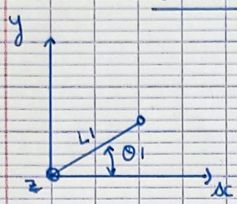
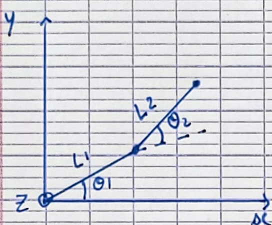
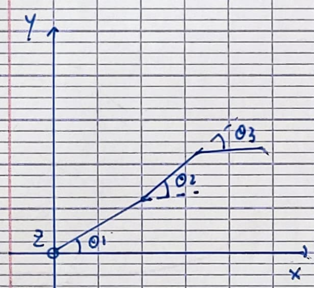
Cinématique Direct



$$z_1 = 0$$
$$x_1 = L_1 \times \cos(\theta_1)$$
$$y_1 = L_1 \times \sin(\theta_1)$$

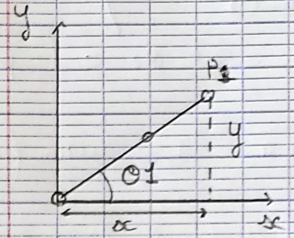


$$z_2 = L_2 \sin(\theta_2)$$
$$x_2 = \cos(\theta_1)\left(L_1 + L_2 \cos(\theta_2)\right)$$
$$y_2 = \sin(\theta_1)\left(L_1 + L_2 \cos(\theta_2)\right)$$



$$z_3 = \sin(\theta_2)\left(L_2 + \sin(\theta_3 . L_3)\right)$$
$$x_3 = \cos(\theta_1)\left(L_1 + L_2 \cos(\theta_2) + L_3 \cos(\theta_2 + \theta\right.$$
$$y_3 = \sin(\theta_1)\left(L_1 + L_2 \cos(\theta_2) + L_3 \cos(\theta_2 + \theta_3)\right)$$

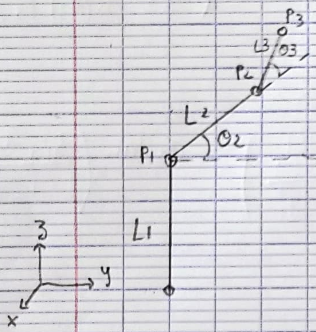
Cinématique inverse :



On trouve à l'aide des formule trigonométriques ;

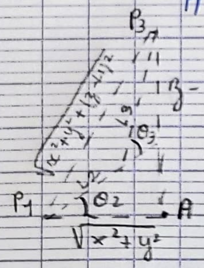
$$\theta_1 = \arctan\left(\frac{x}{y}\right)$$

On cherche θ_2, θ_3 :



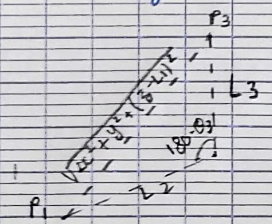
Exemple d'un schéma du bras :

Voici les différents triangle que l'on obtient entre p1, p2 et p3



on se place dans le triangle P1 P2 P3 :

$\Rightarrow$



On utilise les formules de Al-Kashi, on trouve :

$$x^2 + y^2 + (z - L_1)^2 = L_2^2 + L_3^2 - 2 L_2 L_3 \cos(180 - \theta_3)$$

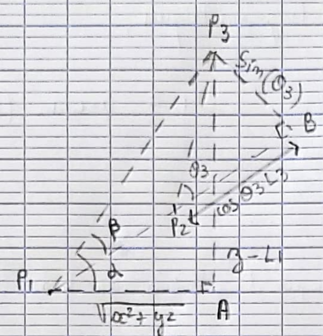
or, $\cos(\pi - x) = -\cos(x)$

$$\Leftrightarrow \quad = L_2^2 + L_3^2 + 2 L_2 L_3 \cos(\theta_3)$$

$$\Leftrightarrow \quad \cos(\theta_3) = \frac{x^2 + y^2 + (z - L_1)^2 - L_2^2 - L_3^2}{2 L_2 L_3}$$

$$\Rightarrow \theta_3 = \arccos\left(\frac{x^2+y^2+(z-L_1)^2-L_2^2-L_3^2}{2L_2L_3}\right)$$

Il nous reste à trouver θ_2, Soit $\theta_2 = \alpha - \beta$, on a :



on a : $\alpha = \arctan\left(\frac{z-L_1}{\sqrt{x^2+y^2}}\right)$

Avec le triangle P_1P_3B on obtient :

$$\beta = \arctan\left(\frac{\sin(\theta_3)L_3}{L_2+\cos(\theta_3).L_3}\right)$$

Et donc $\theta_2 = \arctan\left(\frac{z-L_1}{\sqrt{x^2+y^2}}\right) - \arctan\left(\frac{\sin(\theta_3-L_3)}{L_2+\cos(\theta_3)L_3}\right)$