HW1_Sol

$\boxed{2}$

(a) $xy = \begin{pmatrix} a_{11}y_1 + a_{12}y_2 \\ a_{21}y_1 + a_{22}y_2 \end{pmatrix}$ $xy + z = \begin{pmatrix} a_{11}y_1 + a_{12}y_2 + z_1 \\ a_{21}y_1 + a_{22}y_2 + z_2 \end{pmatrix}$

(b) $(y_1, y_2) \cdot \begin{pmatrix} a_{11}y_1 + a_{12}y_2 \\ a_{21}y_1 + a_{22}y_2 \end{pmatrix} = \left((a_{11}y_1 + a_{12}y_2) \cdot y_1 + (a_{21}y_1 + a_{22}y_2) \cdot y_2 \right)$

$\boxed{3}$

(a) $(n \times 1)$

(b) $\{(d \times n) \cdot (n \times d)\}^{-1} = (d \times d)^{-1} = (d \times d)$

(c) $(d \times d) \cdot (d \times n) \cdot (n \times 1) = (d \times n)(n \times 1)$

$= (d \times 1)$

$\boxed{4}$

(a) $$Var(aX+b) = E[(aX+b) - E(aX+b)]^2$$
$$= E[(aX+b)^2 - 2(aX+b)\cdot E(aX+b) + \{E(aX+b)\}^2]$$
$$= E[a^2X^2 + 2abX + b^2 - 2(aX+b)\cdot(aE(X)+b) + \{aE(X)+b\}^2]$$
$$= a^2E(X^2) + 2ab\cdot E(X) + b^2 - \cancel{2}\{aE(X)+b\}^2 + \cancel{\{aE(X)+b\}^2}$$
$$= a^2E(X^2) + \cancel{2abE(X)+b}^2 - \{a^2E(X)^2 + \cancel{2abE(X)+b}^2\}$$
$$= a^2[E(X^2) - \{E(X)\}^2] = \boxed{a^2\cdot Var(X)}$$

(b) $X_1 \cdots X_n$, iid, μ, σ^2

$$\bar{x} = \frac{1}{n}\sum X_i$$

① $$E(\bar{X}) = E\left(\frac{1}{n}\sum X_i\right) = \frac{1}{n}\cdot E(X_1+X_2+\cdots+X_n)$$
$$= \frac{1}{n}(E(X_1)+\cdots+E(X_n)) =$$
$$= \frac{1}{n}(X_1+\cdots+X_n) = \frac{1}{n}\sum X_i = \mu$$

② $$Var(\bar{X}) = E[\bar{X} - E(\bar{X})]^2 = E[\bar{X}^2 - 2\bar{X}\cdot E(\bar{X}) + \{E(\bar{X})\}^2]$$
$$= E(\bar{X}^2) - \cancel{2}\cdot E(\bar{X})\cdot E(\bar{X}) + \cancel{\{E(\bar{X})\}^2}$$
$$= E(\bar{X}^2) - \{E(\bar{X})\}^2$$
$$= \frac{1}{n}\sum \bar{X}_i^2 - \{\mu\}^2$$
$$= \frac{1}{n}(\bar{X}_1^2+\cdots+\bar{X}_n^2) - \mu^2 = \mu^2-\mu^2 = 0$$

$$\bar{x} = \bar{X}_1 = \cdots = \bar{X}_n$$

$\boxed{5}$

(a) $P(X=1 \,\&\, Y=1) + P(X=0 \,\&\, Y=1) = P(Y=1) = \frac{25}{100} = \frac{1}{4}$

(b) $P(Y=1 \mid X=1) = \frac{P(Y=1 \,\&\, X=1)}{P(X=1)} = \frac{P(Y=1 \,\&\, X=1)}{P(Y=0 \,\&\, X=1) + P(Y=1 \,\&\, X=1)} = \frac{\frac{10}{100}}{\frac{15}{100}} = \frac{10}{15} = \frac{2}{3}$

(c) $P(Y=1 \mid X=1) = P(Y=1)$

$P(Y=1 \,\&\, X=1) = P(Y=1)P(X=1)$

$\frac{2}{3} \neq \frac{1}{4} \times \frac{\cancel{15}\,3}{\cancel{100}\,20}$

$\neq \frac{3}{80}$

$\therefore$ <u>Not Independent</u>