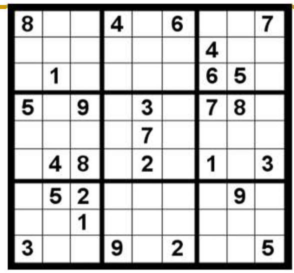
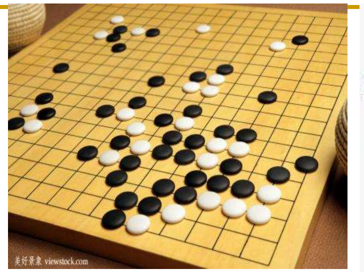
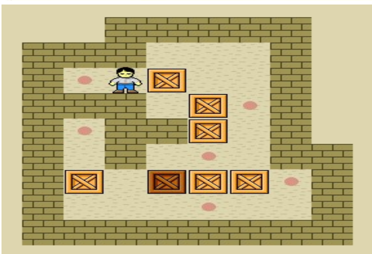
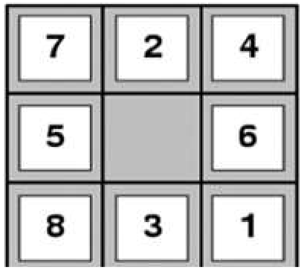
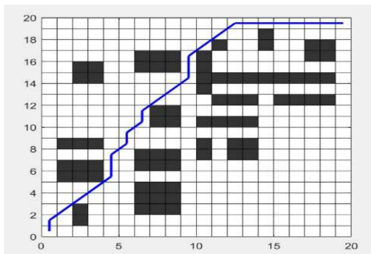
Problem Solving
- by Searching

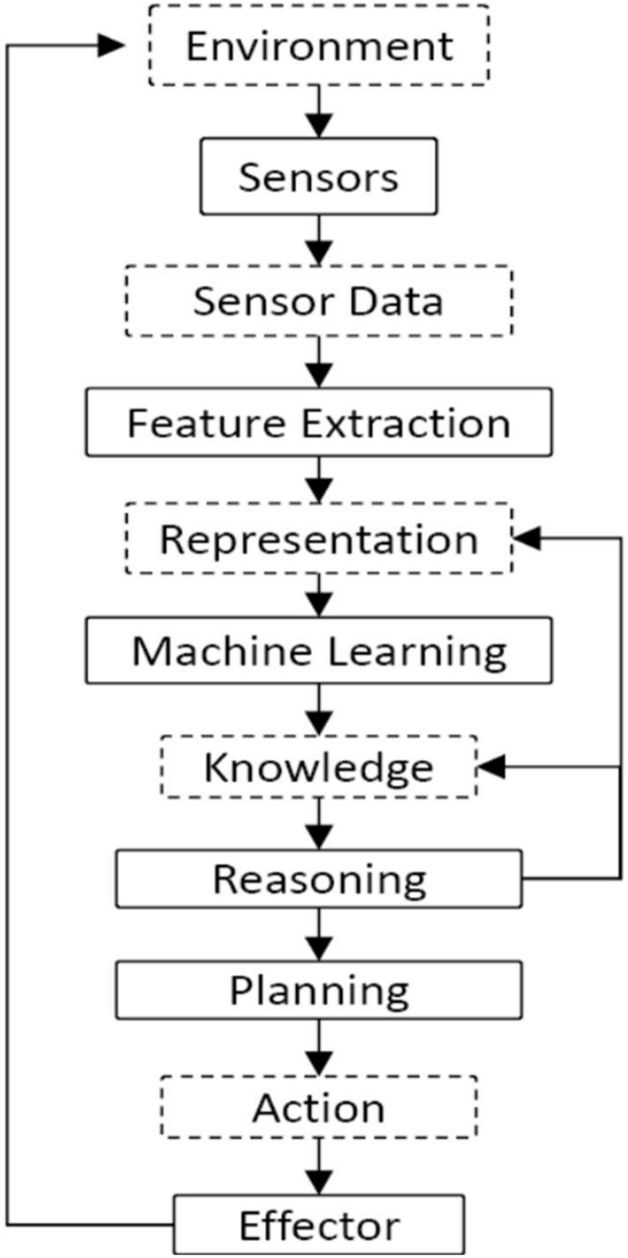



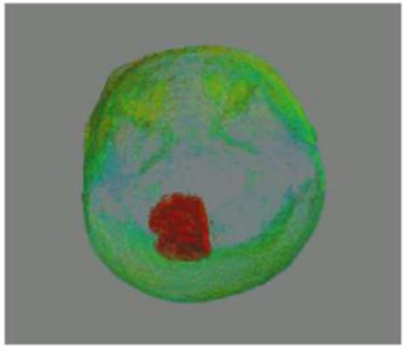


Outline

- Problem Solving by Searching
- Uninformed Search Strategies
- Informed (Heuristic) Search Strategies



Formal tasks: Playing board games, card games. Solving puzzles, mathematical and logic problems.



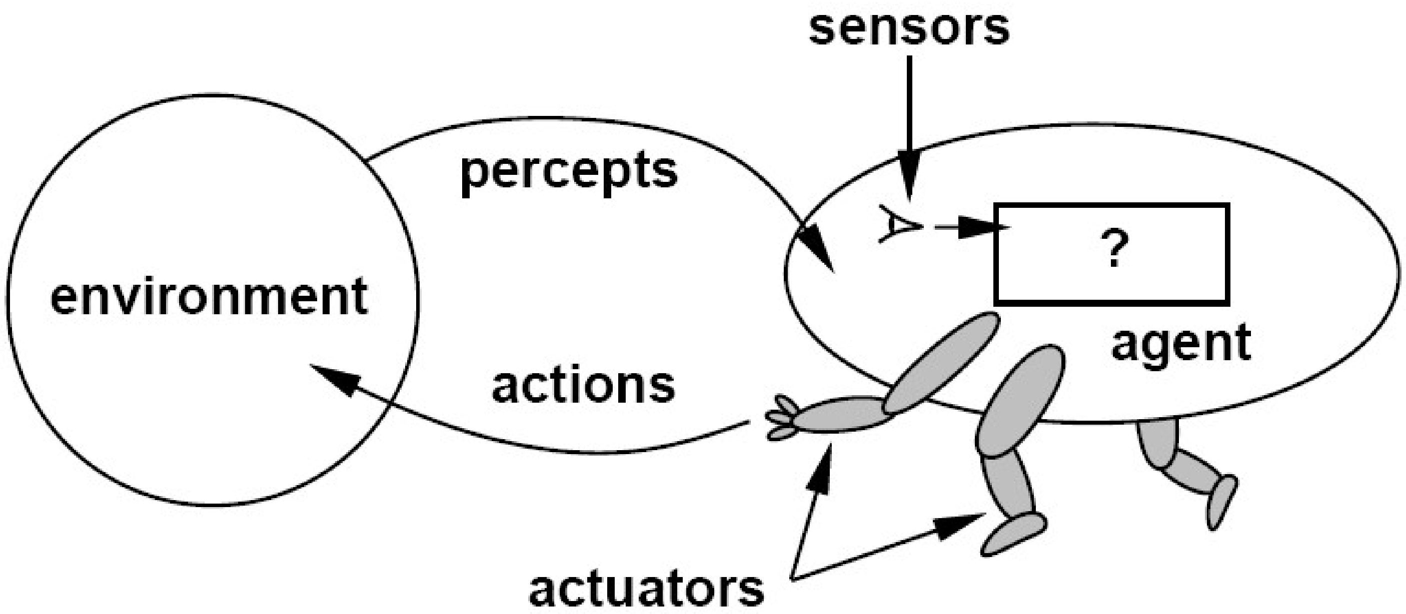
Expert tasks: Medical diagnosis, engineering, scheduling, computer hardware design.

Mundane tasks: Everyday speech, written language, perception, walking, object manipulation.

Human tasks: Awareness of self, emotion, imagination, morality, subjective experience, high-level-reasoning, consciousness.

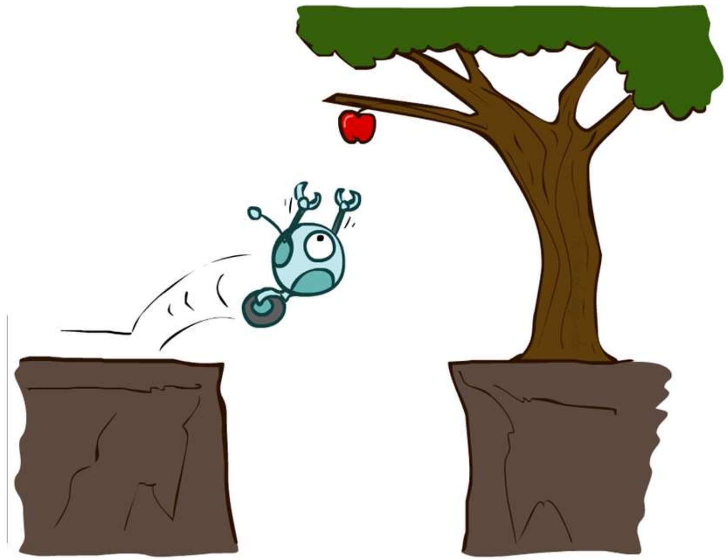
Agent

- An agent is anything that can be viewed as perceiving its environment through sensors and acting upon that environment through actuators.



Types of Agents

- **Reflex agents:**
  - Choose action based on current percept (and maybe memory)
  - May have memory or a model of the world's current state
  - Do not consider the future consequences of their actions
  - Consider how the world IS

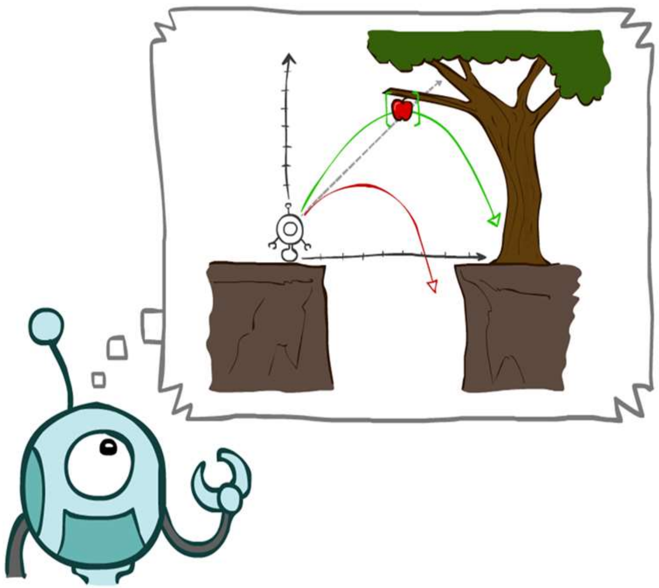


- Can a reflex agent be rational?

Types of Agents

❖ **Planning agents:**

- Ask “what if”
- Decisions based on (hypothesized) consequences of actions
- Must have a model of how the world evolves in response to actions
- Must formulate a goal (test)
- Consider how the world WOULD BE



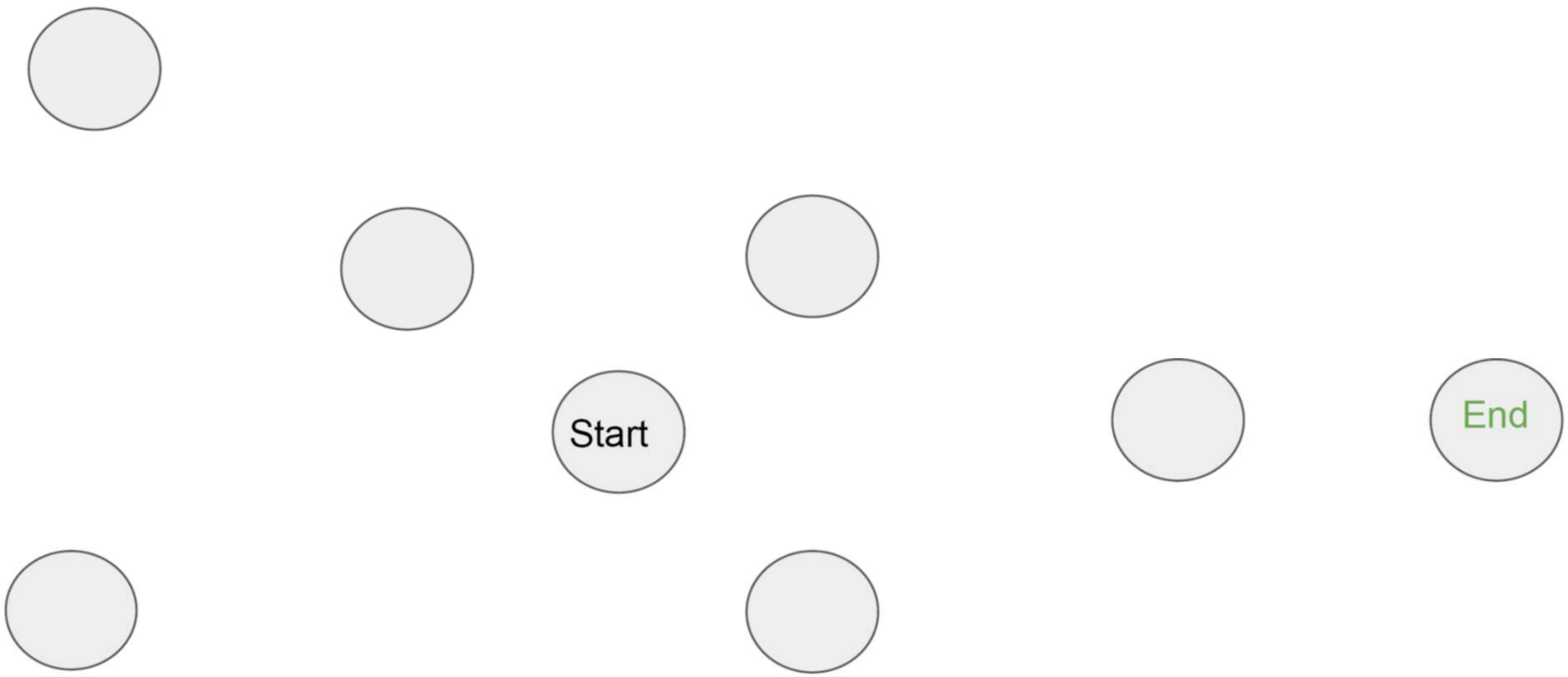
Search

- We will consider the problem of designing goal-based agents in fully observable, deterministic, discrete, static environments
  - The agent must find a *sequence of actions* that reaches the goal
  - The **performance measure** is defined by (a) reaching the goal and (b) how "expensive" the path to the goal is
  - We are focused on the process of finding the solution; while executing the solution, we assume that the agent can safely ignore its percepts (**open-loop system**)

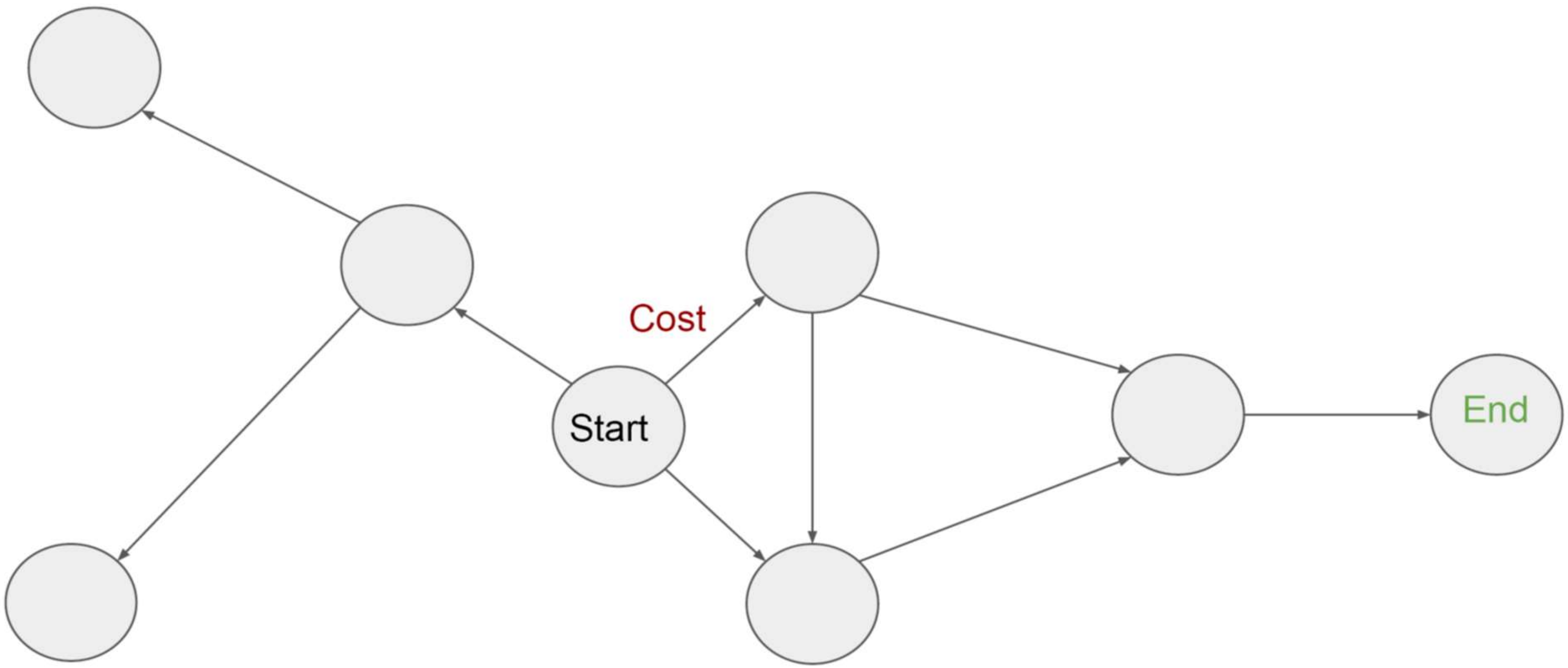
Search


How to find shortest path
to my destination?
HOME
DESTINATION

Search



Search



Search problem components

- Initial state
- Goal state
- Actions
- Transition model
  - What state results from performing a given action in a given state?
- Path cost
  - Assume that it is a sum of nonnegative *step costs*
- The optimal solution is the sequence of actions that gives the *lowest* path cost for reaching the goal

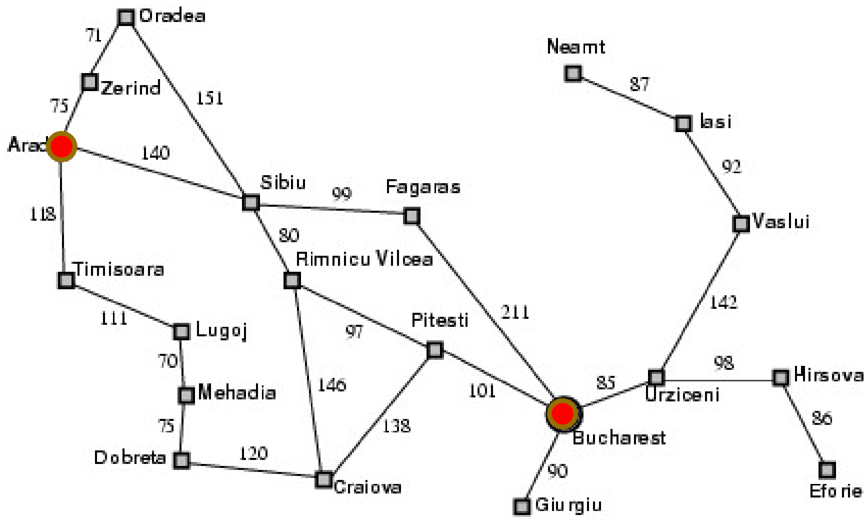
State space

- The initial state, actions, and transition model define the **state space** of the problem.
  - **The set of all states reachable** from the initial state by any sequence of actions
  - Can be represented as a **directed graph** where the nodes are states and the links between nodes are actions.

Example: Romania

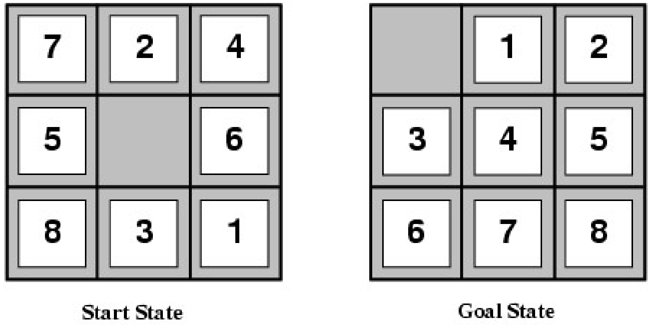
- On vacation in Romania; currently in Arad
- Flight leaves tomorrow from Bucharest

- Initial state
  - Arad
- Actions
  - Go from one city to another
- Transition model
  - If you go from city A to city B, you end up in city B
- Goal state
  - Bucharest
- Path cost
  - Sum of edge costs (total distance traveled)



Example: The 8-puzzle

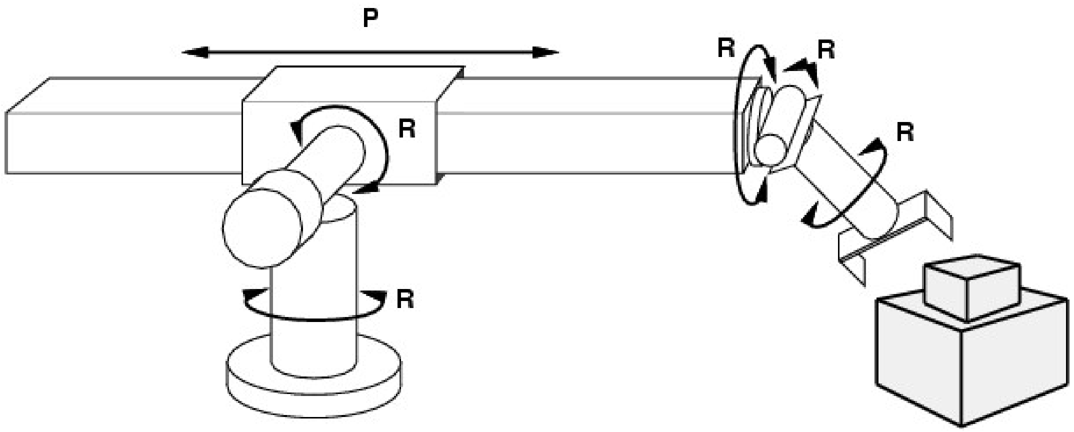
- states? locations of tiles
  - 8-puzzle: 181,440 states (9!/2)
  - 15-puzzle: ~1.3 trillion states
  - 24-puzzle: ~10^{25} states



- actions? move blank left, right, up, down
- Initial and goal states? given
- action costs? 1 per move

- Note: optimal solution of *n*-Puzzle family is NP-hard

Example: Robot motion planning

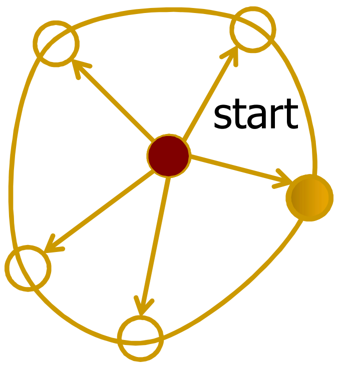


- States
  - Real-valued joint parameters (angles, displacements)
- Actions
  - Continuous motions of robot joints
- Goal state
  - Configuration in which object is grasped
- Path cost
  - Time to execute, smoothness of path, etc.

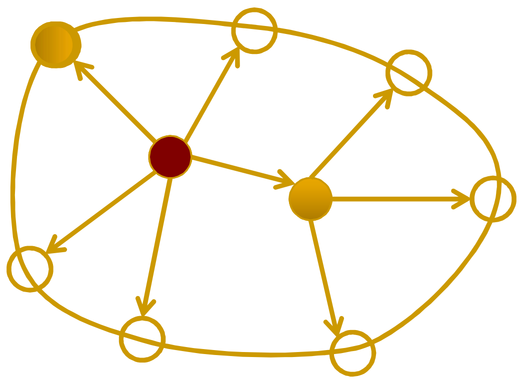
Search: Basic idea

- Let’s begin at the start state and expand it by making a list of all the possible successor states
- Maintain a frontier or a list of unexpanded states
- At each step, pick a state from the frontier to expand
- Keep going until you reach a goal state
- Try to expand as few states as possible

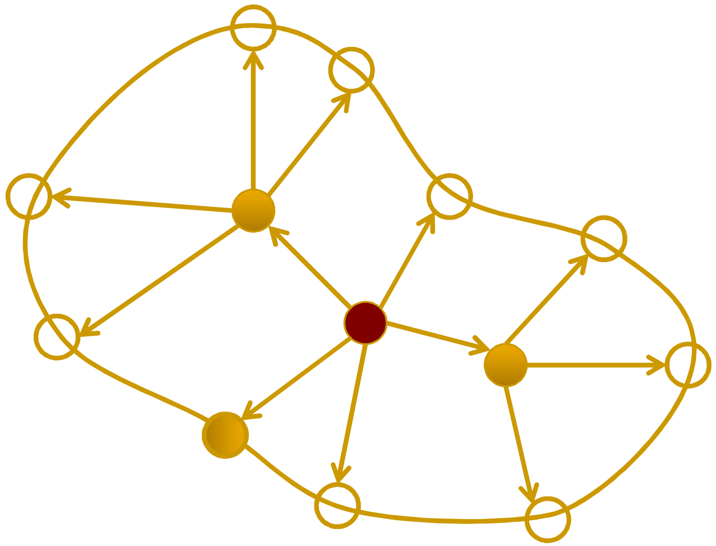
Search: Basic idea



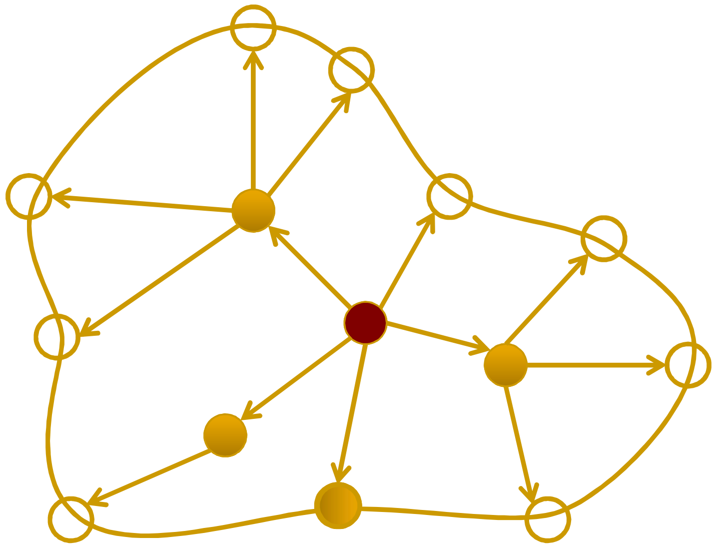
Search: Basic idea



Search: Basic idea



Search: Basic idea



General Tree-like Search

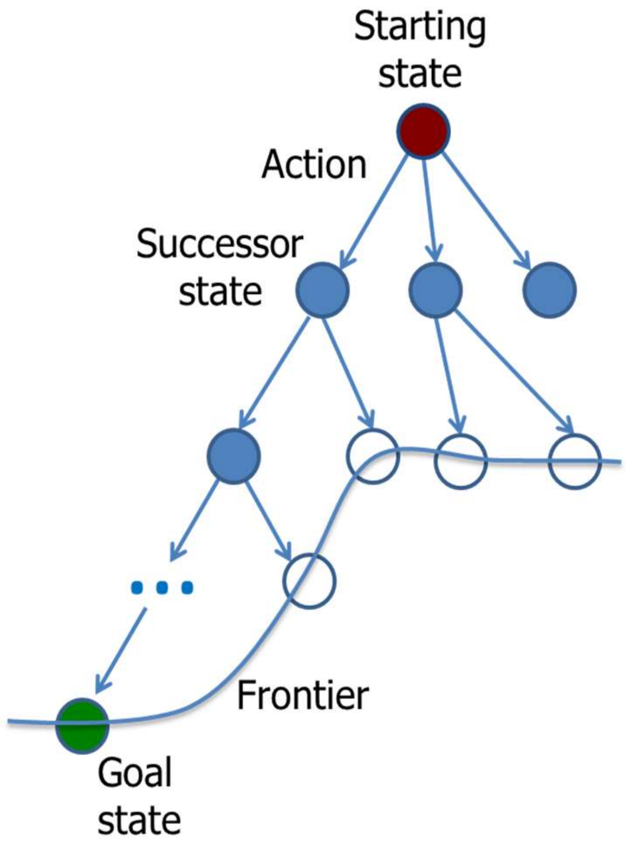
```
function TREE-SEARCH( problem, strategy) returns a solution, or failure
    initialize the search tree using the initial state of problem
    loop do
        if there are no candidates for expansion then return failure
        choose a leaf node for expansion according to strategy
        if the node contains a goal state then return the corresponding solution
        else expand the node and add the resulting nodes to the search tree
    end
```

- Important ideas:
  - Fringe/Frontier/Open List(as queue)
  - Expansion
  - Exploration strategy

- Main question: which fringe nodes to explore?

Search tree

- "What if" tree of sequences of actions and outcomes
  - ✓ The root node corresponds to the starting state
  - ✓ The children of a node correspond to the successor states of that node's state
  - ✓ A path through the tree corresponds to a sequence of actions
  - ✓ A solution is a path ending in the goal state
- Nodes vs. states
  - A state is a representation of the world, while a node is a data structure that is part of the search tree
  - A node has to keep a pointer to its parent, path cost, possibly other info

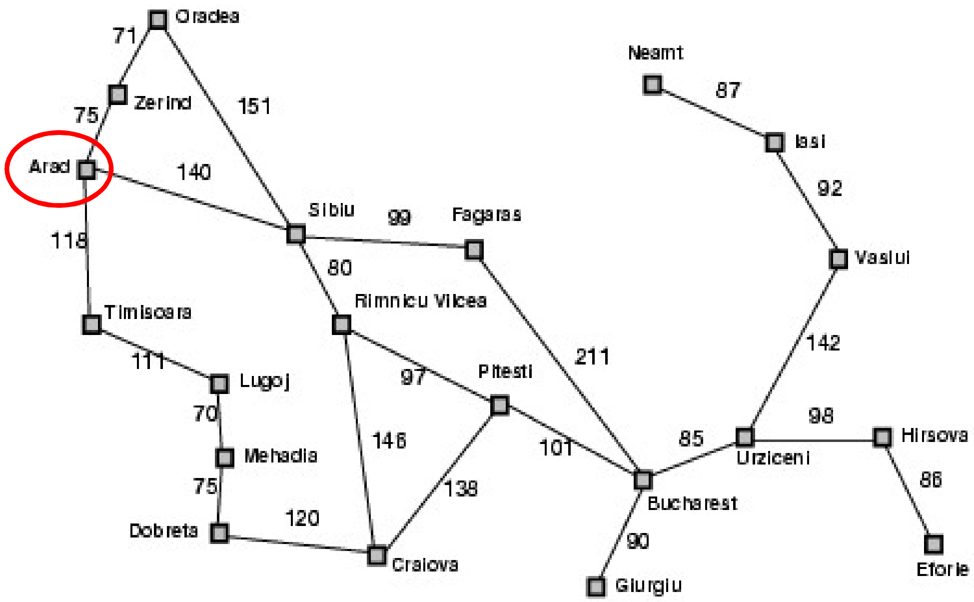


Tree-like Search Algorithm Outline

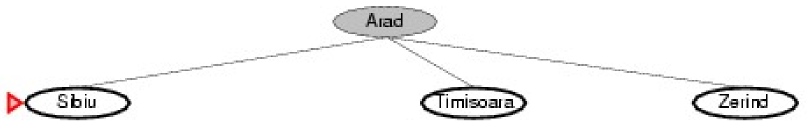
- Initialize the **frontier** using the **starting state**
- While the frontier is not empty
  - Choose a frontier node according to **search strategy** and take it off the frontier
  - If the node contains the **goal state**, return the solution
  - Else **expand** the node and add its children to the frontier

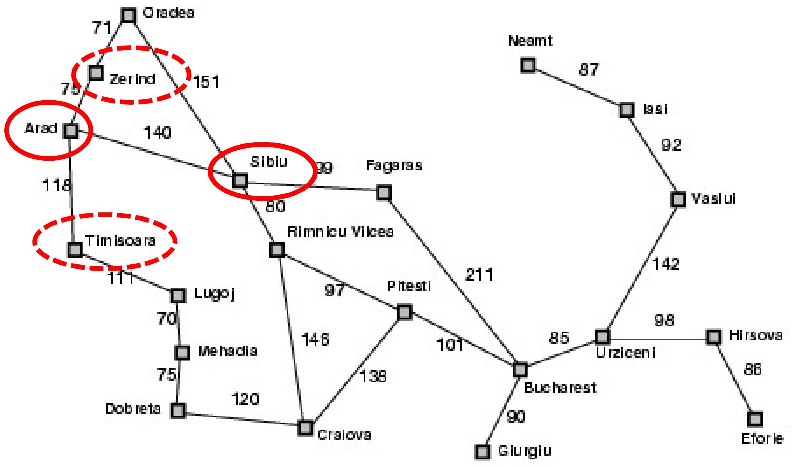
Tree-like search example

Start: Arad
Goal: Bucharest



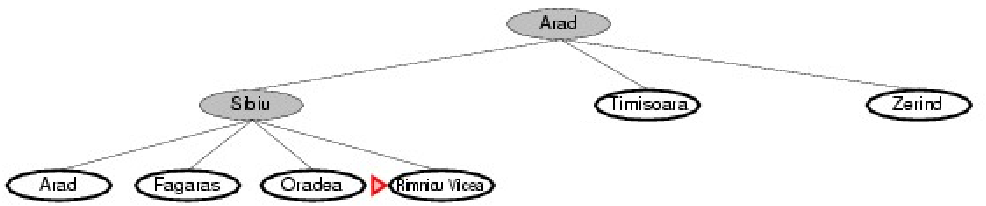
Tree-like search example



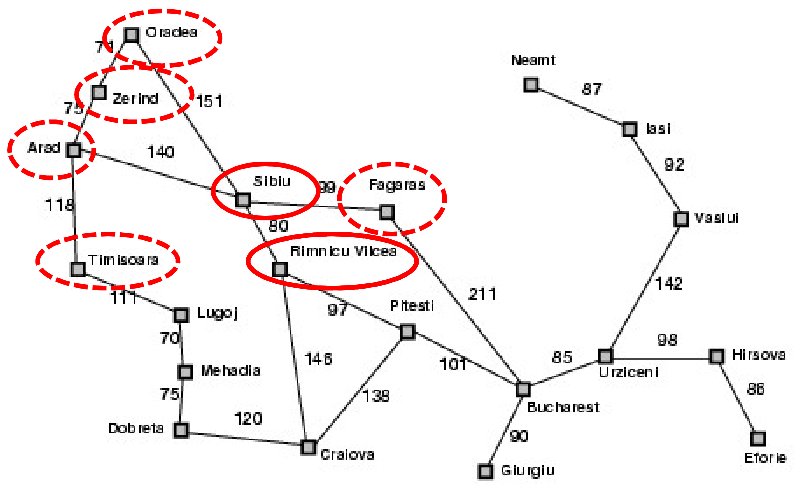


Start: Arad
Goal: Bucharest

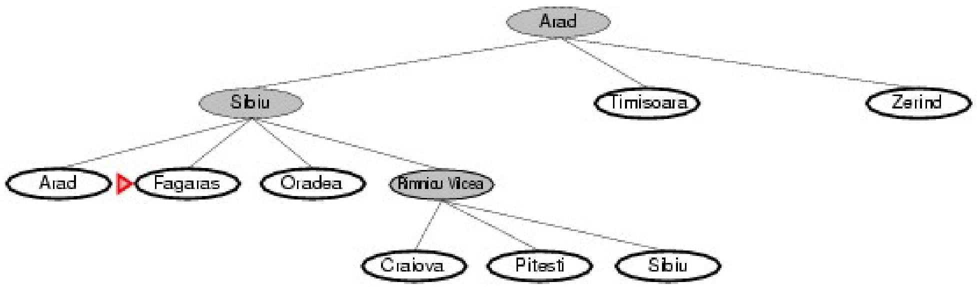
Tree-like search example



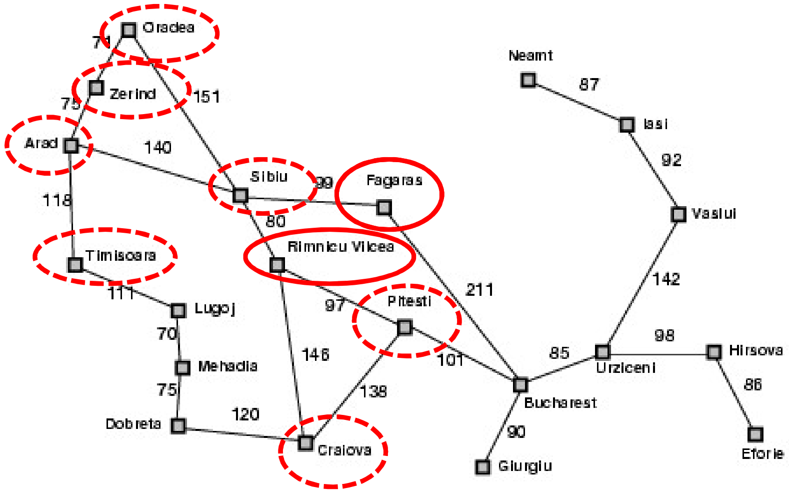
Start: Arad
Goal: Bucharest



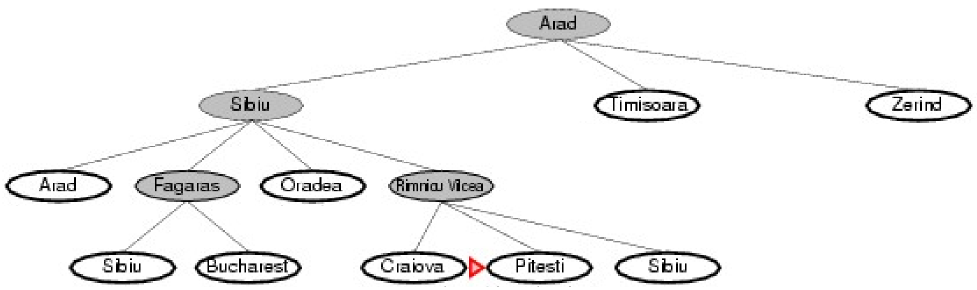
Tree-like search example



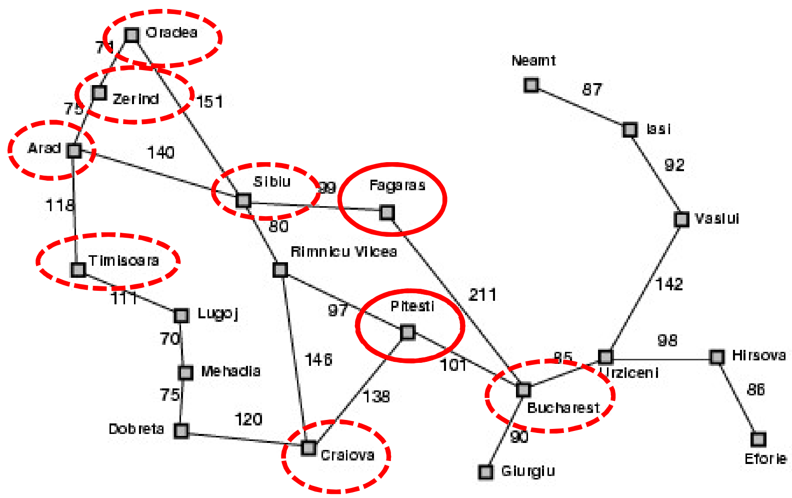
Start: Arad
Goal: Bucharest



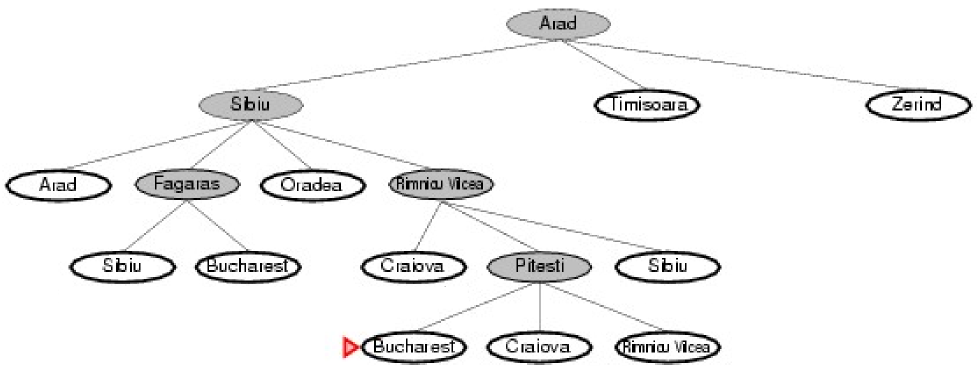
Tree-like search example



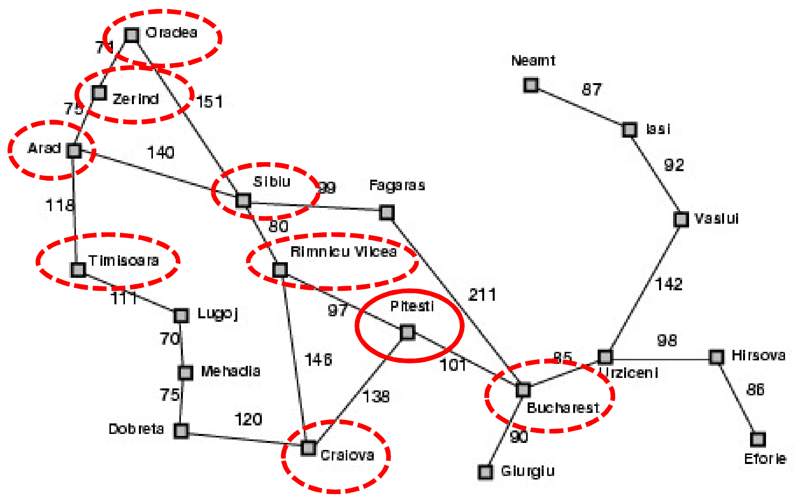
Start: Arad
Goal: Bucharest



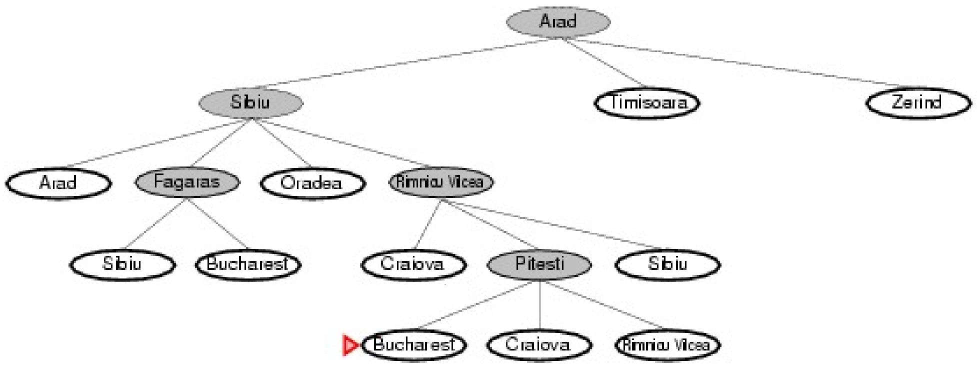
Tree-like search example



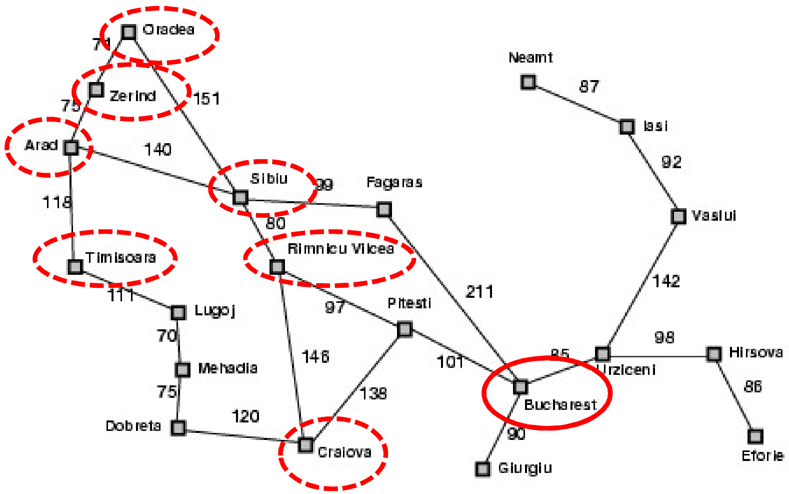
Start: Arad
Goal: Bucharest



Tree-like search example



Start: Arad
Goal: Bucharest

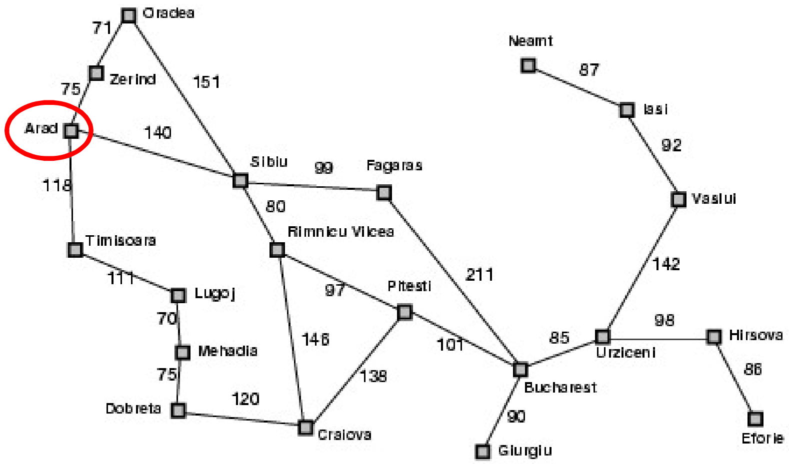


Handling repeated states

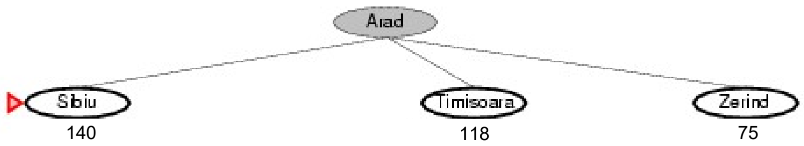
- To handle repeated states:
  - Every time you expand a node, add that state to the **explored set**; do not put explored states on the frontier again.
  - Every time you add a node to the frontier, check whether it has already existed in the frontier with a higher path cost. If yes, replace that node with the new one.

Search without repeated states

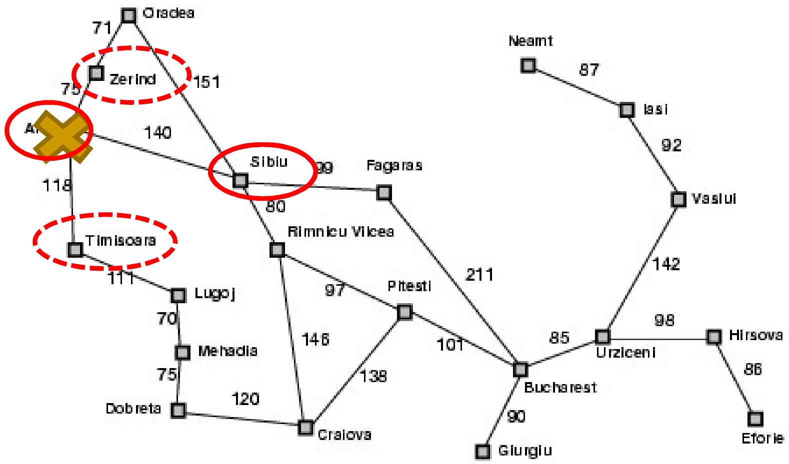
Start: Arad
Goal: Bucharest



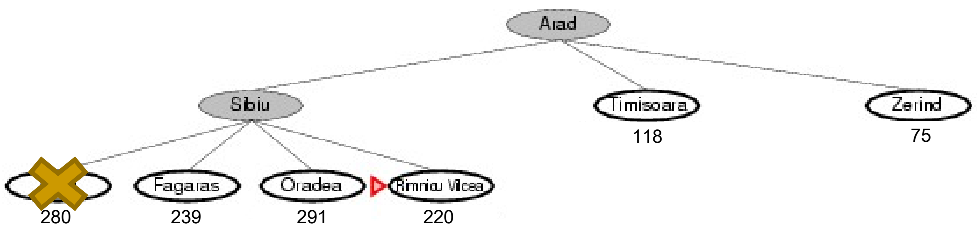
Search without repeated states

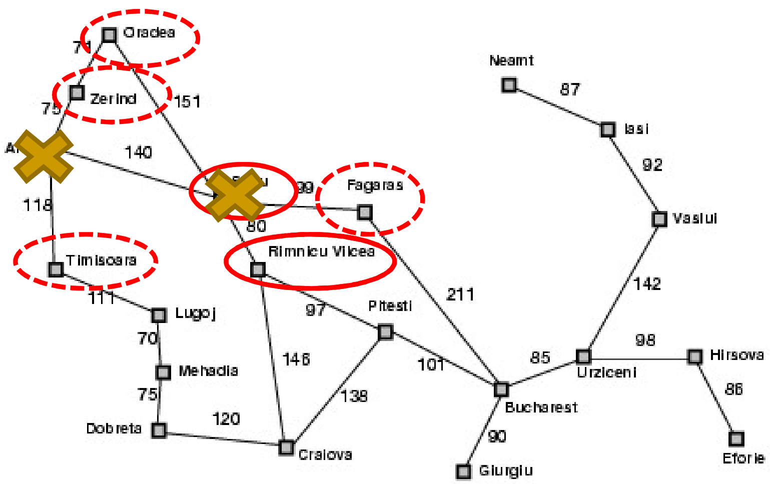


Start: Arad
Goal: Bucharest



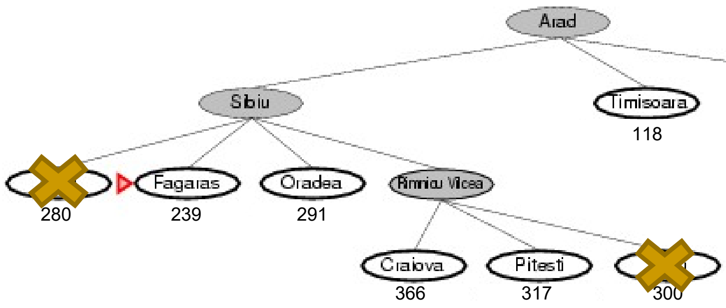
Search without repeated states

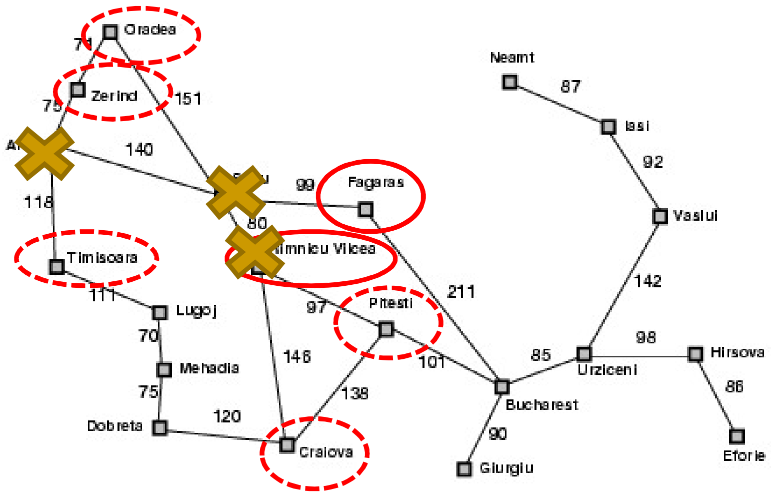




Start: Arad
Goal: Bucharest

Search without repeated states





Start: Arad
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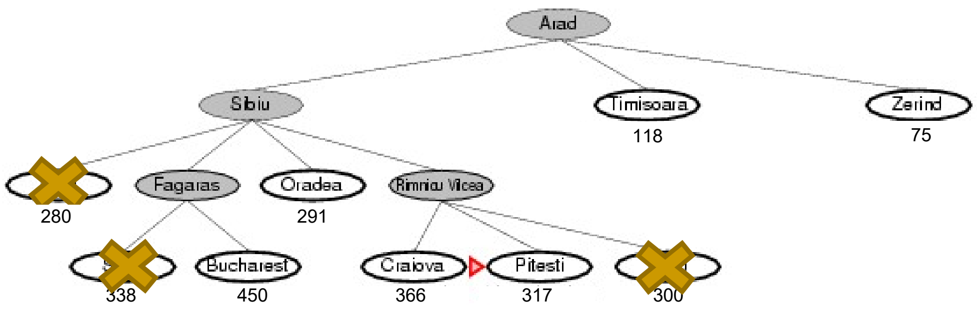
Search without repeated states



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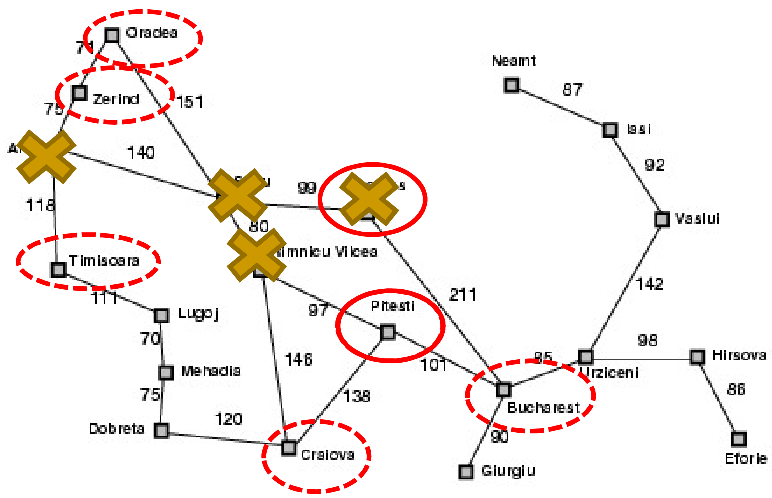
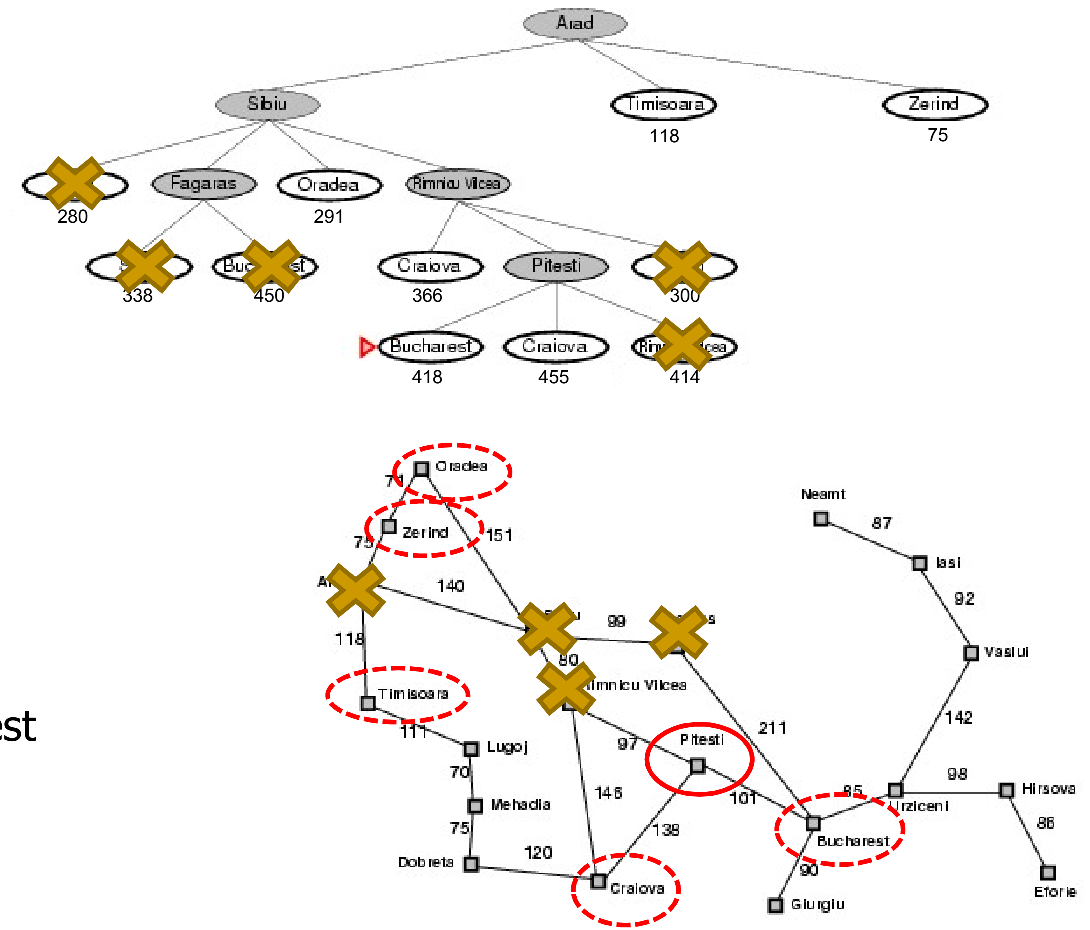
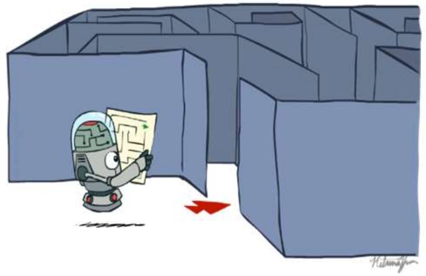


Search without repeated states



Start: Arad
Goal: Bucharest

Search



- **Search problem:**
  - States (configurations of the world)
  - Actions and costs（Plans have costs , sum of action costs)
  - Successor function (world dynamics)
  - Start state and goal test
- **Search algorithm:**
  - Systematically builds a search tree
  - Chooses an ordering of the fringe (unexplored nodes)
  - Optimal: find the least-cost plans

Search Strategy

- A search strategy is defined by picking the order of node expansion
  - **Uninformed** search (or blind search) strategies
    - given no clue about how close a state is to the goal(s)
  - **Informed** search (or heuristic search) strategies
    - use domain-specific hints about the location of goals
    - use an ***evaluation function*** to rank nodes and select the most promising one for expansion

Outline

- Problem-Solving by Searching
- Uninformed Search Strategies
- Informed (Heuristic) Search Strategies

Uninformed search strategies

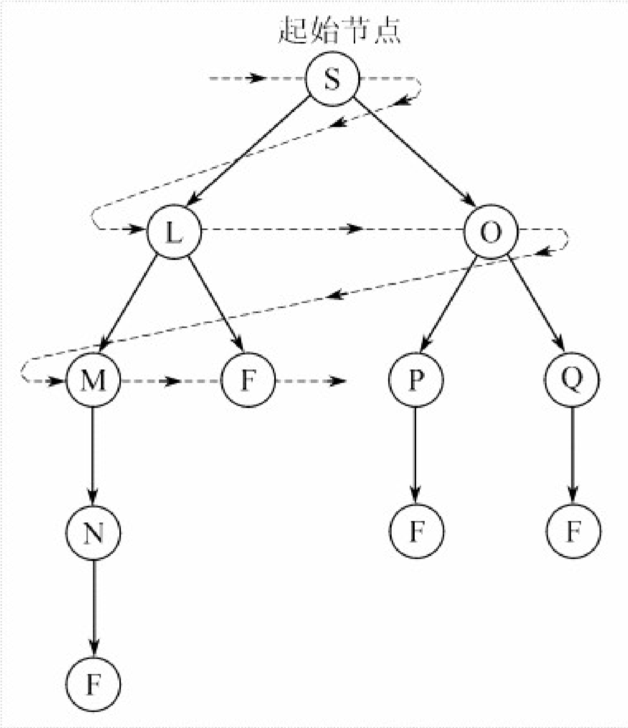
- Breadth-first search
- Depth-first search
- Iterative deepening search
- Uniform-cost search

宽度优先搜索（Breadth-first search， BFS）

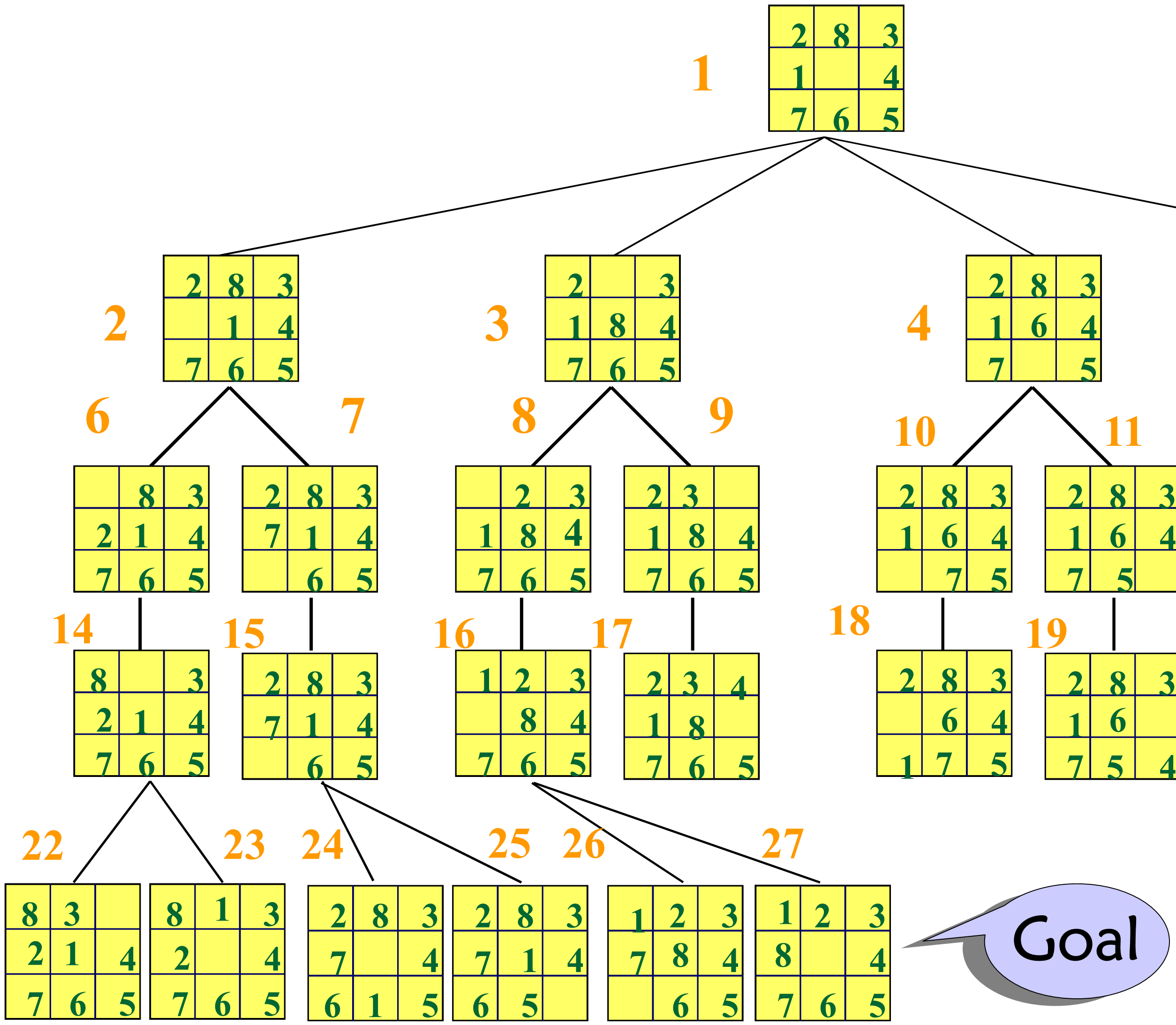
- 基本思想:

  从初始节点出发，逐层对节点进行扩展

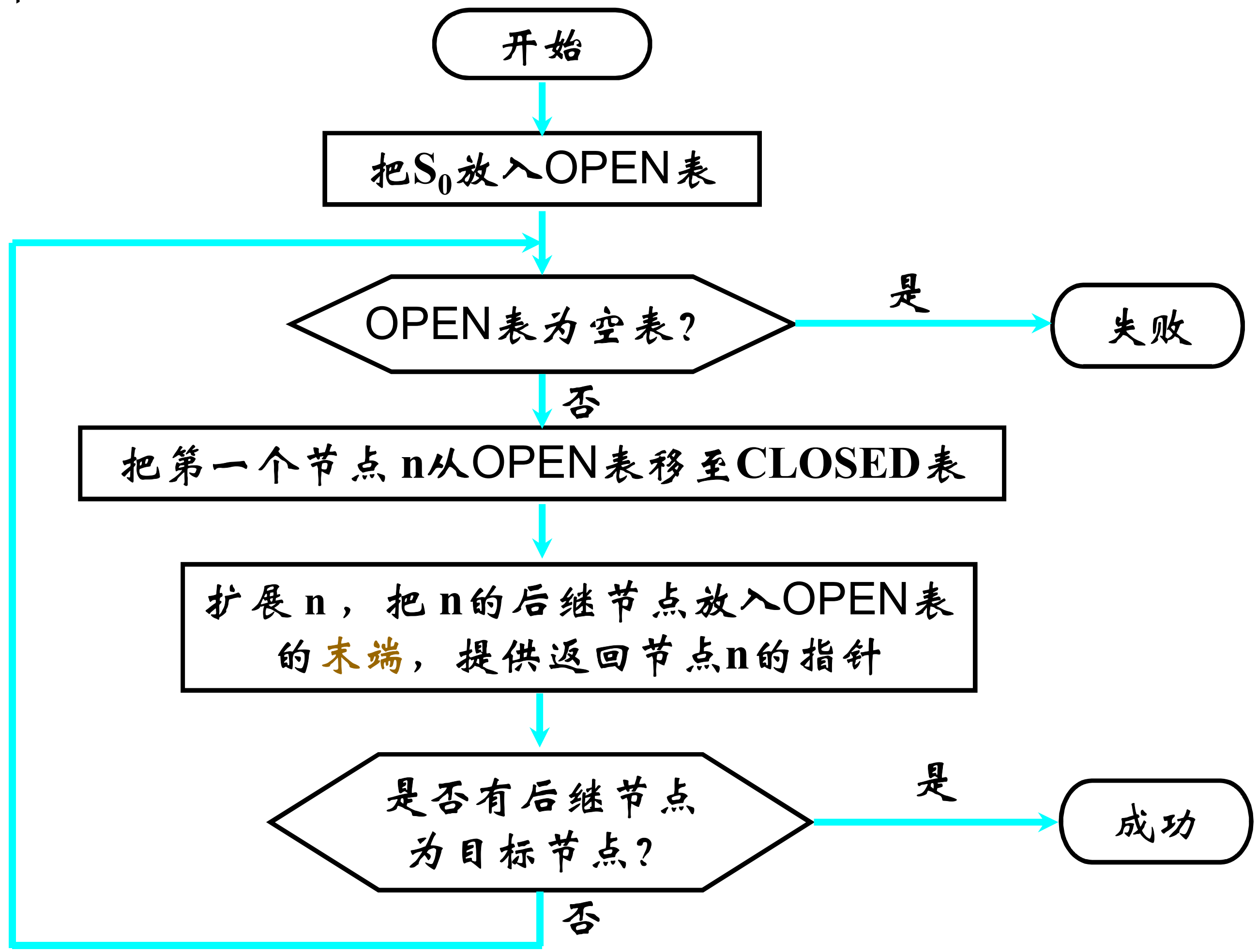
- 特点:
  - OPEN表是一个队列结构，即先进先出的数据结构
  - 搜索代价高
  - 有解时必能找到解



BFS



宽度优先算法框图


开始
把S_0放入OPEN表
OPEN表为空表?
是
失败
否
把第一个节点n从OPEN表移至CLOSED表
扩展n，把n的后继节点放入OPEN表的末端，提供返回节点n的指针
是否有后继节点为目标节点?
是
成功
否

搜索轨迹的记录

- OPEN表：
  用于存放刚生成的节点

状态节点	父节点

- CLOSED表：
  用于存放将要扩展或者已扩展的节点

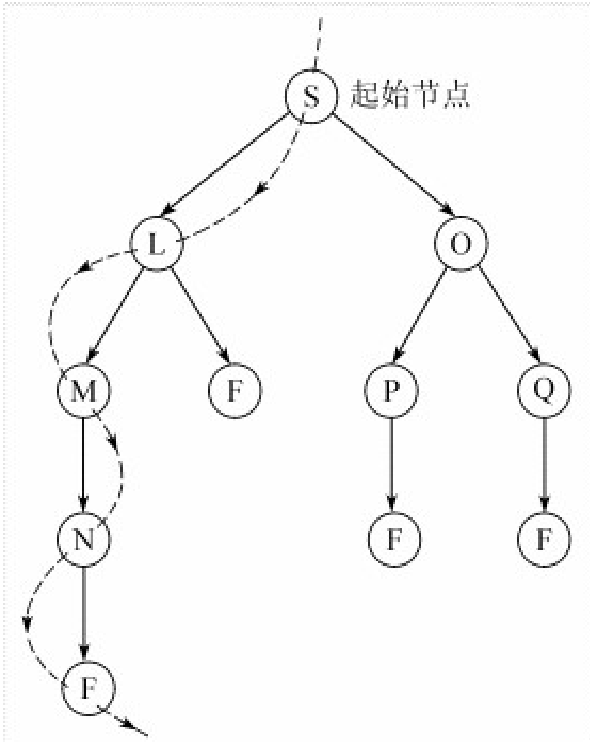
编号	状态节点	父节点

深度优先搜索
（Depth-first search，DFS）

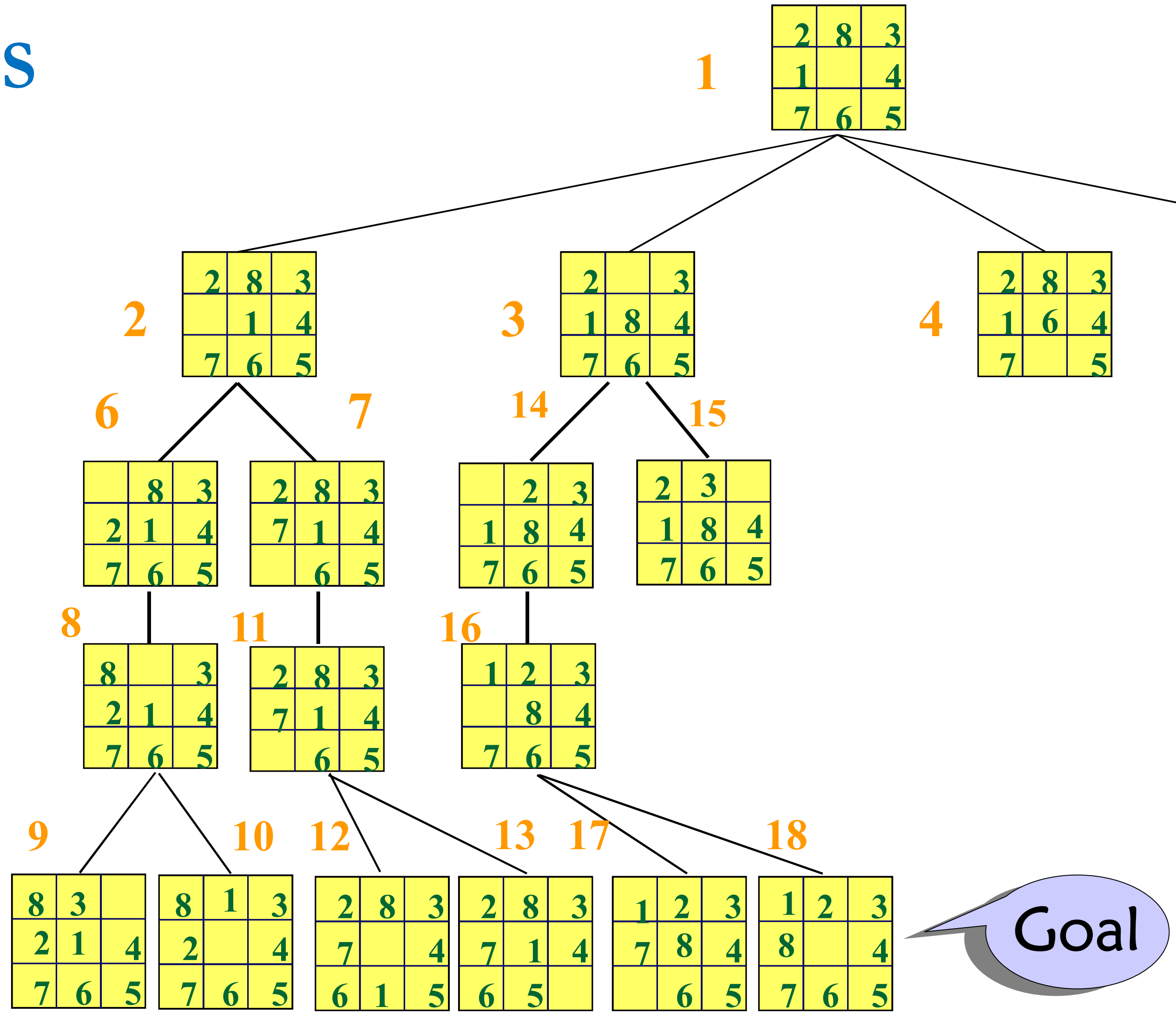
- 基本思想

　　首先扩展最新产生的(即最深的)节点

- 有界深度优先搜索 (Depth-limited search ,DLS)
- 特点
  - OPEN表是一个堆栈结构，即先进后出的数据结构
  - 效率较高
  - 无法保证找到解

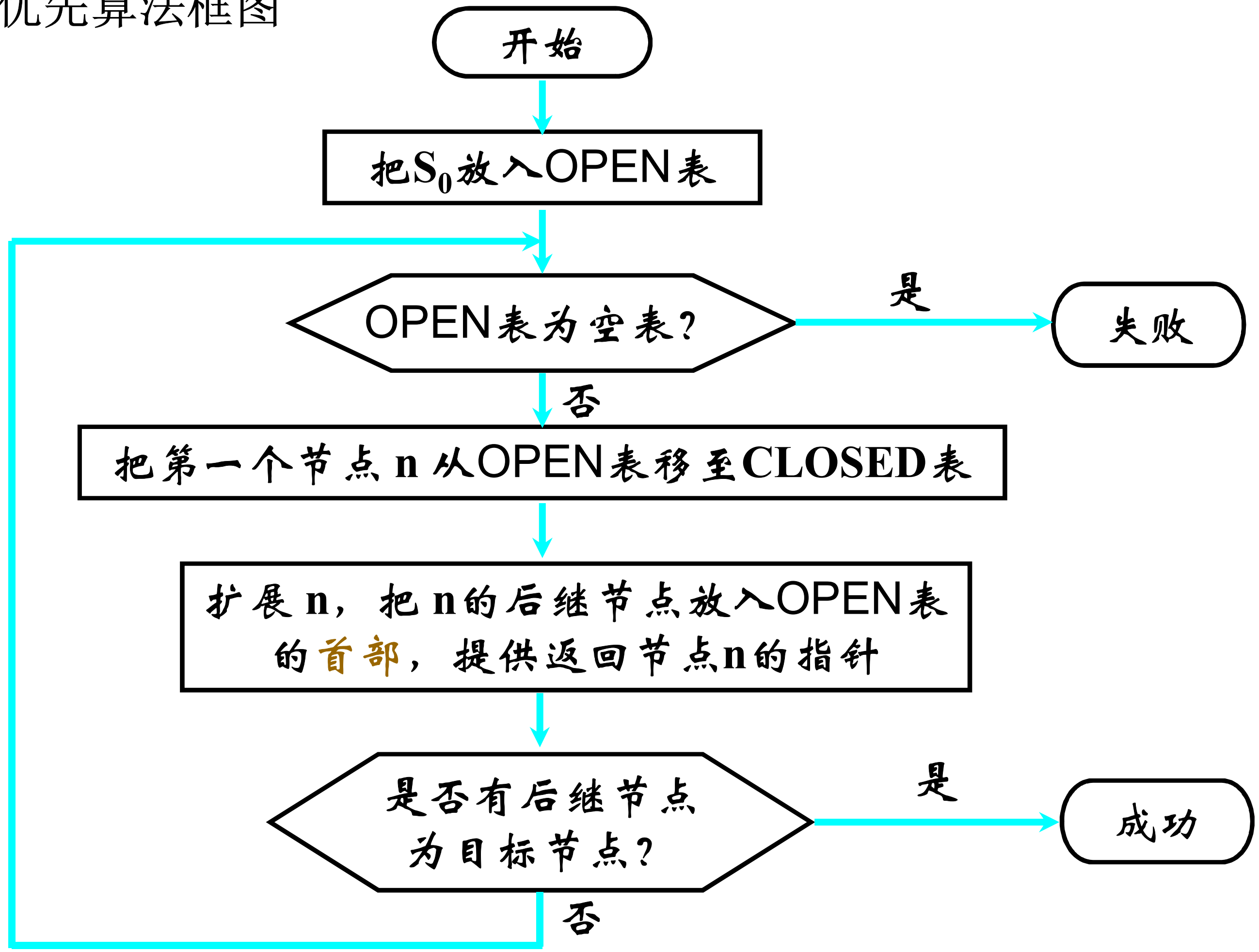


DFS



八数码难题的有界深度搜索树

深度优先算法框图



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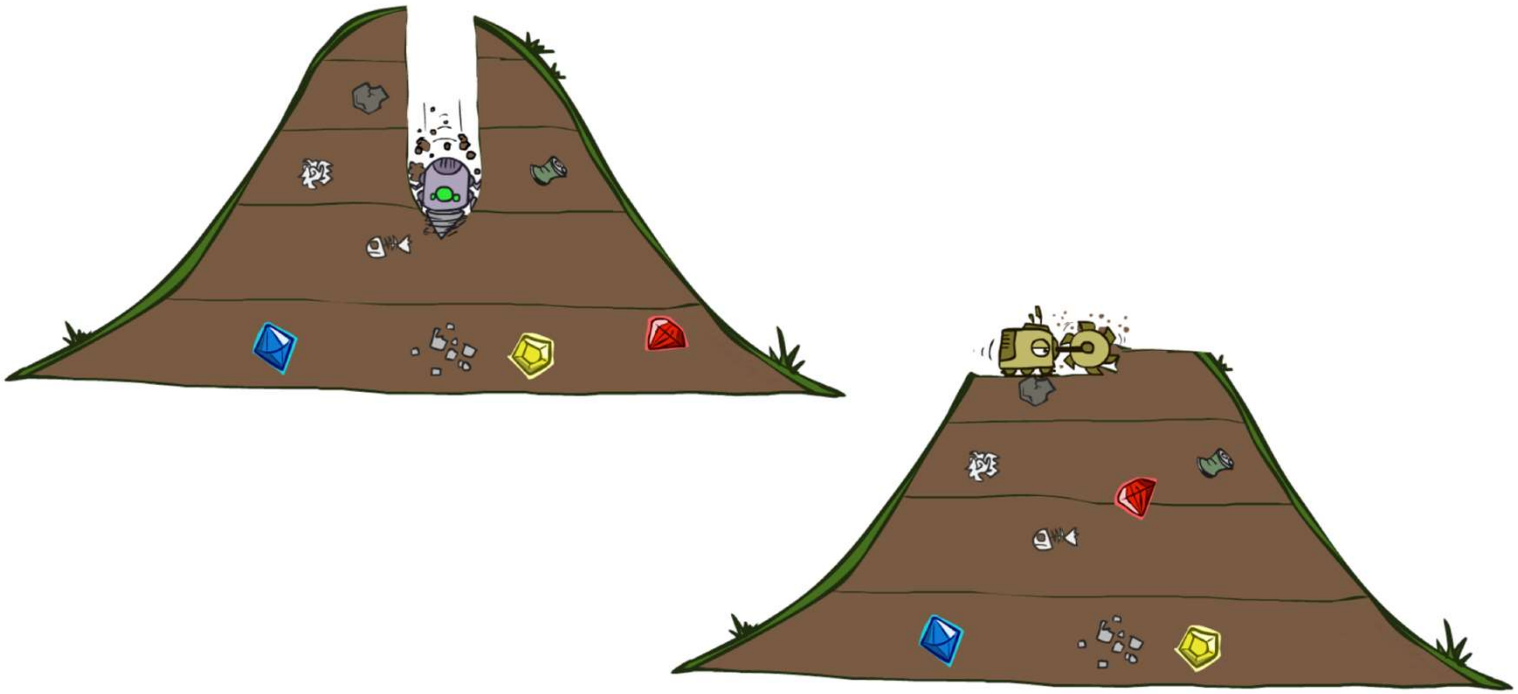
扩展n，把n的后继节点放入OPEN表的首部，提供返回节点n的指针
是否有后继节点为目标节点？
是
成功
否

DFS vs BFS



Measures of performance

Completeness

Guaranteed to find a solution when there is one?

Optimality

Finds an optimal solution?

Time

How long does it take to find a solution?

Space

How much memory is needed to conduct the search?

Search cost is about time cost and space cost.

Domain cost = path cost

Total cost = domain cost + search cost

Analysis of search strategies

- Evaluation criteria of strategies:
  - **Completeness**
  - **Optimality**
  - **Time complexity**
  - **Space complexity**
- Time and space complexity are measured in terms of
  - ***b****:* maximum branching factor of the search tree
  - ***d****:* depth of the optimal solution
  - ***m***: maximum length of any path in the state space (may be infinite)

Properties of breadth-first search

- **Complete?**

  Yes (if branching factor b is finite)
- **Optimal?**

  Yes – if cost = 1 per step
- **Time?**

  Number of nodes in a b-ary tree of depth d: $O(b^d)$

  (d is the depth of the optimal solution)
- **Space?**

  $O(b^d)$

- Space is the bigger problem (more than time)

Properties of depth-first search

- **Complete?**

  Fails in infinite-depth spaces, spaces with loops
- **Optimal?**

  No – returns the first solution it finds
- **Time?**

  Could be the time to reach a solution at maximum depth m: $O(b^m)$

  Terrible if m is much larger than d

  But if there are lots of solutions, may be much faster than BFS
- **Space?**

  $O(bm)$, i.e., linear space!

Iterative deepening search

- Use DFS as a subroutine
  1. Check the root
  2. Do a DFS searching for a path of depth 1
  3. If there is no path of depth 1, do a DFS searching for a path of depth 2
  4. If there is no path of depth 2, do a DFS searching for a path of depth 3…

Properties of iterative deepening search

- **Complete?**

  Yes, when the branching factor is finite

- **Optimal?**

  Yes, when the path cost is a nondecreasing function of the depth of the node

- **Time?**

  $d\,b^1 + (d\text{-}1)b^2 + \ldots + b^d = O(b^d)$

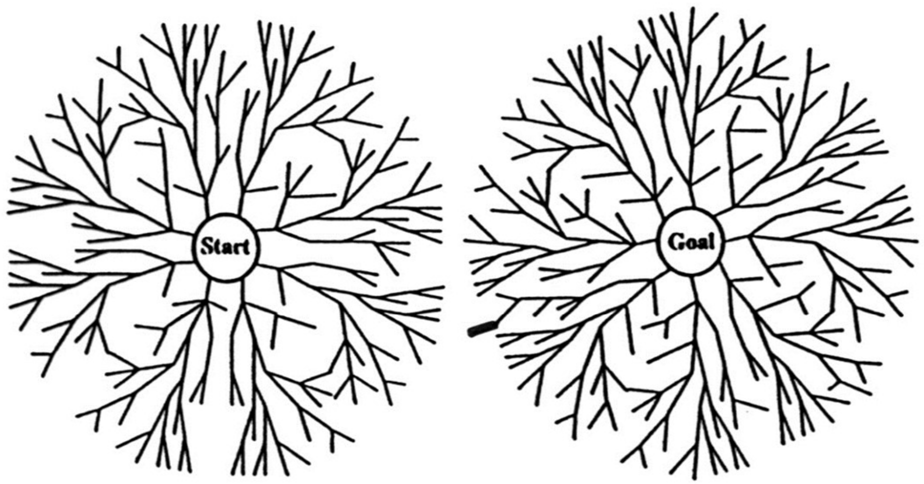
- **Space?**

  $O(bd)$

Quiz

- When to use BFS / DFS / IDS？

Bi-Directional Search



A schematic view of a bidirectional breadth-first search that is about to succeed, when a branch from the start node meets a branch from the goal node.

Bi-Directional Search

- **Complete?**

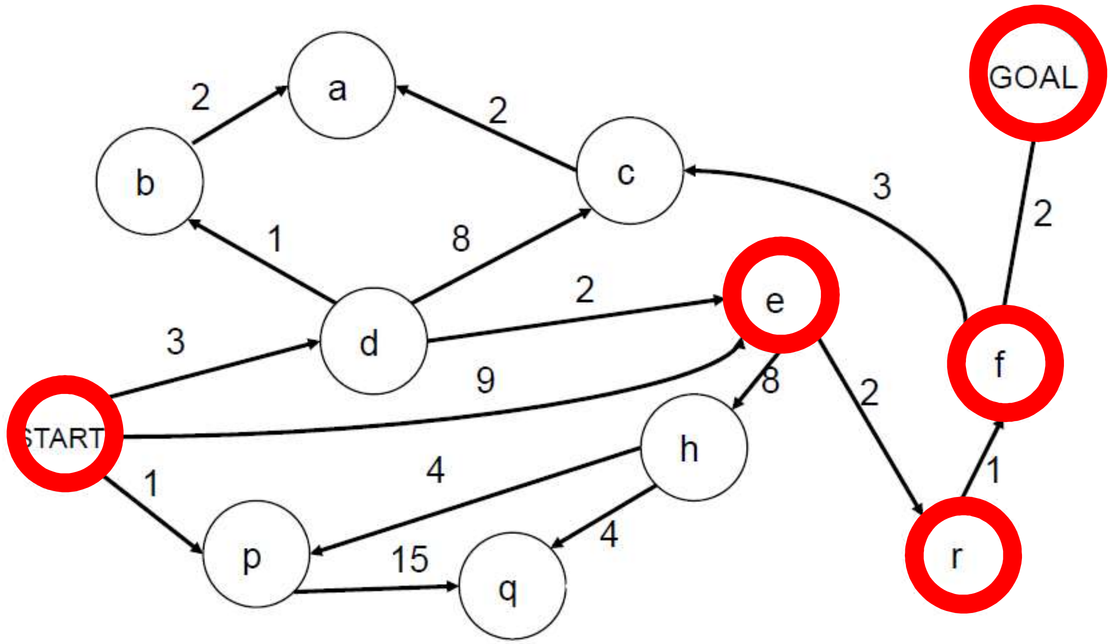
  It depends
- **Optimal?**

  It depends
- **Time?**

  $O(b^{d/2})$
- **Space?**

  $O(b^{d/2})$

Search with varying step costs

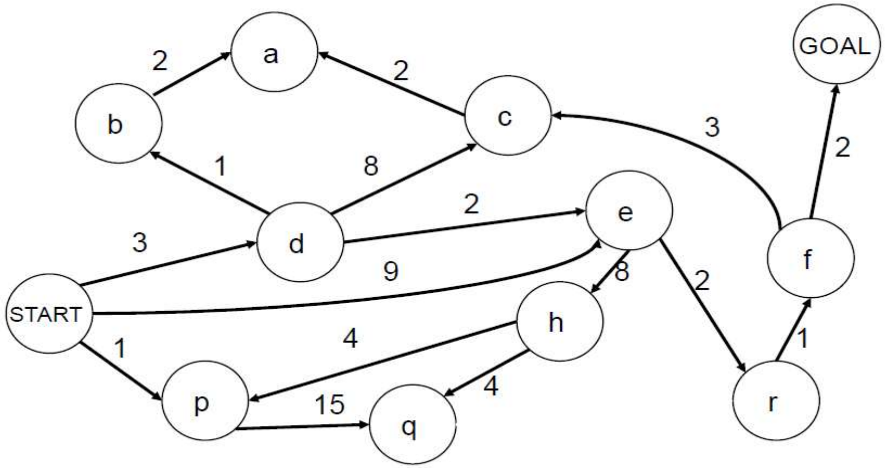


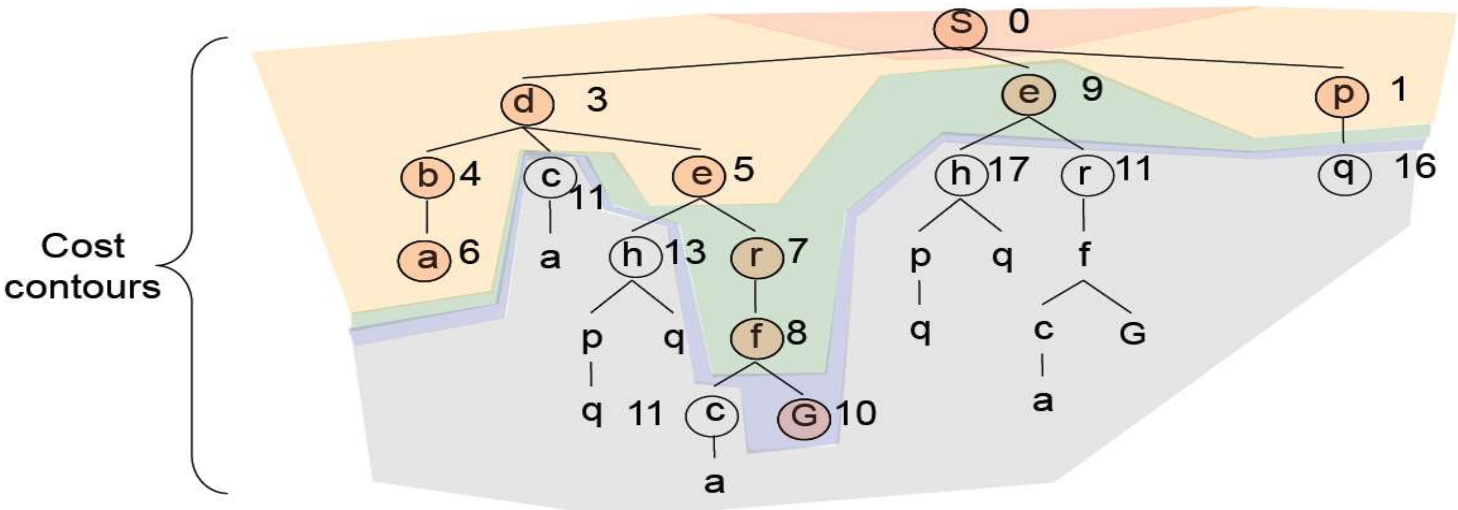
- BFS finds the path with the fewest steps, but does not always find the cheapest path

Uniform-cost search

- For each frontier node, save the total cost of the path from the initial state to that node
- Expand the frontier node with the lowest path cost
- Implementation: *frontier* is a priority queue ordered by the path cost
- Equivalent to Dijkstra's algorithm in general

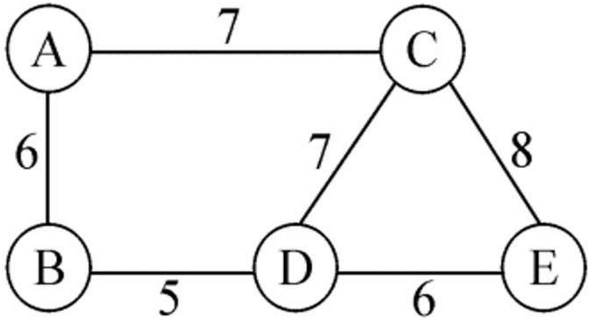
Uniform-cost search example





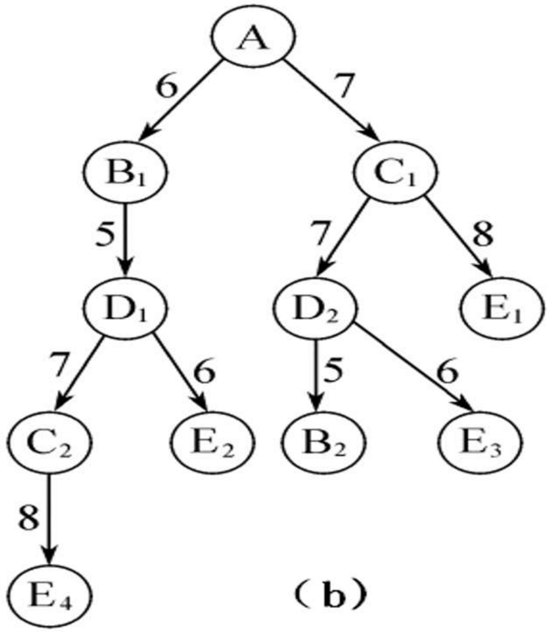
Exp：推销员旅行问题

设A、B、C、D和E是五个城市，推销员从城市A出发到城市E，已知5个城市间的交通图和每两个城市间的旅行费用，问推销员该走怎样的路线费用最省？



(a) 旅行交通图

(b) 旅行交通图的代价树



（b）

Properties of uniform-cost search

- **Complete?**

  Yes, if step cost is greater than some positive constant ε

- **Optimal?**

  Yes – nodes expanded in increasing order of path cost

- **Time?**

  Number of nodes with path cost ≤ cost of optimal solution (C^*), $O(b^{C^*/\varepsilon})$

  This can be greater than $O(b^d)$: the search can explore long paths consisting of small steps before exploring shorter paths consisting of larger steps

- **Space?**

  $O(b^{C^*/\varepsilon})$

Uninformed search strategies

Algorithm	Complete?	Optimal?	Time complexity	Space complexity
BFS	Yes	If all step costs are equal	$O(b^d)$	$O(b^d)$
DFS	No	No	$O(b^m)$	$O(bm)$
IDS	Yes	If all step costs are equal	$O(b^d)$	$O(bd)$
UCS	Yes	Yes	Number of nodes with $g(n) \leq C^*$	

b: maximum branching factor of the search tree
d: depth of the optimal solution
m: maximum length of any path in the state space
C*: cost of optimal solution
g(n): cost of path from start state to node n

Outline

- Problem-Solving by Searching
- Uninformed Search Strategies
- Informed (Heuristic) Search Strategies

棋局的穷举

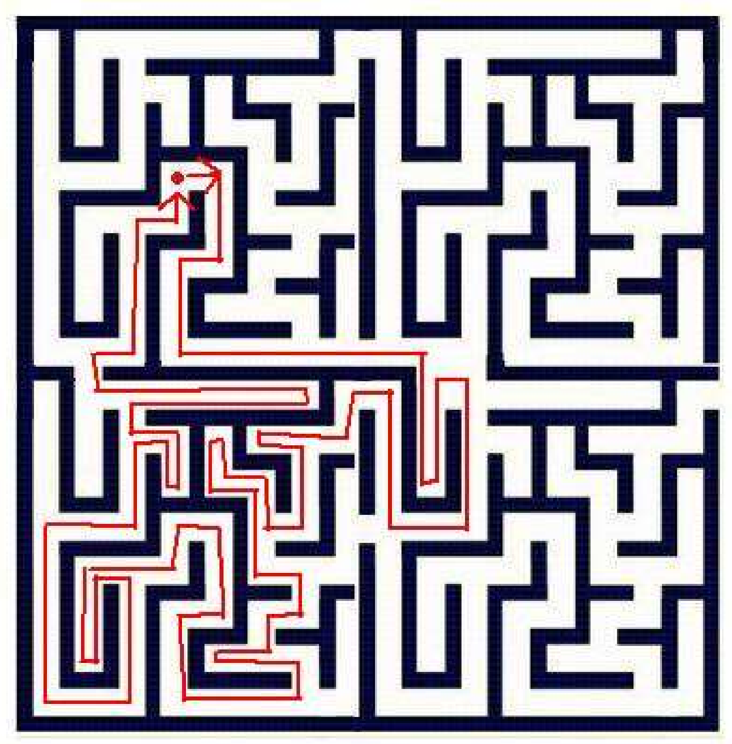
棋局数：

- 一字棋：$9! \approx 3.6 \times 10^5$
- 西洋棋：10^{78}
- 国际象棋：10^{120}
- 围棋：10^{761}

假设每步可以选择一种棋局,用并行速度(10^{-104}秒/步)计算,国际象棋的算法需用10^{16}年,即1亿亿年才可以算完。

How does Theseus find the way out of Minotaur's labyrinth?

Ariadne’s clew:



Informed search strategies

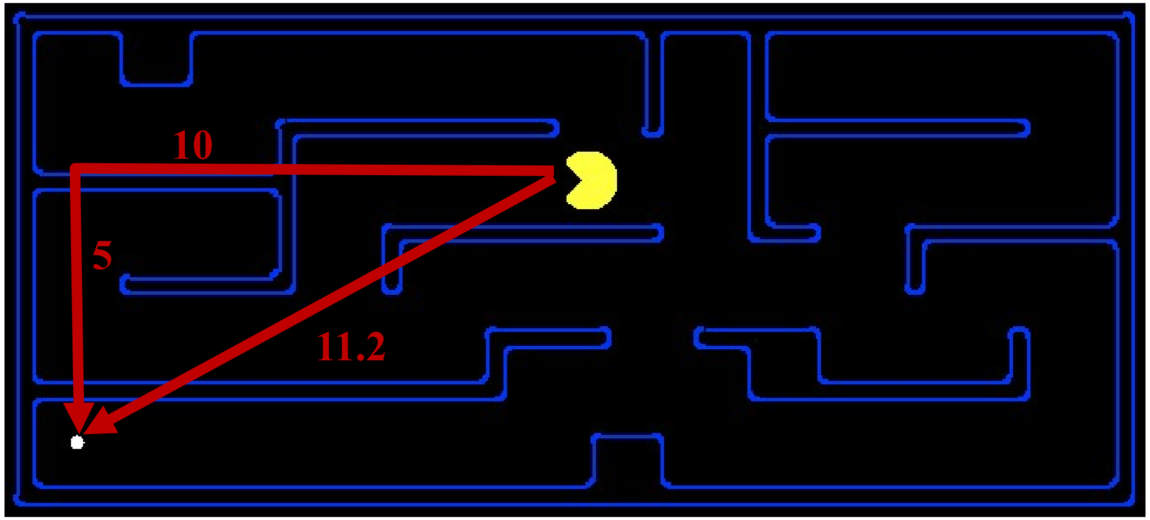
- Idea: give the algorithm “hints” about the desirability of different states
- Use an ***evaluation function*** to rank nodes and select the most promising one for expansion

Informed search strategies

- Greedy best-first search
- A* search

Greedy Best-first Search

- Strategy: expand a node that you think is closest to a goal state
  - **heuristic function**: *estimates* how close a state is to a goal
  - Designed for a particular search problem
  - Examples: Manhattan distance, Euclidean distance for pathing

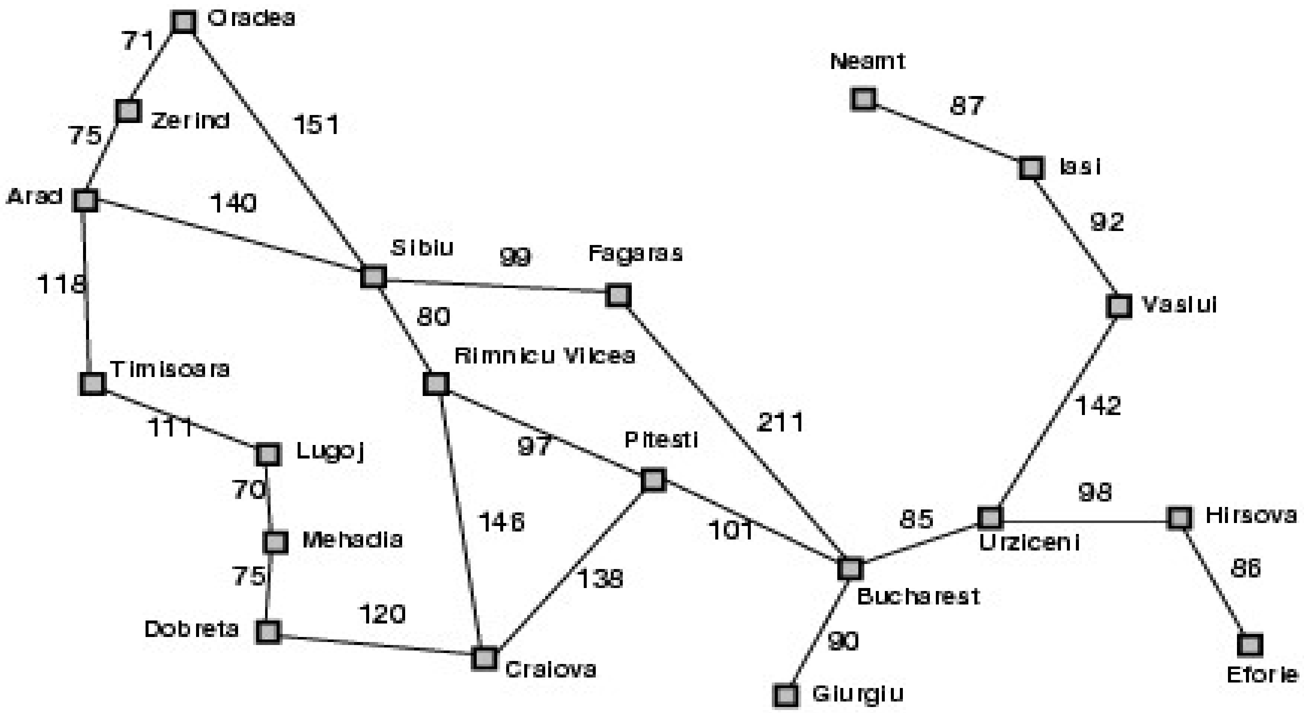


Greedy best-first search

- Expand the node that has the lowest value of the heuristic function $h(n)$
  - Try to expand the node that is closest to the goal, on the grounds that this is likely to lead to a solution quickly.

Heuristic for the Romania problem

h(n)——straight-line distances to Bucharest



Straight–line distance to Bucharest

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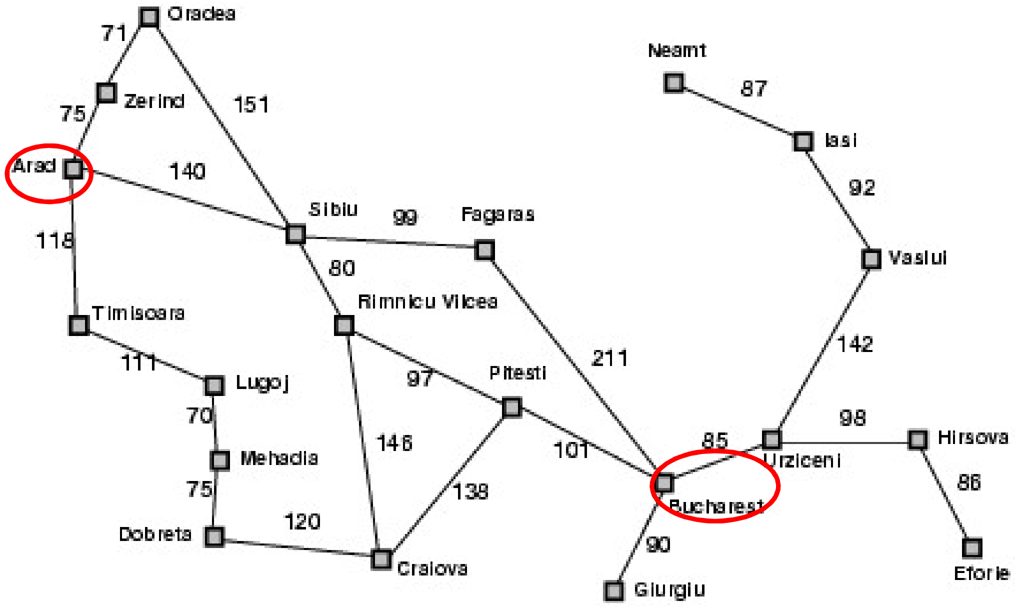
Greedy best-first search example



h(x)

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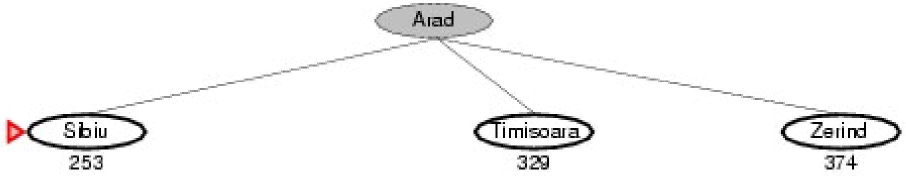
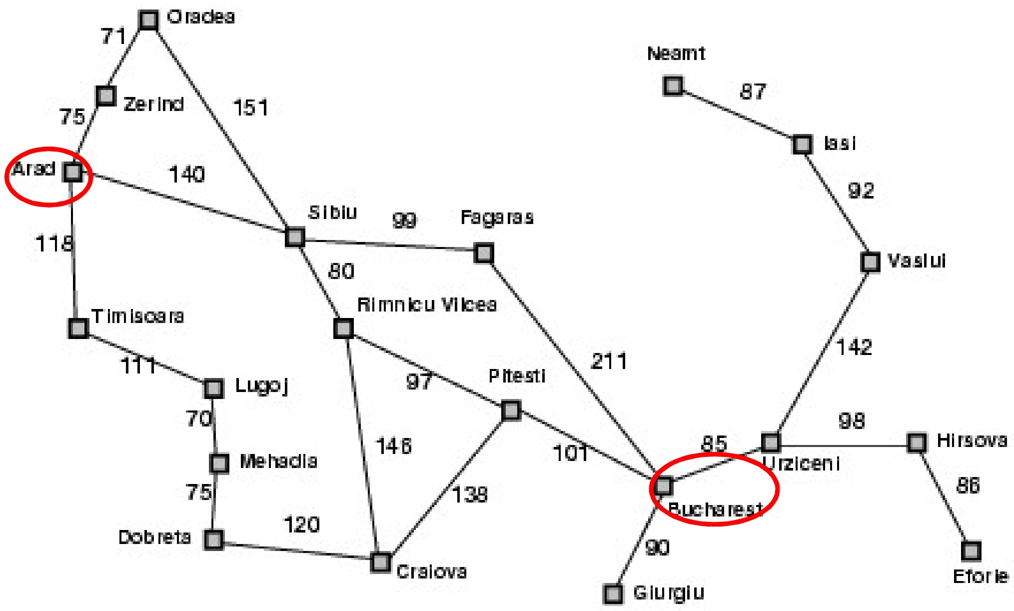
Greedy best-first search example

- Expand the node that seems closest…





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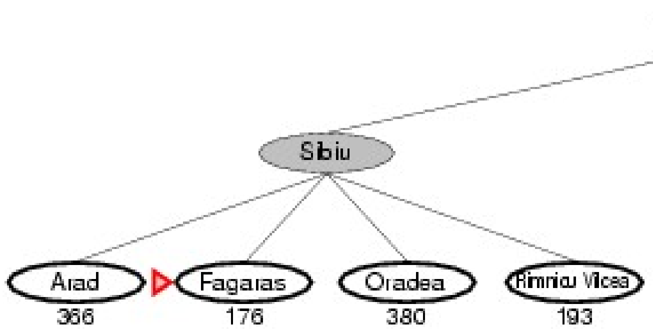
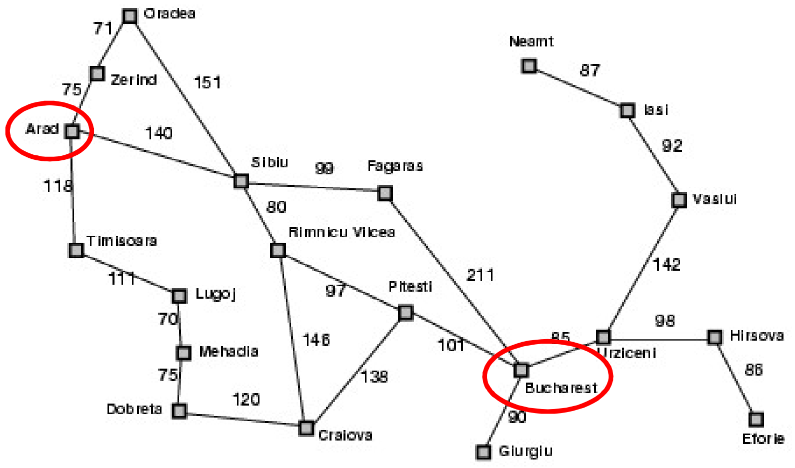
Greedy best-first search example





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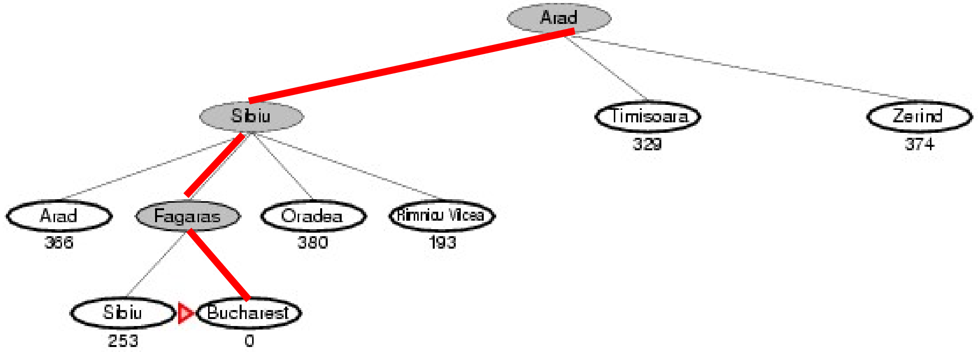
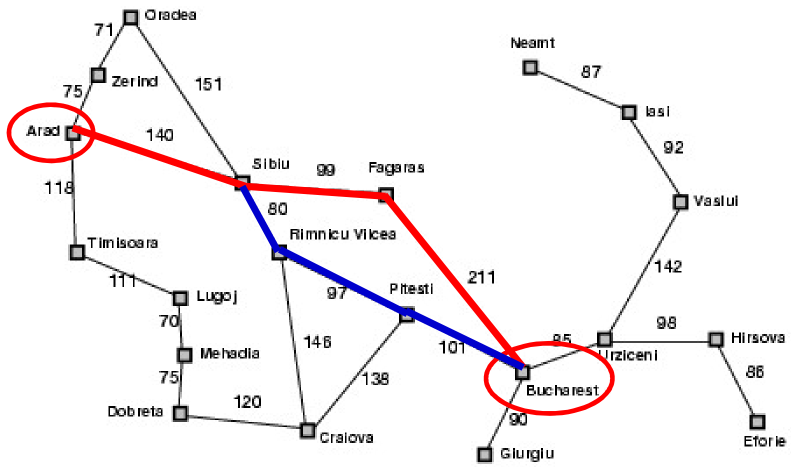
Greedy best-first search example





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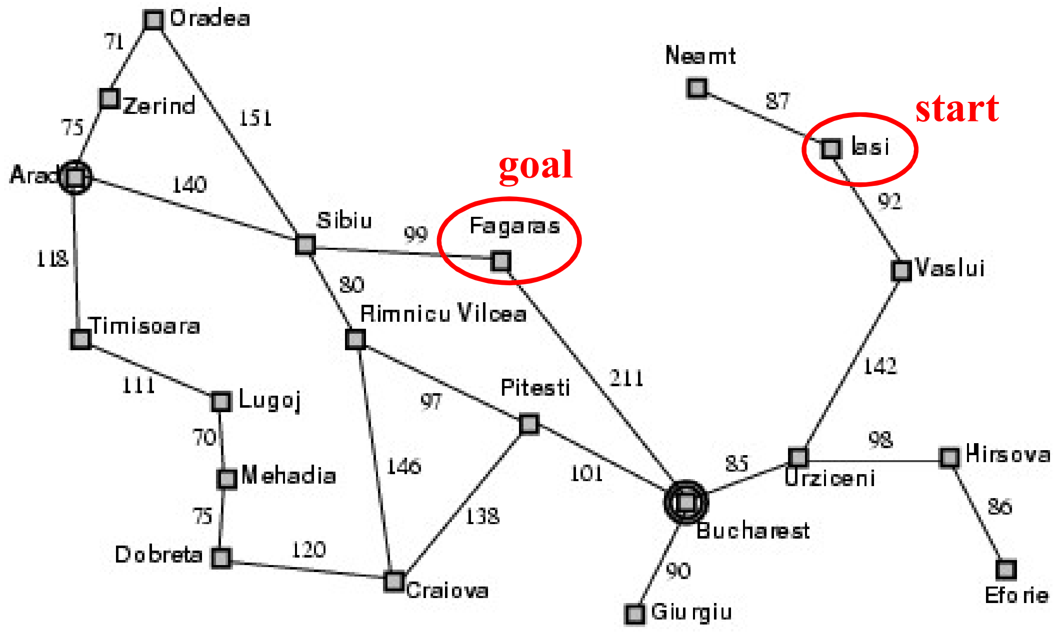
Properties of greedy best-first search

- **Complete?**

  No – can get stuck in loops



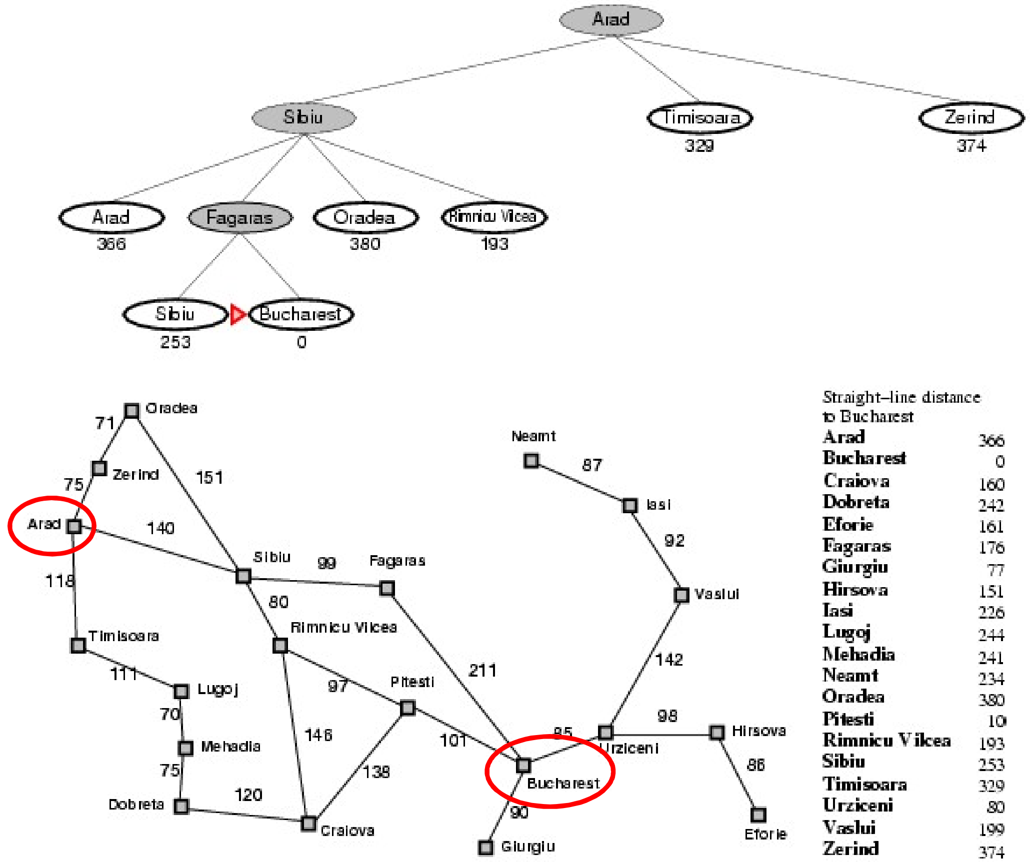
Properties of greedy best-first search

- **Complete?**

  No – can get stuck in loops

- **Optimal?**

  No



Properties of greedy best-first search

- **Complete?**

  No – can get stuck in loops
- **Optimal?**

  No
- **Time?**

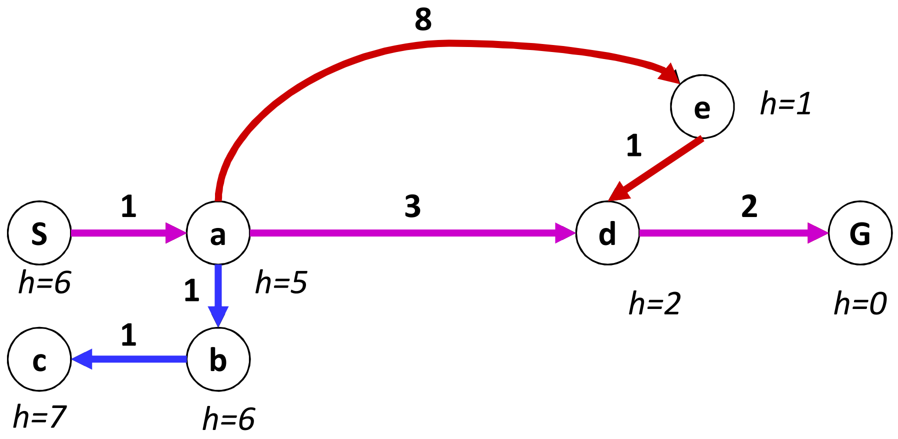
  Worst case: $O(b^m)$

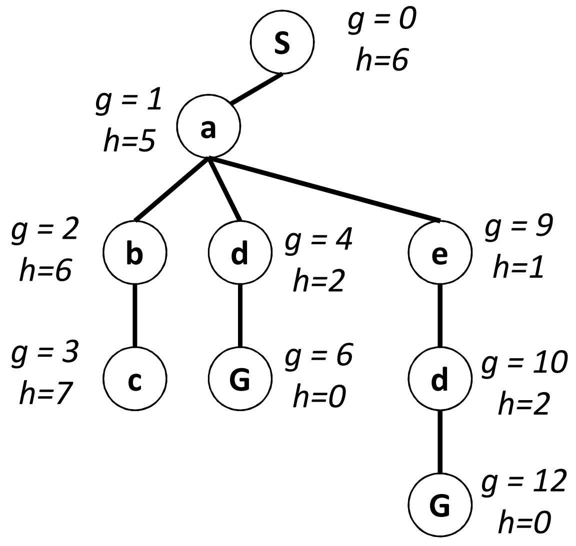
  Can be much better with a good heuristic
- **Space?**

  Worst case: $O(b^m)$

Combining UCS and Greedy

- Uniform-cost orders by path cost, or *backward cost* g(n)
- Greedy orders by goal proximity, or *forward cost* h(n)





- A* Search orders by the sum: f(n) = g(n) + h(n)

Admissible heuristics

- A heuristic $h(n)$ is admissible if

$$0 \leq h(n) \leq h^*(n)$$

where $h^*(n)$ is the true cost to the nearest goal

- An admissible heuristic never overestimates the cost to reach the goal, i.e., it is optimistic.
- Example: straight line distance never overestimates the actual road distance.

A^* search

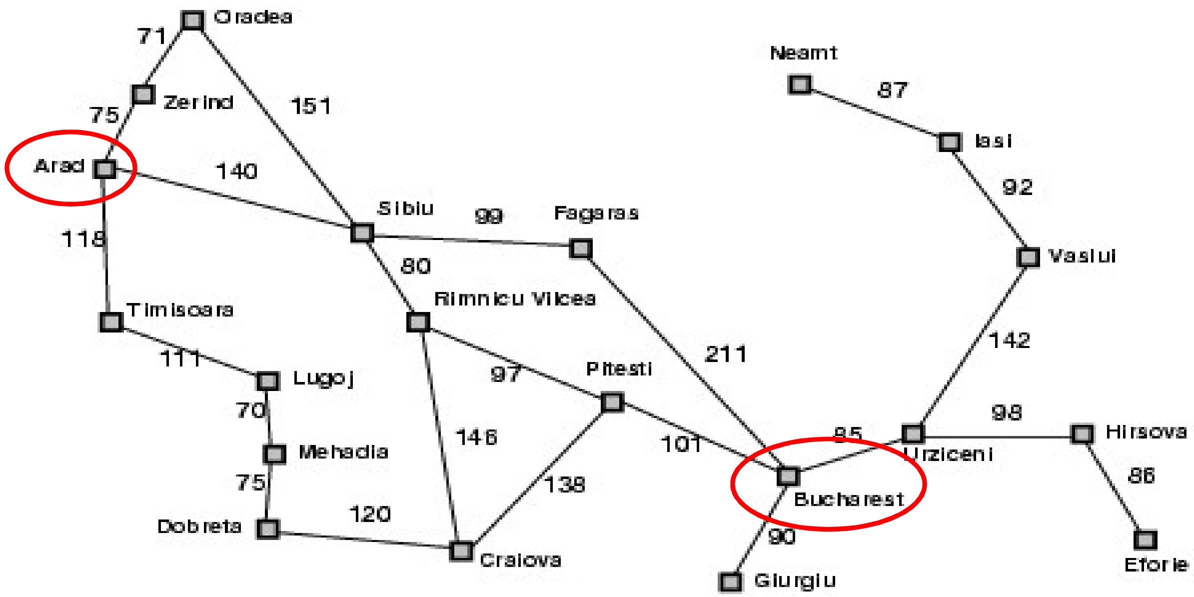
- Idea: avoid expanding paths that are already expensive
- The **evaluation function** $f(n)$ is the estimated total cost of the path through node n to the goal:

$$f(n) = g(n) + h(n)$$

$g(n)$: cost so far to reach n (path cost)

$h(n)$: estimated cost from n to goal (heuristic)

A* search example



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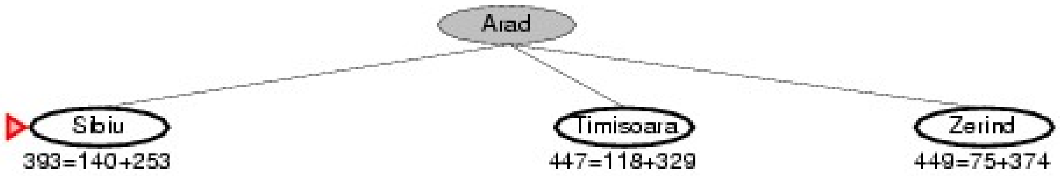
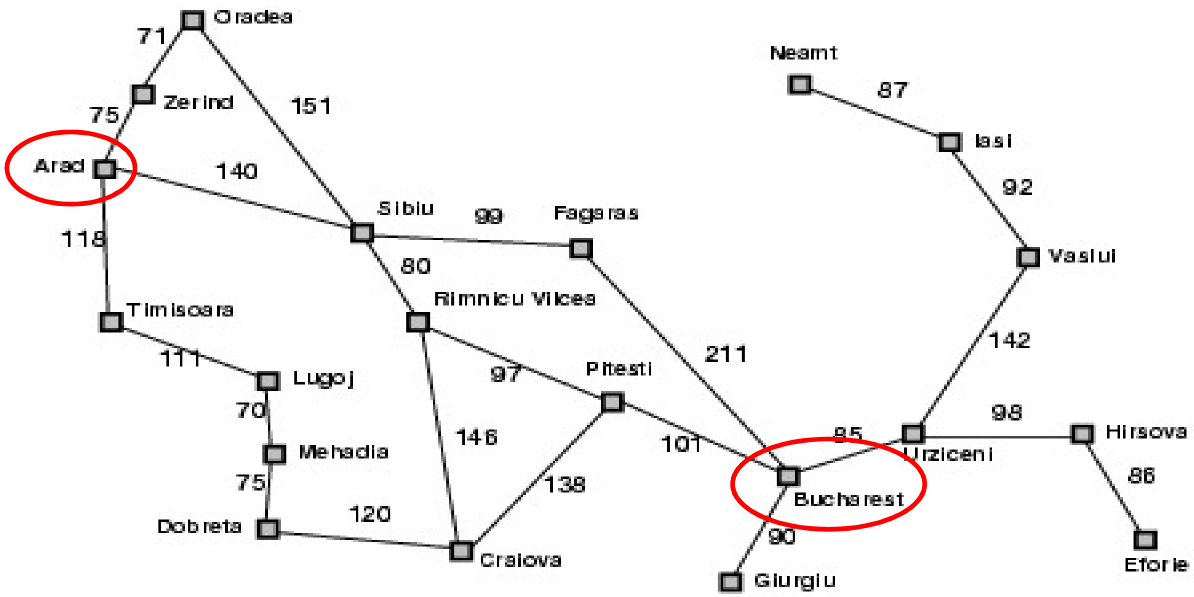
A* search example





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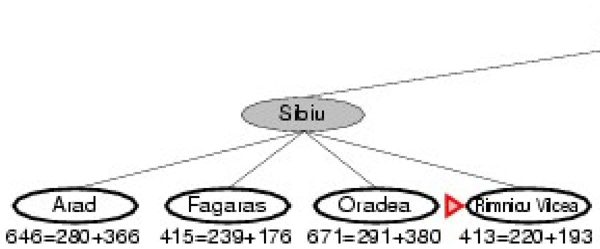

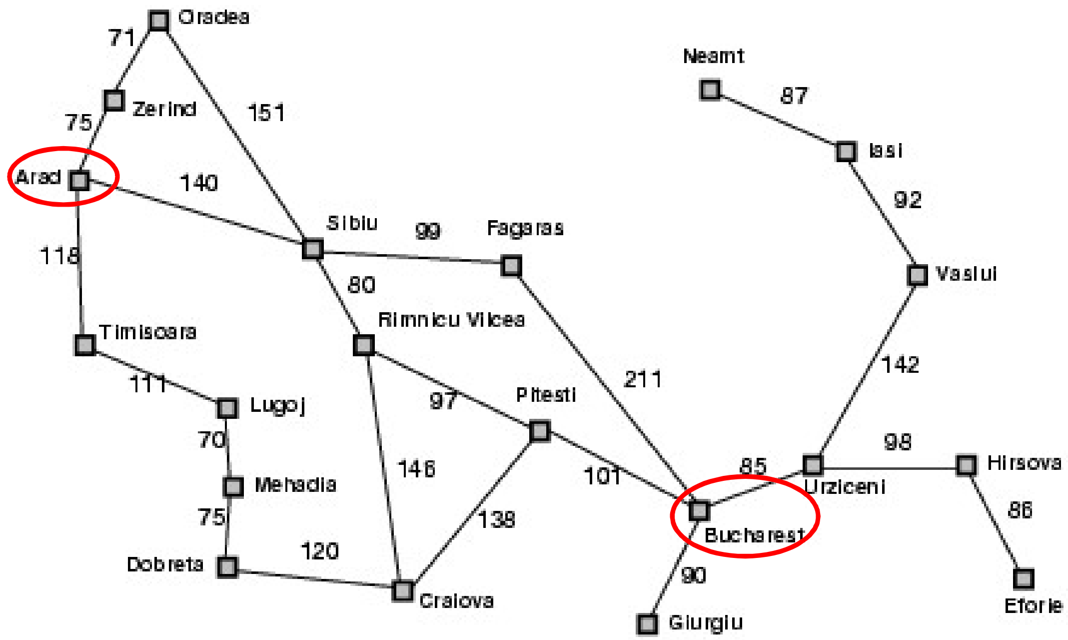

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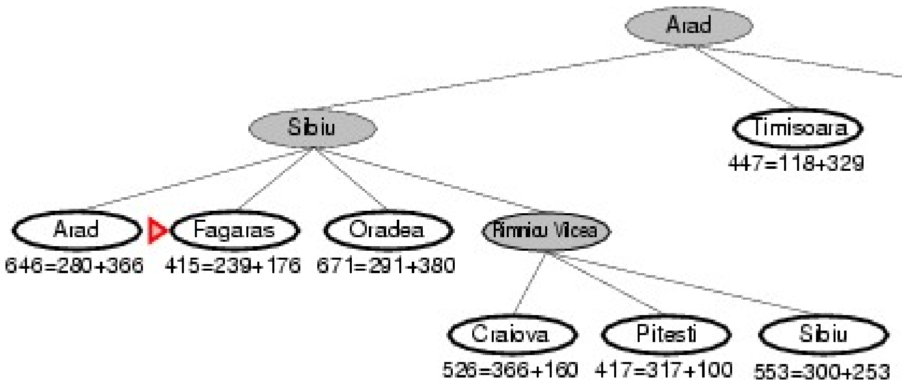

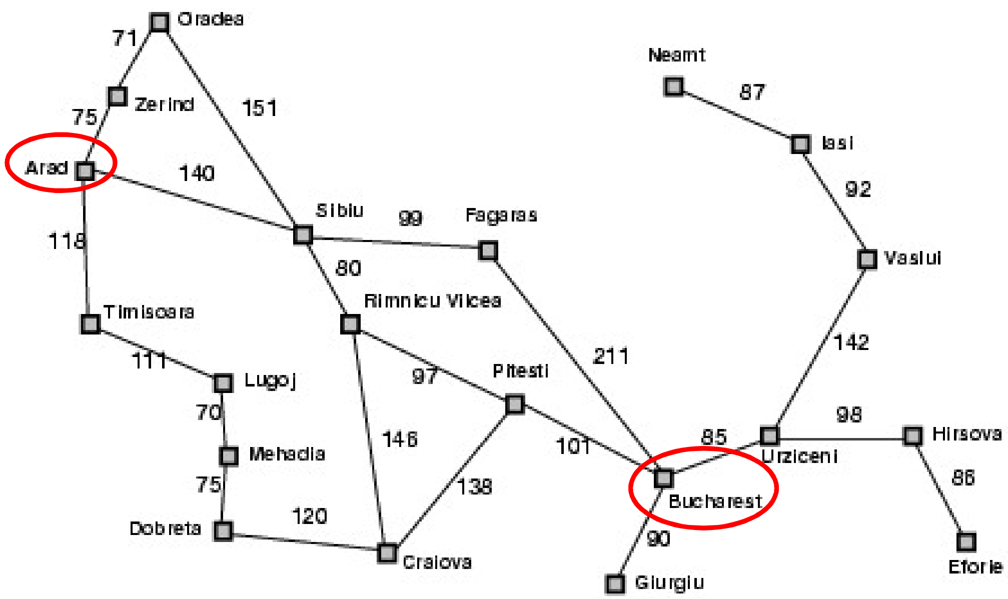

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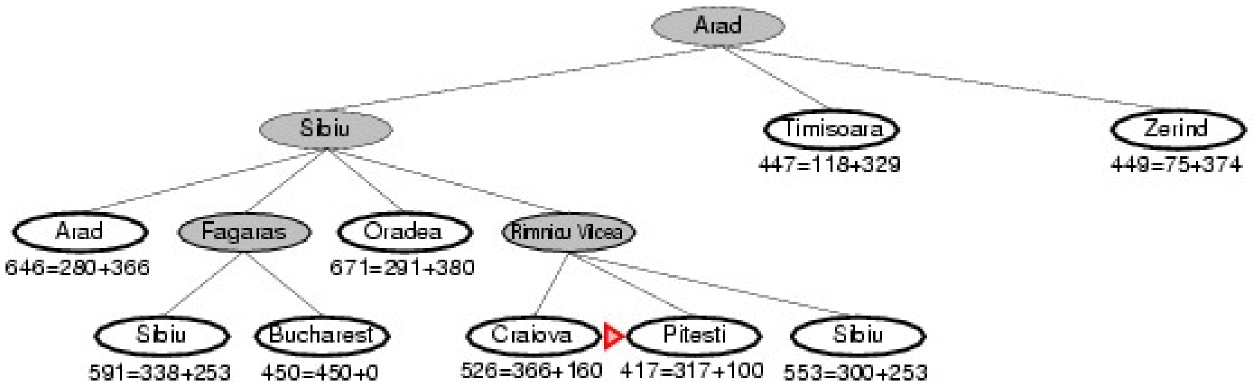

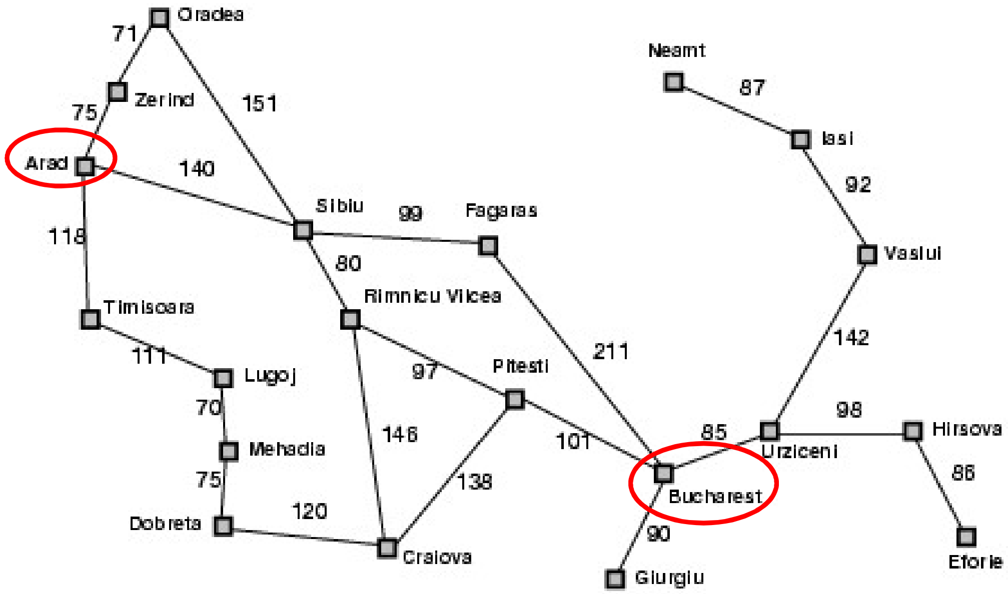

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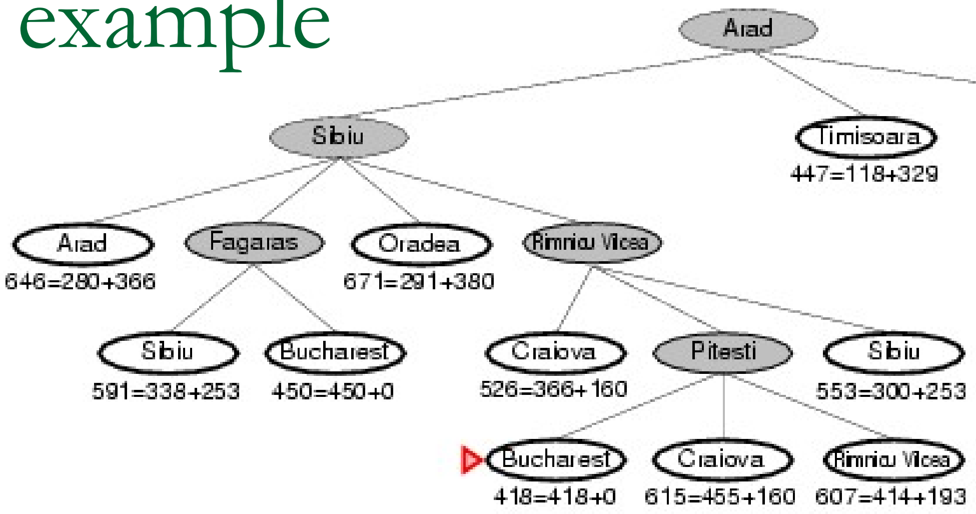
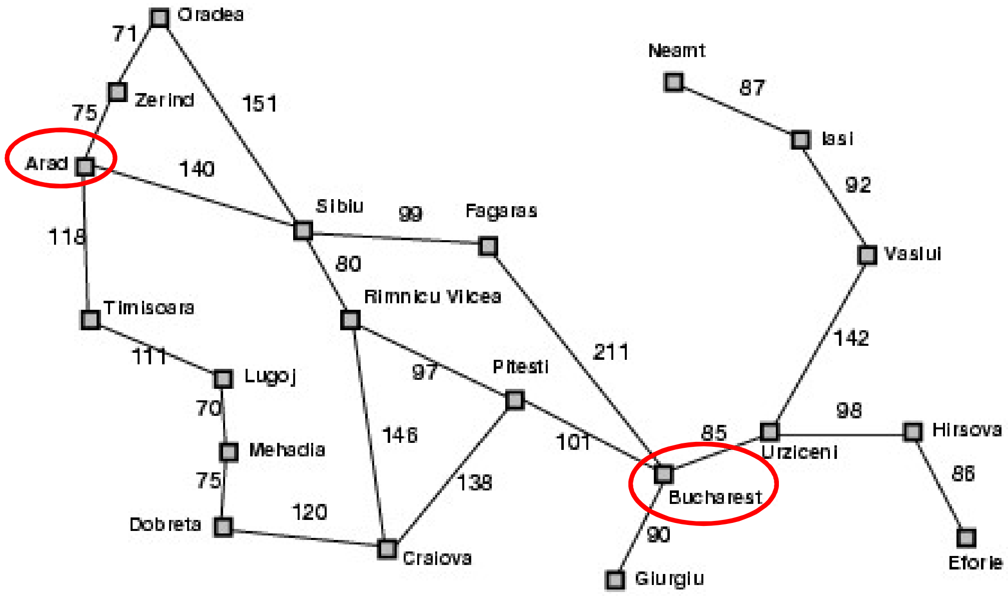
A* search example





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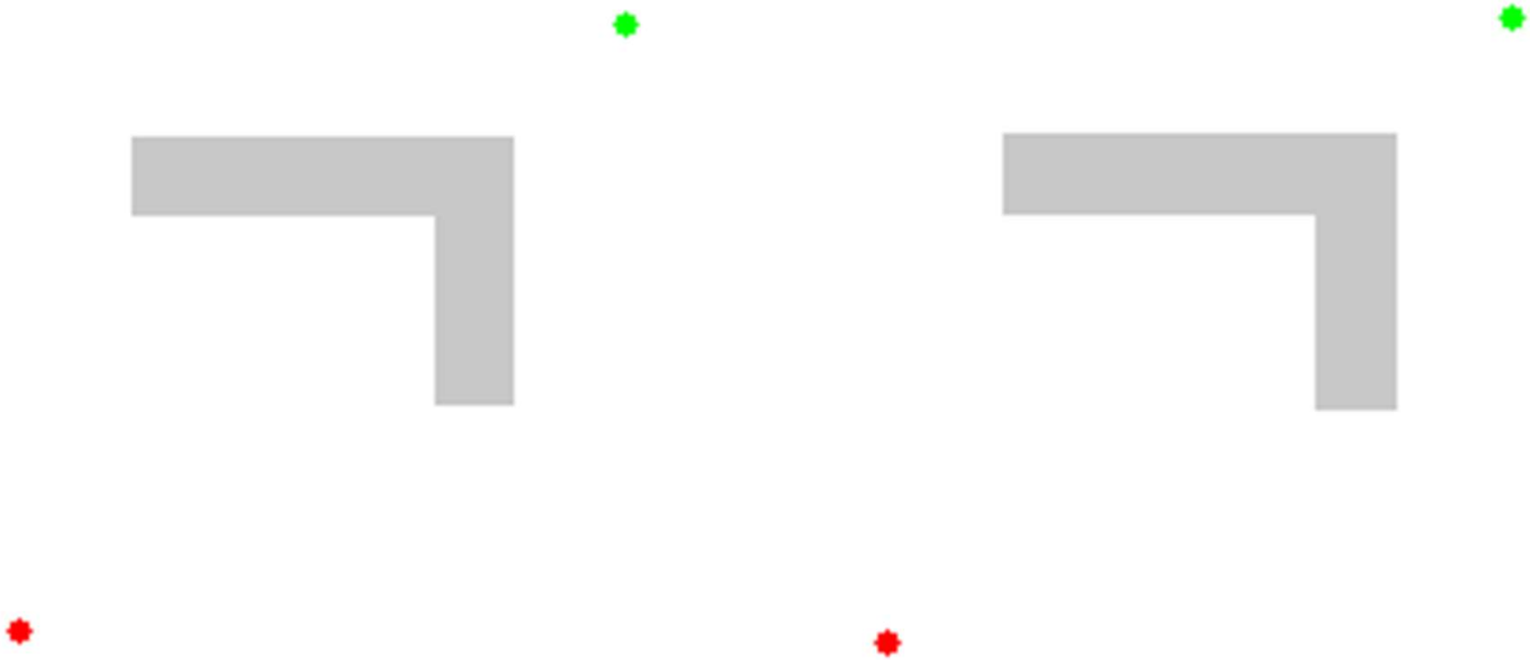
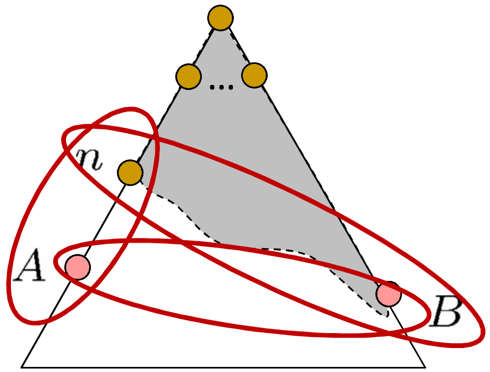
Uniform cost search vs. A* search



Source: Wikipedia

Optimality of A* Tree Search



Proof:

- Imagine B is on the fringe
- Some ancestor *n* of A is on the fringe, too (maybe A!)
- Claim: *n* will be expanded before B
  1. f(n) is less or equal to f(A)
  2. f(A) is less than f(B)
  3. *n* expands before B
- All ancestors of A expand before B
- A expands before B
- A* search is optimal

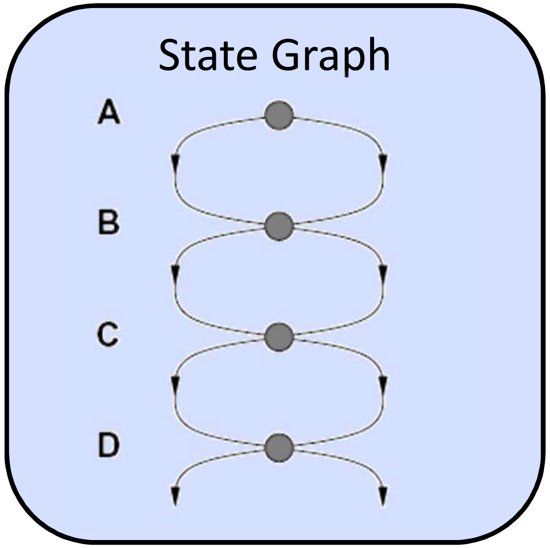
$f(n) = g(n) + h(n)$	Definition of f-cost
$f(n) \leq g(A)$	Admissibility of h
$g(A) = f(A)$	h = 0 at a goal

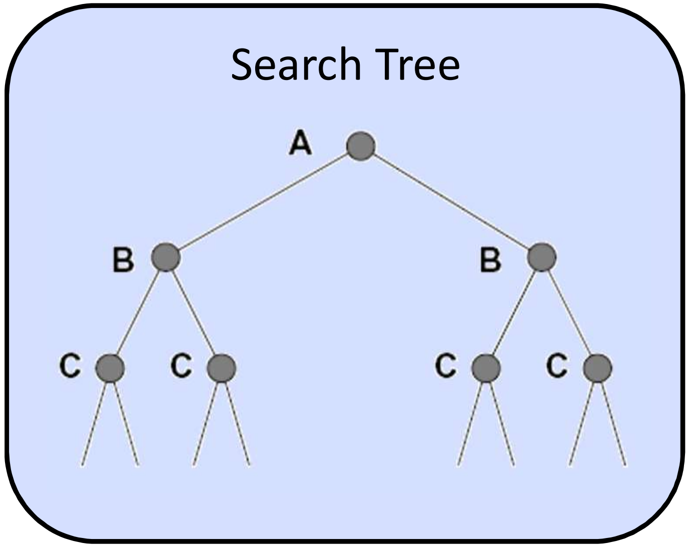
$g(A) < g(B)$	B is suboptimal
$f(A) < f(B)$	h = 0 at a goal

$$f(n) \leq f(A) < f(B)$$

Tree-like Search: Extra Work!

- Failure to detect repeated states can cause exponentially more work.



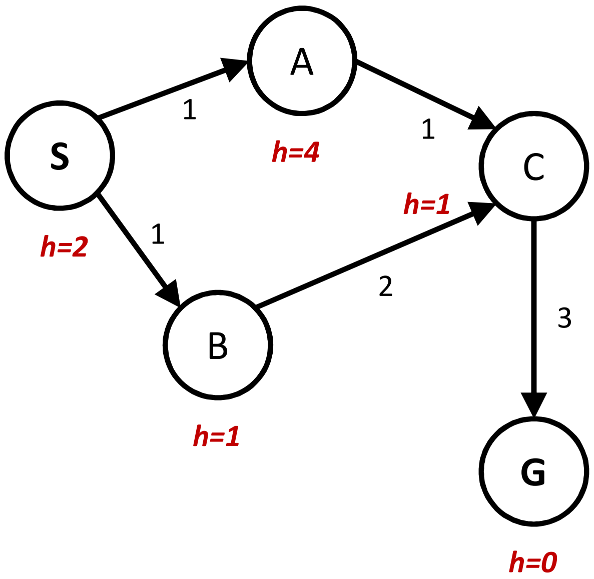


Graph Search

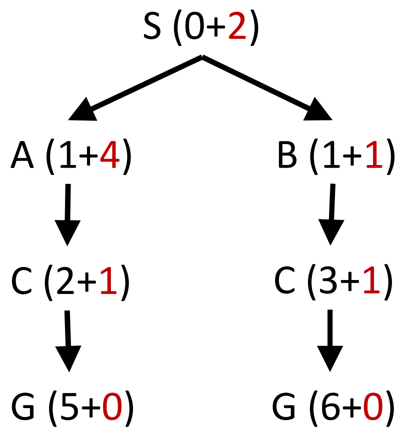
- Idea: never expand a state twice
- How to implement:
  - Tree search + set of expanded states ("closed set")
  - Expand the search tree node-by-node, but…
  - Before expanding a node, check to make sure its state has never been expanded before
- How about optimality?

A* Graph Search Gone Wrong?

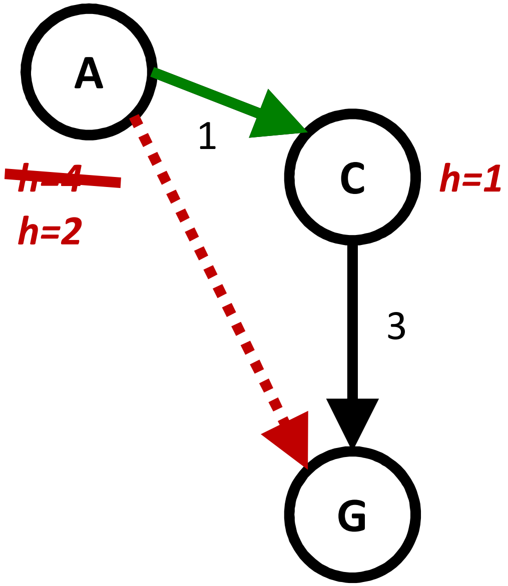
State space graph



Search tree



Consistency of Heuristics



- Main idea: estimated heuristic costs ≤ actual costs
  - Admissibility: heuristic cost ≤ actual cost to goal

    h(A) ≤ actual cost from A to G
  - Consistency: heuristic "arc" cost ≤ actual cost for each arc

    h(A) – h(C) ≤ cost(A to C)
- Consequences of consistency:
  - The f value along a path never decreases

    h(A) ≤ cost(A to C) + h(C)
  - A* graph search is optimal

Optimality of A*

- **Tree-like search** (i.e., search without repeated state detection):
  - A* is optimal if heuristic is ***admissible*** (and non-negative)
- **Graph search** (i.e., search with repeated state detection)
  - A* optimal if heuristic is ***consistent***
- Consistency implies admissibility
  - In general, most natural admissible heuristics tend to be consistent, especially if they come from relaxed problems.

Source: Berkeley CS188x

Properties of A*

- **Complete?**

  Yes – unless there are infinitely many nodes with $f(n) \leq C^*$

- **Optimal?**

  Yes

- **Time?**

  Number of nodes for which $f(n) \leq C^*$ (exponential)

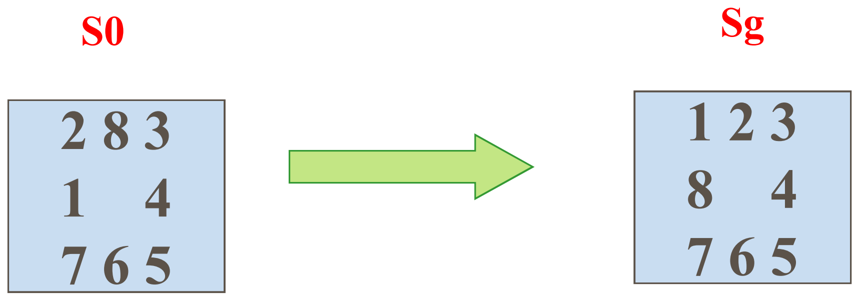
- **Space?**

  Exponential

Example: 8-puzzle

- $h_1(n)$ = number of misplaced tiles
- $h_2(n)$ = total Manhattan distance

  (i.e., no. of squares from desired location of each tile)

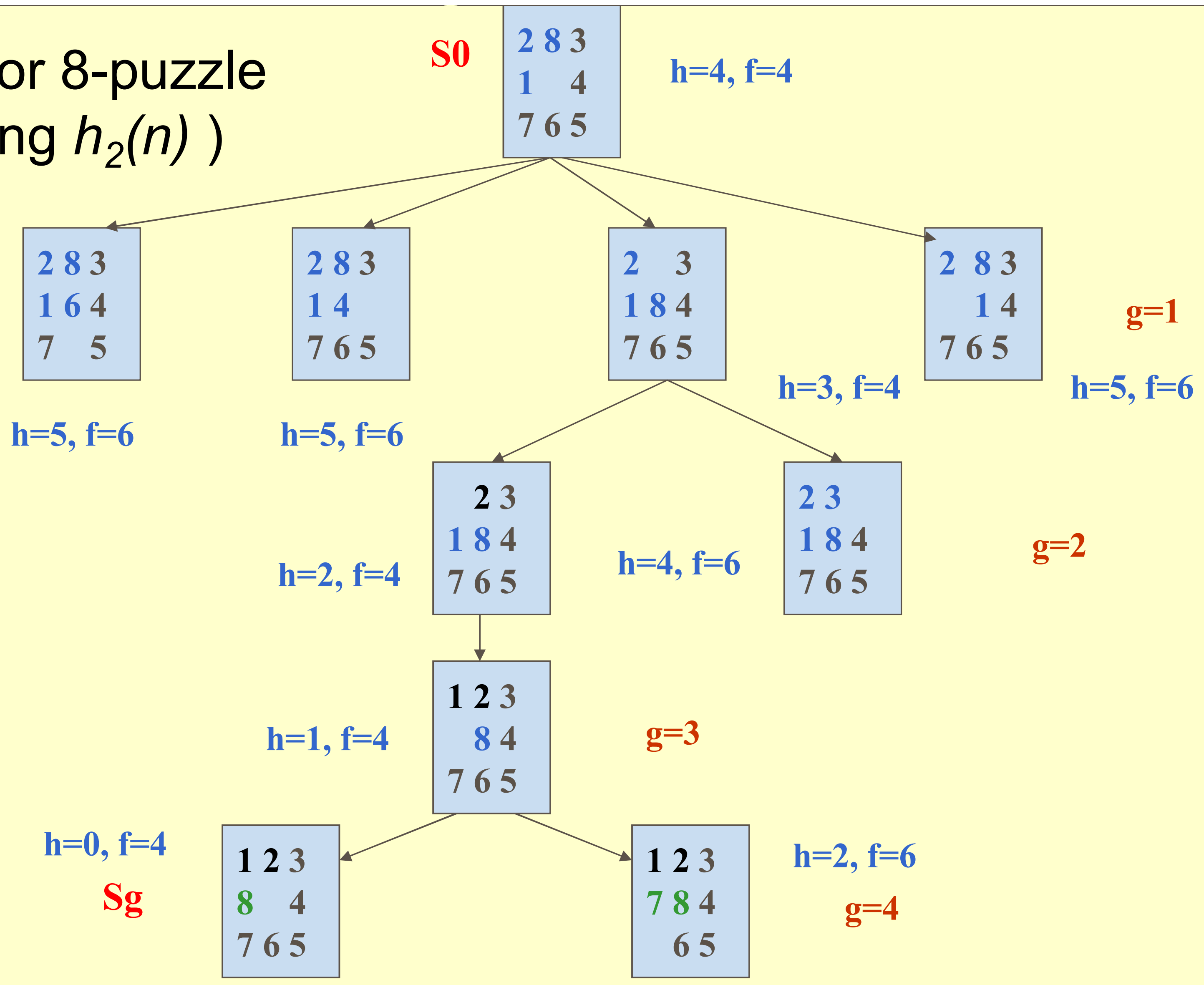


- $h_1(S) = ?$ 3
- $h_2(S) = ?$ 1+1+2 = 4
- If $h_2(n) \geq h_1(n)$ for all n (both admissible), then h_2 dominates h_1 , *and* h_2 is better for search

Dominance

- If h_1 and h_2 are both admissible heuristics and $h_2(n) \geq h_1(n)$ for all n, then h_2 dominates h_1
- Which one is better for search?
  - A* search expands every node with $f(n) < C^*$ or $h(n) < C^* - g(n)$
  - Therefore, A* search with h_1 will expand more nodes

A* for 8-puzzle (using $h_2(n)$)



Heuristics from relaxed problems

- A problem with fewer restrictions on the actions is called a relaxed problem.
- The cost of an optimal solution to a relaxed problem is an admissible heuristic for the original problem.
- If the rules of the 8-puzzle are relaxed so that a tile can move anywhere, then $h_1(n)$ gives the shortest solution.
- If the rules are relaxed so that a tile can move to any adjacent square, then $h_2(n)$ gives the shortest solution.

Combining heuristics

- Suppose we have a collection of admissible heuristics $h_1(n)$, $h_2(n)$, …, $h_m(n)$, but none of them dominates the others
- How can we combine them?

$$h(n) = \max\{h_1(n), h_2(n), \ldots, h_m(n)\}$$

Weighted A* search

- **Idea:** speed up search at the expense of optimality
- Take an admissible heuristic, “inflate” it by a multiple $\alpha > 1$, and then perform A* search as usual.
- Fewer nodes tend to get expanded, but the resulting solution may be suboptimal (its cost will be at most α times the cost of the optimal solution).

Example of weighted A* search

Heuristic: 5 * Euclidean distance from goal

Source: Wikipedia

Compare: Exact A*

All search strategies (different fringe strategies)

Algorithm	Complete?	Optimal?	Time complexity	Space complexity
BFS	Yes	If all step costs are equal	$O(b^d)$	$O(b^d)$
DFS	No	No	$O(b^m)$	$O(bm)$
IDS	Yes	If all step costs are equal	$O(b^d)$	$O(bd)$
UCS	Yes	Yes	Number of nodes with $g(n) \leq C^*$	
Greedy	No	No	Worst case: $O(b^m)$ Best case: $O(bd)$	
A*	Yes	Yes (if heuristic is admissible)	Number of nodes with $g(n)+h(n) \leq C^*$	

A note on the complexity of search

- We said that the worst-case complexity of search is exponential in the length of the solution path
  - But the length of the solution path can be exponential in the number of “objects” in the problem!
- Example: towers of Hanoi

Q & A