Aristotle's Lyceum is the institution considered to be the forerunner of the modern university. Opened in 335 BC, the Lyceum was a center of study and research in both science and philosophy.

The different types of mathematics seen in quantitative finance

The real-world subject of quantitative finance uses tools from many branches of mathematics. And financial modeling can be approached in a variety of different ways. For some strange reason the advocates of different branches of mathematics get quite emotional when discussing the merits and demerits of their methodologies and those of their 'opponents.' Is this a territorial thing, what are the pros and cons of martingales and differential equations, what is all this fuss and will it end in tears before bedtime? Always both informative and objective, Team Wilmott gives you its unbiased analysis of the different camps. This is the first in a series of tutorials on the mathematics of finance.

Financial modeling

Here's a list the various approaches to modeling and a selection of useful tools. The distinction between a 'modeling approach' and a 'tool' will start to become clear.

Modeling approaches:

- Probabilistic
- Deterministic
- Discrete: difference equations
- Continuous: differential equations

Useful tools:

- Simulations
- Approximations
- Asymptotic analysis
- Series solutions
- Discretization methods
- Green's functions

While these are not exactly arbitrary lists, they are certainly open to some criticism or addition. Let's first take a look at the Modeling Approaches.

Probabilistic: One of the main assumptions about the financial markets, at least as far as quantitative finance goes, is that asset prices are random. We tend to think of describing financial variables as following some random path, with parameters describing the growth of the asset and its degree of randomness. We effectively model the asset path via a specified rate of growth, on average, and its deviation from that average. This approach to modeling has had the greatest impact over the last 30 years, leading to the explosive growth of the derivatives markets.

Deterministic: The idea behind this approach is that our model will tell us everything about the future. Given enough data, and a big enough brain, we can write down some equations or an algorithm for predicting the future. Interestingly, the subject of dynamical systems and chaos fall into this category. And, as you know, chaotic systems show such sensitivity to initial conditions that predictability is in practice impossible. This is the 'butterfly effect,' that a butterfly flapping its wings in Brazil will 'cause' rainfall over Manchester. (Like what doesn't!) A topic popular in the early 1990s, this has not lived up to its promises in the financial world.

Discrete/Continuous: Whether probabilistic or deterministic the eventual model you write down can be discrete or continuous. Discrete means that asset prices and/or time can only be incremented in finite chunks, whether a dollar or a cent, a year or a day. Continuous means that no such lower increment exists. For reasons that Team Wilmott has never understood, the mathematics of continuous processes is often easier than that of discrete ones. But then when it comes to number crunching you have to anyway turn a continuous model into a discrete one.

In discrete models we end up with difference equations. An example of this is the binomial model for asset pricing. Time progresses in finite amounts, the timestep. In continuous models we end up with differential equations. The equivalent of the binomial model in discrete space is the Black-Scholes model, which has continuous asset price and continuous time. Whether binomial or Black-Scholes both of these models come from the probabilistic assumptions about the financial world.

Now let's take a look at some of the tools available.

Simulations: If the financial world is random then we can experiment with the future by running simulations. For example, an asset price may be represented by its average growth and its risk, so let's simulate what could happen in the future to this random asset. If we were to take such an approach we would want to run many, many simulations. There'd be little point in running just the one, we'd like to see a range of possible future scenarios.

Simulations can also be used for non-probabilistic problems. Just because of the similarities between mathematical equations a model derived in a deterministic framework may have a probabilistic interpretation.

Discretization methods: The complement to simulation methods, there are many types of these. The best known of these are the finite-difference methods which are discretizations of continuous models such as Black-Scholes. Depending on the problem you are solving, and unless it's very simple, you will probably go down the simulation or finite-difference routes for your number crunching.

Approximations: In modeling we aim to come up with a solution representing something meaningful and useful, such as an option price. Unless the model is really simple, we may not be able to solve it easily. This is where approximations come in. A complicated model may have approximate solutions. And these approximate solutions might be good enough for our purposes.

Asymptotic analysis: This is an incredibly useful technique, used in most branches of applicable mathematics, but almost unknown in finance. The idea is simple, find approximate solutions to a complicated problem by exploiting parameters or variables that are either large or small, or special in some way. For example, there are simple approximations for vanilla option values close to expiry.

Green's functions: This is a very special technique that only works in certain situations. The idea is that solutions to some difficult problems can be built up from solutions to special solutions of a similar problem. That's not a very helpful definition, but we'll be seeing enough of this later on.

All of the above ideas will be explained in detail over the coming months. One of the aims of this educational series is to make the mathematics of finance as straightforward as possible. We will be using simple examples, and understandable analogies, to achieve this.