Aristotle's Lyceum is the institution considered to be the forerunner of the modern university. Opened in 335 BC, the Lyceum was a center of study and research in both science and philosophy.

The Kelly Criterion

We like to draw comparisons between the worlds of finance and of gambling, you may have noticed. Our philosophy is that gambling is just a particular form of investing, one that is often technically simpler to understand than the world of stocks and shares, convertible bonds and options. If you can't cope with the mathematics of roulette, or the emotional rollercoaster of Blackjack, then you certainly shouldn't be working in a bank ... except maybe shining shoes. We are now going to explore a very simple idea, known to all professional gamblers but still not known by everyone in finance.

Risk and return

We saw in the last Lyceum piece how expectations and standard deviations are important when investing over and over again, in bet after bet on the toss of a coin for example. In the long run you'll make a tidy profit if your expectation is positive, but perhaps with some nasty drawdown early on. If you've got bottomless pockets then that drawdown is not going to worry you. More typically, however, a short run of bad luck may wipe you out and wipe out your chances of the long-run profit.

Let's address the issue of the 'long run' and see if there's any way we can get to our long-run profit by clever money management.

The long run

You've got $1,000 in your pocket and an opportunity to play some game of chance over and over again. Your analysis of the game, the possible outcomes, and your probability of winning each time, suggest that you've got an edge.

The outcome of a $1 bet is the random variable ϕ, and the positive edge means that the average payoff for that bet is μ, with $\mu > 0$.

$$E[\phi] = \mu.$$

The standard deviation of the outcome σ is given by

$$E[\phi^2] - (E[\phi])^2 = \sigma^2.$$

Later on we'll be making an assumption about the relative size of μ and σ, but for the moment let's keep things as general as possible.

Bet a fraction of your wealth each time

The thousand dollars won't last long if we bet it all in one go. Let's instead bet a fraction f. And we'll keep betting the same fraction f every game, whether we're up to $10,000 or down to $10.

After one bet we have

$$1000\ (1 + f\phi).$$

After two bets we have

$$1000\ (1 + f\phi_1)(1 + f\phi_2).$$

The subscripts simply acknowledge the fact that the ϕs are random, and different each game.

After the evening's session of N games we've got

$$1000\ \prod_{i=1}^{N}(1 + f\phi_i).$$

(A product of N terms.)

We now ask the question "What is the expected growth rate?" That is, if we compared the money we've got at the end of the evening with the exponential growth we'd get from putting the money in the bank for a while, what is the equivalent interest rate?

To calculate such a thing we need to take logarithms of this expression.

Now we can state our investment goal, we will invest a fraction f of our wealth each game in such a way as to maximize the expected return. This means maximize the following expression:

$$E\left[\log(1000) + \sum_{i=1}^{N}\log(1 + f\phi_i)\right].$$

Since the ϕs come from the same distribution each time, this amounts to maximizing

$$E[\log(1 + f\phi)]. \qquad (1)$$

by the optimal choice of f.

Now we come to the fun bit.

Optimal investment fraction

Typically the bet is at best only going to give us a slight edge, so that the mean μ is going to be much smaller than the standard deviation σ. If that is the case we can approximate expression (1) by

$$f\mu - \frac{1}{2}f^2\phi^2.$$

This is what we want to maximize.

We can find the maximum by differentiating with respect to f. So, the choice of fraction which maximizes our long-term growth rate is just

$$f = \frac{\mu}{\sigma^2}.$$

This is the famous Kelly criterion, beloved of all pro gamblers. The idea behind it and the result are important for money management in finance as well. Don't invest too much, it's too risky; don't invest too little, you have to wait forever to get some payback.

Indeed, some of the results from asset allocation models show remarkable similarity to this. Even the Sharpe ratio, the measure of performance for funds and traders etc. is related to this simple concept.