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Machine Learning
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Rappel Calcul matriciel

Produit Matriciel

$$A \cdot B = C$$

$$\underbrace{d1\times d2 \quad d2\times d3}_{} \qquad d1\times d3$$

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \quad 2\times 2 \qquad B = \begin{bmatrix} e & f & g \\ h & i & j \end{bmatrix} \quad 2\times 3$$

$ae + bh$

$$A \cdot B = \begin{bmatrix} - & - & - \\ - & - & - \end{bmatrix} \quad 2\times 3$$

Transposé

$$A = \begin{bmatrix} a & b \\ c & d \\ e & f \end{bmatrix} \quad 3\times 2$$

$$A^T = \begin{bmatrix} a & c & e \\ b & d & f \end{bmatrix} \quad 2\times 3$$

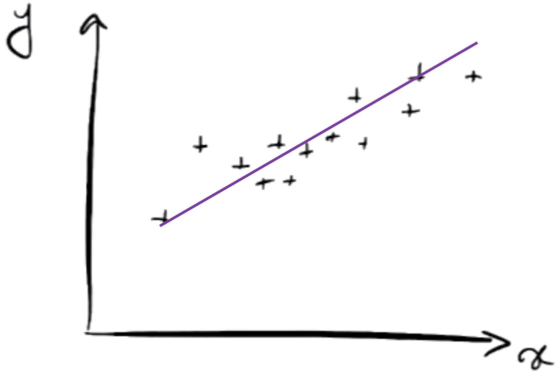
Transformation matricielle

Modèle (Régression linéaire simple)

$$f(x) = a\,x + b$$

$$F = X \cdot \Theta$$

$$\underbrace{\begin{bmatrix} f(x^{(1)}) \\ f(x^{(2)}) \\ \vdots \\ f(x^{(m)}) \end{bmatrix}}_{m \times 1} \quad \underbrace{\begin{bmatrix} x^{(1)} & 1 \\ \vdots & \vdots \\ x^{(m)} & 1 \end{bmatrix}}_{m \times 2} \quad \underbrace{\begin{bmatrix} a \\ b \end{bmatrix}}_{2 \times 1}$$



x	y
$x^{(1)}$	$y^{(1)}$
$\vdots$	$\vdots$
$x^{(m)}$	$y^{(m)}$

m

$n = 1$

Transformation matricielle

Fonction de coût

$$\underbrace{J(a,b)}_{1\times 1} = \frac{1}{2m}\sum_{i=1}^{m}\Big(\underbrace{\underbrace{a x^{(i)} + b}_{X\,.\,\Theta \atop m\times 1} - \underbrace{y^{(i)}}_{Y \atop m\times 1}}_{m\times 1}\Big)^2$$

$$\frac{1}{2m} \times \begin{bmatrix} - \\ - \\ - \\ - \\ - \\ - \end{bmatrix}^2$$

$$J(\Theta) = \frac{1}{2m}\sum (X\Theta - Y)^2$$

Le Gradient

$$\begin{bmatrix} \dfrac{\partial J(a,b)}{\partial a} \\[2ex] \dfrac{\partial J(a,b)}{\partial b} \end{bmatrix} \begin{matrix} = \dfrac{1}{m}\sum x(ax+b-y) \\[2ex] = \dfrac{1}{m}\sum 1\times(ax+b-y) \end{matrix}$$

$$\frac{\partial J(\Theta)}{\partial \Theta} = \frac{1}{m} X^T (X\Theta - Y)$$

$$X = \begin{bmatrix} x^{(1)} & 1 \\ \vdots & \vdots \\ x^{(m)} & 1 \end{bmatrix}$$

$$X^T = \begin{bmatrix} x^{(1)} & x^{(2)} & \dots & x^{(m)} \\ 1 & 1 & \dots & 1 \end{bmatrix}$$

Transformation matricielle

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Descente de gradient

$$\left\{ \begin{bmatrix} a \\ b \end{bmatrix} \begin{matrix} = a - \alpha \dfrac{\partial J}{\partial a} \\ = b - \alpha \dfrac{\partial J}{\partial b} \end{matrix} \right\}$$

$$\Downarrow$$

$$\left\{ \theta = \theta - \alpha \frac{\partial J}{\partial \theta} \right\}$$

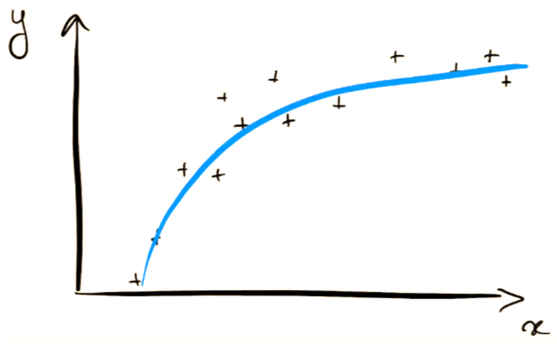
$$X = \underbrace{\begin{bmatrix} x^{(1)} & 1 \\ \dots & \dots \\ x^{(m)} & 1 \end{bmatrix}}_{m \times (n+1)} \qquad \theta = \underbrace{\begin{bmatrix} a \\ b \end{bmatrix}}_{(n+1) \times 1} \qquad Y = \underbrace{\begin{bmatrix} y^{(1)} \\ \dots \\ y^{(m)} \end{bmatrix}}_{m \times 1}$$

$$\underbrace{F = X.\theta}_{m \times 1} \qquad \underbrace{J(\theta)}_{1 \times 1} = \frac{1}{2m} \sum (X.\theta - Y)^2$$

$$\underbrace{\frac{\partial J(\theta)}{\partial \theta}}_{(n+1) \times 1} = \frac{1}{m} X^T.(X.\theta - Y)$$

Transformation matricielle

Modèle (Régression polynomiale)



$$f(x) = ax^2 + bx + c$$

$$F = X \cdot \Theta$$

$$\underbrace{\begin{bmatrix} x^{2(1)} & x^{(1)} & 1 \\ \vdots & \vdots & \vdots \\ x^{2(m)} & x^{(m)} & 1 \end{bmatrix}}_{m \times 3} \cdot \underbrace{\begin{bmatrix} a \\ b \\ c \end{bmatrix}}_{3 \times 1}$$

Modèle (Régression multiple)

$$f(x) = a x_1 + b x_2 + c$$

$$F = X \cdot \theta$$

$$\underbrace{\begin{bmatrix} x_1^{(1)} & x_2^{(1)} & 1 \\ \vdots & \vdots & \vdots \\ x_1^{(m)} & x_2^{(m)} & 1 \end{bmatrix}}_{m \times 3} \quad \underbrace{\begin{bmatrix} a \\ b \\ c \end{bmatrix}}_{3 \times 1}$$

Evaluation

Coefficient de détermination

$$R^2 = 1 - \frac{\sum(y - f(x))^2}{\sum(y - \bar{y})^2}$$

- Permet d’évaluer les performances d’un modèle de ML
- Plus il est proche de 1 plus le modèle est performant