FitchLearning

Problem-Solving Session

Statistical Essentials for VaR and ES

Breaches in VaR

CQF

Dr Richard Diamond
(r.diamond@cqf.com)

For today, we have **learning outcomes:**

- understand the first principles (probability, notation) for VaR and ES
- be able to read the formulae
- be able to generalize and compute **Portfolio VaR** ← Analytical VaR
- get to know about **Liquidity Adjustment** to VaR ← LVaR

Analytical

Historical

Monte-Carlo

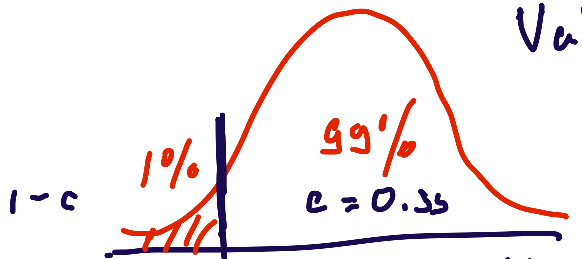
$$VaR(X) = \underset{\text{negligible}}{\mu} + \underset{\text{cond}}{\Phi^{-1}}(1-c)$$

EWMA $\propto 0.7\, r_{t-1}$

GARCH(1,1) $\propto \bar{\sigma}$

$\sigma^2_{t+1} \propto \overset{0.2}{\sigma^2_t} + \overset{0.8}{r^2_t}$

$\sigma^2_t \propto \sigma^2_{t-1} + r^2_{t-1}$



log-returns, Normally Dist.

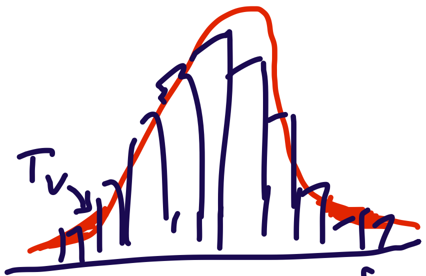
$\Phi^{-1}(0,01)$

-2.32635

Inverse CDF

NORMSINV()

$-\cancel{2.33}$



V is degrees of freedom

Let's start with VaR definition from CQF Lecture on Market Risk EXERCISES

3. Assume that P&L of an investment portfolio is a random variable that follows Normal distribution $X \sim N(\mu, \sigma^2)$. Use the definition of *VaR as a percentile* to derive analytical expression for VaR calculation.

RV

x is value of X $\quad \Pr(x < X) = 0.01$

Solution:

The probability of loss $x < 0$ being worse than VaR < 0 is

$$\begin{aligned} \Pr\left(x \le \mathrm{VaR}(X)\right) &= 1 - c \\ \mathrm{VaR}_c(X) &= \inf\{x \,|\, \Pr(X > x) \le 1 - c\} = \inf\{x \,|\, F_X(x) \ge c\} \end{aligned}$$

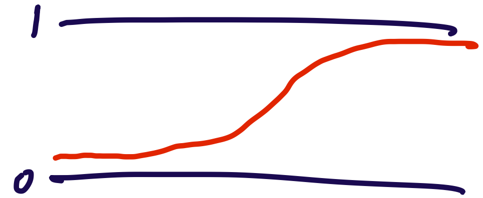
for or 99% confidence, the probability that X ~~above~~ worst loss x is less than $(1 - 0.99) = 0.01$.

If P&L X is a random variable then VaR(X) is also a random variable. In order to use the well-known Normal Distribution functions, we have to work with the Standard Normal variable

$$\frac{X_t - \mu}{\sigma}$$

$$\Phi^{-1}\left[\cancel{\Pr}\left(\phi \le \frac{\mathrm{VaR}(X) - \mu}{\sigma}\right)\right] = \Phi^{-1}\left[1 - c\right] \quad \Longrightarrow$$

Analytical $$\mathrm{VaR}(X) = \mu + \underbrace{\Phi^{-1}(1 - c)}_{\text{Factor (Standard Percentile)}} \times \sigma$$



Inverse CDF for a probability distribution is known as 'percentile function'.

<u>Are we confident to read the formulae?</u>

We will see what we can address in terms of **VaR backtesting**:

- compute rolling standard deviation (analytically) or percentile (empirically) – that will be "**P&L Explained**"

- count and analyse independence of breaches at 99th percentile (too few exceptions was just as problematic as too many)

Backtesting of Normal Analytical VaR can be seen as P&L attribution:

P&L (10-day Return) – Analytical VaR

P&L attribution test (PLAT) is a controversial topic of FRTB (Basel III). It is more readily applied to products (derivatives, structured products, exotic derivatives)

- which have exposure to Expiry Time, Volatility, Interest Rates, and

- where it is possible to compute these exposures (i.e., Greeks)

Breaches from Analytical VaR model -- essentially P&L predicted by Normal factor

Imagine that each morning you calculate 99%/10day VaR from available prior data only. Once ten days pass you compare that VaR number to the realised return and check if your prediction about the worst loss was breached. You are given a dataset of FTSE 100 index levels, continue in Excel.

C.1 Calculate the rolling 99%/10day Value at Risk for an investment in the market index using a sample standard deviation of log-returns, as follows:

- The rolling standard deviation for a sample of 21 is computed for days 1-21, 2-22, ..., there must be 21 observations in the sample. So, you have a time series of σ_t.
- Scale standard deviation to reflect a ten days move $\sigma_{10D} = \sqrt{10 \times \sigma^2}$ (we can add variances) and scale an average daily return as $\mu_{10D} = \mu \times 10$ where μ is a mean return of all data given.
- Calculate Value at Risk for each day t (starting on Day 21) as follows:

$$\text{VaR} = \mu_{10D} + \text{Factor} \times \sigma_{10D} \quad \dagger$$

where Factor is a percentile of the Standard Normal Distribution that 'cuts' 1% on the tail.

In Excel, you will have a final column with VaR_t as a percentage since calculation is done on returns.

Please read the instructions. Are there questions on notation and steps?

VaR Backtesting

- Breach in VaR = Actual Loss is worse than predicted Worst Loss

- VaR is fixed at time t and compared to the realised return at time $t + 10$. A breach occurs when a realised 10-day return $r_{10D,t} = \ln(S_{t+10}/S_t)$ is below the VaR_t quantity (negative scale).

- Either we have **a.** volatility estimation problem or **b.** <u>Normal Factor alone is not a very good predictor for P&L</u> in market index returns, or more specifically, **c.** their left tail, large negative returns.

- Conditional breaches: Pr (not VaR_t+1 | not VaR_t)

- Independence of breaches in VaR:
    - A different problem to one of checking for *iid*-ness/testing for Normality in large samples
    - We need some kind of "distribution of exceedances (breaches)" to test against
    - Christoffersen's test measures the dependency between consecutive days only.

See also https://www.mathworks.com/help/risk/overview-of-var-backtesting.html