Event-Based Two-Step Transmission Mechanism for the Stabilization of Networked T–S Fuzzy Systems With Random Uncertainties

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***Abstract*—This article studies an event-based two-step transmission mechanism (TSTM) in the control design for networked T–S fuzzy systems. The transmission task is achieved in two steps. Consecutive triggering packets are relabeled in the first step by applying a traditional event-triggered mechanism (ETM). Then a probabilistic approach is employed to determine which packet is a real release packet (RRP) in the second step. This event-based TSTM is particularly suitable for scenarios in which traditional ETMs are unable to determine which packets are redundant. By discarding most of the unnecessary data packets, especially when the system is tending toward stability, the burden on the network bandwidth is reduced. To establish a control strategy for T–S fuzzy-based nonlinear systems with random uncertainties, a new timing analysis technique is proposed. Additionally, the necessary conditions for a nonlinear system's mean-square asymptotic stability (MSAS) are derived. Finally, two practical applications demonstrate the effectiveness of the suggested transmission mechanism in networked T–S fuzzy systems.**

***Index Terms*—Event-triggered mechanism (ETM), networked T–S fuzzy systems, two-step transmission mechanism (TSTM).**

I. Introduction

NONLINEAR problems have gained significant attention in recent years due to their broad applications in practical systems, with numerous research results being published on the subject [1], [2]. The T–S fuzzy modeling method is widely used to approximate nonlinear control systems owing to its excellent approximation ability [3], [4], [5]. This method employs linear subsystems with fuzzy rules to approximate the entire nonlinear system. In this way, classical linear system theories can be conveniently applied to analyze and synthesize complex nonlinear control systems. For example, Echreshavi et al. [6] used the T–S fuzzy model to solve the integral sliding mode control problem of nonlinear systems with disturbance. T–S fuzzy-based predictive control for a nonlinear system is investigated in [7]. Song et al. [8] investigated an event-triggered finite-time output feedback control for an intertype-2 (IT-2) T–S fuzzy system. The above results indicate that the T–S fuzzy modeling method offers a solid approximation for nonlinear systems, and the corresponding theoretical outcomes demonstrate satisfactory control performance by using the T–S modeling method.

It is known that the usage of communication networks in networked control systems (NCSs) can bring many advantages. Therefore, NCSs have a wide range of applications, such as in unmanned aerial vehicles, smart grids, and process control. Most of these applications are nonlinear, and there are still many challenges that must be overcome, such as the problem of asynchronous premise variables [9], reduced network bandwidth [10], and the occurrence of cyber attacks [34]. Many research results related to NCSs have been published in recent years to address these challenges. For instance, cloud-aided semi-vehicle suspension systems are studied in [11], in which control information is transmitted over the wireless network under cyber attacks, and the event-triggered communication scheme is considered. The interaction between human operators and the space environment is described using IT-2 T–S fuzzy NCSs in [12], wherein a time-varying threshold for the event-triggered mechanism (ETM) is developed to alleviate the burden of network bandwidth. In [13], batched-network-coding-based communication protocols are introduced to find the optimal balance between the number of released data packets, time delay, and reliability. However, due to the limitation in network bandwidth, the number of data packets to be transmitted over the network should be restricted. Therefore, it is important to design an appropriate mechanism for NCSs to determine which data packet is necessary for control systems.

Compared with a traditional time-triggered mechanism, an ETM occupies less network bandwidth. Under an ETM, a data packet can be transmitted over the network only when the

Manuscript received 15 August 2023; revised 30 October 2023; accepted 26 November 2023. Date of publication 22 December 2023; date of current version 17 January 2024. This work was supported in part by the National Natural Science Foundation of China under Grant 62273183, Grant 62022044, and Grant 62103193; in part by the Natural Science Foundation of Jiangsu Province of China under Grant BK20231288; and in part by the National Research Foundation of Korea (NRF) Grant funded by the Korea Government (Ministry of Science and ICT) under Grant NRF-2020R1A2C1005449. This article was recommended by Associate Editor A. Ferreira de Loza. *(Corresponding authors: Zhou Gu; Choon Ki Ahn.)*

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Color versions of one or more figures in this article are available at https://doi.org/10.1109/TCYB.2023.3337797.

Digital Object Identifier 10.1109/TCYB.2023.3337797

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designed event-triggered condition (ETC) is violated, thereby reducing redundant data and conserving network resources. Meanwhile, control performance is ensured at a certain level by using the appropriate stability theories. Generally, two key objectives must be considered when developing ETMs: 1) a reasonable ETC must be constructed to enable sampling packet release at appropriate instants and 2) a balance between the control strategies and the stability of NCSs should be achieved. Based on these objectives, the ETM has been widely studied in NCSs [14], [15], [16], [17]. There are mainly two types of triggering mechanisms: 1) static ETMs and 2) dynamic ETMs. In [18], a static ETM is designed for nonlinear dynamic systems to decrease the packet release rate, conserving network resources. Attaining mean-square exponential stability and static ETM control of a system with stochastic disturbance is discussed in [19]. Although the above results show that the system using static ETMs performs satisfactorily in both reducing the consumption of network resources and maintaining the system control indicators, the threshold, which is extremely sensitive to the data-releasing rate, is a fixed constant in static ETMs. To address this problem, researchers have developed a dynamic ETM that can adjust the threshold to render the ETC more flexible and adapt to the system. For instance, an adaptive ETM is proposed to dynamically adjust the threshold according to the state variation of the system in [20]. In addition, some novel threshold functions have been developed for nonlinear interconnected systems [21] and T–S fuzzy systems [22], by which lower-data-releasing rates were obtained while ensuring the desired system performance. A distributed dynamic ETM is designed to reduce the communication burden for the secondary frequency control of DC microgrids in [36]. In general, both a static ETM and a dynamic ETM can effectively decrease the release rate of data packets, but in some cases, these two mechanisms still cannot effectively solve the problem of limited bandwidth.

To further improve ETMs, other types of ETMs have been developed, such as integral-based ETMs [23], [24], [25], [26], memory-based ETMs [27], [28], [29], [30], adaptive memory-based ETMs [31], and segment-weighted information-based ETMs [32]. These types of ETMs are improvements to static ETMs and dynamic ETMs. Using these ETMs can improve the reliability of the ETM, especially for systems with random perturbations, despite the lower-data-release rate (DRR). Although the use of the above ETMs alleviates the pressure on the network bandwidth (to a certain degree), there is still redundant data transmitted over the network, particularly when the system tends to be stable. This inherent defect of relative ETMs is a challenging issue, which motivates this current study.

Based on the discussion above, this article develops an event-based two-step transmission mechanism (TSTM) for T–S fuzzy systems. The main contributions of this article are as follows.

1) A novel event-based TSTM is established, allowing for a further reduction in the data-releasing rate. With the traditional ETMs, such as in [33] and [34], although certain sampling packets are "selected" by the ETC as releasing packets, we cannot ascertain that these packets are critical for control performance. Thus, a probabilistic method is used for selecting the real release packets (RRPs) from the packets that violate the ETC.
2) A new timing analysis method has been developed, which involves the grouping of a predefined number of triggering packets and the subsequent relabeling and release of the real triggered packet at the final triggering instant within each group. This innovative approach represents a significant advancement from the traditional analysis method.
3) Unlike previous methods, exemplified in [35] and [36], the proposed event-based TSTM effectively resolves challenges arising from the inherent flaws of relative ETMs, which often result in a considerable number of unexpected triggering events as the system approaches stability. To address this issue, traditional methods often yield UUB results, whereas our method yields mean-square asymptotic stability (MSAS) results. Simulation results validate the superiority of the proposed approach.

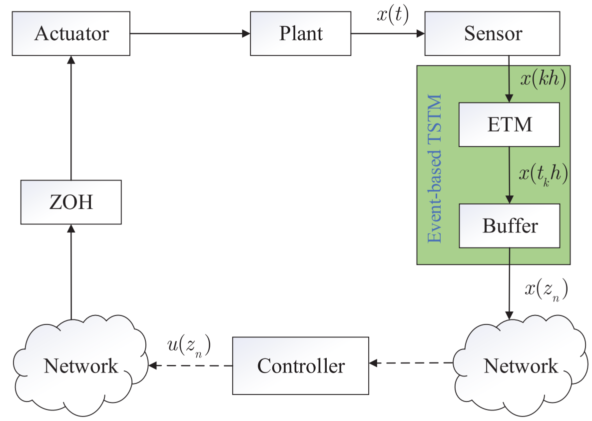

Fig. 1. Structure of NCSs with event-based TSTM.

The remainder of this article is organized as follows: the system model and the event-based TSTM are described in Section II. In Section III, the main results of the T–S fuzzy systems using the event-based TSTM are presented. Two practical applications using the event-based TSTM are presented in Section IV to demonstrate the validity of the theoretical results. Section V concludes this article.

Notation: In this article, let $\text{He}(A) := A + A^T$, $\langle P|f\rangle := f^T Pf$, and $\mathcal{L}^q := \{1, 2, \ldots, q\}$. $\mathcal{C}^l_{s=1}\{X_s\}$ and $\mathcal{D}^l_{s=1}\{Y_s\}(\mathcal{D}^l\{Y\})$ are used to denote the column vector $[X_1^T \ X_2^T \ \cdots \ X_l^T]^T$ and diagonal matrix $\text{diag}\{Y_1 \ Y_2 \ \cdots \ Y_l\}(\text{diag}\{\underbrace{Y \ Y \ \cdots \ Y}_{l}\})$, respectively.

II. System Description

A. System Model

A nonlinear system in [27] that uses the T–S fuzzy modeling method is considered as follows.

System Rule i: If $\varphi_1(t)$ is $\Theta_{i1}, \ldots,$ and $\varphi_p(t)$ is Θ_{ip}, then

$$\dot{x}(t) = (A_i + \Delta A_i(t))x(t) + B_i u(t) \tag{1}$$

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where $i \in \mathcal{L}^q$, $\varphi(t) = [\varphi_1(t), \varphi_2(t), \ldots, \varphi_p(t)]$ is a vector consisting of p premise variables, and $\Theta_{i\epsilon}(t)$ is the ϵth fuzzy set for $i \in \mathcal{L}^q$, $\epsilon \in \mathcal{L}^p$. In (1), the vectors of the states and control inputs are denoted by $x(t) \in \mathbb{R}^n$ and $u(t) \in \mathbb{R}^u$, respectively. $\Delta A_i(t) = \beta(t)\bar{A}_i$ represents random uncertainties with known matrices A_i, B_i, and $\bar{A}_i$ and a random variable $\beta(t)$. The expectation and variance of $\beta(t)$ are denoted by $\mathbb{E}\{\beta(t)\} = \bar{\beta}$ and $\mathrm{Var}\{\beta(t)\} = \hat{\beta}^2$, respectively.

The nonlinear system can be approximated as follows by using the center average defuzzifier removing:

$$\dot{x}(t) = \sum_{i=1}^{q} \alpha_i(\varphi(t))[(A_i + \Delta A_i(t))x(t) + B_i u(t)] \tag{2}$$

and the membership function (MF) can be written as

$$\alpha_i(\varphi(t)) = \frac{\theta_i(\varphi(t))}{\sum_{i=1}^{q} \theta_i(\varphi(t))} \tag{3}$$

where $0 \le \alpha_i(\varphi(t)) \le 1$, $\sum_{i=1}^{q} \alpha_i(\varphi(t)) = 1$, and

$$\theta_i(\varphi(t)) = \prod_{\epsilon=1}^{p} \Theta_{i\epsilon}(\varphi_\epsilon(t)). \tag{4}$$

B. Event-Based TSTM

The ETM is an effective information transmission mechanism that reduces the number of transmission packets. It provides an excellent selection mechanism to exclude unnecessary sampling packets from accessing the network. Nevertheless, in practical applications, packets that meet the releasing condition of the ETM may not contribute to system performance. Moreover, in many cases, we are not sure which one contributes to system performance. To solve this problem, the event-based TSTM shown in Fig. 1 is proposed in this study. Under this communication mechanism, several triggered packets are grouped in a buffer, and one of them is randomly selected to be released over the network, reducing the DRR.

In the traditional ETM, $t_k h$ is denoted as a releasing instant that satisfies

$$t_{k+1}h = \inf\{t > t_k h | \Phi(t) > 0\} \tag{5}$$

where $\Phi(t) = \langle \Omega | e(t) \rangle - \sigma \langle \Omega | x(t_k h) \rangle$; $e(t) = x(t_k h) - x(t_k h + gh)$, $g \in \mathcal{L}^{g_M - 1}$, and h is the sampling period; $\sigma \in [0, 1)$ is a prescribed threshold parameter, and $\Omega > 0$ is a weight matrix to be designed.

Based on [33], we define

$$\eta_m^n(t) = \begin{cases} t - t_{k+m}h, & t \in \mathbb{T}_0 \\ t - t_{k+m}h - gh, & t \in \mathbb{T}_g \\ t - t_{k+m}h - g_M h, & t \in \mathbb{T}_{g_M} \end{cases} \tag{6}$$

$$e_m^n(t) = \begin{cases} 0, & t \in \mathbb{T}_0 \\ x(t_{k+m}h) - x(t_{k+m}h + gh), & t \in \mathbb{T}_g \\ x(t_{k+m}h) - x(t_{k+m}h + g_M h), & t \in \mathbb{T}_{g_M} \end{cases} \tag{7}$$

for $k = nl - n, n \in N, m \in \mathcal{L}^{l-1}$, where $\mathbb{T}_0 = [t_{k+m}h + \eta_{k+m}, t_{k+m}h + h + \bar{\eta})$, $\mathbb{T}_g = [t_{k+m}h + gh + \bar{\eta}, t_{k+m}h + gh + h + \bar{\eta})$, and $\mathbb{T}_{g_M} = [t_{k+m}h + g_M h + \bar{\eta}, t_{k+m+1}h + \eta_{k+m+1})$ with $\bar{\eta} = \max\{\eta_{k+m}\}$.

Next, we develop the event-based TSTM. As shown in Fig. 2, l triggering packets are grouped sequentially, and one of the packets in the group is randomly selected as an RRP. A random variable $\gamma_m \in \{0, 1\}$ that obeys Bernoulli distribution and satisfies $\sum_{m=1}^{l} \gamma_m = 1$ is used to describe the probability of release of the mth triggered packet in the group with $m \in \mathcal{L}^l$. $\gamma_m = 1$ represents the real release data (RRD) at the mth triggering instant. The expectation and variance of γ_m are denoted as $\mathbb{E}\{\gamma_m = 1\} = \bar{\gamma}_m \ge 0$ and $\mathrm{Var}\{\gamma_m = 1\} = \hat{\gamma}_m^2$, respectively.

From the above discussion, the event-based TSTM can be expressed as

$$\langle \Omega | e_m^n(t) \rangle > \sigma \langle \Omega | x(t - \eta_m^n(t)) \rangle \tag{8}$$

for $t \in J_n = [z_n + \iota_n + \tau_n, z_{n+1} + \iota_{n+1} + \tau_{n+1})$ with $z_n = \sum_{m=1}^{l} \gamma_m t_{k+m} h$, $k = (l-1)(n-1)$, $\tau_n = \eta_{nl-n+1}$, and $\iota_n = t_{nl-n+1}h - z_n$, where $n \in N^+$. Here, z_n is the real release instant (RRI), ι_n denotes the length between the last triggered instant and the RRI in a group and is considered the waiting delay, and τ_n denotes the network-induced delay of the last triggered instant in a group.

Suppose

$$\mathrm{Prob}\{z_n = t_{m'}h | \Phi(t) > 0\} = \gamma_{m'} \tag{9}$$

for $m' \in \{(n-1)l - n + 2, (n-1)l - n + 3, \ldots, nl - n + 1\}$, $t \in [t_{(n-1)l-n+2}h, t_{nl-n+1}h]$, $n \in N^+$ and $\sum_{m'=(n-1)l-n+2}^{nl-n+1} \gamma_{m'} = 1$.

Remark 1: To clearly understand the proposed event-based TSTM (8), we take $l = 3$ as an example. The sampling, virtual triggering instants (VTIs), and RRIs are shown in Fig. 2. Based on the above discussion, $z_1 = t_1 h$, $z_2 = t_4 h$, $z_3 = t_7 h$; $\tau_1 = \eta_3$, $\tau_2 = \eta_5$, $\tau_3 = \eta_7$; $\iota_1 = t_l h - z_1$ and $\iota_2 = t_{2l-1}h - z_2$; $J_1 = [z_1 + \iota_1 + \tau_1, z_2 + \iota_2 + \tau_2)$ and $J_2 = [z_2 + \iota_2 + \tau_2, z_3 + \iota_3 + \tau_3)$. Defining $t_0 = z_1 + \iota_1 + \tau_1$, it follows that $\cup_{n=1}^{+\infty} J_n = [t_0, +\infty)$. Therefore, the system can be analyzed in the interval $[t_0, +\infty)$.

Remark 2: In (8), τ_n represents the period from the releasing time to the arrival time of the data packets. The RRI is designed at the last triggering instant of the group. For example, the packets at 5 h, 7 h, and 10 h are grouped as VTIs. As discussed above, only the packet at 7 h is an RRI and is transmitted over the network at 10 h.

Remark 3: If $l = 1$, the proposed event-based TSTM becomes the traditional ETM in [33]. The value of l is selected by experience and the requirement for feasible solutions to the criterion. Generally, the larger the value of l, the fewer packets are transmitted, which could result in system instability. The smaller the value, the more triggered packets, finally degenerating into the general ETM. $l = 3$ is selected in this article because of its representativeness and scalability. If $l = 3$ can render the system stable, it can be successfully extended to a value of more than 3. However, it should be noted that the value of l has an upper bound.

Letting $J_m^n = [t_{k+m}h + \eta_{k+m}, t_{k+m+1}h + \eta_{k+m+1})$, $k = nl - n, n \in N^+, m \in \mathcal{L}^{l-1}$, it follows that:

$$[t_{k+1}h + \eta_{k+1}, t_{k+l}h + \eta_{k+l}) = \sum_{m=1}^{l-1} J_m^n = J_n. \tag{10}$$

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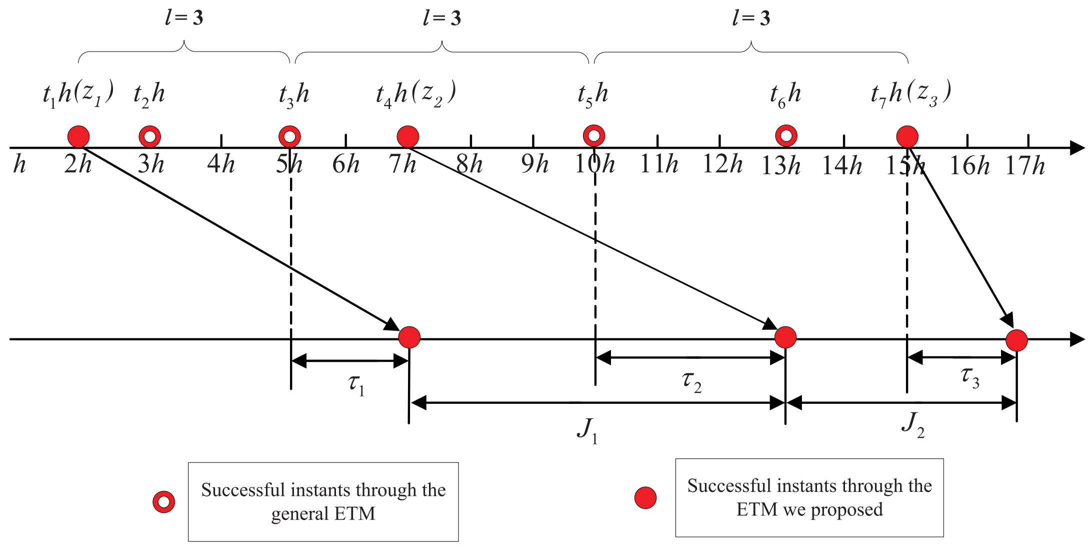

Fig. 2. Example ($l = 3$) of the design of the transmission interval and the sampling, triggering, and releasing instants.

For $t \in J_m^n$, combining (6)–(10) yields

$$x(t_{k+m}h) = x(t - \eta_m^n(t)) + e_m^n(t). \tag{11}$$

Then, the real released state is represented as follows:

$$x(z_n) = \sum_{m=1}^{l-1} \gamma_m \varrho_{1,m}^n(t) + (1 - \sum_{m=1}^{l-1} \gamma_m) \varrho_{2,l}^n(t) \tag{12}$$

where $\varrho_{1,m}^n(t) = x(t - \eta_m^n(t)) + e_m^n(t)$, $\varrho_{2,l}^n(t) = x(t - \eta_l^n(t)) + e_l^n(t)$ for $t \in J_n$.

C. Closed-Loop T–S Fuzzy Control Systems With Event-Based TSTM

Consider the jth fuzzy rule of the state feedback control using the proposed event-based TSTM.

Rules j: If $\varphi_1(z_n)$ is $\Theta_{j1}, \ldots,$ and $\varphi_p(z_n)$ is Θ_{jp}, then

$$u(t) = K_j x(z_n) \tag{13}$$

for $t \in J_n$, where K_j is designed in the following. Applying the above similar method in Section II-A yields

$$u(t) = \sum_{j=1}^{q} \alpha_j(\varphi(z_n)) K_j x(z_n). \tag{14}$$

Similar to [27], the MF is assumed to satisfy

$$\alpha_j(\varphi(z_n)) > \lambda_j \alpha_j(\varphi(t)) \tag{15}$$

for $t \in J_n$, where $\lambda_j > 0$ is a given scalar.

Recalling (12), we can obtain

$$\begin{aligned} u(t) &= \sum_{j=1}^{q} \alpha_j(\varphi(z_n)) K_j x(z_n) \\ &= \sum_{j=1}^{q} \alpha_j(\varphi(z_n)) K_j \left\{ \sum_{m=1}^{l-1} \gamma_m \varrho_{1,m}^n(t) + \left(1 - \sum_{m=1}^{l-1} \gamma_m\right) \varrho_{2,l}^n(t) \right\} \end{aligned} \tag{16}$$

for $t \in J_n$.

Then, from (2) and (16), one can obtain the closed-loop T–S fuzzy system for $t \in J_n$ as

$$\dot{x}(t) = \sum_{i,j=1}^{q} \alpha_i(\varphi(t)) \alpha_j(\varphi(z_n)) \Big[\zeta_{ij}(t) + \phi_{ij}(t) \Big] \tag{17}$$

where

$$\begin{aligned} \zeta_{ij}(t) &= \big(A_i + \bar{\beta} \bar{A}_i\big) x(t) \\ &\quad + \sum_{m=1}^{l-1} \bar{\gamma}_m B_i K_j \varrho_{1,m}^n(t) + \left(1 - \sum_{m=1}^{l-1} \bar{\gamma}_m\right) B_i K_j \varrho_{2,l}^n(t) \\ \phi_{ij}(t) &= \big(\beta(t) - \bar{\beta}\big) \bar{A}_i x(t) \\ &\quad + \sum_{m=1}^{l-1} (\gamma_m - \bar{\gamma}_m) B_i K_j \varrho_{1,m}^n(t) \\ &\quad + \sum_{m=1}^{l-1} (\bar{\gamma}_m - \gamma_m) B_i K_j \varrho_{2,l}^n(t). \end{aligned}$$

The primary goal of this study is to propose the fuzzy control law in (14) and the TSTM in (8) such that the T–S fuzzy system in (17) with the proposed event-based TSTM is MSAS.

III. Main Results

The MSAS of the closed-loop T–S fuzzy system (17) is analyzed in this section. Before this process, denotations and a lemma are given to help us obtain the main results.

For simplicity, $\alpha_i(\varphi(t))$ and $\alpha_j(\varphi(z_n))$ are denoted by α_i^t and $\alpha_j^{z_n}$, respectively. In addition, for $m \in N^l$, $r \in \mathcal{L}^{4l+2}$, we also define

$$\begin{aligned} &\xi_1(t) = \mathcal{C}_{m=1}^{l}\{\eth_m\}, \quad \xi_2(t) = \mathcal{C}_{m=1}^{l}\{\daleth_m^1\} \\ &\xi_3(t) = \mathcal{C}_{m=1}^{l}\{\daleth_m^2\}, \quad \xi_4(t) = \mathcal{C}_{m=1}^{l}\{\hbar_m\} \\ &\xi^T(t) = \big[x^T(t)\ \xi_1^T(t)\ x^T(t - \eta_M)\ \xi_2^T(t)\ \xi_3^T(t)\ \xi_4^T(t)\big] \end{aligned}$$

where $\eth_m = x(t - \eta_m^n(t))$, $\hbar_m = e_m^n(t)$, $\daleth_m^1 = (1/(\eta_m^n(t))) \int_{t-\eta_m^n(t)}^{t} x(t) ds$, $\daleth_m^2 = (1/(\eta_M - \eta_m^n(t)))$

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$\int_{t-\eta_M}^{t-\eta_m^n(t)} x(t)ds$, and $\mathbb{I}_r \triangleq [0_{n\times(r-1)n}\ I_{n\times n}\ 0_{n\times[(4l+2-r)n]}]$ with $\eta_M = h+\bar{\eta}$.

Lemma 1: Given a scalar $\varepsilon \in (0,1)$ and matrices $R \in \mathbb{R}^{n\times n} > 0$ and $F_1, F_2 \in \mathbb{R}^{n\times m}$, then the following inequality holds:

$$\min_{\varepsilon\in(0,1)} \mathbb{F}(\varepsilon, R) \geq \left\langle \begin{bmatrix} R & * \\ H & R \end{bmatrix} \middle| \begin{bmatrix} F_1\xi \\ F_2\xi \end{bmatrix} \right\rangle$$

for the vector $\xi \in \mathbb{R}^m$, where $\mathbb{F}(\varepsilon, R) = (1/\varepsilon)\langle R|F_1\xi\rangle + (1/(1-\varepsilon))\langle R|F_2\xi\rangle$, if there exists a matrix $H \in \mathbb{R}^n$ such that

$$\begin{bmatrix} R & * \\ H & R \end{bmatrix} \geq 0.$$

Theorem 1: For given scalars $\lambda_j, \sigma, \gamma_m, \bar{\beta}$, and l, and matrix K_j, the system (17) with the event-based TSTM in (8) is mean-square asymptotically stable provided that there exist symmetric matrices $P > 0$, $Q > 0$, $R_s > 0$, $\Omega > 0$, and Σ_i, and matrices H_{s1}, H_{s2}, and H_{s3}, such that

$$\Gamma_{ij} - \Sigma_i < 0 \tag{18}$$

$$\Upsilon_{ii} < 0 \tag{19}$$

$$\Upsilon_{ij} + \Upsilon_{ji} < 0, \quad (i<j) \tag{20}$$

$$\begin{bmatrix} \hat{R}_s & * \\ H_s & \hat{R}_s \end{bmatrix} \geq 0 \tag{21}$$

for $s, m \in \mathcal{L}^l$ and $i, j \in \mathcal{L}^q$, where

$$\begin{aligned}\Gamma_{ij} = \sum_{s=1}^{l} \Big[&\mathbf{He}\big(\mathbb{I}_\delta^T \mathscr{A}^T P \mathbb{I}_1\big) + \langle Q|\mathbb{I}_1\rangle - \langle Q|\mathbb{I}_{l+2}\rangle \\ &+ \big\langle R_s^{-1} | \eta_M R_s \mathscr{A}\mathbb{I}_\delta \big\rangle + \big\langle R_s^{-1} | \eta_M \hat{\beta} R_s \bar{A}_i \mathbb{I}_1 \big\rangle \\ &+ \sum_{m=1}^{l-1} \big\langle R_s^{-1} | \eta_M \hat{\gamma}_m R_s B_i K_j \mathbb{I}_\wp^m \big\rangle \\ &- \left\langle \begin{bmatrix} \hat{R}_s & * \\ H_s & \hat{R}_s \end{bmatrix} \middle| \begin{bmatrix} \mathscr{B}_s \\ \mathscr{C}_s \end{bmatrix} \right\rangle - \langle \Omega | \mathbb{I}_{3l+2+s} \rangle \\ &+ \sigma \langle \Omega | (\mathbb{I}_{s+1} + \mathbb{I}_{3l+2+s}) \rangle \Big]\end{aligned}$$

$$\Upsilon_{ij} = \lambda_j(\Gamma_{ij} - \Sigma_i) + \Sigma_i.$$

Proof: Select the following L–K function:

$$V(t) = \sum_{i=1}^{3} V_i(t) \tag{22}$$

where $V_1(t) = \langle P|x(t)\rangle$, $V_2(t) = \int_{t-\eta_M}^{t} \langle Q|x(s)\rangle ds$, and $V_3(t) = \eta_M \sum_{s=1}^{l} \int_{t-\eta_M}^{t} \int_{w}^{t} \langle R_s|\dot{x}(v)\rangle dv dw$.

For $\beta(t)$ and γ_m, we have

$$\mathbb{E}\{\beta(t) - \bar{\beta}\} = 0, \quad \mathbb{E}\{\gamma_m - \bar{\gamma}_m\} = 0. \tag{23}$$

Along with the trajectories of (17) and taking the expectation of $dV(t)/dt$ yields

$$\mathbb{E}\{dV_1(t)/dt\} = \mathbb{E}\big\{\mathbf{He}\big(\dot{x}^T(t)Px(t)\big)\big\}$$

$$\mathbb{E}\{dV_2(t)/dt\} = \mathbb{E}\{\langle Q|x(t)\rangle - \langle Q|x(t-\eta_M)\rangle\}$$

$$\mathbb{E}\{dV_3(t)/dt\} = \sum_{s=1}^{l} \mathbb{E}\left\{\eta_M^2 \langle R_s|\dot{x}(t)\rangle - \eta_M \int_{t-\eta_M}^{t} \langle R_s|\dot{x}(w)\rangle dw\right\}.$$

From (17) and (23), we can deduce that

$$\mathbb{E}\{dV_1(t)/dt\} = \mathbb{E}\left\{\sum_{i,j=1}^{q} \alpha_i^t \alpha_j^{z_n} \big\langle \mathbf{He}\big(\mathbb{I}_\delta^T \mathscr{A}^T P \mathbb{I}_1\big) | \xi(t) \big\rangle \right\} \tag{24}$$

where

$$\mathbb{I}_\delta^T = \Big[\mathbb{I}_1^T\ \mathbb{I}_{2-(l+1)}^T\ \mathbb{I}_{l+2}^T\ \mathbb{I}_{(l+3)-(2l+2)}^T\ \mathbb{I}_{(2l+3)-(3l+2)}^T\ \mathbb{I}_{(3l+3)-(4l+2)}^T\Big]$$

$$\mathbb{I}_{2-(l+1)}^T = \big[\mathbb{I}_2^T\ \mathbb{I}_3^T\ \cdots\ \mathbb{I}_{l+1}^T\big]$$

$$\mathbb{I}_{(l+3)-(2l+2)}^T = \big[\mathbb{I}_{l+3}^T\ \mathbb{I}_{l+4}^T\ \cdots\ \mathbb{I}_{2l+2}^T\big]$$

$$\mathbb{I}_{(2l+3)-(3l+2)}^T = \big[\mathbb{I}_{2l+3}^T\ \mathbb{I}_{2l+4}^T\ \cdots\ \mathbb{I}_{3l+2}^T\big]$$

$$\mathbb{I}_{(3l+3)-(4l+2)}^T = \big[\mathbb{I}_{3l+3}^T\ \mathbb{I}_{3l+4}^T\ \cdots\ \mathbb{I}_{4l+2}^T\big]$$

$$\mathscr{A} = \big[A_i + \bar{\beta}\bar{A}_i\ \iota_l\ 0\ \jmath_l\ \jmath_l\ \iota_l\big], \quad \jmath_l = [0\ 0\ \cdots\ 0]_{n\times ln}$$

$$\iota_l = \left[\bar{\gamma}_1 B_i K_j\ \bar{\gamma}_2 B_i K_j\ \cdots\ \left(1 - \sum_{m=1}^{l-1}\bar{\gamma}_m\right) B_i K_j\right]_{n\times ln}$$

$$\begin{aligned}\mathbb{E}\{dV_2(t)/dt\} &= \mathbb{E}\{\langle Q|x(t)\rangle - \langle Q|x(t-\eta_M)\rangle\} \\ &= \mathbb{E}\left\{\sum_{i,j=1}^{q} \alpha_i^t \alpha_j^{z_n} \big\langle \big(\langle Q|\mathbb{I}_1\rangle - \langle Q|\mathbb{I}_{l+2}\rangle\big) | \xi(t) \big\rangle \right\}.\end{aligned} \tag{25}$$

In $\mathbb{E}\{dV_3(t)/dt\}$

$$\begin{aligned}&\sum_{s=1}^{l} \mathbb{E}\big\{\eta_M^2 \langle R_s|\dot{x}(t)\rangle\big\} \\ &\leq \sum_{s=1}^{l}\sum_{i,j=1}^{q} \alpha_i^t \alpha_j^{z_n} \eta_M^2 \mathbb{E}\big\{\big\langle R_s | (\zeta_{ij}(t) + \phi_{ij}(t)) \big\rangle\big\} \\ &= \sum_{s=1}^{l}\sum_{i,j=1}^{q} \alpha_i^t \alpha_j^{z_n} \eta_M^2 \mathbb{E}\big\{\big\langle R_s|\zeta_{ij}(t)\big\rangle + \big\langle R_s|\phi_{ij}(t)\big\rangle + \mathbf{He}(\phi_{ij}^T(t) R_s \zeta_{ij}(t))\big\} \\ &= \sum_{s=1}^{l}\sum_{i,j=1}^{q} \alpha_i^t \alpha_j^{z_n} \eta_M^2 \mathbb{E}\big\{\big\langle R_s|\zeta_{ij}(t)\big\rangle + \big\langle R_s|\phi_{ij}(t)\big\rangle\big\}.\end{aligned} \tag{26}$$

Notice that

$$\mathbb{E}\big\{\eta_M^2 \big\langle R_s|\zeta_{ij}(t)\big\rangle\big\} = \mathbb{E}\big\{\big\langle R_s^{-1} | \eta_M R_s \mathscr{A}\mathbb{I}_\delta \xi(t) \big\rangle\big\} \tag{27}$$

and

$$\begin{aligned}\mathbb{E}\{\eta_M^2 \langle R_s|\phi_{ij}(t)\rangle\} = \mathbb{E}\Big\{&\big\langle R_s^{-1} | \eta_M \hat{\beta} R_s \bar{A}_i \mathbb{I}_1 \xi(t) \big\rangle \\ &+ \sum_{m=1}^{l-1} \big\langle R_s^{-1} | \eta_M \hat{\gamma}_m R_s B_i K_j \mathbb{I}_\wp^m \xi(t) \big\rangle \Big\}\end{aligned} \tag{28}$$

where $\mathbb{I}_\wp^m = \mathbb{I}_{m+1} + \mathbb{I}_{3l+2+m} - \mathbb{I}_{l+1} - \mathbb{I}_{4l+2}$.

The integral item in $\mathbb{E}\{dV_3(t)/dt\}$ can be derived by

$$\begin{aligned}&-\eta_M \int_{t-\eta_M}^{t} \langle R_s|\dot{x}(w)\rangle dw \\ &\quad = -\eta_M \int_{t-\eta_s^n(t)}^{t} \langle R_s|\dot{x}(w)\rangle dw - \eta_M \int_{t-\eta_M}^{t-\eta_s^n(t)} \langle R_s|\dot{x}(w)\rangle dw.\end{aligned} \tag{29}$$

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Using the Wirtinger-based inequality yields

$$-\eta_M \int_{t-\eta_s^n(t)}^{t} \langle R_s|\dot{x}(w)\rangle dw \le -\frac{1}{\frac{\eta_s^n(t)}{\eta_M}}\left\langle \hat{R}_s\middle|\mathscr{B}_s\xi(t)\right\rangle \quad (30)$$

where

$$\hat{R}_s = \begin{bmatrix} R_s & 0 \\ 0 & 3R_s \end{bmatrix}, \quad \mathscr{B}_s = \begin{bmatrix} \mathbb{I}_1 - \mathbb{I}_{s+1} \\ \mathbb{I}_1 + \mathbb{I}_{s+1} - 2\mathbb{I}_{l+2+s} \end{bmatrix}.$$

Similarly, we can obtain

$$-\eta_M \int_{t-\eta_M}^{t-\eta_s^n(t)} \langle R_s|\dot{x}(w)\rangle dw \le -\frac{1}{1-\frac{\eta_s^n(t)}{\eta_M}}\left\langle \hat{R}_s\middle|\mathscr{C}_s\xi(t)\right\rangle \quad (31)$$

where

$$\mathscr{C}_s = \begin{bmatrix} \mathbb{I}_{s+1} - \mathbb{I}_{l+2} \\ \mathbb{I}_{s+1} + \mathbb{I}_{l+2} - 2\mathbb{I}_{2l+2+s} \end{bmatrix}.$$

Combining (30) and (31) and using Lemma 1 yields

$$-\frac{1}{\frac{\eta_s^n(t)}{\eta_M}}\left\langle \hat{R}_s\middle|\mathscr{B}_s\xi(t)\right\rangle - \frac{1}{1-\frac{\eta_s^n(t)}{\eta_M}}\left\langle \hat{R}_s\middle|\mathscr{C}_s\xi(t)\right\rangle \le -\left\langle \begin{bmatrix} \hat{R}_s & * \\ H_s & \hat{R}_s \end{bmatrix}\middle|\begin{bmatrix} \mathscr{B}_s\xi(t) \\ \mathscr{C}_s\xi(t) \end{bmatrix}\right\rangle \quad (32)$$

where

$$H_s = \begin{bmatrix} H_{s1} & * \\ H_{s2} & H_{s3} \end{bmatrix}.$$

Recalling (8), one has

$$\sum_{s=1}^{l} \langle (\sigma\langle \Omega|(\mathbb{I}_{s+1} + \mathbb{I}_{3l+2+s})\rangle - \langle \Omega|\mathbb{I}_{3l+2+s}\rangle)|\xi(t)\rangle > 0. \quad (33)$$

Combining (24)–(33), we have

$$\begin{aligned}
&\mathbb{E}\{\dot{V}(t)\} \\
&\le \mathbb{E}\Bigg\{ \sum_{i,j=1}^{q} \alpha_i^t \alpha_j^{z_n} \xi^T(t) \sum_{s=1}^{l} \Big(\mathbf{He}(\mathbb{I}_\delta^T \mathscr{A}^T P \mathbb{I}_1) + \langle Q|\mathbb{I}_1\rangle \\
&\quad - \langle Q|\mathbb{I}_{l+2}\rangle + \left\langle R_s^{-1}|\eta_M R_s \mathscr{A}\mathbb{I}_\delta\right\rangle + \left\langle R_s^{-1}|\eta_M \hat{\beta} R_s \bar{A}_i \mathbb{I}_1\right\rangle \\
&\quad + \sum_{m=1}^{l-1} \left\langle R_s^{-1}|\eta_M \hat{\gamma}_m R_s B_i K_j \mathbb{I}_{\wp}^m\right\rangle - \left\langle \begin{bmatrix} \hat{R}_s & * \\ H_s & \hat{R}_s \end{bmatrix}\middle|\begin{bmatrix} \mathscr{B}_s \\ \mathscr{C}_s \end{bmatrix}\right\rangle \\
&\quad - \langle \Omega|\mathbb{I}_{3l+2+s}\rangle + \sigma\langle \Omega|(\mathbb{I}_{s+1} + \mathbb{I}_{3l+2+s})\rangle \Big)\xi(t) \Bigg\} \\
&= \mathbb{E}\left\{ \sum_{i,j=1}^{q} \alpha_i^t \alpha_j^{z_n} \langle \Gamma_{ij}|\xi(t)\rangle \right\}. \quad (34)
\end{aligned}$$

Since the MF has the properties of $\sum_{i=1}^{q} \alpha_i^t = \sum_{j=1}^{q} \alpha_j^{z_n} = 1$, it follows that:

$$\mathbb{E}\left\{ \sum_{i=1}^{q} \alpha_i^t \left(\sum_{j=1}^{q} \alpha_j^t - \sum_{j=1}^{q} \alpha_j^{z_n} \right) \langle \Sigma_i|\xi(t)\rangle \right\} = 0. \quad (35)$$

Then, we have

$$\begin{aligned}
&\mathbb{E}\left\{ \sum_{i,j=1}^{q} \alpha_i^t \alpha_j^{z_n} \langle \Gamma_{ij}|\xi(t)\rangle \right\} \\
&= \mathbb{E}\left\{ \sum_{i,j=1}^{q} \alpha_i^t \alpha_j^{z_n} \langle \Gamma_{ij}|\xi(t)\rangle + \sum_{i,j=1}^{q} \alpha_i^t (\alpha_j^t - \alpha_j^{z_n}) \langle \Sigma_i|\xi(t)\rangle \right\} \\
&= \mathbb{E}\left\{ \sum_{i,j=1}^{q} \alpha_i^t \alpha_j^{z_n} \langle (\Gamma_{ij} - \Sigma_i)|\xi(t)\rangle + \sum_{i,j=1}^{q} \alpha_i^t \alpha_j^t \langle \Sigma_i|\xi(t)\rangle \right\}. \quad (36)
\end{aligned}$$

Combining (15), (18), and (36), one has

$$\begin{aligned}
\mathbb{E}\{\dot{V}(t)\} &= \mathbb{E}\left\{ \sum_{i,j=1}^{q} \alpha_i^t \alpha_j^{z_n} \langle \Gamma_{ij}|\xi(t)\rangle \right\} \\
&\le \mathbb{E}\left\{ \sum_{i,j=1}^{q} \alpha_i^t \alpha_j^t \langle \lambda_j(\Gamma_{ij} - \Sigma_i)|\xi(t)\rangle + \sum_{i,j=1}^{q} \alpha_i^t \alpha_j^t \langle \Sigma_i|\xi(t)\rangle \right\} \\
&\le \mathbb{E}\Bigg\{ \sum_{i,j=1}^{q} \alpha_i^t \alpha_j^t \big\langle \big(\lambda_i(\Gamma_{ii} - \Sigma_i) + \Sigma_i\big)|\xi(t)\big\rangle \\
&\quad + \sum_{i=1}^{q} \sum_{i<j} \alpha_i^t \alpha_j^t \big\langle (\Upsilon_{ij} + \Upsilon_{ji})|\xi(t)\big\rangle \Bigg\} \quad (37)
\end{aligned}$$

where $\Upsilon_{ij} = \lambda_j(\Gamma_{ij} - \Sigma_i) + \Sigma_i$.

According to (18)–(21), we can deduce that $\mathbb{E}\{\dot{V}(t)\} < 0$ for $t \in J_n$. Then, one can conclude that the system (17) is mean-square asymptotically stable. This proof is completed. ■

Sufficient conditions for the system's MSAS (17) are obtained in Theorem 1. Based on the above results, we design the controller gains K_j and the weight matrix of event-based TSTM Ω.

Theorem 2: For given scalars λ_j, σ, $\bar{\beta}$, γ_m, l, and ρ, the system (17) with the event-based TSTM (8) is mean-square asymptotically stable if there exist symmetric matrices $X > 0$, $\tilde{Q} > 0$, $\tilde{R}_s > 0$, $\tilde{\Omega} > 0$, and $\tilde{\Sigma}_i$, and matrices $\tilde{H}_{s1}$, $\tilde{H}_{s2}$, and $\tilde{H}_{s3}$ such that

$$\begin{bmatrix} \tilde{\Gamma}_{ij}^1 - \tilde{\Sigma}_i & * \\ \tilde{\Psi}_{ij} & \tilde{\Pi} \end{bmatrix} < 0 \quad (38)$$

$$\begin{bmatrix} \tilde{\Upsilon}_{ii} & * \\ \lambda_i \tilde{\Psi}_{ii} & \lambda_i \tilde{\Pi} \end{bmatrix} < 0 \quad (39)$$

$$\begin{bmatrix} \tilde{\Upsilon}_{ij} + \tilde{\Upsilon}_{ji} & * & * \\ \lambda_j \tilde{\Psi}_{ij} & \lambda_j \tilde{\Pi} & * \\ \lambda_i \tilde{\Psi}_{ji} & 0 & \lambda_i \tilde{\Pi} \end{bmatrix} < 0, \quad (i < j) \quad (40)$$

$$\begin{bmatrix} \tilde{\hat{R}}_s & * \\ \tilde{H}_s & \tilde{\hat{R}}_s \end{bmatrix} \ge 0 \quad (41)$$

for $s, m \in \mathcal{L}^l$ and $i, j \in \mathcal{L}^q$, where

$$\begin{aligned}
\tilde{\Gamma}_{ij}^1 = \sum_{s=1}^{l} \Bigg\{ &\mathbf{He}(\mathbb{I}_\delta^T \tilde{\mathscr{A}}^T \mathbb{I}_1) + \left\langle \tilde{Q}\middle|\mathbb{I}_1\right\rangle - \left\langle \tilde{Q}\middle|\mathbb{I}_{l+2}\right\rangle \\
&- \left\langle \tilde{\Omega}\middle|\mathbb{I}_{3l+2+s}\right\rangle + \sigma\left\langle \tilde{\Omega}\middle|(\mathbb{I}_{s+1} + \mathbb{I}_{3l+2+s})\right\rangle \\
&- \left\langle \begin{bmatrix} \tilde{\hat{R}}_s & * \\ \tilde{H}_s & \tilde{\hat{R}}_s \end{bmatrix}\middle|\begin{bmatrix} \mathscr{B}_s \\ \mathscr{C}_s \end{bmatrix}\right\rangle \Bigg\}
\end{aligned}$$

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$$\tilde{\mathscr{V}}_{ij}^{2s} = \begin{bmatrix} \eta_M \tilde{\mathscr{A}} \mathbb{I}_\delta \\ \eta_M \hat{\beta} \bar{A}_i X \mathbb{I}_1 \end{bmatrix}, \quad \tilde{\mathscr{V}}_{ij}^{3s} = \begin{bmatrix} \eta_M \hat{\gamma}_1 B_i Y_j \mathbb{I}_\wp^1 \\ \eta_M \hat{\gamma}_2 B_i Y_j \mathbb{I}_\wp^2 \\ \vdots \\ \eta_M \hat{\gamma}_{l-1} B_i Y_j \mathbb{I}_\wp^{l-1} \end{bmatrix}$$

$$\tilde{\mathscr{V}}_{ij}^2 = \mathcal{C}_{s=1}^l \left\{ \tilde{\mathscr{V}}_{ij}^{2s} \right\}, \quad \tilde{\mathscr{V}}_{ij}^3 = \mathcal{C}_{s=1}^l \left\{ \tilde{\mathscr{V}}_{ij}^{3s} \right\}, \quad \tilde{\Psi}_{ij} = \mathcal{C}_{w=2}^3 \left\{ \tilde{\mathscr{V}}_{ij}^w \right\}$$

$$\tilde{\mathscr{R}}_1^s = \mathcal{D}^2 \left\{ \rho^2 \tilde{R}_s - 2\rho X \right\}, \quad \tilde{\mathscr{R}}_2^s = \mathcal{D}^{l-1} \left\{ \rho^2 \tilde{R}_s - 2\rho X \right\}$$

$$\tilde{\mathscr{R}}_1 = \mathcal{D}_{s=1}^l \left\{ \tilde{\mathscr{R}}_1^s \right\}, \quad \tilde{\mathscr{R}}_2 = \mathcal{D}_{s=1}^l \left\{ \tilde{\mathscr{R}}_2^s \right\}, \quad \tilde{\Pi} = \mathcal{D}_{r=1}^2 \left\{ \tilde{\mathscr{R}}_r \right\}$$

$$\tilde{\mathscr{A}} = [A_i X + \bar{\beta} \bar{A}_i X \ \tilde{\imath}_l \ 0 \ \jmath_l \ \jmath_l \ \tilde{\imath}_l]$$

$$\tilde{\imath}_l = \left[\bar{\gamma}_1 B_i Y_j \ \bar{\gamma}_2 B_i Y_j \ \cdots \ \left(1 - \sum_{m=1}^{l-1} \bar{\gamma}_m \right) B_i Y_j \right]$$

$$\jmath_l = [0 \ 0 \ \cdots \ 0], \quad \tilde{\Upsilon}_{ij} = \lambda_j \left(\tilde{\Gamma}_{ij}^1 - \tilde{\Sigma}_i \right) + \tilde{\Sigma}_i.$$

Proof: Define $X = P^{-1}$, $\tilde{Q} = \langle Q|X \rangle$, $\tilde{\Omega} = \langle \Omega|X \rangle$, $\tilde{R}_s = \langle R_s|X \rangle$, $\tilde{H}_s = \langle H_s|\mathcal{D}^2\{X\} \rangle$, and $\Im = \text{diag}\{X, \mathscr{G}_1^{-1}, \mathscr{G}_2^{-1}\}$, where $\mathscr{G}_1^{-1} = \mathcal{D}_{s=1}^l \{\mathcal{D}^2\{R_s^{-1}\}\}$ and $\mathscr{G}_2^{-1} = \mathcal{D}_{s=1}^l \{\mathcal{D}^{l-1}\{R_s^{-1}\}\}$.

Using the Schur complement for (18) in Theorem 1, one can obtain

$$\begin{bmatrix} \Gamma_{ij}^1 - \Sigma_i & * \\ \Psi_{ij} & \Pi \end{bmatrix} < 0 \tag{42}$$

where

$$\begin{aligned} \Gamma_{ij}^1 = \sum_{s=1}^l \Bigg\{ & \mathbf{He}\left(\mathbb{I}_\delta^T \mathscr{A}^T P \mathbb{I}_1 \right) + \langle Q|\mathbb{I}_1 \rangle - \langle Q|\mathbb{I}_{l+2} \rangle \\ & - \langle \Omega|\mathbb{I}_{3l+2+s} \rangle + \sigma \langle \Omega|(\mathbb{I}_{s+1} + \mathbb{I}_{3l+2+s}) \rangle \\ & - \left\langle \begin{bmatrix} \hat{R}_s & * \\ H_s & \hat{R}_s \end{bmatrix} \middle| \begin{bmatrix} \mathscr{B}_s \\ \mathscr{C}_s \end{bmatrix} \right\rangle \Bigg\} \end{aligned}$$

$$\mathscr{V}_{ij}^{2s} = \begin{bmatrix} \eta_M R_s \mathscr{A} \mathbb{I}_\delta \\ \eta_M \hat{\beta} R_s \bar{A}_i \mathbb{I}_1 \end{bmatrix}, \quad \mathscr{V}_{ij}^{3s} = \begin{bmatrix} \eta_M \hat{\gamma}_1 R_s B_i K_j \mathbb{I}_\wp^1 \\ \eta_M \hat{\gamma}_2 R_s B_i K_j \mathbb{I}_\wp^2 \\ \vdots \\ \eta_M \hat{\gamma}_{l-1} R_s B_i K_j \mathbb{I}_\wp^{l-1} \end{bmatrix}$$

$$\mathscr{V}_{ij}^2 = \mathcal{C}_{s=1}^l \left\{ \mathscr{V}_{ij}^{2s} \right\}, \quad \mathscr{V}_{ij}^3 = \mathcal{C}_{s=1}^l \left\{ \mathscr{V}_{ij}^{3s} \right\}, \quad \Psi_{ij} = \mathcal{C}_{w=2}^3 \left\{ \mathscr{V}_{ij}^w \right\}$$

$$\mathscr{R}_1^s = \mathcal{D}^2\{-R_s\}, \quad \mathscr{R}_2^s = \mathcal{D}^{l-1}\{-R_s\}, \quad \mathscr{R}_1 = \mathcal{D}_{s=1}^l \left\{ \mathscr{R}_1^s \right\}$$

$$\mathscr{R}_2 = \mathcal{D}_{s=1}^l \left\{ \mathscr{R}_2^s \right\}, \quad \Pi = \mathcal{D}_{r=1}^2 \{ \mathscr{R}_r \}.$$

Defining $Y_j = K_j X$ and premultiplying and postmultiplying (42) with $\Im$ generates (38). Considering the property of $-X\tilde{R}_s^{-1}X \leq \rho^2 \tilde{R}_s - 2\rho X$, we can obtain that (38)–(41) hold by using the same method. ■

If one sets $l = 1$, the event-based TSTM is then reduced to the traditional ETM, such as the ETM in [33], as discussed in Remark 3.

Corollary 1: Given $\bar{\beta}$, λ_j, σ, and ρ, the system (17) is mean-square asymptotically stable with the event-based TSTM (8) if there exist matrices $X > 0$, $\tilde{Q} > 0$, $\tilde{R} > 0$, $\tilde{\Omega} > 0$, $\tilde{H}_1$, $\tilde{H}_2$, and $\tilde{H}_3$ and symmetric matrices $\tilde{\Sigma}_i$ such that

$$\begin{bmatrix} \tilde{\Gamma}_{ij}^1 - \tilde{\Sigma}_i & * \\ \tilde{\mathscr{V}}_{ij}^2 & \tilde{\mathscr{R}}_1 \end{bmatrix} < 0$$

$$\begin{bmatrix} \tilde{\Upsilon}_{ii} & * \\ \lambda_i \tilde{\mathscr{V}}_{ii}^2 & \lambda_i \tilde{\mathscr{R}}_1 \end{bmatrix} < 0$$

$$\begin{bmatrix} \tilde{\Upsilon}_{ij} + \tilde{\Upsilon}_{ji} & * & * \\ \lambda_j \tilde{\mathscr{V}}_{ij}^2 & \lambda_j \tilde{\mathscr{R}}_1 & * \\ \lambda_i \tilde{\mathscr{V}}_{ji}^2 & 0 & \lambda_i \tilde{\mathscr{R}}_1 \end{bmatrix} < 0, \quad (i < j)$$

$$\begin{bmatrix} \tilde{\hat{R}} & * \\ \tilde{H} & \tilde{\hat{R}} \end{bmatrix} \geq 0$$

for $i, j \in \mathcal{L}^q$, where

$$\begin{aligned} \tilde{\Gamma}_{ij}^1 = & \mathbf{He}\left(\mathbb{I}_1^T \tilde{\mathscr{A}} \right) + \left\langle \tilde{Q}|\mathbb{I}_1 \right\rangle - \left\langle \tilde{Q}|\mathbb{I}_3 \right\rangle - \left\langle \tilde{\Omega}|\mathbb{I}_6 \right\rangle \\ & + \sigma \left\langle \tilde{\Omega}|(\mathbb{I}_2 + \mathbb{I}_6) \right\rangle \\ & - \left\langle \begin{bmatrix} \tilde{\hat{R}} & * \\ \tilde{H} & \tilde{\hat{R}} \end{bmatrix} \middle| \begin{bmatrix} \mathscr{B} \\ \mathscr{C} \end{bmatrix} \right\rangle \end{aligned}$$

$$\mathbb{I}_r = [0_{n \times (r-1)n} \ I_{n \times n} \ 0_{n \times [(6-r)n]}],$$

$$\mathscr{B} = \begin{bmatrix} \mathbb{I}_2 - \mathbb{I}_3 \\ \mathbb{I}_2 + \mathbb{I}_3 - 2\mathbb{I}_5 \end{bmatrix}, \quad \mathscr{C} = \begin{bmatrix} \mathbb{I}_1 - \mathbb{I}_2 \\ \mathbb{I}_1 + \mathbb{I}_2 - 2\mathbb{I}_4 \end{bmatrix}$$

$$\tilde{H} = \begin{bmatrix} \tilde{H}_1 & * \\ \tilde{H}_2 & \tilde{H}_3 \end{bmatrix}, \quad \tilde{\mathscr{V}}_{ij}^2 = \begin{bmatrix} \eta_M \tilde{\mathscr{A}} \\ \eta_M \hat{\beta} \bar{A}_i X \mathbb{I}_1 \end{bmatrix}$$

$$\tilde{\mathscr{R}}_1 = \begin{bmatrix} \rho^2 \tilde{R} - 2\rho X & * \\ 0 & \rho^2 \tilde{R} - 2\rho X \end{bmatrix}$$

$$\tilde{\mathscr{A}} = [A_i X + \bar{\beta} \bar{A}_i X \ B_i Y_j \ 0 \ 0 \ 0 \ B_i Y_j]$$

$$\tilde{\Upsilon}_{ij} = \lambda_j (\tilde{\Gamma}_{ij}^1 - \tilde{\Sigma}_i) + \tilde{\Sigma}_i.$$

Proof: The proof is similar to the one for Theorems 1 and 2 and is completed here. ■

IV. Application Example

Example 1: The following nonlinear mass-spring-damper system [37] is considered an example to show the efficacy of the outcomes obtained:

$$m\ddot{y} + f(y, \dot{y}) + z_1(y) = z_2(y, \dot{y})u \tag{43}$$

where m is the mass, y is the displacement, u is the force, and $f(y, \dot{y})$ and $z_2(y, \dot{y})$ are the nonlinear terms of the damper and force, respectively. $z_1(y)$ is the uncertain term concerning spring.

Choose $m = 1$ and define $f(y, \dot{y}) = -0.75\dot{y}$, $z_1(y) = 0.67y^3 - 0.05y^2$, $z_2(y) = 1 - 0.1y^2$, and $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} y \\ \dot{y} \end{bmatrix}$. We have

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -0.67x_1^3 + 0.05x_1 + 0.75x_2 + (1 - 0.1x_1^2)u. \end{cases} \tag{44}$$

Similar to the method proposed in [38], the above nonlinear system can be approximated as a T–S fuzzy system in the form of (1). The system matrices are

$$A_1 = \begin{bmatrix} 0 & 1 \\ 0.05 & 0.75 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 0 & 1 \\ -1.4575 & 0.75 \end{bmatrix}$$

$$B_1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0 \\ 0.775 \end{bmatrix}.$$

For the uncertain matrices $\Delta A_i(t)$, we select $\bar{\beta} = 0.2$ and

$$\bar{A}_1 = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}, \quad \bar{A}_2 = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}.$$

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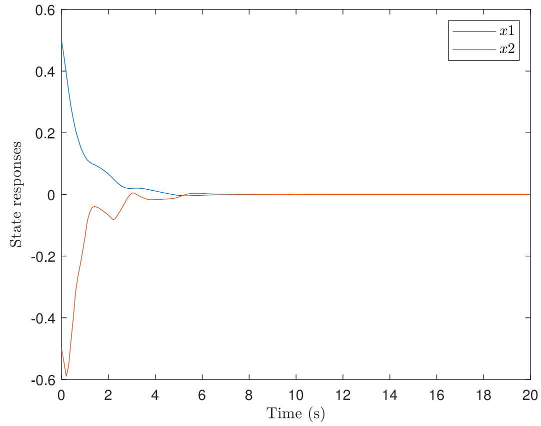

Fig. 3. State responses of the system (43) under the event-based TSTM.

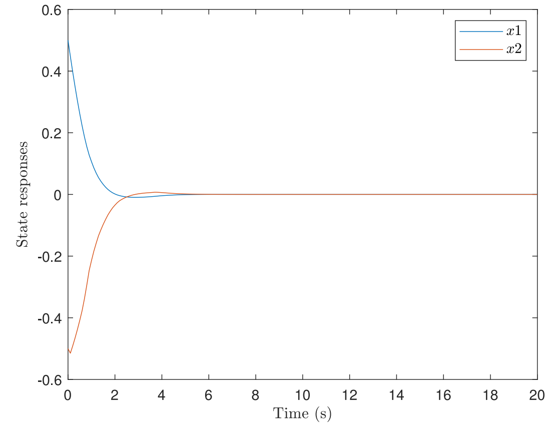

Fig. 5. State responses of the system (43) under the traditional ETM.

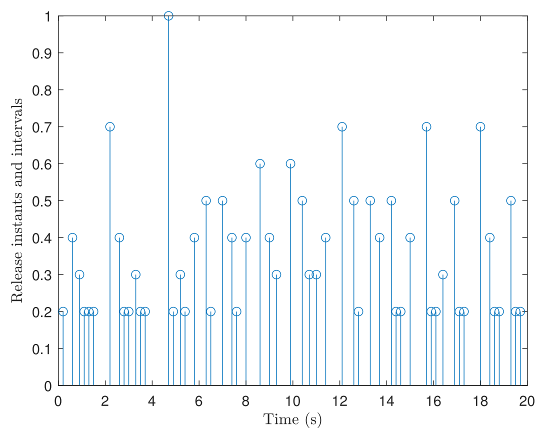

Fig. 4. RIIs of the system (43) under the event-based TSTM.

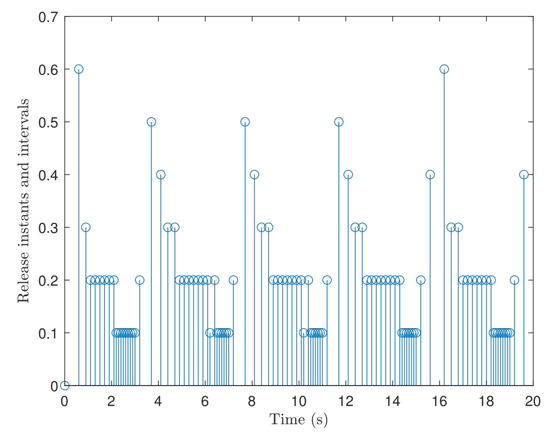

Fig. 6. RIIs under the traditional ETM.

In addition, the normalized MFs are $\alpha_2^t = x_1^2(t)/2.25$ and $\alpha_1^t = 1 - \alpha_2^t$. Choose $h = 0.1$, $\rho = 0.4$, $\sigma = 0.1$, $\lambda_1 = 0.8$, $\lambda_2 = 0.9$, $l = 3$, $\bar{\gamma}_m = 1/3$, and $\eta_M = 0.17$.

By applying Theorem 2, the T–S fuzzy controller gain and weight matrix under the event-based TSTM can be calculated as follows:

$$\begin{aligned} K_1 &= [-1.4120 \;\; -2.8198] \\ K_2 &= [-0.2640 \;\; -3.2884] \\ \Omega &= \begin{bmatrix} 0.0220 & -0.0281 \\ -0.0281 & 0.2311 \end{bmatrix}. \end{aligned} \tag{45}$$

We can draw the state trajectories and releasing intervals with the initial state $x(0) = [0.5 \;\; -0.5]^T$ using the controller in (14) and the event-based TSTM in (8). Fig. 3 depicts the system's state trajectories (43), demonstrating that the fuzzy controller in (14) can guarantee the system's MSAS with the proposed event-based TSTM. The releasing instants and intervals (RIIs) of the T–S fuzzy systems under the event-based TSTM are plotted in Fig. 4, which shows that many redundant data points are discarded, reducing network bandwidth load.

To more clearly show the benefits of the suggested event-based TSTM, we choose $l = 1$, and then the communication mechanism becomes the traditional ETM as discussed in Remark 3. The controller gains and the weight matrix can be obtained by applying Corollary 1

$$\begin{aligned} K_1 &= [-1.4220 \;\; -2.6018] \\ K_2 &= [-0.3580 \;\; -3.1647] \\ \Omega &= \begin{bmatrix} 0.0165 & -0.0204 \\ -0.0204 & 0.1896 \end{bmatrix}. \end{aligned} \tag{46}$$

The state responses and RIIs of the system (43) with the parameters in (46) under the traditional ETM and controller are presented in Figs. 5 and 6, respectively. From Figs. 3–6, it can be observed that the system using the designed event-based TSTM shows a good control performance with fewer triggered signals than that using the traditional ETM. Furthermore, by comparing Fig. 4 with Fig. 6, it can be observed that the phenomenon of successive triggering data is alleviated, greatly reducing the number of unnecessary triggered packets and conserving the network bandwidth while the stability of the system is guaranteed under the event-based TSTM. Additionally, the control performance of the system (44) under the event-based TSTM is similar to the system using the traditional ETM, even if there are fewer than half of the triggering packets.

Denote the numbers of RRP and DRR as $\mathscr{T}$ and $\mathscr{P}$, respectively. For a sampling period $h = 0.1$, the number of sampling packets is 200 during [0, 20] s. Table I shows that the numbers of RRPs for the traditional ETM and the event-based TSTM

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TABLE I
NUMBERS OF RRP AND DRR IN [0, 20] S

	$\mathscr{T}$	$\mathscr{P}$
The method in [33]	101	50.5%
Our proposed method	55	27.5%

are 101 and 55, respectively, and the DRRs are 50.5% and 27.5%, respectively. By combining Fig. 3 and Table I, one can observe that the proposed event-based TSTM in (8) can greatly reduce the number of RRPs and decrease the DRR while ensuring the stability of the system.

Example 2: Consider a truck-trailer system [39]

$$\begin{cases} \dot{x}_1(t) = -\frac{v\bar{t}}{Lt_0}x_1(t) + \frac{v\bar{t}}{lt_0}u(t) \\ \dot{x}_2(t) = \frac{v\bar{t}}{Lt_0}x_1(t) \\ \dot{x}_3(t) = \frac{v\bar{t}}{t_0}\sin\left(x_2(t) + \frac{v\bar{t}}{2L}x_1(t)\right) \end{cases} \tag{47}$$

where $x_1(t)$, $x_2(t)$, and $x_3(t)$ are the angular difference between the truck and trailer, the angle of the trailer, and the vertical position of the rear of the trailer, respectively. By employing the T–S fuzzy modeling method, we can approximate (47) as follows:

$$x(t) = \sum_{i=1}^{2} \alpha_i(\theta(t))\left((A_i + \Delta A_i(t))x(t) + B_i u(t)\right) \tag{48}$$

where

$$A_1 = \begin{bmatrix} -\frac{v\bar{t}}{Lt_0} & 0 & 0 \\ \frac{v\bar{t}}{Lt_0} & 0 & 0 \\ \frac{v^2\bar{t}^2}{2Lt_0} & \frac{v\bar{t}}{t_0} & 0 \end{bmatrix}, \quad A_2 = \begin{bmatrix} -\frac{v\bar{t}}{Lt_0} & 0 & 0 \\ \frac{v\bar{t}}{Lt_0} & 0 & 0 \\ \frac{dv^2\bar{t}^2}{2Lt_0} & \frac{dv\bar{t}}{t_0} & 0 \end{bmatrix}$$

$$B_1 = \begin{bmatrix} \frac{v\bar{t}}{lt_0} \\ 0 \\ 0 \end{bmatrix}, \quad B_2 = \begin{bmatrix} \frac{v\bar{t}}{lt_0} \\ 0 \\ 0 \end{bmatrix}$$

with $t_0 = 0.5$, $d = 5/\pi$, $L = 5.5$, $\bar{t} = 2$, $l = 2.8$, and $v = -1$, and the MFs are given as

$$\alpha_1(\theta(t)) = \left(1 - \frac{1}{1 + e^{-3(\theta(t)-0.5\pi)}}\right)\left(1 + \frac{1}{1 + e^{-3(\theta(t)-0.5\pi)}}\right)$$

$$\alpha_2(\theta(t)) = 1 - \alpha_1(\theta(t)), \quad \theta(t) = x_2(t) - \frac{2}{11}x_1(t).$$

For the uncertain matrices $\Delta A_i(t)$, we select $\bar{\beta} = 0.2$ and

$$\bar{A}_1 = \begin{bmatrix} 0.5 & 0 & 0 \\ 0 & 0.5 & 0 \\ 0 & 0 & 0.5 \end{bmatrix}, \quad \bar{A}_2 = \begin{bmatrix} 0.5 & 0 & 0 \\ 0 & 0.5 & 0 \\ 0 & 0 & 0.5 \end{bmatrix}.$$

Applying Theorem 2 with the given parameters: $h = 0.1$, $\rho = 0.53$, $\sigma = 0.1$, $\lambda_1 = 0.8$, $\lambda_2 = 0.9$, $l = 3$, $\bar{\gamma}_m = 1/3$, $m = 1, 2, 3$, and $\eta_M = 0.17$, we can obtain the control gains and the weight matrix as follows:

$$K_1 = \begin{bmatrix} 2.2684 & -2.1581 & 0.1891 \end{bmatrix}$$

$$K_2 = \begin{bmatrix} 2.3059 & -2.2791 & 0.2013 \end{bmatrix}$$

$$\Omega = 10^{-4} \times \begin{bmatrix} 8.4505 & 0.8904 & -1.3062 \\ 0.8904 & 0.3774 & 0.6606 \\ -1.3062 & 0.6606 & 5.2640 \end{bmatrix}. \tag{49}$$

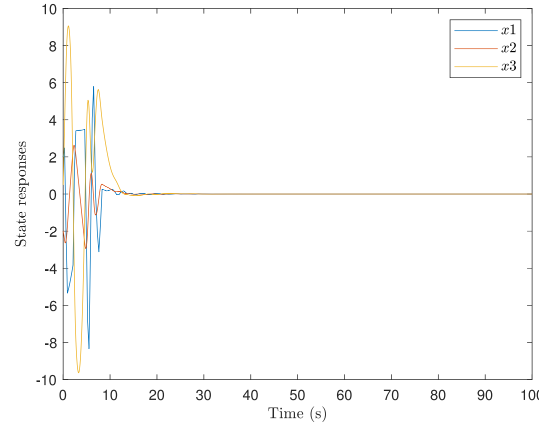

Fig. 7. State responses of the system (48) under the event-based TSTM.

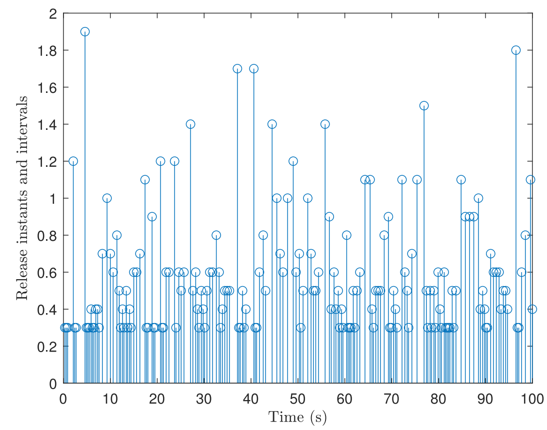

Fig. 8. RIIs of the system (48) under the event-based TSTM.

Under the initial state $x(0) = [2 \quad -2 \ 0.5]^T$ and the parameters in (49), the state trajectories and releasing intervals of the system are depicted in Figs. 7 and 8, respectively. From Fig. 7, it is evident that the proposed method ensures asymptotic stability in the mean square for the system (48). Furthermore, a count from Fig. 8 reveals that 173 data packets are released within 100 s, indicating that only 17.3% of the data packets are sufficient to guarantee the MSAS of the system.

V. CONCLUSION

The event-based TSTM for T–S fuzzy nonlinear NCSs has been studied in this article. This communication mechanism involves two steps. In the first step, the traditional ETM is applied. In the second step, l consecutively triggered packets are grouped, and only one packet in this group is released randomly over the communication channel, thus reducing the number of unnecessary triggering packets, especially when systems tend to be stable. The validity of the event-based TSTM is demonstrated through examples of the mass-spring-damper system and the truck-trailer system. Future research

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will focus on the extension of these results to nonlinear systems under cyber attacks and the development of optimal algorithms to determine the appropriate parameters in complex systems.

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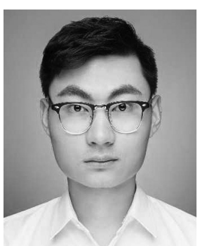

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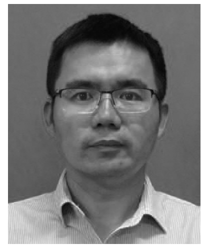

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