Short Papers

Weighted Memory H_∞ Stabilization of Time-Varying Delayed Takagi–Sugeno Fuzzy Systems

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***Abstract*—This note proposes a novel weighted memory H_∞ controller for time-varying delayed Takagi–Sugeno fuzzy systems via a distributed delay method. In contrast to the traditional memory controller based on instantaneous historical data, the continuous historical information over a given period is utilized to construct the fuzzy-based memory controller, in which a weighting function is employed for the first time to describe the importance of historical information. Then, the weighted historical information is expressed as a distributed delay and the weights are represented as delay kernel. By using a less conservative integral inequality with respect to the weighted historical information, original sufficient conditions are deduced to solve the fuzzy weighted memory H_∞ controller gains. Finally, simulations are carried out to reveal the merits of the presented controller.**

***Index Terms*—Delayed fuzzy systems, H_∞ stabilization, weighted memory control.**

I. Introduction

During last decades, Takagi–Sugeno (T-S) fuzzy method is recognized as an efficient manner to treat various practical nonlinear systems [1], [2], [3], [4], [5], [6]. By formulating the complicated nonlinearities of practical plants through a set of linear subsystems, the connection between the linear system and nonlinear systems will be established. By means of T-S fuzzy model, various technologies developed for linear system are able to analyze systems with nonlinearities, such as wind turbine systems, autonomous vehicles, and bioeconomic system[7], [8], [9], [10].

For the control problem of systems, the control performance of the system with a memory-based control strategy is generally better than the one with a memoryless control strategy, especially for time-delay systems. In recent years, the memory controllers dependent on the constant or time-varying delay have been addressed for nonlinear time-delay systems [11], [12], [13] and linear time-delay systems [14], [15]. However, most of the abovementioned memory controllers only use the instantaneously past system information. Different from this memory-based controller, an adaptive distributed-delay-dependent memory controller including varying gains is constructed for linear systems with faulty actuator [16]. In [17], a distributed-delay-dependent memory-based controller is designed for Markov jump systems, where the continuous past information of system over a time interval is utilized. It is noted that the same weight is utilized to describe the historical states in the memory-based controller in [16] and [17]. In fact, the importance of states at different instants is different. Usually, the earlier the historical state is, the less significant it is, which means smaller weight it has. Therefore, it is natural and more practical to design the memory-based controller by considering a weighting function (WF) for different historical information. To the best of authors' knowledge, no attention has been paid to investigate the fuzzy weighted memory control issue for time-varying delayed T-S fuzzy systems.

This note contributes to develop a novel fuzzy weighted memory control strategy for T-S fuzzy systems under time-varying delay. The novelties are highlighted as follows.

1) A new fuzzy weighted memory controller consisting of the continuous past states and a WF is presented. The controller is formulated by a distributed delay containing a delay kernel to represent the WF. Compared with the conventional memory controller without continuous historical information and WF, the weighted past states are utilized and then it is potential to improve control performance.
2) Some novel sufficient conditions for designing the fuzzy weighted memory controller gains are provided by using an integral inequality dependent on the WF. Contrasted to Legendre polynomials-based method in [18] to treat the distributed delay with kernel, the presented strategy gets rid of the error term induced by Legendre polynomials.

The rest of this article is organized as follows. Section II gives the system description and control problem formulation. The stability analysis and controller design conditions are produced in Section III. Simulation results provided in Section IV illustrate the validity of the proposed approach. Finally, Section V concludes the article.

Notation: In the work, $\otimes$ stands for Kronecker product. $He(X)$ represents $X + X^\top$ and $X^\top$ denotes the transpose of X. $\mathcal{L}_2[0, \infty)$ refers to the space of square-integrable vector functions over $[0, \infty)$.

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II. System Description and Problem Formulation

Consider the next time-varying delayed T-S fuzzy system.

Rule i: **IF** $w_1(t)$ is $\mathscr{G}_1^i, \dots$ and $w_\varphi(t)$ is $\mathscr{G}_\varphi^i$, **THEN**

$$\begin{cases}\dot{x}(t) = A_i x(t) + C_i x(t-\chi(t)) + B_i u(t) + D_i f(t)\\ z(t) = Lx(t)\end{cases} \tag{1}$$

in which $w_1(t), \dots, w_\varphi(t)$ represent premise variables, $\mathscr{G}_\varphi^i (i = 1, \dots, l)$ means fuzzy set, $x(t)$ and $x(t-\chi(t)) \in \mathbb{R}^n$ mean the state and delayed state, respectively, $\chi(t)$ stands for time-varying delay with upper bound χ, $u(t) \in \mathbb{R}^g$ denotes control input, $f(t) \in \mathbb{R}^{n_f}$ is external disturbance that belongs to $\mathcal{L}_2[0, \infty)$, $z(t) \in \mathbb{R}^{n_z}$ means performance output, L is the control performance output matrix, and the matrices A_i, B_i, C_i, D_i, and L are with constant parameters.

The following overall fuzzy model can be deduced:

$$\begin{cases}\dot{x}(t) = \sum_{i=1}^{l} \kappa_i(w(t)) \left[A_i x(t) + C_i x(t-\chi(t)) \right.\\ \qquad \left. + B_i u(t) + D_i f(t)\right]\\ z(t) = Lx(t)\end{cases} \tag{2}$$

where $\kappa_i(w(t))$ represents the normalized membership function (MF)

$$\kappa_i(w(t)) = \frac{\nu_i(w(t))}{\sum_{i=1}^{\varphi} \nu_i(w(t))},\ \nu_i(w(t)) = \prod_{a=1}^{\varphi} \mathscr{G}_a^i(w_a(t))$$

$$\kappa_i(w(t)) \geq 0,\ \sum_{i=1}^{l} \kappa_i(w(t)) = 1$$

$\mathscr{G}_a^i(w_a(t))$ means the grade of membership of $w_a(t)$ in $\mathscr{G}_a^i$.

Following the abovementioned way, the fuzzy weighted memory controller is:

$$u(t) = \sum_{j=1}^{l} \kappa_j(w(t)) \left(K_j x(t) + F_j \int_{-\chi}^{0} r(s) x(t+s) ds\right) \tag{3}$$

in which K_j and F_j are controller gains to be solved, and $r(s)$ is the WF satisfying $\int_{-\chi}^{0} r(s) ds = 1$. For simplifying the following presentation, $\kappa_i(w(t))$ and $\kappa_j(w(t))$ are abbreviated as κ_i and κ_j, respectively.

Combining the controller (3) and system (2) results in the next closed-loop fuzzy system

$$\begin{cases}\dot{x}(t) = \sum_{i=1}^{l} \sum_{j=1}^{l} \kappa_i \kappa_j \Big[(A_i + B_i K_j) x(t) + D_i f(t)\\ \qquad + C_i x(t-\chi(t)) + B_i F_j \int_{-\chi}^{0} r(s) x(t+s) ds\Big]\\ z(t) = Lx(t).\end{cases} \tag{4}$$

By defining

$$\mathbf{r}(s) = \begin{bmatrix} r(s) & r_1(s) & \cdots & r_q(s)\end{bmatrix}^\top \tag{5}$$

$$R(s) = \mathbf{r}(s) \otimes I_n,\ \mathcal{I} = \begin{bmatrix} I_n & 0_{n \times qn}\end{bmatrix} \tag{6}$$

$$\mathscr{R}(t) = \int_{-\chi}^{0} R(s) x(t+s) ds \tag{7}$$

where $\mathbf{r}(s)$ is with the property $\dot{\mathbf{r}}(s) = \mathcal{R}\mathbf{r}(s)$ and $\mathcal{R} \in \mathbb{R}^{n(q+1)\times n(q+1)}$.

Remark 1: The WF $r(s)$ is introduced to describe the importance of the historical data. In fact, the earlier data are less important. Then, $r(s)$ is usually chosen as an increasing function for $s \in [t-\chi, t]$. In addition, the selected $r(s), r_1(s), \dots, r_q(s)$ need to make the vector $\mathbf{r}(s)$ defined in (5) satisfy the property $\dot{\mathbf{r}}(s) = \mathcal{R}\mathbf{r}(s)$. This means the elements $r(s), r_1(s), \dots, r_q(s)$ are the solutions of linear homogeneous differential equations with constant coefficient matrix $\mathcal{R}$.

In terms of the definitions in (5)–(7), the fuzzy system (4) can be reexpressed as follows:

$$\begin{cases}\dot{x}(t) = \sum_{i=1}^{l} \sum_{j=1}^{l} \kappa_i \kappa_j \left[(A_i + B_i K_j) x(t) + D_i f(t)\right.\\ \qquad \left. + C_i x(t-\chi(t)) + B_i F_j \mathcal{I}\mathscr{R}(t)\right]\\ z(t) = Lx(t)\end{cases} \tag{8}$$

where $\int_{-\chi}^{0} r(s) x(t+s) ds = \mathcal{I}\mathscr{R}(t)$.

Remark 2: The memory controller based on delay $\chi(t)$ in [11] can be formulated as

$$u(t) = \sum_{j=1}^{l} \kappa_j \left[K_j x(t) + F_j x(t-\chi(t))\right] \tag{9}$$

for fuzzy case. Based on this controller, it yields

$$\begin{cases}\dot{x}(t) = \sum_{i=1}^{l} \sum_{j=1}^{l} \kappa_i \kappa_j \left[(A_i + B_i K_j) x(t) + D_i f(t)\right.\\ \qquad \left. + (C_i + B_i F_j) x(t-\chi(t))\right]\\ z(t) = Lx(t).\end{cases} \tag{10}$$

For the conventional memory controller in [11], it is supposed that the exact value of $\chi(t)$ is known in real time. However, in some practical systems, it is difficult or costly to detect the time delay online, which reduces the feasibility of its practical application. Without measuring the time-varying delay $\chi(t)$ in real time, our weighted memory controller (3) only depends on the delay upper bound χ. Therefore, it is more feasible and practical than the time-varying delay dependent memory controller.

Remark 3: If we choose the matrix $F_j = 0$, then the proposed fuzzy weighted memory controller reduces to

$$u(t) = \sum_{j=1}^{l} \kappa_j K_j x(t) \tag{11}$$

which equals to the traditional memoryless controller in [2]. Then, we have

$$\begin{cases}\dot{x}(t) = \sum_{i=1}^{l} \sum_{j=1}^{l} \kappa_i \kappa_j \left[(A_i + B_i K_j) x(t)\right.\\ \qquad \left. + C_i x(t-\chi(t)) + D_i f(t)\right]\\ z(t) = Lx(t).\end{cases} \tag{12}$$

For further proceeding, the next lemma is provided.

Lemma 1 ([19]): For $\Theta > 0 \in \mathbb{R}^{n\times n}$, $\Theta = \Theta^\top$ and $\mathbf{r}(s)$ defined in (5), it leads to

$$\int_{-\chi}^{0} x^\top(t+s) \Theta x(t+s) ds$$

$$\geq \left(\int_{-\chi}^{0} R(s) x(t+s) ds\right)^\top (\mathfrak{R} \otimes \Theta) \int_{-\chi}^{0} R(s) x(t+s) ds \tag{13}$$

with $\mathfrak{R}^{-1} = \int_{-\chi}^{0} \mathbf{r}(s)\mathbf{r}^\top(s) ds > 0$.

III. Main Theorems

In this section, Theorem 1 is obtained first to analyze the stability of systems for given controller gains. Then, in terms of the stability conditions in Theorem 1, the corresponding controller design conditions are derived in Theorem 2.

Theorem 1: For given scalars χ, ρ_1, and ρ_2, the H_∞ stability with prescribed γ of system (8) under the fuzzy weighted memory controller (3) with given gains K_j and F_j is guaranteed, if there exist matrices M, $Q > 0$, $\Theta > 0$, $\mathcal{J} = \begin{bmatrix} J & S\\ S^\top & J\end{bmatrix} > 0$, $J > 0$, and matrix H such that

$$\mathscr{M} > 0 \tag{14}$$

$$\Psi_{ii} < 0,\ i = j \tag{15}$$

$$\Psi_{ij} + \Psi_{ji} < 0,\ i < j \tag{16}$$

where

$$\mathscr{M} = M + diag\{0, \mathfrak{R} \otimes Q\},\ \mathfrak{R}^{-1} = \int_{-\chi}^{-0} \mathbf{r}(s)\mathbf{r}^\top(s)ds$$

$$\begin{aligned}\Psi_{ij} = &He(\Gamma_0^\top M\Gamma_1) + He(\mathbf{HS}_{ij})\\ &+ \chi^2\epsilon_1^\top J\epsilon_1 - \epsilon_{\mathcal{J}}^\top \mathcal{J}\epsilon_{\mathcal{J}}\\ &+ \epsilon_2^\top(Q + \chi\Theta)\epsilon_2 - \epsilon_4^\top Q\epsilon_4 - \epsilon_{\mathbb{R}}^\top(\mathfrak{R}\otimes\Theta)\epsilon_{\mathscr{R}}\\ &- \gamma^2\epsilon_{6+q}^\top I\epsilon_{6+q} - \epsilon_{7+q}^\top I\epsilon_{7+q} + He(\epsilon_{7+q}^\top L\epsilon_2)\end{aligned}$$

$$\Gamma_0 = \begin{bmatrix}\epsilon_2\\ \epsilon_{\mathscr{R}}\end{bmatrix},\ \epsilon_{\mathscr{R}} = \begin{bmatrix}\epsilon_5\\ \vdots\\ \epsilon_{5+q}\end{bmatrix},\ \epsilon_{\mathcal{J}} = \begin{bmatrix}\epsilon_2 - \epsilon_3\\ \epsilon_3 - \epsilon_4\end{bmatrix}$$

$$\Gamma_1 = \begin{bmatrix}\epsilon_1\\ R(0)\epsilon_2 - R(-\chi)\epsilon_4 - \mathcal{R}\epsilon_{\mathscr{R}}\end{bmatrix}, \mathbf{H} = \rho_1\epsilon_1^\top H + \rho_2\epsilon_2^\top H$$

$$\mathbf{S}_{ij} = -I\epsilon_1 + (A_i + B_iK_j)\epsilon_2 + C_i\epsilon_3 + B_iF_j\mathcal{I}\epsilon_{\mathscr{R}} + D_i\epsilon_{6+q}$$

$$\epsilon_b = \begin{cases}\left[0_{n,n(b-1)}\ I_n\ 0_{n,n(5+q-b)+n_f+n_z}\right], & b = 1,\dots,5+q\\ \left[0_{n_f,n(5+q)}\quad I_{n_f}\quad 0_{n_f,n_z}\right], & b = 6+q\\ \left[0_{n_z,n(5+q)+n_f}\quad I_{n_z}\right], & b = 7+q.\end{cases}$$

Proof: Consider the Lyapunov–Krasovskii functional (LKF)

$$\begin{aligned}\mathbb{L}(t) = &\zeta^\top(t)M\zeta(t) + \int_{\chi}^{0}\int_{t-\sigma}^{t} \dot{x}^\top(\sigma)J\dot{x}(\sigma)d\sigma ds\\ &+ \int_{-\chi}^{0} x^\top(t+s)(Q + (s+\chi)\Theta)x(t+s)ds\end{aligned} \tag{17}$$

where $\zeta(t) = \begin{bmatrix}x(t)\\ \mathscr{R}(t)\end{bmatrix}$, $J > 0$, $Q > 0$, and $\Theta > 0$.

By using Lemma 1, one has

$$\int_{-\chi}^{0} x^\top(t+s)Qx(t+s)ds \geq \mathscr{R}^\top(t)[\mathfrak{R}\otimes Q]\mathscr{R}(t). \tag{18}$$

Based on (18), $J > 0$, $Q > 0$, and $\Theta > 0$, the positiveness of $\mathbb{L}(t)$ can be ensured by

$$\mathbb{L}(t) \geq \zeta^\top(t)\mathscr{M}\zeta(t) > 0 \tag{19}$$

for $\mathscr{M} = M + \mathrm{diag}\{0, \mathfrak{R}\otimes Q\} > 0$.

Next, $\dot{\mathbb{L}}(t)$ is solved as

$$\begin{aligned}\dot{\mathbb{L}}(t) = &He(\varsigma^\top(t)\Gamma_0^\top M\Gamma_1\varsigma(t)) + x^\top(t)(Q + \chi\Theta)x(t)\\ &- x^\top(t-\chi)Qx(t-\chi) - \int_{-\chi}^{0} x^\top(t+s)\Theta x(t+s)ds\\ &+ \chi^2\dot{x}^\top(t)J\dot{x}(t) - \int_{-\chi}^{0}\dot{x}^\top(t+s)J\dot{x}(t+s)ds.\end{aligned} \tag{20}$$

Define

$$\begin{aligned}\varsigma^\top(t) = &\left[\dot{x}^\top(t),\ x^\top(t),\ x^\top(t-\chi(t))\right.\\ &\left. x^\top(t-\chi),\ \mathscr{R}^\top(t),\ f^\top(t),\ z^\top(t)\right].\end{aligned} \tag{21}$$

With the help of Lemma 1, it results in

$$-\int_{-\chi}^{0} x^\top(t+s)\Theta x(t+s)ds \leq -\mathscr{R}^\top(t)(\mathfrak{R}\otimes\Theta)\mathscr{R}(t). \tag{22}$$

From the definition of $\varsigma^\top(t)$ in (21) and $\epsilon_{\mathscr{R}}$ defined in Theorem 1, one can rewrite $\mathscr{R}(t)$ as

$$\mathscr{R}(t) = \epsilon_{\mathscr{R}}\varsigma(t) \tag{23}$$

which further leads to

$$-\int_{-\chi}^{0} x^\top(t+s)\Theta x(t+s)ds \leq -\varsigma^\top(t)\epsilon_{\mathscr{R}}^\top(\mathfrak{R}\otimes\Theta)\epsilon_{\mathscr{R}}\varsigma(t). \tag{24}$$

Resorting to well-known Jensen inequality and reciprocally convex technique, one gets

$$\begin{aligned}&-\int_{-\chi}^{0}\dot{x}^\top(t+s)J\dot{x}(t+s)ds\\ &\leq -\varsigma^\top(t)\begin{bmatrix}\epsilon_2-\epsilon_3\\ \epsilon_3-\epsilon_4\end{bmatrix}^\top \mathcal{J}\begin{bmatrix}\epsilon_2-\epsilon_3\\ \epsilon_3-\epsilon_4\end{bmatrix}\varsigma(t)\\ &= -\varsigma^\top(t)\epsilon_{\mathcal{J}}^\top\mathcal{J}\epsilon_{\mathcal{J}}\varsigma(t).\end{aligned} \tag{25}$$

According to the property $\dot{\mathbf{r}}(s) = \mathcal{R}\mathbf{r}(s)$, we have

$$\frac{d}{dt}\mathscr{R}(t) = R(0)x(t) - R(-\chi)x(t-\chi) - \mathcal{R}\mathscr{R}(t). \tag{26}$$

From (21) and (26), it yields

$$\zeta(t) = \Gamma_0\varsigma(t),\ \dot{\zeta}(t) = \Gamma_1\varsigma(t). \tag{27}$$

Based on the definition of system, it yields

$$\mathbf{S}_{ij}\varsigma(t) = 0. \tag{28}$$

Then, one can get

$$\sum_{i=1}^{l}\sum_{j=1}^{l}\kappa_i\kappa_j\left[\varsigma^\top(t)\mathbf{HS}_{ij}\varsigma(t)\right] = 0. \tag{29}$$

From (20), the fuzzy system (8) satisfies H_∞ stability once the next condition holds.

$$\begin{aligned}&\dot{\mathbb{L}}(t) - \gamma^2 f^\top(t)f(t) + z^\top(t)z(t)\\ &\leq \sum_{i=1}^{l}\sum_{j=1}^{l}\kappa_i\kappa_j\left[\varsigma^\top(t)\left(He\left(\Gamma_0^\top M\Gamma_1\right) + He(\mathbf{HS}_{ij})\right.\right.\\ &\quad + \chi^2\epsilon_1^\top J\epsilon_1 - \epsilon_{\mathcal{J}}^\top\mathcal{J}\epsilon_{\mathcal{J}} + \epsilon_2^\top(Q+\chi\Theta)\epsilon_2 - \epsilon_4^\top Q\epsilon_4\\ &\quad - \epsilon_{\mathscr{R}}^\top(\mathfrak{R}\otimes\Theta)\epsilon_{\mathscr{R}} - \gamma^2\epsilon_{6+q}^\top I\epsilon_{6+q} - \epsilon_{7+q}^\top I\epsilon_{7+q}\\ &\quad \left.\left. + He\left(\epsilon_{7+q}^\top L\epsilon_2\right)\right)\varsigma(t)\right] < 0.\end{aligned} \tag{30}$$

Then, the condition (30) can be ensured by (15) and (16), which fulfills the proof. ∎

Remark 4: By utilizing the weighted memory controller, a new distributed delay model with the kernel representing the WF is established. Nevertheless, the existing approach [18] based on Legendre polynomials to cope with this model will lead to approximation error. To overcome this challenging problem, our proposed method utilizing integral inequality in Lemma 1 to handle the WF $r(s)$ will not result in approximation error, which further generates less conservative stability conditions.

Theorem 2: For given scalars χ, ρ_1, and ρ_2, the H_∞ stability with prescribed γ of system (8) under the fuzzy weighted memory controller

Algorithm 1: Steps for Implementing the Proposed Weighted Memory Control Strategy.

Step 1. For given scalar χ, select a WF $r(s)$. For given q, to ensure $\mathbf{r}(s)$ obey $\dot{\mathbf{r}}(s) = \mathcal{R}\mathbf{r}(s)$, choose the rest term $r_1(s), \ldots, r_q(s)$ and calculate the matrix $\mathcal{R}$.
Step 2. For given scalars ρ_1, ρ_2 and the H_∞ stability with prescribed γ, obtain the fuzzy controller gains K_j and F_j by solving the LMI conditions (29)–(31) in **Theorem 2**.
Step 3. Substitute the parameters $r(s)$, K_j and F_j to the fuzzy weighted memory control law (3).
Step 4. Utilize the control law (3) to ensure the stability of the time-varying delayed fuzzy system (2).

(3) with designed gains $K_j = Z_{Kj}X^{-\top}$ and $F_j = Z_{Fj}X^{-\top}$ is guaranteed, if there exist matrices $\hat{M}, \hat{Q} > 0, \hat{\Theta} > 0, \hat{\mathcal{J}} = \begin{bmatrix} \hat{J} & \hat{S} \\ \hat{S}^\top & \hat{J} \end{bmatrix} > 0$, $\hat{J} > 0$, and matrix X such that

$$\hat{\mathscr{M}} > 0 \tag{31}$$

$$\hat{\Psi}_{ii} < 0, \ i = j \tag{32}$$

$$\hat{\Psi}_{ij} + \hat{\Psi}_{ji} < 0, \ i < j \tag{33}$$

where

$$\begin{aligned}
\hat{\mathscr{M}} &= \hat{M} + diag\{0, \mathfrak{R} \otimes \hat{Q}\} \\
\hat{\Psi}_{ij} &= He(\Gamma_0^\top \hat{M}\Gamma_1) + He(\hat{\mathbf{H}}\hat{\mathbf{S}}_{ij}) \\
&\quad + \chi^2\epsilon_1^\top \hat{J}\epsilon_1 - \begin{bmatrix} \epsilon_2 - \epsilon_3 \\ \epsilon_3 - \epsilon_4 \end{bmatrix}^\top \hat{\mathcal{J}} \begin{bmatrix} \epsilon_2 - \epsilon_3 \\ \epsilon_3 - \epsilon_4 \end{bmatrix} \\
&\quad + \epsilon_2^\top(\hat{Q} + \chi\hat{\Theta})\epsilon_2 - \epsilon_4^\top \hat{Q}\epsilon_4 - \epsilon_{\mathscr{R}}^\top(\mathfrak{R} \otimes \hat{\Theta})\epsilon_{\mathscr{R}} \\
&\quad - \gamma^2\epsilon_{6+q}^\top I\epsilon_{6+q} - \epsilon_{7+q}^\top I\epsilon_{7+q} + He(\epsilon_{7+q}^\top LX\epsilon_2) \\
\hat{\mathbf{H}} &= \rho_1\epsilon_1^\top I + \rho_2\epsilon_2^\top I \\
\hat{\mathbf{S}}_{ij} &= -X\epsilon_1 + (A_iX + B_iZ_{Kj})\epsilon_2 + C_i\epsilon_3 + B_iZ_{Fj}\mathcal{I}\epsilon_{\mathscr{R}} + D_i\epsilon_{6+q}.
\end{aligned}$$

Proof: Define $X = H^{-1}$, $\hat{M} = (I_{n(q+1)} \otimes X)M(I_{n(q+1)} \otimes X^\top)$, $\hat{Q} = XWX^\top$, $\hat{\Theta} = X\Theta X^\top$, $\hat{J} = XJX^\top$, $\hat{S} = XSX^\top$, $K_jX^\top = Z_{Kj}$, and $F_jX^\top = Z_{Fj}$.

By left- and right-hand side multiplying (15) with matrix diag$\{X, X, X, X, I_{n(q+1)} \otimes X, I, I\}$ and its transpose, it leads to $\hat{\Psi}_{ii} < 0$ in (32). By following the similar way, the conditions (31) and (33) can be obtained.

Thus, the proof is completed. ■

In order to show the details of executing the proposed weighted memory control strategy, the specific steps are presented in the following Algorithm 1.

IV. Simulation Studies

Example 1: The parameter values of fuzzy system are chosen as follows:

$$A_1 = \begin{bmatrix} -1 & 1 \\ 2 & 3 \end{bmatrix}, C_1 = \begin{bmatrix} 0.5 & 1 \\ 0.2 & -1 \end{bmatrix}, B_1 = \begin{bmatrix} 0.5 \\ 1 \end{bmatrix}, D_1 = \begin{bmatrix} 0 \\ 2 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} 0 & 1 \\ 3 & -2 \end{bmatrix}, C_2 = \begin{bmatrix} 0.2 & 0.5 \\ 1 & -1.5 \end{bmatrix}, B_2 = \begin{bmatrix} 1 \\ 0.5 \end{bmatrix}, D_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

TABLE I
Optimized H_∞ Index γ

γ	χ/s 0.2	0.5	0.8
Existing memoryless controller (11) in [2]	0.5866	0.7689	1.5014
Existing memory controller (9) in [11]	0.5846	0.7454	1.0595
Our weighted memory controller (3)	0.5576	0.6007	0.6320

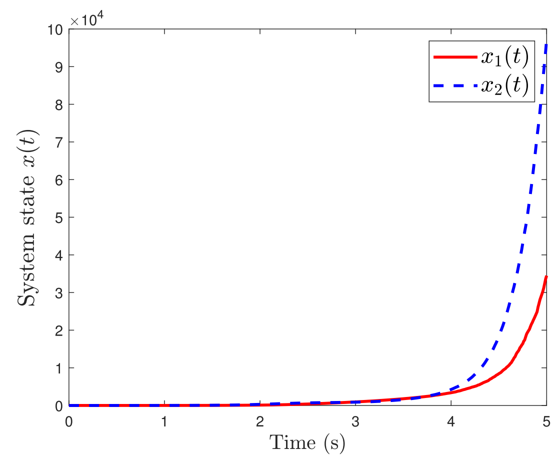

Fig. 1. $x(t)$ of open-loop system.

$$L = I_2, \ \kappa_1(x(t)) = \sin^2(x_1(t)), \ \text{and} \ \ \kappa_2(x(t)) = 1 - \kappa_1(x(t)).$$

A WF $r(s) = \dfrac{\pi}{2\chi}\cos(\frac{\pi}{2\chi}s)$ is selected. For $q = 1$, to ensure $\mathbf{r}(s)$ obey $\dot{\mathbf{r}}(s) = \mathcal{R}\mathbf{r}(s)$, another term $r_1(s) = \frac{\pi}{2\chi}\sin(\frac{\pi}{2\chi}s)$ is considered, which gives

$$\mathbf{r}(s) = \begin{bmatrix} \frac{\pi}{2\chi}\cos\left(\frac{\pi}{2\chi}s\right) \\ \frac{\pi}{2\chi}\sin\left(\frac{\pi}{2\chi}s\right) \end{bmatrix}, \mathcal{R} = \begin{bmatrix} 0 & -\frac{\pi}{2\chi} \\ \frac{\pi}{2\chi} & 0 \end{bmatrix}.$$

In order to reveal the effectiveness of the presented weighted memory controller over some existing memoryless controller and memory controller, the optimized γ derived by these approaches for different delay upper bound χ are produced in Table I. In order to make fair comparison, for the memoryless controller (11) in [2] and the memory controller (9) in [11], the corresponding controller design conditions are deduced by using the same method in this work to deal with time-varying delay term $x(t - \chi(t))$. For these methods, the parameters $\rho_1 = 1$ and $\rho_2 = 5$ are chosen. It tells that smaller γ can be calculated via our weighted memory controller than the memoryless controller (11) and the time-varying delay-based memory controller (9). This shows that the WF and designable matrix F_j are useful for leading to less conservative outcomes.

By choosing $\chi = 0.8\,s$, $\rho_1 = 1$, $\rho_2 = 5$, and $\gamma = 1$, the gains of fuzzy controller solved via Theorem 2 are given as follows:

$$K_1 = \begin{bmatrix} -1.6560 & -9.8722 \end{bmatrix}, \ F_1 = \begin{bmatrix} -0.2557 & -0.0868 \end{bmatrix}$$

$$K_2 = \begin{bmatrix} -2.5655 & -9.2105 \end{bmatrix}, \ F_2 = \begin{bmatrix} -0.6959 & 0.1729 \end{bmatrix}.$$

The external disturbance and initial state are considered as $f(t) = 3e^{-5t}$ and $x(0) = [5; -6]$, respectively. Fig. 1 depicts the open-loop state responses, which are unstable without control. The curves of system state controlled by our fuzzy weighted memory controller are shown in Fig. 2. It indicates that the constructed weighted memory

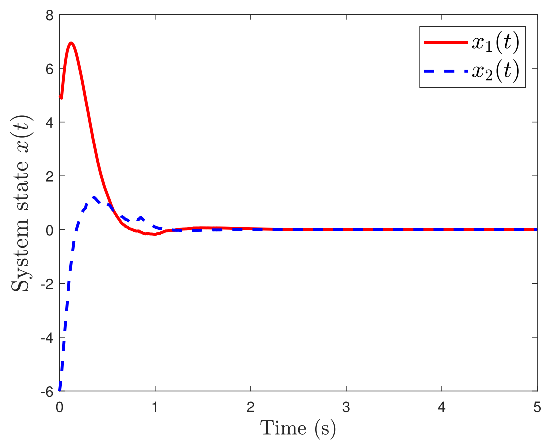

Fig. 2. $x(t)$ under the fuzzy weighted memory controller.

control mechanism is efficient to guarantee the time-varying delayed fuzzy system to be stable.

Example 2: A practical delayed truck–trailer model established in [20] is shown as

$$\begin{cases} \dot{x}_1(t) = -\mu\frac{\alpha\beta}{\delta_1\sigma}x_1(t) - (1-\mu)\frac{\alpha\beta}{\delta_1\sigma}x_1(t-\chi(t)) + \frac{\alpha\beta}{\delta_2\sigma}u(t) \\ \dot{x}_2(t) = \mu\frac{\alpha\beta}{\delta_1\sigma}x_1(t) + (1-\mu)\frac{\alpha\beta}{\delta_1\sigma}x_1(t-\chi(t)) \\ \dot{x}_3(t) = -\frac{\alpha\beta}{\delta_1\sigma}\sin\Big(x_2(t) + \mu\frac{\alpha\beta}{2\delta_1}x_1(t) \\ \qquad + (1-\mu)\frac{\alpha\beta}{2\delta_1}x_1(t-\chi(t))\Big) \end{cases} \tag{34}$$

where $x_1(t)$, $x_2(t)$, and $x_3(t)$ stands for the system states and the scalars are $\alpha = -1$, $\beta = 2$, $\delta_1 = 5.5$, $\delta_2 = 2.8$, $\sigma = 0.5$, and $\mu = 0.7$.

According to [20], the MFs are chosen as

$$\kappa_1(w(t)) = \begin{cases} \frac{\sin(w(t)) - \theta w(t)}{w(t)(1-\theta)}, & w(t) \neq 0 \\ 1, & w(t) = 0 \end{cases}$$

$$\kappa_2(w(t)) = 1 - \kappa_1(w(t)) \tag{35}$$

where $w(t) = x_2(t) + \mu\frac{\alpha\beta}{2\delta_1}x_1(t) + (1-\mu)\frac{\alpha\beta}{2\delta_1}x_1(t-\chi(t))$ and $\theta = 0.01/\pi$.

Then, the nonlinear system (34) considering external disturbance is formulated by a T-S fuzzy system with the following parameters:

$$A_1 = \begin{bmatrix} -\mu\frac{\alpha\beta}{\delta_1\sigma} & 0 & 0 \\ \mu\frac{\alpha\beta}{\delta_1\sigma} & 0 & 0 \\ \mu\frac{\alpha^2\beta^2}{2\delta_1\sigma} & \frac{\alpha\beta}{\sigma} & 0 \end{bmatrix}, \quad C_1 = \begin{bmatrix} -(1-\mu)\frac{\alpha\beta}{\delta_1\sigma} & 0 & 0 \\ (1-\mu)\frac{\alpha\beta}{\delta_1\sigma} & 0 & 0 \\ (1-\mu)\frac{\alpha^2\beta^2}{2\delta_1\sigma} & 0 & 0 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} -\mu\frac{\alpha\beta}{\delta_1\sigma} & 0 & 0 \\ \mu\frac{\alpha\beta}{\delta_1\sigma} & 0 & 0 \\ \mu\frac{\theta\alpha^2\beta^2}{2\delta_1\sigma} & \frac{\theta\alpha\beta}{\sigma} & 0 \end{bmatrix}, \quad C_2 = \begin{bmatrix} -(1-\mu)\frac{\alpha\beta}{\delta_1\sigma} & 0 & 0 \\ (1-\mu)\frac{\alpha\beta}{\delta_1\sigma} & 0 & 0 \\ (1-\mu)\frac{\theta\alpha^2\beta^2}{2\delta_1\sigma} & 0 & 0 \end{bmatrix}$$

$$B_1 = B_2 = \begin{bmatrix} \frac{\alpha\beta}{\delta_2\sigma} & 0 & 0 \end{bmatrix}^\top, \quad D_1 = D_2 = \begin{bmatrix} 0.1 & 0.1 & 0 \end{bmatrix}^\top$$

and $L = I_3$ is chosen. The same WF with Example 1 is considered here.

Then, the corresponding comparisons of H_∞ performance γ obtained by selecting $\rho_1 = 20$ and $\rho_2 = 5$ are given in Table II . On basis of this table, one can obtain the same conclusion with Example 1 that our weighted memory controller with WF and designable matrix F_j is able to generate better H_∞ performance γ than the memoryless controller (11) and the time-varying delay-based memory controller (9).

TABLE II
OPTIMIZED H_∞ INDEX γ

γ	χ/s		
	1	2	3
Existing memoryless controller (11) in [2]	1.0420	2.7142	4.9486
Existing memory controller (9) in [11]	1.0411	2.7135	4.5951
Our weighted memory controller (3)	0.2854	0.5698	0.6760

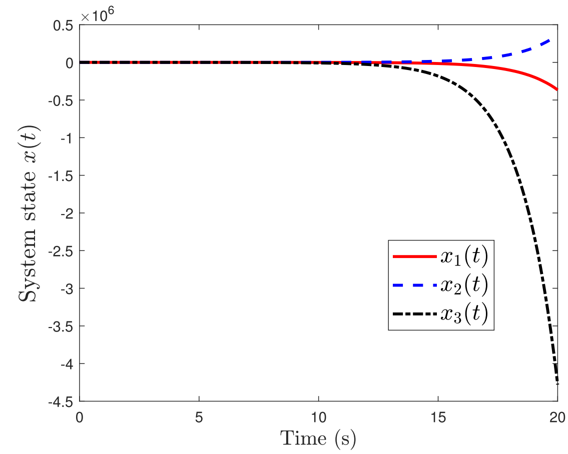

Fig. 3. $x(t)$ of open-loop system (34).

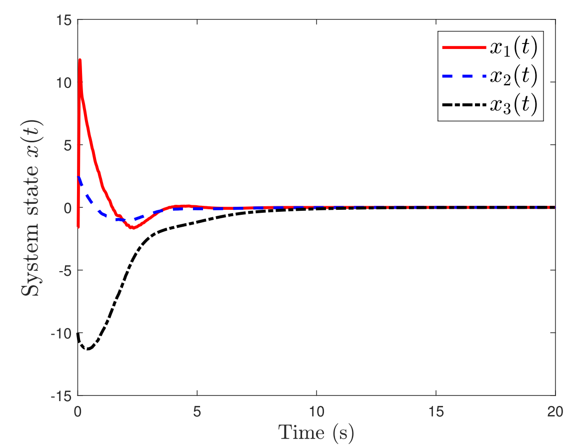

Fig. 4. $x(t)$ of system (34) under the fuzzy weighted memory controller.

By setting $\chi = 2\,s$, $\rho_1 = 20$, $\rho_2 = 5$, and $\gamma = 1$, the gains of fuzzy controller computed by Theorem 2 are shown as follows:

$$K_1 = \begin{bmatrix} 17.0437 & -48.8433 & 5.2263 \end{bmatrix}$$

$$F_1 = \begin{bmatrix} 0.2942 & 0.4336 & -0.1607 \end{bmatrix}$$

$$K_2 = \begin{bmatrix} 17.2495 & -49.7677 & 5.3403 \end{bmatrix}$$

$$F_2 = \begin{bmatrix} 0.2544 & 0.5401 & -0.2219 \end{bmatrix}.$$

The external disturbance and initial state are considered as $f(t) = 5\cos(0.5t)e^{-t}$ and $x(0) = [-0.5\pi; 0.75\pi; -10]$, respectively. The system state trajectories without and with control are depicted in Figs. 3 and 4, respectively. They manifest that the stability of the practical delayed truck–trailer is ensured well via constructed weighted memory controller.

V. Conclusion

This article addressed the weighted memory H_∞ controller design for T-S fuzzy systems under time-varying delay. A more general distributed-delay-dependent controller model involving a WF was introduced to describe the memory control information. By applying the weighting-function-dependent LKF and integral inequality, new sufficient criteria for designing the fuzzy weighted memory H_∞ controller were derived. At last, two examples were simulated to show the superiorities of the presented strategy. For some real nonlinear systems, the probabilistic delay [21], actuator fault [22], and actuator saturation [23] are common and practical problems, which are not considered in our study. These could limit the applications of the proposed weighted memory control strategy to practical systems. In the future, how to extend the weighted memory control approach to deal with the issues deserves further research.

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