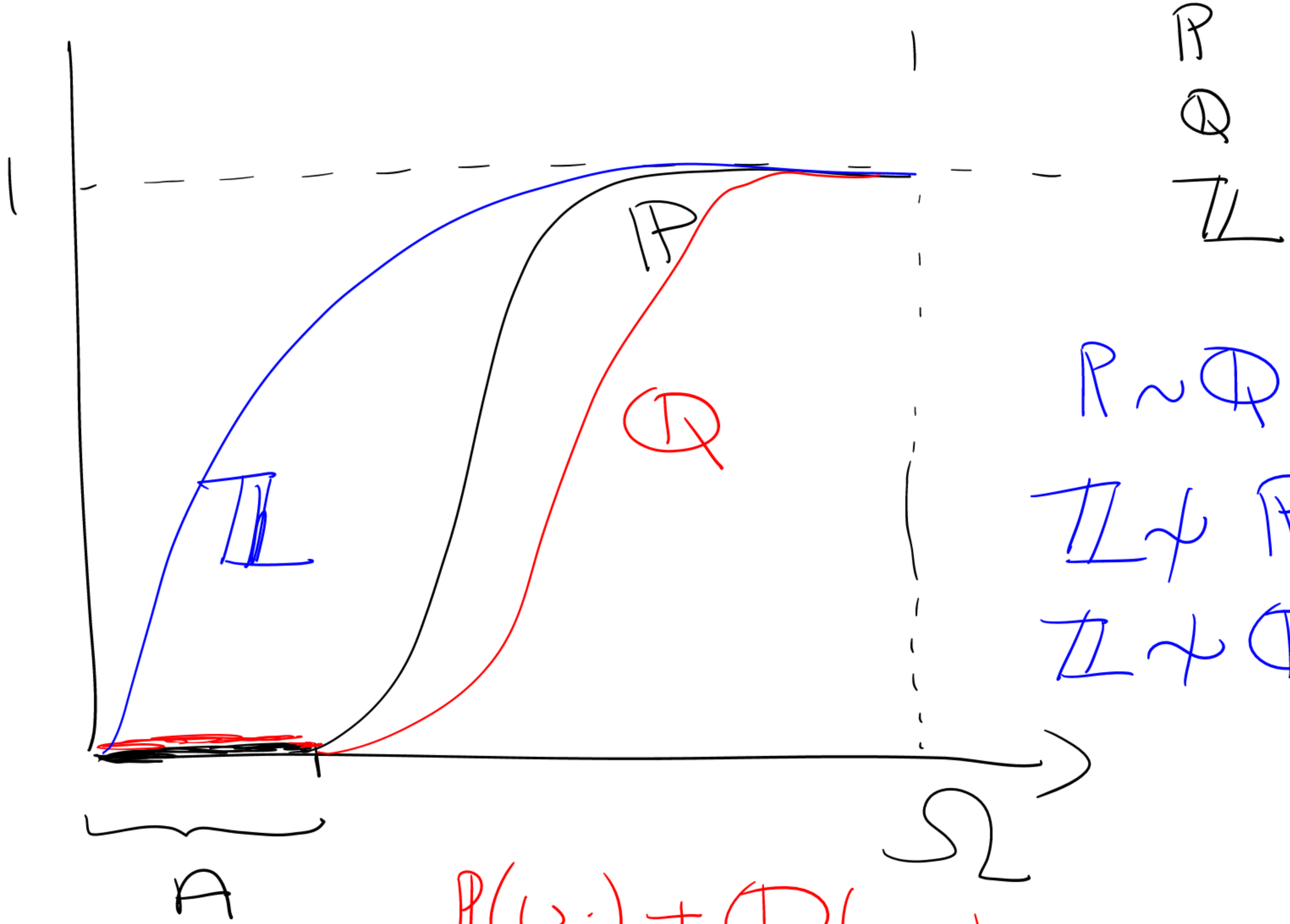
$$p(x) = \frac{dP(x)}{dx}$$

$$\int_{-\infty}^{x} dP(s) = \int_{-\infty}^{x} p(s)\, ds$$

$$P(x) - \underbrace{P(-\infty)}_{0} = \int_{-\infty}^{x} p(s)\, ds$$

$$P(x) = \int_{-\infty}^{x} p(s)\, ds$$



$\mathbb{P} \sim \mathbb{Q}$

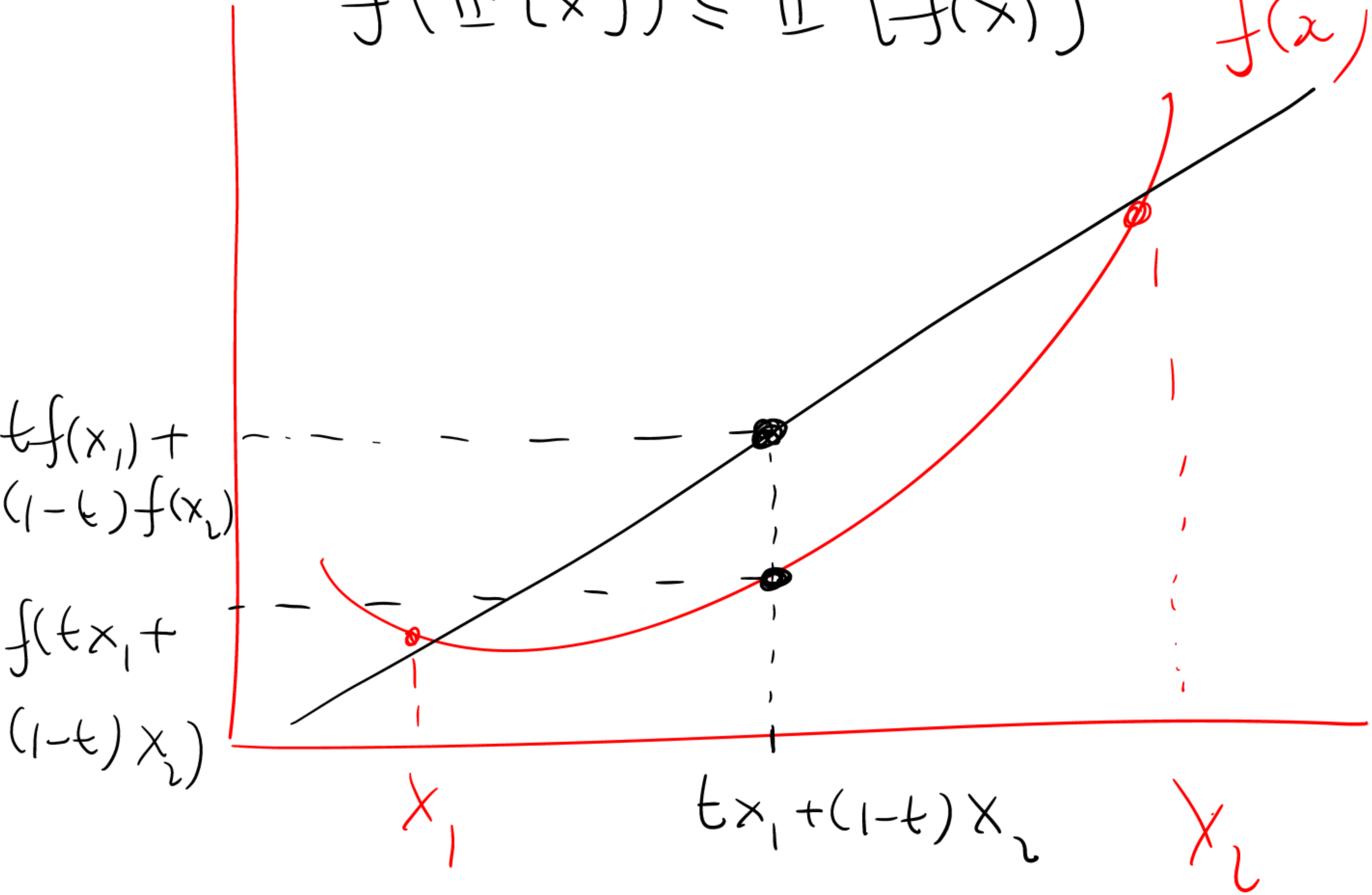
$\mathbb{L} \not\sim \mathbb{P}$

$\mathbb{L} \not\sim \mathbb{Q}$

$\mathbb{P}(\omega_i) \neq \mathbb{Q}(\omega_j)$

$$f(\mathbb{E}[x]) \le \mathbb{E}[f(x)]$$

$0 \le t \le 1$



$$f(tx_1 + (1-t)x_2) \le t f(x_1) + (1-t) f(x_2)$$

Adapted Process | $\mathcal{F}_t$ measurable.

$$\mathbb{E}_t[M_T | \mathcal{F}_t] \qquad t < T$$

$$\mathbb{E}[X_t \mid \underbrace{X_s}_{\mathcal{F}_s}]$$

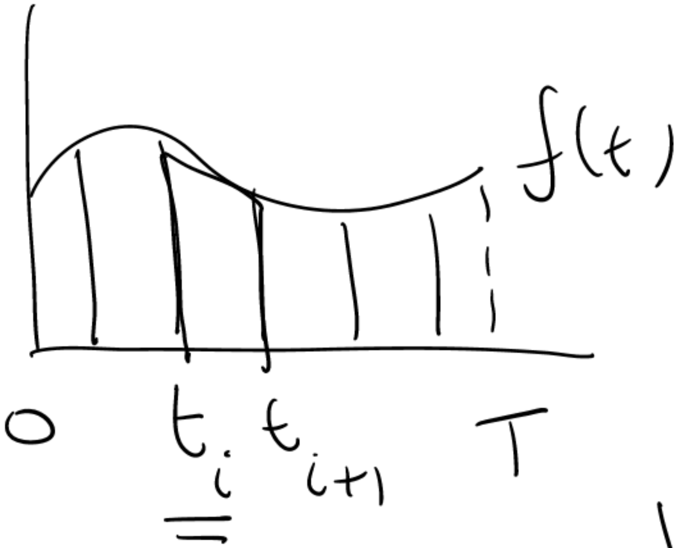
$$\mathbb{E}[X_t - X_s + X_s \mid X_s] =$$

$$\underbrace{\mathbb{E}[X_t - X_s \mid X_s]}_{N(0,|t-s|)} + \mathbb{E}[X_s \mid X_s]$$

$$= X_s$$

Riemann Integ.

$$\int_0^T f(t)\,dt = \lim_{N\to\infty} \sum_{i=0}^{N-1} f(t_i)\,(t_{i+1} - t_i)$$ Left hand rectangle rule.



$N \to \infty \quad (t_{i+1} - t_i) \to 0$

$$= \lim_{N\to\infty} \sum_{i=0}^{N-1} f(t_{i+1})\,(t_{i+1} - t_i)$$ RH Rule.

$$= \lim_{N\to\infty} \sum_{i=0}^{N-1} \frac{1}{2}\left(f(t_{i+1}) + f(t_i)\right)(t_{i+1} - t_i)$$

$$\int_0^T f(t, X_t)\, dX_t$$

① $\lim\limits_{N \to \infty} \sum\limits_{i=0}^{N-1} f(t_i, X_{t_i})(X_{t_{i+1}} - X_{t_i})$ Itô Integral

② $\lim\limits_{N \to \infty} \sum\limits_{i=0}^{N-1} f(t_{i+1}, X_{t_{i+1}})(X_{t_{i+1}} - X_{t_i})$

③ $\lim\limits_{N \to \infty} \sum\limits_{i=0}^{N-1} \frac{1}{2}\left(f(t_{i+1}, X_{t_{i+1}}) + f(t_i, X_{t_i})\right)$

$(X_{t_{i+1}} - X_{t_i})$

$$\mathbb{E}\int_0^T f(t,X_t)\,dX_t = \left[\lim_{n\to\infty}\sum_{i=0}^{n-1} f(t_i,X_{t_i})\,\mathbb{E}\left(X_{t_{i+1}}-X_{t_i}\right)\right]$$

$$N\left(0,\ |t_{i+1}-t_i|\right)$$

Itô Product Rule

X, Y processes satisfying

$$dX = A(X,t)\,dt + B(X,t)\,dX_1$$

$$dY = a(Y,t)\,dt + b(Y,t)\,dX_2$$

Write $F = XY$, what is $dF = ?$

$$\frac{\partial F}{\partial X} = Y \qquad \frac{\partial F}{\partial Y} = X \qquad \frac{\partial^2 F}{\partial X^2} = 0 = \frac{\partial^2 F}{\partial Y^2} \qquad \frac{\partial^2 F}{\partial X \partial Y} = 1 = \frac{\partial^2 F}{\partial Y \partial X}$$

$$dF = \frac{\partial F}{\partial X} dX + \frac{\partial F}{\partial Y} dY + \frac{1}{2} \frac{\partial^2 F}{\partial X^2} dX^2 + \frac{1}{2} \frac{\partial^2 F}{\partial Y^2} dY^2 + \frac{\partial^2 F}{\partial X \partial Y} dX\,dY$$

$$d(XY) = Y\,dX + X\,dY + dX\,dY$$