Discrete and Algorithmic Geometry

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Lattice polygons and the number $2i + 7$

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Referee report

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Summary

The aim of this paper is to work on the relation between the area a, the lattice points on the boundary b, and those in the interior of a lattice polygon, i. The main bound announced is the already known $b \leq 2i + 7$; and they indicate three different ways to get to prove it: as it was first proved by Scoot (see references in the paper), by induction on the number of interior lattice points together with a technique to relate a polygon with an other one contained in it (which is called *clipping off vertices*), and with algebraic geometry -in particular, toric geometry.

It also introduces the notion of *level* of a polygon: the number of *onion skins* it has, defined as the number of inner polygons that it contains (the first onion skin of a polygon P is defined as $P^{(1)} = \text{conv}(\text{int}(P))$, and the set of onion skins is the succession of recursively finding them). With this new concept it offers an improvement on the bounds of the relation between the parameters (a, b, i), by taking this parameter l also into account. These new bounds, which clearly improve the existing ones, are proved in two ways analogous to the first two proofs of the original bound. It is said that the algebraic geometry proof is being developed by exploding the relation between lattice geometry and algebraic geometry that toric geometry defines.

Personal opinion

In my opinion, this paper is presented in an interesting, clear way, with a helpful structure and transparent line of development.

Already from the introduction it is clear for the reader where is it going and which tools will be used. There are even some suggested exercises for the reader to get used to the least intuitive concepts used, such as lattice equivalence of polygons.

The theory on *onion skins* appears very interesting to me, and it is really intuitive to follow, and the figures are really helpful to get the idea.

At some points the deduction of the inequalities that follow from a bunch of them may be hard to see at a glance, but it helps to keep the paper shorter, so that it seems correct in most cases.

The only thing I miss in the paper is further information on toric geometry if the paper is faced to a reader who might be unfamiliar with the topic, since the paper is almost self-contained in every other field used.

Minor comments

There are, however, some misspellings and formal mistakes to take into account:

- In the paper title, maybe the formula $2i + 7$ should be in *math mode*.
- In the abstract, when it says *"... with area a, and p respectively..."* that p should instead be a b.
- In the statement of *exercice 3*, broken formulas and out-of-margin formulas should be fixed.
- In section 0.4, page 3, there is a missing reference.
- In page 9, when the procedure of clipping off vertices is explained, it might be useful to reference the corresponding figures when necessary, specially since figure 13 is not in the same page.
- Still in page 9, the heading of theorem 7 should be in the following page.
- In the first reference, the title of the book should be *... du group de Cremona*, with the corresponding capital letter.