Call-by-name, Call-by-value, Call-by-need, and the Linear Lambda Calculus

Short Talk

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Idea

Problem

We want to understand how to evaluate expressions in the lambda calculus.

- Call by Name

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Examples

```ocaml
let rec dumb_works n = if false then dumb_works n else 42

let () = print_int (dumb_works 0)
```

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let rec dumb_works n = if false then dumb_works n else 42

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let branch b l r = if b then l else r

let rec dumb_breaks n = branch false (dumb_breaks n) 42

let () = print_int (dumb_breaks 0)
```

Example

```haskell
dumb n = if (n == 0 && n == 1) then dumb n else 42

forceEvalBranch b l r = if b then l else r

dumb2 n = forceEvalBranch (n == 0 && n == 1) (dumb2 n) 42

main = do
    print $ dumb 0
    print $ dumb 1
    print $ dumb2 0
    print $ dumb2 1
```

Example

```haskell
dumb n = if (n == 0 && n == 1) then dumb n else 42

forceEvalBranch b l r = if b then l else r

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main = do
    print $ dumb 0
    print $ dumb 1
    print $ dumb2 0
    print $ dumb2 1
```

Example

```haskell
project x y = x

loop x = loop x

main = print $ project 2 (loop 3)
```

Is there a call by name programming language?

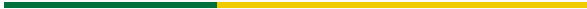Yes ... kind of (e.g. Algol 60)

Example

Consider the following:

```
let rec h n = if n = 0 then 1 else n * h (n - 1) * h (n - 1)

let printalot m =
  let () = print_endline (string_of_int m) in
  let () = print_endline (string_of_int m) in
  let () = print_endline (string_of_int m) in
  let () = print_endline (string_of_int m)

let () = printNTimes (h 40)
```

PRELIMINARIES

Lambda Calculus

- model of computation
- basis for functional programming languages

Linear Logic

- a resource sensitive logic
- can be used to priority in evaluation of proof terms

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- a resource sensitive logic
- can be used to priority in evaluation of proof terms

A massive leap of faith

- build a linear lambda calculus
  [Maraist et al.(1995)Maraist, Odersky, Turner, and Wadler,]
- show that it can be used to model execution strategies

Why do we care?

- learn about the underlying structure of evaluation strategies
- learn how to improve our current programming languages (e.g. Rust)
- learn about them as models for complexity theory [Forster et al.(2019)Forster, Kunze, and Roth,]

Call by Value

- evaluate the argument before the function
- the argument is evaluated at most once

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Call by Value via "suspending computation"

- suspend computation of all the innermost functions calls
- evaluate from last to first

Example

```
let f x y = x + y
let () = print_int (f (1 + 2) (3 + 4))
```

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- suspend f (1 + 2) (3 + 4)

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- suspend f (1 + 2) (3 + 4)
- suspend 1 + 2

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let f x y = x + y
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- suspend f (1 + 2) (3 + 4)
- suspend 1 + 2
- suspend 3 + 4

Example

```
let f x y = x + y
let () = print_int (f (1 + 2) (3 + 4))
```

- suspend f (1 + 2) (3 + 4)
- suspend 1 + 2
- suspend 3 + 4
- evaluate 3 + 4

```
let f x y = x + y
let () = print_int (f (1 + 2) (3 + 4))
```

- suspend f (1 + 2) (3 + 4)
- suspend 1 + 2
- suspend 3 + 4
- evaluate 3 + 4
- evaluate 1 + 2

Example

```
let f x y = x + y
let () = print_int (f (1 + 2) (3 + 4))
```

- suspend f (1 + 2) (3 + 4)
- suspend 1 + 2
- suspend 3 + 4
- evaluate 3 + 4
- evaluate 1 + 2
- evaluate f 3 7

Conclusion

In short

- Call by Name
- Call by Value
- Call by Need

For the next time

- Model Call by Name and Call by Need in this model
- Show interpretations have nice properties (soundness, completeness, etc.)

THANK YOU

📄 Yannick Forster, Fabian Kunze, and Marc Roth.
The weak call-by-value lambda-calculus is reasonable for both time and space.
Proc. ACM Program. Lang., 4(POPL), dec 2019.
doi: 10.1145/3371095.
URL https://doi.org/10.1145/3371095.

📄 John Maraist, Martin Odersky, David N. Turner, and Philip Wadler.

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Electron. Notes Theor. Comput. Sci., 1:370–392, January 1995.
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