Find the Second Derivative

$$\frac{1}{(x^2 - 4x)}$$

Find the first derivative.

Tap for fewer steps...

Rewrite $\frac{1}{x^2 - 4x}$ as $\left(x^2 - 4x\right)^{-1}$.

$$\frac{d}{dx}\left[\left(x^2 - 4x\right)^{-1}\right]$$

Differentiate using the chain rule, which states that $\frac{d}{dx}[f(g(x))]$ is $f'(g(x))\,g'(x)$ where $f(x) = x^{-1}$ and $g(x) = x^2 - 4x$.

Tap for fewer steps...

To apply the Chain Rule, set u as $x^2 - 4x$.

$$\frac{d}{du}\left[u^{-1}\right]\frac{d}{dx}\left[x^2 - 4x\right]$$

Differentiate using the Power Rule which states that $\frac{d}{du}[u^n]$ is nu^{n-1} where $n = -1$.

$$-u^{-2}\frac{d}{dx}\left[x^2 - 4x\right]$$

Replace all occurrences of u with $x^2 - 4x$.

$$-\left(x^2 - 4x\right)^{-2}\frac{d}{dx}\left[x^2 - 4x\right]$$

Differentiate.

Tap for fewer steps...

By the Sum Rule, the derivative of $x^2 - 4x$ with respect to x is $\frac{d}{dx}\left[x^2\right] + \frac{d}{dx}[-4x]$.

$$-\left(x^2 - 4x\right)^{-2}\left(\frac{d}{dx}\left[x^2\right] + \frac{d}{dx}[-4x]\right)$$

Differentiate using the Power Rule which states that $\frac{d}{dx}[x^n]$ is nx^{n-1} where $n = 2$.

$$-\left(x^2 - 4x\right)^{-2}\left(2x + \frac{d}{dx}[-4x]\right)$$

Since -4 is constant with respect to x, the derivative of $-4x$ with respect to x is $-4\frac{d}{dx}[x]$.

$$-\left(x^2 - 4x\right)^{-2}\left(2x - 4\frac{d}{dx}[x]\right)$$

Differentiate using the Power Rule which states that $\frac{d}{dx}[x^n]$ is nx^{n-1} where $n = 1$.

$$-\left(x^2 - 4x\right)^{-2}(2x - 4 \cdot 1)$$

Multiply -4 by 1.

$$-\left(x^2 - 4x\right)^{-2}(2x - 4)$$

Simplify.

Tap for fewer steps...

Rewrite the expression using the negative exponent rule $b^{-n} = \frac{1}{b^n}$.

$$-\frac{1}{\left(x^2 - 4x\right)^2}(2x - 4)$$

Reorder the factors of $-\frac{1}{\left(x^2 - 4x\right)^2}(2x - 4)$.

$$-(2x - 4)\frac{1}{\left(x^2 - 4x\right)^2}$$

Apply the distributive property.

$$(-(2x) - -4)\frac{1}{\left(x^2 - 4x\right)^2}$$

Multiply 2 by -1.

$$(-2x - -4)\frac{1}{\left(x^2 - 4x\right)^2}$$

Multiply -1 by -4.

$$(-2x+4)\frac{1}{(x^2-4x)^2}$$

Simplify the denominator.

Tap for fewer steps...

Factor x out of x^2-4x.

Tap for fewer steps...

Factor x out of x^2.

$$(-2x+4)\frac{1}{(x\cdot x-4x)^2}$$

Factor x out of $-4x$.

$$(-2x+4)\frac{1}{(x\cdot x+x\cdot -4)^2}$$

Factor x out of $x\cdot x+x\cdot -4$.

$$(-2x+4)\frac{1}{(x(x-4))^2}$$

Apply the product rule to $x(x-4)$.

$$(-2x+4)\frac{1}{x^2(x-4)^2}$$

Multiply $-2x+4$ by $\frac{1}{x^2(x-4)^2}$.

$$\frac{-2x+4}{x^2(x-4)^2}$$

Factor 2 out of $-2x+4$.

Tap for fewer steps...

Factor 2 out of $-2x$.

$$\frac{2(-x)+4}{x^2(x-4)^2}$$

Factor 2 out of 4.

$$\frac{2(-x)+2(2)}{x^2(x-4)^2}$$

Factor 2 out of $2(-x)+2(2)$.

$$\frac{2(-x+2)}{x^2(x-4)^2}$$

Factor -1 out of $-x$.

$$\frac{2(-(x)+2)}{x^2(x-4)^2}$$

Rewrite 2 as $-1(-2)$.

$$\frac{2(-(x)-1(-2))}{x^2(x-4)^2}$$

Factor -1 out of $-(x)-1(-2)$.

$$\frac{2(-(x-2))}{x^2(x-4)^2}$$

Rewrite $-(x-2)$ as $-1(x-2)$.

$$\frac{2(-1(x-2))}{x^2(x-4)^2}$$

Move the negative in front of the fraction.

$$f'(x) = -\frac{2(x-2)}{x^2(x-4)^2}$$

Find the second derivative.

Tap for fewer steps...

Since -2 is constant with respect to x, the derivative of $-\frac{2(x-2)}{x^2(x-4)^2}$ with respect to x is $-2\frac{d}{dx}\left[\frac{x-2}{x^2(x-4)^2}\right]$.

$$-2\frac{d}{dx}\left[\frac{x-2}{x^2(x-4)^2}\right]$$

Differentiate using the Quotient Rule which states that $\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right]$ is $\frac{g(x)\frac{d}{dx}[f(x)] - f(x)\frac{d}{dx}[g(x)]}{g(x)^2}$ where $f(x) = x-2$ and $g(x) = x^2(x-4)^2$.

$$-2\frac{x^2(x-4)^2\frac{d}{dx}[x-2] - (x-2)\frac{d}{dx}\left[x^2(x-4)^2\right]}{\left(x^2(x-4)^2\right)^2}$$

Differentiate.

Tap for fewer steps...

By the Sum Rule, the derivative of $x-2$ with respect to x is $\frac{d}{dx}[x] + \frac{d}{dx}[-2]$.

$$-2\frac{x^2(x-4)^2\left(\frac{d}{dx}[x] + \frac{d}{dx}[-2]\right) - (x-2)\frac{d}{dx}\left[x^2(x-4)^2\right]}{\left(x^2(x-4)^2\right)^2}$$

Differentiate using the Power Rule which states that $\frac{d}{dx}[x^n]$ is nx^{n-1} where $n=1$.

$$-2\frac{x^2(x-4)^2\left(1 + \frac{d}{dx}[-2]\right) - (x-2)\frac{d}{dx}\left[x^2(x-4)^2\right]}{\left(x^2(x-4)^2\right)^2}$$

Since -2 is constant with respect to x, the derivative of -2 with respect to x is 0.

$$-2\frac{x^2(x-4)^2(1+0) - (x-2)\frac{d}{dx}\left[x^2(x-4)^2\right]}{\left(x^2(x-4)^2\right)^2}$$

Simplify the expression.

Tap for fewer steps...

Add 1 and 0.

$$-2\frac{x^2(x-4)^2\cdot 1 - (x-2)\frac{d}{dx}\left[x^2(x-4)^2\right]}{\left(x^2(x-4)^2\right)^2}$$

Multiply x^2 by 1.

$$-2\frac{x^2(x-4)^2-(x-2)\frac{d}{dx}\left[x^2(x-4)^2\right]}{\left(x^2(x-4)^2\right)^2}$$

Differentiate using the Product Rule which states that $\frac{d}{dx}[f(x)\,g(x)]$ is $f(x)\frac{d}{dx}[g(x)]+g(x)\frac{d}{dx}[f(x)]$ where $f(x)=x^2$ and $g(x)=(x-4)^2$.

$$-2\frac{x^2(x-4)^2-(x-2)\left(x^2\frac{d}{dx}\left[(x-4)^2\right]+(x-4)^2\frac{d}{dx}\left[x^2\right]\right)}{\left(x^2(x-4)^2\right)^2}$$

Differentiate using the chain rule, which states that $\frac{d}{dx}[f(g(x))]$ is $f'(g(x))\,g'(x)$ where $f(x)=x^2$ and $g(x)=x-4$.

Tap for fewer steps...

To apply the Chain Rule, set u as $x-4$.

$$-2\frac{x^2(x-4)^2-(x-2)\left(x^2\left(\frac{d}{du}\left[u^2\right]\frac{d}{dx}[x-4]\right)+(x-4)^2\frac{d}{dx}\left[x^2\right]\right)}{\left(x^2(x-4)^2\right)^2}$$

Differentiate using the Power Rule which states that $\frac{d}{du}[u^n]$ is nu^{n-1} where $n=2$.

$$-2\frac{x^2(x-4)^2-(x-2)\left(x^2\left(2u\frac{d}{dx}[x-4]\right)+(x-4)^2\frac{d}{dx}\left[x^2\right]\right)}{\left(x^2(x-4)^2\right)^2}$$

Replace all occurrences of u with $x-4$.

$$-2\frac{x^2(x-4)^2-(x-2)\left(x^2\left(2(x-4)\frac{d}{dx}[x-4]\right)+(x-4)^2\frac{d}{dx}\left[x^2\right]\right)}{\left(x^2(x-4)^2\right)^2}$$

Differentiate.

Tap for fewer steps...

By the Sum Rule, the derivative of $x-4$ with respect to x is $\frac{d}{dx}[x]+\frac{d}{dx}[-4]$.

$$-2\frac{x^2(x-4)^2-(x-2)\left(x^2\left(2\left(x-4\right)\left(\frac{d}{dx}[x]+\frac{d}{dx}[-4]\right)\right)+(x-4)^2\frac{d}{dx}\left[x^2\right]\right)}{\left(x^2(x-4)^2\right)^2}$$

Differentiate using the Power Rule which states that $\frac{d}{dx}[x^n]$ is nx^{n-1} where $n=1$.

$$-2\frac{x^2(x-4)^2-(x-2)\left(x^2\left(2\left(x-4\right)\left(1+\frac{d}{dx}[-4]\right)\right)+(x-4)^2\frac{d}{dx}\left[x^2\right]\right)}{\left(x^2(x-4)^2\right)^2}$$

Since -4 is constant with respect to x, the derivative of -4 with respect to x is 0.

$$-2\frac{x^2(x-4)^2-(x-2)\left(x^2\left(2\left(x-4\right)(1+0)\right)+(x-4)^2\frac{d}{dx}\left[x^2\right]\right)}{\left(x^2(x-4)^2\right)^2}$$

Simplify the expression.

Tap for fewer steps...

Add 1 and 0.

$$-2\frac{x^2(x-4)^2-(x-2)\left(x^2\left(2\left(x-4\right)\cdot 1\right)+(x-4)^2\frac{d}{dx}\left[x^2\right]\right)}{\left(x^2(x-4)^2\right)^2}$$

Multiply 2 by 1.

$$-2\frac{x^2(x-4)^2-(x-2)\left(x^2\left(2\left(x-4\right)\right)+(x-4)^2\frac{d}{dx}\left[x^2\right]\right)}{\left(x^2(x-4)^2\right)^2}$$

Differentiate using the Power Rule which states that $\frac{d}{dx}[x^n]$ is nx^{n-1} where $n=2$.

$$-2\frac{x^2(x-4)^2-(x-2)\left(x^2\left(2\left(x-4\right)\right)+(x-4)^2(2x)\right)}{\left(x^2(x-4)^2\right)^2}$$

Combine fractions.

Tap for fewer steps...

Move 2 to the left of $(x-4)^2$.

$$-2\frac{x^2(x-4)^2-(x-2)\left(x^2\left(2\left(x-4\right)\right)+2\cdot(x-4)^2x\right)}{\left(x^2(x-4)^2\right)^2}$$

Combine -2 and $\frac{x^2(x-4)^2-(x-2)\left(x^2\left(2\left(x-4\right)\right)+2(x-4)^2x\right)}{\left(x^2(x-4)^2\right)^2}$.

$$\frac{-2\left(x^2(x-4)^2-(x-2)\left(x^2\left(2\left(x-4\right)\right)+2(x-4)^2x\right)\right)}{\left(x^2(x-4)^2\right)^2}$$

Move the negative in front of the fraction.

$$-\frac{(2)\left(x^2(x-4)^2-(x-2)\left(x^2\left(2\left(x-4\right)\right)+2(x-4)^2x\right)\right)}{\left(x^2(x-4)^2\right)^2}$$

Simplify.

Tap for fewer steps...

Apply the product rule to $x^2(x-4)^2$.

$$-\frac{2\left(x^2(x-4)^2-(x-2)\left(x^2\left(2\left(x-4\right)\right)+2(x-4)^2x\right)\right)}{(x^2)^2\left((x-4)^2\right)^2}$$

Apply the distributive property.

$$-\frac{2\left(x^2(x-4)^2+(-x--2)\left(x^2\left(2\left(x-4\right)\right)+2(x-4)^2x\right)\right)}{(x^2)^2\left((x-4)^2\right)^2}$$

Apply the distributive property.

$$-\frac{2\left(x^2(x-4)^2+(-x--2)\left(x^2\left(2x+2\cdot-4\right)+2(x-4)^2x\right)\right)}{(x^2)^2\left((x-4)^2\right)^2}$$

Apply the distributive property.

$$-\frac{2\left(x^2(x-4)^2+(-x--2)\left(x^2\left(2x\right)+x^2\left(2\cdot-4\right)+2(x-4)^2x\right)\right)}{\left(x^2\right)^2\left((x-4)^2\right)^2}$$

Apply the distributive property.

$$-\frac{2\left(x^2(x-4)^2\right)+2\left((-x--2)\left(x^2\left(2x\right)+x^2\left(2\cdot-4\right)+2(x-4)^2x\right)\right)}{\left(x^2\right)^2\left((x-4)^2\right)^2}$$

Simplify the numerator.

Tap for fewer steps...

Factor 2 out of $2x^2(x-4)^2+2\left(-x--2\right)\left(x^2\left(2x\right)+x^2\cdot2\cdot-4+2(x-4)^2x\right)$.

Tap for more steps...

$$-\frac{2\left(x^2(x-4)^2+(-x--2)\left(x^2\left(2x\right)+x^2\cdot2\cdot-4+2(x-4)^2x\right)\right)}{\left(x^2\right)^2\left((x-4)^2\right)^2}$$

Rewrite $(x-4)^2$ as $(x-4)\left(x-4\right)$.

$$-\frac{2\left(x^2\left((x-4)\left(x-4\right)\right)+(-x--2)\left(x^2\left(2x\right)+x^2\cdot2\cdot-4+2(x-4)^2x\right)\right)}{\left(x^2\right)^2\left((x-4)^2\right)^2}$$

Expand $(x-4)\left(x-4\right)$ using the FOIL Method.

Tap for more steps...

$$-\frac{2\left(x^2\left(x\cdot x+x\cdot-4-4x-4\cdot-4\right)+(-x--2)\left(x^2\left(2x\right)+x^2\cdot2\cdot-4+2(x-\right.\right.}{\left(x^2\right)^2\left((x-4)^2\right)^2}$$

Simplify and combine like terms.

Tap for more steps...

$$-\frac{2\left(x^2\left(x^2-8x+16\right)+(-x--2)\left(x^2\left(2x\right)+x^2\cdot2\cdot-4+2(x-4)^2x\right)\right)}{\left(x^2\right)^2\left((x-4)^2\right)^2}$$

Apply the distributive property.

$$-\frac{2\left(x^2x^2+x^2\left(-8x\right)+x^2\cdot 16+\left(-x--2\right)\left(x^2\left(2x\right)+x^2\cdot 2\cdot -4+2(x-4)^2x\right)\right)}{\left(x^2\right)^2\left(\left(x-4\right)^2\right)^2}$$

Simplify.

Tap for more steps...

$$-\frac{2\left(x^4-8x^2x+16\cdot x^2+\left(-x--2\right)\left(x^2\left(2x\right)+x^2\cdot 2\cdot -4+2(x-4)^2x\right)\right)}{\left(x^2\right)^2\left(\left(x-4\right)^2\right)^2}$$

Multiply x^2 by x by adding the exponents.

Tap for more steps...

$$-\frac{2\left(x^4-8x^3+16x^2+\left(-x--2\right)\left(x^2\left(2x\right)+x^2\cdot 2\cdot -4+2(x-4)^2x\right)\right)}{\left(x^2\right)^2\left(\left(x-4\right)^2\right)^2}$$

Multiply -1 by -2.

$$-\frac{2\left(x^4-8x^3+16x^2+\left(-x+2\right)\left(x^2\left(2x\right)+x^2\cdot 2\cdot -4+2(x-4)^2x\right)\right)}{\left(x^2\right)^2\left(\left(x-4\right)^2\right)^2}$$

Simplify each term.

Tap for more steps...

$$-\frac{2\left(x^4-8x^3+16x^2+\left(-x+2\right)\left(2x^3-8x^2+2x^3-16x^2+32x\right)\right)}{\left(x^2\right)^2\left(\left(x-4\right)^2\right)^2}$$

Add $2x^3$ and $2x^3$.

$$-\frac{2\left(x^4-8x^3+16x^2+\left(-x+2\right)\left(4x^3-8x^2-16x^2+32x\right)\right)}{\left(x^2\right)^2\left(\left(x-4\right)^2\right)^2}$$

Subtract $16x^2$ from $-8x^2$.

$$-\frac{2\left(x^4-8x^3+16x^2+(-x+2)\left(4x^3-24x^2+32x\right)\right)}{\left(x^2\right)^2\left((x-4)^2\right)^2}$$

Expand $(-x+2)\left(4x^3-24x^2+32x\right)$ by multiplying each term in the first expression by each term in the second expression.

$$-\frac{2\left(x^4-8x^3+16x^2-x\left(4x^3\right)-x\left(-24x^2\right)-x\left(32x\right)+2\left(4x^3\right)+2\left(-24x^2\right)+2\right.}{\left(x^2\right)^2\left((x-4)^2\right)^2}$$

Simplify each term.

Tap for more steps...

$$-\frac{2\left(x^4-8x^3+16x^2-4x^4+24x^3-32x^2+8x^3-48x^2+64x\right)}{\left(x^2\right)^2\left((x-4)^2\right)^2}$$

Add $24x^3$ and $8x^3$.

$$-\frac{2\left(x^4-8x^3+16x^2-4x^4+32x^3-32x^2-48x^2+64x\right)}{\left(x^2\right)^2\left((x-4)^2\right)^2}$$

Subtract $48x^2$ from $-32x^2$.

$$-\frac{2\left(x^4-8x^3+16x^2-4x^4+32x^3-80x^2+64x\right)}{\left(x^2\right)^2\left((x-4)^2\right)^2}$$

Subtract $4x^4$ from x^4.

$$-\frac{2\left(-3x^4-8x^3+16x^2+32x^3-80x^2+64x\right)}{\left(x^2\right)^2\left((x-4)^2\right)^2}$$

Add $-8x^3$ and $32x^3$.

$$-\frac{2\left(-3x^4+24x^3+16x^2-80x^2+64x\right)}{\left(x^2\right)^2\left((x-4)^2\right)^2}$$

Subtract $80x^2$ from $16x^2$.

$$-\frac{2\left(-3x^4+24x^3-64x^2+64x\right)}{(x^2)^2\left((x-4)^2\right)^2}$$

Rewrite $-3x^4+24x^3-64x^2+64x$ in a factored form.

Tap for more steps...

$$-\frac{2x(x-4)\left(-3x^2+12x-16\right)}{(x^2)^2\left((x-4)^2\right)^2}$$

Combine terms.

Tap for fewer steps...

Multiply the exponents in $(x^2)^2$.

Tap for fewer steps...

Apply the power rule and multiply exponents, $(a^m)^n = a^{mn}$.

$$-\frac{2x(x-4)\left(-3x^2+12x-16\right)}{x^{2\cdot 2}\left((x-4)^2\right)^2}$$

Multiply 2 by 2.

$$-\frac{2x(x-4)\left(-3x^2+12x-16\right)}{x^4\left((x-4)^2\right)^2}$$

Multiply the exponents in $\left((x-4)^2\right)^2$.

Tap for fewer steps...

Apply the power rule and multiply exponents, $(a^m)^n = a^{mn}$.

$$-\frac{2x(x-4)\left(-3x^2+12x-16\right)}{x^4(x-4)^{2\cdot 2}}$$

Multiply 2 by 2.

$$-\frac{2x(x-4)\left(-3x^2+12x-16\right)}{x^4(x-4)^4}$$

Cancel the common factor of x and x^4.

Tap for fewer steps...

Factor x out of $2x(x-4)\left(-3x^2+12x-16\right)$.

$$-\frac{x\left((2(x-4))\left(-3x^2+12x-16\right)\right)}{x^4(x-4)^4}$$

Cancel the common factors.

Tap for more steps...

$$-\frac{(2(x-4))\left(-3x^2+12x-16\right)}{x^3(x-4)^4}$$

Cancel the common factor of $x-4$ and $(x-4)^4$.

Tap for fewer steps...

Factor $x-4$ out of $2(x-4)\left(-3x^2+12x-16\right)$.

$$-\frac{(x-4)\left(2\left(-3x^2+12x-16\right)\right)}{x^3(x-4)^4}$$

Cancel the common factors.

Tap for more steps...

$$-\frac{2\left(-3x^2+12x-16\right)}{x^3(x-4)^3}$$

Factor -1 out of $-3x^2$.

$$-\frac{2\left(-\left(3x^2\right)+12x-16\right)}{x^3(x-4)^3}$$

Factor -1 out of $12x$.

$$-\frac{2\left(-\left(3x^2\right)-(-12x)-16\right)}{x^3(x-4)^3}$$

Factor -1 out of $-\left(3x^2\right)-(-12x)$.

$$-\frac{2\left(-\left(3x^2-12x\right)-16\right)}{x^3(x-4)^3}$$

Rewrite -16 as $-1(16)$.

$$-\frac{2\left(-\left(3x^2-12x\right)-1(16)\right)}{x^3(x-4)^3}$$

Factor -1 out of $-\left(3x^2-12x\right)-1(16)$.

$$-\frac{2\left(-\left(3x^2-12x+16\right)\right)}{x^3(x-4)^3}$$

Rewrite $-\left(3x^2-12x+16\right)$ as $-1\left(3x^2-12x+16\right)$.

$$-\frac{2\left(-1\left(3x^2-12x+16\right)\right)}{x^3(x-4)^3}$$

Move the negative in front of the fraction.

$$--\frac{2\left(3x^2-12x+16\right)}{x^3(x-4)^3}$$

Multiply -1 by -1.

$$1\frac{2\left(3x^2-12x+16\right)}{x^3(x-4)^3}$$

Multiply $\frac{2\left(3x^2-12x+16\right)}{x^3(x-4)^3}$ by 1.

$$f''(x)=\frac{2\left(3x^2-12x+16\right)}{x^3(x-4)^3}$$