Aristotle's Lyceum is the institution considered to be the forerunner of the modern university. Opened in 335 BC, the Lyceum was a center of study and research in both science and philosophy.

Discretization

In this Tutorial we are going to look at discretization. How can we represent the value of an option that depends on an underlying asset, price S, and time, t, in a computer program, and how can we determine the values of the key Greeks, delta, gamma and theta?

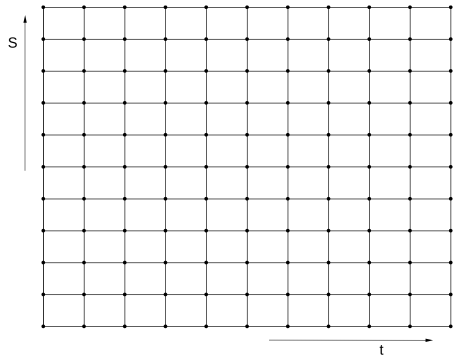

Figure 1: The finite-difference grid

In the world of finite differences we use the grid or mesh shown in Figure 1. This grid usually has equal time steps, the time between nodes, and equal asset steps.

Let's introduce some notation. The time step will be dt and the asset step dS, both of which are constant. Thus the grid is made up of the points at asset values $S = i * dS$ and times $t = T - k * dt$ where i and k are integers. Notice how we've changed the direction of time, as k increases so real time decreases. This is because when we solve the Black-Scholes equation we must work backwards in time from known option values at expiry. We will write the option value at each of these grid points as $V(i, k)$.

Suppose that we know the option value at each of the grid points, can we use this information to find the sensitivities of the option value with respect to S and t?

Approximating theta

Our approximation for the first time-derivative of V is simply

```
Theta(i, k) = _
    ( V(i, k) – V(i, k + 1) ) / dt
```

It uses the option value at the two points marked in Figure 2. The error in this approximation is $O(dt)$.

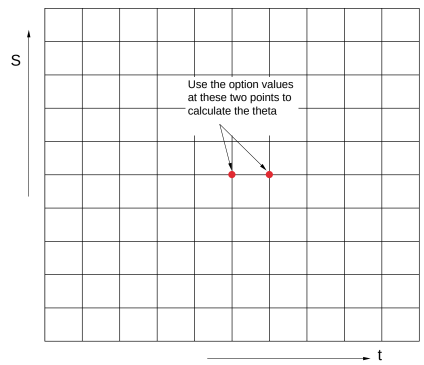

Figure 2: An approximation to the theta.

Approximating delta

Let's examine a cross section of our grid at one of the time steps.

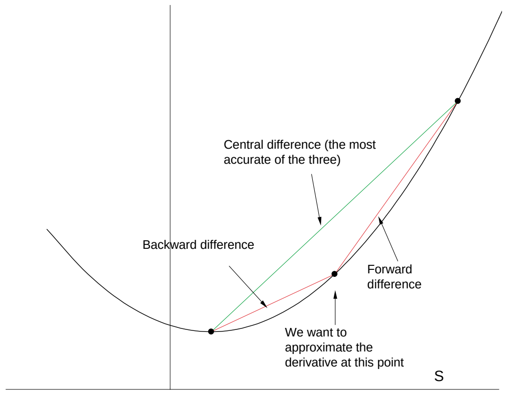

Figure 3: Three approximations to the delta.

In Figure 3 is shown this cross-section. The figure shows three things: the function we are approximating (the option curve), the values of the function at the grid points (the dots) and three possible approximations to the first derivative (the three straight lines). These are called a forward difference, a backward difference and a central difference, respectively.

One of these approximations is better than the others, and it is obvious from the diagram which it is. The central difference has an error of $O(dS^2)$. Our approximation for the first S-derivative of V is simply

```
Delta(i, k) = _
( V(i + 1, k) – V(i - 1, k) ) / dS
```

Approximating gamma

The gamma of an option is the second derivative of the option with respect to the underlying.

The natural approximation for this is

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Gamma(i, k) = _
( V(i + 1, k) – 2 * V(i, k) + _
    V(i - 1, k) ) / dS / dS
```

Exercise

Write a VB program that takes the function $V = (T - t) \log(S)$ and numerically calculates the delta, gamma and theta at different asset values and times.