$$u_t = u_{xx}$$

$$u = \underbrace{X(x)}\,\underbrace{T(t)}$$

$$f(x+dx,\, y+dy) = f + f_x\, dx + f_y\, dy + \frac{1}{2} f_{xx}\, dx^2 + \frac{1}{2} f_{yy}\, dy^2 + \cdots$$

$$\boxed{df = \frac{\partial f}{\partial x}\, dx + \frac{\partial f}{\partial y}\, dy}$$

Total change

Differential of f

$$M\,dx + N\,dy = 0$$

$\exists\, G(x,y)$? s.t

$$dG = \underbrace{M\,dx + N\,dy}_{=0}$$

$$\therefore\ dG = 0 \implies G(x,y) = \text{const}$$

$$dG = M\,dx + N\,dy \longrightarrow$$

$$\boxed{\frac{\partial G}{\partial x}}\,dx + \boxed{\frac{\partial G}{\partial y}}\,dy \equiv M\,dx + N\,dy$$

$$\boxed{\begin{aligned}\frac{\partial G}{\partial x} &= M \\ \frac{\partial G}{\partial y} &= N\end{aligned}} \longrightarrow G(x,y)$$

$$u_t = u_{xx}$$

$$\underbrace{\frac{T'}{T}}_{\text{indep of } x} = \underbrace{\frac{X''}{X}}_{\text{indep of } t} = \underline{\underline{\text{Const}}}$$

$$z = \frac{1}{y^{n-1}} = y^{1-n}$$

$$\frac{dz}{dx} = (1-n)\, y^{-n} \cdot \frac{dy}{dx}$$

$$\sum_{n=0}^{N} a_n y^{(n)} = g(x)$$

$$v'x + v = x^{k} f(1, v) \qquad k \to k=0 \qquad k=2$$

$$x\frac{dv}{dx} = f(1,v) - v$$

$$\boxed{\frac{dx}{x} = \frac{dv}{f(1,v)-v}}$$

Var. sep.

$$\frac{dy}{dx} = h(x)\, g(y)$$

$$= \frac{x+y}{x-y}$$

$$u = 2x + y \longrightarrow$$

$$y = u - 2x$$

$$\frac{dy}{dx} = \frac{du}{dx} - 2 = y' \equiv u' - 2$$

$$ay'' + by' + y = 0$$

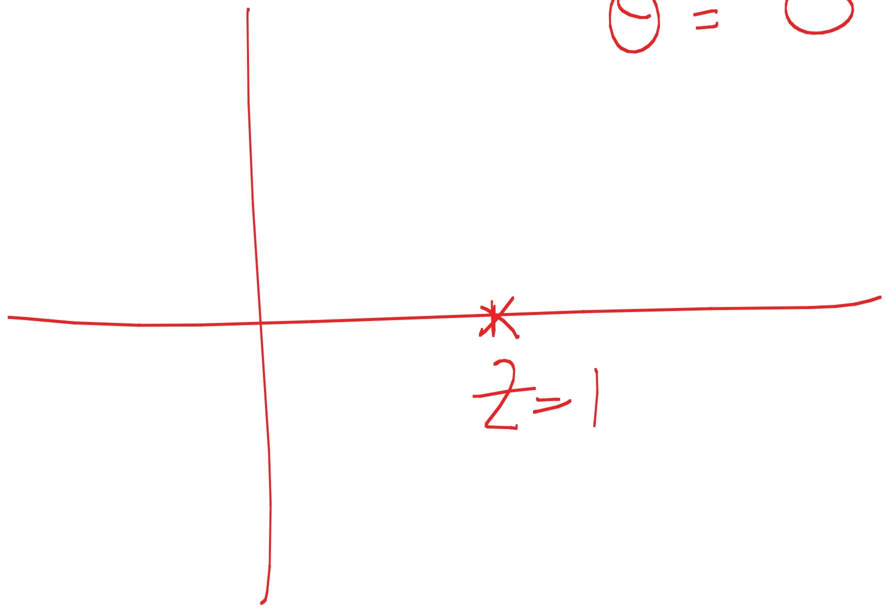
$$|e^{i\theta}| = 1$$

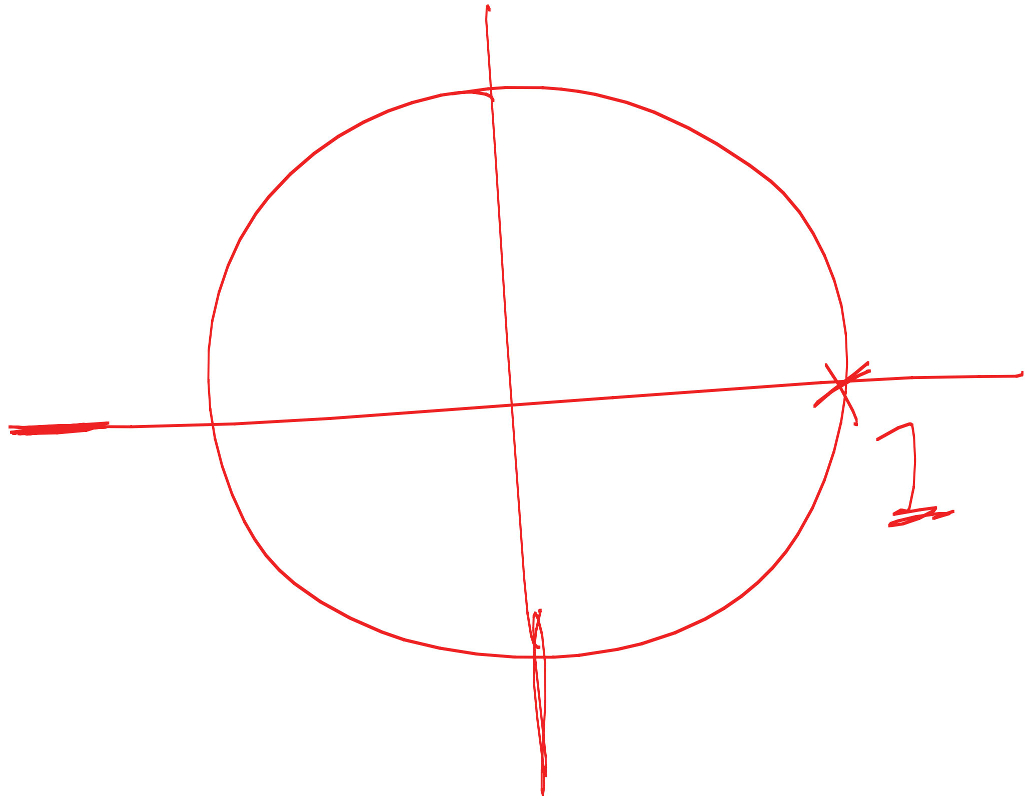
φ

$$z = 1 + i0$$

$$|z| = 1$$

$$\theta = 0 + 2k\pi$$





$r = 1$

4^{y}_{n}

$$S_n = a + ar + ar^2 + \cdots + ar^n \quad ①$$

$$rS_n = ar + ar^2 + ar^3 + \cdots + ar^n + ar^{n+1} \quad ②$$

① − ②

$$S_n(1-r) = a - ar^{n+1}$$

$$\boxed{S_n = \frac{a(1-r^{n+1})}{1-r}}$$

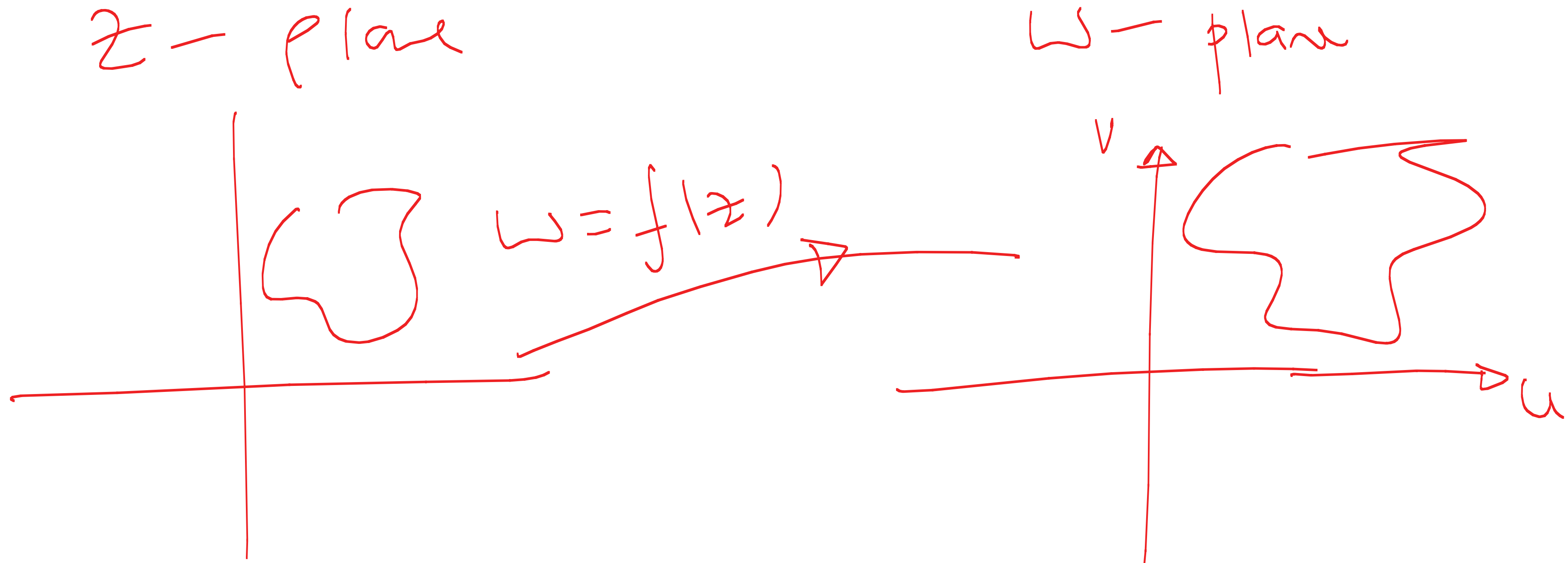
$$|r| < 1$$

$$\lim_{n \to \infty} r^{n+1} \to 0$$

fn. diff^{ble} — regular
— analytic

Holomorphic H

$$f(z) \in H(\mathbb{C})$$



$z = x + iy$

f - fn of a $\mathbb{C}$ var

$$w = u + iv = f(x+iy)$$
$$= u(x,y) + i\,v(x,y)$$

Cauchy – Riemann Eqns

$$f(z) = u + iv$$

$$u = e^x \cos y$$

$$v = e^x \sin y$$

$$\left.\begin{aligned} u_x &= e^x \cos y \\ v_y &= e^x \cos y \end{aligned}\right\} =$$

$$\left.\begin{aligned} u_y &= -e^x \sin y \\ v_x &= e^x \sin y \end{aligned}\right\}$$

$$\boxed{\begin{aligned} \frac{\partial u}{\partial x} &= \frac{\partial v}{\partial y} \\ \frac{\partial u}{\partial y} &= -\frac{\partial v}{\partial x} \end{aligned}}$$

C-R eqns

$$u = \mathrm{Re}(f)$$

$$v = \mathrm{Im}(f)$$

If $\left.\begin{bmatrix} u_{xx} + u_{yy} = 0 \\ v_{xx} + v_{yy} = 0 \end{bmatrix}\right\}$ Real & Imag. part of $f(z)$ are <u>harmonic</u>

& Laplace's Eq^{n}

<u>Ex:</u> Test $\sin z$, $\cos z$

$$\sin(x+iy) = \sin x \cos iy + \cos x \sin iy$$

$$= \sin x \cosh y + i \cos x \sinh y$$

$$\left.\begin{aligned} u &= \mathrm{Re}(f(z)) = \sin x \cosh y \\ v &= \mathrm{Im}(f(z)) = \cos x \sinh y \end{aligned}\right\}$$

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