Quasi-Consensus Control for Stochastic Multiagent Systems: When Energy Harvesting Constraints Meet Multimodal FDI Attacks

Bin Wei, Engang Tian, *Member, IEEE*, Zhou Gu, *Member, IEEE*, Junyong Zhai, *Senior Member, IEEE*, and Dong Liang

***Abstract*—In this article, the quasi-consensus control problem is investigated for a class of stochastic nonlinear time-varying multiagent systems (MASs). The innovation points of this research can be highlighted as follows: first of all, the dynamics of the plant are stochastic, nonlinear, and time varying, which resembles the natural systems in practice closely. Meanwhile, an energy harvesting protocol is put forward to collect adequate energy from the external environment. Second, as a generalization of the existing result, the ultimate control objective is quasi-consensus in a probabilistic sense, that is, designing a distributed control protocol in order that the probability of centering the allowable region for the states of each agent is larger than some predetermined values. Third, the MASs are subject to false data-injection (FDI) attacks, and a more general multimodal FDI model is proposed. On the basis of the probabilistic-constrained analysis technique and the recursive linear matrix inequalities (RLMIs), sufficient conditions are provided to guarantee the probabilistic quasi-consensus property. To derive the controller gains, an optimal probabilistic-constrained algorithm is designed by solving a convex optimization problem. Finally, two examples are provided to substantiate the validity of the proposed framework.**

***Index Terms*—Energy harvesting constraints (EHCs), false data injection (FDI), nonlinear multiagent systems (MASs), quasi-consensus.**

I. Introduction

IN THE past few decades, the group motion behaviors, such as consensus, rendezvous, and flocking, etc., have been paid numerous attention due to their expansive real-world applications [1], [2], [3], [4], [5], [6], [7]. For example, time-varying formation control of unmanned aerial vehicles (UAVs) [8], formation learning control of autonomous underwater vehicles [9], and motion synchronization of multiple surface vessels [10] have been investigated. Frankly speaking, the main objective of consensus is to design specific control protocols to drive all agents to achieve an agreement via co-sharing local neighbor information subject to communication topologies. Commonly, a rich body of research outcomes have been reported concerning linear multiagent systems (MASs) under different network conditions (see [11] and [12]). Recently, the quasi-consensus problem has stirred initial interest [13], [14]. However, related references concentrating on nonlinear MASs are still scarce. Hence, it is still a challenging and knotty issue to propose valid control approaches for nonlinear MASs.

First of all, it is noticeable to point out that, up to now, in the majority of the existing literature, the considered MASs are assumed to be time invariant [15], [16], the essentially important problem is to guarantee the state error between adjoin subsystems remains within a fixed set (the size of the set depends on explicit design parameters, bounded uncertainties, and external disturbances). However, for stochastic systems, it is impossible to confine the state error by means of designing the corresponding distributed quasi-consensus controller since the existence of unpredictable factors, such as sudden temperature fluctuation, operation point shifting, and gradual aging of system components. To date, some previous studies for time-varying systems have been conducted [17], [18], [19]. In [18], the recursive filtering problem for a kind of time-varying systems has been investigated. Nevertheless, when it comes to the distributed quasi-consensus problem of *time-varying* MASs, the result is quite limited. Moreover, it should be mentioned that the aforementioned systems are deterministic ones. When *stochastic* factors are taken into consideration, the quasi-consensus control methods become more complicated than deterministic counterparts. In [19], the consensus issue in the mean square sense has been studied for a type of discrete-time stochastic MASs. To sum up, there exist limited results on the quasi-consensus problem for stochastic nonlinear time-varying MASs.

Second, for stochastic systems, it is usually required that the variance of the state error between adjacent agents should be bounded or minimized in the mean square sense [20],

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[21], that is, the state error between adjacent agents satisfies *variance-constrained* performance indicator. However, variance-constrained requirement still has some drawbacks, for instance, when there exist random occurring large or unbounded noises in systems, it is hard to find the exact variance bounds. In this case, an alleged *probabilistic-constrained* method utilized in [22], [23], and [24] is preferred. The objective is to design the controller such that the probability of some variables staying within a confined region is bigger than a fixed value. For example, Wei et al. [22] have designed an elegant tracking controller for time-varying systems and it can be guaranteed that the probability of the tracking error falling into a given sphere is larger than a given constant. In view of the foregoing discussion, the probabilistic quasi-consensus control problem has not been paid considerable attention for the targeted MASs, and this constitutes the main motivation of the current investigation.

Third, it is well-known that agents interact mutually through a predetermined communication topology in MASs. Undoubtedly, the communication channel brings not only convenience for stream transmission, but also adds security threats inevitably, including, but not limited to DoS attacks [25], [26], [27], [28], deception attacks [29], and replay attacks [30]. Without loss of generality, false data-injection (FDI) attacks are one of the noteworthy cyber attacks where the adversaries hijack and tamper with the transmitted signal from its correct value to the undermined signal. It should be emphasized that in the bulk of the established attack models, the manipulated signal is usually replaced by a bounded nonlinear term. Regretfully, two significant factors are mostly overlooked: 1) the attacker should adopt diverse attack types smartly in order to bypass the examination of the embedded detector and 2) the energy confinement for finite numbers of attack behaviors have been ignored in most of the previous investigation. In the current research, in comparison with the single deception attack, a novel deception attack type called multimodal FDI (MFDI) model is proposed whose purpose is to puzzle the detection of the detector by launching modified signals of different modals. In addition, the energy limitation of such attack manner is also taken into consideration. Consequently, it is vital and sensible to create the MFDI model for the considered MASs, which encourages us to analyze profoundly.

In the system monitoring and controlling process, energy supply is an obligatory module because the information delivery truly requests ample energy depletion, and it is also particularly indispensable for MASs where the information exchange is fairly frequent and intricate requiring large quantities of energy. Therefore, it is natural to put forward an energy-replenishing scenario for the sake of maintaining the operation over the network. Hence, an energy harvesting protocol is put forward whose goal is to collect adequate energy from the external environment through the harvester [31], [32]. To address the aforementioned issue, the energy harvesting techniques in filtering and estimation have raised much attention initially [33], [34], [35]. In [35], the power control schemes of harvesting sensors have been discussed in the background of remote state estimation, and the distributed filters fusion topic has been examined for time-varying multisensor systems with energy harvesting constraints (EHCs) and parameter uncertainty.

In terms of the discussion made above, we are set about addressing the finite-horizon quasi-consensus control issue for nonlinear MASs with multienergy harvesting sensors and MFDI. The main contributions of this article can be highlighted from the following four aspects.

1) In this article, the formulated quasi-consensus problem is quite comprehensive that takes into consideration the stochastic nonlinearities, the randomly occurring MFDI attacks, and the energy harvesting protocol, which better reflects the engineering reality.
2) Different from [20] and [21] investigating the traditional consensus control issue of the stochastic MASs, the quasi-consensus concept in the probability sense is introduced, which demands that the probability of the state error of arbitrary two agents falling into a predefined region is bigger than a given constant. For the stochastic factors of the concerned systems, in contrast to the conventional control strategies, the probabilistic-constrained method utilized in this article is obviously more suitable.
3) For the sake of increasing the diversity and stealthiness of the studied attacks, a dual stochastic MFDI attack model is developed in this article, whose objective is to enrich attack means by generating aggressive information containing different energy upper bounds. Meanwhile, the restriction of finite energy for attackers is analyzed quantitatively.
4) The dynamic of the energy amount of the sensor of the ith agent is comprehensively conducted and the probability distribution of the current energy level at each step is computed iteratively, which makes it more challenging in the design of the distributed quasi-consensus controller.

The remainder of this article is scheduled as follows. Section II elaborates the MASs model, the MFDI structure, and the energy harvested protocol in a unified framework. In Section III, sufficient conditions are derived to ensure the quasi-consensus in the probability sense, and the minimized constraint set is provided. A numerical example and an experimental example are conducted in Section IV. Eventually, this article is summarized in Section V with some concluding remarks.

Notations: The notation used, here, is standard except when otherwise stated. $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ denote the n dimensional Euclidean space and the set of all $n \times m$ real matrices. A^{T} denotes the transpose of matrix A. $X \geq Y$ represents X-Y is positive semidefinite, where X and Y are symmetric matrices. $\mathrm{tr}(R)$ denotes the trace of the matrix R and $\Pr\{\cdot\}$ means the occurrence probability of the event "." $\mathbb{E}\{z\}$ and $\mathbb{E}\{z|x\}$, respectively, denote expectation of z and expectation of z conditional on x. The functions h_k and φ_k are given matrices. In symmetric block matrices, "$*$" is adopted to imply an ellipsis for terms that are induced by symmetry. $\mathrm{diag}\{\cdots\}$ stands for a block-diagonal matrix. $[0, N]$ denotes that the set $\{0, 1, \ldots, N\}$, and the symbol "$\otimes$" implies the Kronecker product.

II. Problem Formulation

Without loss of generality, MASs are composed of N agents that co-sharing the network through a fixed communication topology denoted by a directed graph $\aleph = (\Im, \wp, \mathcal{A})$, where $\Im = \{1, 2, \ldots, N\}$ stands for the collection of agent nodes, $\wp \in \Im \times \Im$ represents the gathering of agent edges, and $\mathcal{A} = [a_{ij}]_{N\times N}$ refers to the weighted adjacency matrix with non-negative adjacency element a_{ij}. An edge of $\aleph$ is denoted by the ordered pair $(i, j) \in \wp$. The elements of $\mathcal{A}$ are connected with the information communication. Concretely, $a_{ij} > 0 \Longleftrightarrow (i, j) \in \wp$, which means the agent j can broadcast information to agent i. On the contrary, $a_{ij} = 0 \Longleftrightarrow (i, j) \notin \wp$. The neighborhood of agent t, that is, $\mathcal{N}_t = \{j \in \Im, (j, t) \in \wp\}$ is called a neighbor of agent t, and self-loops (i, i) are not allowed, that is, $a_{ii} = 0$. The in-degree of the mth agent is defined as $d_m = \sum_{j\in \mathcal{N}_m} a_{mj}$ and the in-degree matrix is given as $\mathcal{D} = \mathrm{diag}\{d_1, d_2, \ldots, d_N\}$. The Laplacian matrix of $\aleph$ is given as $\mathcal{L} = \mathcal{D} - \mathcal{A}$.

Consider a class of time-varying MASs with N agents in the finite horizon $[0, \Gamma]$

$$\begin{cases} x_{i,k+1} = A_k x_{i,k} + B_k u_{i,k} + D_k \nu_{i,k} + f_i(k, x_k, \vartheta_k) \\ y_{i,k} = C_k x_{i,k} \end{cases} \tag{1}$$

where $x_{i,k} \in \mathbb{R}^{n_x} (i = 1, 2, \ldots, N)$ and $u_{i,k} \in \mathbb{R}^{n_u}$ are the sate vector and the control input of the ith agent, respectively; $y_{i,k} \in \mathbb{R}^{n_y}$ is the measurement output of the ith agent; $\nu_{i,k}$ denotes N mutually independent noise sequences satisfying $\mathbb{E}\{\nu_{i,k}\} = 0$ and $\mathbb{E}\{\nu_{i,k}^2\} = V_i$. A_k, B_k, C_k, and D_k are known time-varying matrices with proper dimensions. $\nu_{i,k}$ and ϑ_k are mutually uncorrelated. For simplicity of expression, $f_i(k, x_k)$ is used to imply $f_i(k, x_k, \vartheta_k)$ in this article.

$f_i(k, x_k)$ represents stochastic nonlinearities which satisfy the following statistical traits:

$$\begin{aligned} &\mathbb{E}\{f_i(k, x_k)\} = 0 \\ &\mathbb{E}\Big\{f_i(k, x_k) f_j^{\mathrm{T}}(m, x_m)|x_m\Big\} = 0, \quad k \neq m, i \neq j \\ &\mathbb{E}\big\{f_i(k, x_k) f_i^{\mathrm{T}}(k, x_k)|x_k\big\} = \sum_{r=1}^{p} \varrho_{ir}\varrho_{ir}^{\mathrm{T}} x_{i,k}^{\mathrm{T}} T_{ir} x_{i,k} \end{aligned} \tag{2}$$

where ϱ_{ir} and $T_{ir} (r = 1, 2, \ldots, p)$ are known matrices with compatible dimensions. $p > 0$ is a given integer.

Remark 1: The stochastic nonlinear function $f_i(k, x_k)$ stands for some types of stochastic nonlinearities specially. For instance:

1) state-multiplicative noises, that is, $f_i(k, x_k) = \sum_{s=1}^{r} h_s^{\mathrm{T}} x_k \delta_k$, where δ_k is a vector denoting the noise with zero mean, $h_s \in \mathbb{R}^n$;
2) random sequences whose powers depend on sector-bounded nonlinear function of the state, such as $f_i(k, x_k) = \phi(x_k)\Im_k$, where $\|\phi(x_k)\| \leq h_k\|x_k\|$ and $\Im_k$ is a white noise sequence;
3) random vectors whose powers are decided by the symbol of a nonlinear function, for instance, $f_i(k, x_k) = h^{\mathrm{T}}(\mathrm{sign}(\varphi(x_k)))^{\mathrm{T}} x_k \varphi_k$, where $\mathrm{sign}(\cdot)$ represents the signum function.

A. Description of MFDI

In this article, we consider that the measurement output $y_{i,k}$ on the ith sensor, to be transmitted to the ith controller, is prone to be manipulated by false signals which occur in a random fashion. To this end, an innovative MFDI attack building is constructed to describe the concerned attack phenomenon. To utilize the random binary sequence to facilitate the description of the switching property of the concerned attacks, we introduce the following definition:

$$\gamma_{jk}^{i} = \begin{cases} 1, & \sigma(k) = j \\ 0, & \sigma(k) \neq j \end{cases} \tag{3}$$

where γ_{jk}^{i} is a set of stochastic variables to characterize the randomness of MFDI attacks, $\sigma(k)$ is a switching variable which belongs to the set $\{1, 2, \ldots, j, \ldots, m\}$.

Then, the aforementioned MFDI attack model can be formulated by

$$\bar{y}_{i,k} = \alpha_{i,k} y_{i,k} + \big(1 - \alpha_{i,k}\big) \sum_{j=1}^{m} \gamma_{jk}^{i} \Re_j\big(y_{i,k}\big) \tag{4}$$

where $\alpha_{i,k} (i = 1, 2, \ldots, N)$ and $\gamma_{jk}^{i} (j = 1, 2, \ldots, m)$ follow Bernoulli distribution law and satisfy:

$$\begin{aligned} &\mathbb{E}\big\{\alpha_{i,k}\big\} = \bar{\alpha}_i, \mathbb{E}\Big\{\gamma_{jk}^{i}\Big\} = \bar{\gamma}_j^{i}, \mathbb{E}\big\{\gamma_k^{i}\big\} = \bar{\gamma}^{i} \\ &\sum_{j=1}^{m} \gamma_{jk}^{i} = 1, \sum_{j=1}^{m} \bar{\gamma}_j^{i} = 1, 0 \leq \bar{\alpha}_i, \bar{\gamma}_j^{i} \leq 1. \end{aligned} \tag{5}$$

The falsified signals $\Re_j(y_{i,k})$ are assumed to be known as

$$\Re_j^{\mathrm{T}}\big(y_{i,k}\big)\Re_j\big(y_{i,k}\big) \leq y_{i,k}^{\mathrm{T}} \mathcal{H}_j^{\mathrm{T}} \mathcal{H}_j y_{i,k} \tag{6}$$

where $\mathcal{H}_j (j = 1, 2, \ldots, m)$ is a set of known matrices with suitable dimensions indicating the upper bound of the nonlinearity $\Re_j(\cdot)$.

For the convenience of presentation, (4) can be rewritten as the compact form

$$\bar{y}_{i,k} = \alpha_{i,k} y_{i,k} + \big(1 - \alpha_{i,k}\big)\gamma_k^{i} \Re\big(y_{i,k}\big) \tag{7}$$

where

$$\begin{aligned} \gamma_k^{i} &= \big[\gamma_{1k}^{i} I, \gamma_{2k}^{i} I, \ldots, \gamma_{mk}^{i} I\big] \\ \Re^{\mathrm{T}}\big(y_{i,k}\big) &= \big[\Re_1^{\mathrm{T}}\big(y_{i,k}\big), \Re_2^{\mathrm{T}}\big(y_{i,k}\big), \ldots, \Re_m^{\mathrm{T}}\big(y_{i,k}\big)\big]^{\mathrm{T}}. \end{aligned}$$

Assumption 1 [23]: For fixed scalars $\mathcal{F}_i$ and $\bar{\alpha}_i$, the following inequality is satisfied:

$$(1 - \bar{\alpha}_i) \sum_{j=1}^{m} \bar{\gamma}_j^{i} y_{i,k}^{\mathrm{T}} \mathcal{H}_j^{\mathrm{T}} \mathcal{H}_j y_{i,k} \leq \mathcal{F}_i y_{i,k}^{\mathrm{T}} y_{i,k} \tag{8}$$

then we can obtain

$$(1 - \bar{\alpha}_i) \sum_{j=1}^{m} \bar{\gamma}_j^{i} \mathrm{tr}\Big(\mathcal{H}_j^{\mathrm{T}} \mathcal{H}_j\Big) \leq \mathcal{F}_i \tag{9}$$

where $\mathcal{F}_i$ represents the energy level of the MFDI (4).

Remark 2: In order to enhance the confusability and diversity of the concerned attacks, a dual stochastic MFDI attack model is put forward, where the occurrence of attack and the selection of the pretransmitted modified data are both random. In (4), $\alpha_{i,k}(i = 1, \dots, N)$ and $\gamma_{jk}^{i}(j = 1, \dots, m)$ are two stochastic variables to characterize the randomness of such attack behavior. When $\alpha_{i,k} = 0$, the real output of the ith agent is $\sum_{j=1}^{m} \gamma_{jk}^{i} \Re_j(y_{i,k})$, and when $\gamma_{jk}^{i} = 1$, the attacker launches an attack signal randomly, and each attack signal has a different energy upper bound. When $\alpha_{i,k} = 1$, the system operates normally. It can be noted that the foregoing attack model enriches the attack means and the stealthiness of the MFDI attack is increased relatively compared to the single modal case. In addition, the feature of the boundness of the falsified signals $\Re_j(y_{i,k})$ helps the concealment of the studied attack behavior [36].

Remark 3: From Assumption 1, the attack energy is taken into consideration in this model. The energy level of each subsystem $\mathcal{F}_i$ is totally determined by $\bar{\alpha}_i$, $\bar{\gamma}_j^i$ and matrices $\mathcal{H}_j$. Apparently, the adversaries possess limited energy, if the energy level is predetermined beforehand, then parameters $\bar{\alpha}_i$, $\bar{\gamma}_j^i$, and $\mathcal{H}_j$ can be regulated properly to enrich attack means.

B. Description of EHCs

As depicted in Fig. 1, after the measured output is modified by the attacker, it would be transmitted to its neighboring agents over the shared communication channel. Before that, an energy harvesting scheduling protocol is employed to arrange the data transmission. The actual measured output through such a mechanism is denoted by

$$\tilde{y}_{i,k} = \lambda_{i,k} \bar{y}_{i,k} \tag{10}$$

where $\lambda_{i,k}$ is defined as follows:

$$\lambda_{i,k} = \begin{cases} 1, & e_{i,k} > 0 \\ 0, & e_{i,k} = 0 \end{cases} \tag{11}$$

and the current energy level of the ith sensor at instant k is indicated by $e_{i,k} \in \{0, 1, 2, \dots, B_i\}$. It is assumed that the random variables $\alpha_{i,k}$, γ_{jk}^{i}, and $\lambda_{i,k}$ are mutually independent.

For the purpose of describing the energy collection process preferably, three rules are given as follows.

Rule 1: B_i is the maximum number of units of energy that the sensor of the ith agent can store. Then, extra energy will be discarded if the sum of energy harvested exceeds B_i.

Rule 2: At each instant k, the ith sensor is able to broadcast information to its adjacent agent if and only if it reserves nonzero unit of energy. Meanwhile, consume one unit of energy to support such information scheduling.

Rule 3: The total energy gathered at instant k of the ith sensor is represented by $g_{i,k}$, which obeys the following probability distribution [37]:

$$\Pr\{g_{i,k} = t\} \triangleq p_t, \quad t = 0, 1, 2, \dots, \bar{S} \tag{12}$$

where $0 \leq p_t \leq 1$ reflects the availability of the sensor of the ith agent harvesting energy from the context and $\bar{S}$ represents the maximum energy that the harvester can collect and $\sum_{t=0}^{\bar{S}} p_t = 1$. $g_{i,k}$ and aforementioned random variables are mutually irrelevant.

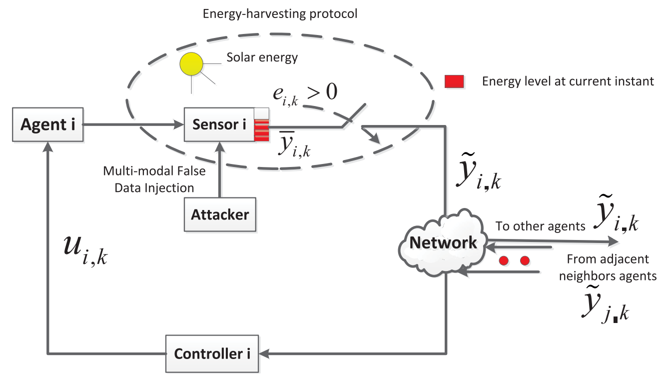

Fig. 1. Outline of stochastic nonlinear MASs.

Thus, the dynamics of the energy level of the sensor of agent i can be represented by

$$\begin{cases} e_{i,k+1} = \min\{e_{i,k} + g_{i,k} - \lambda_{i,k}, B_i\} \\ e_{i,0} \quad \leq B_i. \end{cases} \tag{13}$$

Remark 4: To date, there exist a bunch of literature concerning the energy harvesting protocol [31], [32], [37], [38], [39]. It should be noted that the energy production process for the deployed energy harvester with a single module is described by the Bernoulli model, which is encouraged by some time-uncorrelated sources (e.g., wind and water energy). On the other side, considering the physical constraints of the sensor, each energy harvesting processing unit includes limited submodules, therefore, the net energy collected can be modeled as a binomial process [32]. In addition, the energy harvesting procedure can also be described as state-independent Markov Processes, whose transition probabilities are decided by the current energy level. It should be noticed that the above probability distribution models can be reduced into the Poisson distribution structure when the number of submodules turns to infinity [38].

C. Problems of Interests

In this part, based on the above description, the control objectives can be summarized as solving Problems 1 and 2 shown below.

Problem 1: For all MASs (1) with a given directed graph $\aleph$ and a preset constant p, if there exist a group post-definite matrices $P_k(0 \leq k \leq \Gamma)$ such that any two agent states $x_{i,k}$ and $x_{j,k}(i, j \in \Im)$ at instant k satisfy

$$\begin{aligned} &\Pr\{(x_{i,k} - x_{j,k}) \in \Upsilon_k\} \geq p \\ &\quad \Upsilon_k \triangleq \left\{(x_{i,k} - x_{j,k}) | (x_{i,k} - x_{j,k})^{\mathrm{T}} P_k^{-1} (x_{i,k} - x_{j,k}) \leq 1\right\} \end{aligned} \tag{14}$$

then quasi-consensus in the probability sense is reached.

Problem 2: As far as (14) is satisfied, the following cost function:

$$\mathcal{G}_k^{\mathrm{opt}} = \min_{\mathscr{F}} \mathrm{tr}(P_k) \tag{15}$$

is minimized and the related parameters are available online.

Remark 5: In this article, a novel indicator is first proposed to value the consensus performance of the stochastic MASs, and unlike the literature concerning the quasi-consensus [21], [40], the $\mathcal{H}_\infty$ consensus [41], the bounded consensus [20], [42] for deterministic MASs, the quasi-consensus defined in Problem 1 demands that the probability of the difference between arbitrary agents centering in a fixed scope is larger than a preset constant, which is an original yet moderate evaluation standard of the consensus behavior. In conclusion, the new index can be adopted to reasonably characterize the dynamics in the probability sense.

D. Preliminaries

In this section, some mathematical symbols are defined beforehand, which facilitates the subsequent analysis

$$\begin{aligned}
\mathcal{C} &= \mathrm{diag}\{C_k, \ldots, C_k\}\\
\mathcal{H} &= \mathrm{diag}\{\mathcal{H}_1, \ldots, \mathcal{H}_m\}\\
\bar{\mathcal{H}} &= \mathrm{diag}\{\mathcal{H}, \ldots, \mathcal{H}\}\\
\varrho_r &= \mathrm{diag}\{\varrho_{1r}, \ldots, \varrho_{Nr}\}\\
T_r &= \mathrm{diag}\{T_{1r}, \ldots, T_{Nr}\}\\
\bar{V} &= \mathrm{diag}\{V_1, \ldots, V_N\}\\
P_k &= \mathrm{diag}\big\{\bar{P}_k, \ldots, \bar{P}_k\big\}.
\end{aligned}$$

III. Main Results

A. Calculation of the Probability Distribution of $\lambda_{i,k}$

As can be seen from (11) that the probability distribution $\lambda_{i,k}$ is related to the current energy level $e_{i,k}$. Thus, the calculation of the probability distribution of $e_{i,k}$ is shown as follows.

Let the probability distribution of the current energy level $e_{i,k}$ be $\mathcal{Q}_{i,k} \triangleq [q_{i,k}^0, q_{i,k}^1, \ldots, q_{i,k}^{B_i}]^{\mathrm{T}}$, where $q_{i,k}^s = \Pr\{e_{i,k} = s\}(s = 0, 1, \ldots, B_i)$. Thanks to the independence of $g_{i,k}$, for $s = 0, 1, \ldots, B_i - 1$, it can be seen that

$$\begin{aligned}
q_{i,k+1}^s &= \Pr\big\{e_{i,k+1} = s\big\}\\
&= \Pr\big\{\min\big\{e_{i,k} + g_{i,k} - \lambda_{i,k}, B_i\big\} = s\big\}\\
&= \Pr\big\{e_{i,k} + g_{i,k} - \lambda_{i,k} = s\big\}\\
&= \Pr\big\{e_{i,k} = 0, g_{i,k} = s\big\}\\
&\quad + \sum_{\iota=1}^{s+1} \Pr\big\{e_{i,k} = \iota, g_{i,k} = s + 1 - \iota\big\}\\
&= \Pr\big\{e_{i,k} = 0\big\}\Pr\big\{g_{i,k} = s\big\}\\
&\quad + \sum_{\iota=1}^{s+1} \Pr\big\{e_{i,k} = \iota\big\}\Pr\big\{g_{i,k} = s + 1 - \iota\big\}\\
&= q_{i,k}^0 p_s + \sum_{\iota=1}^{s+1} q_{i,k}^\iota p_{s+1-\iota}.
\end{aligned} \tag{16}$$

Hence, the probability $q_{i,k+1}^{B_i}$ can be computed as follows:

$$\begin{aligned}
q_{i,k+1}^{B_i} &= 1 - \sum_{s=0}^{B_i-1} q_{i,k+1}^s\\
&= 1 - q_{i.k}^0 \sum_{s=0}^{B_i-1} p_s - \sum_{s=0}^{B_i-1}\sum_{\iota=1}^{s+1} q_{i,k}^\iota p_{s+1-\iota}.
\end{aligned} \tag{17}$$

From (16) and (17), the form of the probability distribution $\mathcal{Q}_{i,k}$ is attained recursively as follows:

$$\mathcal{Q}_{i,k+1} = R_i + \mathcal{M}_i \mathcal{Q}_{i,k} \tag{18}$$

where $R_i = [\underbrace{0 \cdots 0}_{B_i} \quad 1]^{\mathrm{T}}$ and

$$\mathcal{M}_i = \begin{bmatrix}
p_0 & p_0 & 0 & \cdots & 0\\
p_1 & p_1 & p_0 & \cdots & 0\\
p_2 & p_2 & p_1 & \cdots & 0\\
\vdots & \vdots & \vdots & \ddots & \vdots\\
p_{B_i-1} & p_{B_i-1} & p_{B_i-2} & \cdots & p_0\\
-\sum_{s=0}^{B_i-1} p_s & -\sum_{s=0}^{B_i-1} p_s & -\sum_{s=0}^{B_i-2} p_s & \cdots & -p_0
\end{bmatrix}.$$

According to (18), the computation procedure of the probability distribution $\lambda_{i,k}$ is revealed in Proposition 1.

Proposition 1: The random variable $\lambda_{i,k}$ representing the harvested energy abides by the following probability distribution:

$$\Pr\big\{\lambda_{i,k} = 1\big\} = \bar{\lambda}_{i,k}, \Pr\big\{\lambda_{i,k} = 0\big\} = 1 - \bar{\lambda}_{i,k} \tag{19}$$

in which

$$\bar{\lambda}_{i,k} \triangleq 1 - d_i \mathcal{Q}_{i,k}, d_i \triangleq \begin{bmatrix} 1 & \underbrace{0 \quad \cdots \quad 0}_{B_i} \end{bmatrix}.$$

Proof: One can be possessed from (11) that

$$\begin{aligned}
\Pr\big\{\lambda_{i,k} = 0\big\} &= \Pr\big\{e_{i,k} = 0\big\} = d_i \mathcal{Q}_{i,k}\\
\Pr\big\{\lambda_{i,k} = 1\big\} &= 1 - \Pr\big\{e_{i,k} = 0\big\} = 1 - d_i \mathcal{Q}_{i,k}.
\end{aligned}$$

■

Therefore, the aforementioned proposition is proven.

B. Deviation Error System Model

In this portion, the deviation error systems model is established and analyzed. Moreover, some lemmas and definitions are given ahead.

In stochastic nonlinear MASs (1), the control inputs $u_{i,k}(i = 1, 2, \ldots, N)$ are designed as follows:

$$u_{i,k} = K_k \sum_{j\in \mathcal{N}_i} a_{ij}\big(\tilde{y}_{j,k} - \tilde{y}_{i,k}\big) \tag{20}$$

where K_k is the controller gain to be designed.

The nonlinear systems (1) with the distributed quasi-consensus controller can be rewritten as follows:

$$\begin{aligned}
x_{k+1} &= ((I \otimes A_k) - (\Lambda_k \Xi_k \mathcal{L} \otimes B_k K_k C_k))x_k\\
&\quad - \Big(\Lambda_k (I - \Xi_k)\gamma_k^0 (\mathcal{L} \otimes I_m) \otimes (B_k K_k)\Big)\Re(y_k)\\
&\quad + F_k + (I \otimes D_k)\nu_k
\end{aligned} \tag{21}$$

where

$$\begin{aligned}
x_k &= \big[x_{1,k}^{\mathrm{T}}, \ldots, x_{N,k}^{\mathrm{T}}\big]^{\mathrm{T}}\\
\nu_k &= \big[\nu_{1,k}^{\mathrm{T}}, \ldots, \nu_{N,k}^{\mathrm{T}}\big]^{\mathrm{T}}\\
\Xi_k &= \mathrm{diag}\big\{\alpha_{1,k}, \ldots, \alpha_{N,k}\big\}\\
\Lambda_k &= \mathrm{diag}\big\{\lambda_{1,k}, \ldots, \lambda_{N,k}\big\}\\
\gamma_k^0 &= \mathrm{diag}\Big\{\gamma_k^1, \ldots, \gamma_k^N\Big\}
\end{aligned}$$

$$\begin{aligned}\bar{\Xi} &= \mathrm{diag}\{\bar{\alpha}_1, \ldots, \bar{\alpha}_N\}\\ \bar{\Lambda} &= \mathrm{diag}\{\bar{\lambda}_1, \ldots, \bar{\lambda}_N\}\\ \bar{\gamma}^0 &= \mathrm{diag}\left\{\bar{\gamma}^1, \ldots, \bar{\gamma}^N\right\}\\ F_k &= \left[f_1^{\mathrm{T}}(k, x_k), \ldots, f_N^{\mathrm{T}}(k, x_k)\right]^{\mathrm{T}}\\ \Re(y_k) &= \left[\Re^{\mathrm{T}}(y_{1,k}), \ldots, \Re^{\mathrm{T}}(y_{N,k})\right]^{\mathrm{T}}.\end{aligned}$$

For the sake of analyzing the quasi-consensus control design, some proper transformations are needed concerning on the traits of the stochastic MASs. Define the average state $\bar{x}_k$ as follows:

$$\bar{x}_k \triangleq \mathbb{E}\left\{\frac{1}{N}\sum_{i=1}^{N} x_{i,k}\right\} = \mathbb{E}\left\{\frac{1}{N}(\mathbf{1}^{\mathrm{T}} \otimes I)x_k\right\} \tag{22}$$

and the deviation error of each state from the average state is given by

$$\hat{x}_{i,k} \triangleq x_{i,k} - \bar{x}_k. \tag{23}$$

From (22) and (23), by utilizing the feature of the Laplacian matrix, that is, $\mathbf{1}^{\mathrm{T}}\mathcal{L} = \mathcal{L}\mathbf{1} = \mathbf{0}$, one has

$$\begin{aligned}\bar{x}_{k+1} &= \mathbb{E}\left\{\frac{1}{N}(\mathbf{1}^{\mathrm{T}} \otimes I)x_{k+1}\right\}\\ &= \frac{1}{N}(\mathbf{1}^{\mathrm{T}} \otimes I)\big(-(\bar{\Lambda}\bar{\Xi}\mathcal{L} \otimes B_k K_k C_k)x_k\\ &\quad - \Big(\bar{\Lambda}\big(I - \bar{\Xi}\big)\bar{\gamma}^0(\mathcal{L} \otimes I_m) \otimes B_k K_k)\Re(y_k)\Big) + A_k\bar{x}_k\\ &= A_k\bar{x}_k.\end{aligned} \tag{24}$$

Then, we can obtain that

$$\begin{aligned}\hat{x}_{k+1} &= x_{k+1} - (\mathbf{1} \otimes I)\bar{x}_{k+1}\\ &= (I \otimes A_k)\big(\hat{x}_k + (\mathbf{1} \otimes I)\bar{x}_k\big) + F_k\\ &\quad - (\Lambda_k\Xi_k\mathcal{L} \otimes B_k K_k C_k)\big(\hat{x}_k + (\mathbf{1} \otimes I)\bar{x}_k\big)\\ &\quad - \Big(\Lambda_k(I - \Xi_k)\gamma_k^0(\mathcal{L} \otimes I_m) \otimes (B_k K_k)\Big)\Re(y_k)\\ &\quad + (I \otimes D_k)v_k - (\mathbf{1} \otimes I)\bar{x}_{k+1}\\ &= (I \otimes A_k)\hat{x}_k - (\Lambda_k\Xi_k\mathcal{L} \otimes B_k K_k C_k)\hat{x}_k + F_k\\ &\quad - \Big(\Lambda_k(I - \Xi_k)\gamma_k^0(\mathcal{L} \otimes I_m) \otimes (B_k K_k)\Big)\Re(y_k)\\ &\quad + (I \otimes D_k)v_k\\ &= \big((I \otimes A_k) - \big(\bar{\Lambda}\bar{\Xi}\mathcal{L} \otimes B_k K_k C_k\big)\big)\hat{x}_k + F_k\\ &\quad - \Big(\bar{\Lambda}(I - \bar{\Xi})\bar{\gamma}^0(\mathcal{L} \otimes I_m) \otimes (B_k K_k)\Big)\Re(y_k)\\ &\quad + \big(\big(\bar{\Lambda}\bar{\Xi} - \Lambda_k\Xi_k\big)\mathcal{L} \otimes B_k K_k C_k\big)\hat{x}_k + (I \otimes D_k)v_k\\ &\quad + \Big(\Big(\bar{\Lambda}(I - \bar{\Xi})\bar{\gamma}^0 - \Lambda_k(I - \Xi_k)\gamma_k^0\Big)(\mathcal{L} \otimes I_m)\otimes\\ &\quad (B_k K_k)\Big)\Re(y_k)\\ &\triangleq \eta_{0k}\Gamma_k + \eta_{1k}\Gamma_k + \eta_{2k}\Gamma_k + F_k + (I \otimes D_k)v_k\end{aligned} \tag{25}$$

where

$$\begin{aligned}\Gamma_k &= \begin{bmatrix}1 & \hat{x}_k^{\mathrm{T}} & \Re^{\mathrm{T}}(y_k)\end{bmatrix}^{\mathrm{T}}\\ \eta_{0k} &= \Big[0 \quad I \otimes A_k - \bar{\Lambda}\bar{\Xi}\mathcal{L} \otimes B_k K_k C_k \quad -\bar{\Lambda}(I - \bar{\Xi})\\ &\qquad \bar{\gamma}^0(\mathcal{L} \otimes I_m) \otimes (B_k K_k)\Big]\\ \eta_{1k} &= \begin{bmatrix}0 & \Omega_{1k}\mathcal{L} \otimes B_k K_k C_k & 0\end{bmatrix}\\ \eta_{2k} &= \begin{bmatrix}0 & 0 & \Omega_{2k}(\mathcal{L} \otimes I_m) \otimes (B_k K_k)\end{bmatrix}\\ \Omega_{1k} &= \big(\bar{\Lambda}\bar{\Xi} - \Lambda_k\Xi_k\big),\ \Omega_{2k} = \bar{\Lambda}(I - \bar{\Xi})\bar{\gamma}^0 - \Lambda_k(I - \Xi_k)\gamma_k^0.\end{aligned}$$

The following lemmas are useful for the derivation of our main results.

Lemma 1 [22]: For a fixed matrix $B > 0$ and a given vector $\mathcal{Y}$ with suitable dimensions, a constraint set $\mathcal{O}$ is generated as follows:

$$\mathcal{O} := \left\{\mu \middle| (\mu - \mathcal{Y})^{\mathrm{T}}B(\mu - \mathcal{Y}) \le 1\right\}$$

where μ is a stochastic variable. If the following inequality:

$$\mathbb{E}\left\{(\mu - \mathcal{Y})^{\mathrm{T}}B(\mu - \mathcal{Y})\right\} \le 1 - p \tag{26}$$

holds, we have

$$\Pr(\mu \in \mathcal{O}) \ge p. \tag{27}$$

Lemma 2: For a given scalar $p \in [0, 1]$, positive-definite matrix P_k and the state of stochastic MAS x_i, x_j, Problem 1, that is, (14) is satisfied if

$$\mathbb{E}\left\{\hat{x}_k^{\mathrm{T}}P_k^{-1}\hat{x}_k\right\} \le 1 - p \tag{28}$$

holds.

Proof: On the basis of the connection between of (14) and (23), it is not difficult to see that

$$\begin{aligned}\left\|x_{i,k} - x_{j,k}\right\|_2 &= \left\|x_{i,k} - \bar{x}_k + \bar{x}_k - x_{j,k}\right\|_2\\ &\le \left\|x_{i,k} - \bar{x}_k\right\|_2 + \left\|x_{j,k} - \bar{x}_k\right\|_2\\ &\le \left\|\hat{x}_k\right\|_2.\end{aligned} \tag{29}$$

■

C. Design of the Distributed Quasi-consensus Controller

In this section, sufficient conditions are provided to ensure Problem 1 be solved. First, Assumption 2 is shown below.

Assumption 2: The initial condition concentrating on the deviation error at the beginning satisfies the following constraint:

$$\mathbb{E}\left\{\hat{x}_0^{\mathrm{T}}P_0^{-1}\hat{x}_0\right\} \le 1 - p \tag{30}$$

where $\hat{x}_0 = x_0 - (\mathbf{1} \otimes I_N)\bar{x}_0$, in which x_0 denotes the augmented initial state of (1) and $\bar{x}_0$ represents the initial average state, respectively. P_0 is a given positive-definite matrix.

Theorem 1: Under Assumptions 1 and 2, consider the deviation error (25) and the predetermined scalar $p \in [0, 1]$, if there exist a family of matrices P_{k+1}, K_k, and positive scalars π_{rk}, μ_{ik}, κ_1, and κ_2 such that

$$\begin{bmatrix}\Upsilon & * & * & *\\ \eta_{0k} & -P_{k+1} & * & *\\ \bar{\eta}_{1k} & 0 & -P_{k+1} & *\\ \bar{\eta}_{2k} & 0 & 0 & -P_{k+1}\end{bmatrix} \le 0 \tag{31}$$

$$\begin{bmatrix}-\pi_{rk} & \varrho_r^{\mathrm{T}}\\ * & -P_{k+1}\end{bmatrix} \le 0 \tag{32}$$

$$\begin{bmatrix}\mu_{ik} & D_k^{\mathrm{T}}\\ * & -\bar{P}_{k+1}\end{bmatrix} \le 0 \tag{33}$$

where

$$\begin{aligned}\Upsilon &= \sum_{r=1}^{p}\mathrm{tr}\Big(P_{k+1}^{-1}\varrho_r\varrho_r^{\mathrm{T}}\Big)\Delta_{1k} + \Delta_{2k} - \kappa_1\Sigma_{1k} - \kappa_2\Sigma_{2k}\\ \bar{\eta}_{1k} &= \begin{bmatrix}0 & \Upsilon_{1k}\mathcal{L} \otimes B_k K_k C_k & 0\end{bmatrix}\end{aligned}$$

$$\bar{\eta}_{2k}=[0\quad 0\quad \Upsilon_{1k}(\mathcal{L}\otimes I_m)\otimes B_kK_k]$$
$$\Upsilon_{2k}=\sqrt{\bar{\Lambda}(I-\bar{\Xi})\bar{\gamma}^0\big(I-\bar{\Lambda}(I-\bar{\Xi})\bar{\gamma}^0\big)},\ \Upsilon_{1k}=\sqrt{\bar{\Lambda}\bar{\Xi}(I-\bar{\Lambda}\bar{\Xi})}.$$

Then, the inequality (14) holds.

Proof: The proof is carried out by using *the mathematical induction* approach.

In the first place, (30) is met initially from Assumption 2. Next, assuming (28) is also satisfied at instant k, which means $\mathbb{E}\{\hat{x}_k^{\mathrm{T}}P_k^{-1}\hat{x}_k\}\le 1-p$. Then, our objective is to prove (28) is true at the next instant $k+1$.

For the stochastic MASs (1), computing (28) at $k+1$ and we acquire

$$\Pi=\mathbb{E}\Big\{\hat{x}_{k+1}^{\mathrm{T}}P_{k+1}^{-1}\hat{x}_{k+1}\Big\}-(1-p).\tag{34}$$

From (24), the above (34) can be rewritten as follows:

$$\begin{aligned}\Pi=\mathbb{E}\Big\{&\Gamma_k^{\mathrm{T}}\eta_{0k}^{\mathrm{T}}P_{k+1}^{-1}\eta_{0k}\Gamma_k+\Gamma_k^{\mathrm{T}}\eta_{1k}^{\mathrm{T}}P_{k+1}^{-1}\eta_{1k}\Gamma_k\\&+\Gamma_k^{\mathrm{T}}\eta_{2k}^{\mathrm{T}}P_{k+1}^{-1}\eta_{2k}\Gamma_k+F_k^{\mathrm{T}}P_{k+1}^{-1}F_k\\&+\nu_k^{\mathrm{T}}(I\otimes D_k)^{\mathrm{T}}P_{k+1}^{-1}(I\otimes D_k)\nu_k\Big\}-(1-p).\end{aligned}\tag{35}$$

Notice that

$$\begin{aligned}\mathbb{E}\Big\{F_k^{\mathrm{T}}P_{k+1}^{-1}F_k\Big\}&=\mathbb{E}\Big\{\mathrm{tr}\Big\{P_{k+1}^{-1}F_kF_k^{\mathrm{T}}\Big\}\Big\}\\&=\sum_{i=1}^{N}\sum_{r=1}^{p}\mathrm{tr}\Big(\bar{P}_{k+1}^{-1}\varrho_{ir}\varrho_{ir}^{\mathrm{T}}\Big)x_{i,k}^{\mathrm{T}}T_{ir}x_{i,k}\\&=\sum_{r=1}^{p}\mathrm{tr}\Big(P_{k+1}^{-1}\varrho_r\varrho_r^{\mathrm{T}}\Big)\big(\hat{x}_k+(\mathbf{1}\otimes I)\bar{x}_k\big)^{\mathrm{T}}\\&\quad\times T_r\big(\hat{x}_k+(\mathbf{1}\otimes I)\bar{x}_k\big)\\&=\sum_{r=1}^{p}\mathrm{tr}\Big(P_{k+1}^{-1}\varrho_r\varrho_r^{\mathrm{T}}\Big)\Gamma_k^{\mathrm{T}}\Delta_{1k}\Gamma_k\end{aligned}$$

where

$$\Delta_{1k}=\begin{bmatrix}\bar{x}_k^{\mathrm{T}}\big(\mathbf{1}^{\mathrm{T}}\otimes I\big)T_r(\mathbf{1}\otimes I)\bar{x}_k & * & *\\ T_r(\mathbf{1}\otimes I)\bar{x}_k & T_r & 0\\ 0 & 0 & 0\end{bmatrix}$$

and

$$\begin{aligned}&\mathbb{E}\Big\{\nu_k^{\mathrm{T}}(I\otimes D_k)^{\mathrm{T}}P_{k+1}^{-1}(I\otimes D_k)\nu_k\Big\}\\&=\mathbb{E}\Big\{\mathrm{tr}\Big\{\nu_k^{\mathrm{T}}(I\otimes D_k)^{\mathrm{T}}P_{k+1}^{-1}(I\otimes D_k)\nu_k\Big\}\Big\}\\&=\mathrm{tr}\Big\{(I\otimes D_k)^{\mathrm{T}}P_{k+1}^{-1}(I\otimes D_k)\nu_k\nu_k^{\mathrm{T}}\Big\}\\&=\mathrm{tr}\Big\{\Big(I\otimes D_k^{\mathrm{T}}\bar{P}_{k+1}^{-1}D_k\Big)\bar{V}\Big\}\\&=\Gamma_k^{\mathrm{T}}\bar{\Delta}_{2k}\Gamma_k\end{aligned}$$

where

$$\bar{\Delta}_{2k}=\mathrm{diag}\Big\{\mathrm{tr}\Big\{\Big(I\otimes D_k^{\mathrm{T}}\bar{P}_{k+1}^{-1}D_k\Big)\bar{V}\Big\},0,0\Big\}.$$

In addition, it can be seen from (6) and (28), the aforementioned constraints can be converted into the following form:

$$\Pi_k^{\mathrm{T}}\Sigma_{1k}\Pi_k\le 0\tag{36}$$

$$\Pi_k^{\mathrm{T}}\Sigma_{2k}\Pi_k\le 0\tag{37}$$

where

$$\Sigma_{1k}=\mathrm{diag}\Big\{-1+p,P_k^{-1},0\Big\}$$
$$\Sigma_{2k}=\begin{bmatrix}-\bar{x}_k^{\mathrm{T}}\big(\mathbf{1}^{\mathrm{T}}\otimes I\big)\mathcal{C}^{\mathrm{T}}\bar{\mathcal{H}}^{\mathrm{T}}\bar{\mathcal{H}}\mathcal{C}(\mathbf{1}\otimes I)\bar{x}_k & * & *\\ -\mathcal{C}^{\mathrm{T}}\bar{\mathcal{H}}^{\mathrm{T}}\bar{\mathcal{H}}\mathcal{C}(\mathbf{1}\otimes I)\bar{x}_k & \Sigma_0 & 0\\ 0 & 0 & I\end{bmatrix}$$
$$\Sigma_0=-\mathcal{C}^{\mathrm{T}}\bar{\mathcal{H}}^{\mathrm{T}}\bar{\mathcal{H}}\mathcal{C}.$$

Then, (35) can be rewritten as follows:

$$\begin{aligned}\Pi=\Gamma_k^{\mathrm{T}}\Big\{&\eta_{0k}^{\mathrm{T}}P_{k+1}^{-1}\eta_{0k}+\bar{\eta}_{1k}^{\mathrm{T}}P_{k+1}^{-1}\bar{\eta}_{1k}+\bar{\Delta}_{2k}\\&+\bar{\eta}_{2k}^{\mathrm{T}}P_{k+1}^{-1}\bar{\eta}_{2k}+\sum_{r=1}^{p}\mathrm{tr}\Big(P_{k+1}^{-1}\varrho_r\varrho_r^{\mathrm{T}}\Big)\Delta_{1k}\Big\}\Gamma_k\\&-(1-p).\end{aligned}\tag{38}$$

By utilizing the Schur complement Lemma [43], one can be found that according to (32) and (33)

$$\mathrm{tr}\Big(P_{k+1}^{-1}\varrho_r\varrho_r^{\mathrm{T}}\Big)\le\pi_{rk}\tag{39}$$
$$\mathrm{tr}\Big(D_k^{\mathrm{T}}\bar{P}_{k+1}^{-1}D_k\Big)\le\mu_{ik}.\tag{40}$$

Then, we can summarize that

$$\begin{aligned}\Pi\le\Gamma_k^{\mathrm{T}}\Big\{&\eta_{0k}^{\mathrm{T}}P_{k+1}^{-1}\eta_{0k}+\bar{\eta}_{1k}^{\mathrm{T}}P_{k+1}^{-1}\bar{\eta}_{1k}\\&+\bar{\eta}_{2k}^{\mathrm{T}}P_{k+1}^{-1}\bar{\eta}_{2k}+\sum_{r=1}^{p}\pi_{rk}\Delta_{1k}+\Delta_{2k}\Big\}\Gamma_k\end{aligned}\tag{41}$$

where

$$\Delta_{2k}=\mathrm{diag}\Big\{\mathrm{tr}\Big\{(I\otimes\mu_{ik})\bar{V}\Big\}-1+p,0,0\Big\}.$$

Moreover, according to (34)–(41), and using S-Procedure Lemma [43], we can conclude that if there exist non-negative constants κ_1 and κ_2 render

$$\begin{aligned}&\eta_{0k}^{\mathrm{T}}P_{k+1}^{-1}\eta_{0k}+\bar{\eta}_{1k}^{\mathrm{T}}P_{k+1}^{-1}\bar{\eta}_{1k}+\bar{\eta}_{2k}^{\mathrm{T}}P_{k+1}^{-1}\bar{\eta}_{2k}\\&+\sum_{r=1}^{p}\pi_{rk}\Delta_{1k}+\Delta_{2k}-\kappa_1\Sigma_{1k}-\kappa_2\Sigma_{2k}\le 0\end{aligned}\tag{42}$$

then (34) is ensured. By using the Schur complement Lemma again, (42) is equal to (31). Hence, we can attain that $\Pi\le 0$, which signifies that the objective in this proof is achieved, that is, $\mathbb{E}\{\hat{x}_{k+1}^{\mathrm{T}}P_{k+1}^{-1}\hat{x}_{k+1}\}\le 1-p$. Till now, the proof is finished. ■

On the basis of Theorem 1, we are ready to solve the minimization issue in Problem 2. For this purpose, a theorem and an algorithm are demonstrated in this part. First, a new theorem centering on the optimization is provided for Problem 2.

Theorem 2: For the time-varying stochastic MASs (1) and the distributed controller (20), suppose Assumption 2 holds for fixed x_0, $\bar{x}_0$, and P_0. Hence, the quasi-consensus condition $\Pr\{(x_{i,k}-x_{j,k})\in\Upsilon_k\}\ge p$ holds for the given scalar p if there exist positive-definite matrices $P_{k+1}=\mathrm{diag}_N\{\bar{P}_{k+1}\}$, K_k and

Algorithm 1 OPCDCDA

Step 1. Set $k = 0$. Fix the initial value of the confidence probability p, the coefficient ϱ_{ir} and Γ_{ir} in (2), the agent state $x_{i,0}$, and the parameter matrix P_0 satisfying (30), calculate the initial average state $\bar{x}_0$;

Step 2. Settle the optimal problem (43) in Theorem 2, calculate $\mathcal{G}_k^{opt}$ and attain corresponding $\mathscr{F} = \{P_{k+1}, \pi_{rk}, \mu_{ik}, \kappa_1, \kappa_2\}$, Next, let $k = k + 1$ and update $P_k = P_{k+1}$;

Step 3. if $k < N$, go back to Step 2; or else forward to Step 4;

Step 4. Stop.

positive constants π_{rk}, μ_{ik}, κ_1, and κ_2 to resolve the following minimization problem

$$\mathcal{G}_k^{\text{opt}} = \min_{\mathscr{F}} \text{tr}(P_{k+1}) \tag{43}$$

where $\mathscr{F} = \{P_{k+1}, \pi_{rk}, \mu_{ik}, \kappa_1, \kappa_2\}$, then the convex optimization problem in Problem 2 is addressed.

Remark 6: The main criteria in Theorem 2, that is, (31)–(33), are recursive linear matrix inequalities (RLMIs). The LMI Control Toolbox implements state-of-the-art interior-point LMI solvers. The LMIs have a polynomial time complexity. The number $N(\varepsilon)$ of flops needed to compute an ε-accurate solution is bounded by $O(MN^3)$, where M is the full row size of the LMIs and N is the number of scalar decision variables. In this article, the dimensions of the variables are $x_k \in \mathbb{R}^{n_x}$, $r = 1, 2, \ldots, p$, and $i = 1, 2, \ldots, s$. Therefore, the total rows of (31)–(33) are $M = 30n_x + 1$, and the scalar decision variables amount to $N = n_x^2 + s + p + 3$. As such, the computation complexity of the proposed designed scheme can be represented as $O(30n_x^7)$.

For the purpose of exhibiting the realization of Theorem 2 and deriving the gain of the designed distributed controller, the optimal probabilistic-constrained distributed controller design algorithm (OPCDCDA, Algorithm 1) is proposed in this section.

IV. Two Illustrative Examples

In this part, the effectiveness and merit of the developed strategy can be verified through two examples, including a practical example.

A. Example 1

Consider a type of nonlinear stochastic MASs consisting of five agents with the following parameters:

$$A_k = \begin{bmatrix} 0.8 + 0.08\sin(0.12k) & 0.4 \\ 0.15 & -0.75 + 0.2\cos(0.1k) \end{bmatrix}$$
$$B_k = \begin{bmatrix} 0.8 + 0.2\sin(0.4k) \\ 0.5 \end{bmatrix}$$
$$C_k = \begin{bmatrix} 0.64 + 0.05\sin(k) & 0.2 \\ 0 & 0.64 + 0.1\sin(k) \end{bmatrix}$$
$$D_k = \begin{bmatrix} 0.16 + 0.05\cos(0.32k) \\ 0.18 \end{bmatrix}$$
$$\bar{\alpha} = \text{diag}\{0.80, 0.82, 0.78.0.7.0.65\}$$
$$\bar{\gamma}^0 = \text{diag}\left\{\bar{\gamma}^1, \bar{\gamma}^2, \bar{\gamma}^3, \bar{\gamma}^4, \bar{\gamma}^5\right\}$$
$$\bar{\gamma}^1 = [0.46, 0.54], \bar{\gamma}^2 = [0.5, 0.5], \bar{\gamma}^3 = [0.3, 0.7]$$
$$\bar{\gamma}^4 = [0.34, 0.66], \bar{\gamma}^5 = [0.5, 0.5]$$
$$B_i = 4(i = 1, 2, 3, 4, 5), \eta = 1.2.$$

Five agents connect through the following communication topology and the Laplacian matrix is set by:

$$\mathcal{L} = \begin{bmatrix} 0.8 & 0 & 0 & 0 & -0.8 \\ -0.5 & 1 & 0 & -0.5 & 0 \\ -0.6 & 0 & 0.6 & 0 & 0 \\ -0.6 & 0 & 0.6 & 0 & 0 \\ 0 & -0.8 & 0 & 0 & 0.8 \end{bmatrix}.$$

Predefine the initial states of five agents, that is, $x_{1,0} = [1; 0]$, $x_{2,0} = [-0.5; 1]$, $x_{3,0} = [0; 1]$, $x_{4,0} = [-0.1; 1]$, $x_{5,0} = [0.2; 0.2]$, and set the positive-definite matrix $P_0 = I_{10\times 10}$ and the confidence probability $p = 0.9$. And, the stochastic nonlinearity function of each agent is identical and given in the following form:

$$f_i(k, x_k) = \begin{bmatrix} 0.1 \\ 0.2 \end{bmatrix} \Big[\sqrt{0.1}\text{sign}(x_{1k})x_{1k}\varphi_{1k} + \sqrt{0.2}\text{sign}(x_{2k})x_{2k}\varphi_{2k}\Big]$$

where $\varphi_{1k} \sim \mathcal{N}(0, 1)$ and $\varphi_{2k} \sim \mathcal{N}(0, 1)$.

Define the average measurement y_k^{avg} as follows:

$$y_k^{\text{avg}} = \frac{1}{5}\sum_{i=1}^{5} \bar{y}_{i,k}.$$

The modified signal $\Re(y_{i,k})(i = 1, 2, 3, 4, 5)$ launched by attackers is expressed as follows:

$$\Re(y_{i,k}) = \left[\Re_1^{\text{T}}(y_{i,k}), \Re_2^{\text{T}}(y_{i,k}), \Re_3^{\text{T}}(y_{i,k}), \Re_4^{\text{T}}(y_{i,k}), \Re_5^{\text{T}}(y_{i,k})\right]^{\text{T}}$$

where

$$\Re_1^{\text{T}}(y_{i,k}) = \left[0.7\sin(y_k^{\text{avg}}), 0.5\sin(y_k^{\text{avg}})\right]^{\text{T}}$$
$$\Re_2^{\text{T}}(y_{i,k}) = \left[0.75\sin(y_k^{\text{avg}}), 0.5\sin(y_k^{\text{avg}})\right]^{\text{T}}$$
$$\Re_3^{\text{T}}(y_{i,k}) = \left[0.45\sin(y_k^{\text{avg}}), 0.55\sin(y_k^{\text{avg}})\right]^{\text{T}}$$
$$\Re_4^{\text{T}}(y_{i,k}) = \left[0.75\sin(y_k^{\text{avg}}), 0.5\sin(y_k^{\text{avg}})\right]^{\text{T}}$$
$$\Re_5^{\text{T}}(y_{i,k}) = \left[0.45\sin(y_k^{\text{avg}}), 0.55\sin(y_k^{\text{avg}})\right]^{\text{T}}.$$

In the simulation, running the proposed OPCDCDA 10 times, the attained results are included in Figs. 2–10. The elements of the state of agents are shown in Figs. 2 and 3. It can be seen that the trajectory of each agent gradually approximates each other and the quasi-consensus of five agents is achieved immediately, which demonstrates the efficiency of the designed distributed controller. Relatively, the deviation error measuring the controller performance is illustrated in Figs. 4 and 5. The red lines indicate the deviation error of each agent, the blue dashed lines denote each upper bound of the deviation error with $p = 0.9$. It can be noted that the state of each agent can track the average state preferably, which

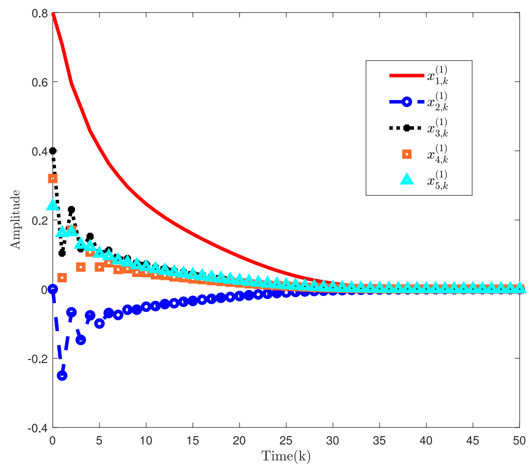

Fig. 2. First row of the state of five agents.

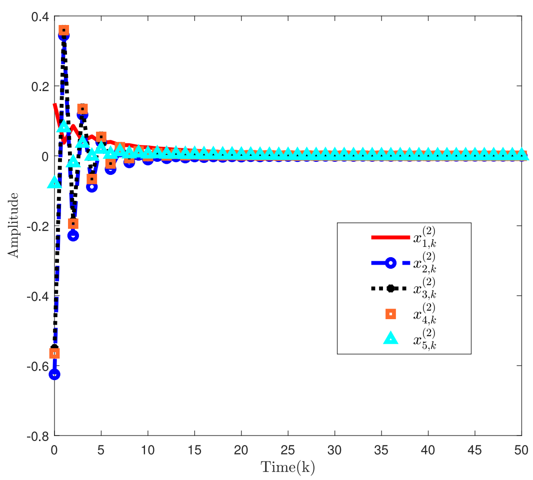

Fig. 3. Second row of the state of five agents.

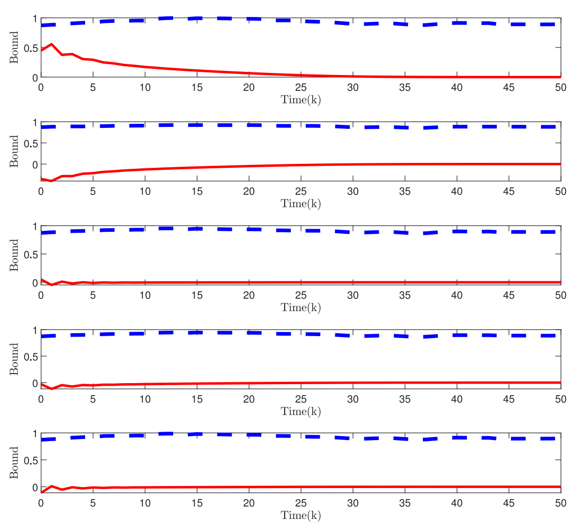

Fig. 4. First row of the derivation error of five agents.

manifests that the agreement of five agents can be reached in the finite horizon. Besides, the deviation error stays below its upper bound in every subplot.

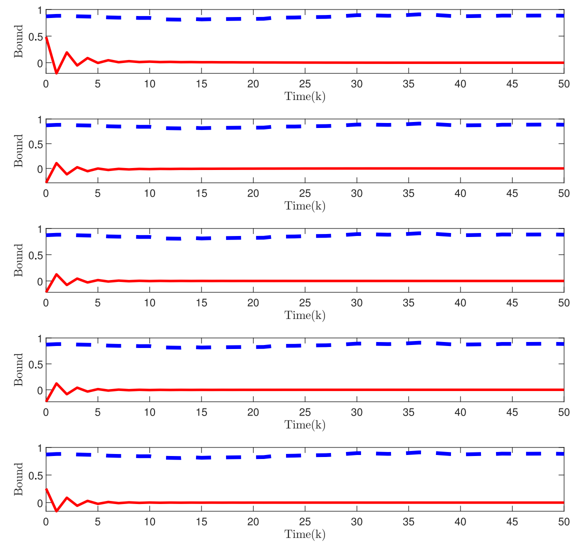

Fig. 5. Second row of the derivation error of five agents.

TABLE I
VALUE OF $\mathcal{F}_i (i = 1, 2, 3, 4, 5)$ UNDER DIFFERENT VALUES OF $\bar{\gamma}$

$\bar{\gamma}_1^i$	$\bar{\gamma}_2^i$	$\mathcal{F}_1$	$\mathcal{F}_2$	$\mathcal{F}_3$	$\mathcal{F}_4$	$\mathcal{F}5$
0.01	0.99	0.0504	0.0456	0.0660	0.0759	0.1055
0.11	0.89	0.0553	0.0512	0.0641	0.0853	0.1020
0.21	0.79	0.0600	0.0568	0.0619	0.0947	0.0985
0.31	0.69	0.0649	0.0624	0.0597	0.1041	0.0950
...	...	...	...	...	...	...
0.91	0.09	0.09368	0.0962	0.0465	0.1617	0.0740

The MFDI attack imposing on each agent is depicted as $(1-\alpha_{i,k}) \sum_{j=1}^{2} \bar{\gamma}_j^i \Re_j(y_{i,k}) (i = 1, 2, 3, 4, 5)$, adopting the aforementioned parameters, the corresponding energy constraint level can be computed as $F_1 = 0.0721$, $F_2 = 0.0731$, $F_3 = 0.0600$, $F_4 = 0.1069$, and $F_5 = 0.0884$. In addition, the attacked instants of agents 1–5 are displayed in Fig. 6, the frequency of attack is decided by the value of $\bar{\alpha}_i$. Next, for the purpose of evaluating the impact of $\bar{\gamma}^i$, multiple sets of parameters $\bar{\gamma}^i$ of five agents and the corresponding $\mathcal{F}_i$ are given in Table I, respectively.

Determine the primal energy levels $e_{1,0} = 0$, $e_{2,0} = 1$, $e_{3,0} = 2$, $e_{4,0} = 4$, $e_{5,0} = 3$, and the maximum amount of energy that each sensor can store is four units. There is an assumption that in five agents, the amount of energy collected $g_{i,k}$ obeys the Poisson distribution [44], that is, $\text{Prob}(g_{i,k} = t) = ([\eta^t \exp(-\eta)]/t!)$, with the identical Poisson parameter η. The energy levels of five agents are reflected in Fig. 7.

Eventually, for different confidence probabilities $p = 0.7$, 0.8, and 0.9, Fig. 8 illustrates the optimized $\mathcal{G}_k^{\text{opt}}$ of the underlined MASs, respectively. One summary can be found that the smaller the probability is, the smaller $\mathcal{G}_k^{\text{opt}}$ can be obtained, which means a smaller violation probability leads a heaver limitation on the solved ellipsoid.

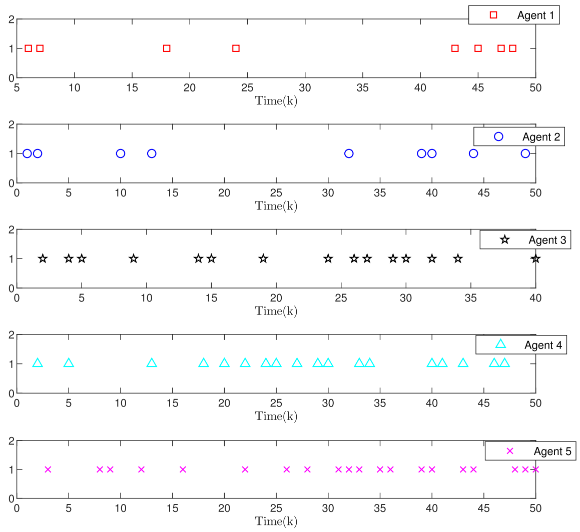

Fig. 6. Attacked instants of five agents.

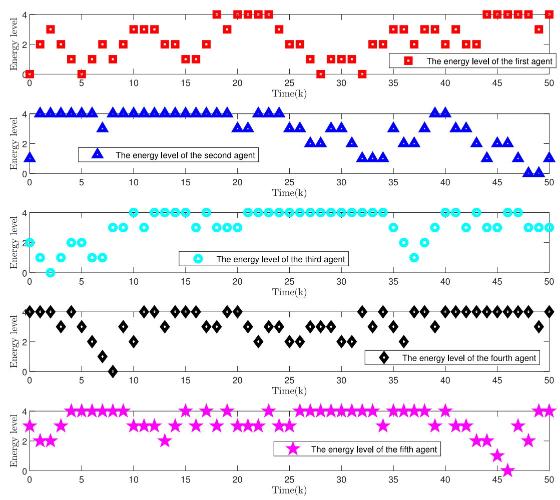

Fig. 7. Energy level of five agents.

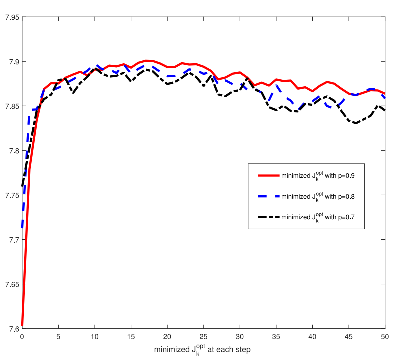

Fig. 8. Optimized ellipsoid trace (P_k) with different confident probabilities.

Define the cumulative deviation error index $J_k = \sum_{k=0}^{\Gamma} ||\hat{x}_k||$. In order to demonstrate the superiority of the proposed method, we compare our attained results with those in [21], in which the conventional mean-square consensus method is investigated. In Figs. 9 and 10, the cumulative deviation error of four agents are displayed, respectively, where the red dashed lines denote the cumulative deviation error via the method in [21], and blue counterparts represent the cumulative deviation error using the proposed approach in this article. It can be seen that the blue dashed lines are basically below the red ones among four subplots, which illustrates that using the concerned method in this article can gain the improved system performance when there exist unpredictable factors in system.

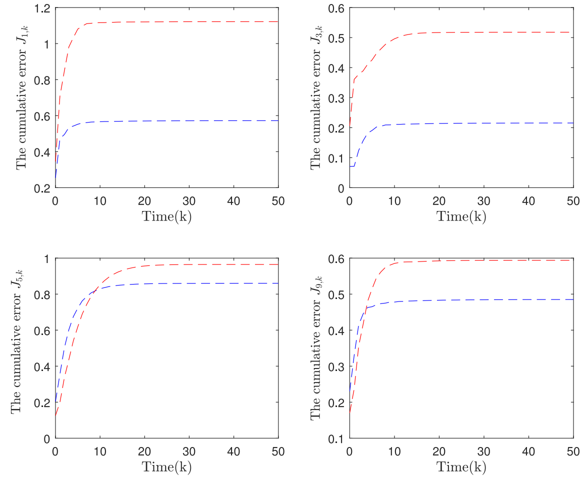

Fig. 9. First element of the cumulative deviation error using different approaches.

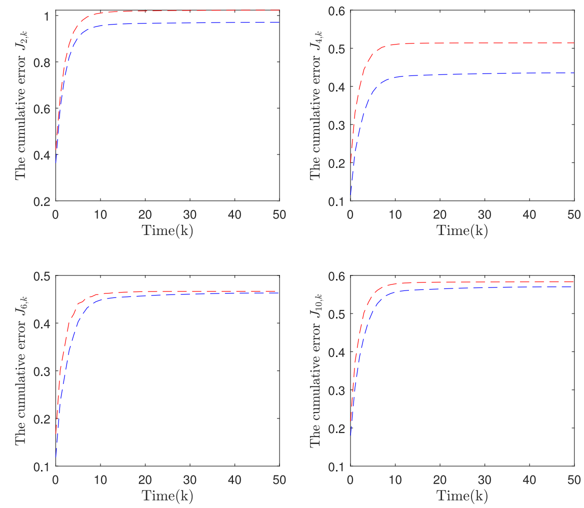

Fig. 10. Second element of the cumulative deviation error using different approaches.

B. Example 2

In this part, we consider a continuous stirred tank reactor (SCTR) [45], depicted in Fig. 11. By utilizing modern sensing technology, we first provide the parameter matrices of the proposed CSTR model as follows:

$$A_k = \begin{bmatrix} 0.3872 + 0.01\sin(0.2k) & 0.0222 & 0.01823 \\ 0.2444 & 0.3897 & 0.0007102 \\ -0.6849 & 0.9711 & 0.4008 \end{bmatrix}$$

$$B_k = \left[0.8 + 0.2\sin(0.4k), 0.5, 0.4\right]^{\mathrm{T}}$$

$$D_k = \left[0.16 + 0.05\cos(0.32k), 0.18, 0.2\right]^{\mathrm{T}}$$

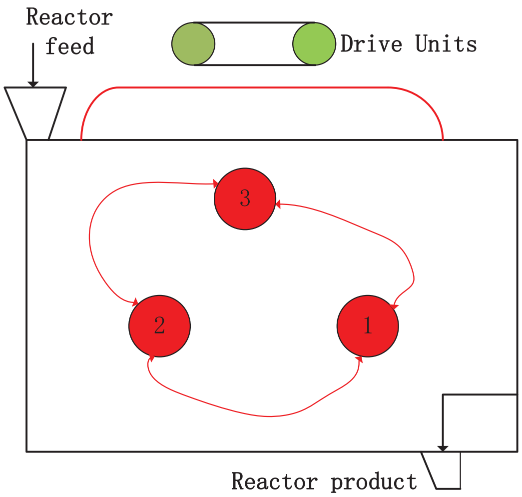

Fig. 11. Structure of the CSTR.

$$C_k = \begin{bmatrix} 0.64 + 0.05\sin(k) & 0.2 & 0 \\ 0 & 0.64 + 0.1\sin(k) & 0.2 \\ 0.5 & 0.1 & 0.1 \end{bmatrix}$$

$$\bar{\alpha} = \mathrm{diag}\{0.70, 0.8, 0.78\},\ \bar{\gamma}^0 = \mathrm{diag}\left\{\bar{\gamma}^1, \bar{\gamma}^2, \bar{\gamma}^3\right\}$$
$$\bar{\gamma}^1 = [0.2, 0.4, 0.4],\ \bar{\gamma}^2 = [0.5, 0.25, 0.25]$$
$$\bar{\gamma}^3 = [0.6, 0.15, 0.25],\ V_i = 0.5 (i = 1, 2, 3).$$

The determined Laplacian matrix is set by

$$\mathcal{L} = \begin{bmatrix} -1 & 0 & 1 \\ 0 & -1 & 1 \\ 1 & 1 & -2 \end{bmatrix}.$$

Preset the initial conditions of state of three subsystems $x_{1,0} = [1; 0; 1]$, $x_{2,0} = [-0.5; 1; -0.5]$, $x_{3,0} = [0; 1; 0.2]$, and the aggressive information is given as follows:

$$\Re(y_{i,k}) = \left[\Re_1^{\mathrm{T}}(y_{i,k}), \Re_2^{\mathrm{T}}(y_{i,k}), \Re_3^{\mathrm{T}}(y_{i,k})\right]^{\mathrm{T}}$$

where

$$\Re_1^{\mathrm{T}}(y_{i,k}) = \left[0.7\sin\left(y_k^{\mathrm{avg}}\right), 0.5\sin\left(y_k^{\mathrm{avg}}\right)\right]^{\mathrm{T}}$$
$$\Re_2^{\mathrm{T}}(y_{i,k}) = \left[0.75\sin\left(y_k^{\mathrm{avg}}\right), 0.5\sin\left(y_k^{\mathrm{avg}}\right)\right]^{\mathrm{T}}$$
$$\Re_3^{\mathrm{T}}(y_{i,k}) = \left[0.45\sin\left(y_k^{\mathrm{avg}}\right), 0.55\sin\left(y_k^{\mathrm{avg}}\right)\right]^{\mathrm{T}}.$$

The other system parameters remain unchanged. After operating the proposed algorithm ten times. The simulation results are included in Figs. 12–14 and Table II. The parameters of EHCs remain the same with Example 1 except for the Poisson parameter $\eta = 1.0$. According to (18) and (19), the probability distribution of the sensor energy level $\mathcal{Q}_{i,k}$ and the expectation of the measurement transmission $\bar{\lambda}_{i,k}$ are computed recursively as shown in Table II. In addition, from Figs. 12–14, it can be found that the state trajectories of three agents reach an agreement via the interagent information change subject to MFDI, which demonstrates the usefulness of the designed quasi-consensus controller.

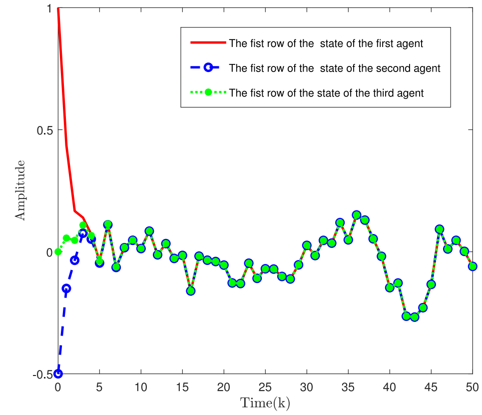

Fig. 12. First element of states of three agents.

TABLE II
PROBABILITY DISTRIBUTION OF THE SENSOR ENERGY LEVEL AND THE EXPECTATION OF THE MEASUREMENT TRANSMISSION ($\eta = 1.0$)

k	0	1	2	$\cdots$	50	51
$q_{i,k}^0$	0	0.3679	0.2707	$\cdots$	0.1154	0.1154
$q_{i,k}^1$	1	0.3679	0.3383	$\cdots$	0.1983	0.1983
$q_{i,k}^2$	0	0.1839	0.2256	$\cdots$	0.2253	0.2253
$q_{i,k}^3$	0	0.0613	0.1085	$\cdots$	0.2303	0.2303
$q_{i,k}^4$	0	0.0190	0.0569	$\cdots$	0.2308	0.2308
$\bar{\lambda}_{i,k}$	1	0.6321	0.7293	$\cdots$	0.8846	0.8846

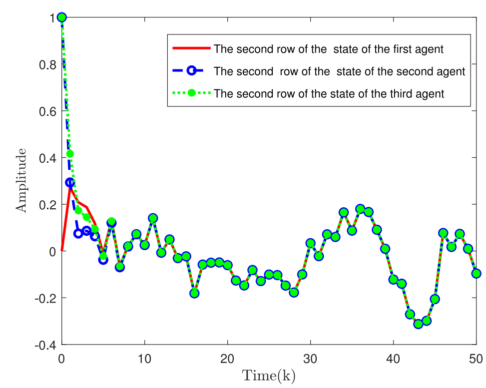

Fig. 13. Second element of states of three agents.

V. CONCLUSION

In this article, the distributed consensus protocol has been designed to solve the finite-horizon quasi-consensus problem for a class of time-varying stochastic nonlinear MASs with energy-constrained sensors subject to MFDI. The concerned attacks are assumed to randomly occur in data exchanges among agents. Besides, a random variable is used to describe energy amounts of the deployed harvester. In the light of the phenomena aforementioned, a new quasi-consensus concept has been proposed and realized by establishing sufficient conditions for the addressed MASs. Then, the semidefinite program method is utilized in the design of distributed controller. Finally, the effectiveness of the

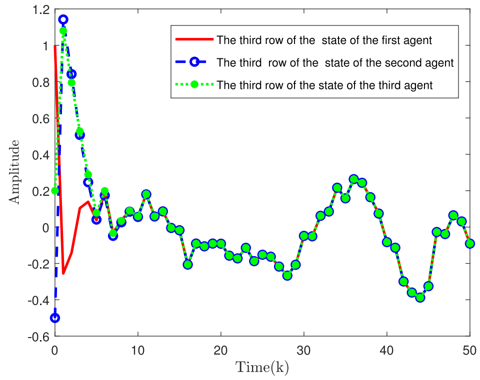

Fig. 14. Third element of states of three agents.

attained results is illustrated by blue two simulation examples. Furthermore, other possible future research directions include the probabilistic-constrained leader-follower formation control problem [46] and the protocol-based probabilistic-constrained control/filtering problem [47], [48], [49].

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