Lyceum

Aristotle's Lyceum is the institution considered to be the forerunner of the modern university. Opened in 335 BC, the Lyceum was a center of study and research in both science and philosophy.

Optimality in Horse Racing

Previously in the Lyceum…

We saw recently how odds are established by bookies. We even saw how to spot arbitrage opportunities. In practice, of course, you could spend a lifetime looking for arbitrage opportunities that rarely occur in real life. In this Lyceum we are going to see if we can exploit the difference between the odds as set by the bookie and the odds that you estimate. Remember, the odds set by the bookie are really determined by the wagers placed, which are more to do with irrational sentiment ("I'm going to bet on this horse 'cos it's got the same name as the pet rat I had when I was a child") than with a cold-hearted estimation of the probabilities.

We need some more notation. Let's use p_i as the probability of the ith horse winning the race. This is supposed to be the real probability, not the bookie's probability. Obviously, the odds must sum to one:

$$\sum_{i=1}^{N} p_i = 1 .$$

If we wager w_i on the ith horse then we expect to make

$$m = \left(\sum_{i=1}^{N} p_i w_i (q_i + 1) \right) - 1 . \qquad (1)$$

In this the qs are the bookies odds (see last issue). This is under the assumption that the total wager, the sum of all the ws, is one. An obvious goal is to make this quantity positive, we want to get a positive return on average. But there may be many ways to make this positive. How do we decide which way is best?

Another quantity we might want to look at is the standard deviation of winnings. This is given by

$$\sqrt{\sum_{i=1}^{N} p_i \left(w_i (q_i + 1) - 1 - m \right)^2} . \qquad (2)$$

This measures the dispersion of winnings about the average, and is often interpreted as a measure of risk. If this were zero our profit or loss would be a sure thing.

Here's an example.

Horse	Bookie's odds	Your Estimate of Probability	Wager
Nijinsky	5	0.2	
Red Rum	6	0.2	
Oxo	1	0.1	
Red Marauder	1	0.1	
Gay Lad	2	0.1	
Roquefort	2	0.1	
Red Alligator	2	0.1	
Shergar	2	0.1	

How should you bet? The following calculations are easily done on a spreadsheet.

Scenario 1: Maximize expected return

Since you place no premium on reducing risk you should bet everything on the horse that maximizes

$$p_i (q_i + 1) .$$

In this case, that is Red Rum. The expected return is 40% with a standard deviation of 280%. A very risky bet!

Horse	Bookie's odds	Your Estimate of Probability	Wager
Nijinsky	5	0.2	0
Red Rum	6	0.2	1
Oxo	1	0.1	0
Red Marauder	1	0.1	0
Gay Lad	2	0.1	0
Roquefort	2	0.1	0
Red Alligator	2	0.1	0
Shergar	2	0.1	0

Scenario 2: Minimize standard deviation

An interesting strategy. The solution is given below.

Horse	Bookie's odds	Your Estimate of Probability	Wager
Nijinsky	5	0.2	0.063062
Red Rum	6	0.2	0.054068
Oxo	1	0.1	0.189203
Red Marauder	1	0.1	0.189246
Gay Lad	2	0.1	0.126108
Roquefort	2	0.1	0.126108
Red Alligator	2	0.1	0.126108
Shergar	2	0.1	0.126097

I say 'interesting' because this strategy results in zero standard deviation, and a return of -62 per cent. In other words, a guaranteed loss!

Scenario 3: Maximize return divided by standard deviation

A strategy that seeks to benefit from a positive expectation but with a smaller risk. For mathematical reasons (the Central Limit Theorem) this is a natural strategy. The solution is given below.

The expected return is now 31% with a standard deviation of 164%.

Horse	Bookie's odds	Your Estimate of Probability	Wager
Nijinsky	5	0.2	0.459016
Red Rum	6	0.2	0.540984
Oxo	1	0.1	0
Red Marauder	1	0.1	0
Gay Lad	2	0.1	0
Roquefort	2	0.1	0
Red Alligator	2	0.1	0
Shergar	2	0.1	0