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Robust H_∞ control for nonlinear systems over network: A piecewise analysis method

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Abstract

A piecewise analysis method (PAM) is proposed to investigate the H_∞ performance analysis and H_∞ control design for nonlinear networked control systems (NNCSs), which are presented in the form of T–S fuzzy model with time-varying input delay. Different from the existing method in dealing with the time-varying delay, the whole variation interval of the delay is divided into two subintervals with equal length. Respecting for the delay belonging each subinterval, new criteria on H_∞ performance analysis of the NNCSs are obtained by checking the variation of the derivative of the Lyapunov functional in the two subintervals. Then, criteria for the H_∞ controller design are obtained by using the convexity properties of the matrix inequality and some other new analysis techniques, which are shown in terms of nonlinear matrix inequalities and can be solved by using a cone complementarity liberalization method. As application of the derived results, H_∞ controller design is carried out for the nonlinear mass-spring system. Discussion shows that the proposed method is less conservative than the existing references.

Keywords: Nonlinear networked control systems; Piecewise analysis method; Interval time-varying delay; Lyapunov–Krasovskii functional

1. Introduction

Networked control systems (NCSs) are spatially distributed systems in which the communication between sensors, actuators and controllers occur through a shared band limited digital communication network [1]. In recent years, increasing investigations have been focused on the stability analysis and controller synthesis of the NCSs, see [2–8] and references therein.

In these research works, modeling the NCSs as time delay systems with some constrains [4,5] have been widely accepted by many researchers. In these NCSs models, the network-induced delay and/or packet dropout are depicted as constant or time-varying delays, some special characters of the NCSs are denoted by the property of the delay, for example, the time-varying delay in the NCSs is piecewise constant, the derivation of the delay equals to 1 and the lower bound of the delay is often bigger than zero [4,9].

General speaking, most of the modeling method and control scheme concerning NCSs are only suitable for linear networked control systems (LNCSs), and only a few of results are about nonlinear networked control systems (NNCSs)

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[10,11]. The reason is that the system modeling and analysis for nonlinear system is much more complex than linear system, especially when delay exists in the systems.

T–S fuzzy model method can represent the local dynamics in different state-space regions by many linear models, which present a way to utilize the mature theories and analysis methods to study the nonlinear systems [7,12–16]. As described above, the LNCSs can be modeled as linear time delay systems with constraints on the delay and many good results have been obtained in the past decade. The above analysis provides a way to investigate the NNCSs by using T–S fuzzy model method: (1) by using so-called local approximation in fuzzy partition spaces, the nonlinear NCS plant is represented by the Takagi–Sugeno fuzzy model, which is described by fuzzy IF–THEN rules. The main feature of a Takagi–Sugeno fuzzy model is to express the local dynamics of each fuzzy implication (rule) by a linear system model. (2) The fuzzy controller is designed by using a PDC (parallel distributed compensation) method, each control rule is designed from the corresponding rule of a NCS T-S fuzzy model. Different from the common PDC method, the premise variable in the network-based fuzzy control rules is different from those in plant rules, which contains the information of network-induced delay. Furthermore, the networked delay and packet dropout is modeled as the input delay in this paper. (3) Combining (1) and (2), the nonlinear NCS can be represented by the T–S fuzzy model with random input delay. More recently, the robust stabilization, H_∞ control design and guaranteed cost networked control for NNCSs have been considered based on T–S fuzzy model in [17–20]. However, since the limitation on the employed method to treat the time-varying delay, the derived results in the existing references [4,18,19,17] appear much conservativeness, therefore, further improvements on the existing results should be carried out for future use.

The main contribution of this paper includes:

(1) An innovative nonlinear NCS modeling is presented. Different from common T–S fuzzy model, the control signal is a piecewise constant function because of the sampling behavior and a logic ZOH (zero-order holder) is adopted at the actuator to choose the newest signal as input signal. Then based on the T–S fuzzy modeling method, the nonlinear NCS can be modeled as special kinds of T–S fuzzy model with fast time-varying input delay. In this way, some well-established analysis method in linear NCS theory can be applied for the nonlinear NCS.
(2) A new Lyapunov functional analysis method is proposed, which can reduce the conservatism effectively. In the existing method in dealing with fast time-varying delay, only the information of delay bound are utilized. In this paper, besides the delay bounds, the inner information of the time-varying delay is used, which is achieved by partitioning the delay range into two subinterval and using different analysis method for delay falling into different subintervals.
(3) By using the convexity of the matrix inequality, the conservatism caused by enlarging $\eta(t)$ to its upper bound η_M can be avoided.

This paper considers the robust H_∞ control for nonlinear systems over network. The NNCSs are approximated by the T–S fuzzy models with uncertainties and input delay, where the networked induced delay and packet dropout in both forward and backward channel are considered, which are combined as one index in the systems. The main contribution of this paper lies on its less conservative, which is achieved by using a new Lyapunov functional approach and convexity property of the matrix inequality. Firstly, the variation range of the networked delay is equally divided into two subintervals and a new Lyapunov functional is constructed. Concerning the time-varying delay following into different subintervals, different techniques are used in the derivation of the Lyapunov functional and novel criteria for the asymptotically stable are obtained. Since more information of the time delay is employed, it can be expected that the concluded results will be much less conservative than the existing ones, which will be shown by some practical examples.

2. System description

Consider a continuous nonlinear time-delay system, which is represented by the following T–S fuzzy model

Plant rule i :

$$\begin{aligned} &\text{If } \theta_1(t) \text{ is } F_{i1}, \ldots, \theta_r(t) \text{ is } F_{ir} \\ &\text{then } \dot{x}(t) = (A_i + \Delta A_i)x(t) + (B_i + \Delta B_i)u(t) + B_{\omega i}\omega(t) \\ &\quad z(t) = C_i x(t) + D_i u(t) \end{aligned}$$

where A_i, B_i, $B_{\omega i}$, C_i and D_i are matrices with appropriate dimensions. $x(t) \in \mathbb{R}^n$, $u(t) \in \mathbb{R}^m$ and $z(t) \in \mathbb{R}^p$ are the state vector, control input vector and controlled output vector, respectively. $\omega(t) \in \mathfrak{L}_2[0, \infty)$ is the external disturbance and modeling error signal. $i \in \{1, 2, \ldots, r\} \triangleq \mathbb{S}$ and r is the member of IF–THEN rules. $\Delta A_i(t)$ and $\Delta B_i(t)$ are unknown matrices of appropriate dimensions satisfying

$$[\Delta A_i(t)\ \Delta B_i(t)] = H_i F_i(t)[E_{1i}\ \ E_{2i}] \tag{1}$$

where H_i, E_{1i} and $E_{2i}(i \in \mathbb{S})$ are known constant matrices of appropriate dimensions and $F_i(t)$ is an unknown matrix function with Lebesgue measurable elements satisfying

$$F_i^T(t)F_i(t) \leq I$$

By using center-average defuzzifier, product interference and singleton fuzzifier, the T–S fuzzy systems can be inferred as

$$\begin{cases} \dot{x}(t) = (A(\mu_i) + \Delta A(\mu_i))x(t) + (B(\mu_i) + \Delta B(\mu_i))u(t) + B_\omega(\mu_i)\omega(t) \\ z(t) = C(\mu_i)x(t) + D(\mu_i)u(t), t \in [i_k h + \tau_{i_k}, i_{k+1}h + \tau_{i_{k+1}}) \end{cases} \tag{2}$$

where τ_{i_k} and $\tau_{i_{k+1}}$ stand for the transmitted delay of the packets sampling at the instants $i_k h$ and $i_{k+1}h$, respectively. $A(\mu_i) = \sum_{i=1}^r \mu_i A_i$, $B(\mu_i)$, $C(\mu_i)$ and $D_i(\mu_i)$ can be similarly defined. $\Delta A(\mu_i) = \sum_{i=1}^r \mu_i \Delta A_i$, $\Delta B(\mu_i) = \sum_{i=1}^r \mu_i \Delta B_i(t)$:

$$\mu_i(\theta(t)) = \frac{\omega_i(\theta(t))}{\sum_{i=1}^r \omega_i(\theta(t))}, \quad \omega_i(\theta(t)) = \prod_{j=1}^g W_j^i(\theta_j(t))$$

$W_j^i(\theta_j(t))$ is the grade membership value of $\theta_j(t)$ in $W_j{}^i$ and $\mu_i(\theta(t))$ satisfies

$$\mu_i(\theta(t)) \geq 0, \quad \sum_{i=1}^r \mu_i(\theta(t)) = 1$$

For notational simplicity, we use μ_i to represent $\mu_i(\theta(t))$ in the following description.

In this paper, the nonlinear plant and controller are assumed to be connected through the shared network, therefore, the communication time of the data from sensor to controller and from controller to actuator should be taken into consideration. According to the modeling method in [4], the following assumption is needed.

Assumption 1. The sensor and actuator are time-driven and the controller is event-driven. The sampling period of the sensor is h. The sequence of the packet sampled at the sensor which can be successful arrive at the actuator is denoted as $i_1, i_2, \ldots$

Similar to [4,19], the controller has the following form:

Control rule i :

If $\theta_1(i_k h)$ is $F_{i1}, \ldots, \theta_r(i_k h)$ is F_{ir}

Then $u(t^+) = K_i x(i_k h), \quad t \in \{i_k h + \tau_{i_k}, k = 1,2, \ldots\}$

The inferred fuzzy controller is given by

$$u(t) = K(\mu_j^k)x(i_k h) \tag{3}$$

where $K(\mu_j^k) = \sum_{j=1}^r \mu_j^k K_j$, $K_j (j \in \mathbb{S})$ is the fuzzy control feedback gain to be determined and $i_k h$ is the sampling instant of the state vector. Define $\eta(t) = t - i_k h$, we can rewrite Eq. (3) as

$$u(t) = K(\mu_j^k)x(t - \eta(t)) \tag{4}$$

Combining Eqs. (2) and (4), the closed-loop nonlinear NCS can be expressed as

$$\begin{aligned}\dot{x}(t) &= (A(\mu_i) + \Delta A(\mu_i))x(t) + (B(\mu_i) + \Delta B(\mu_i))K(\mu_j^k)x(t - \eta(t)) + B_\omega(\mu_i)\omega(t)\\ z(t) &= C(\mu_i)x(t) + D(\mu_i)K(\mu_j^k)x(t - \eta(t)), \quad t \in [i_k h + \tau_{i_k}, i_{k+1}h + \tau_{i_{k+1}})\end{aligned} \tag{5}$$

Remark 1. $\{i_1, i_2, i_3, \ldots\}$ is subset of $\{1, 2, 3, \ldots\}$, which contains the information of packet losses and wrong packet sequence. If $\{i_1, i_2, i_3, \ldots\} = 1, 2, 3, \ldots\}$, $i_{k+1} = i_k + 1$, which means no packet losses and wrong packet sequence happen. If $i_{k+1} - i_k = n(\geq 2)$, it means $n-1$ continuous packets are lost. If $i_{k+1} < i_k$, it means wrong packet sequence happens. It can be easily seen that $\cup_{k=1}^{\infty}[i_k h + \tau_{i_k},\ i_{k+1}h + \tau_{i_{k+1}}) = [0,\ \infty)$. In this paper we assume that $u(t)$=0 before the first control signal reaches the plant and a constant $\eta_2 > 0$ exists such that $(i_{k+1} - i_k)h + \tau_{i_{k+1}} \leq \eta_2$ $(k = 1, 2, \ldots)$.

Remark 2. As pointed out in [4], when $i_{k+1} < i_k$, that is, the new packet reaches the destination before the old one, discarding the old packet may result in a less conservative result. Therefore we assume $i_{k+1} > i_k$ in the following discussion.

Remark 3. Some differences between NNCS and normal nonlinear system should be noted in the modeling process: (1) Due to the sampling behavior, the control signal is not a continuous function as a result of the existence of the zero-order holder (ZOH), but a piecewise constant function, which updates its data at the instants $\{i_k h + \tau_{i_k}\}$. (2) The time stamp is used, which invariably keeps in the packet transmission. (3) The actuator is event-driven with a logic ZOH to accept the arrived control input signal only the time stamp of control input packet is the newest.

The following lemmas are necessary in the proof of the main results.

Lemma 1 (*Han [21]*). *For any constant matrix* $W \in \mathbb{R}^{n\times n}$, $W = W^T > 0$, *constant* $\gamma > 0$ *and vector function* $\dot{x} : [-\gamma, 0] \to \mathbb{R}^n$ *such that the following integration is well defined, then*

$$-\gamma \int_{t-\gamma}^{t} \dot{x}^T(s)W\dot{x}(s)\,ds \leq \begin{bmatrix} x(t) \\ x(t-\gamma) \end{bmatrix}^T \begin{bmatrix} -W & W \\ W & -W \end{bmatrix} \begin{bmatrix} x(t) \\ x(t-\gamma) \end{bmatrix} \tag{6}$$

Lemma 2 (*Tian et al. [22]*). Ξ_1, Ξ_2 *and* Ω *are constant matrices of appropriate dimensions and* $0 \leq \tau_m \leq \tau(t) \leq \tau_M$, *then*

$$(\tau(t) - \tau_m)\Xi_1 + (\tau_M - \tau(t))\Xi_2 + \Omega < 0 \tag{7}$$

if and only if

$$(\tau_M - \tau_m)\Xi_1 + \Omega < 0 \tag{8}$$

and

$$(\tau_M - \tau_m)\Xi_2 + \Omega < 0 \tag{9}$$

hold.

The purpose of this work is to design a robust H_∞ controller equation (4) for the T–S fuzzy model equation (5) such that the following requirements are satisfied.

1. The T–S fuzzy system equation (5) with the controller equation (4) and $\omega(t) = 0$ is asymptotically stable for all admissible parameter uncertainties.
2. Under zero initial condition, system equation (5) with the controller equation (4) satisfies $\|z(t)\|_2 < \gamma\|\omega(t)\|_2$ for any non-zero $\omega(t) \in \mathfrak{L}_2[0,\ \infty)$.

3. Main results

In [23], the central point of variation of the delay was firstly employed to study the stability and stabilization for systems with interval time delay, which is called DCP method. Furthermore, the DCP method has been used to study the stability and control design of a class of networked control system [9].

As an extension of the DCP method, we divide the variation interval of the delay into two parts with equal length and define constant $\delta=(\tau_M-\tau_m)/2$ and $\tau_0=\tau_m+\delta$. Define two sets as

$$\Phi_1=\{t|\tau(t)\in[\tau_m,\tau_0]\}$$

$$\Phi_2=\{t|\tau(t)\in(\tau_0,\tau_M]\}$$

we can see that $\Phi_1\cup\Phi_2=R^+$ and $\Phi_1\cap\Phi_2=\phi$ (empty set). In the following analysis, we will discuss the variation of $\dot V(x_t)(\dot V(x_t)$ is the derivation of the Lyapunov functional) under two cases, that is, $\tau(t)\in[\tau_m,\tau_0]$ and $\tau(t)\in(\tau_0,\tau_M]$, in each case, different analysis technique is used to obtain the sufficient conditions, the criteria for the stability of system equation (5) is the sum of the conditions in each case.

Theorem 1. *For a prescribed $\gamma>0$, constants τ_m, τ_M and given matrices K_j, system equation (5) is asymptotically stable if there exist matrices $P>0$, $Q=\left[\begin{smallmatrix}Q_{11} & Q_{21}^T\\ Q_{21} & Q_{22}\end{smallmatrix}\right]>0$, $Q_0>0$, $R_0>0$, $R_1>0$, $R_2>0$, $N_{ij}\in\mathbb{R}^{5n\times n}$, $M_{ij}\in\mathbb{R}^{5n\times n}$, $T_{ij}\in\mathbb{R}^{5n\times n}$, $S_{ij}\in\mathbb{R}^{5n\times n}(i,j\in\mathbb{S})$ such that the following LMIs hold:*

$$\Xi_a^{ij}(l)+\Xi_a^{ji}(l)<0 \tag{10}$$

$$\Xi_b^{ij}(l)+\Xi_b^{ji}(l)<0 \tag{11}$$

$$l=1,2,\quad i\le j\in\mathbb{S}$$

where

$$\Xi_a^{ij}(l)=\begin{bmatrix}\Xi_{a1}^{ij}+W_a^{ij}+(W_a^{ij})^T & * & * & *\\ \Xi_{a2}^{ij}(l) & -R_1 & * & *\\ \Xi_{31}^{ij} & 0 & \Xi_{33} & *\\ \Xi_{41}^{ij} & 0 & \Xi_{43}^{i} & \Xi_{44}\end{bmatrix}$$

$$\Xi_b^{ij}(l)=\begin{bmatrix}\Xi_{b1}^{ij}+W_b^{ij}+(W_b^{ij})^T & * & * & *\\ \Xi_{b2}^{ij}(l) & -R_2 & * & *\\ \Xi_{31}^{ij} & 0 & \Xi_{33} & *\\ \Xi_{41}^{ij} & 0 & \Xi_{43}^{i} & \Xi_{44}\end{bmatrix}$$

$$\Xi_{a1}^{ij}=\begin{bmatrix}\Gamma_1^i & * & * & * & *\\ K_j^TB_i^TP & 0 & * & * & *\\ R_0 & 0 & -Q_0-R_0+Q_{11} & * & *\\ 0 & 0 & Q_{21} & Q_{22}-Q_{11}-R_2/\delta & *\\ 0 & 0 & 0 & -Q_{21}+R_2/\delta & -Q_{22}-R_2/\delta\end{bmatrix}$$

$$\Xi_{b1}^{ij}=\begin{bmatrix}\Gamma_1^i & * & * & * & *\\ K_j^TB_i^TP & 0 & * & * & *\\ R_0 & 0 & -Q_0-R_0+Q_{11}-R_1/\delta & * & *\\ 0 & 0 & Q_{21}+R_1/\delta & Q_{22}-Q_{11}-R_1/\delta & *\\ 0 & 0 & 0 & -Q_{21} & -Q_{22}\end{bmatrix}$$

$$\Xi_{a2}^{ij}(1) = \sqrt{\delta} N_{ij}^T, \quad \Xi_{a2}^{ij}(2) = \sqrt{\delta} M_{ij}^T, \quad \Xi_{b2}^{ij}(1) = \sqrt{\delta} T_{ij}^T, \quad \Xi_{b2}^{ij}(2) = \sqrt{\delta} S_{ij}^T$$

$$\Xi_{31}^{ij} = \begin{bmatrix} C_i & D_i K_j & 0 & 0 & 0 \\ B_{\omega i}^T P & 0 & 0 & 0 & 0 \end{bmatrix}, \quad \Xi_{33} = \mathrm{diag}(-I \quad -\gamma^2 I)$$

$$\Xi_{41}^{ij} = \begin{bmatrix} \tau_m R_0 \mathcal{A}_{ij} \\ \sqrt{\delta} R_1 \mathcal{A}_{ij} \\ \sqrt{\delta} R_2 \mathcal{A}_{ij} \end{bmatrix}, \quad \Xi_{43}^{i} = \begin{bmatrix} 0 & \tau_m R_0 B_{\omega i} \\ 0 & \sqrt{\delta} R_1 B_{\omega i} \\ 0 & \sqrt{\delta} R_2 B_{\omega i} \end{bmatrix}, \quad \Xi_{44} = \mathrm{diag}(-R_0 \quad -R_1 \quad -R_2)$$

$$W_a^{ij} = [0 \quad -N_{ij} + M_{ij} \quad N_{ij} \quad -M_{ij} \quad 0]$$

$$W_b^{ij} = [0 \quad -T_{ij} + S_{ij} \quad 0 \quad T_{ij} \quad -S_{ij}]$$

$$\Gamma_1^i = P A_i + A_i^T P + Q_0 - R_0$$

$$\mathcal{A}_{ij} = [A_i \quad B_i K_j \quad 0 \quad 0 \quad 0]$$

Proof. Choose the Lyapunov–Krasovskii functional candidate as

$$\begin{aligned} V(x_t) = {} & x^T(t) P x(t) + \int_{t-\tau_m}^{t} x^T(s) Q_0 x(s)\, ds \\ & + \int_{t-\delta}^{t} \eta^T(s) Q \eta(s)\, ds + \tau_m \int_{t-\tau_m}^{t} \int_{s}^{t} \dot{x}^T(v) R_0 \dot{x}(v)\, dv\, ds \\ & + \int_{t-\tau_0}^{t-\tau_m} \int_{s}^{t} \dot{x}^T(v) R_1 \dot{x}(v)\, dv\, ds + \int_{t-\tau_M}^{t-\tau_0} \int_{s}^{t} \dot{x}^T(v) R_2 \dot{x}(v)\, dv\, ds \end{aligned} \tag{12}$$

where

$$\eta^T(t) = [x^T(t-\tau_m) \ x^T(t-\tau_0)]$$

Taking the derivation of $V(x_t)$, we have

$$\begin{aligned} \dot{V}(x_t) \le {} & 2\dot{x}^T(t) P x(t) + x^T(t) Q_0 x(t) - x^T(t-\tau_m) Q_0 x(t-\tau_m) \\ & + \eta^T(t) Q \eta(t) - \eta^T(t-\delta) Q \eta(t-\delta) \\ & + \dot{x}^T(t)(\tau_m^2 R_0 + \delta R_1 + \delta R_2)\dot{x}(t) - \tau_m \int_{t-\tau_m}^{t} \dot{x}^T(s) R_0 \dot{x}(s)\, ds \\ & - \int_{t-\tau_m}^{t-\tau_0} \dot{x}^T(s) R_1 \dot{x}(s)\, ds - \int_{t-\tau_M}^{t-\tau_0} \dot{x}^T(s) R_2 \dot{x}(s)\, ds \\ & + z^T(t) z(t) - \gamma^2 \omega^T(t)\omega(t) - z^T(t) z(t) + \gamma^2 \omega^T(t)\omega(t) \end{aligned} \tag{13}$$

For all $t \in R^+$, we can obtain that $\tau(t) \in \Phi_1$ or $\tau(t) \in \Phi_2$, that is, $\tau(t) \in [\tau_m, \tau_0]$ or $\tau(t) \in (\tau_0, \tau_M]$. In the following, concerning $\tau(t) \in \Phi_1$ or $\tau(t) \in \Phi_2$, we will discuss the variation $\dot{V}(x_t)$ for two cases.

Case 1: For $\tau(t) \in \Phi_1$, that is, $\tau(t) \in [\tau_m, \tau_0]$.

By using Lemma 1, we can obtain that

$$-\tau_m \int_{t-\tau_m}^{t} \dot{x}^T(s) R_0 \dot{x}(s)\, ds \le \begin{bmatrix} x(t) \\ x(t-\tau_m) \end{bmatrix}^T \begin{bmatrix} -R_0 & R_0 \\ R_0 & -R_0 \end{bmatrix} \begin{bmatrix} x(t) \\ x(t-\tau_m) \end{bmatrix} \tag{14}$$

$$-\int_{t-\tau_M}^{t-\tau_0} \dot{x}^T(s) R_2 \dot{x}(s)\, ds \le \begin{bmatrix} x(t-\tau_0) \\ x(t-\tau_M) \end{bmatrix}^T \begin{bmatrix} -R_2/\delta & R_2/\delta \\ R_2/\delta & -R_2/\delta \end{bmatrix} \begin{bmatrix} x(t-\tau_0) \\ x(t-\tau_M) \end{bmatrix} \tag{15}$$

Combining Eqs. (13)–(15) and employing free-weighing matrices N_{ij}, M_{ij}, we obtain

$$\begin{aligned}\dot{V}(t) \le & \sum_{i=1}^{r}\sum_{j=1}^{r}\mu_i\mu_j^k\{\zeta^T(t)\Xi_{a1}^{ij}\zeta(t)+x^T(t)PB_{\omega i}\omega(t)\}\\ & +\dot{x}^T(t)(\tau_m^2R_0+\delta R_1+\delta R_2)\dot{x}(t)-\int_{t-\tau_0}^{t-\tau_m}\dot{x}^T(s)R_1\dot{x}(s)\,ds\\ & +2\sum_{i=1}^{r}\sum_{j=1}^{r}\mu_i\mu_j^k\zeta^T(t)N_{ij}\left[x(t-\tau_m)-x(t-\tau(t))-\int_{t-\tau(t)}^{t-\tau_m}\dot{x}(s)\,ds\right]\\ & +2\sum_{i=1}^{r}\sum_{j=1}^{r}\mu_i\mu_j^k\zeta^T(t)M_{ij}\left[x(t-\tau(t))-x(t-\tau_0)-\int_{t-\tau_0}^{t-\tau(t)}\dot{x}(s)\,ds\right]\\ & +z^T(t)z(t)-\gamma^2\omega^T(t)\omega(t)-z^T(t)z(t)+\gamma^2\omega^T(t)\omega(t)\end{aligned} \tag{16}$$

where N_{ij} and M_{ij} are slack matrices defined in Theorem 1. Note that

$$z^T(t)z(t)\le\sum_{i=1}^{r}\sum_{j=1}^{r}\mu_i\mu_j^k\zeta^T(t)\mathcal{C}_{ij}^T\mathcal{C}_{ij}\zeta(t) \tag{17}$$

$$\dot{x}^T(t)R_s\dot{x}(t)\le\sum_{i=1}^{r}\sum_{j=1}^{r}\mu_i\mu_j^k\begin{bmatrix}\mathcal{A}_{ij}^T\\ B_{\omega i}^T\end{bmatrix}R_s\begin{bmatrix}\mathcal{A}_{ij}^T\\ B_{\omega i}^T\end{bmatrix}^T,\quad s=0,1,2 \tag{18}$$

where

$$\mathcal{C}_{ij}=[C_i\ \ D_iK_j\ \ 0\ \ 0\ \ 0]$$

The following inequalities can been obtained:

$$\begin{aligned}&-2\sum_{i=1}^{r}\sum_{j=1}^{r}\mu_i\mu_j^k\zeta^T(t)N_{ij}\int_{t-\tau(t)}^{t-\tau_m}\dot{x}(s)\,ds\\ &\le(\tau(t)-\tau_m)\sum_{i=1}^{r}\sum_{j=1}^{r}\mu_i\mu_j^k\zeta^T(t)N_{ij}^TR_1^{-1}N_{ij}\zeta(t)+\int_{t-\tau(t)}^{t-\tau_m}\dot{x}^T(s)R_1\dot{x}(s)\,ds\end{aligned} \tag{19}$$

$$\begin{aligned}&-2\sum_{i=1}^{r}\sum_{j=1}^{r}\mu_i\mu_j^k\zeta^T(t)M_{ij}\int_{t-\tau_0}^{t-\tau(t)}\dot{x}(s)\,ds\\ &\le(\tau_0-\tau(t))\sum_{i=1}^{r}\sum_{j=1}^{r}\mu_i\mu_j^k\zeta^T(t)M_{ij}^TR_1^{-1}M_{ij}\zeta(t)+\int_{t-\tau_0}^{t-\tau(t)}\dot{x}^T(s)R_1\dot{x}(s)\,ds\end{aligned} \tag{20}$$

Substituting Eqs. (17)–(20) into Eq. (16), we obtain

$$\begin{aligned}&\dot{V}(x_t)+z^T(t)z(t)-\gamma^2\omega^T(t)\omega(t)\\ &\le\sum_{i=1}^{r}\sum_{j=1}^{r}\mu_i\mu_j^k\begin{bmatrix}\zeta(t)\\ \omega(t)\end{bmatrix}^T\left\{\begin{bmatrix}\Lambda_a^{ij} & PB_{\omega i}\\ B_{\omega i}^TP & -\gamma^2I\end{bmatrix}+\begin{bmatrix}\mathcal{A}_{ij}^T\\ B_{\omega i}^T\end{bmatrix}(\tau_m^2R_0+\delta R_1+\delta R_2)\begin{bmatrix}\mathcal{A}_{ij}^T\\ B_{\omega i}^T\end{bmatrix}^T\right.\\ &\quad\left.+(\tau(t)-\tau_m)\begin{bmatrix}N_{ij}\\ 0\end{bmatrix}R_1^{-1}\begin{bmatrix}N_{ij}\\ 0\end{bmatrix}^T+(\tau_0-\tau(t))\begin{bmatrix}M_{ij}\\ 0\end{bmatrix}R_1^{-1}\begin{bmatrix}M_{ij}\\ 0\end{bmatrix}^T\right\}\begin{bmatrix}\zeta(t)\\ \omega(t)\end{bmatrix}\\ &=\sum_{i,j=1}^{r}\sum_{i\le j}^{r}\mu_i\mu_j^k\begin{bmatrix}\zeta(t)\\ \omega(t)\end{bmatrix}^T\left\{\Sigma(i,j)+(\tau(t)-\tau_m)\begin{bmatrix}N_{ij}\\ 0\end{bmatrix}R_1^{-1}\begin{bmatrix}N_{ij}\\ 0\end{bmatrix}^T\right.\end{aligned}$$

$$+(\tau_0-\tau(t))\begin{bmatrix} M_{ij} \\ 0 \end{bmatrix} R_1^{-1} \begin{bmatrix} M_{ij} \\ 0 \end{bmatrix}^T$$

$$+\Sigma(j,i)+(\tau(t)-\tau_m)\begin{bmatrix} N_{ji} \\ 0 \end{bmatrix} R_1^{-1} \begin{bmatrix} N_{ji} \\ 0 \end{bmatrix}^T$$

$$+(\tau_0-\tau(t))\begin{bmatrix} M_{ji} \\ 0 \end{bmatrix} R_1^{-1} \begin{bmatrix} M_{ji} \\ 0 \end{bmatrix}^T \Bigg\} \begin{bmatrix} \zeta(t) \\ \omega(t) \end{bmatrix} \tag{21}$$

where $\Lambda_a^{ij}=\Xi_{a1}^{ij}+W_a^{ij}+(W_a^{ij})^T+\mathcal{C}_{ij}^T\mathcal{C}_{ij}$ and $\Sigma(i,j)=\left[\begin{smallmatrix} \Lambda_a^{ij} & PB_{\omega i} \\ B_{\omega i}^T P & -\gamma^2 I \end{smallmatrix}\right]+\left[\begin{smallmatrix} \mathcal{A}_{ij}^T \\ B_{\omega i}^T \end{smallmatrix}\right](\tau_m^2 R_0+\delta R_1+\delta R_2)\left[\begin{smallmatrix} \mathcal{A}_{ij}^T \\ B_{\omega i}^T \end{smallmatrix}\right]^T$. By using Lemma 2 and Schur complement, we can conclude that Eq. (10) is the sufficient condition to guarantee

$$\Sigma(i,j)+(\tau(t)-\tau_m)\begin{bmatrix} N_{ij} \\ 0 \end{bmatrix} R_1^{-1} \begin{bmatrix} N_{ij} \\ 0 \end{bmatrix}^T$$

$$+(\tau_0-\tau(t))\begin{bmatrix} M_{ij} \\ 0 \end{bmatrix} R_1^{-1} \begin{bmatrix} M_{ij} \\ 0 \end{bmatrix}^T$$

$$+\Sigma(j,i)+(\tau(t)-\tau_m)\begin{bmatrix} N_{ji} \\ 0 \end{bmatrix} R_1^{-1} \begin{bmatrix} N_{ji} \\ 0 \end{bmatrix}^T$$

$$+(\tau_0-\tau(t))\begin{bmatrix} M_{ji} \\ 0 \end{bmatrix} R_1^{-1} \begin{bmatrix} M_{ji} \\ 0 \end{bmatrix}^T < 0 \tag{22}$$

Combining Eqs. (21) and (22), we can show that for $t \in \Phi_1$

$$\dot{V}(x_t)+z^T(t)z(t)-\gamma^2\omega^T(t)\omega(t)<0 \tag{23}$$

Case 2: For $t \in \Phi_2$, that is, $\tau(t) \in (\tau_0, \tau_M]$.
By using Lemma 1, we can obtain that

$$-\int_{t-\tau_0}^{t-\tau_m} \dot{x}^T(s)R_1\dot{x}(s)\,ds \le \begin{bmatrix} x(t-\tau_m) \\ x(t-\tau_0) \end{bmatrix}^T \begin{bmatrix} -R_1/\delta & R_1/\delta \\ R_1/\delta & -R_1/\delta \end{bmatrix} \begin{bmatrix} x(t-\tau_m) \\ x(t-\tau_0) \end{bmatrix} \tag{24}$$

By employing free-weighing matrices T_{ij}, S_{ij}, we can obtain the following inequality from Eq. (13):

$$\dot{V}(t) \le \sum_{i=1}^{r}\sum_{j=1}^{r}\mu_i\mu_j^k\{\zeta^T(t)\Xi_{b1}^{ij}\zeta(t)+x^T(t)PB_{\omega i}\omega(t)\}$$

$$+\dot{x}^T(t)(\tau_m^2R_0+\delta R_1+\delta R_2)\dot{x}(t)-\int_{t-\tau_M}^{t-\tau_0}\dot{x}^T(s)R_2\dot{x}(s)\,ds$$

$$+2\sum_{i=1}^{r}\sum_{j=1}^{r}\mu_i\mu_j^k\zeta^T(t)T_{ij}\left[x(t-\tau_0)-x(t-\tau(t))-\int_{t-\tau(t)}^{t-\tau_0}\dot{x}(s)\,ds\right]$$

$$+2\sum_{i=1}^{r}\sum_{j=1}^{r}\mu_i\mu_j^k\zeta^T(t)S_{ij}\left[x(t-\tau(t))-x(t-\tau_M)-\int_{t-\tau_M}^{t-\tau(t)}\dot{x}(s)\,ds\right]$$

$$+z^T(t)z(t)-\gamma^2\omega^T(t)\omega(t)-z^T(t)z(t)+\gamma^2\omega^T(t)\omega(t) \tag{25}$$

where T_{ij} and S_{ij} are slack matrices defined in Theorem 1. Following a similar proof in Case 1, we can obtain that Eq. (10) is sufficient condition to guarantee:

$$\dot{V}(x_t) + z^T(t)z(t) - \gamma^2\omega^T(t)\omega(t) < 0 \tag{26}$$

Combing Eqs. (23) and (26), we can obtain that for all $t \in R^+$, $\dot{V}(x_t) + z^T(t)z(t) - \gamma^2\omega^T(t)\omega(t) < 0$. Integrating both sides of Eq. (23) from 0 to t yields

$$V(t) - V(0) + \int_0^t z^T(s)z(s)\,ds - \int_0^t \gamma^2\omega^T(s)\omega(s)\,ds < 0 \tag{27}$$

Letting $t \to \infty$ and under zero initial condition, we can show from Eq. (27) that

$$\int_0^\infty z^T(s)z(s)\,ds < \int_0^\infty \gamma^2\omega^T(s)\omega(s)\,ds \tag{28}$$

that is, $\|z(t)\|_2 < \gamma\|\omega(t)\|_2$. Under the condition that $\omega(t) = 0$, we can conclude from Eqs. (10) and (11) that

$$\begin{bmatrix} \Xi_{a1}^{ij} + W_a^{ij} + (W_a^{ij})^T & * & * \\ \Xi_{a2}^{ij}(l) & -R_1 & * \\ \Xi_{41}^{ij} & 0 & \Xi_{44} \end{bmatrix} + \begin{bmatrix} \Xi_{a1}^{ji} + W_a^{ji} + (W_a^{ji})^T & * & * \\ \Xi_{a2}^{ji}(l) & -R_1 & * \\ \Xi_{41}^{ji} & 0 & \Xi_{44} \end{bmatrix} < 0 \tag{29}$$

$$\begin{bmatrix} \Xi_{b1}^{ij} + W_b^{ij} + (W_b^{ij})^T & * & * \\ \Xi_{b2}^{ij}(l) & -R_2 & * \\ \Xi_{41}^{ij} & 0 & \Xi_{44} \end{bmatrix} + \begin{bmatrix} \Xi_{b1}^{ji} + W_b^{ji} + (W_b^{ji})^T & * & * \\ \Xi_{b2}^{ji}(l) & -R_2 & * \\ \Xi_{41}^{ji} & 0 & \Xi_{44} \end{bmatrix} < 0 \tag{30}$$

Similar to the above proof, $\dot{V}(x_t) < 0$ can be concluded from Eqs. (29) and (30). This completes the proof. □

Remark 4. Throughout the proof of Theorem 1, it can be seen that we need not enlarge $\tau(t)(\cdot)$ to $\tau_M(\cdot)$, therefore the common existed conservatism caused by this kind of enlargement in [24–28] can be avoided. The less conservativeness of the obtained criteria can be seen from the examples.

By a commonly used analysis method for parameter uncertainties, the following result can be obtained for the robust stability of the system equation (5).

Theorem 2. *For a prescribed* $\gamma > 0$, *constants* τ_m, τ_M *and given matrices* K_j, *system equation* (5) *is asymptotically stable if there exist matrices* $P > 0$, $Q = \left[\begin{smallmatrix} Q_{11} & Q_{21}^T \\ Q_{21} & Q_{22} \end{smallmatrix}\right] > 0$, $Q_0 > 0$, $R_0 > 0$, $R_1 > 0$, $R_2 > 0$, $N_{ij} \in \mathbb{R}^{5n\times n}$, $M_{ij} \in \mathbb{R}^{5n\times n}$, $T_{ij} \in \mathbb{R}^{5n\times n}$, $S_{ij} \in \mathbb{R}^{5n\times n}$ $(i, j \in \mathbb{S})$ *and scalars* ε_i *such that the following LMIs hold*:

$$\bar{\Xi}_a^{ij}(l) + \bar{\Xi}_a^{ji}(l) < 0 \tag{31}$$

$$\bar{\Xi}_b^{ij}(l) + \bar{\Xi}_b^{ji}(l) < 0 \tag{32}$$

$$l = 1,2, \quad i \le j \in \mathbb{S}$$

where

$$\bar{\Xi}_a^{ij}(l) = \begin{bmatrix} \Xi_{a1}^{ij} + W_a^{ij} + (W_a^{ij})^T & * & * & * & * \\ \Xi_{a2}^{ij}(l) & -R_1 & * & * & * \\ \Xi_{31}^{ij} & 0 & \Xi_{33} & * & * \\ \Xi_{41}^{ij} & 0 & \Xi_{43}^{i} & \Xi_{44} & * \\ \Xi_{51}^{i} & 0 & 0 & 0 & \Xi_{55} \end{bmatrix}$$

$$\bar{\Xi}_b^{ij}(l)=\begin{bmatrix} \Xi_{b1}^{ij}+W_b^{ij}+(W_b^{ij})^T & * & * & * & * \\ \Xi_{b2}^{ij}(l) & -R_2 & * & * & * \\ \Xi_{31}^{ij} & 0 & \Xi_{33} & * & * \\ \Xi_{41}^{ij} & 0 & \Xi_{43}^{i} & \Xi_{44} & * \\ \Xi_{51}^{i} & 0 & 0 & 0 & \Xi_{55} \end{bmatrix}$$

$$\Xi_{51}^{i}=\begin{bmatrix} H_i^T P & 0 & 0\ 0\ 0 \\ \varepsilon_i E_{1i} & \varepsilon_i E_{2i} K_j & 0\ 0\ 0 \end{bmatrix},\quad \Xi_{55}=\mathrm{diag}(-\varepsilon_i I - \varepsilon_i I)$$

Based on Theorems 1 and 2, the sufficient conditions for the asymptotically stable of the following system can be obtained:

$$\dot{x}(t)=(A(\mu_i)+\Delta A(\mu_i))x(t)+(B(\mu_i)+\Delta B(\mu_i))K(\mu_j^k)x(t-\eta(t)) \tag{33}$$

Corollary 1. *For constants τ_m, τ_M and given matrices K_j, system equation (33) is asymptotically stable if there exist matrices $P>0$, $Q=[\begin{smallmatrix} Q_{11} & Q_{21}^T \\ Q_{21} & Q_{22}\end{smallmatrix}]>0$, $Q_0>0$, $R_0>0$, $R_1>0$, $R_2>0$, $N_{ij}\in\mathbb{R}^{5n\times n}$, $M_{ij}\in\mathbb{R}^{5n\times n}$, $T_{ij}\in\mathbb{R}^{5n\times n}$, $S_{ij}\in\mathbb{R}^{5n\times n}$ $(i,j\in\mathbb{S})$ and scalars ε_i such that the following LMIs hold*:

$$\Sigma_a^{ij}(l)+\Sigma_a^{ji}(l)<0 \tag{34}$$

$$\Sigma_b^{ij}(l)+\Sigma_b^{ji}(l)<0 \tag{35}$$

$$l=1,2,\quad i\le j\in\mathbb{S}$$

where

$$\Sigma_a^{ij}(l)=\begin{bmatrix} \Xi_{a1}^{ij}+W_a^{ij}+(W_a^{ij})^T & * & * & * \\ \Xi_{a2}^{ij}(l) & -R_1 & * & * \\ \Xi_{41}^{ij} & 0 & \Xi_{44} & * \\ \Xi_{51}^{i} & 0 & 0 & \Xi_{55} \end{bmatrix}$$

$$\Sigma_b^{ij}(l)=\begin{bmatrix} \Xi_{b1}^{ij}+W_b^{ij}+(W_b^{ij})^T & * & * & * \\ \Xi_{b2}^{ij}(l) & -R_2 & * & * \\ \Xi_{41}^{ij} & 0 & \Xi_{44} & * \\ \Xi_{51}^{i} & 0 & 0 & \Xi_{55} \end{bmatrix}$$

where Ξ_{a1}^{ij}, Ξ_{b1}^{ij}, $\Xi_{a2}^{ij}(l)$, $\Xi_{b2}^{ij}(l)$, W_a^{ij}, W_b^{ij}, Ξ_{41}^{ij}, Ξ_{44}, Ξ_{51}^{i}, Ξ_{55} *are as defined in Theorem 2.*

4. Robust H_∞ controller design

Now, we are in a position to state the delay-dependent robust H_∞ control result for the system equation (5).

Theorem 3. *For a prescribed $\gamma>0$ and constants τ_m, τ_M, system equation (5) is asymptotically stable if there exist matrices $X>0$, $\tilde{Q}=[\begin{smallmatrix} \tilde{Q}_{11} & \tilde{Q}_{21}^T \\ \tilde{Q}_{21} & \tilde{Q}_{22}\end{smallmatrix}]>0$, $\bar{R}_s>0(s=0,1,2)$, $\tilde{Q}_0>0$, $\tilde{N}_{ij}$, $\tilde{M}_{ij}$, $\tilde{T}_{ij}$, $\tilde{S}_{ij}$ and Y_j of appropriate dimensions and $\mu_i(l=1,2,i,j\in\mathbb{S})$ such that the following LMIs hold*:

$$\tilde{\Xi}_a^{ij}(l)+\tilde{\Xi}_a^{ji}(l)<0 \tag{36}$$

$$\tilde{\Xi}_b^{ij}(l)+\tilde{\Xi}_b^{ji}(l)<0 \tag{37}$$

$$-X\tilde{R}_s^{-1}X < -\bar{R}_s, \quad s = 0,1,2 \qquad (38)$$
$$l = 1,2, \quad i < j \in \mathbb{S}$$

where

$$\tilde{\Xi}_a^{ij}(l) = \begin{bmatrix} \tilde{\Xi}_{a1}^{ij} + \tilde{W}_a^{ij} + (\tilde{W}_a^{ij})^T & * & * & * & * \\ \tilde{\Xi}_{a2}^{ij}(l) & -\tilde{R}_1 & * & * & * \\ \tilde{\Xi}_{31}^{ij} & 0 & \tilde{\Xi}_{33} & * & * \\ \tilde{\Xi}_{41}^{ij} & 0 & \tilde{\Xi}_{43}^{i} & \tilde{\Xi}_{44} & * \\ \tilde{\Xi}_{51}^{i} & 0 & 0 & 0 & \tilde{\Xi}_{55} \end{bmatrix}$$

$$\tilde{\Xi}_b^{ij}(l) = \begin{bmatrix} \tilde{\Xi}_{b1}^{ij} + \tilde{W}_b^{ij} + (\tilde{W}_b^{ij})^T & * & * & * & * \\ \tilde{\Xi}_{b2}^{ij}(l) & -\tilde{R}_1 & * & * & * \\ \tilde{\Xi}_{31}^{ij} & 0 & \tilde{\Xi}_{33} & * & * \\ \tilde{\Xi}_{41}^{ij} & 0 & \tilde{\Xi}_{43}^{i} & \tilde{\Xi}_{44} & * \\ \tilde{\Xi}_{51}^{i} & 0 & 0 & 0 & \tilde{\Xi}_{55} \end{bmatrix}$$

$$\tilde{\Xi}_{a1}^{ij} = \begin{bmatrix} \tilde{\Gamma}_1^i & * & * & * & * \\ Y_j^T B_i^T & 0 & * & * & * \\ \tilde{R}_0 & 0 & -\tilde{Q}_0 - \tilde{R}_0 + \tilde{Q}_{11} & * & * \\ 0 & 0 & \tilde{Q}_{21} & \tilde{Q}_{22} - \tilde{Q}_{11} - \tilde{R}_2/\delta & * \\ 0 & 0 & 0 & -\tilde{Q}_{21} + \tilde{R}_2/\delta & -\tilde{Q}_{22} - \tilde{R}_2/\delta \end{bmatrix}$$

$$\tilde{\Xi}_{b}^{ij}(l) = \begin{bmatrix} \tilde{\Gamma}_1^i & * & * & * & * \\ Y_j^T B_i^T & 0 & * & * & * \\ \tilde{R}_0 & 0 & -\tilde{Q}_0 - \tilde{R}_0 + \tilde{Q}_{11} - \tilde{R}_1/\delta & * & * \\ 0 & 0 & \tilde{Q}_{21} + \tilde{R}_1/\delta & \tilde{Q}_{22} - \tilde{Q}_{11} - \tilde{R}_1/\delta & * \\ 0 & 0 & 0 & -\tilde{Q}_{21} & -\tilde{Q}_{22} \end{bmatrix}$$

$$\tilde{\Xi}_{a2}^{ij}(1) = \sqrt{\delta}\tilde{N}_{ij}^T, \quad \tilde{\Xi}_{a2}^{ij}(2) = \sqrt{\delta}\tilde{M}_{ij}^T, \quad \tilde{\Xi}_{b2}^{ij}(1) = \sqrt{\delta}\tilde{T}_{ij}^T, \quad \tilde{\Xi}_{b2}^{ij}(2) = \sqrt{\delta}\tilde{S}_{ij}^T$$

$$\tilde{\Xi}_{31}^{ij} = \begin{bmatrix} C_i X & D_i Y_j & 0 & 0 & 0 \\ B_{\omega i}^T & 0 & 0 & 0 & 0 \end{bmatrix}, \quad \tilde{\Xi}_{33} = \mathrm{diag}(-I - \gamma^2 I)$$

$$\tilde{\Xi}_{41}^{ij} = \begin{bmatrix} \tau_m \tilde{\mathcal{A}}_{ij} \\ \sqrt{\delta}\tilde{\mathcal{A}}_{ij} \\ \sqrt{\delta}\tilde{\mathcal{A}}_{ij} \end{bmatrix}, \quad \tilde{\Xi}_{43}^{ij} = \begin{bmatrix} 0 & \tau_m B_{\omega i} \\ 0 & \sqrt{\delta} B_{\omega i} \\ 0 & \sqrt{\delta} B_{\omega i} \end{bmatrix}, \quad \tilde{\Xi}_{44} = \mathrm{diag}(-\bar{R}_0 - \bar{R}_1 - \bar{R}_2)$$

$$\tilde{W}_a^{ij} = [0 \ \ -\tilde{N}_{ij} + \tilde{M}_{ij} \ \ \tilde{N}_{ij} \ \ -\tilde{M}_{ij} \ \ 0]$$

$$\tilde{W}_b^{ij} = [0 \ \ -\tilde{T}_{ij} + \tilde{S}_{ij} \ \ 0 \ \ \tilde{T}_{ij} \ \ -\tilde{S}_{ij}]$$

$$\tilde{\Gamma}_1^i = A_i X + X A_i^T + \tilde{Q}_0 - \tilde{R}_0$$

$$\tilde{\mathcal{A}}_{ij} = [A_i X \ \ B_i Y_j \ \ 0 \ \ 0 \ \ 0]$$

$$\Xi_{51}^i = \begin{bmatrix} \mu_i H_i^T & 0 & 0\ 0\ 0 \\ E_{1i}X & E_{2i}Y_j & 0\ 0\ 0 \end{bmatrix}, \quad \Xi_{55} = \mathrm{diag}(-\mu_i I \quad -\mu_i I)$$

and the controller feedback gain $K_j = Y_j X^{-T} (j \in \mathbb{S})$.

Proof. Defining $X = P^{-1}$, pre and post-multiplying Eqs. (31) and (32) with

$$\mathrm{diag}(X\ X\ X\ X\ X\ X\ X\ I\ I\ R_0^{-1}\ R_1^{-1}\ R_2^{-1}\ I\ I)$$

and its transpose, respectively, and defining $\tilde{Q} = [X\ X]Q[\begin{smallmatrix} X \\ X \end{smallmatrix}]$, $\tilde{Q}_i = XQ_iX$, $\tilde{R}_i = XR_iX$ $(i = 0, 1)$, $\tilde{R}_2 = XR_2X$, $J = \mathrm{diag}(X, X, X, X, X, X)$, $\tilde{N}_{ij} = JN_{ij}X$, $\tilde{M}_{ij} = JM_{ij}X$, $\tilde{T}_{ij} = JT_{ij}X$, $\tilde{S}_{ij} = JS_{ij}X$, $Y_j = K_jX$ and $\mu_i = \varepsilon_i^{-1}$ $(l = 1, 2, i, j \in \mathbb{S})$, Eqs. (36)′ and (37)′ can be obtained from Eqs. (31) and (32), where Eqs. (36)′ and (37)′ are obtained from Eqs. (36) and (37) by replacing $\bar{R}_s$ with $X\tilde{R}_s^{-1}X$. Combining Eq. (38), we can obtain Eqs. (36) and (37) from Eqs. (36)′ and (37)′. This completes the proof. □

It is worth mentioning that the criteria in Theorem 3 are not strict LMI conditions due to Eq. (38). In the following, we will propose an algorithm to solve this nonconvex feasibility problem by formulating it into a sequential optimization problem subject to LMI constraints.

Define $L_s = \tilde{R}_s^{-1}$, $V_s = \bar{R}_s^{-1} (s = 0, 1, 2)$, $\bar{X} = X^{-1}$, Eq. (38) can be approximately translated into Eq. (39) by using Schur complement

$$\begin{bmatrix} L_s & \bar{X} \\ \bar{X} & V_s \end{bmatrix} > 0 \tag{39}$$

The following linearization algorithm is proposed for the solvability of Theorem 3.

1. Given values of τ_m, τ_M and γ.
2. Find a group of solutions to Eqs. (36), (37) and (39) and

$$\begin{bmatrix} \bar{R}_s & I \\ I & V_s \end{bmatrix} > 0, \quad \begin{bmatrix} \tilde{R}_s & I \\ I & L_s \end{bmatrix} > 0, \quad \begin{bmatrix} X & I \\ I & \bar{X} \end{bmatrix} > 0, \quad s = 0,1,2 \tag{40}$$

   if there is no feasible solution, exit.
3. Set k=1 and solve the minimum problem

$$\min \quad \mathrm{tr}\left(\sum_{i=0}^{2}(V_{ik}\bar{R}_i + \bar{R}_{ik}V_i + L_{ik}\tilde{R}_i + \tilde{R}_{ik}L_i) + X_k\bar{X} + \bar{X}_kX\right)$$

$$\text{s.t.} \quad \text{Eqs. (36), (37) and Eqs. (39) and (40).}$$

4. If $-X\tilde{R}_s^{-1}X < -\bar{R}_s$ holds, solve the matrix variables Y_j and X, exit. Else, set k=k+1 and return Step 2.
5. If k=c (c is the given iterative steps), exit. Output the feedback gain K_j=Y_jX^{-1}.

5. Simulation examples

Example 1. Consider system equation (33) with parameters

$$A_1 = \begin{bmatrix} -2 & 0 \\ 0 & -0.9 \end{bmatrix}, \quad A_2 = \begin{bmatrix} -1 & 0.5 \\ 0 & -1 \end{bmatrix}$$

$$B_1 = \begin{bmatrix} -1 & 0 \\ -1 & -1 \end{bmatrix}, \quad B_2 = \begin{bmatrix} -1 & 0 \\ 0.1 & -1 \end{bmatrix}$$

$$E_{1i} = E_{2i} = \begin{bmatrix} a & 0 \\ 0 & a \end{bmatrix}, \quad H_i = K_i = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad i = 1,2 \tag{41}$$

Table 1

a	[29]	[26,30]	Corollary 1	Over [26,30] (%)
0	0.78	0.83	1.38	66.3
0.3	0.53	0.58	0.96	65.5
0.5	0.37	0.48	0.72	50.0

By using the method in [29,26,30] and Corollary 1 in this paper, the result can be seen in Table 1. From the table, it can be seen that by using the method in this paper, the allowable upper bound of the delay can be greatly improved.

Example 2. Consider the nonlinear mass-spring system

$$\dot{x}_1 = x_2 \tag{42}$$

$$\dot{x}_2 = -0.01x_1 - 0.67x_1^3 + \omega + u \tag{43}$$

where $x_1 \in [-1, 1]$ and $\omega = 0.2\sin(2\pi t)\exp(-t)$ is external disturbance. The following conditions hold obviously for $x_1 \in [-1, 1]$:

$$-0.67x_1 \le -0.67x_1^3 \le 0 \quad \text{when } x_1 \ge 0$$

$$0 \le -0.67x_1^3 \le -0.67x_1 \quad \text{when } x_1 \ge 0$$

Hence it can be represented by the convex combination of the upper bound and the lower bound:

$$-0.67x_1^3 = \mu_1(x_1)\cdot 0 + (1-\mu_1(x_1))\cdot 0.67x_1$$

where $\mu_1(x_1) \in [0, 1]$. By solving the above equation, the membership functions $\mu_1(x_1)$ and $\mu_2(x_1)(= 1 - \mu_1(x_1))$ representing fuzzy sets zero and nonzero are obtained as follows: $\mu_1(x_1) = 1 - x_1^2$ and $\mu_2(x_1) = 1 - \mu_1(x_1) = x_1^2$. By using $\mu_1(x_1)$ and $\mu_2(x_1)$, the original nonlinear system equations (42) and (43) can be represented by the following fuzzy model:

Rule 1 : If x_1 is μ_1 then

$$\dot{x}(t) = A_1x(t) + B_1u(t) + B_{\omega 1}\omega(t)$$

$$z(t) = C_1x(t) + D_1u(t)$$

Rule 2 : If x_1 is μ_2 then

$$\dot{x}(t) = A_2x(t) + B_2u(t) + B_{\omega 2}\omega(t)$$

$$z(t) = C_2x(t) + D_2u(t)$$

where

$$A_1 = \begin{bmatrix} 0 & 1 \\ -0.01 & 0 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 0 & 1 \\ -0.68 & 0 \end{bmatrix}, \quad B_1 = B_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$B_{\omega 1} = B_{\omega 2} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad C_1 = C_2 = [1 \;\; 0], \quad D_1 = D_2 = 0.$$

By using the method in [19], for given $\tau_m = 0$, $\tau_M = 0.2$, the minimum γ is obtained as $\gamma_{\min} = 1.2448$. In this paper, by using Theorem 3, for given $\tau_m = 0$ and $\gamma = 1.2448$, the maximum allowable bound τ_M is obtained as 1.21, the corresponding feedback gains are obtained as

$$K_1 = [-8.3485 \;\; -13.7065]$$

$$K_2 = [-6.3756 \;\; -11.3869]$$

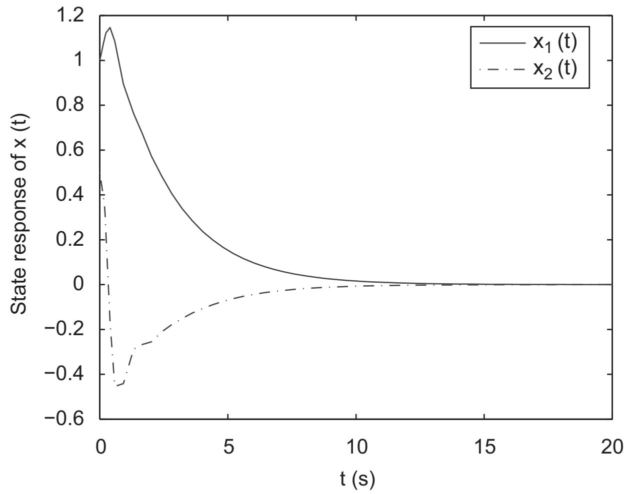

Fig. 1. State response of the nonlinear systems.

For given $\tau_m = 0$, $\tau_M = 0.2$, the minimum γ is obtained as $\gamma_{\min} = 1.01$, the corresponding feedback gains are obtained as

$$K_1 = [-0.9927 \quad -2.6309]$$

$$K_2 = [-0.4123 \quad -2.9079]$$

Choose the membership function as $\mu_1(x_1) = 1 - x_1^2$ and $\mu_2(x_1) = x_1^2$ and choose the initial value of the state as $x(0) = [1, 0.5]'$, the state response can be seen in Fig. 1.

6. Conclusion

This paper studies the problem of H_∞ control design for nonlinear networked control systems (NNCSs). The NNCSs are presented in the form of T–S fuzzy model with input time-varying delay. By using the piecewise analysis method, new sufficient conditions for the asymptotical stability of the NNCS are obtained. Firstly, the whole variation interval of the delay is divided into two subintervals with equal length, respecting for the delay belongs to different subintervals, new criteria on H_∞ performance analysis of the NNCSs are obtained by checking the variation of the derivative of the Lyapunov functional in the two subintervals. Then, new criteria for the H_∞ controller design are obtained by using the convexity properties of the matrix inequality. A numerical example and a practical example have been given to demonstrate the effectiveness and less conservatism of the proposed method.

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