Call-by-name, Call-by-value, Call-by-need, and the Linear Lambda Calculus

Short Talk

Quentin Schroeder

MPRI - Université Paris-Cité

Idea

Motivation

Problem

We want to understand how to evaluate expressions in the lambda calculus.

- Call by Name

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- Call by Need

Call by value (Ocaml)

Examples

```
let rec dumb_works n = if (n = 0 && n = 1) then dumb n else 42

let () = print_int (dumb_works 0)
```

Call by value (Ocaml)

Examples

```
let rec dumb_works n = if (n = 0 && n = 1) then dumb n else 42

let () = print_int (dumb_works 0)
```

```
let branch b l r = if b then l else r

let rec dumb_breaks n = branch (n = 0 && n = 1) (dumb_breaks n) 42

let () = print_int (dumb_breaks 0)
```

Call by need (Haskell)

Example

```
dumb n = if (n == 0 && n == 1) then dumb n else 42

forceEvalBranch b l r = if b then l else r

dumb2 n = forceEvalBranch (n == 0 && n == 1) (dumb2 n) 42

main = do
    print $ dumb 0
    print $ dumb 1
    print $ dumb2 0
    print $ dumb2 1
```

Call by need (Haskell)

Example

```
dumb n = if (n == 0 && n == 1) then dumb n else 42

forceEvalBranch b l r = if b then l else r

dumb2 n = forceEvalBranch (n == 0 && n == 1) (dumb2 n) 42

main = do
    print $ dumb 0
    print $ dumb 1
    print $ dumb2 0
    print $ dumb2 1
```

Example

```
project x y = x

loop x = loop x

main = print $ project 2 (loop 3)
```

Call by Name (???)

Is there a call by name programming language?

Yes . . . kind of (e.g. Algol 60)

Example

Consider the following:

```
let rec horrible n = if n = 0 then 1 else n * horrible (n - 1) * horrible (

let printalot m =
  let () = print_endline (string_of_int m) in
  let () = print_endline (string_of_int m) in
  let () = print_endline (string_of_int m) in
  let () = print_endline (string_of_int m)

let () = printNTimes (horrible 40)
```

Preliminaries

Lambda Calculus

- model of computation
- basis for functional programming languages

Linear Logic

- a resource sensitive logic
- can be used to priority in evaluation of proof terms

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Linear Logic

- a resource sensitive logic
- can be used to priority in evaluation of proof terms

A massive leap of faith

- build a linear lambda calculus
  [Maraist et al.(1995)Maraist, Odersky, Turner, and Wadler,]
- show that it can be used to model execution strategies

Why do we care?

- learn about the underlying structure of evaluation strategies
- learn how to improve our current programming languages (e.g. Rust)
- learn about them as models for complexity theory [Forster et al.(2019)Forster, Kunze, and Roth,]

Interpretation of Call by Value

Call by Value

- evaluate the argument before the function
- the argument is evaluated at most once

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Call by Value via ”suspending computation”

- suspend computation of all the innermost functions calls
- evaluate from last to first

Interpretation of Call by Value

Example

```
let f x y = x + y
let () = print_int (f (1 + 2) (3 + 4))
```

Interpretation of Call by Value

Example

```
let f x y = x + y
let () = print_int (f (1 + 2) (3 + 4))
```

- evaluate `f (1 + 2) (3 + 4)`

Interpretation of Call by Value

Example

```
let f x y = x + y
let () = print_int (f (1 + 2) (3 + 4))
```

- evaluate `f (1 + 2) (3 + 4)`
- evaluate `1 + 2`

Interpretation of Call by Value

Example

```
let f x y = x + y
let () = print_int (f (1 + 2) (3 + 4))
```

- evaluate `f (1 + 2) (3 + 4)`
- evaluate `1 + 2`
- evaluate `3 + 4`

Interpretation of Call by Value

Example

```
let f x y = x + y
let () = print_int (f (1 + 2) (3 + 4))
```

- evaluate `f (1 + 2) (3 + 4)`
- evaluate `1 + 2`
- evaluate `3 + 4`
- evaluate `f 3 7`

Interpretation of Call by Value

Example

```
let f x y = x + y
let () = print_int (f (1 + 2) (3 + 4))
```

- evaluate `f (1 + 2) (3 + 4)`
- evaluate `1 + 2`
- evaluate `3 + 4`
- evaluate `f 3 7`
- evaluate `3 + 7`

Interpretation of Call by Value

Example

```
let f x y = x + y
let () = print_int (f (1 + 2) (3 + 4))
```

- evaluate `f (1 + 2) (3 + 4)`
- evaluate `1 + 2`
- evaluate `3 + 4`
- evaluate `f 3 7`
- evaluate `3 + 7`
- print `10`

Interpretation of Call by Value

Example

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let f x y = x + y
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- evaluate `f (1 + 2) (3 + 4)`
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Interpretation of Call by Value

Example

```
let f x y = x + y
let () = print_int (f (1 + 2) (3 + 4))
```

- evaluate `f (1 + 2) (3 + 4)`
- evaluate `1 + 2`
- evaluate `3 + 4`
- evaluate `f 3 7`
- evaluate `3 + 7`
- print `10`

For the suspend interpretation: we would view this as blocking evaluation of `f` until the arguments are evaluated.

Conclusion

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In short

- Call by Name
- Call by Value
- Call by Need

For the next time

- Model Call by Name and Call by Need in this model
- Show interpretations have nice properties (soundness, completeness, etc.)

Thank you

References I

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