

αομ. 39

$$Z = X + Y - |X - Y|$$

α)

X \ Y	1	2	3	4	5	$P_X(X)$
3	$\frac{2}{15}$	$\frac{2}{15}$	$\frac{1}{15}$	$\frac{1}{15}$	$\frac{1}{30}$	$\frac{13}{30}$
4	$\frac{2}{15}$	$\frac{1}{15}$	$\frac{1}{15}$	$\frac{1}{15}$	$\frac{1}{30}$	$\frac{11}{30}$
5	$\frac{1}{15}$	$\frac{1}{30}$	$\frac{1}{30}$	$\frac{1}{30}$	$\frac{1}{30}$	$\frac{6}{30}$
$P_Y(Y)$	$\frac{5}{15}$	$\frac{4}{30}$	$\frac{5}{30}$	$\frac{5}{30}$	$\frac{3}{30}$	

$$\text{COV}(X, Y) = E[(Y - E(Y))(X - E(X))] \quad (1)$$

$$E(X) = 3 \cdot \frac{13}{30} + 4 \cdot \frac{11}{30} + 5 \cdot \frac{6}{30} = \frac{39 + 44 + 30}{30} = \frac{113}{30}$$

$$E(Y) = 1 \cdot \frac{10}{30} + 2 \cdot \frac{7}{30} + 3 \cdot \frac{5}{30} + 4 \cdot \frac{5}{30} + 5 \cdot \frac{3}{30} = \frac{10 + 14 + 15 + 20 + 15}{30} = \frac{74}{30}$$

$$\begin{aligned} (1) \Rightarrow & \frac{2}{15} \left(1 - \frac{74}{30}\right) \left(3 - \frac{113}{30}\right) + \frac{2}{15} \left(2 - \frac{74}{30}\right) \left(3 - \frac{113}{30}\right) + \frac{1}{15} \left(3 - \frac{74}{30}\right) \left(3 - \frac{113}{30}\right) \\ & + \frac{1}{15} \left(4 - \frac{74}{30}\right) \left(3 - \frac{113}{30}\right) + \frac{1}{30} \left(5 - \frac{74}{30}\right) \left(3 - \frac{113}{30}\right) + \frac{2}{15} \left(1 - \frac{74}{30}\right) \left(4 - \frac{113}{30}\right) \\ & + \frac{1}{15} \left(3 - \frac{74}{30}\right) \left(4 - \frac{113}{30}\right) + \frac{1}{30} \left(5 - \frac{74}{30}\right) \left(4 - \frac{113}{30}\right) + \frac{1}{15} \left(1 - \frac{74}{30}\right) \left(5 - \frac{113}{30}\right) \\ & + \frac{1}{30} \left(2 - \frac{74}{30}\right) \left(5 - \frac{113}{30}\right) + \frac{1}{30} \left(3 - \frac{74}{30}\right) \left(5 - \frac{113}{30}\right) + \frac{1}{30} \left(4 - \frac{74}{30}\right) \left(5 - \frac{113}{30}\right) \\ & + \frac{1}{30} \left(5 - \frac{74}{30}\right) \left(5 - \frac{113}{30}\right) = \frac{218}{900} \Rightarrow \boxed{\text{COV}(X, Y) = \frac{109}{450}} \end{aligned}$$

X \ Y	1	2	3	4	5
3	$3+1- 3-1 =2$	$3+2- 3-2 =4$	$3+3- 3-3 =6$	$3+4- 3-4 =6$	$3+5- 3-5 =6$
4	$4+1- 4-1 =2$	$4+2- 4-2 =4$	$4+3- 4-3 =6$	$4+4- 4-4 =8$	$4+5- 4-5 =8$
5	$5+1- 5-1 =2$	$5+2- 5-2 =4$	$5+3- 5-3 =6$	$5+4- 5-4 =8$	$5+5- 5-5 =10$

$$P(Z=2) = P_{YX}(1,3) + P_{YX}(1,4) + P_{YX}(1,5) = \frac{2}{15} + \frac{2}{15} + \frac{1}{15} = \frac{5}{15} = \frac{1}{3}$$

$$P(Z=4) = P_{YX}(2,3) + P_{YX}(2,4) + P_{YX}(2,5) = \frac{2}{15} + \frac{1}{15} + \frac{1}{30} = \frac{4+2+1}{30} = \frac{7}{30}$$

$$P(Z=6) = P_{YX}(3,3) + P_{YX}(3,4) + P_{YX}(3,5) + P_{YX}(4,3) + P_{YX}(5,3) = \frac{2+2+1+2+1}{30} = \frac{8}{30} = \frac{4}{15}$$

$$P(z=8) = P_{YX}(3,4) + P_{YX}(4,4) + P_{YX}(5,4) = \frac{1}{30} + \frac{1}{15} + \frac{1}{30} = \frac{4}{30} = \frac{2}{15}$$

$$P(z=10) = P_{YX}(5,5) = \frac{1}{30}$$

$$\textcircled{8} E(z) = 2 \cdot \frac{1}{3} + 4 \cdot \frac{7}{30} + 6 \cdot \frac{4}{15} + 8 \cdot \frac{2}{15} + 10 \cdot \frac{1}{30} = \frac{20 + 4 \cdot 7 + 8 \cdot 6 + 4 \cdot 8 + 10}{30} = \frac{138}{30} = \frac{23}{5}$$

$$= \frac{23}{5} \Rightarrow E(z) = \frac{23}{5}$$

200.40

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X \ Y	1	2	3	4	P <sub>Y</sub>
1	1/10	1/10	1/10	1/10	2/5
2	1/10	3/10	1/10	0	3/10
3	1/10	1/10	0	0	1/5
4	1/10	0	0	0	1/10
P <sub>X</sub>	2/5	3/10	1/5	1/10	

$$P(X=1, Y=1) = \frac{2}{5} \cdot \frac{1}{4} = \frac{1}{20} = \frac{1}{10}$$

$$P(X=2, Y=1) = \frac{3}{5} \cdot \frac{2}{4} \cdot \frac{1}{3} = \frac{6}{60} = \frac{1}{10}$$

$$P(X=3, Y=1) = \frac{3}{5} \cdot \frac{2}{4} \cdot \frac{2}{3} \cdot \frac{1}{2} = \frac{12}{120} = \frac{1}{10}$$

$$P(X=4, Y=1) = \frac{3}{5} \cdot \frac{2}{4} \cdot \frac{1}{3} \cdot \frac{2}{2} \cdot \frac{1}{1} = \frac{6}{60} = \frac{1}{10}$$

$$P(X=1, Y=2) = \frac{2}{5} \cdot \frac{3}{4} \cdot \frac{1}{3} = \frac{2}{20} = \frac{1}{10}$$

$$P(X=1, Y=3) = \frac{2}{5} \cdot \frac{3}{4} \cdot \frac{2}{3} \cdot \frac{1}{2} = \frac{2}{20} = \frac{1}{10}$$

$$P(X=1, Y=4) = \frac{2}{5} \cdot \frac{3}{4} \cdot \frac{2}{3} \cdot \frac{1}{2} \cdot 1 = \frac{2}{20} = \frac{1}{10}$$

$$P(X=2, Y=2) = \frac{3}{5} \cdot \frac{2}{4} \cdot \frac{2}{3} \cdot \frac{1}{2} = \frac{2}{20} = \frac{1}{10}$$

$$P(X=3, Y=2) = \frac{3}{5} \cdot \frac{2}{4} \cdot \frac{2}{3} \cdot \frac{1}{2} \cdot 1 = \frac{1}{10}$$

$$P(X=4, Y=2) = 0$$

$$P(X=2, Y=3) = \frac{3}{5} \cdot \frac{2}{4} \cdot \frac{2}{3} \cdot \frac{1}{2} \cdot 1 = \frac{1}{10}$$

$$P(X=3, Y=3) = 0, P(X=4, Y=3) = 0, P(X=2, Y=4) = 0, P(X=3, Y=4) = 0, P(X=4, Y=4) = 0$$

$$\textcircled{6} E(X) = 1 \cdot \frac{2}{5} + 2 \cdot \frac{3}{10} + 3 \cdot \frac{1}{5} + 4 \cdot \frac{1}{10} = \frac{4 + 6 + 6 + 4}{10} = 2$$

$$E(Y) = 1 \cdot \frac{2}{5} + 2 \cdot \frac{3}{10} + 3 \cdot \frac{1}{5} + 4 \cdot \frac{1}{10} = \frac{4 + 6 + 6 + 4}{10} = 2$$

$$\begin{aligned} \text{cov}(X, Y) &= \frac{1}{10}(1-2)^2 + \frac{1}{10}(2-2)(1-2) \\ &+ \frac{1}{10}(3-2)(1-2) + \frac{1}{10}(4-2)(1-2) + \\ &\frac{1}{10}(1-2)(2-2) + \frac{1}{10}(2-2)^2 + \frac{1}{10}(3-2)(2-2) \\ &+ \frac{1}{10}(3-2)(1-2) + \frac{1}{10}(2-2)(3-2) + \\ &\frac{1}{10}(4-2)(1-2) = \frac{1}{10} - \frac{1}{10} - \frac{1}{10} - \frac{1}{10} - \frac{1}{10} - \frac{1}{10} - \frac{1}{10} - \frac{1}{10} - \frac{1}{10} - \frac{1}{10} = -\frac{5}{10} = -\frac{1}{2} \end{aligned}$$

$$\boxed{\text{cov}(X, Y) = -\frac{1}{2}}$$

ασκ. 40

η διαφορετικά κουπόνια

Έστω  $X_i$  ο αριθμός των δοκιμών που θα γίνουν μέχρι να εμφανιστεί το  $n$ -οστό διαφορετικό κουπόνι.  $H \quad X \sim \text{Geom}(p)$ .

Έχουμε ότι στη  $i-1$  προσπάθεια έχουμε δει και τα  $n-1$  διαφορετικά κουπόνια, και ψάχνουμε πόσες ακόμα προσπάθειες πρέπει να γίνουν ώστε να εμφανιστεί το  $n$ -οστό.  $\therefore p_i = \frac{n-(i-1)}{n}$

Από τη θεωρία έχουμε ότι η μέση τιμή είναι

$$\frac{1}{p_i} = \frac{n}{n-(i-1)} = \frac{n}{n-i+1}$$

Άρα

$$E(X) = \sum_{i=1}^n \frac{n}{n-i+1}$$

ασκ. 41  $n=30$  άτομα η Α διαλέγει 10, με  $\binom{30}{10}$  τρόπους.

η Β διαλέγει 5, με  $\binom{30}{5}$  τρόπους.

(α) Έστω  $E_i$  το ενδεχόμενο το άτομο  $u$  να επιλεγεί  $i$ -φορές, με  $i=0, 1, 2$ .

$$P(E_0) = \frac{\binom{29}{10} \binom{29}{5}}{\binom{30}{10} \binom{30}{5}}$$

$$P(E_1) = \frac{\binom{29}{9} \binom{29}{6}}{\binom{30}{10} \binom{30}{5}} + \frac{\binom{29}{10} \binom{29}{4}}{\binom{30}{10} \binom{30}{5}}$$

$$P(E_2) = \frac{\binom{29}{8} \binom{29}{7}}{\binom{30}{10} \binom{30}{5}}$$

(β) Ο γενικός τύπος είναι:

$$P(X=x) = 1 - \frac{\binom{30-x}{10}}{\binom{30}{10}} \cdot \frac{\binom{30-x}{5}}{\binom{30}{5}}, \quad x=10, 11, 12, 13, 14, 15$$

Επειδή  $P(X=x) = 1 - P(\text{οι παίκτες να επιλέξουν αριθμούς 0 φορές})$

(γ) Έτσι επιτυχία = να μην επιλεγεί και αποτυχία = επιλεχθεί