Observer-Based Multi-Instant Fuzzy State Estimation of Discrete-Time Nonlinear Circuits via a New Slack Variables Technique

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***Abstract*—This brief investigates the problem of relaxed observer-based multi-instant fuzzy state estimation of discrete-time nonlinear circuits. In contrast with the recent result whose observer gain matrices may be too many to be applied in practice, there is only an unique group of observer gain matrices required in this study while the conservatism can be reduced markedly by developing a new slack variables technique. The proposed slack variables technique is constructed on the basis of multi-instant homogenous polynomials so that much more recessive algebraic properties of the normalized fuzzy weighting functions can be excavated in order to remove those redundant positive definite constraints which bring about conservatism. Especially, those previous slack variables techniques belong to special cases of ours. Finally, a benchmark example is given to validate the progressiveness of the developed result.**

***Index Terms*—State estimation, observer design, nonlinear systems, Takagi-Sugeno fuzzy systems, slack variables.**

I. Introduction

FUZZY control methods have been showing the superiority in handling nonlinear control problems and a large number of literature are reported in recent years, such as, [1]–[3]. Impressively, Takagi–Sugeno(T–S) fuzzy systems [4] have been applied in various occasions with nonlinear dynamics [5]–[7]. However, most of those T–S fuzzy-model-based results are often too conservative because their used Lyapunov functions belong to the single Lyapunov functions [8]. For obtaining less conservative results, several kinds of slack variables techniques have been applied in [9]–[10] and the literature therein. More recently, the so-called homogenous polynomially parameter-dependent Lyapunov functions [11] have been expanded ceaselessly in order to bring in much freedom for further reducing the conservatism, e.g., [12]–[13]. Nevertheless, it should be noted that those previous slack variables techniques can't be fully adapted to this more advanced analytical framework of homogenous polynomials and thus there still is much conservatism required to be removed by means of developing some new slack variables techniques.

On the other hand, a considerable number of state variables in realistic circuits systems cannot be measured directly owing to either the objective restriction or the economic constraints [14]–[15]. Therefore, it must be fairly meaningful for estimating the unmeasurable state variables based on the available information of the measurable system outputs [16]. So far, the problem of observer-based fuzzy state estimation has been investigated in [17]–[20] with the help of T-S fuzzy approximations [21], [22]. But, the main disadvantages of the mentioned methods are with much conservatism to be lifted [23]. To solve this problem, the multi-instant fuzzy observer has been firstly proposed in [24], which is homogenous polynomially parameter-dependent on both the current-time and the past-time normalized fuzzy weighting functions(NFWFs) with a group of adjustable degrees. More recently, the result of [24] has been relaxed by proposing an improved multi-instant fuzzy switching observer in [25] while the computational burden becomes more heavy as a tradeoff. More specifically, the result of [25] is limited by two aspects: The first issue is that its used slack variables technique belongs to a conventional one with much conservatism; and the second issue is that the computational burden (including the off-line solution time T_{offline} and the number of observer gain matrix scalars to be on-line written in the control CPU N_{online}) may become too heavy to be applied in practical engineering.

In this brief, the problem of relaxed observer-based multi-instant fuzzy state estimation of discrete-time nonlinear circuits will be further investigated. In contrast with the recent result [25], an unique group of observer gain matrices is required in this brief while the conservatism can be reduced markedly by proposing a new slack variables technique which is fully adapted to the analytical framework of homogenous polynomials. As a result, much more recessive algebraic properties of the normalized fuzzy weighting functions can be excavated in order to remove those redundant positive definite constraints which bring about conservatism in essence. Finally, a benchmark example will also been used to validate the progressiveness of the developed approach.

Manuscript received September 29, 2021; revised November 6, 2021; accepted November 17, 2021. Date of publication November 22, 2021; date of current version March 28, 2022. This work was supported in part by the National Natural Science Foundation of China under Grant 62022044 and Grant 61773221, and in part by the Jiangsu Natural Science Foundation for Distinguished Young Scholars under Grant BK20190039. This brief was recommended by Associate Editor S. Li. *(Corresponding author: Xiangpeng Xie.)*

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Color versions of one or more figures in this article are available at https://doi.org/10.1109/TCSII.2021.3129463.

Digital Object Identifier 10.1109/TCSII.2021.3129463

TABLE I
THE NOMENCLATURE TABLE FOR USED NOTATIONS

notation	meaning
r-tuple w	$w_1 \cdots w_r$
$\mathcal{K}(s)$	all w such that $\sum_{j=1}^{r} w_j = s,\ w_j \in \mathbb{N}$
ϱ^k	$\varrho_1^{k_1} \cdots \varrho_r^{k_r}$
$J(s)$	$\frac{(r+s-1)!}{s!(r-1)!}$
χ_j	$\underbrace{0}_{1-\text{st element}} \cdots \underbrace{1}_{j-\text{th element}} \cdots \underbrace{0}_{r-\text{th element}}$
$\pi(w)$	$(w_1!) \times \cdots \times (w_r!)$

Notations: Throughout the remainder of this brief, all the notations are used by standard. Such as, the set of positive integers is denoted as $\mathbb{Z}_+$. $m!$ represents the factorial, i.e., $m! = (\prod_{j=1}^{m} j)$ as $m \in \mathbb{N}$.

II. PRELIMINARIES AND BACKGROUNDS

A. Preliminaries

The notations of homogeneous polynomials are recalled in Table I, which should be the same as the literature [25].

Furthermore, for $w \in \mathcal{K}(s_1)$ and $q \in \mathcal{K}(s_2)$, we write $w \geq q$ if $w_j \geq q_j$ holds in true for all $j \in \{1, \ldots, r\}$. The operations of summation $(w+q)$, and subtraction $(k-k')$, are both operated componentwise.

In this brief, all the elements in $w \in \mathcal{K}(s)$ are reordered as $w^1, w^2, \ldots, w^{J(s)}$ in descending order of converted decimal values. For example, if $w \in \mathcal{K}(2)$ and $r = 2$, then we have $w^1 = 20, w^2 = 11, w^3 = 02$, and if $w \in \mathcal{K}(3)$, then we have $w^1 = 30, w^2 = 21, w^3 = 12, w^4 = 03$. Based on this, the augmented matrix consists of all the matrices R_{wq} such that $w, q \in \mathcal{K}(s)$ can be defined as follows:

$$[R_{wq}]_{J(s)\times J(s)} = \begin{bmatrix} R_{w^1 q^1} & \cdots & R_{w^1 q^{J(s)}} \\ \vdots & \ddots & \vdots \\ R_{w^{J(s)} q^1} & \cdots & R_{w^{J(s)} q^{J(s)}} \end{bmatrix}.$$

For instance, if $w, q \in \mathcal{K}(2)$, we have

$$\left[R_{wq}\right]_{J(2)\times J(2)} = \begin{bmatrix} R_{2020} & R_{2011} & R_{2002} \\ R_{1120} & R_{1111} & R_{1102} \\ R_{0220} & R_{0211} & R_{0202} \end{bmatrix}.$$

For simplicity, for $w = w_1 \cdots w_r$ and $j \in \{1, \ldots, r\}$, the following shortenings are used throughout the remainder:

$$\begin{cases} h_j(t) = h_j(\phi(t)),\ \ h_j(t-1) = h_j(\phi(t-1)), \\ h(t) = (h_1(\phi(t)), \ldots, h_r(\phi(t)))^T, \\ h(t-1) = (h_1(\phi(t-1)), \ldots, h_r(\phi(t-1)))^T, \\ h(t)^w = \prod_{j=1}^{r} h_j(t)^{w_j},\ \ h(t-1)^w = \prod_{j=1}^{r} h_j(t-1)^{w_j}. \end{cases} \tag{1}$$

B. Backgrounds

Consider a class of discrete-time T-S fuzzy systems:

$$\begin{cases} \mathrm{x}(t+1) = \sum_{j=1}^{r} h_j(\phi(t))(A_j \mathrm{x}(t) + B_j \mathrm{u}(t)) \\ \mathrm{y}(t) = \sum_{j=1}^{r} h_j(\phi(t)) C_j \mathrm{x}(t). \end{cases} \tag{2}$$

where the system state vector $x(t) \in \mathbb{R}^{n_x}$ to be estimated, $u(t) \in \mathbb{R}^{n_u}$ is known input vector, and the measurable system output vector is represented by $y(t) \in \mathbb{R}^{n_y}$. $h_j(\phi(t))$ is the j-th NFWF such that $h_j(\phi(t)) \geq 0$ and $\sum_{j=1}^{r} h_j(\phi(t)) = 1$.

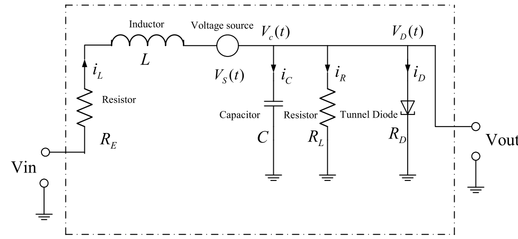

Fig. 1. A typical tunnel diode circuit given in [1].

Remark 1: The above T-S fuzzy systems (2) have been applied to represent a class of tunnel diode circuits with nonlinear dynamics, such as the typical tunnel diode circuit shown in Fig. 1, which is characterized as $C\dot{V}_C(t) = i_L(t) - \frac{V_C(t)}{R_L} - \frac{V_C(t)}{R_D}$ with $\frac{1}{R_D} = a_1 + a_2 V_D^2(t)$. While $V_D(t)$ belongs to a measurable variable, the nonlinear terms $V_D^2(t)$ can be represented by the T-S fuzzy model with $\phi(t) = V_D(t)$. Therefore, the other unmeasurable variables ($V_C(t)$ and $i_L(t)$) can be estimated by using the fuzzy observer.

The multi-instant fuzzy observer has been firstly proposed in [24], which is homogenous polynomially parameter-dependent on both the current-time and the past-time NFWFs with a group of adjustable degrees of $g_1, g_2, g_3, g_4 \in \mathbb{Z}_+$:

$$\begin{cases} \hat{\mathrm{x}}(t+1) = A_{\phi(t)}\hat{\mathrm{x}}(t) + B_{\phi(t)}\mathrm{u}(t) + S^{-1}_{\phi_{g_3}(t-1)\phi_{g_4}(t)} \\ \qquad\qquad \times L_{\phi_{g_1}(t-1)\phi_{g_2}(t)}(\mathrm{y}(t) - \hat{\mathrm{y}}(t)), \\ \hat{\mathrm{y}}(t) = C_{\phi(t)}\hat{\mathrm{x}}(t). \end{cases} \tag{3}$$

and we define

$$L_{\phi_{g_1}(t-1)\phi_{g_2}(t)} = \sum_{\binom{p\in\mathcal{K}(g_1)}{k\in\mathcal{K}(g_2)}} \{h(t-1)^p h(t)^k L_{pk}\}, \tag{4}$$

$$S_{\phi_{g_3}(t-1)\phi_{g_4}(t)} = \sum_{\binom{p\in\mathcal{K}(g_3)}{k\in\mathcal{K}(g_4)}} \{h(t-1)^p h(t)^k S_{pk}\}, \tag{5}$$

where $L_{pk} \in \mathbb{R}^{n_x \times n_y}$ and $S_{pk} \in \mathbb{R}^{n_x \times n_x}$ are observer gain matrices.

For $\phi(-1) = \phi(0)$, the fuzzy estimation error system becomes as follows:

$$\mathrm{e}(t+1) = (A_{z(t)} - S^{-1}_{\phi_{g_3}(t-1)\phi_{g_4}(t)} L_{\phi_{g_1}(t-1)\phi_{g_2}(t)} C_{\phi(t)})\mathrm{e}(t). \tag{6}$$

Remark 2: Owing to the fact that there is still much conservatism in the result of [24], the so-called deep division method has been developed in the recent literature [25]. However, the computational burden of [25], including either T_{offline} or N_{online}, becomes too heavy to be applied in practice. In order to further enhancing the quality of fuzzy state estimation, a new slack variables technique is proposed in this brief.

III. MAIN RESULTS

Theorem 1: The fuzzy estimation error system (6) becomes globally asymptotically stable, if there are observer gain matrices $L_{pk} \in \mathbb{R}^{n_x \times n_y}$ $(p \in \mathcal{K}(g_1), k \in \mathcal{K}(g_2))$; $S_{pk} \in \mathbb{R}^{n_x \times n_x}$ $(p \in \mathcal{K}(g_3), k \in \mathcal{K}(g_4))$; symmetric matrices $P_p \in \mathbb{R}^{n_x \times n_x}$ $(p \in \mathcal{K}(g_5))$; and slack matrices $R_{wq} \in \mathbb{R}^{2n_x \times 2n_x}$ with

$R_{wq} = \left(R_{qw}\right)^T$ and their augmented matrices $[R_{wq}]_{J(s)\times J(s)} < 0$ for $w, q \in \mathcal{K}(s)$; such that the LMIs of (7) are satisfied:

$$\sum_{\substack{w,q\in\mathcal{K}(s)\\ k'\geq w+q}} \frac{g_7!}{\pi(k'')}\frac{(g_6-2s)!}{\pi(k'-w-q)}R_{wq} + \begin{bmatrix} \Upsilon^{11}_{k''k'} & * \\ \Upsilon^{21}_{k''k'} & \Upsilon^{22}_{k''k'} \end{bmatrix} > 0,$$
$$\forall\ k'' \in \mathcal{K}(g_7), k' \in \mathcal{K}(g_6); \tag{7}$$

where $g_6 = \max\{1+g_2, 1+g_4, 2s, g_5\}$, $g_7 = \max\{g_1, g_3, g_5\}$,

$$\Upsilon^{11}_{k''k'} = \sum_{\substack{k''\in\mathcal{K}(g_7),k'\in\mathcal{K}(g_6)\\ p\in\mathcal{K}(g_5),k''\geq p}} \left\{\frac{(g_7-g_5)!}{\pi(k''-p)}\frac{g_6!}{\pi(k'')}P_p\right\}, \tag{8}$$

$$\Upsilon^{21}_{k''k'} = \sum_{\substack{k''\in\mathcal{K}(g_7),k'\in\mathcal{K}(g_6),p\in\mathcal{K}(g_3)\\ j\in\{1,\ldots,r\},k\in\mathcal{K}(g_4),k'\geq k+\chi_j,k''\geq p}} \left\{\frac{(g_7-g_3)!}{\pi(k''-p)} \times \frac{(g_6-g_4-1)!}{\pi(k'-k-\chi_j)}S_{pk}A_j\right\}$$
$$- \sum_{\substack{k''\in\mathcal{K}(g_7),k'\in\mathcal{K}(g_6),p\in\mathcal{K}(g_1)\\ j\in\{1,\ldots,r\},k\in\mathcal{K}(g_2),k'\geq k+\chi_j,k''\geq p}} \left\{\frac{(g_7-g_1)!}{\pi(k''-p)} \times \frac{(g_6-g_2-1)!}{\pi(k'-k-\chi_j)}L_{pk}C_j\right\}, \tag{9}$$

$$\Upsilon^{22}_{k''k'} = \sum_{\substack{k''\in\mathcal{K}(g_7),k'\in\mathcal{K}(g_6)\\ p\in\mathcal{K}(g_3),k''\geq p,k\in\mathcal{K}(g_4),k'\geq k}} \left\{\frac{(g_7-g_3)!}{\pi(k''-p)} \times \frac{(g_6-g_4)!}{\pi(k'-k)}(S_{pk}+S^T_{pk})\right\}$$
$$+ \sum_{\substack{k''\in\mathcal{K}(g_7),k'\in\mathcal{K}(g_6)\\ p\in\mathcal{K}(g_5),k'\geq p}} \left\{\frac{g_7!}{\pi(k'')}\frac{(g_6-g_5)!}{\pi(k''-p)}P_p\right\}. \tag{10}$$

Proof: Different from [24] and [25], we choose the following Lyapunov function candidate in this brief:

$$V(\mathrm{e},\phi) = \mathrm{e}^T(t)(P_{\phi_{g_5}(t-1)})\mathrm{e}(t), \tag{11}$$

where $P_{\phi_{g_5}(t-1)} = \sum_{p\in\mathcal{K}(g_5)} h(t-1)^p P_p$.

Thus, the first difference $\Delta V(\mathrm{e},\phi)$ along the solution to the fuzzy estimation error system (6) becomes as follows:

$$\Delta V(\mathrm{e},\phi) = \mathrm{e}^T(t)(\Theta^T P_{\phi_{g_5}(t)}\Theta - P_{\phi_{g_5}(t-1)})\mathrm{e}(t), \tag{12}$$

where $P_{\phi_{g_5}(t)} = \sum_{p\in\mathcal{K}(g_5)}\{h(t)^p P_p\}$,

$$\Theta = \left(A_{\phi(t)} - S^{-1}_{\phi_{g_3}(t-1)\phi_{g_4}(t)}L_{\phi_{g_1}(t-1)\phi_{g_2}(t)}C_{\phi(t)}\right).$$

The fuzzy estimation error system (6) becomes globally asymptotically stable if we have:

$$\Theta^T P_{\phi_{g_5}(t)}\Theta - P_{\phi_{g_5}(t-1)} < 0. \tag{13}$$

On the other hand, (13) can be ensured by another inequality (14):

$$\begin{bmatrix} P_{\phi_{g_5}(t-1)} & * \\ \Phi_{21} & \Phi_{22} \end{bmatrix} > 0, \tag{14}$$

where $\Phi_{21} = S_{\phi_{g_3}(t-1)\phi_{g_4}(t)}A_{\phi(t)} - L_{\phi_{g_1}(t-1)\phi_{g_2}(t)}C_{\phi(t)}$, $\Phi_{22} = \mathrm{He}(S_{\phi_{g_3}(t-1)\phi_{g_4}(t)}) - P_{\phi_{g_5}(t)}$.

TABLE II
FEASIBLE INTERVALS OF b FOR THREE METHODS GIVEN IN [24], [25] AND THIS BRIEF

a	1.00	1.25	1.50	1.75
[24]	[-0.4,157]	[-0.4,95]	[-0.4,64]	[-0.4,46]
[25]	[-0.4,260]	[-0.4,140]	[-0.4,88]	[-0.4,60]
This brief	[-0.4,346]	[-0.4,183]	[-0.4,113]	[-0.4,76]
increase rate	33.08%	30.71%	28.41%	26.67%

Further, we obtain

$$\begin{bmatrix} P_{z_{g_5}(t-1)} & * \\ \Phi_{21} & \Phi_{22} \end{bmatrix} + \sum_{w,q\in\mathcal{K}(s)} h(t)^{(w+q)}R_{wq}$$
$$= \sum_{\substack{k''\in\mathcal{K}(g_7)\\ k'\in\mathcal{K}(g_6)}} h(t-1)^{k''}h(t)^{k'}\mathrm{Left}(7) > 0. \tag{15}$$

Meanwhile, we also get

$$\sum_{w,q\in\mathcal{K}(s)} h(t)^{(w+q)}R_{wq} = \begin{pmatrix} h(t)^{w^1}I \\ \vdots \\ h(t)^{w^{J(s)}}I \end{pmatrix}^T [R_{wq}]_{J(s)\times J(s)} \begin{pmatrix} h(t)^{w^1}I \\ \vdots \\ h(t)^{w^{J(s)}}I \end{pmatrix}$$
$$< 0, \tag{16}$$

where $w \in \mathcal{K}(s)$ and $I \in \mathbb{R}^{2n_x\times 2n_x}$.

Combining both (15) and (16), the inequality (14) can be ensured by the LMIs of (7). As a result, the fuzzy estimation error system (6) becomes globally asymptotically stable if the LMIs of (7) are satisfied. ■

Remark 3: If $w \neq q$, the positive definite constraint on $R_{wq} < 0$ isn't required in the united $[R_{wq}]_{J(s)\times J(s)} < 0$ of our newly developed tool. In other words, much more recessive algebraic properties of the NFWFs have been excavated in order to remove those redundant positive definite constraints which bring about conservatism. In particular, those previous slack variables techniques given in [24], [25] belong to special cases of the proposed one with $s = 1$.

IV. NUMERICAL SIMULATION

Example: The benchmark example of (2) which has also been applied in [24], [25] is given with: $A_1 = \begin{bmatrix} 2.5 & 1 \\ 0.5 & 2 \end{bmatrix}$, $A_2 = \begin{bmatrix} 0.5 & 0 \\ 2.5 & a \end{bmatrix}$, $C_1 = \begin{bmatrix} b & 1 \end{bmatrix}$, $C_2 = \begin{bmatrix} 1 & 1 \end{bmatrix}$.

It is worth noting that there exist two variable parameters (a in A_2 and b in C_1, respectively) whose feasible intervals can be compared with different methods, e.g., [24], [25] and Theorem 1 of this brief. For fairness, the same group of adjustable degrees of $g_1 - g_5$ are chosen, i.e., $g_1 = 1$, $g_2 = 2$, $g_3 = 1, g_4 = 2, g_5 = 2$ for [24], [25] and Theorem 1 of this brief. When the first variable parameter a is chosen from $a \in \{1, 1.25, 1.5, 1.75\}$ like [24], [25], the feasible interval of b can be tested via the MATLAB function '[tmin, xfeas] = feasp(lmis)' for the underlying methods, respectively. For ease of presentation, all the feasible intervals have been given in Table II. It is evident that Theorem 1 of this brief is capable of giving the maximum feasible interval of b for

TABLE III
N_{ONLINE} AND T_{OFFLINE} FOR [24], [25] AND THIS BRIEF

Methods	[24]	[25]	This brief
N_{online}	36	216	36
T_{offline}	0.0957s	68.3476s	0.4970s

each chosen a. In other words, the conservatism has been well reduced by using the proposed slack variables technique.

On the other hand, the computational burden for [24], [25] and Theorem 1 of this brief (N_{online} and T_{offline}) is also discussed in Table III. Seen from Table III, [24] and Theorem 1 of this brief need the same number of observer gain matrix scalars to be online written in the RAM of the CPU, which is only 1/6 of the number for [25]. It does means that the first issue encountered in [25] has been well addressed in this brief. Moreover, T_{offline} of this brief is far less than T_{offline} of [25]. And this means that the second issue encountered in [25] has also been resolved in this brief.

When we choose $a = 1.75$ and $b = 76$, it is noticed from Table II that $(a = 1.75, b = 76)$ is out of the feasible intervals of two existing methods given in [24], [25]. Then, using Theorem 1 of this brief with $g_1 = 1, g_2 = 2, g_3 = 1, g_4 = 2, g_5 = 2$, the following observer gain matrices can be obtained via offline solving the LMIs of (7):

$$L_{1020} = \begin{bmatrix} 0.0488 \\ -0.0162 \end{bmatrix}, L_{1011} = \begin{bmatrix} 0.2315 \\ -0.0558 \end{bmatrix},$$
$$L_{1002} = \begin{bmatrix} -0.0067 \\ 0.0302 \end{bmatrix}, L_{0120} = \begin{bmatrix} 0.0443 \\ -0.0153 \end{bmatrix},$$
$$L_{0111} = \begin{bmatrix} 0.2349 \\ -0.0593 \end{bmatrix}, L_{0102} = \begin{bmatrix} -0.0507 \\ 0.0449 \end{bmatrix},$$
$$S_{1020} = \begin{bmatrix} 1.3028 & -0.5225 \\ -0.4821 & 0.2238 \end{bmatrix},$$
$$S_{1011} = \begin{bmatrix} 6.5966 & -2.3370 \\ -1.8401 & 0.7032 \end{bmatrix},$$
$$S_{1002} = \begin{bmatrix} 2.7903 & -0.1815 \\ -0.4032 & 0.0448 \end{bmatrix},$$
$$S_{0120} = \begin{bmatrix} 1.1767 & -0.4730 \\ -0.4697 & 0.1987 \end{bmatrix},$$
$$S_{0111} = \begin{bmatrix} 5.8031 & -1.8208 \\ -1.6718 & 0.5215 \end{bmatrix},$$
$$S_{0102} = \begin{bmatrix} 2.2576 & -0.1420 \\ -0.3368 & 0.0358 \end{bmatrix}.$$

Using the obtained observer gain matrices and presetting the initial conditions as $\mathrm{x}(0) = (0.5, \quad -0.6)^T$ and $\hat{\mathrm{x}}(0) = (-0.5, \quad -4.0)^T$, the trajectories of two estimation errors ($\mathrm{e}_1(t)$ and $\mathrm{e}_2(t)$) have been given in Fig. 2, where the fuzzy estimation error system with $(a = 1.75, b = 76)$ can be globally asymptotically stable by utilizing the proposed method.

Further, when we choose $a = 2$ and $b = 38$, it has been reported in [24] that there exists a feasible solution to design the multi-instant fuzzy observer (3). Here, for the same plant, using Theorem 1 of this brief with $g_1 = 1, g_2 = 2, g_3 = 1, g_4 = 2, g_5 = 2$, another group of observer gain matrices are

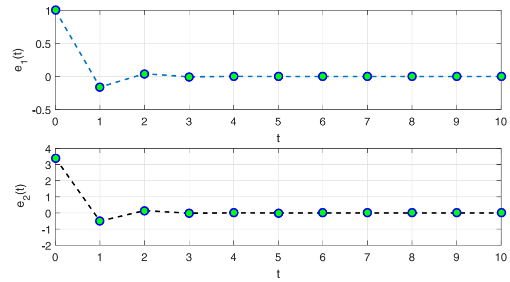

Fig. 2. The trajectories of two estimation errors ($\mathrm{e}_1(t)$ and $\mathrm{e}_2(t)$).

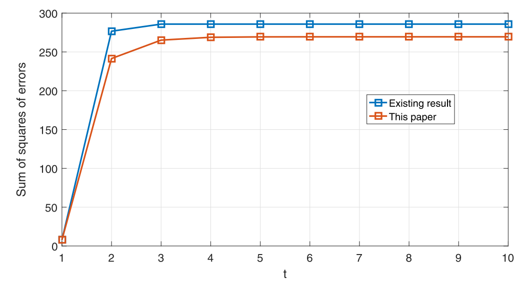

Fig. 3. The trajectories of different $f_{\text{cost}}(t)$ for [24] and this brief.

obtained via offline solving the LMIs of (7):

$$L_{1020} = \begin{bmatrix} 1.2728 \\ -0.3759 \end{bmatrix}, L_{1011} = \begin{bmatrix} 3.5653 \\ -0.6366 \end{bmatrix},$$
$$L_{1002} = \begin{bmatrix} 0.3493 \\ 0.7074 \end{bmatrix}, L_{0120} = \begin{bmatrix} 1.0955 \\ -0.3230 \end{bmatrix},$$
$$L_{0111} = \begin{bmatrix} 3.9590 \\ -0.7654 \end{bmatrix}, L_{0102} = \begin{bmatrix} -0.3664 \\ 0.9044 \end{bmatrix},$$
$$S_{1020} = \begin{bmatrix} 18.8145 & -7.8588 \\ -6.5937 & 3.2780 \end{bmatrix},$$
$$S_{1011} = \begin{bmatrix} 63.0044 & -21.6155 \\ -17.5209 & 7.2073 \end{bmatrix},$$
$$S_{1002} = \begin{bmatrix} 34.5216 & -1.7289 \\ -5.0435 & 0.6768 \end{bmatrix},$$
$$S_{0120} = \begin{bmatrix} 16.8377 & -7.3298 \\ -6.2756 & 3.0271 \end{bmatrix},$$
$$S_{0111} = \begin{bmatrix} 62.0505 & -20.1778 \\ -16.7599 & 5.6397 \end{bmatrix},$$
$$S_{0102} = \begin{bmatrix} 32.4552 & -1.8418 \\ -4.8072 & 0.6008 \end{bmatrix}.$$

Then, using the previous multi-instant fuzzy observer reported in [24] and the newly developed one of this brief, respectively, an important specification of estimation accuracy is defined as $f_{\text{cost}}(t) = \sum_{i=1}^{t} \mathrm{e}^T(i)\mathrm{e}(i)$ and compared together in Fig. 3. Seen from Fig. 3, the $f_{\text{cost}}(t)$ for this brief is always smaller than the $f_{\text{cost}}(t)$ for [24] and this means that much better estimation accuracy has been obtained in this brief. In some detail, we get $f_{\text{cost}}(10) = 285.8619$ for [24] and $f_{\text{cost}}(10) = 269.4681$ for this brief, i.e., the estimation accuracy has been enhanced by 5.73% in this brief.

V. CONCLUSION

This brief has proposed considerable relaxations of observer-based multi-instant fuzzy state estimation of discrete-time nonlinear circuits. In contrast with the recent result given in [25] with too many observer gain matrices, the unique group of observer gain matrices is required in this study while the conservatism has also been reduced markedly by developing a new slack variables technique. Indeed, the proposed slack variables technique is constructed on the basis of multi-instant homogenous polynomials so that much more recessive algebraic properties of the normalized fuzzy weighting functions have been excavated in order to remove those redundant positive definite constraints which bring about conservatism. As a result, less conservative results can be obtained at the expense of more economical computational burden. Finally, a benchmark example has been applied to fully validate the progressiveness of this brief. Considering that our further study on observer-based control synthesis belongs to a non-convex optimization problem which is difficult to solve, the future research direction may include related developments based on meta-heuristic algorithms, such as [26]–[28].

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