Factor Analysis - Measurement Invariance

Factor Analysis ED 216B - Instructor: Karen Nylund-Gibson

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DATA SOURCE: This lab exercise utilizes the NCES public-use dataset: Education Longitudinal Study of 2002 (Lauff & Ingels, 2014) See website: nces.ed.gov

```
# load packages
library(MplusAutomation)
library(haven)
library(rhdf5)
library(tidyverse)
library(here)
library(corrplot)
library(kableExtra)
library(reshape2)
library(semPlot)
```

Lab 9 - Begin

Read in data

```
lab_data <- read_csv(here("data", "els2002_data_subset3.csv"))
```

Preparations: subset,reorder, rename, and recode data

```
invar_data <-  lab_data %>%
  select(bystlang, freelnch, byincome,                    # covariates
         stolen, t_hurt, p_fight, hit, damaged, bullied,  # factor 1 (indicators)
         safe, disrupt, gangs, rac_fght,                  # factor 2 (indicators)
         late, skipped, mth_read, mth_test, rd_test) %>%
  rename("unsafe" = "safe") %>%
  mutate(
    freelnch = case_when(    # Grade 10, percent free lunch - transform to binary
        freelnch <  3 ~ 0,   # school has less than 11%
        freelnch >= 3 ~ 1))  # school has greater than or equal to 11%
```

```r
table(invar_data$freelnch) # reasonably balanced groups
```

Take a quick look at variable distributions

```r
melt(invar_data[,4:13]) %>%
  ggplot(., aes(x=value, label=variable)) +
  geom_histogram(bins = 15) +
  facet_wrap(~variable, scales = "free")
```

Reverse code factor for ease of interpretation

```r
cols = c("unsafe", "disrupt", "gangs", "rac_fght")

invar_data[ ,cols] <-  5 - invar_data[ ,cols]
```

factor names and interpretation:

- VICTIM: student reports being a victim of injury to self or property
  - scale range: Never, Once or twice, More than twice
  - higher values indicate greater frequency of victimization events
- NEG_CLIM: Student reports on negative school climate attributes
  - scale range: Strongly Disagree - Strongly Agree
  - higher values indicate a more negative climate

check correct coding, explore correlations

```r
cor_matrix <- cor(invar_data[4:13], use = "pairwise.complete.obs")

corrplot(cor_matrix,
         method = "circle",
         type = "upper")
```

Step 0: Estimate the Unconditional Confirmatory Factor Analysis (CFA) model

---

Number of parameters = 31

- 10 item loadings (10items*2groups)
- 10 intercepts
- 10 residual variances
- 01 factor co-variances

```
cfa_m0  <- mplusObject(
  TITLE = "model0 - unconditional CFA model",
  VARIABLE =
    "usevar = stolen-rac_fght;",

  ANALYSIS =
    "estimator = mlr;",

  MODEL =
    "VICTIM by stolen* t_hurt p_fight hit damaged bullied;
     VICTIM@1; ! UVI identification

     NEG_CLIM by unsafe* disrupt gangs rac_fght;
     NEG_CLIM@1; ",

  PLOT = "type = plot3;",
  OUTPUT = "sampstat standardized residual modindices (3.84);",

  usevariables = colnames(invar_data),
  rdata = invar_data)

cfa_m0_fit <- mplusModeler(cfa_m0,
                           dataout=here("invar_mplus", "lab9_invar_data.dat"),
                           modelout=here("invar_mplus", "M0_CFA_fullsample.inp"),
                           check=TRUE, run = TRUE, hashfilename = FALSE)
```

Step 1: Run separate CFA models for each sub-sample

Group freelnch = 0 (low) CFA

```
cfa_m1  <- mplusObject(
  TITLE = "CFA model1 - group is 0 for freelnch",
  VARIABLE =
    "usevar = stolen-rac_fght;

     !freelnch (0 = school proportion is less than 11 percent)
     USEOBS = freelnch == 0; ",

  ANALYSIS =
    "estimator = mlr;",

  MODEL =
    "VICTIM by stolen* t_hurt p_fight hit damaged bullied;
     VICTIM@1; ! UVI identification

     NEG_CLIM by unsafe* disrupt gangs rac_fght;
     NEG_CLIM@1; ",
```

```
  PLOT = "type = plot3;",
  OUTPUT = "sampstat standardized residual modindices (3.84);",

  usevariables = colnames(invar_data),
  rdata = invar_data)

cfa_m1_fit <- mplusModeler(cfa_m1,
                           dataout=here("invar_mplus", "lab9_invar_data.dat"),
                           modelout=here("invar_mplus", "M1_CFA_freelnch_0.inp"),
                           check=TRUE, run = TRUE, hashfilename = FALSE)
```

Group freelnch = 1 (moderate to high) CFA

```
cfa_m2  <- mplusObject(
  TITLE = "CFA model2 - group is 1 for freelnch",
  VARIABLE =
    "usevar = stolen-rac_fght;

     !freelnch (1 = school proportion is greater than or equal to 11 percent)
     USEOBS = freelnch == 1; ",

  ANALYSIS =
    "estimator = mlr;",

  MODEL =
    "VICTIM by stolen* t_hurt p_fight hit damaged bullied;
     VICTIM@1; ! UVI identification

     NEG_CLIM by unsafe* disrupt gangs rac_fght;
     NEG_CLIM@1; ",

  PLOT = "type = plot3;",
  OUTPUT = "sampstat standardized residual modindices (3.84);",

  usevariables = colnames(invar_data),
  rdata = invar_data)

cfa_m2_fit <- mplusModeler(cfa_m2,
                           dataout=here("invar_mplus", "lab9_invar_data.dat"),
                           modelout=here("invar_mplus", "M2_CFA_freelnch_1.inp"),
                           check=TRUE, run = TRUE, hashfilename = FALSE)
```

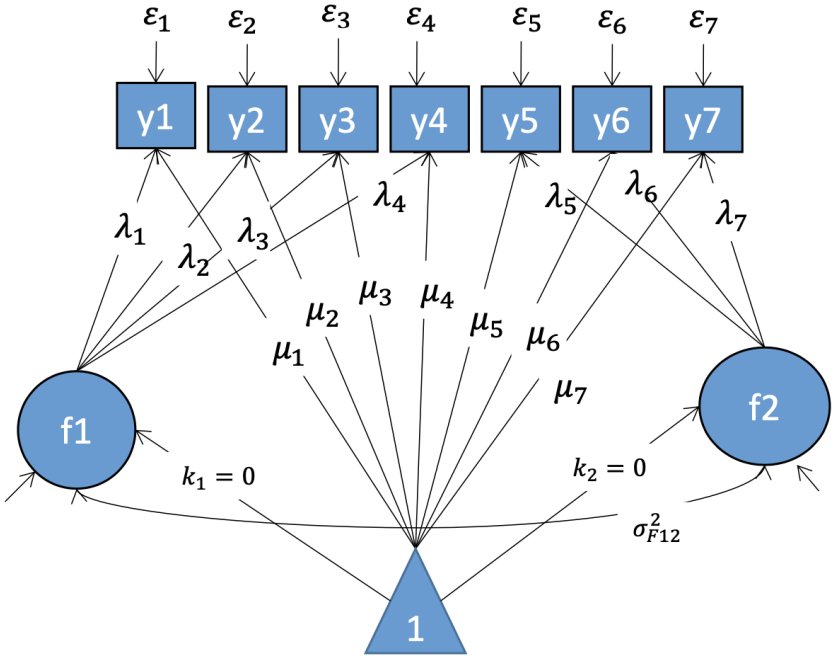

***Figure*: Picture depicting mean structure from slide by Dr. Karen Nylund-Gibson**

Step 2: Configural invariance

- free item loadings, intercepts, and residuals
- factor means fixed to zero

Number of parameters = 62

- 20 item loadings (10items*2groups)
- 20 intercepts
- 20 residual variances
- 02 factor co-variances (1 for each group)

```
cfa_m3  <- mplusObject(
  TITLE = "CFA model3 - configural invariance",
  VARIABLE =
    "usevar = stolen-rac_fght;

     grouping = freelnch (0=freelnch_0 1=freelnch_1); ",
```

```
  ANALYSIS =
    "estimator = mlr;",

  MODEL =
    "VICTIM by stolen* t_hurt p_fight hit damaged bullied;
     VICTIM@1; ! UVI identification

     NEG_CLIM by unsafe* disrupt gangs rac_fght;
     NEG_CLIM@1;

     [VICTIM-NEG_CLIM@0]; !factor means set to zero

     MODEL freelnch_1:

     VICTIM by stolen* t_hurt p_fight hit damaged bullied;
     VICTIM@1;

     [stolen t_hurt p_fight hit damaged bullied]; !free intercepts

     NEG_CLIM by unsafe* disrupt gangs rac_fght;
     NEG_CLIM@1;

     [unsafe disrupt gangs rac_fght]; !free intercepts

     [VICTIM-NEG_CLIM@0]; ",

  PLOT = "type = plot3;",
  OUTPUT = "sampstat standardized residual modindices (3.84);",

  usevariables = colnames(invar_data),
  rdata = invar_data)

cfa_m3_fit <- mplusModeler(cfa_m3,
                           dataout=here("invar_mplus", "lab9_invar_data.dat"),
                           modelout=here("invar_mplus", "M3_configural.inp"),
                           check=TRUE, run = TRUE, hashfilename = FALSE)
```

Step 3: Metric invariance

- item loadings (set to equal)
- free intercepts and residuals
- factor means fixed to zero
- free factor variances in group 2

Number of parameters = 54

- 10 item loadings (set to equal)
- 20 intercepts
- 20 residual variances
- 02 factor variances
- 02 factor co-variances

```
cfa_m4  <- mplusObject(
  TITLE = "CFA model4 - metric invariance",
  VARIABLE =
    "usevar = stolen-rac_fght;

     grouping = freelnch (0=freelnch_0 1=freelnch_1); ",

  ANALYSIS =
    "estimator = mlr;",

  MODEL =
    "VICTIM by stolen* t_hurt p_fight hit damaged bullied;
     VICTIM@1; ! UVI identification

     NEG_CLIM by unsafe* disrupt gangs rac_fght;
     NEG_CLIM@1;

     [VICTIM-NEG_CLIM@0];

     MODEL freelnch_1:

     VICTIM; ! free factor variances for group 2

     [stolen t_hurt p_fight hit damaged bullied];

     NEG_CLIM;

     [unsafe disrupt gangs rac_fght];

     [VICTIM-NEG_CLIM@0]; ",

  PLOT = "type = plot3;",
  OUTPUT = "sampstat standardized residual modindices (3.84);",

  usevariables = colnames(invar_data),
  rdata = invar_data)

cfa_m4_fit <- mplusModeler(cfa_m4,
                           dataout=here("invar_mplus", "lab9_invar_data.dat"),
                           modelout=here("invar_mplus", "M4_metric.inp"),
                           check=TRUE, run = TRUE, hashfilename = FALSE)
```

Step 4: Scalar invariance

- item loadings (set to equal)
- intercepts (set to equal)
- free residuals
- factor means fixed to zero
- free factor variances in group 2

Number of parameters = 46

- 10 item loadings (set to equal)
- 10 intercepts (set to equal)
- 20 residual variances
- 02 factor variances
- 02 factor co-variances
- 02 factor means

```
cfa_m5  <- mplusObject(
  TITLE = "model5 - scalar invariance",
  VARIABLE =
    "usevar = stolen-rac_fght;

     grouping = freelnch (0=freelnch_0 1=freelnch_1); ",

  ANALYSIS =
    "estimator = mlr;",

  MODEL =
    "VICTIM by stolen* t_hurt p_fight hit damaged bullied;
     VICTIM@1;

     NEG_CLIM by unsafe* disrupt gangs rac_fght;
     NEG_CLIM@1;

     [VICTIM-NEG_CLIM@0];

     MODEL freelnch_1:

     VICTIM; ! free factor variances for group 2

     NEG_CLIM;

     [VICTIM-NEG_CLIM]; ! free factor means",

  PLOT = "type = plot3;",
  OUTPUT = "sampstat standardized residual modindices (3.84);",

  usevariables = colnames(invar_data),
```

```
  rdata = invar_data)

cfa_m5_fit <- mplusModeler(cfa_m5,
                           dataout=here("invar_mplus", "lab9_invar_data.dat"),
                           modelout=here("invar_mplus", "M5_scalar.inp"),
                           check=TRUE, run = TRUE, hashfilename = FALSE)
```

Step 5: Strict invariance

- item loadings (set to equal)
- intercepts (set to equal)
- residuals (set to equal)
- factor means fixed to zero
- free factor variances in group 2

Number of parameters = 36

- 10 item loadings (set to equal)
- 10 intercepts (set to equal)
- 10 residual variances
- 02 factor variances
- 02 factor co-variances
- 02 factor means

```
cfa_m6  <- mplusObject(
  TITLE = "model6 - strict invariance",
  VARIABLE =
    "usevar = stolen-rac_fght;

     grouping = freelnch (0=freelnch_0 1=freelnch_1); ",

  ANALYSIS =
    "estimator = mlr;",

  MODEL =
    "VICTIM by stolen* t_hurt p_fight hit damaged bullied;
     VICTIM@1;

     NEG_CLIM by unsafe* disrupt gangs rac_fght;
     NEG_CLIM@1;

     [VICTIM-NEG_CLIM@0];

     stolen-rac_fght(1-10); ! set residuals to be equal across groups

     MODEL freelnch_1:
```

```
    VICTIM; ! free factor variances for group 2

    NEG_CLIM;

    [VICTIM-NEG_CLIM]; ! free factor means

    stolen-rac_fght(1-10); ",

  PLOT = "type = plot3;",
  OUTPUT = "sampstat standardized residual modindices (3.84);",

  usevariables = colnames(invar_data),
  rdata = invar_data)

cfa_m6_fit <- mplusModeler(cfa_m6,
                           dataout=here("invar_mplus", "lab9_invar_data.dat"),
                           modelout=here("invar_mplus", "M6_strict.inp"),
                           check=TRUE, run = TRUE, hashfilename = FALSE)
```

Step 6: Structural invariance A (fixed factor variances)

Demonstration of structural invariance using the Scalar model

- item loadings (set to equal)
- intercepts (set to equal)
- free residuals (Scalar)
- factor means free in group 2
- factor variances (set to 1)
- free factor covariances

Number of parameters = 44

- 10 item loadings (set to equal)
- 10 intercepts (set to equal)
- 20 residual variances
- 00 factor variances
- 02 factor co-variances
- 02 factor means

```
# fixed factor variances
cfa_m7  <- mplusObject(
  TITLE = "model7 - structural invariance A" ,
  VARIABLE =
    "usevar = stolen-rac_fght;
```

```
    grouping = freelnch (0=freelnch_0 1=freelnch_1); ",

  ANALYSIS =
    "estimator = mlr;",

  MODEL =
    "VICTIM by stolen* t_hurt p_fight hit damaged bullied;
     VICTIM@1;

     NEG_CLIM by unsafe* disrupt gangs rac_fght;
     NEG_CLIM@1;

     [VICTIM-NEG_CLIM@0];

     MODEL freelnch_1:

     [VICTIM-NEG_CLIM]; ! free factor means

     VICTIM@1; NEG_CLIM@1; ! fix factor variance to 1",

  PLOT = "type = plot3;",
  OUTPUT = "sampstat standardized residual modindices (3.84);",

  usevariables = colnames(invar_data),
  rdata = invar_data)

cfa_m7_fit <- mplusModeler(cfa_m7,
                           dataout=here("invar_mplus", "lab9_invar_data.dat"),
                           modelout=here("invar_mplus", "M7_structuralA.inp"),
                           check=TRUE, run = TRUE, hashfilename = FALSE)
```

Step 7: Structural invariance B (fixed factor variances and equal covariances)

Demonstration of structural invariance using the Scalar model

- item loadings (set to equal)
- intercepts (set to equal)
- free residuals (Scalar)
- factor means free in group 2
- factor variances (set to equal)
- factor covariances (set to equal)

Number of parameters = 43

- 10 item loadings (set to equal)
- 10 intercepts (set to equal)

- 20 residual variances
- 00 factor variances
- 01 factor co-variances
- 02 factor means

```
# equal factor variances and covariances
cfa_m8  <- mplusObject(
  TITLE = "model8 - structural invariance B" ,
  VARIABLE =
    "usevar = stolen-rac_fght;

     grouping = freelnch (0=freelnch_0 1=freelnch_1); ",

  ANALYSIS =
    "estimator = mlr;",

  MODEL =
    "VICTIM by stolen* t_hurt p_fight hit damaged bullied;
     VICTIM@1;

     NEG_CLIM by unsafe* disrupt gangs rac_fght;
     NEG_CLIM@1;

     [VICTIM-NEG_CLIM@0];

     VICTIM with NEG_CLIM (11) ! set covariances to equal;

     MODEL freelnch_1:

     [VICTIM-NEG_CLIM]; ! free factor means

     VICTIM@1; NEG_CLIM@1; ! fix factor variance to 1

     VICTIM with NEG_CLIM (11); ! set covariances to equal",

  PLOT = "type = plot3;",
  OUTPUT = "sampstat standardized residual modindices (3.84);",

  usevariables = colnames(invar_data),
  rdata = invar_data)

cfa_m8_fit <- mplusModeler(cfa_m8,
                           dataout=here("invar_mplus", "lab9_invar_data.dat"),
                           modelout=here("invar_mplus", "M8_structuralB.inp"),
                           check=TRUE, run = TRUE, hashfilename = FALSE)
```

Latent Factor Means differences:

(model: Step_07_STRUCTURAL)

Mean differences: Students in sub-sample `freelnch_1` have...

```
VICTIM     -0.026      0.091     -0.291      0.771  (not significant)
NEG_CLIM    0.632      0.104      6.104      0.000  (higher scores for "NEG_CLIM")
```

Comparing Fit Across Models:

Guidlines for loadings & fit indices

- **Simple structure:** "0.4 - 0.3 - 0.2" rule Howard (2016) (primary loadings > 0.4 / cross-loadings < 0.3 / minimum difference = 0.2)
- **RMSEA:** < .05 indicates "good" fit Brown (2015)
- **CFI:** > .95 indicates "good" fit Brown (2015)
- **SRMR:** < .08 indicates "good" fit Hu and Bentler (1999)
- **Invariance:** Changes in **CFI** less than or equal to **-0.01** are acceptable

read into R summary of all 10 invariance models

```
# read all of the output files
all_models <- readModels(here("invar_mplus"))
```

extract fit statistics, sort by `Filename`

```
invar_summary <- LatexSummaryTable(all_models,
                 keepCols=c("Filename", "Parameters","ChiSqM_Value", "CFI","TLI",
                            "SRMR", "RMSEA_Estimate", "RMSEA_90CI_LB", "RMSEA_90CI_UB"),
                 sortBy = "Filename")
```

Create table and rename columns

```
invar_summary %>%
  kable(booktabs = T,
        col.names = c("Model",
                      "Par",
                      "ChiSq",
                      "CFI",
                      "TLI",
                      "SRMR",
                      "RMSEA",
                      "Lower CI",
                      "Upper CI")) %>%
  kable_styling(latex_options = c("striped", "scale_down", linesep = ""),
                full_width = F,
                position = "left")
```

Calculate Satora-Bentler scaled Chi-square difference test (use with MLR estimator)

See website: stats.idre.ucla.edu

- SB0 = null model Chi-square value
- SB1 = alternate model Chi-square value
- c0 = null model scaling correction factor
- c1 = alternate model scaling correction factor
- d0 = null model degrees of freedom
- d1 = alternate model degrees of freedom
- df = Chi-square test degrees of freedom

```
# compare configural to metric

SB0 <- all_models[["M4_metric.out"]][["summaries"]][["ChiSqM_Value"]]
SB1 <- all_models[["M3_configural.out"]][["summaries"]][["ChiSqM_Value"]]
c0  <- all_models[["M4_metric.out"]][["summaries"]][["ChiSqM_ScalingCorrection"]]
c1  <- all_models[["M3_configural.out"]][["summaries"]][["ChiSqM_ScalingCorrection"]]
d0  <- all_models[["M4_metric.out"]][["summaries"]][["ChiSqM_DF"]]
d1  <- all_models[["M3_configural.out"]][["summaries"]][["ChiSqM_DF"]]
df  <- abs(d0-d1)

# Satora-Bentler scaled Difference test equations
cd <- (((d0*c0)-(d1*c1))/(d0-d1))
t  <- (((SB0*c0)-(SB1*c1))/(cd))

# Chi-square and degrees of freedom
t
df

# Significance test
pchisq(t, df, lower.tail=FALSE)
```

```
# compare metric to scalar

SB0 <- all_models[["M5_scalar.out"]][["summaries"]][["ChiSqM_Value"]]
SB1 <- all_models[["M4_metric.out"]][["summaries"]][["ChiSqM_Value"]]
c0  <- all_models[["M5_scalar.out"]][["summaries"]][["ChiSqM_ScalingCorrection"]]
c1  <- all_models[["M4_metric.out"]][["summaries"]][["ChiSqM_ScalingCorrection"]]
d0  <- all_models[["M5_scalar.out"]][["summaries"]][["ChiSqM_DF"]]
d1  <- all_models[["M4_metric.out"]][["summaries"]][["ChiSqM_DF"]]
df  <- abs(d0-d1)

# Satora-Bentler scaled Difference test equations
cd <- (((d0*c0)-(d1*c1))/(d0-d1))
t  <- (((SB0*c0)-(SB1*c1))/(cd))

# Chi-square and degrees of freedom
t
df

# Significance test
pchisq(t, df, lower.tail=FALSE)
```

Invariance short-cut

```
mx  <- mplusObject(
  TITLE = "INVARIANCE SHORT_CUT - LAB 9 DEMO",
  VARIABLE =
    "usevar = stolen-rac_fght;

     grouping = freelnch (0=freelnch_0 1=freelnch_1); ",

  ANALYSIS =
  "Estimator = MLR;
   MODEL= CONFIG METRIC SCALAR;",

     MODEL =
    "VICTIM by stolen* t_hurt p_fight hit damaged bullied;
     VICTIM@1;

     NEG_CLIM by unsafe* disrupt gangs rac_fght;
     NEG_CLIM@1;" ,

  PLOT = "",
  OUTPUT = "sampstat residual;",

  usevariables = colnames(invar_data),
  rdata = invar_data)

mx_fit <- mplusModeler(mx,
                           dataout=here("invar_short", "Invar_short_cut.dat"),
                           modelout=here("invar_short", "Invar_short_cut.inp"),
                           check=TRUE, run = TRUE, hashfilename = FALSE)
```

Invariance Testing (Chi-square values - Chi-Square difference p-values are biased)

Model	Number of Parameters	Chi-Square	Degrees of Freedom	P-Value
Configural	62	149.315	68	0.0000
Metric	54	163.312	76	0.0000
Scalar	46	179.176	84	0.0000

Models Compared	Chi-Square	Degrees of Freedom	P-Value
Metric against Configural	14.759	8	0.0640
Scalar against Configural	30.022	16	0.0179
Scalar against Metric	15.444	8	0.0511

End of Lab 9

References

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