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journal homepage: www.elsevier.com/locate/fss





## Dynamic sum-based event-triggered $H_{\infty}$ filtering for networked T-S fuzzy wind turbine systems with deception attacks

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#### ARTICLE INFO

# Keywords: T-S fuzzy wind turbine system Dynamic sum-based event-triggered scheme $H_{\infty}$ filtering Deception attacks

#### ABSTRACT

This article is concerned with the dynamic sum-based event-triggered  $H_{\infty}$  filtering issue for networked wind turbine systems subject to deception attacks and communication delays. By considering the time-varying wind power case rather than the existing maximum power case, a more general T-S fuzzy system with two premise variables is modeled for nonlinear wind turbine system. In order to save the communication cost, a novel dynamic sum-based event-triggered scheme is proposed, which has the following three merits. First, some past sampled measurements are utilized to reduce redundant transmissions. Second, an auxiliary dynamic variable based on the past sampled measurements is introduced in the triggering condition to further enlarge the triggering intervals. Third, a dynamic triggering threshold is designed, which can be regulated adaptively along with the system evolution. With the help of Lyapunov method and linear matrix inequality technique, some sufficient co-design conditions of  $H_{\infty}$  filter and triggering matrices are derived for the T-S fuzzy filtering error system with deception attacks and communication delays. Lastly, some simulations are carried out to illustrate the advantages of the proposed strategy.

#### 1. Introduction

In recent decades, the global energy landscape has been witnessing a profound shift towards sustainable and renewable sources of power generation. As a representative type of renewable energy, wind energy stands out as a vital player in the efforts to combat climate change and reduce greenhouse gas emissions. The growing urgency to transition from fossil fuels to clean energy sources has propelled substantial research into the optimization, efficiency enhancement, and cost reduction of wind turbine systems (WTSs). Recently, increasing attention has been devoted to analyze and design of WTSs to improve the power output and grid integration capability, and many research results and Overview papers about WTSs can be found in [1–5].

For practical WTSs, their inherent nonlinear and uncertain dynamics lead to challenge problems to control system design. There are two typical kinds of wind turbine generator: permanent magnetic synchronous generator (PMSG) and doubly-fed induction generator (DFIG). Regarding the DFIG-based WTSs integrating an energy store system considered in [6], a robust control strategy based on integral sliding mode control is developed to manage the powers of WTSs. In [7], an advanced backstepping integral action control method is proposed for the DFIG-based WTSs to monitor the rotor and the network side converters under normal network operations, while an improved protection scheme is added to the controller to overcome the negative effect caused by network fault. Compared

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https://doi.org/10.1016/j.fss.2024.109084

Received 22 October 2023; Received in revised form 30 May 2024; Accepted 15 July 2024

Available online 26 July 2024

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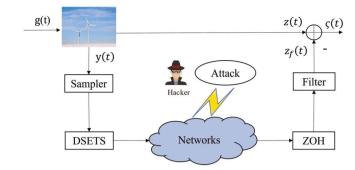


Fig. 1. The framework of  $H_{\infty}$  filtering for networked WTS against deception attacks under DSETS.

with DFIG, PMSG is more widely studied and used due to the features like direct drive, low rotational velocity and cheap maintenance fee. In order to handle the nonlinear dynamics, a T-S fuzzy modeling approach is utilized to establish a group of linear systems related to membership functions (MFs) for nonlinear switched systems [8]. Based on the T-S fuzzy model, various investigations about WTSs have been reported in [9–12]. To name a few, the sampled-data fuzzy controller is designed in [9] for T-S fuzzy PMSG-based WTSs affected by actuator faults. In [10], a disturbance observer is constructed to attenuate the negative effect of external disturbance and a sliding mode controller is designed to stabilize the T-S fuzzy PMSG-based WTSs. With the development of network technique, networked WTSs has attracted more and more attention [13–16]. The security of data communication is an important issue for networked systems [17,18]. If the exchanged signals are attacked in network channels, the expected performance, even stability, could be degraded. By considering cyber-attacks, many studies about secure control problems for WTSs can be found in [19,20,14]. Generally speaking, in [19], an adaptive resilient control strategy for the variable-speed WTSs working at low-speed area is proposed to deal with the deception attacks. A dual-triggered resilient torque control method for the variable-speed WTSs subject to denial-of-service (DoS) attacks is investigated in [20]. Considering DoSs and deception attacks simultaneously, the authors in [14] address the resilient control problem for WTSs.

On the other research line, as a high effective manner to mitigate network congestion and save limited network resource, event-triggered scheme (ETS) has been viewed as a replacement for the traditional time-triggered scheme [21–26]. In order to decrease the redundant transmissions, some control problems of a fuzzy WTS with back to back converter and a fuzzy WTS with actuator faults via ETSs are studied in [27] and [28], respectively. Note that the above outcomes are mostly based on the full measurable system states, which is difficult to be realized in engineering applications. It is known that  $H_{\infty}$  filtering is a powerful tool to estimate system states when the system disturbance is bounded. Meanwhile, the abrupt disturbances and noises in practical systems could result in unnecessary triggering events under the above ETSs [27,28], which only utilize the current sampled data. Recently, an improved integral ETS using some historical state information is proposed in [29] to further reduce communication burden. Although it has some advantages over some conventional ETSs, there exists the difficulty of analyzing Zeno behavior in the integral ETS [29]. In order to avoid Zeno behavior, a discrete version of the ETS using historical sampled data is addressed in [30] for the synchronization of delayed neural networks. In addition, the triggering thresholds of ETSs have significant effect on data releasing rate. However, the triggering thresholds in [27–30] are constant parameters and cannot be adjusted adaptively along with the system evolution.

Motivated by the previous observations, a dynamic sum-based event-triggered  $H_{\infty}$  fuzzy filter is designed for networked T-S fuzzy PMSG-based WTSs against deception attacks. The main contributions of this paper can be summarized as follows:

- 1) A new T-S fuzzy PMSG-based model is established by considering the time-varying wind power case, where an extra premise variable is introduced to handle the nonlinearity induced by time-varying wind power. Different from some existing results [4,10] considering a constant value at the maximum wind power case, our proposed model does not require that the generator always operates at the maximum power point.
- 2) A novel dynamic sum-based ETS (DSETS) is presented, which adopts the sum of some historical sampled outputs, a dynamic variable and some dynamic triggering thresholds. Compared with the traditional sampled-data ETS [21] only dependent on current sampled data, the network resource can be notably saved by the designed DSETS. In addition, the introduced dynamic variable in our DSETS is potential to generate larger triggering intervals than the existing integral ETS [30].
- 3) Some sufficient co-design conditions of fuzzy filter gains and triggering matrices are accomplished to ensure the stability of the filtering error WTS with given  $H_{\infty}$  index, where the proposed DSETS, deception attacks and communication delays are considered in a united framework.

**Notation:** In this article, \* is used to represent  $[*]XY = Y^TXY$  or  $\begin{bmatrix} X & Y \\ Y^T & X \end{bmatrix} = \begin{bmatrix} X & Y \\ * & X \end{bmatrix}$ , where  $Y^T$  is the transpose of Y.  $\mathbb{E}\{\cdot\}$  denotes the mathematical expectation operation.

#### 2. Preliminaries

This section introduces the T–S fuzzy model of a nonlinear PMSG-based WTS. The framework of  $H_{\infty}$  filtering for networked WTS subject to deception attacks by using the proposed DSETS is drawn in Fig. 1.

The mechanical power output of the wind is formulated as:

$$\mathfrak{P}(t) = 0.5 \, \Im \pi \, R^2 \, \Theta_n(\chi(t), \nu) \, v_1^3(t), \tag{1}$$

in which  $v_1(t)$  and  $v_2(t)$  are wind and rotor velocities, respectively,  $\Im$  is air density, and the length of blade is represented by R.  $\Theta_n(\chi(t), \nu)$  stands for power factor,  $\chi(t) = \nu_2(t)R/\nu_1(t)$  denotes tip velocity ratio,  $\nu$  means pitch angle.

According to [31], it yields

$$\Theta_p(\chi(t), \nu) = 0.22(\frac{116}{\chi_i} - 0.4\nu - 5)e^{-\frac{12.5}{\chi_i}},\tag{2}$$

where  $\frac{1}{\chi_i} = \frac{1}{\chi(t) + 0.08\nu} - \frac{0.035}{\nu^3 + 1}$ . Then, the aerodynamic torque is given as:

$$\mathfrak{T}(t) = \frac{\mathfrak{P}(t)}{v_2(t)} = \frac{\mathfrak{T}\pi R^3 \Theta_p(\chi(t), \nu) v_1^2(t)}{2\chi(t)}.$$
(3)

The following dynamic equations of PMSG-based WTS are inferred as [32]:

$$\begin{cases} \mathfrak{F}_{d}(t) = \mathfrak{R}_{s}\mathfrak{I}_{d}(t) + \mathfrak{L}_{d}\frac{d\mathfrak{I}_{d}(t)}{dt} - w_{e}(t)\mathfrak{L}_{q}\mathfrak{I}_{q}(t) + b_{1}g(t) \\ \mathfrak{F}_{q}(t) = \mathfrak{R}_{s}\mathfrak{I}_{q}(t) + \mathfrak{L}_{q}\frac{d\mathfrak{I}_{q}(t)}{dt} + w_{e}(t)\mathfrak{L}_{d}\mathfrak{I}_{d}(t) \\ + w_{e}(t)Y_{f} + b_{2}g(t) \end{cases} ,$$

$$\dot{w}_{g}(t) = \frac{1}{J}(\mathfrak{T}_{e}(t) - \mathfrak{T}(t) - \epsilon w_{g}(t)) + b_{3}g(t)$$

$$\mathfrak{T}_{e}(t) = \frac{3}{2}P_{n}\mathfrak{I}_{q}(t)Y_{f}$$

$$(4)$$

in which  $\mathfrak{F}_d(t)$  and  $\mathfrak{I}_d(t)$  represent the voltage and current of d-axis,  $\mathfrak{F}_d(t)$  and  $\mathfrak{I}_d(t)$  are the voltage and current of q-axis,  $\mathfrak{L}_d$  and  $\mathcal{L}_q$  denote the stator inductance on d axis and q axis, respectively, g(t) is external disturbance in  $\mathcal{L}_2[0,\infty)$ ,  $P_n$ ,  $w_g(t) = v_2(t)$  are the amount of poles and the rotor velocity,  $\mathfrak{R}_s$  means the stator resistance, the permanent magnet flux is denoted by  $Y_f$ ,  $\mathfrak{T}_e(t)$  means the torque of generator,  $w_e(t) = P_n w_g(t)$ , J means the moment of inertia and  $\epsilon$  represents the friction coefficient, respectively.

By choosing the state vector  $\zeta(t) = \begin{bmatrix} \zeta_1(t) & \zeta_2(t) & \zeta_3(t) \end{bmatrix}^T = \begin{bmatrix} w_{\varrho}(t) & \Im_{\varrho}(t) & \Im_{\varrho}(t) \end{bmatrix}^T$ , control input  $u(t) = \begin{bmatrix} \mathfrak{F}_{\varrho}(t) & \mathfrak{F}_{\varrho}(t) \end{bmatrix}^T$  and using (1)-(4), the nonlinear state space of PMSG-based WTS can be expressed as follows:

$$\dot{\mathcal{L}}(t) = \mathcal{A}_0 \mathcal{L}(t) + \mathcal{B}_0 u(t) + Bg(t), \tag{5}$$

where  $\zeta(t)$  and u(t) denote the state and control input, and

$$\mathcal{A}_0 = \begin{bmatrix} -\frac{\epsilon + T_0(t)w_g(t)}{2J_f} & \frac{3P_nY_f}{2J} & 0 \\ -\frac{P_n^JY_f}{\mathfrak{L}_q} & -\frac{\mathfrak{R}_s}{\mathfrak{L}_q} & -\frac{P_nw_g(t)\mathfrak{L}_d}{\mathfrak{L}_d} \\ 0 & \frac{P_nw_g(t)\mathfrak{L}_q}{\mathfrak{L}_d} & -\frac{\mathfrak{R}_s}{\mathfrak{L}_d} \end{bmatrix}, \quad \mathcal{B}_0 = \begin{bmatrix} 0 & 0 \\ \frac{1}{\mathfrak{L}_q} & 0 \\ 0 & \frac{1}{\mathfrak{L}_d} \end{bmatrix}, \quad B = \begin{bmatrix} b_3 \\ -\frac{b_2}{\mathfrak{L}_q} \\ -\frac{b_1}{\mathfrak{L}_d} \end{bmatrix}, \quad T_0(t) = \frac{\Theta_p(\chi(t), \nu)\mathfrak{T}\pi R^5}{2\chi^3(t)}.$$

In order to construct the T-S fuzzy model of wind turbine system with PMSG, two premise variables  $\theta_1(t) = w_v(t)$  and  $\theta_2(t) = w_v(t)$  $T_0(t)w_g(t)$  are selected. It is assumed that  $\theta_1(t) \in [\theta_{1\,\text{min}} \quad \theta_{1\,\text{max}}] = [M_1 \quad M_2]$  and  $\theta_2(t) \in [\theta_{2\,\text{min}} \quad \theta_{2\,\text{max}}] = [M_3 \quad M_4]$ . Then, one can

Plant rule i: IF  $\theta_1(t)$  is  $x_i^1$ ,  $\theta_2(t)$  is  $x_i^2$ , THEN

$$\dot{\zeta}(t) = \mathcal{A}_i \zeta(t) + \mathcal{B}_0 u(t) + B_i g(t), \tag{6}$$

where  $B_i = B$ ,  $x_i^s(s = 1, ..., m)$  means the fuzzy set for the *i*th rule (i = 1, ..., r), m = 2 and r = 4 are the amount of premise variables and fuzzy rules, and

$$\mathcal{A}_{1} = \begin{bmatrix} -\frac{e+M_{3}}{P_{n}^{J}Y_{f}} & \frac{3P_{n}Y_{f}}{2J} & 0 \\ -\frac{P_{n}Y_{f}}{\Sigma_{q}} & -\frac{\Re_{s}}{\Sigma_{q}} & -\frac{P_{n}M_{1}\Sigma_{d}}{\Sigma_{q}} \\ 0 & \frac{P_{n}M_{1}\Sigma_{q}}{L_{d}} & -\frac{\Re_{s}}{\Sigma_{d}} \end{bmatrix}, \quad \mathcal{A}_{2} = \begin{bmatrix} -\frac{e+M_{4}}{I} & \frac{3P_{n}Y_{f}}{2J} & 0 \\ -\frac{P_{n}Y_{f}}{I} & \frac{\Im_{n}Y_{f}}{\Sigma_{q}} & -\frac{\Im_{s}}{\Sigma_{q}} & -\frac{P_{n}M_{1}\Sigma_{d}}{\Sigma_{q}} \\ 0 & \frac{P_{n}M_{1}\Sigma_{q}}{L_{d}} & -\frac{\Re_{s}}{\Sigma_{d}} \end{bmatrix},$$

$$\mathcal{A}_{3} = \begin{bmatrix} -\frac{e+M_{3}}{I} & \frac{3P_{n}Y_{f}}{2J} & 0 \\ -\frac{P_{n}Y_{f}}{\Sigma_{q}} & -\frac{\Im_{s}}{\Sigma_{q}} & -\frac{P_{n}M_{2}\Sigma_{d}}{\Sigma_{q}} \\ 0 & \frac{P_{n}M_{2}\Sigma_{q}}{\Sigma_{d}} & -\frac{\Im_{s}}{\Sigma_{d}} \end{bmatrix}, \quad \mathcal{A}_{4} = \begin{bmatrix} -\frac{e+M_{4}}{I} & \frac{3P_{n}Y_{f}}{I} & 0 \\ -\frac{P_{n}Y_{f}}{I} & \frac{\Im_{n}Y_{f}}{I} & 0 \\ -\frac{P_{n}Y_{f}}{I} & -\frac{\Im_{s}}{I} & -\frac{P_{n}M_{2}\Sigma_{d}}{I} \\ 0 & \frac{P_{n}M_{2}\Sigma_{q}}{I} & -\frac{\Im_{s}}{I} \end{bmatrix}.$$

By choosing a state feedback controller  $u(t) = \mathcal{K}\zeta(t)$  to stabilize the WTS, the closed-loop T-S fuzzy WTS is given as:

$$\begin{cases} \dot{\zeta}(t) = \sum_{i=1}^{r} \mu_i(t)(A_i\zeta(t) + B_ig(t)) \\ y(t) = C\zeta(t) \\ z(t) = L\zeta(t) \end{cases}, \tag{7}$$

where  $y(t) \in \mathbb{R}^p$  is the measured output,  $z(t) \in \mathbb{R}^q$  is the signal to be estimated,  $A_i = \mathcal{A}_i + \mathcal{B}_0 \mathcal{K}$ ,  $B_i$ , C and L are known parameter matrices with appropriate dimensions,  $\mu_i(t)$  is the normalized membership function satisfying

$$\begin{split} \sum_{i=1}^{4} \mu_{i}(t) &= 1, \quad \theta_{1 \min}(t) = \frac{\theta_{1}(t) - M_{1}}{M_{2} - M_{1}}, \quad \theta_{1 \max}(t) = 1 - \theta_{1 \min}(t), \\ \theta_{2 \min}(t) &= \frac{\theta_{2}(t) - M_{3}}{M_{4} - M_{3}}, \quad \theta_{2 \max}(t) = 1 - \theta_{2 \min}(t), \\ \mu_{1}(t) &= \theta_{1 \min}(t)\theta_{2 \min}(t), \quad \mu_{2}(t) = \theta_{1 \min}(t)\theta_{2 \max}(t), \\ \mu_{3}(t) &= \theta_{1 \max}(t)\theta_{2 \min}(t), \quad \mu_{4}(t) = \theta_{1 \max}(t)\theta_{2 \max}(t). \end{split}$$

$$(8)$$

**Remark 1.** In some existing results [4,10], the T-S fuzzy PMSG-based WTS model is usually established by assuming the maximum wind power case with a constant value of  $T_0(t)$ . However, in this paper, a new T-S fuzzy model is derived as (7), where the time-varying wind power case is considered and a premise variable  $\vartheta_2(t) = T_0(t)w_g(t)$  is added. Compared with [4,10], it is not necessary for the presented model that the generator is always maintained at the maximum power point with higher control cost.

By considering the limited capacity of the communication channel, an event generator is established between the sensor and the filtering to determine whether the current sampled output  $y(l_k h)$  is released to the filtering or not. When the last triggering time is  $t_k h$ , the next triggering time  $t_{k+1} h$  is determined by the following DSETS:

$$t_{k+1}h = t_k h + \min_{l \ge 1} \left\{ lh | \sum_{n=0}^N \psi_n(t) \ge \lambda \phi(t) \right\},\tag{9}$$

where

$$\begin{split} & \psi_n(t) = [*] \Phi_n e_n(t) - \delta_n(lh) [*] \Phi_n y(l_k h - nh), \\ & e_n(t) = y(t_k h - nh) - y(l_k h - nh), \ n = 0, 1, \cdots, N, \\ & \dot{\phi}(t) = -\rho \phi(t) - \sum_{n=0}^{N} \psi_n(t), \end{split}$$

and h is the sampling period,  $l_k h = t_k h + lh, l \in \mathbb{N}$ ,  $\Phi_n$  are positive weighting matrices to be determined,  $\phi(t)$  is an auxiliary dynamic variable,  $\lambda$  and  $\rho$  are positive scalars,  $\delta_n(lh) \in [\delta_{nm}, \ \delta_{nM}](0 < \delta_{nm} \le \delta_{nM} < 1)$  are dynamic triggering thresholds, which are adjusted by

$$\delta_n(lh) = \delta_{nM} - (\delta_{nM} - \delta_{nm}) \frac{2}{\pi} \tanh(\|y(l_k h)\|). \tag{10}$$

**Remark 2.** If we choose  $\delta_{nm} = \delta_{nM} = \delta_n$  and  $\lambda = 0$ , the proposed DSETS reduces to the existing ETS developed in [30]:

$$t_{k+1}h = t_k h + \min_{l \ge 1} \left\{ lh | \sum_{n=0}^{N} \psi_n(t) \ge 0 \right\}. \tag{11}$$

In addition, if N=0,  $\delta_{0m}=\delta_{0M}=\delta_0$  and  $\lambda=0$  are considered, our DSETS is further reduced to the conventional normal ETS studied in [21]:

$$t_{k+1}h = t_k h + \min_{t \ge 1} \left\{ lh | \psi_0(t) \ge 0 \right\}. \tag{12}$$

Note that our proposed DSETS covers the existing ETSs [21,30] as the special cases, which means it is more general and practical. Moreover, compared with the existing ETSs in [21,30], the proposed DSETS utilizes not only the historical sampled outputs but also an auxiliary dynamic variable, which is potential to decrease the data triggering frequency and save more limited network resources.

**Remark 3.** In terms of the adaptive law in (10), the dynamic triggering thresholds are related to the sampled output  $y(l_k h)$ . The idea behind the regulation rule is that more data need to be triggered to improve the filtering performance when  $y(l_k h)$  is away from the steady state. On the other hand, if  $y(l_k h)$  converges to the equilibrium point, the triggered signals can be reduced to mitigate communication burden.

Given the function of a zero-order-holder (ZOH), the latest triggered signal  $y(t_k h)$  will be maintained as the input of filter until a new data is accepted, that is

$$\tilde{y}(t) = y(t_k h), \quad t \in [t_k h + d_k, \ t_{k+1} h + d_{k+1}),$$
(13)

where  $d_k$  means the transmission delay induced by communication network at time  $t_k h$ . Similar to [33], the interval  $t \in [t_k h + d_k, t_{k+1} h + d_{k+1})$  is divided into  $t_{k+1} - t_k + 1$  segments, and each segment is denoted by  $\varpi_l = [t_k h + lh + d_k^l, t_k h + lh + h + d_{k+1}^{l+1})$ . Obviously,  $t \in [t_k h + d_k, t_{k+1} h + d_{k+1}) = \bigcup_{l=0}^{t_{k+1} - t_k + 1} \varpi_l$  with  $d_k^0 = d_k$ ,  $d_k^{t_{k+1} - t_k + 1} = d_{k+1}$ . By defining  $d_n(t) = t - t_k h - lh + nh$ , it yields

$$nh = d_{nm} \le d_n(t) \le (n+1)h + \bar{d} = d_{nM},$$
 (14)

where delay upper bound is  $\bar{d} = \max \{d_k\}$ . And we can rewrite  $y(t_k h - nh)$  as

$$y(t_k h - nh) = e_n(t) + y(t - d_n(t)).$$
(15)

In addition, it is critical to consider the impact of cyber-attacks since the triggered data packets are vulnerable to malicious attacks and change while being transported via the communication network. This work considers the deception attacks which intends to change the data packets by injecting a false signal. In this manner, the actual input to the filter can be described as

$$\hat{\mathbf{y}}(t) = \tilde{\mathbf{y}}(t) + \alpha(t)f(t),\tag{16}$$

where  $\alpha(t)$  is a Bernoulli distributed variable, f(t) is the deception signal and meets the following constraint

$$|| f(t) ||_2 \le || Fy(t) ||_2,$$
 (17)

with a constant matrix F. The feature of deception attacks is described by the stochastic variable  $\alpha(t)$ , whose probabilities are  $Prob\{\alpha(t)=1\}=\mathbb{E}\{\alpha(t)\}\triangleq \bar{\alpha}_1,\ Prob\{\alpha(t)=0\}=1-\bar{\alpha}_1=\bar{\alpha}_2.$ 

Based on the DSETS, the T-S fuzzy filter is constructed as

$$\begin{cases} \dot{\zeta}_{f}(t) = \sum_{j=1}^{r} \mu_{j}(t_{k}h)(A_{fj}\zeta_{f}(t) + B_{fj}\hat{y}(t)) \\ z_{f}(t) = C_{f}\zeta_{f}(t) \end{cases}, \tag{18}$$

in which  $A_{fj}$ ,  $B_{fj}$ ,  $C_f$  are filter gains going to be designed.

Define  $\tilde{\zeta}(t) = [\zeta^T(t) \ \zeta_f^T(t)]^T$  and  $\zeta(t) = z(t) - z_f(t)$ . By combining (7), (16) and (18), the filtering error system is deduced as

$$\begin{cases} \dot{\bar{\zeta}}(t) = \sum_{i=1}^{r} \sum_{j=1}^{r} \mu_i \mu_j^k \left( \mathscr{A}_{ij} \tilde{\zeta}(t) + \mathscr{B}_j \hat{y}(t) + \mathscr{B}_{gi} g(t) \right) \\ \zeta(t) = \mathscr{L}_0 \tilde{\zeta}(t) \end{cases}, \tag{19}$$

where  $\mu_i$ ,  $\mu_i^k$  are the abbreviations of  $\mu_i(t)$  and  $\mu_i^k(t_k h)$ , and

$$\mathcal{A}_{ij} = \begin{bmatrix} A_i & 0 \\ 0 & A_{fj} \end{bmatrix}, \ \mathcal{B}_j = \begin{bmatrix} 0 \\ B_{fj} \end{bmatrix}, \ \mathcal{B}_{gi} = \begin{bmatrix} B_i \\ 0 \end{bmatrix}, \ \mathcal{L}_0 = \begin{bmatrix} L & -C_f \end{bmatrix}.$$

To deal with the asynchronous MFs  $\mu_i$  and  $\mu_j^k$ , the assumption  $\mu_j^k \ge \rho_j \mu_j$  with  $\rho_j (0 < \rho_j \le 1)$  borrowing the method in [33] is utilized.

Substituting (15) and (16) into (19), we establish the filtering error system as the following delay model:

$$\begin{cases} \dot{\tilde{\zeta}}(t) = \sum_{i=1}^{r} \sum_{j=1}^{r} \mu_{i} \mu_{j}^{k} \left( \mathcal{A}_{ij} \tilde{\zeta}(t) + \mathcal{B}_{dj} \tilde{\zeta}(t - d_{0}(t)) + \mathcal{B}_{j} e_{0}(t) + \mathcal{B}_{gi} g(t) + \alpha(t) \mathcal{B}_{j} f(t) \right) \\ \zeta(t) = \mathcal{L}_{0} \tilde{\zeta}(t) \end{cases}, \tag{20}$$

where

$$\mathcal{B}_{dj} = \begin{bmatrix} 0 & 0 \\ B_{fj}C & 0 \end{bmatrix}.$$

To end this section, a helpful lemma is introduced as below.

**Lemma 1.** [34]For given positive scalars  $p, q, \varepsilon \in (0, 1)$ , matrix  $\mathcal{W} \in \mathbb{R}^p > 0$ , matrices  $\mathcal{J}_1 \in \mathbb{R}^{p \times q}$  and  $\mathcal{J}_2 \in \mathbb{R}^{p \times q}$ . For all vector  $x \in \mathbb{R}^q$ , define:

$$\mathscr{F}(\varepsilon, \mathscr{W}) = \frac{1}{\varepsilon} \ \varkappa^T \mathcal{J}_1^T \mathscr{W} \mathcal{J}_1 \varkappa + \frac{1}{1 - \varepsilon} \ \varkappa^T \mathcal{J}_2^T \mathscr{W} \mathcal{J}_2 \varkappa. \tag{21}$$

Then, if there exists a matrix  $\mathcal{U} \in \mathbb{R}^{p \times p}$  such that  $\begin{bmatrix} \mathcal{W} & \mathcal{U}^T \\ * & \mathcal{W} \end{bmatrix} > 0$ , one has

 $\Xi^{ij} = \begin{bmatrix} \Xi^{ij}_1 & * \\ \Xi^{ij}_2 & \Xi_3 \end{bmatrix}, \; \Psi = \begin{bmatrix} -\Phi_0 & * & * \\ 0 & -\Phi_1 & * \\ 0 & 0 & -\Phi_2 \end{bmatrix},$ 

$$\min_{\varepsilon \in (0,1)} \mathcal{F}(\varepsilon, \mathcal{W}) \geqslant \begin{bmatrix} \mathcal{F}_1 \mathsf{x} \\ \mathcal{F}_2 \end{bmatrix}^T \begin{bmatrix} \mathcal{W} & \mathcal{U}^T \\ * & \mathcal{W} \end{bmatrix} \begin{bmatrix} \mathcal{F}_1 \mathsf{x} \\ \mathcal{F}_2 \end{bmatrix}. \tag{22}$$

#### 3. Main results

In this section, sufficient conditions to ensure the stability of the filtering error system with given  $H_{\infty}$  index in the presence of deception attacks are provided Theorem 1. Furthermore, the co-design conditions of triggering matrices and filter gains are produced in Theorem 2.

**Theorem 1.** For given scalars h,  $\bar{d}$ ,  $d_{0m}$ ,  $d_{0m}$ ,  $d_{1m}$ ,  $d_{1m}$ ,  $d_{2m}$ ,  $d_{2m}$ ,  $d_{2m}$ ,  $\bar{\alpha}_1$ ,  $\beta$ ,  $\gamma$ ,  $\delta_{0m}$ ,  $\delta_{1m}$ ,  $\delta_{2m}$ , under the DSETS (9), the filtering error system (20) is stable in mean square sense with prescribed  $H_{\infty}$  performance index  $\gamma$  if there exist symmetric matrices P > 0,  $R_{1n} > 0$ ,  $R_{2n} > 0$ ,  $R_{3n} > 0$ ,  $R_{4n} > 0$ ,  $\Phi_n > 0$ ,  $R_{4n} > 0$ ,  $R_$ 

$$\Xi^{ij} - \Gamma_i < 0, \tag{23}$$

$$\rho_i \Xi^{(i)} - \rho_i \Gamma_i + \Gamma_i < 0 (i = j), \tag{24}$$

$$\rho_i \Xi^{ij} + \rho_i \Xi^{ji} - \rho_i \Gamma_i - \rho_i \Gamma_i + \Gamma_i + \Gamma_i \le 0 \ (i < j), \tag{25}$$

where

$$\begin{split} \Xi_{43} = & R_{40} - X_0^T, \Xi_{54} = -X_0^T + R_{40}, \Xi_{55} = -R_{20} - R_{40}, \\ \Xi_{44} = & -2R_{40} + X_0^T + X_0 + \delta_{0M}(\rho + \lambda)H^TC^T\Phi_0CH, \\ \Xi_{77} = & -R_{11} - R_{31} - R_{41}, \ \Xi_{87} = R_{41} - X_1^T, \\ \Xi_{88} = & -2R_{41} + X_1^T + X_1 + \delta_{1M}(\rho + \lambda)H^TC^T\Phi_1CH, \\ \Xi_{98} = & -X_1^T + R_{41}, \ \Xi_{99} = -R_{21} - R_{41}, \\ \Xi_{1010} = & -R_{12} - R_{32} - R_{42}, \ \Xi_{1110} = R_{42} - X_2^T \\ \Xi_{1111} = & -2R_{42} + X_2^T + X_2 + \delta_{2M}(\rho + \lambda)H^TC^T\Phi_2CH, \\ \Xi_{1211} = & -X_2^T + R_{42}, \ \Xi_{1212} = -R_{22} - R_{42}, \ H = \begin{bmatrix} I & 0 \end{bmatrix}. \end{split}$$

Proof. Select the Lyapunov-Krasovskii functional as

$$V(t) = \tilde{\zeta}^T(t)P\tilde{\zeta}(t) + \sum_{n=0}^{n=2} V_n(t) + \lambda \phi(t), \tag{26}$$

where

$$\begin{split} V_{0}(t) &= \int\limits_{t-d_{0m}}^{t} \tilde{\zeta}^{T}(s) R_{10} \tilde{\zeta}(s) ds + \int\limits_{t-d_{0M}}^{t} \tilde{\zeta}^{T}(s) R_{20} \tilde{\zeta}(s) ds \\ &+ (d_{0M} - d_{om}) \int\limits_{-d_{0M}}^{-d_{om}} \int\limits_{t+s}^{t} \tilde{\zeta}^{T}(v) R_{40} \tilde{\zeta}(v) dv ds + d_{0m} \int\limits_{-d_{0m}}^{0} \int\limits_{t+s}^{t} \tilde{\zeta}^{T}(v) R_{30} \tilde{\zeta}(v) dv ds, \\ V_{1}(t) &= \int\limits_{t-d_{1m}}^{t} \tilde{\zeta}^{T}(s) R_{11} \tilde{\zeta}(s) ds + \int\limits_{t-d_{1M}}^{t} \tilde{\zeta}^{T}(s) R_{21} \tilde{\zeta}(s) ds \\ &+ (d_{1M} - d_{1m}) \int\limits_{-d_{1M}}^{-d_{1m}} \int\limits_{t+s}^{t} \tilde{\zeta}^{T}(v) R_{41} \tilde{\zeta}(v) dv ds + d_{1m} \int\limits_{-d_{1m}}^{0} \int\limits_{t+s}^{t} \tilde{\zeta}^{T}(v) R_{31} \tilde{\zeta}(v) dv ds, \\ V_{2}(t) &= \int\limits_{t-d_{2m}}^{t} \tilde{\zeta}^{T}(s) R_{12} \tilde{\zeta}(s) ds + \int\limits_{t-d_{2M}}^{t} \tilde{\zeta}^{T}(v) R_{42} \tilde{\zeta}(v) dv ds + d_{2m} \int\limits_{-d_{2m}}^{0} \int\limits_{t+s}^{t} \tilde{\zeta}^{T}(v) R_{32} \tilde{\zeta}(v) dv ds. \end{split}$$

By following the similar proof process in Lemma 2.2 in [25], the positiveness of  $\phi(t)$  can be derived. Then, it is easy to ensure V(t) > 0.

Computing the derivative of V(t), one can obtain

$$\dot{V}(t) = \dot{\xi}^{T}(t)P\tilde{\xi}(t) + \tilde{\xi}^{T}(t)P\dot{\tilde{\xi}}(t) + \sum_{n=0}^{2} \dot{V}_{n}(t) + \lambda\dot{\phi}(t) 
= \dot{\tilde{\xi}}^{T}(t)P\tilde{\xi}(t) + \tilde{\xi}^{T}(t)P\dot{\tilde{\xi}}(t) + \sum_{n=0}^{2} \dot{V}_{n}(t) + \lambda\left(-\rho\phi(t) - \sum_{n=0}^{N} \psi_{n}(t)\right),$$
(27)

where

$$\begin{split} \dot{V}_{0}(t) = & \tilde{\xi}^{T}(t) R_{10} \tilde{\xi}(t) + \tilde{\xi}^{T}(t) R_{20} \tilde{\xi}(t) + d_{0m}^{2} \dot{\tilde{\xi}}^{T}(t) R_{30} \dot{\tilde{\xi}}(t) - d_{0m} \int_{t-d_{0m}}^{t} \dot{\tilde{\xi}}^{T}(s) R_{30} \dot{\tilde{\xi}}(s) ds - (d_{0M} - d_{0m}) \int_{t-d_{0M}}^{t-d_{0m}} \dot{\tilde{\xi}}^{T}(s) R_{40} \dot{\tilde{\xi}}(s) ds \\ & - \tilde{\xi}^{T}(t-d_{0m}) R_{10} \tilde{\xi}(t-d_{0m}) - \tilde{\xi}^{T}(t-d_{0M}) R_{20} \tilde{\xi}(t-d_{0M}) + (d_{0M} - d_{0m})^{2} \dot{\tilde{\xi}}^{T}(t) R_{40} \dot{\tilde{\xi}}(t), \\ \dot{V}_{1}(t) = & \tilde{\xi}^{T}(t) R_{11} \tilde{\xi}(t) + \tilde{\xi}^{T}(t) R_{21} \tilde{\xi}(t) + d_{1m}^{2} \dot{\tilde{\xi}}^{T}(t) R_{31} \dot{\tilde{\xi}}(t) - d_{1m} \int_{t-d_{1m}}^{t} \dot{\tilde{\xi}}^{T}(s) R_{31} \dot{\tilde{\xi}}(s) ds - (d_{1M} - d_{1m}) \int_{t-d_{1M}}^{t-d_{1m}} \dot{\tilde{\xi}}^{T}(s) R_{41} \dot{\tilde{\xi}}(s) ds \end{split}$$

$$-\tilde{\zeta}^T(t-d_{1m})R_{11}\tilde{\zeta}(t-d_{1m}) - \tilde{\zeta}^T(t-d_{1M})R_{21}\tilde{\zeta}(t-d_{1M}) + (d_{1M}-d_{1m})^2\dot{\tilde{\zeta}}^T(t)R_{41}\dot{\tilde{\zeta}}(t),$$

$$\dot{V}_{2}(t) = \tilde{\zeta}^{T}(t)R_{12}\tilde{\zeta}(t) + \tilde{\zeta}^{T}(t)R_{22}\tilde{\zeta}(t) + d_{2m}^{2}\dot{\zeta}^{T}(t)R_{32}\dot{\zeta}(t) - d_{2m}\int_{t-d_{2m}}^{t}\dot{\zeta}^{T}(s)R_{32}\dot{\zeta}(s)ds - (d_{2M} - d_{2m})\int_{t-d_{2M}}^{t-d_{1m}}\dot{\zeta}^{T}(s)R_{42}\dot{\zeta}(s)ds$$

$$-\tilde{\zeta}^T(t-d_{2m})R_{12}\tilde{\zeta}(t-d_{2m}) - \tilde{\zeta}^T(t-d_{2M})R_{22}\tilde{\zeta}(t-d_{2M}) + (d_{2M}-d_{2m})^2\dot{\tilde{\zeta}}^T(t)R_{42}\dot{\tilde{\zeta}}(t).$$

According to the well-known Jensen inequality, for n = 0, 1, 2, it leads to

$$-d_{nm} \int_{t-d_{nm}}^{t} \dot{\xi}^{T}(s) R_{3n} \dot{\xi}(s) ds \le -[*] R_{3n} [\tilde{\xi}(t) - \tilde{\xi}(t - d_{nm})]. \tag{28}$$

With the aid of Lemma 1, for n = 0, 1, 2, we have

$$-(d_{nM} - d_{nm}) \int_{t-d_{nM}}^{t-d_{nm}} \dot{\xi}^{T}(s) R_{4n} \dot{\xi}(s) ds \le -\iota_{n}^{T}(t) \Omega_{n} \iota_{n}(t), \tag{29}$$

where

$$\begin{split} & \iota_n(t) = \begin{bmatrix} \tilde{\zeta}^T(t-d_{nm}) & \tilde{\zeta}^T(t-d_n(t)) & \tilde{\zeta}^T(t-d_{nM}) \end{bmatrix}^T, \\ & \Omega_n = \begin{bmatrix} R_{4n} & * & * \\ -R_{4n} + X_n^T & 2R_{4n} - X_n^T - X_n & * \\ -X_n^T & X_n^T - R_{4n} & R_{4n} \end{bmatrix}. \end{split}$$

According to the presented DSETS (9), it leads to

$$\sum_{n=0}^{2} \psi_n(t) < \lambda \phi(t). \tag{30}$$

By combining (30) and (27), it gives

$$\dot{V}(t) \le \dot{\tilde{\zeta}}^{T}(t)P\tilde{\zeta}(t) + \tilde{\zeta}^{T}(t)P\dot{\tilde{\zeta}}(t) + \sum_{n=0}^{2} \dot{V}_{n}(t) - (\rho + \lambda) \sum_{n=0}^{N} \psi_{n}(t), \tag{31}$$

which further yields

$$\dot{V}(t) \le \dot{\bar{\zeta}}^{T}(t)P\tilde{\zeta}(t) + \tilde{\zeta}^{T}(t)P\dot{\bar{\zeta}}(t) + \sum_{n=0}^{2} \dot{V}_{n}(t) + (\rho + \lambda) \sum_{n=0}^{2} \left(\delta_{nM}[*]\Phi_{n}y(l_{k}h - nh) - [*]\Phi_{n}e_{n}(t)\right)$$
(32)

for  $\delta_n(lh) \leq \delta_{nM}$ .

Define the following augmented vector

$$\xi(t) = [\tilde{\xi}^{T}(t), \tilde{\xi}^{T}(t), \tilde{\xi}^{T}(t - d_{0m}), \tilde{\xi}^{T}(t - d_{0}(t)), \tilde{\xi}^{T}(t - d_{0M}), g^{T}(t), \tilde{\xi}^{T}(t - d_{1m}), \tilde{\xi}^{T}(t - d_{1}(t)), \\ \tilde{\xi}^{T}(t - d_{1M}), \tilde{\xi}^{T}(t - d_{2m}), \tilde{\xi}^{T}(t - d_{2}(t)), \tilde{\xi}^{T}(t - d_{2M}), e_{0}^{T}(t), e_{1}^{T}(t), e_{1}^{T}(t), f^{T}(t)]^{T}.$$

$$(33)$$

For the filtering error system (19) and the constructed relax matrix  $\mathcal{G}$ , it yields

$$\sum_{i=1}^{r} \sum_{j=1}^{r} \mu_{i} \mu_{j}^{k} \xi^{T}(t) (\mathcal{G}S_{ij} + S_{ij}^{T} \mathcal{G}^{T}) \xi(t) = 0,$$
(34)

where

Taking the mathematical expectation of (34) results in

$$\mathbb{E}\left\{ \ \sum_{i=1}^r \sum_{i=1}^r \mu_i \mu_j^k \xi^T(t) (\mathcal{C}S_{ij} + S_{ij}^T \mathcal{C}^T) \xi(t) \right\}$$

$$= \mathbb{E}\left\{\sum_{i=1}^{r}\sum_{j=1}^{r}\mu_{i}\mu_{j}^{k}\xi^{T}(t)(\mathscr{G}\mathcal{S}_{ij} + \mathcal{S}_{ij}^{T}\mathscr{G}^{T})\xi(t)\right\} = 0,$$
(35)

where

$$\mathcal{S}_{ij} = \begin{bmatrix} -I, \ \mathcal{A}_{ij}, \ 0, \ \mathcal{B}_{dj}, \ 0, \ \mathcal{B}_{gi}, \\ \\ 0, \ 0, \ 0, \ 0, \ 0, \ 0, \ \mathcal{B}_{j}, \ 0, \ 0, \ 0, \ \bar{\alpha}_{1} \mathcal{B}_{j} \end{bmatrix}.$$

From (32) and (35), we have

$$\mathbb{E}\left\{\dot{V}(t) + \varsigma^{T}(t)\varsigma(t) - \gamma^{2}g^{T}(t)g(t)\right\} \leq \mathbb{E}\left\{\sum_{i=1}^{r} \sum_{j=1}^{r} \mu_{i}\mu_{j}^{k}\xi^{T}(t)\Xi^{ij}\xi(t)\right\}. \tag{36}$$

In terms of  $\sum_{j=1}^{r} \mu_j = \sum_{j=1}^{r} \mu_j^k = 1$ , it yields

$$\sum_{i=1}^{r} \sum_{j=1}^{r} \mu_i (\mu_j - \mu_j^k) \Gamma_i = \sum_{i=1}^{r} \mu_i (\sum_{j=1}^{r} \mu_j - \sum_{j=1}^{r} \mu_j^k) \Gamma_i = 0.$$
(37)

By combining (36) and (37), one gets

$$\mathbb{E}\left\{\sum_{i=1}^{r}\sum_{j=1}^{r}\mu_{i}\mu_{j}^{k}\xi^{T}(t)\Xi^{ij}\xi(t)\right\} = \mathbb{E}\left\{\sum_{i=1}^{r}\sum_{j=1}^{r}[\mu_{i}\mu_{j}^{k}\xi^{T}(t)\Xi^{ij}\xi(t) + \mu_{i}(\mu_{j} - \mu_{j}^{k})\xi^{T}(t)\Gamma_{i}\xi(t)]\right\} \\
= \mathbb{E}\left\{\sum_{i=1}^{r}\sum_{j=1}^{r}[\mu_{i}\mu_{j}^{k}\xi^{T}(t)(\Xi^{ij} - \Gamma_{i})\xi(t) + \mu_{i}\mu_{j}\xi^{T}(t)\Gamma_{i}\xi(t)]\right\}.$$
(38)

By considering  $\mu_i^k \ge \rho_j \mu_j$  and  $\Xi^{ij} - \Gamma_i < 0$ , it gives

$$\mathbb{E}\left\{\sum_{i=1}^{r}\sum_{j=1}^{r}\mu_{i}\mu_{j}^{k}\xi^{T}(t)\Xi^{ij}\xi(t)\right\} = \mathbb{E}\left\{\sum_{i=1}^{r}\sum_{j=1}^{r}[\mu_{i}\mu_{j}^{k}\xi^{T}(t)\Xi^{ij}\xi(t) + \mu_{i}(\mu_{j} - \mu_{j}^{k} + \rho_{j}\mu_{j} - \rho_{j}\mu_{j})\xi^{T}(t)\Gamma_{i}\xi(t)]\right\} \\
= \mathbb{E}\left\{\sum_{i=1}^{r}\sum_{j=1}^{r}[\mu_{i}\mu_{j}\xi^{T}(t)(\Gamma_{i} - \rho_{j}\Gamma_{i} + \rho_{j}\Xi^{ij})\xi(t) + (-\mu_{i}\mu_{j}^{k} + \mu_{i}\mu_{j}\rho_{j})\xi^{T}(t)\Gamma_{i}\xi(t) + (\mu_{i}\mu_{j}^{k} - \mu_{i}\mu_{j}\rho_{j})\xi^{T}(t)\Xi^{ij}\xi(t)]\right\} \\
= \mathbb{E}\left\{\sum_{i=1}^{r}\sum_{j=1}^{r}[\mu_{i}\mu_{j}\xi^{T}(t)(\Gamma_{i} - \rho_{j}\Gamma_{i} + \rho_{j}\Xi^{ij})\xi(t) + \mu_{i}(\mu_{j}^{k} - \rho_{j}\mu_{j})\xi^{T}(t)(\Xi^{ij} - \Gamma_{i})\xi(t)]\right\} \\
\leq \mathbb{E}\left\{\sum_{i=1}^{r}\mu_{i}^{2}\xi^{T}(t)(\rho_{i}\Xi^{ii} - \rho_{i}\Gamma_{i} + \Gamma_{i})\xi(t) + \sum_{i=1}^{r}\mu_{i}(\mu_{j}^{k} - \rho_{j}\mu_{j})\xi^{T}(t)(\Xi^{ij} - \Gamma_{i})\xi(t) + \sum_{i=1}^{r}\sum_{j=1}^{r}\xi^{T}(t)(\rho_{j}\Xi^{ij} + \rho_{i}\Xi^{ji} - \rho_{j}\Gamma_{i} - \rho_{i}\Gamma_{j} + \Gamma_{i} + \Gamma_{j})\xi(t)\right\}. \tag{39}$$

According to the conditions (23)-(25), it leads to

$$\mathbb{E}\left\{\dot{V}(t) + \varsigma^{T}(t)\varsigma(t) - \gamma^{2}g^{T}(t)g(t)\right\} < 0,\tag{40}$$

which ensures  $\mathbb{E}\{\dot{V}(t)\}<0$  for g(t)=0.

By integrating (40) over  $[0, +\infty)$ , one has

$$\mathbb{E}\{V(\infty) - V(0)\} < \mathbb{E}\Big\{\int_{0}^{\infty} (\gamma^2 g^T(t)g(t) - \varsigma^T(t)\varsigma(t))dt\Big\}. \tag{41}$$

In addition, for  $g(t) \neq 0$  and zero initial state, it yields  $\mathbb{E}\left\{\int_0^\infty \varsigma^T(t)\varsigma(t)dt\right\} \leq \mathbb{E}\left\{\int_0^\infty \gamma^2 g^T(t)g(t)\right\}$  based on (41). Then, the filtering error system is stable in mean square sense with given  $H_\infty$  index  $\gamma$ .  $\square$ 

**Remark 4.** It is noted that the complexity of the conditions in the above theorem are related to the number of historical sampled outputs N. In order to simplify the derivations, N=2 is considered in this work to show the effectiveness of the proposed DSETS. Meanwhile, the similar results for different N can be obtained easily based on the above theorem.

Based on the results of Theorem 1, we are in position to design the filter gains and triggering matrices.

**Theorem 2.** For given scalars h,  $\bar{d}$ ,  $d_{0m}$ ,  $d_{0m}$ ,  $d_{1m}$ ,  $d_{1m}$ ,  $d_{2m}$ ,  $d_{2m}$ ,  $d_{2m}$ ,  $\kappa$ ,  $\bar{\alpha}_1$ ,  $\beta$ ,  $\gamma$ ,  $\delta_{0M}$ ,  $\delta_{1M}$ ,  $\delta_{2M}$ , under the DSETS (9), the filtering error system (20) is stable in mean square sense with prescribed  $H_{\infty}$  performance index  $\gamma$  if there exist symmetric matrices P > 0,  $R_{1n} > 0$ ,  $R_{2n} >$ 

$$R_{3n} > 0$$
,  $R_{4n} > 0$ ,  $\Phi_n > 0$ ,  $\begin{bmatrix} R_{4n} & X_n \\ * & R_{4n} \end{bmatrix} > 0$  and matrices  $\tilde{\Gamma}_i$  ( $i = 1, 2, ..., r$ ),  $X_n$ ,  $n = 0, 1, 2$ ,  $G_1$ ,  $G_2$  and  $G_3$  such that

$$\tilde{\Xi}^{ij} - \tilde{\Gamma}_i < 0, \tag{42}$$

$$\rho_i \tilde{\Xi}^{(i)} - \rho_i \tilde{\Gamma}_i + \tilde{\Gamma}_i < 0 \ (i = j), \tag{43}$$

$$\rho_i \tilde{\Xi}^{ij} + \rho_i \tilde{\Xi}^{ji} - \rho_i \tilde{\Gamma}_i - \rho_i \tilde{\Gamma}_i + \tilde{\Gamma}_i + \tilde{\Gamma}_i \le 0 \ (i < j), \tag{44}$$

where

Then, the filter gains are computed as  $A_{fj} = G_2^{-1} \mathscr{G}_{Aj}$ ,  $B_{fj} = G_2^{-1} \mathscr{G}_{Bj}$ ,  $C_f = \mathscr{C}_f$ .

**Proof.** On the basis of Theorem 1, the matrix G is constructed as

$$G = \begin{bmatrix} G_1 & G_2 \\ G_3 & \kappa G_2 \end{bmatrix}. \tag{45}$$

Define two new variables  $\mathscr{G}_{Aj} = G_2 A_{fj}$  and  $\mathscr{G}_{Bj} = G_2 B_{fj}$ . Then, some matrices in the conditions in Theorem 1 are rewritten as

$$G\mathscr{A}_{ij} = \begin{bmatrix} G_1 A_i & \mathscr{G}_{Aj} \\ G_3 A_i & \kappa \mathscr{G}_{Aj} \end{bmatrix}, G\mathscr{B}_{dj} = \begin{bmatrix} \mathscr{G}_{Bj} C & 0 \\ \kappa \mathscr{G}_{Bj} C & 0 \end{bmatrix},$$

$$G\mathscr{B}_{gi} = \begin{bmatrix} G_1 B_i \\ G_3 B_j \end{bmatrix}, G\mathscr{B}_j = \begin{bmatrix} \mathscr{G}_{Bj} \\ \kappa \mathscr{G}_{Bi} \end{bmatrix}.$$

$$(46)$$

Substituting the above matrices (46) into the conditions (23)-(25), one can obtain the conditions (42)-(44). Moreover, the filtering gains are calculated by

Table 1 WTS Parameters.

Parameter	Value	Parameter	Value
J	$5 \times 10^{-3} N \cdot m$	R	0.5m
$\mathfrak{F}$	$1.2kg/m^2$	$Y_f$	0.16Wb
$\epsilon$	$6^{-3}N \cdot m \cdot s/rad$	$\mathfrak{R}_s$	$1.13\Omega$
$\mathfrak{L}_d = \mathfrak{L}_a$	$2.7 \times 10^{-3}$ H	$M_1$	-10
$P_n$	8	$M_2$	10

$$A_{fj} = G_2^{-1} \mathcal{G}_{Aj}, \quad B_{fj} = G_2^{-1} \mathcal{G}_{Bj}, \quad C_f = \mathcal{C}_f. \quad \Box$$

#### 4. Simulation example

In the simulation, the system parameters are chosen the same with [4] and shown in Table 1.

In addition, different from the existing T-S fuzzy WTSs modeled under the maximum power case of generator, the time-varying power output related to  $\Theta_p(\chi(t), \nu)$  and  $\chi(t)$  is considered. According to relationship between  $\Theta_p(\chi(t), \nu)$  and  $\chi(t)$  shown in [35], their values belong to  $\Theta_p(\chi(t), \nu) \in [0.14, 0.48]$  and  $\chi(t) \in [5.5, 8.1]$ . Therefore, the bounds of  $\vartheta_2(t) = T_0(t)w_g(t)$  can be computed as  $M_3 = -1.55 \times 10^{-4}$  and  $M_4 = 1.7 \times 10^{-3}$ .

According to the system parameters, and the stabilization controller  $\mathcal{K} = \begin{bmatrix} 0.1 & -0.2 & 0.3 \\ -0.1 & -0.1 & 0.1 \end{bmatrix}$ , the resulting closed-loop system parameters are derived and selected as:

$$\begin{split} A_1 &= \begin{bmatrix} -1.1690 & 384 & 0 \\ -437.037 & -492.5926 & 191.1111 \\ -37.037 & 42.963 & -381.4815 \end{bmatrix}, B_1 = \begin{bmatrix} 10 \\ 5 \\ 0 \end{bmatrix}, \\ A_2 &= \begin{bmatrix} -0.8601 & 384 & 0 \\ -437.037 & -492.5926 & 191.1111 \\ -37.037 & 42.9630 & -381.4815 \end{bmatrix}, B_2 = B_1, \\ A_3 &= \begin{bmatrix} -1.2310 & 384 & 0 \\ -437.037 & -492.5926 & 31.1111 \\ -37.037 & -117.037 & -381.4815 \end{bmatrix}, B_3 = B_1, \\ A_4 &= \begin{bmatrix} -1.5399 & 384 & 0 \\ -437.037 & -492.5926 & 31.1111 \\ -37.037 & -117.037 & -381.4815 \end{bmatrix}, B_4 = B_1, \\ C &= \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}, L &= \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}. \end{split}$$

The upper bound of communication delay d(t) is  $\bar{d}=0.01s$ , and then we have  $d_{0m}=0$ ,  $d_{0M}=0.01$ ,  $d_{1m}=0.01$ ,  $d_{1M}=0.02$ ,  $d_{2m}=0.02$ ,  $d_{2M}=0.03$ . The deception attack is considered as  $f(t)=0.4 \tanh(y(t))$  with the upper bound F=0.4 and the attack probability  $\bar{\alpha}_1=0.4$ . By selecting the  $H_{\infty}$  performance index  $\gamma=0.5$ , the sampling period h=0.001s,  $\rho_1=\rho_2=0.9$ ,  $\rho_3=\rho_4=0.8$ ,  $\kappa=0.8$ ,  $\beta=100$ ,  $\delta_{0M}=0.2$ ,  $\delta_{1M}=0.25$ ,  $\delta_{2M}=0.3$ ,  $\delta_{0m}=\delta_{1m}=\delta_{2m}=0.01$ ,  $\rho=10$  and  $\lambda=1$ , the filter gains and triggering matrices are solved by the conditions in Theorem 2 as:

Theorem 2 as: 
$$A_{f1} = \begin{bmatrix} -81.7118 & 382.2605 & 45.9453 \\ -276.8115 & -470.0202 & 118.4243 \\ -21.3586 & -33.6373 & -229.1419 \end{bmatrix}, \ B_{f1} = \begin{bmatrix} -0.1217 \\ 0.0807 \\ 0.0220 \end{bmatrix},$$
 
$$A_{f2} = \begin{bmatrix} -81.4213 & 382.8825 & 43.9002 \\ -277.1802 & -471.2154 & 114.9926 \\ -21.2805 & -34.1347 & -230.6150 \end{bmatrix}, \ B_{f2} = \begin{bmatrix} -0.1195 \\ 0.0697 \\ 0.0150 \end{bmatrix},$$
 
$$A_{f3} = \begin{bmatrix} -79.3965 & 382.8308 & 24.8882 \\ -276.3782 & -468.5242 & 91.9536 \\ -21.8027 & -36.7193 & -230.6663 \end{bmatrix}, \ B_{f3} = \begin{bmatrix} -0.1521 \\ 0.0585 \\ -0.0002 \end{bmatrix},$$
 
$$A_{f4} = \begin{bmatrix} -80.6052 & 383.9292 & 28.5569 \\ -275.9081 & -469.2693 & 85.6995 \\ -21.8062 & -37.2931 & -232.7477 \end{bmatrix}, \ B_{f4} = \begin{bmatrix} -0.1309 \\ 0.0285 \\ -0.0091 \end{bmatrix},$$
 
$$C_{f} = \begin{bmatrix} -0.4263 & 0.2481 & -0.0280 \end{bmatrix}, \ \Phi_{0} = 0.0080, \ \Phi_{1} = 0.1496, \ \Phi_{2} = 0.1247.$$

The external disturbance is considered as  $g(t) = \begin{cases} \mathscr{U}(t), & 0 < t < 0.2s \\ 0, & t \geq 0.2s \end{cases}$ , and  $\mathscr{U}(t)$  is a random variable obeying  $|\mathscr{U}(t)| \leq 10$ . The initial state of the system is  $\zeta(0) = \begin{bmatrix} 5 & -2 & 3 \end{bmatrix}^T$ , and the initial state of filter is  $\zeta_f(0) = \begin{bmatrix} 4 & -1 & 2 \end{bmatrix}^T$ . In the case where deception

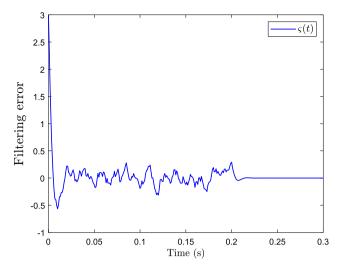


Fig. 2. The filtering error under DSETS.

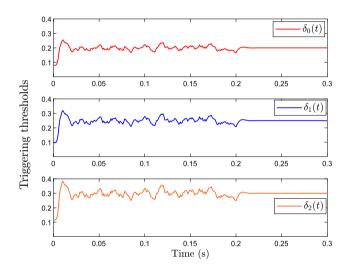


Fig. 3. The dynamic triggering thresholds  $\delta_u(t)$  under DSETS. (For interpretation of the colors in the figure(s), the reader is referred to the web version of this article.)

Table 2
The amount of triggering times.

Methods	Amount	
Our DSETS	39	
Existing ETS [30]	109	
Normal ETS [21]	120	

attacks happening, the curve of filtering error is depicted in the Fig. 2, and the dynamic triggering thresholds, the triggering instants are drawn in Fig. 3 and Fig. 4, respectively. These results denote that the proposed event-triggered filtering method is effective to stabilize the filtering error WTS once the system is invaded by deception attacks.

In the following, the advantage of the presented DSETS over the existing ETSs [30,21] is illustrated. According to the above chosen parameters and derived filter gains, the filtering errors and triggering instants obtained by three ETSs are shown in Fig. 5, Fig. 6 and Table 2, respectively.

It is observed from Fig. 5 and Fig. 6 that the responses of filtering error under three different ETSs are very similar, while the triggering times generated by our DSETS are decreased significantly. As shown in Table 2, the proposed DSETS decreases 64.2% and 67.5% transmissions compared to the existing ETS [30] and normal ETS [21], respectively. This indicates that the presented DSETS utilizing an auxiliary dynamic variable and some past measured outputs has better ability to decrease unnecessary triggering events and save more network resources than some existing ETSs.

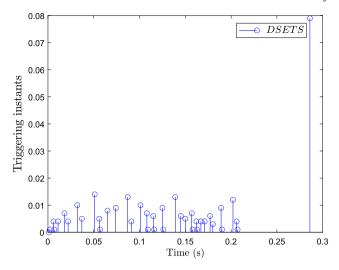
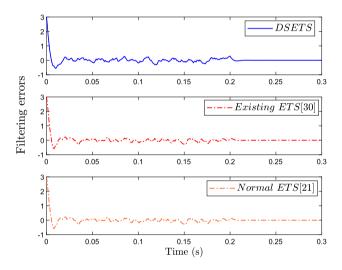


Fig. 4. The triggering intervals under DSETS.



 $\textbf{Fig. 5.} \ \ \textbf{The filtering error under different ETSs}.$ 

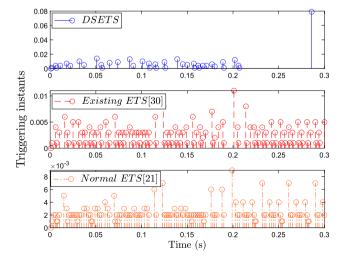


Fig. 6. The triggering intervals under different ETSs.

#### 5. Conclusion

This paper concentrates on the event-triggered  $H_{\infty}$  filter design for T-S fuzzy WTS against deception attacks and communication delays. In order to reduce communication and computation load, an innovative DSETS has been presented whose triggering rule is constructed by using some previous sampled measurements and an auxiliary dynamic variable. Meanwhile, compared with some ETSs with constant triggering thresholds, our DSETS includes dynamic triggering thresholds, which can be adjusted along with system outputs adaptively. Resorting to the Lyapunov–Krasovskii method, some novel co-design conditions for solving  $H_{\infty}$  filter gains and triggering matrices are deduced. The merits of the provided theory are demonstrated via some simulations.

#### CRediT authorship contribution statement

Shen Yan: Writing – review & editing, Writing – original draft, Supervision, Investigation, Funding acquisition, Conceptualization. Xinyi Yang: Writing – original draft, Methodology, Investigation. Zhou Gu: Writing – review & editing, Methodology, Investigation. Xiangpeng Xie: Writing – review & editing, Investigation, Formal analysis. Fan Yang: Writing – review & editing, Software, Investigation.

#### **Declaration of competing interest**

The authors declare that no conflicts of interest exist in the submission of this manuscript.

#### Data availability

No data was used for the research described in the article.

#### Acknowledgements

This work was supported in part by the National Natural Science Foundation of China under Grant 62103193, Grant 62273183, Grant 62373196 and Grant 62303227, in part by the Natural Science Foundation of Jiangsu Province of China under Grant BK20200769, Grant BK20231288, Grant BK20231286, in part by the Startup Funding of Anhui Polytechnic University under Grant 2024YQQ001.

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