

Exercise 1, Question 6a

$$\begin{array}{c}
 \frac{\frac{\frac{}{A_x}}{a: \forall x(A \wedge B), x \vdash a: \forall z(A \wedge B)} \forall_E}{a: \forall x(A \wedge B), x \vdash a x: A \wedge B} \wedge_E^1 \\
 \frac{a: \forall x(A \wedge B), x \vdash \pi_1(a x): A}{a: \forall x(A \wedge B) \vdash \lambda x. p_1': \forall x A} \forall_I
 \end{array}
 \quad
 \begin{array}{c}
 \frac{\frac{\frac{}{A_x}}{a: \forall x(A \wedge B), x \vdash a: \forall z(A \wedge B)} \forall_E}{a: \forall x(A \wedge B), x \vdash a x: A \wedge B} \wedge_E^2 \\
 \frac{a: \forall x(A \wedge B), x \vdash \pi_2(a x): B}{a: \forall x(A \wedge B) \vdash \lambda x. p_2': \forall x B} \forall_I
 \end{array}
 \quad
 \begin{array}{c}
 \frac{\frac{\frac{\frac{}{A_x}}{\Gamma \vdash a: (\forall x A) \wedge (\forall x B)} \wedge_E^1}{\Gamma \vdash \pi_1(a): \forall x A} \wedge_E^2}{\Gamma \vdash (\pi_1(a)) x: A} \forall_E \\
 \frac{\frac{\frac{\frac{}{A_x}}{\Gamma \vdash a: (\forall x A) \wedge (\forall x B)} \wedge_E^1}{\Gamma \vdash \pi_2(a): \forall x B} \wedge_E^2}{\Gamma \vdash (\pi_2(a)) x: B} \forall_E \\
 \frac{}{\Gamma \vdash (\pi_1(a)) x: A \wedge (\pi_2(a)) x: B} \wedge_I
 \end{array}$$

$$\begin{array}{c}
 \frac{a: \forall x(A \wedge B) \vdash \lambda x. p_1': \forall x A \quad a: \forall x(A \wedge B) \vdash \lambda x. p_2': \forall x B}{a: \forall x(A \wedge B) \vdash \lambda x. (p_1, p_2): (\forall x A) \wedge (\forall x B)} \wedge_I \\
 \frac{a: \forall x(A \wedge B) \vdash \lambda x. (p_1, p_2): (\forall x A) \wedge (\forall x B)}{\vdash \lambda a. p': \forall x(A \wedge B) \Rightarrow (\forall x A) \wedge (\forall x B)} \Rightarrow_I \\
 \vdash (p, q) : \forall x(A \wedge B) \Leftrightarrow (\forall x A) \wedge (\forall x B)
 \end{array}$$

$$\begin{array}{c}
 \frac{\frac{\frac{\frac{}{A_x}}{\Gamma \vdash a: (\forall x A) \wedge (\forall x B)} \wedge_E^1}{\Gamma \vdash \pi_1(a): \forall x A} \wedge_E^2}{\Gamma \vdash (\pi_1(a)) x: A} \forall_E \\
 \frac{\frac{\frac{\frac{}{A_x}}{\Gamma \vdash a: (\forall x A) \wedge (\forall x B)} \wedge_E^1}{\Gamma \vdash \pi_2(a): \forall x B} \wedge_E^2}{\Gamma \vdash (\pi_2(a)) x: B} \forall_E \\
 \frac{}{\Gamma \vdash (\pi_1(a)) x: A \wedge (\pi_2(a)) x: B} \wedge_I
 \end{array}$$

$$\begin{array}{c}
 \frac{a: \forall x(A \wedge B), x \vdash (\pi_1(a) x, (\pi_2(a) x)): A \wedge B}{a: \forall x(A \wedge B) \vdash \lambda x. (\pi_1(a) x, (\pi_2(a) x)): \forall x(A \wedge B)} \forall_I \\
 \frac{a: \forall x(A \wedge B) \vdash \lambda x. (\pi_1(a) x, (\pi_2(a) x)): \forall x(A \wedge B)}{\vdash \lambda a. q': (\forall x(A \wedge B) \Rightarrow \forall x(A \wedge B))} \Rightarrow_I \\
 \vdash (p, q) : \forall x(A \wedge B) \Leftrightarrow (\forall x A) \wedge (\forall x B)
 \end{array}$$

Exercice 1, Question 6b (solution partielle)

[illegible]

Exercice 1, Question 6b (suite de la solution)

[illegible]

Exercise 2, Question 1a

$(\lambda a. \lambda y. \lambda x. a x y, \lambda a. \lambda x. \lambda y. a y x)$

Show Proof.

Qnd

Print ...

$$\begin{array}{c}
 \frac{}{\Gamma \vdash a : \forall x \forall y A} A_x \\
 \frac{}{\Gamma \vdash a x : \forall y A} \forall_E \\
 \frac{}{\Gamma \vdash a x y : A} \forall_E \\
 \frac{}{a : \forall x \forall y A, y, x \vdash a x y : A} \forall_I \\
 \frac{}{a : \forall x \forall y A, y \vdash \lambda x. p''' : \forall x A} \forall_I \\
 \frac{}{a : \forall x \forall y A \vdash \lambda y. p'' : \forall y \forall x A} \Rightarrow_I \\
 \frac{}{\vdash \lambda a. p' : (\forall x \forall y A) \Rightarrow (\forall y \forall x A)} \Rightarrow_I \\
 \frac{}{\vdash \lambda a. q' : (\forall y \forall x A) \Rightarrow (\forall x \forall y A)} \Rightarrow_I \\
 \vdash (p, q) : (\forall x \forall y A) \Leftrightarrow (\forall y \forall x A)
 \end{array}$$

Exercise 2, Question 1b

$\vdash (\lambda a.p, \lambda a.q).(\exists x \exists y A) \Leftrightarrow (\exists y \exists x A)$

Exercise 2, Question 2a

$$\begin{array}{c}
 \frac{\frac{\frac{}{Ax}}{\Gamma' \vdash a' : \forall x A} \forall_E}{\Gamma' \vdash a' x : A} \exists_I}{\Gamma' \vdash a : \exists y \forall x A} \exists_E \\
 \frac{\Gamma' \vdash a : \exists y \forall x A, x \vdash \text{dest } a \text{ as } (y, a') \text{ in } \Gamma' : \exists y A}{\Gamma' \vdash a : \exists y \forall x A \vdash \lambda x. p : \forall x \exists y A} \Rightarrow_I \\
 \vdash \lambda a. p : \exists y \forall x A \Rightarrow \forall x \exists y A
 \end{array}$$