CHAPTER 1

PRODUCTS AND MARKETS: EQUITIES, COMMODITIES, EXCHANGE RATES, FORWARDS AND FUTURES

1. **A company makes a 3-for-1 stock split. What effect does this have on the share price?**

   In absence of arbitrage opportunities, the total value of the shares in the company does not change. If there were N shares, with a share price of S each, then the total value of the shares was NS. After the stock split, there are $4N$ shares, and they are worth S' each. The total value of the company remains unchanged, hence the new share price is

$$S' = \frac{NS}{4N} = \tfrac{1}{4}S.$$

2. **A company whose stock price is currently S, pays out a dividend DS, where $0 \leq D \leq 1$. What is the price of the stock just after the dividend date?**

   In absence of arbitrage opportunities, the value of the share after the dividend date, S', plus the value of the dividend, DS, must be equal to the value of the share before the dividend, S. Therefore

$$S' = S - DS = S(1 - D).$$

3. **The dollar-sterling exchange rate (colloquially known as 'cable') is 1.83, £1 $=$ \$1.83. The sterling-euro exchange rate is 1.41, £1 $=$ EUR 1.41. The dollar-euro exchange rate is 0.77, \$1 $=$ EUR 0.77. Is there an arbitrage, and if so, how does it work?**

   Exchange £1 for EUR 1.41. Now exchange EUR 1.41 for $1.41/0.77 = 1.8312$ US dollars. Now exchange \$1.8312 for $1.8312/1.83 = 1.0006$ pounds sterling. You have just made 0.06 pence profit. So there is an arbitrage. Of course, you would have to start with a much larger amount that £1 if you want to make any serious money.

4. You put \$1000 in the bank at a continuously compounded rate of 5% for one year. At the end of this first year rates rise to 6%. You keep your money in the bank for another eighteen months. How much money do you now have in the bank including the accumulated, continuously compounded, interest?

After the first year you will have

$$£1000 \times e^{0.05\times 1} = £1051.27.$$

After the next 18 months you will have

$$£1051.27 \times e^{0.06\times 1.5} = £1150.27.$$

5. **A spot exchange rate is currently 2.350. The one-month forward is 2.362. What is the one-month interest rate assuming there is no arbitrage?**

In a continuously compounded sense we must solve

$$2.350 \times e^{r/12} = 2.362.$$

So

$$r = 6.11\%.$$

6. **A particular forward contract costs nothing to enter into at time t and obliges the holder to buy the asset for an amount F at expiry, T. The asset pays a dividend DS at time t_d, where $0 \le D \le 1$ and $t \le t_d \le T$. Use an arbitrage argument to find the forward price, $F(t)$.**

Hint: Consider the point of view of the writer of the contract when the dividend is re-invested immediately in the asset.

The writer must deliver the asset at time T for a price F. He borrows an amount of cash $\lambda S(t)$ at time t and buys λ of the asset, where $0 \le \lambda \le 1$. At time t_d, he receives a dividend of $\lambda DS(t_d)$ and he immediately reinvests this in the asset. This enables him to buy a further

$$\frac{\lambda DS}{S(1-D)}$$

of the asset (where $S(1-D)$ is the asset price after the dividend date). The writer must then deliver one asset at time T. We must therefore choose λ such that

$$\lambda + \frac{\lambda DS}{S(1-D)} = 1,$$

i.e. $\lambda = 1 - D$. The money (F) received at expiry is then used to pay off the loan. In absence of arbitrage opportunities, we must therefore have

$$F(t) = \lambda S(t)e^{r(T-t)} = (1-D)S(t)e^{r(T-t)}.$$