CHAPTER 15

SWAPS

1. Consider a swap with the following specification:

The floating payment is at the six-month rate, and is set six months before the payment (swaplet) date. The swap expires in 5 years, and payments occur every six months on a principal of \$1. Zero-coupon bond prices are known for all maturities up to 10 years. What is the 'fair' level for the fixed rate side of the swap, so that initially the swap has no value?

The fixed rate side of the swap has value

$$\tfrac{1}{2} r_s \sum_{i=1}^{10} Z(t, i/2),$$

where the fixed rate of interest is r_s. The $\frac{1}{2}$ is there because the tenor is six months, half a year.

There is a relationship between the floating leg of a swap and zero-coupon bonds. A single floating leg of a swap at time T_i is equal to a deposit of \$1 at time $T_i - \frac{1}{2}$ and a withdrawal of \$1 at time T_i.

Adding up all the floating legs, we find that the floating rate side of the swap has value

$$1 - Z(t; 5).$$

The value of the swap to the receiver of the fixed rate side is therefore

$$\tfrac{1}{2} r_s \sum_{i=1}^{10} Z(t, i/2) - 1 + Z(t; 5).$$

The fair level of the fixed rate so that the swap has no initial value is then

$$r_s = 2 \frac{1 - Z(t; 5)}{\sum_{i=1}^{10} Z(t, i/2)}.$$

2. An index amortizing rate swap has a principal which decreases at a rate dependent on the interest rate at settlement dates. Over a payment date, the principal changes from P to $g(r)P$, where $g(r)$

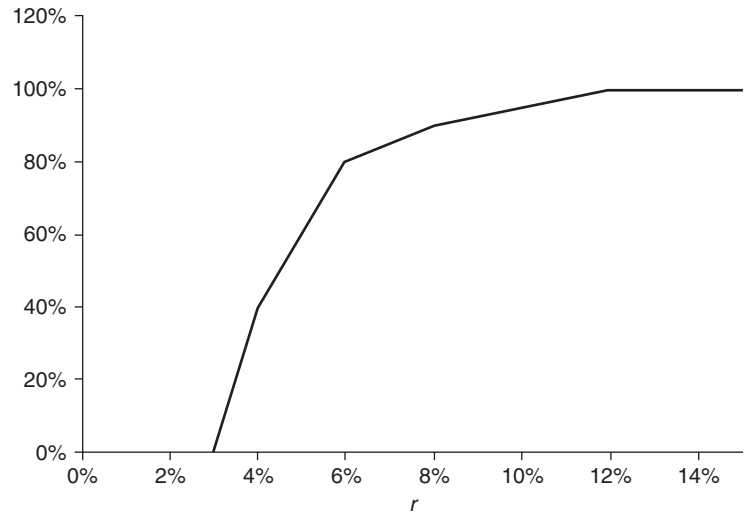

Figure 15.1 A typical amortizing schedule.

is a function specified in the swap contract, and $0 \leq g(r) \leq 1$. How will this affect the level of the fixed rate if the swap initially has no value?

We consider the amortizing schedule given by Figure 15.1.

The party receiving the fixed rate payments will suffer if rates rise because he will pay out a rising floating rate while the principal does not decrease. If rates fall the principal amortizes and so his lower floating rate payments are unfortunately on a lower principal. Again, he suffers. Consequently, the fair level of the fixed rate will be higher than for a swap without amortization.

3. **A swap allows the side receiving floating to close out the position before maturity. How does the 'fair' value for the fixed rate side of the swap compare to that for a swap with no call/put features?**

   This feature gives more rights to the side paying fixed and receiving floating. The fair value of the fixed rate should therefore be higher than for a swap without call/put features (assuming both swaps initially have no value).