Discrete and Algorithmic Geometry

Julian Pfeifle, UPC, 2013

Sheet 2

due on Monday, November 18, 2013

Reading

(1)

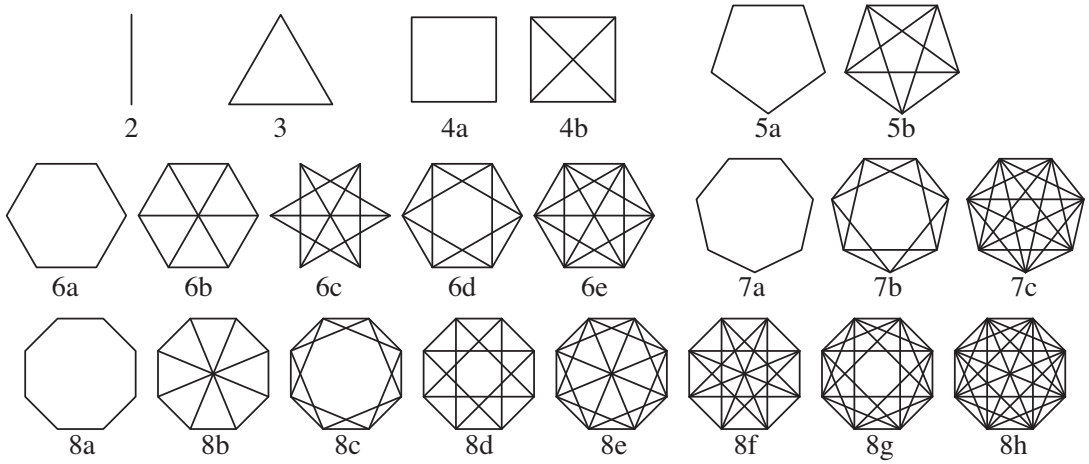
Writing

(1) Show that all induced cycles of length 3, 4 and 5 in the graph of a simple d-polytope P are graphs of 2-faces of P. Conclude that the Petersen graph is not the graph of any polytope (of any dimension).

(*Hint for* 5*-cycles:* First show this for $d = 3$, then prove that any 5-cycle in a simple polytope is contained in some 3-face, and use that a face of a simple polytope is simple.)

(2) Let $n \in \mathbb{N}$ be an integer and S denote a subset of $\{1, 2, \ldots, \lfloor \frac{n}{2} \rfloor\}$. The *circulant graph* $\Gamma_n(S)$ is the graph whose vertex set is $\mathbb{Z}_n$, and whose edge set is the set of pairs of vertices whose difference lies in $S \cup (-S)$.

The following figure collects all connected circulant graphs on up to 8 vertices. Determine the *polytopality range* of each of these graphs, i.e., the set of integers d such that the graph in question is the graph of a d-dimensional polytope.



Software

(1)