Stock $S(t)$

$$\frac{dS}{S} = \mu\, dt + \sigma\, dX$$

$dX \sim \phi \sqrt{dt}$

$dX \sim O(\sqrt{dt})$

Consider $V(S)$ $\quad S \to S + dS$

$$V(S+dS) = V(S) + \frac{dV}{dS} dS + \frac{1}{2} \frac{d^2V}{dS^2} dS^2$$

dV

$$dV = \frac{dV}{dS} dS + \frac{1}{2} \frac{d^2V}{dS^2} dS^2$$

$$dS^2 = \underbrace{\mu^2 S^2 dt^2}_{\to 0} + \underbrace{2\mu\sigma S^2 dX dt}_{\to 0} + \underbrace{\sigma^2 S^2 dt}_{\to \text{keep}}$$

$\therefore O(dt^{3/2})$

$$dS^2 = \sigma^2 S^2 dt$$

$$\therefore \quad dV = \underbrace{\left(\mu S \frac{dV}{dS} + \frac{1}{2}\sigma^2 S^2 \frac{d^2V}{dS^2} \right)}_{\text{drift}} dt + \underbrace{\sigma S \frac{dV}{dS}}_{\text{diffusion}} dX$$

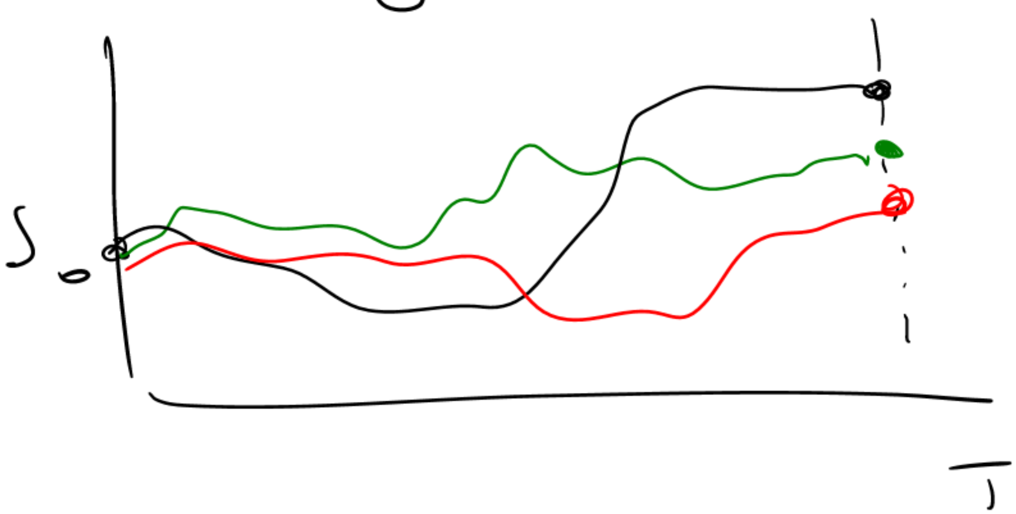
Itô III

Case 2: Integrate over $[t, t+\delta t]$

$$S_{t+\delta t} = S_t \, e^{(\mu - \frac{1}{2}\sigma^2)\delta t + \sigma \phi \sqrt{\delta t}}$$

Case 3: Integrate over $[0, T]$

$$S_T = S_0 \, e^{(\mu - \frac{1}{2}\sigma^2)T + \sigma \phi \sqrt{T}}$$



$$du = -\gamma u\, dt + \sigma\, dX$$

$$du + \gamma u\, dt = \sigma\, dX$$

$\times$ thro by I.F $\quad e^{\gamma t}$

$$\underbrace{e^{\gamma t}(du + \gamma u\, dt)}_{d(e^{\gamma t} u)} = \sigma e^{\gamma t} dX$$

Integ both sides over $[0, t]$

$$\int_0^t d(e^{\gamma s} u_s) = \sigma \int^t e^{\gamma s} dX$$

$$e^{\gamma s} u_s \Big|_0^t = \sigma \int_0^t e^{\gamma s} dX$$

$$d\left(e^{rt}u\right)$$

$$e^{rt}\,d(u) + u\,\underbrace{d\left(e^{rt}\right)}$$

$$\frac{d}{dt}\left(e^{rt}\right) = r e^{rt}$$

$$d\left(e^{rt}\right) = r e^{rt}\,dt$$

$$e^{\gamma s} v_s \Big|_0^t = \sigma \int_0^t e^{\gamma s} \, dX_s$$

$$e^{\gamma t} v_t - v_0 = \sigma \int_0^t e^{\gamma s} \, dX_s$$

$$v_t = v_0 e^{-\gamma t} + \sigma \int_0^t e^{\gamma (s-t)} \, dX_s$$

$$U_t = r_t - \bar{r}$$

$$dS = A(S,t)\,dt + B(S,t)\,dX$$

$$\mathbb{E}[dS] = A\,dt$$

$$\mathbb{V}[dS] = \mathbb{V}[B\,dX]$$

$$= B^2\,\mathbb{V}[dX]$$

$$= B^2\,dt$$

$$dy = A(y,t)\,dt + D(y,t)\,dX$$

$$y_{t+1} - y_t = A(y,t)\,dt + D(y,t)\left[X_{t+1} - X_t\right)$$

$$\frac{1}{2}\sigma^2 \frac{d^2 p}{dr^2} - \gamma \frac{d}{dr}\left((\bar{r}-r)p\right) = 0$$

$$\frac{1}{2}\sigma^2 \frac{dp}{dr} - \gamma(\bar{r}-r)p = \text{Const} = 0$$

$$\frac{1}{2}\sigma^2 \frac{dp}{dr} = -\gamma(r-\bar{r})p$$

$$\int \frac{dp}{p} = -\frac{2\gamma}{\sigma^2}\int \frac{1}{2}\frac{d}{dr}(r-\bar{r})^2 \quad dr$$

$$N(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{1}{2}s^2}\, ds$$

$$= P(X < x)$$

$$\sum_{i=1}^{n} \text{RAND}()$$

① $\mathbb{E}\left[\sum_{i=1}^{n} \text{RAND}()\right] = \sum_{i=1}^{n} \mathbb{E}(\text{RAND}()] = \frac{n}{2}$

To update, let's write

$$\sum_{i=1}^{n} \text{RAND}() - \frac{n}{2}$$

② $\mathbb{V}\left[\sum_{i=1}^{n} \text{RAND}() - n/2\right] = \sum_{i=1}^{n} \mathbb{V}(\text{RAND}()] = \frac{n}{12} \neq 1$

$\mathbb{V}\left[\alpha\left(\sum_{i=1}^{n} \text{RAND}() - n/2\right)\right] = 1$

$\alpha^2 \, n/12 = 1 \longrightarrow \alpha = \sqrt{\frac{12}{n}}$

$$\sqrt{\frac{12}{n}}\left[\sum_{i=1}^{n} RAND() - \frac{n}{2}\right) \sim N(0,1)$$

$$\lim_{n\to\infty} \frac{\sum_{i=1}^{n} RAND() - n \times \frac{1}{2}}{\sqrt{\frac{1}{12}} \times \sqrt{n}} \equiv \frac{\sum_{i=1}^{n} Y_i - n\mu}{\sigma\sqrt{n}}$$

$$\varepsilon_1, \varepsilon_2 \sim N(0,1)$$

$$\left.\begin{aligned} &\mathbb{E}[\varepsilon_1] = 0 = \mathbb{E}[\varepsilon_2] \\ &\mathbb{E}[\varepsilon_1^2] = 1 = \mathbb{E}[\varepsilon_2^2] \end{aligned}\right\} \quad \mathbb{E}(\varepsilon_1 \varepsilon_2) = 0$$

want $\quad \phi_1, \phi_2 \sim N(0,1) \quad$ s.t. $\quad \mathbb{E}(\phi_1 \phi_2) = \rho$

1. $\phi_1 = \varepsilon_1$

2. $\phi_2 = \alpha \varepsilon_1 + \beta \varepsilon_2$

$$\alpha \underbrace{\mathbb{E}(\varepsilon_1^2)}_{=1} + \beta \underbrace{\mathbb{E}[\varepsilon_1 \varepsilon_2]}_{=0} = \rho$$

$$\boxed{\alpha = \rho}$$

$$\mathbb{E}[\phi_1 \phi_2] = \rho = \mathbb{E}[\varepsilon_1(\alpha \varepsilon_1 + \beta \varepsilon_2)]$$

$$\mathbb{E}(\alpha \varepsilon_1^2 + \beta \varepsilon_1 \varepsilon_2) = \rho$$

$$\mathbb{E}[\phi_2^2] = 1$$

$$\mathbb{E}[(\alpha \varepsilon_1 + \beta \varepsilon_2)^2] = 1$$

$$\mathbb{E}[\underbrace{\alpha^2}_{\rho^2} \varepsilon_1^2 + \beta^2 \varepsilon_2^2 + 2\alpha\beta\, \varepsilon_1 \varepsilon_2] = 1$$

$$\rho^2 + \beta^2 + 2\alpha\beta \times 0 = 1$$

$$\beta = \sqrt{1-\rho^2}$$

$$V = V(S, t) \qquad dS = \mu S\, dt + \sigma S\, dX$$

$$t \to t + dt \qquad S \to S + dS \qquad \boxed{dS^2 = \sigma^2 S^2 dt}$$

T.S.E $\quad V(S + dS,\ t + dt) =$

$$V(S, t) + \frac{\partial V}{\partial t} dt + \frac{\partial V}{\partial S} dS + \frac{1}{2} \frac{\partial^2 V}{\partial S^2} dS^2$$

$$dV = V(S + dS, t + dt) - V(S, t) =$$

$$dV = \left(\frac{\partial V}{\partial t} + \mu S \frac{\partial V}{\partial S} + \frac{1}{2} \sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} \right) dt + \sigma S \frac{\partial V}{\partial S} dX$$

Itô $\quad \underline{\overline{\text{IV}}}$