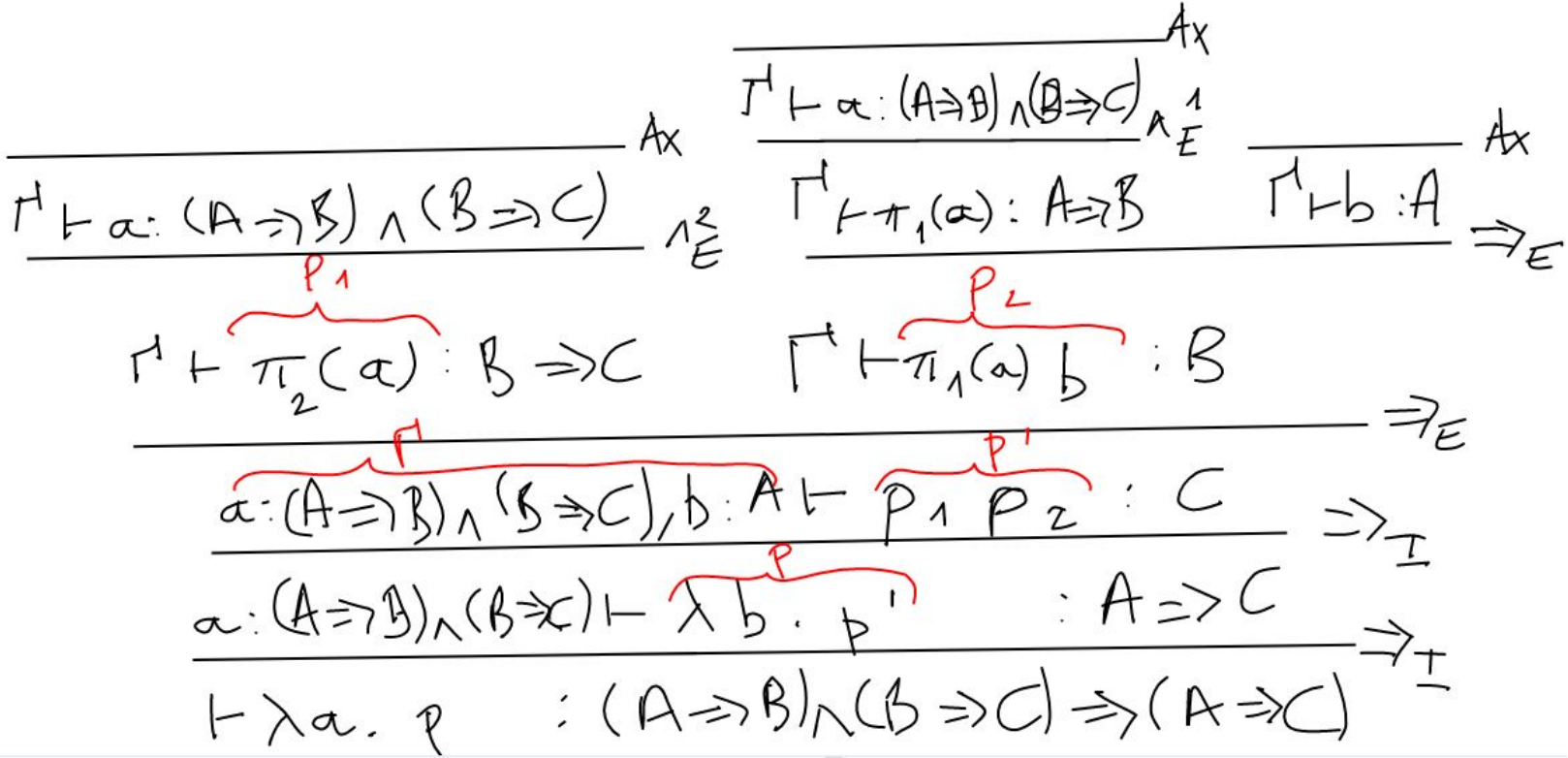
# Exercice 1, Question 1a

$$\dfrac{\dfrac{}{A \vdash A}\,Ax}{\vdash A \Rightarrow A}\,\Rightarrow_I$$

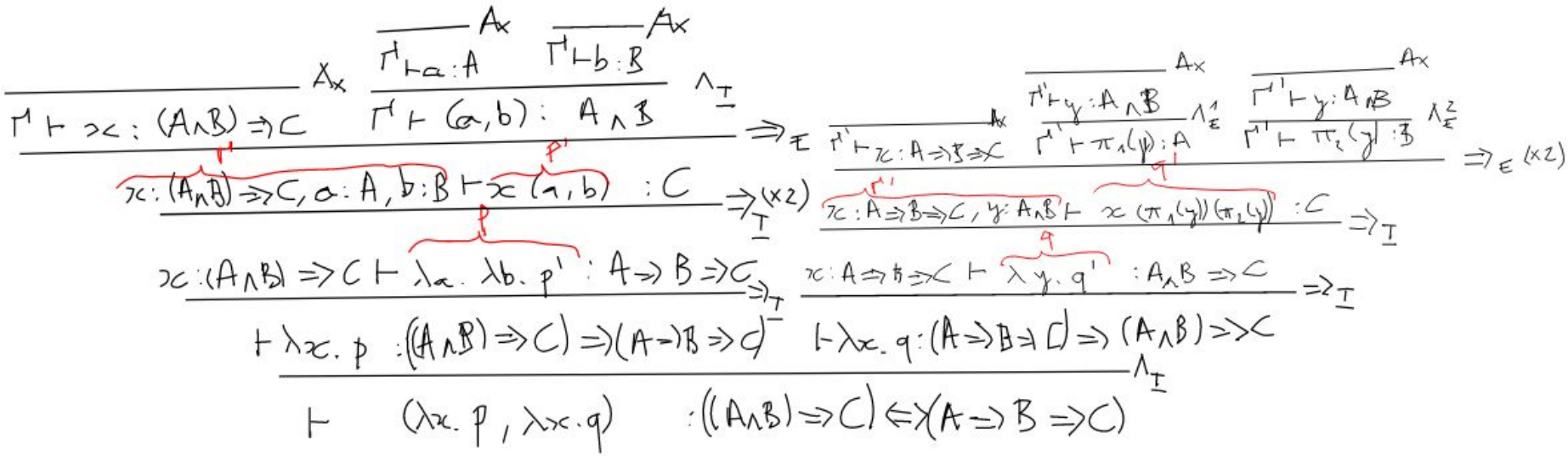
$$\dfrac{\dfrac{}{a : A \vdash a : A}\,Ax}{\vdash \lambda a . a : A \Rightarrow A}\,\Rightarrow_I$$

# Exercice 1, Question 1b

$\lambda a.\ \lambda b.\ (\pi_2(a))\,((\pi_1(a))\,b)$

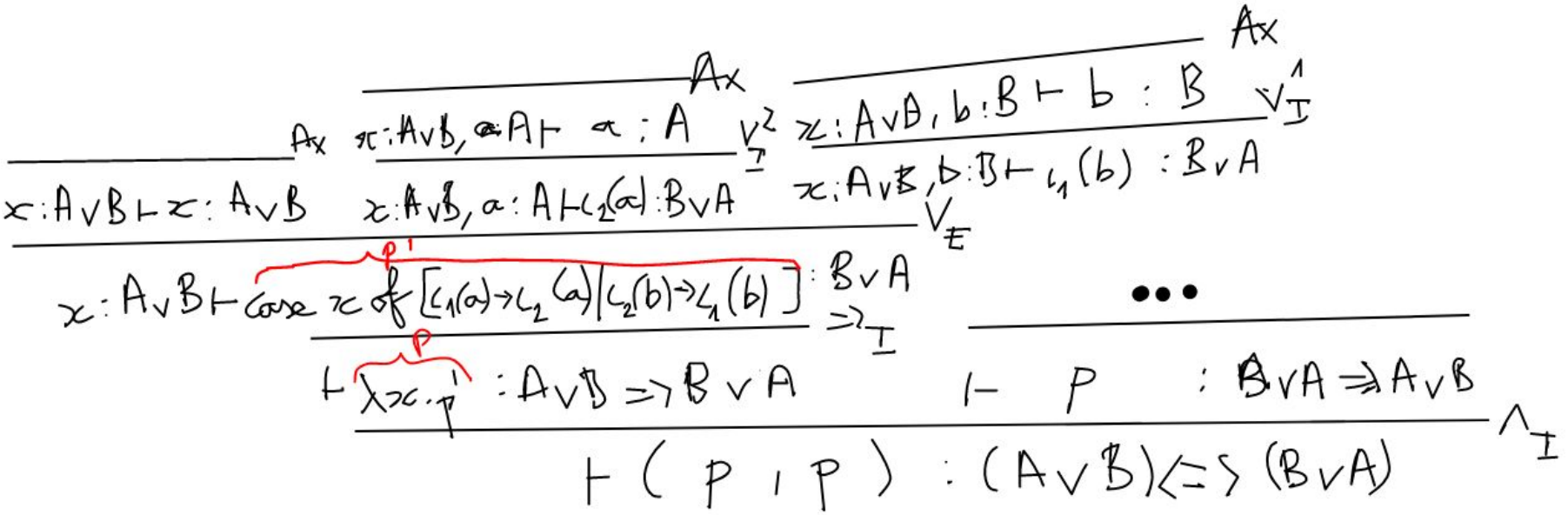


# Exercice 1, Question 2



$$(\lambda x.\lambda a.\lambda b.\, x\,(a,b),\ \lambda x.\lambda y.\, x(\pi_1(y))(\pi_2(y)))$$

# Exercice 1, Question 3d



# Exercice 1, Question 4a

$$\left(\lambda x.\, \pi_1(x),\ \lambda x.\, (x, ())\right)$$

$$\dfrac{\dfrac{\dfrac{}{x : A \wedge T \vdash x : A \wedge T}\,Ax}{x : A \wedge T \vdash \pi_1(x) : A}\,\wedge_E^1}{\vdash \lambda x.\, \pi_1(x) : A \wedge T \Rightarrow A}\,\Rightarrow_I \quad p \qquad \dfrac{\dfrac{\dfrac{}{x : A \vdash x : A}\,Ax \qquad \dfrac{}{x : A \vdash () : T}\,T_I}{x : A \vdash (x, ()) : A \wedge T}\,\wedge_I}{\vdash \lambda x.\, (x, ()) : A \Rightarrow A \wedge T}\,\Rightarrow_I \quad q$$

$$\dfrac{\vdash \lambda x.\, \pi_1(x) : A \wedge T \Rightarrow A \qquad \vdash \lambda x.\, (x, ()) : A \Rightarrow A \wedge T}{\vdash (p, q) : A \wedge T \Leftrightarrow A}\,\wedge_I$$

# Exercice 1, Question 4c