

Exercise 1, Question 1a

$$\frac{\frac{}{A \vdash A} A_x}{\vdash A \Rightarrow A} \Rightarrow_I$$

$$\frac{\frac{}{a:A \vdash a:A} A_x}{\vdash \lambda a. a : A \Rightarrow A} \Rightarrow_I$$

Exercise 1, Question 1b

$$\lambda a. \lambda b. (\pi_2(a)) ((\pi_1(a)) b)$$

$$\begin{array}{c}
 \frac{\Gamma \vdash a : (A \Rightarrow B) \wedge (B \Rightarrow C)}{\Gamma \vdash \pi_2(a) : B \Rightarrow C} \wedge^2_E \quad \frac{\frac{\Gamma \vdash a : (A \Rightarrow B) \wedge (B \Rightarrow C)}{\Gamma \vdash \pi_1(a) : A \Rightarrow B} \wedge^1_E \quad \frac{\Gamma \vdash b : A}{\Gamma \vdash \pi_1(a) b : B} \Rightarrow_E}{\Gamma \vdash \pi_2(a), b : A \vdash p_1 p_2 : C} \Rightarrow_I \\
 \frac{\Gamma \vdash \pi_2(a), b : A \vdash p_1 p_2 : C}{\Gamma \vdash a : (A \Rightarrow B) \wedge (B \Rightarrow C) \vdash \lambda b. p_1' : A \Rightarrow C} \Rightarrow_I \\
 \frac{\Gamma \vdash a : (A \Rightarrow B) \wedge (B \Rightarrow C) \vdash \lambda b. p_1' : A \Rightarrow C}{\vdash \lambda a. p : (A \Rightarrow B) \wedge (B \Rightarrow C) \Rightarrow (A \Rightarrow C)} \Rightarrow_I
 \end{array}$$

Exercise 1, Question 2

$$\begin{array}{c}
 \frac{\Gamma \vdash x : (A \wedge B) \Rightarrow C}{\Gamma \vdash x : (A \wedge B) \Rightarrow C} A_x \quad \frac{\frac{}{\Gamma \vdash a : A} A_x \quad \frac{}{\Gamma \vdash b : B} A_x}{\Gamma \vdash (a, b) : A \wedge B} \wedge_I \\
 \hline
 \Gamma \vdash x : (A \wedge B) \Rightarrow C \quad \Gamma \vdash (a, b) : A \wedge B \Rightarrow E \\
 \hline
 x : (A \wedge B) \Rightarrow C, a : A, b : B \vdash x(a, b) : C \Rightarrow_{(x2)} \\
 \hline
 x : (A \wedge B) \Rightarrow C \vdash \lambda a. \lambda b. p' : A \Rightarrow B \Rightarrow C \Rightarrow_I \\
 \hline
 \vdash \lambda x. p : ((A \wedge B) \Rightarrow C) \Rightarrow (A \Rightarrow B \Rightarrow C) \\
 \hline
 \vdash (\lambda x. p, \lambda x. q) : ((A \wedge B) \Rightarrow C) \Leftrightarrow (A \Rightarrow B \Rightarrow C)
 \end{array}$$

$$(\lambda x. \lambda a. \lambda b. \neg c(a, b), \lambda x c. \lambda y. \neg c(\pi_1(y), \pi_2(y)))$$

Exercise 1, Question 3d

$$\begin{array}{c}
 \frac{\frac{\frac{}{Ax} \quad \frac{}{x:A \vee B, a:A \vdash a:A} \vee_2 \quad \frac{}{x:A \vee B, b:B \vdash b:B} \vee_1}{x:A \vee B, a:A \vdash \lambda c. [c_1(a) \rightarrow c_2(a)] [c_2(b) \rightarrow c_1(b)] : B \vee A} \vee_E}{x:A \vee B \vdash \text{case } x \text{ of } [\lambda c. [c_1(a) \rightarrow c_2(a)] [c_2(b) \rightarrow c_1(b)]] : B \vee A} \vee_I \\
 \frac{x:A \vee B \vdash \text{case } x \text{ of } [\lambda c. [c_1(a) \rightarrow c_2(a)] [c_2(b) \rightarrow c_1(b)]] : B \vee A}{\vdash \lambda x. \text{case } x \text{ of } [\lambda c. [c_1(a) \rightarrow c_2(a)] [c_2(b) \rightarrow c_1(b)]] : A \vee B \Rightarrow B \vee A} \Rightarrow_I \\
 \frac{\vdash \lambda x. \text{case } x \text{ of } [\lambda c. [c_1(a) \rightarrow c_2(a)] [c_2(b) \rightarrow c_1(b)]] : A \vee B \Rightarrow B \vee A \quad \vdash P : B \vee A \Rightarrow A \vee B}{\vdash (P, P) : (A \vee B) \Leftrightarrow (B \vee A)} \wedge_I
 \end{array}$$