
# Call-by-name, Call-by-value, Call-by-need, and the Linear Lambda Calculus

## Short Talk

Quentin Schroeder

MPRI - Université Paris-Cité

# Idea

# Motivation

## Goal

Study evaluation strategies via the linear lambda calculus

## Why?

- found linearity is relevant when studying Call by Need
- noticed it also applies for other strategies

# Overview

1. Linear Lambda Calculus
2. Call by Name
3. Call by Value
4. Notes on Call by Need
5. Results
6. Conclusion

# Simply Typed Lambda Calculus (Syntax)

**Types :** $A, B, C ::= \text{basic types} \mid A \to B$

**Terms :** $L, M, N ::= V \mid M\ N$

**Values :** $V, W ::= x \mid \lambda x.t$

$$\text{Id}\ \frac{}{x : A \vdash x : A}$$

$$\text{Contraction}\ \frac{\Gamma, y : A, z : A \to M : B}{\Gamma, x : A \vdash M[y := x, z := x] : B} \qquad \text{Weakening}\ \frac{\Gamma \vdash M : B}{\Gamma, x : A \vdash M : B}$$

$$\to -\textit{Intro}\ \frac{\Gamma, x : A \vdash M : B}{\Gamma \vdash \lambda x.M : A \to B} \qquad \to -\textit{Elim}\ \frac{\Gamma \vdash M : A \to B \qquad \Delta \vdash N : A}{\Gamma, \Delta \vdash M\ N : B}$$

# Simply Typed Lambda Calculus (Evaluation Strategies)

## Call by Name

Reduces on terms, not values

$(\beta_{name}) : (\lambda x.M)\ N \rightsquigarrow M[x := N]$

## Call by Value

Reduces on values, not terms

$(\beta_{value}) : (\lambda x.M)\ V \rightsquigarrow M[x := V]$

## Example

From [sta24, SE:101670]

$$(\lambda p.\lambda q.p)\ (\lambda a.\lambda b.a)\ (\lambda a.\lambda b.b)$$

# Linear Lambda Calculus (Syntax)

**Types :** $A, B, C ::= \text{basic types} \mid !A \mid A \multimap A$

**Terms :** $L, M, N ::= x \mid !M \mid \text{let } !x = M \text{ in } N \mid \lambda x.M \mid M\ N$

$$\text{Id}\ \frac{}{x : A \vdash x : A} \qquad \text{Dereliction}\ \frac{\Gamma, x : A \vdash M : B}{\Gamma, !x : !A \vdash M : B}$$

$$!\text{-Intro}\ \frac{!\Gamma \vdash M : A}{!\Gamma \vdash !M : !A} \qquad \text{Contraction}\ \frac{\Gamma, !y : !A, !z : !A \multimap M : B}{\Gamma, !x : !A \vdash M[y := x, z := x] : B}$$

$$!\text{-Elim}\ \frac{!\Gamma \vdash M : !A \qquad \Delta, !x : !A \vdash N : B}{\Gamma, \Delta \vdash \text{let } !x = M \text{ in } N : B}$$

## Linear Logic

- a resource sensitive logic
- can be used to priority in evaluation of proof terms

# Preliminaries

## Linear Logic

- a resource sensitive logic
- can be used to priority in evaluation of proof terms

## A massive leap of faith

- build a linear lambda calculus [MOTW95, ]
- show that it can be used to model execution strategies

# References i


- John Maraist, Martin Odersky, David N. Turner, and Philip Wadler, *Call-by-name, Call-by-value, Call-by-need, and the Linear Lambda Calculus*, Electron. Notes Theor. Comput. Sci. **1** (1995), 370–392.
- *Lambda Calculus - Call-by-name AND call-by-value reduction*, March 2024, [Online; accessed 24. Mar. 2024].