

$$F = \begin{bmatrix} 0 & 0 & 0 & 0.6026 \end{bmatrix}$$

$$K = \begin{bmatrix} 0.0417 & 0.8493 & 0 & 0 \\ 3.0320 & 0.5667 & 0 & 0 \\ 0 & 0 & -73.6111 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

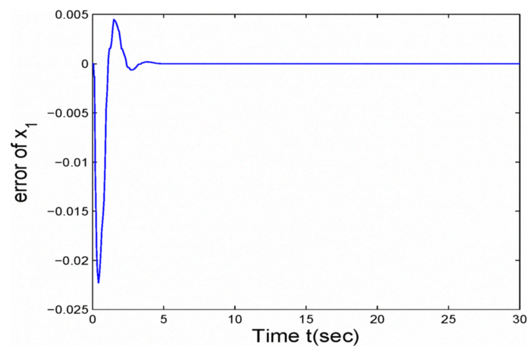

Fig. 2. Error between x_1 and its estimation

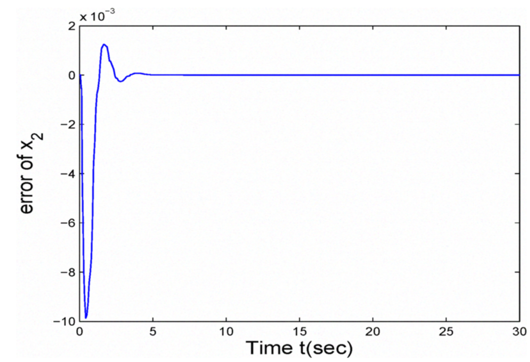

Fig. 3. Error between x_2 and its estimation

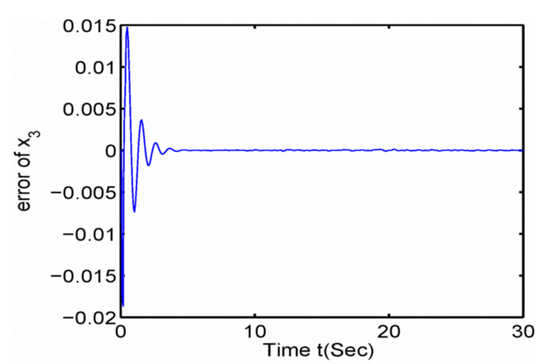

Fig. 4. Error between x_3 and its estimation

In Fig.2-4, we can see there are little difference between the real signals and theirs estimations, it denotes the proposed observer tracks the system well. Fig.6 describes the output

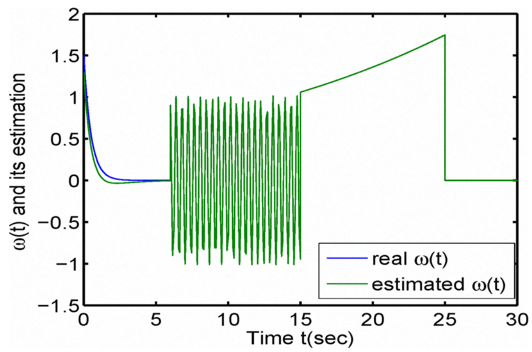

Fig. 5. $\omega(t)$ and its estimation

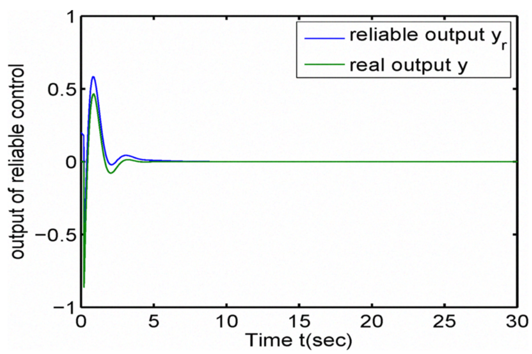

Fig. 6. Output without sensor failures

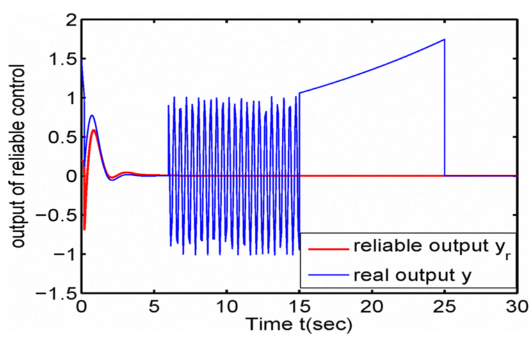

Fig. 7. Real output and reliable output with sensor failures

without sensor fault, and after some faults are introduced into the system, we can get the simulation results as the Fig.7, from which we can see that the real output fails completely with the sensor failure, whereas the reliable output works well. Noting that the sensor failure $\omega(t)$ is set as 4 intervals to demonstrate the result of the reliable output. In interval $t \in [6 \quad 15]$ stands for high-frequency signal, and in $t \in [15 \quad 25]$ represents unbounded sensor failure, after 25s the fault is removed.

VI. CONCLUSION

In this paper, we have investigated an observer-based reliable control for NCS. Attention is focus on the design of a observer, whose estimations are used to design a feedback controller, rather than the state information due to the network-induced time delay in the close-loop system. It has been shown that the gain of observer and feedback controller can be solved in terms of the feasibility of LMIs, A numerical example has been provided to illustrate the effectiveness of the proposed approach even if the sensor failure is unbounded.

VII. ACKNOWLEDGMENT

The authors would like to acknowledge the Natural Science Foundation of China for its support under grant numbers 60704024 , 60774060 and the Key Natural Science Foundation of China (60834002).

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