$P2.$

(a) $\begin{pmatrix} x_{11} & x_{12} \\ x_{21} & x_{22} \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} + \begin{pmatrix} z_1 \\ z_2 \end{pmatrix}$

$= \cancel{x_{11} y_1 + x_{12} y_2 + x_{21} y_1 + x_{22} y_2}$

$\begin{pmatrix} x_{11} y_1 + x_{12} y_2 + z_1 \\ x_{21} y_1 + x_{22} y_2 + z_2 \end{pmatrix}$

(b) $(y_1 \; y_2) \begin{pmatrix} x_{11} & x_{12} \\ x_{21} & x_{22} \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = x_{11} y_1^2 + x_{12} y_1 y_2$
$+ x_{21} y_1 y_2 + x_{22} y_2^2$

$Pb3$

(a) $n \times 1$

(b) $d \times d$

(c) $(d \times d) \times (d \times n) \times (2 \times 1)$

$\rightarrow d \times 1$

$P3 \; 4$

(a) $\frac{1}{n} \left((ax+b) - \overline{(ax+b)} \right)^2 = \frac{a^2}{n} (x - \bar{x})^2 = a^2 Var(x)$

(b) $E(\bar{x}) = \frac{1}{n}(x_1 + \cdots + x_n) = \mu$

$Var(\bar{x}) = \frac{1}{n} (\bar{x} - E(\bar{x}))^2 = \frac{1}{n} \cancel{E(\bar{x}}$

$= \frac{1}{n} (\bar{x}^2 - 2\mu \bar{x} + \mu^2)$

$= \frac{\bar{x}^2}{n} - \left(\frac{\bar{x}}{n}\right)^2 = \frac{1}{n}\left(\bar{x}^2 - \frac{\bar{x}^2}{n}\right)$

$= Var\left(\frac{x_{1+\cdots} + x_n}{n}\right) = \frac{1}{n^2}(\sigma^2 + \sigma^2 + \cdots \sigma^2) = \frac{\sigma^2}{n}$

5

(a) $\dfrac{25}{100}$, $P = \dfrac{25}{2}$

(b) $\dfrac{P(Y=1, X=1)}{P(X=1)} = \dfrac{\frac{10}{100}}{\frac{15}{100}} = \dfrac{2}{3}$

(c)

$$P(X \cap Y) = P(X) P(Y)$$

$$\dfrac{10}{100} \neq \dfrac{15}{100} \times \dfrac{25}{100} \implies \text{not independent}$$