Certificate in Quantitative Finance

Asset Returns: Key, Empirical Stylized Facts

Lecture notes provided by:

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Relevant chapters from Stephen Taylor's book on:

Asset Price Dynamics, Volatility, and Prediction

Today's lecture	Covered in Chapters
Volatility clustering	2
Properties of daily returns	4
Properties of high-frequency returns	12
Tomorrow's lecture	
Volatility concepts	8
GARCH models	9 & 10
Background material	
Random variables and stochastic processes	3

After today's class you will:

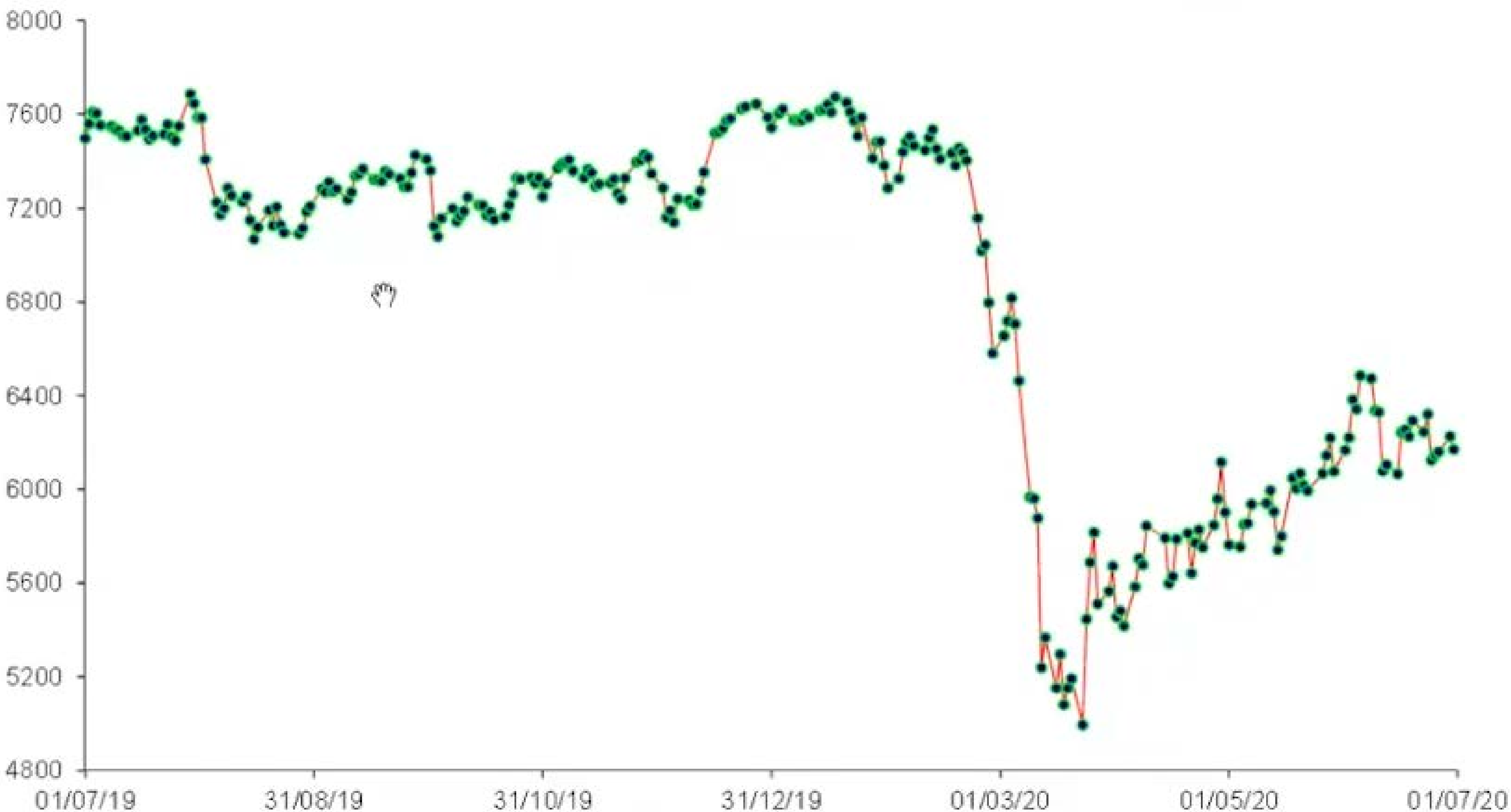
- Know about the most important empirical properties of asset returns,
- Know that changes in volatility explain many empirical effects,
- Have seen several examples of time series,
- Have seen several examples of autocorrelations.

A recent example

The graph on the next page shows the time series of daily closing levels for the FTSE 100-share index from ~~January 2018~~ July 2019 to June 2020.

The subsequent examples are taken from the textbook and are consequently more dated.

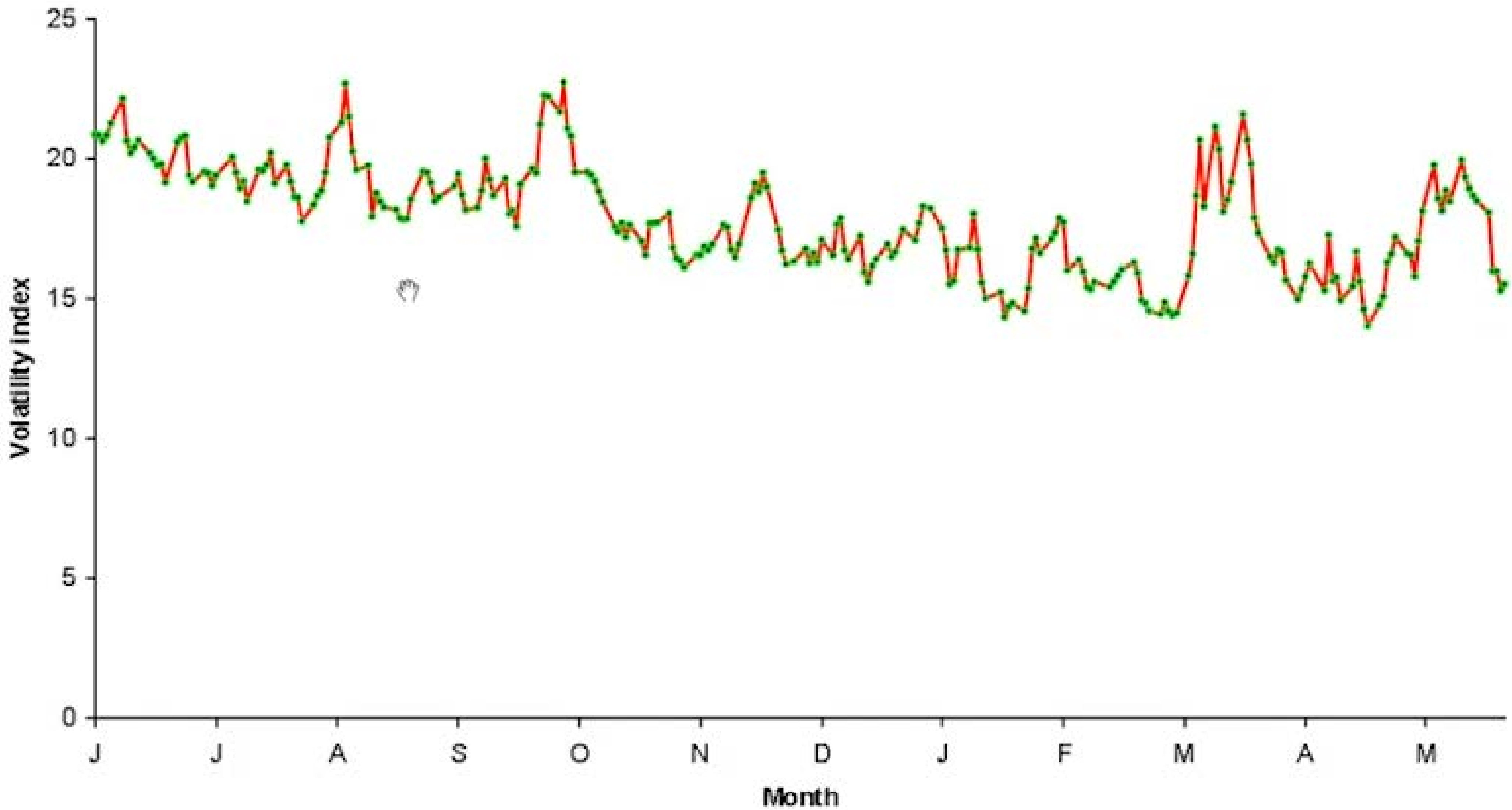
One year of FTSE 100 index levels, July 2019 to June 2020



We will encounter time series for:

- Prices,
- Returns,
- Squared returns, and other functions of returns,
- Realized variance,
- Historical and implied volatility.

Figure 1.2 A year of VIX observations



Prices and returns

The word *price* may refer to a stock price, a stock index level, an exchange rate, a commodity price, a futures price, etc.

Let p_t be a representative *price* at the end of period t. Suppose we:

- buy an asset at time $t-1$,
- receive a dividend d_t at time t, and
- sell the asset at time t.

The *simple* return on our investment is then

$$r_t' = \frac{p_t + d_t - p_{t-1}}{p_{t-1}},$$

ignoring any transaction costs.

The *continuously compounded* return r_t is related to the simple return r_t' by

$$e^{r_t} = 1 + r_t'$$

and it equals the change in the logarithms of prices, adjusted for any dividend payments:

$$r_t = \log(p_t + d_t) - \log(p_{t-1}).$$

The return measures r_t and r_t' are very similar numbers, whenever returns are close to zero, because

$$1 + r_t' = 1 + r_t + \tfrac{1}{2} r_t^2 + \ldots.$$

It would be surprising if an important conclusion depended on the choice between the definitions r_t and r_t'.

We generally prefer r_t and call it the *return* for period t.

We do so because the "continuous" definition of r_t has the advantage that a multi-period return is exactly the sum of single-period returns.

Dividend payments are often:

- zero in most periods,
- ignored when returns are calculated, so then

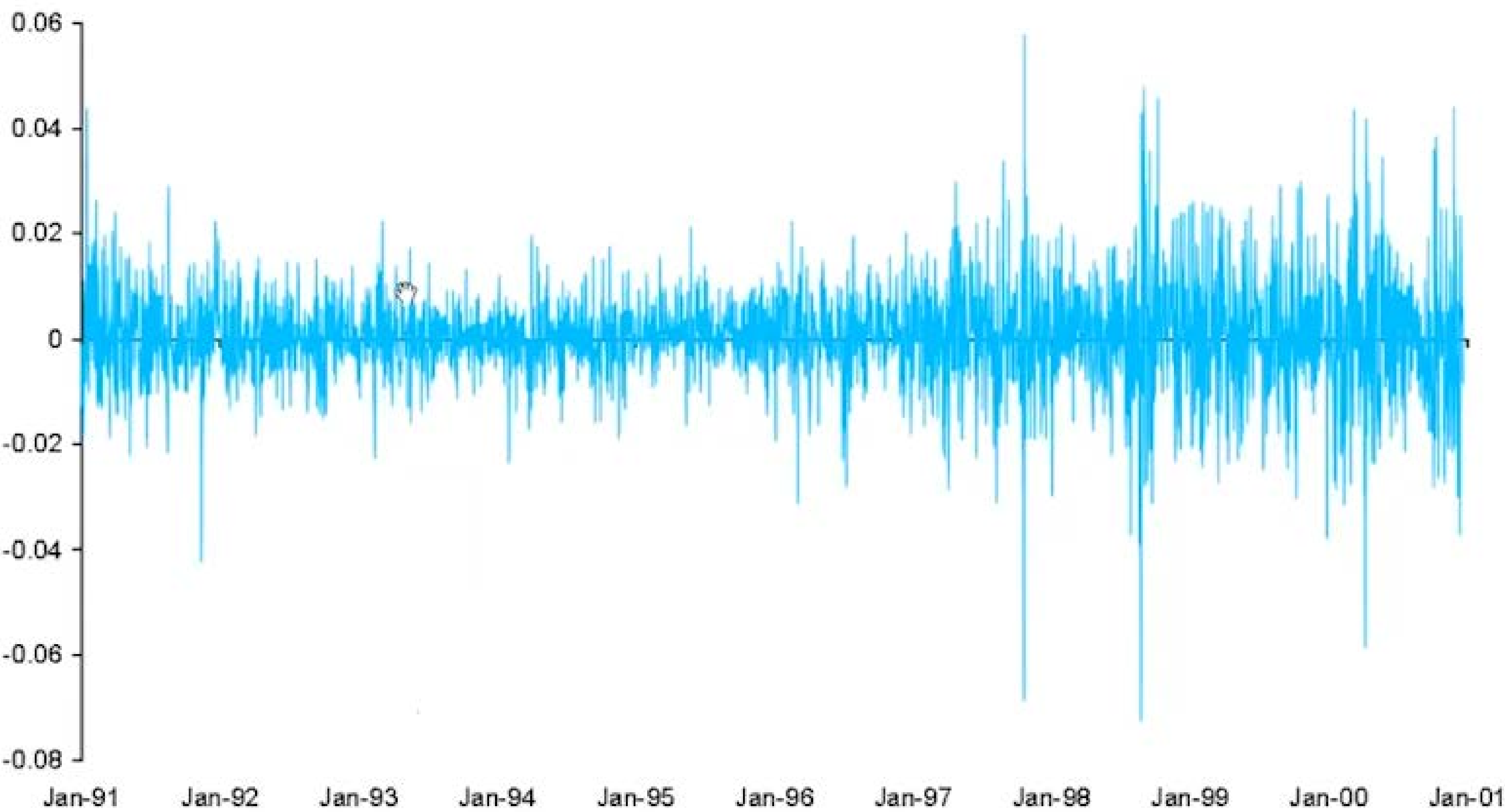
$$r_t = \log(p_t) - \log(p_{t-1}).$$

Example

The textbook contains several results for the S & P 100-share index, from 1991 to 2000:

- The time series contains daily index closing levels,
- No dividend payments are included in the index, or in the return calculations,
- See Figure 2.3 for a plot of the daily returns.

Figure 2.3 Ten years of S & P 100 daily returns



Volatility has many definitions, but they all relate to the variability of returns, usually quantified by a standard deviation.

Long time series of returns always display ***volatility clustering***:

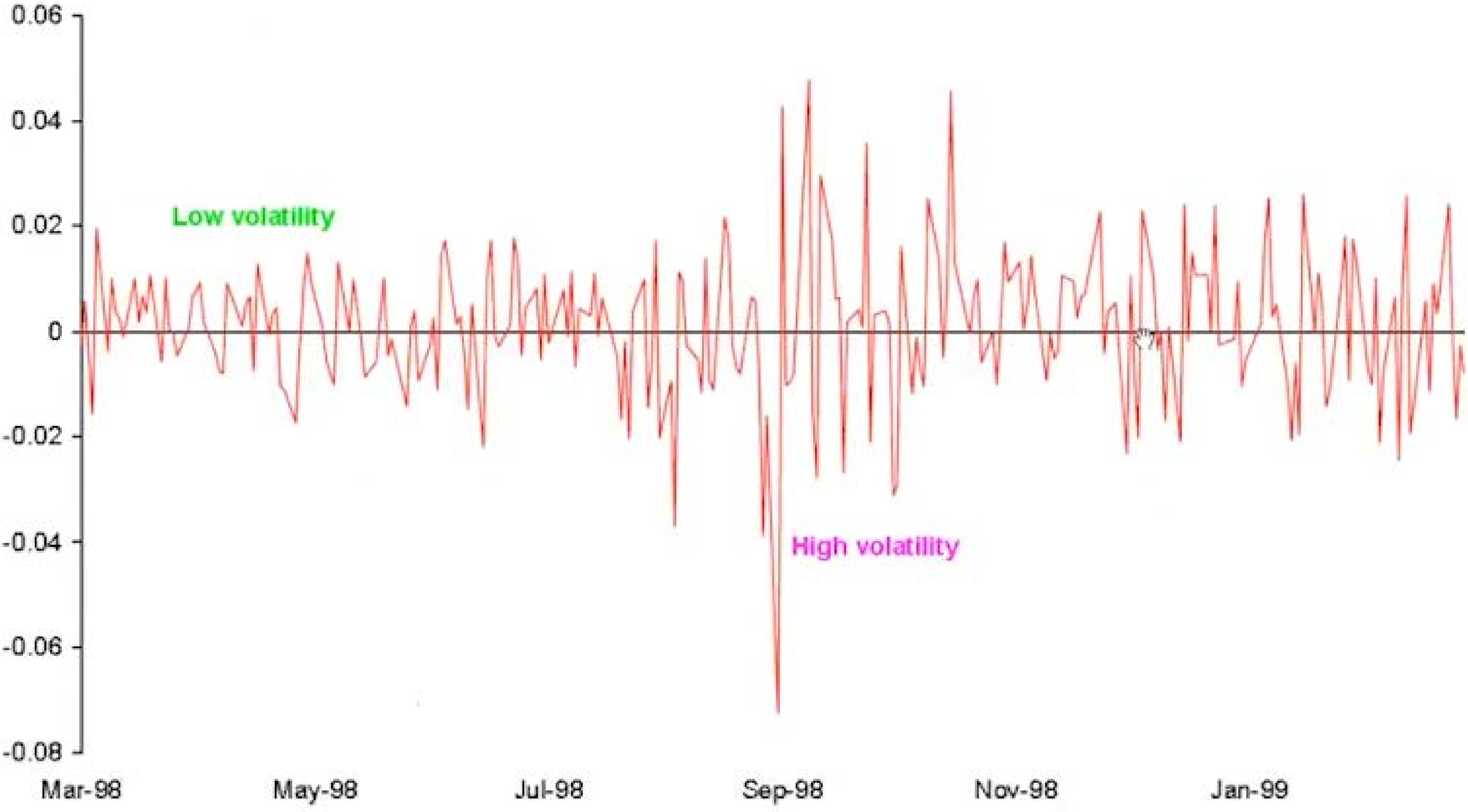
- There are visible periods of high volatility and other periods of low volatility.
- Mandelbrot: "Large price changes tend to be followed by large changes – of either sign – and small price changes tend to be followed by small changes."

It may be difficult to see *volatility clustering* on Figure 2.3.

The effect is clearly seen during 1998, on Figure 2.5:

- High volatility cluster, late-August to mid-October.
- Low volatility before August, relative to volatility after October.

Figure 2.5 One year of S & P 100 daily returns



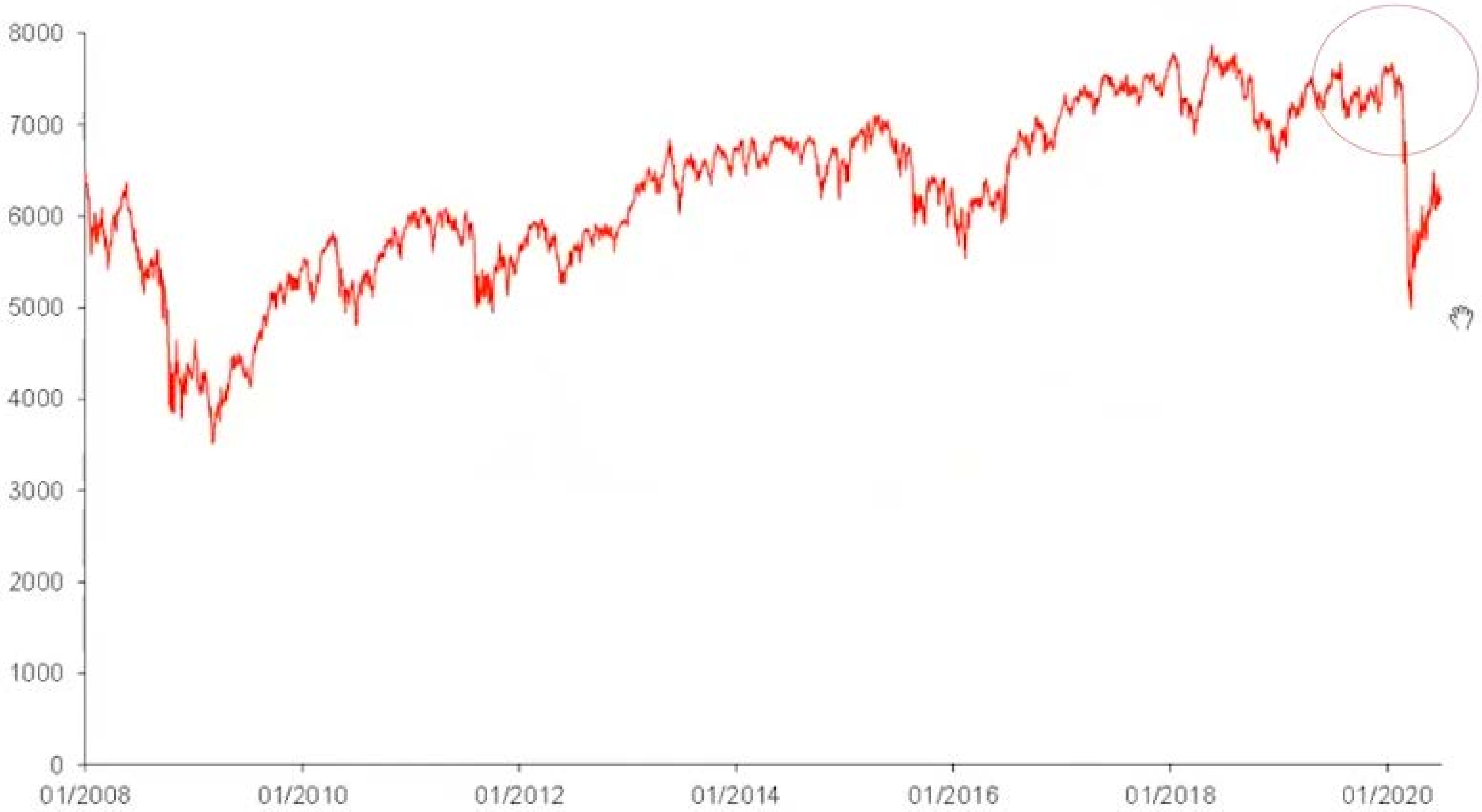
A recent example of prices, returns and volatility clustering

Twelve-and-a-half years of FTSE 100 index levels:

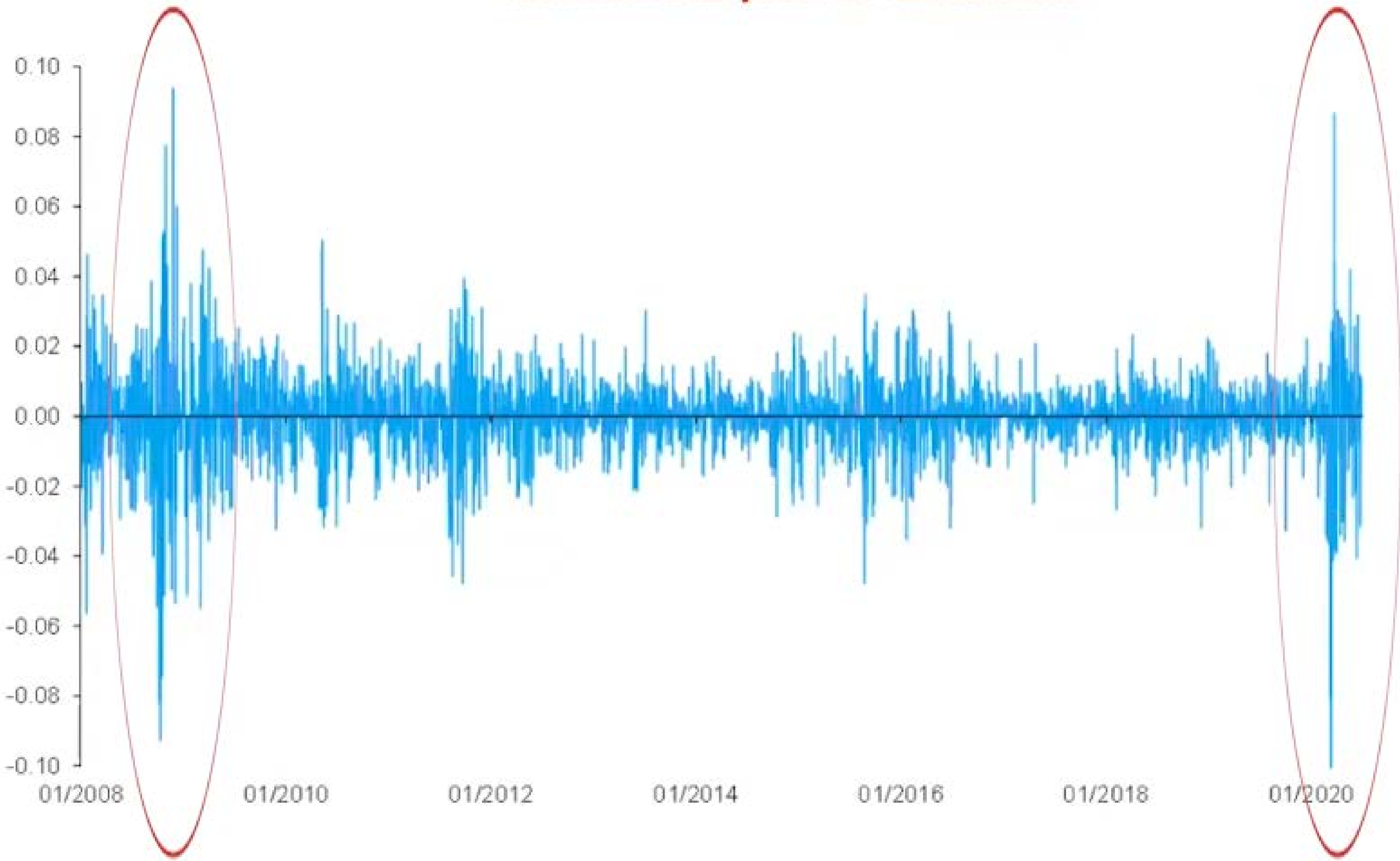
- Daily closing levels, ending in June 2020.
- Volatility is generally high for 4 years, after which it is generally lower for 7 of the next 8 years.
- Volatility is exceptional from October 2008 until December 2008, and likewise during March and April 2020.

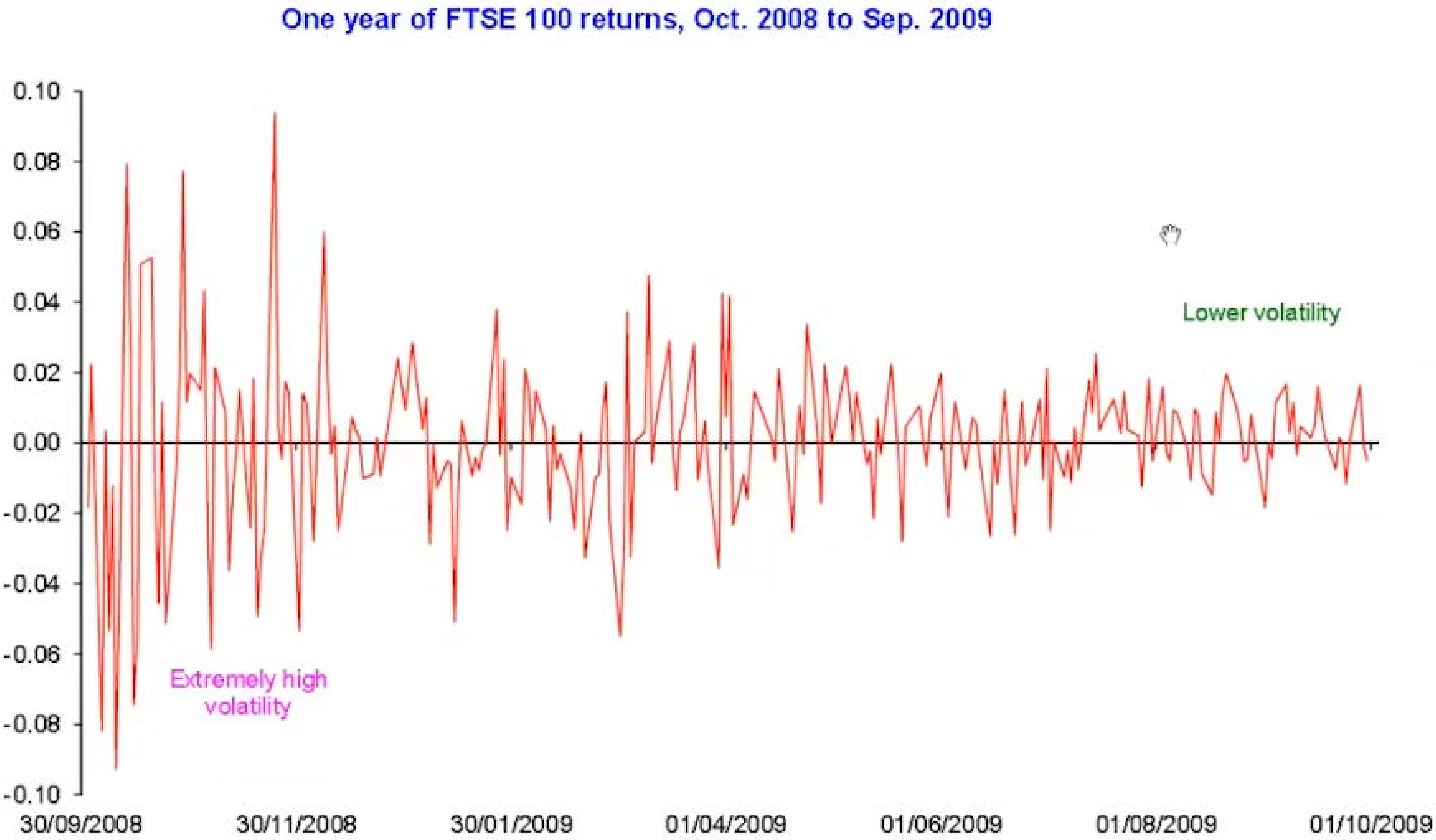
Recommended reading: Chapter 2, excluding Section 2.6.

12.5 years of FTSE 100 index levels, Jan. 2008 to June 2020



Twelve-and-a-half years of FTSE 100 returns





One year of FTSE 100 returns, Oct. 2008 to Sep. 2009
0.10
0.08
0.06
0.04
0.02
0.00
-0.02
-0.04
-0.06
-0.08
-0.10
30/09/2008
30/11/2008
30/01/2009
01/04/2009
01/06/2009
01/08/2009
01/10/2009
Extremely high volatility
Lower volatility

One year of FTSE 100 returns, July 2019 to June 2020
Exceptional volatility commences in March
0.10
0.08
0.06
0.04
0.02
0.00
-0.02
-0.04
-0.06
-0.08
-0.10
-0.12
01/07/19
31/08/19
31/10/19
31/12/19
01/03/20
01/05/20
01/07/20

Properties of daily asset returns

1. Means

The average return is small measured over one day, so consider averages over one year. These are:

- Positive for stocks in the long run.
- Typically estimated from 20th century data as 6% to 8% per annum more than the risk-free rate for U.S. & U.K. stock market indices.
- Near zero for foreign exchange investments.

Properties of daily asset returns

1. Means

The average return is small measured over one day, so consider averages over one year. These are:

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- Near zero for foreign exchange investments.

Many calendar anomalies have been reported for average returns, sometimes based on 100 years of U.S. stock index returns.

Some anomalies have disappeared in recent years and we generally **ignore them**.

U.S. anomalies include:

Day-of-the-week – Negative Monday means, perhaps 0.2% less than for other days of the week.

Day-of-the-month – Means are much higher in the first half than the second half of the month.

Month-of-the-year – Means are much higher in January than other months, especially for small firms.

Holidays – Returns on days before holidays have much higher means than for other days.

2. Standard deviations

Estimates vary substantially for short time series because of volatility clustering. Long time series provide the following approximate ranges for standard deviations:

	One day return	One year return
Currencies	0.6% to 0.9%	10% to 14%
Stock indices	0.7% to 1.3%	11% to 21%
Large US firms	1.2% to 2.0%	19% to 32%
Commodities	1.0% to 2.0%	16% to 32%

Generally the standard deviation (s.d.) decreases as firm size increases and the s.d. for a portfolio decreases as the number of constituents increases.

There is evidence for calendar effects in standard deviations.

The s.d. is a volatility measure and volatility can be expected to partially depend on news. News released when markets are closed must be reflected when they next open, which can explain calendar effects.

Thus the s.d. may be higher:

1. On Mondays than on other days,
2. On trading days that follow holidays.

Minor effects have been documented but they indicate that a relatively small amount of relevant information is announced when markets are closed.

Relevant reading: Sections 4.3 and 4.4.

Calendar effects are covered at length in Section 4.5, but they can be ignored.

3. The distribution of daily returns

A *stylized fact* is an empirical property that is observed for:

- (almost) all markets,
- (almost) all large datasets,
- (almost) all time periods.

Daily returns have **three** important stylized facts.

Stylized fact 1: The distribution of returns is not normal.

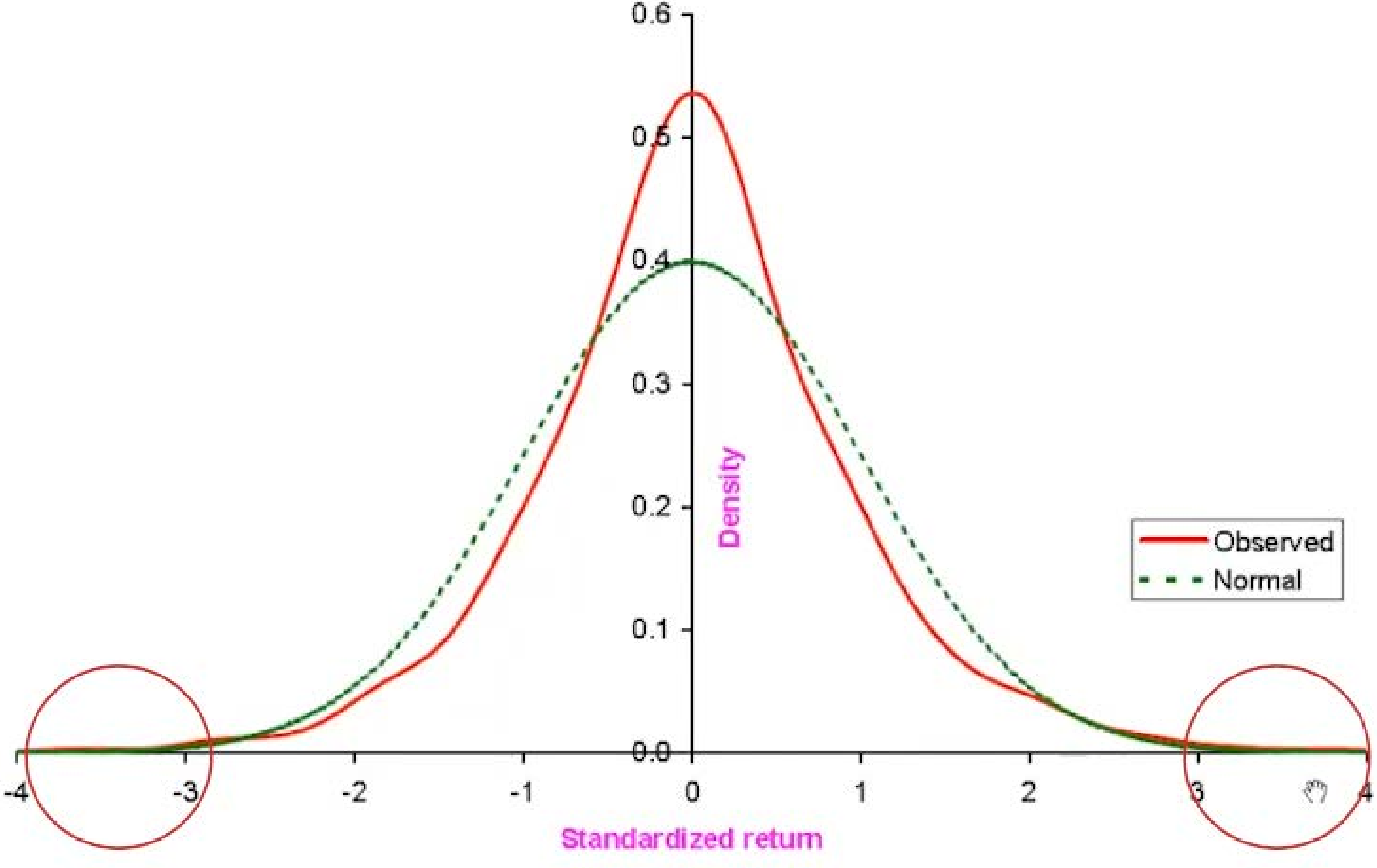
Instead, we can say of the distribution:

- It is approximately symmetric.
- It has *fat tails*.
- It has a *high peak*.

Figure 4.1 illustrates the density for S & P-500 returns.

We all see the high peak, but can we see the fat tails?

Figure 4.1
S&P-500 returns distribution



Compared with a normal distribution, estimates of the probability density of daily returns have:

- More probability within half a standard deviation (s.d.) of the mean,
- More probability three or more s.d. away from the mean.
- Less overall probability elsewhere.

Observations more than 3 s.d. away from the mean are often called *outliers* and sometimes called *extreme values*.

They occur on average, very approximately, 3 to 4 times a year for daily returns.

How big is a typical outlier?

- If the unconditional daily s.d. is 1.2%, from 12.5 recent years of FTSE-100 returns,
- And the FTSE-100 index is at 6000,
- Then 3 s.d. away from the mean is the same as a close-to-close rise or fall equal to approximately 216 points.

The frequency of daily returns more than 4 s.d. from the mean is around one or two a year; it is one every sixty years for a normal distribution.

Evidence for non-normality is often obtained from the *sample kurtosis* of returns, defined for n returns by :

$$k = \frac{\frac{1}{n-1}\sum_{t=1}^{n}(r_t - \bar{r})^4}{\left(\frac{1}{n-1}\sum_{t=1}^{n}(r_t - \bar{r})^2\right)^2}.$$

All normal distributions have kurtosis equal to 3. Sample values of k often exceed 6 for daily returns, but they are nearer to 3 for monthly returns (a consequence of the central limit theorem).

The null hypothesis that returns are both i.i.d. and come from a normal distribution is sometimes tested by evaluating

$$z = \frac{k-3}{\sqrt{24/n}}$$

and comparing z with the standard Normal distribution. Usually large values of z are obtained, which reject the null hypothesis.

Non-normality has important implications for:

- Risk management – exceptional losses are more likely than predicted by normal theory.
- Option pricing – standard pricing formulae need to be improved.

Volatility clustering explains why the distribution of daily returns is not normal.

- Some returns in a sample come from higher volatility periods when the standard deviation of returns is relatively high.
- Other returns come from lower volatility periods.
- The complete sample is then obtained from a mixture distribution.
- Whenever returns come from a mixture of normal distributions, that have different variances, the mixture will have theoretical kurtosis > 3.

  For a proof, see Section 8.4.

We can **not** say which distribution best describes returns.

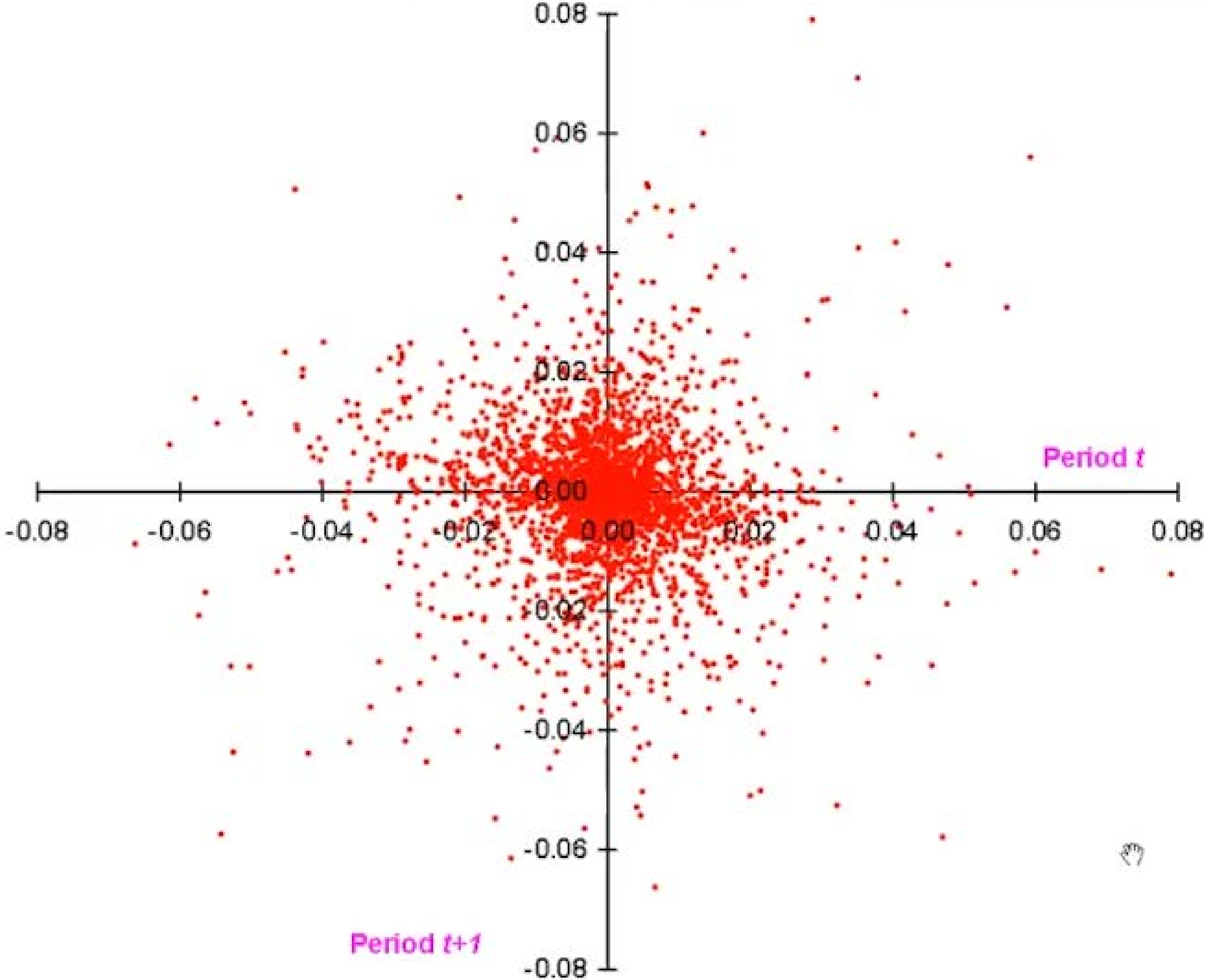
Many plausible distributions are defined by mixtures of Normal distributions, e.g. generalised Student-t, generalised error, generalised hyperbolic and variance-gamma.

Some people have argued for a distribution which has infinite variance but the empirical evidence is solidly against this idea.

Recommended reading: Sections 4.6 and 4.7.

Section 4.8 includes a discussion of specific distributions for returns.

Figure 4.5
Gold returns in consecutive periods



The linear dependence within a sample of n returns is measured using the *sample autocorrelations*:

$$\hat{\rho}_\tau = \frac{\sum_{t=1}^{n-\tau} \underline{(r_t - \bar{r})}\,\underline{(r_{t+\tau} - \bar{r})}}{\sum_{t=1}^{n} (r_t - \bar{r})^2}$$

for a set of positive lags τ.

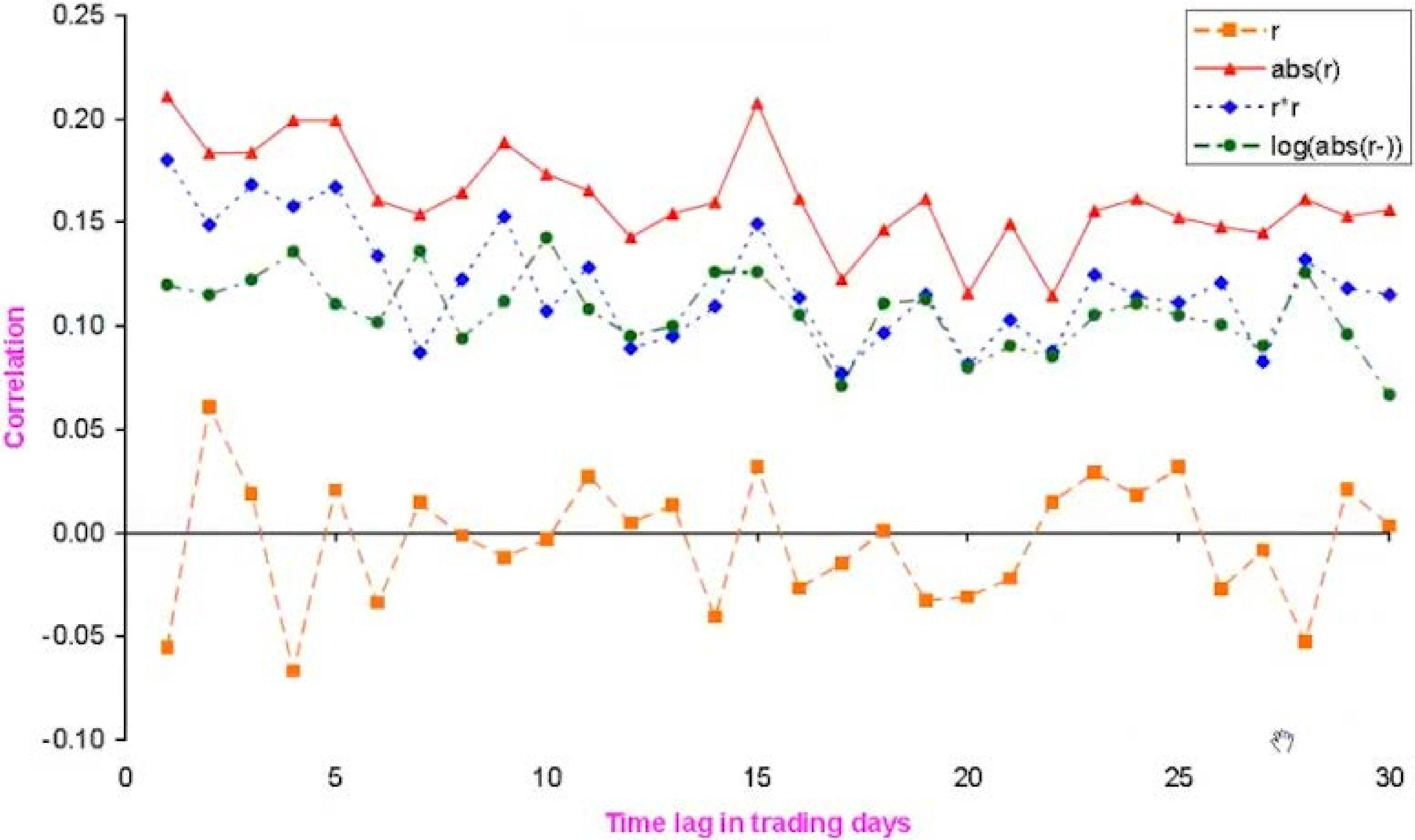
The hat symbol (^) indicates that each sample autocorrelation can be interpreted as an estimate of a correlation term $\rho_\tau = \mathrm{cor}(R_t, R_{t+\tau})$ for a *stochastic process* $\{R_t\}$ which is assumed to be stationary; $\{R_t\}$ generates the observed returns, $\{r_t, 1 \le t \le n\}$.

There are other formulae that provide almost identical results, for example the Excel function CORREL can be used to find one autocorrelation.

Example: Figure 4.8 - The autocorrelations of the gold returns, at lags 1 to 30, are nearly all within the range from -0.05 to 0.05 for a series of 2522 returns.

The sample autocorrelation $\hat{\rho}_\tau$ can be considered to be an estimate of a population parameter ρ_τ. There are many tests of the null hypothesis of **no correlation**, $H_0 : \rho_\tau = 0$, but we will not discuss these. *If curious then see Chapters 5 and 6.*

Figure 4.8
Gold autocorrelations



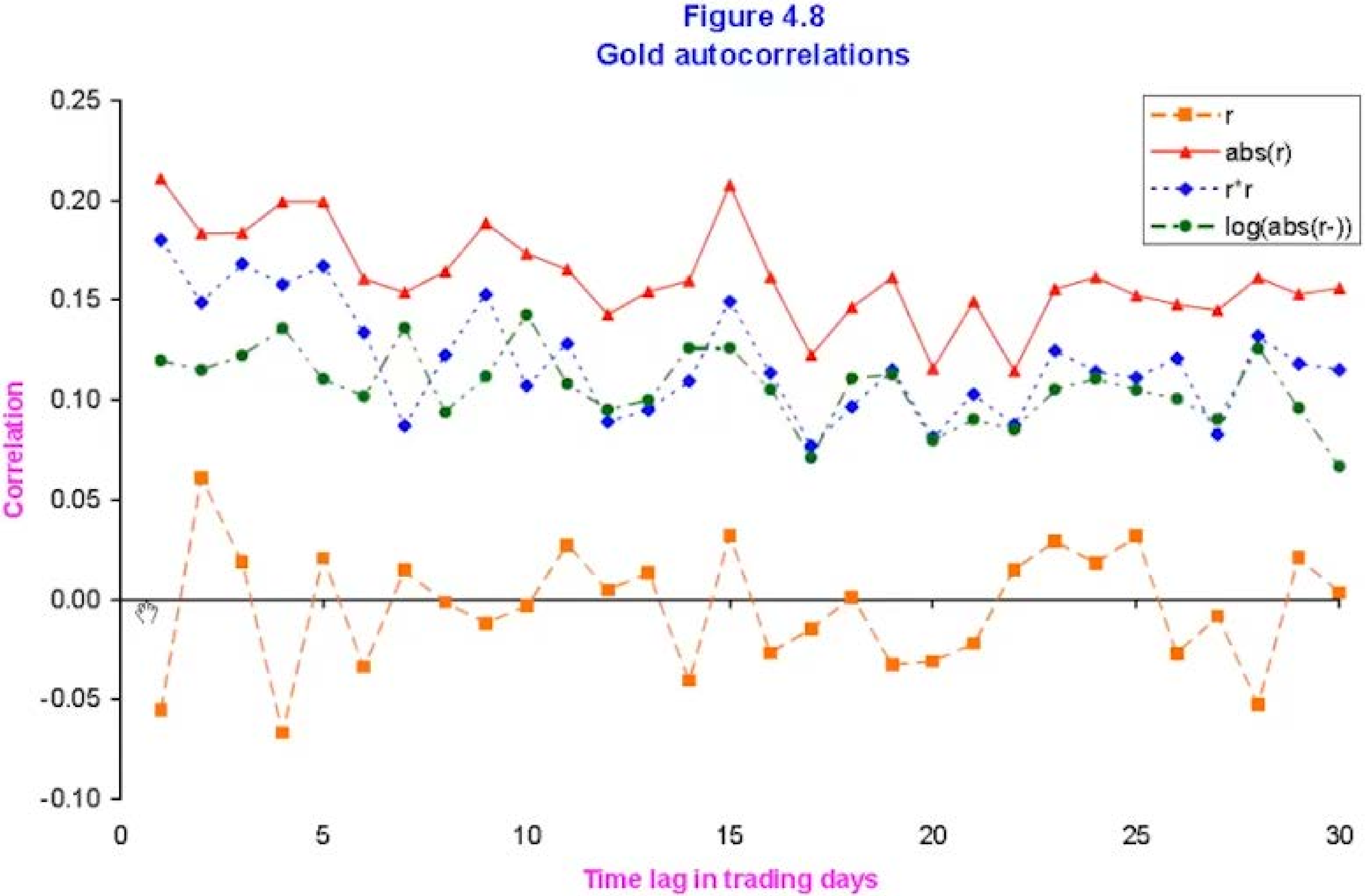


Figure 4.8
Gold autocorrelations
r
abs(r)
r*r
log(abs(r-))
Correlation
Time lag in trading days
0.25
0.20
0.15
0.10
0.05
0.00
-0.05
-0.10
0
5
10
15
20
25
30

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The more specific hypothesis that r_t and $r_{t+\tau}$ are observations from **independent and identically distributed** random variables can be tested more easily, as follows:

- Null hypothesis: the stochastic process is i.i.d.
- Sampling theory: $\hat{\rho}_\tau \sim N(0, 1/n)$, approximately, for an i.i.d. process.
- Test statistic: calculate $z_\tau = \sqrt{n}\hat{\rho}_\tau$ from n observations.
- Null distribution: for large n, can use $z_\tau \sim N(0,1)$.
- Test result: for a 5% significance level, reject the null hypothesis if either $z_\tau < -1.96$ or $z_\tau > 1.96$.
- Here and elsewhere $N(\mu, \sigma^2)$ denotes the Normal distribution having mean equal to μ and variance equal to σ^2.

The autocorrelations from lags 1 to k can be combined into the Box-Pierce test statistic, which is routinely calculated by time series software:

$$Q_k = n\sum_{\tau=1}^{k}\hat{\rho}_\tau^2 = \sum_{\tau=1}^{k} z_\tau^2 .$$

The null hypothesis is then rejected if Q_k exceeds a critical point given by the chi-squared distribution with k degrees-of-freedom, χ_k^2.

This test applies the result that the sample autocorrelations for lags 1 to k are almost independent random variables for an i.i.d. process.

For the gold series, whose autocorrelations are shown on Figure 4.8, $n \cong 2500$,

$\hat{\rho}_\tau \sim N(0, 0.02^2)$ and the null hypothesis is rejected at lags 1, 2 and 4 but not at the other lags (significance level 5%).

Also, $Q_{30} = 67.9$ and this exceeds the 5% critical point which is 43.8.

Recommended reading: Section 4.9.

5. Correlation between functions of returns

A scatter diagram for absolute returns on consecutive days, $|r_t|$ and $|r_{t+1}|$, provides an indication of positive correlation between consecutive absolute returns. Figure 4.6 is an example for absolute gold returns.

Sample autocorrelations can be calculated for several functions of returns, such as:

- absolute returns $|r_t|$,
- squared returns r_t^2,
- logarithms of mean-adjusted absolute returns $\log(|r_t - \bar{r}|)$.

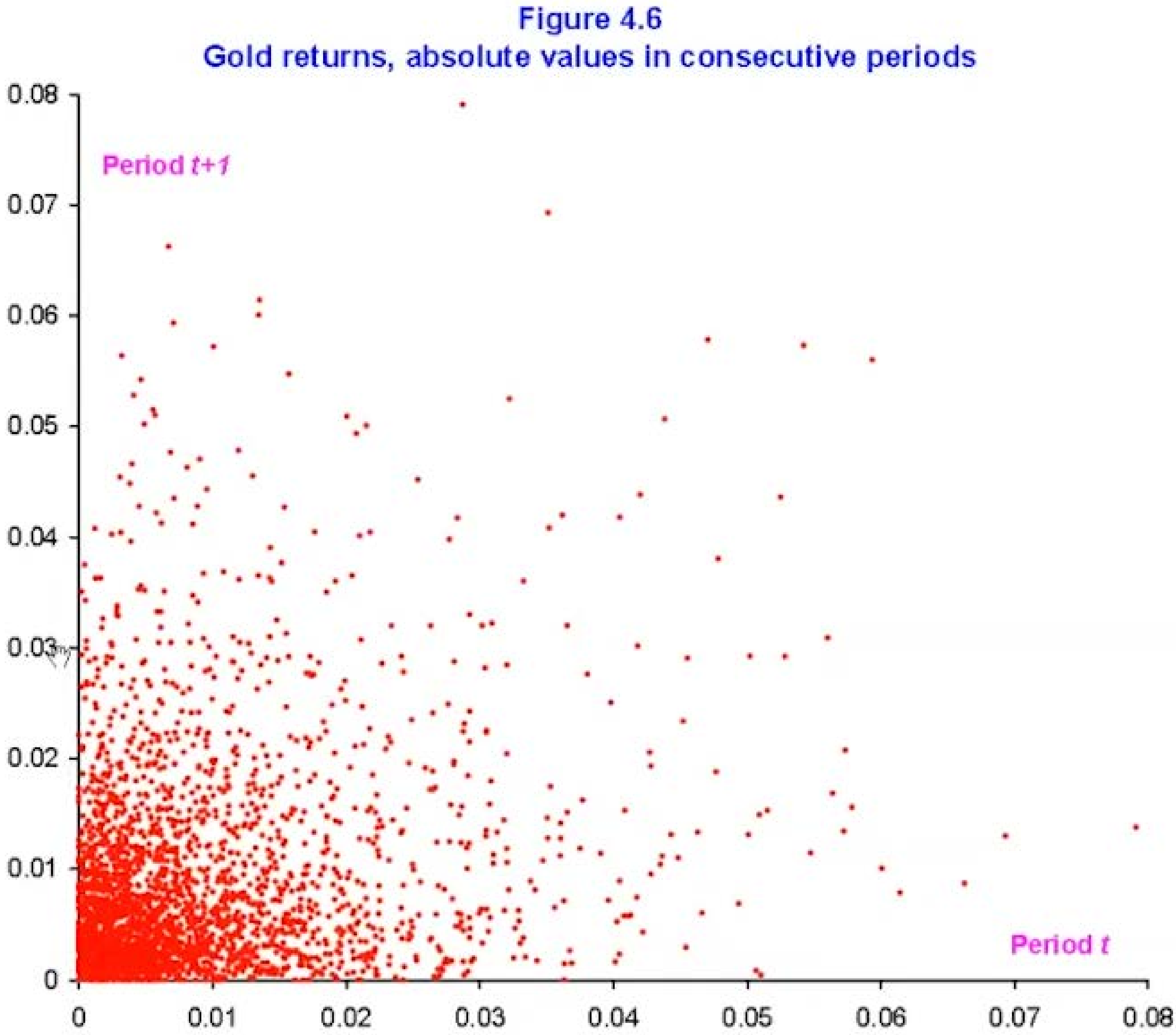
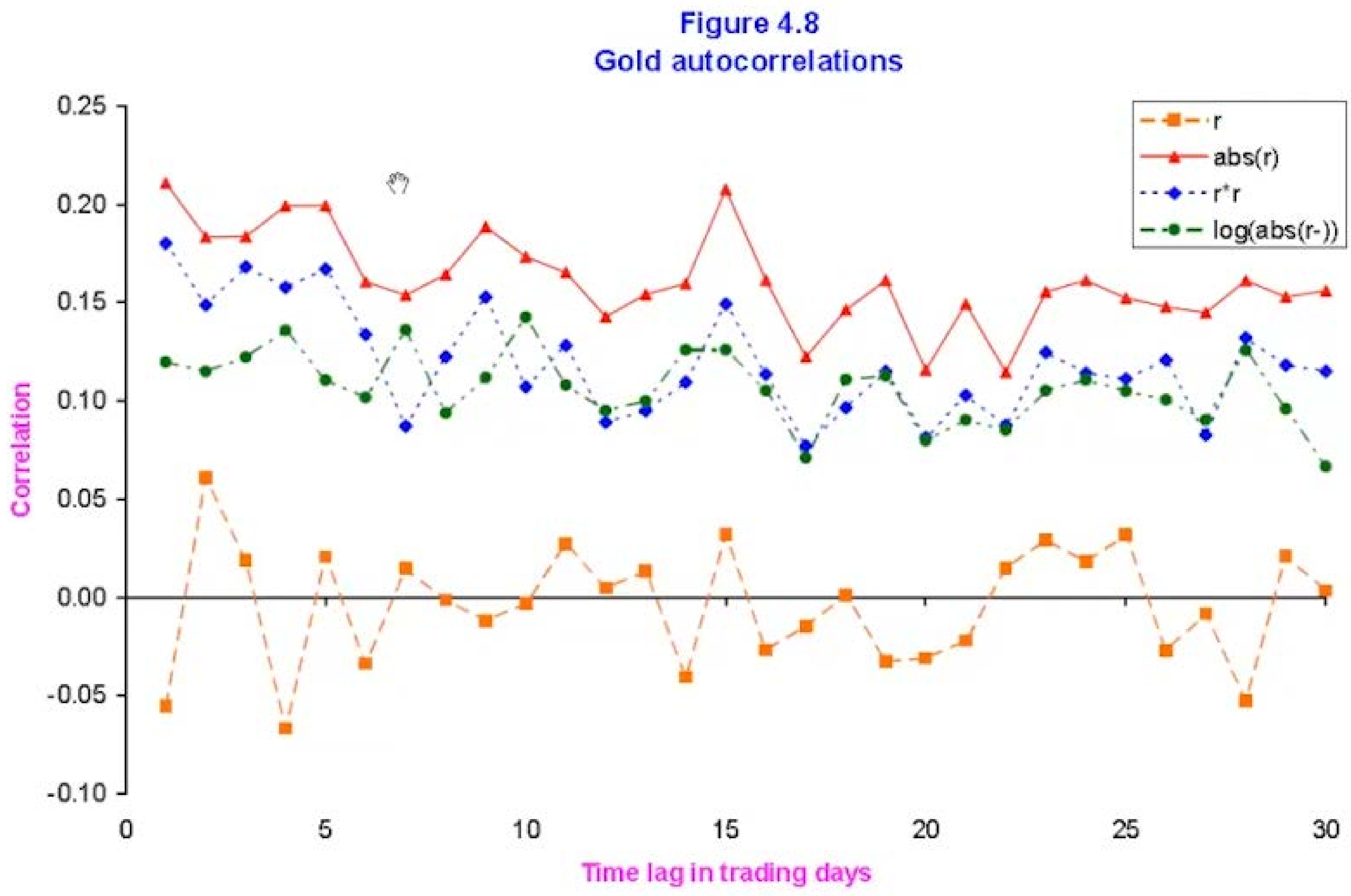


Figure 4.6
Gold returns, absolute values in consecutive periods
Period t+1
Period t
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
0
0.01
0.02
0.03
0.04
0.05
0.06
0.07
0.08

These calculations produce positive sample autocorrelations which support:

Stylized fact 3: There is positive dependence between absolute returns on nearby days, and likewise for squared returns.

See Figure 4.8 again for the gold series. All the autocorrelations for functions of absolute returns exceed the maximum autocorrelation for returns.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0.25
0.20
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0.10
0.05
0.00
-0.05
-0.10
0
5
10
15
20
25
30

Large positive correlations for absolute returns:

- reject the hypothesis that absolute returns come from an i.i.d. process,
- consequently (and easily) show that **the returns process is not i.i.d.**

Gold example - Each sample autocorrelation for any of $|r_t|$, r_t^2, $\log(|r_t - \bar{r}|)$, with lag between 1 and 30, rejects the i.i.d. hypothesis at the 5% level. Most reject at much lower levels. The values of the Q_{30} statistic are 1999, 1139 and 885 for these three functions of returns, compared with 68 for returns.

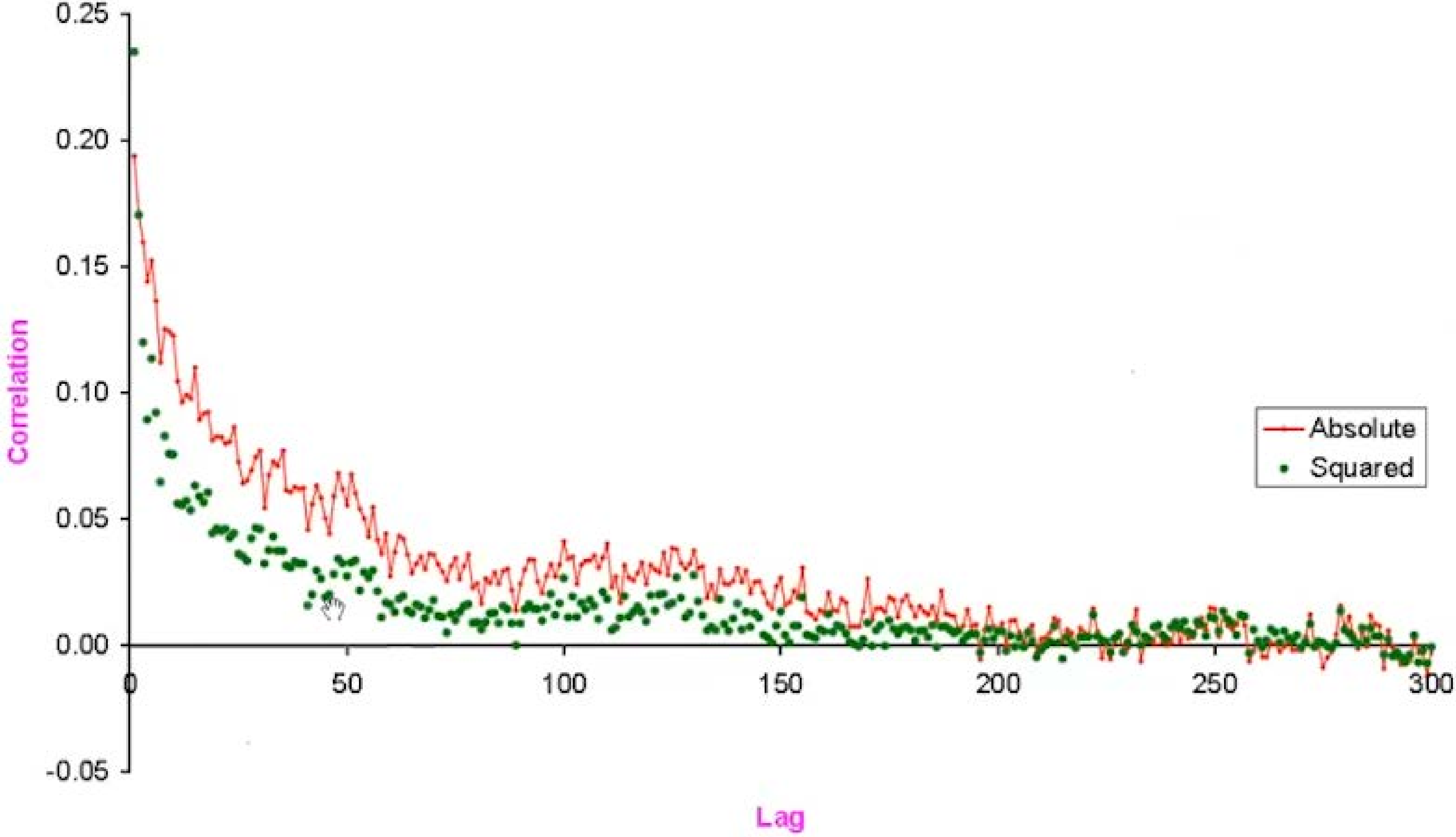
How are we to interpret "nearby" in SF3?

We can say the positive dependence in functions of returns persists for many lags.

Figure 4.12 shows averages of sample autocorrelations for 20 series. Positive dependence is visible for more than 125 lags, hence there is positive dependence between those daily absolute returns which are separated by less than six months.

Ding, Granger and Engle consider the autocorrelations of $\left|r_t\right|^{\lambda}$ for very many lags and all positive λ. There is some evidence that the most linear dependence occurs when λ is near to 1.

Figure 4.12
Autocorrelations of absolute and squared returns, averages across 20 series



Volatility clustering explains the positive dependence among absolute and squared returns.

- High volatility on consecutive days ensures a relatively high probability that both $|r_t|$ and $|r_{t+1}|$ are high.
- Conversely, if volatility is low on consecutive days then it is more likely that both $|r_t|$ and $|r_{t+1}|$ will be low.
- Then, if volatility changes slowly, returns data will have positive correlation between $|r_t|$ and $|r_{t+1}|$. *For a proof, again see Section 8.4.*

The consequences of positive dependence between absolute returns are the same as those of volatility clustering.

They include:

- Volatility changes and can be predicted fairly accurately.
- In continuous time, prices do not follow geometric Brownian motion.
- Option traders should apply improvements on the Black-Scholes formula.

Recommended reading: Section 4.10.

Properties of high-frequency asset returns

Prices can be analysed using observations:

- Once a day, or less often. We may call this a *low* frequency.
- For every trade and/or quotation. This is often complicated, because :
  - Buy and sell quotes differ, with trades at bid, ask or some intermediate level.
  - Times between price observations are variable.
  - Often have thousands of observations a day, sometimes there are millions.
- At some intermediate *high* frequency, such as one price for *each minute*, or one every *five minutes*, or one every *thirty minutes*. The extra observations improve volatility estimates and forecasts.

Textbook example: FTSE-100 futures prices, March 2000 contract on 22 December 1999. The market was then electronic, order-driven and open from 08:00 to 17:30.

Figure 12.1 shows *transaction* (= *trade*) prices.

- One trade, on average, every 20 seconds.
- … but, median inter-trade time 5 seconds.
- More frequent trading after markets open in the U.K. (197 from 08:00 to 08:30) and the U.S. (323 from 14:30 to 15:30).
- 45% of the trade-to-trade price changes are zero,
- … and 17% are one *tick* = 0.5 index points.

Figure 12.2 covers the 15 minutes from 15:45 to 16:00.

- Buy (= *bid*) and sell (= *ask*) *quotes* are shown by small dots, connected by lines.
- Transaction prices are shown by large dots.
- Similar numbers of bid, ask and trade prices are shown (but can't be sure everything is recorded).
- Trades occur at either the most recent bid or the most recent ask.

Figure 12.3 shows the *spread* (= ask minus bid), for contemporaneous quotes.

- Minimum 0.5, maximum 12, mode 1, median 2, average 2.2.

Recommended reading: Sections 12.2 and 12.3.

Figure 12.2 Bids, asks and trades for fifteen minutes

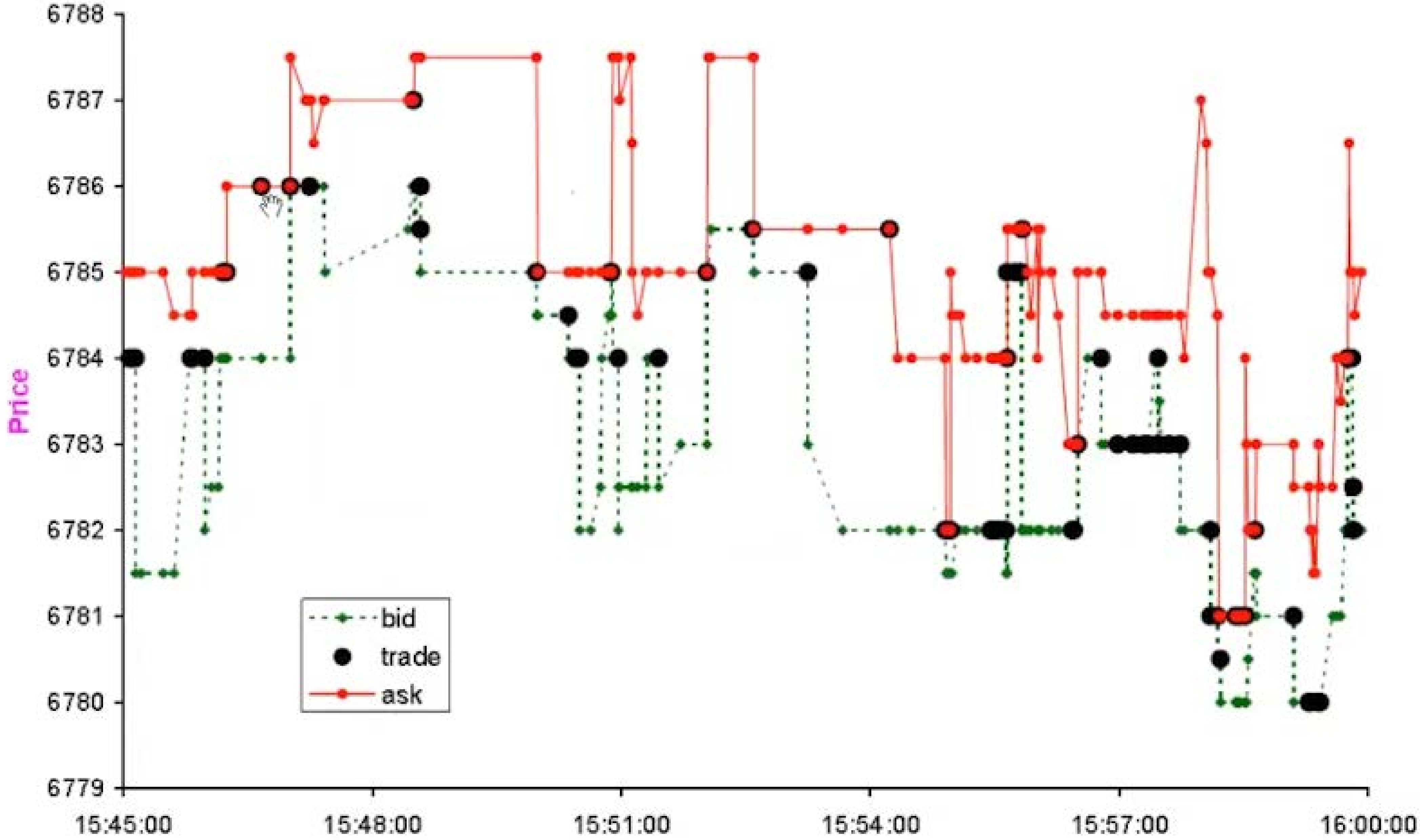
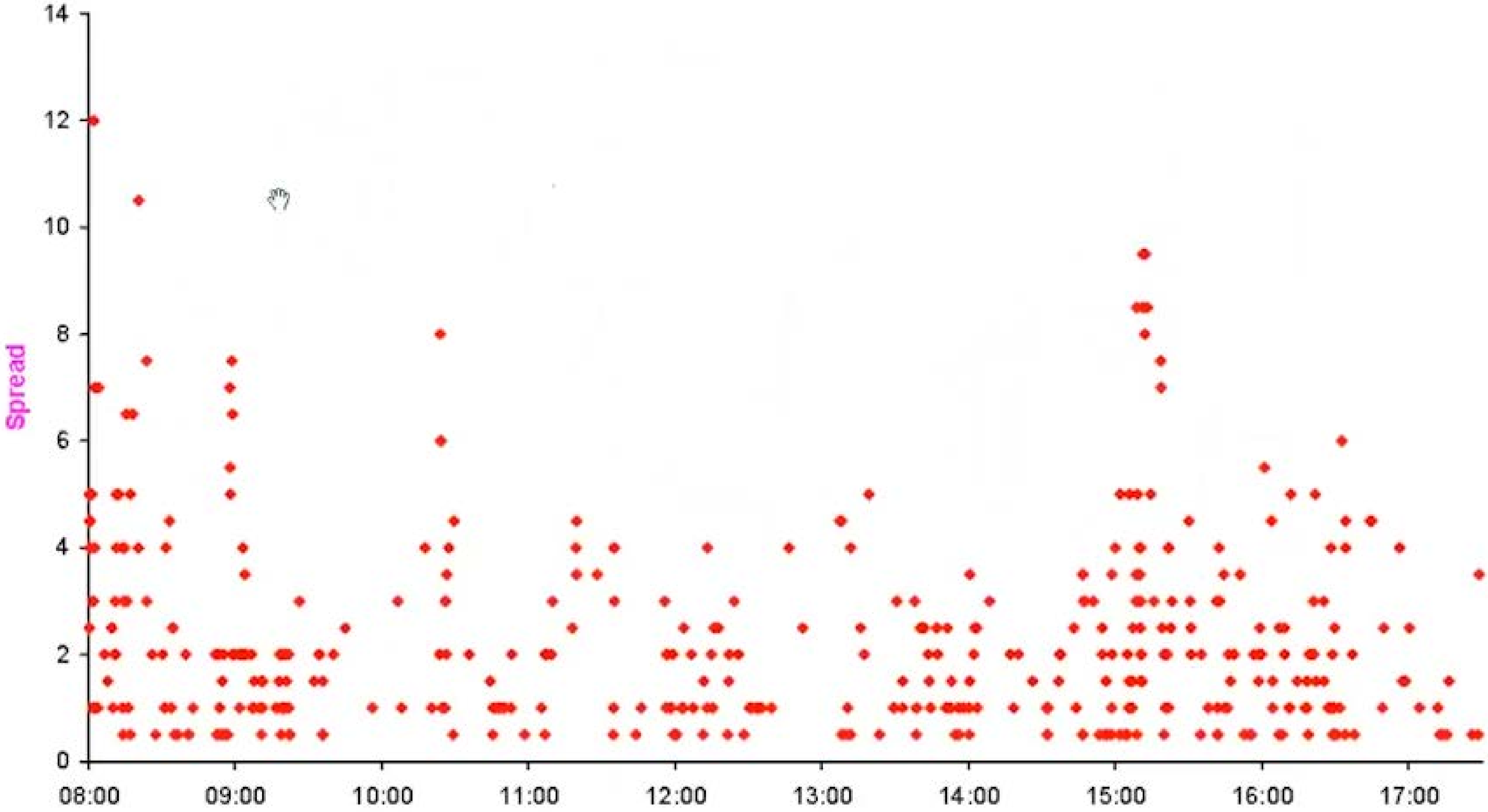


Figure 12.3 Bid/ask spreads

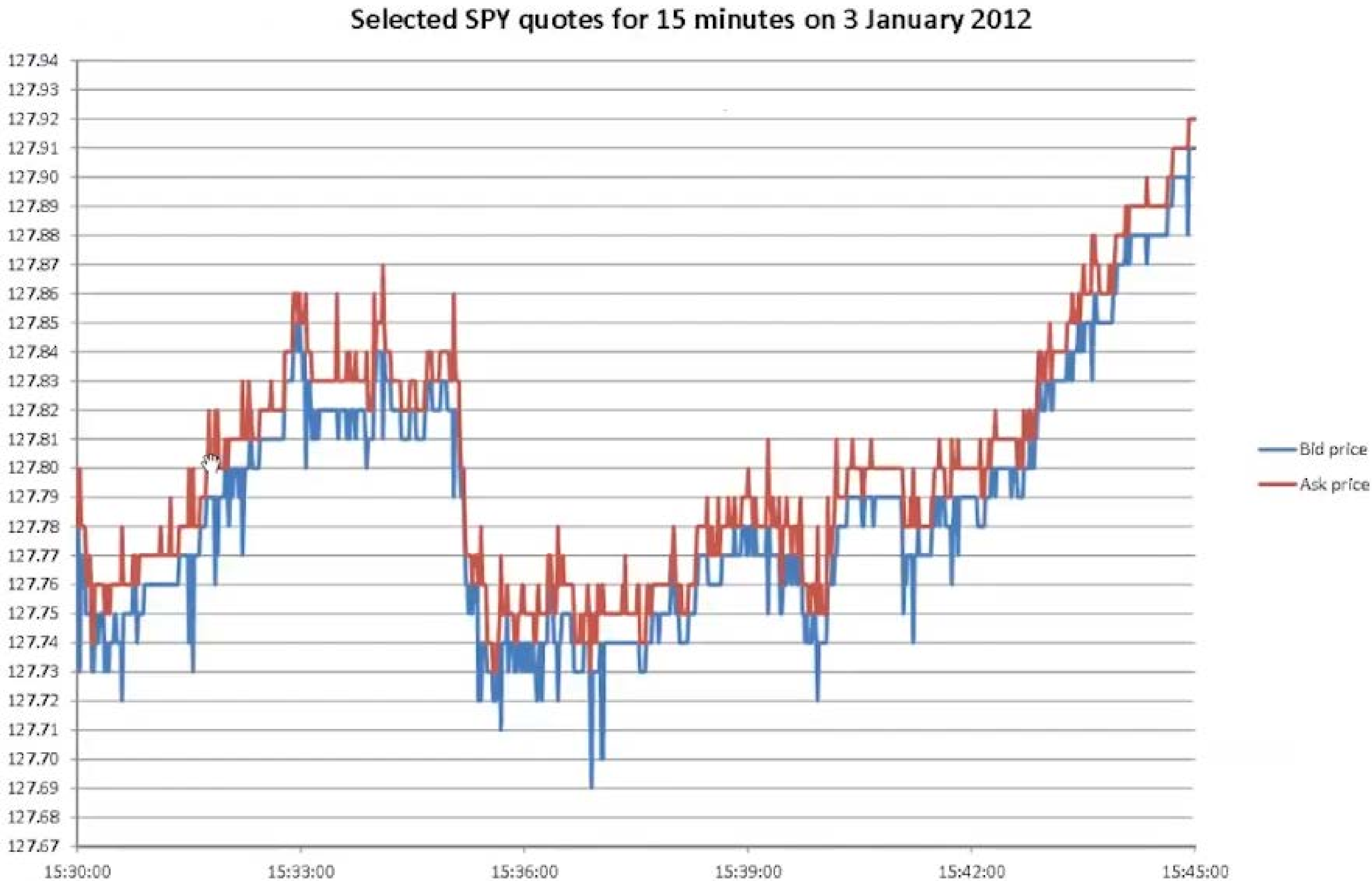


Another example: Prices for the S&P 500 ETF (SPY) on 3 January 2012, extracted from the TAQ database.

SPY trades on several U.S. exchanges. The most competitive bid-ask spread is usually one tick, which equals one cent.

From 15:30 to 15:45,

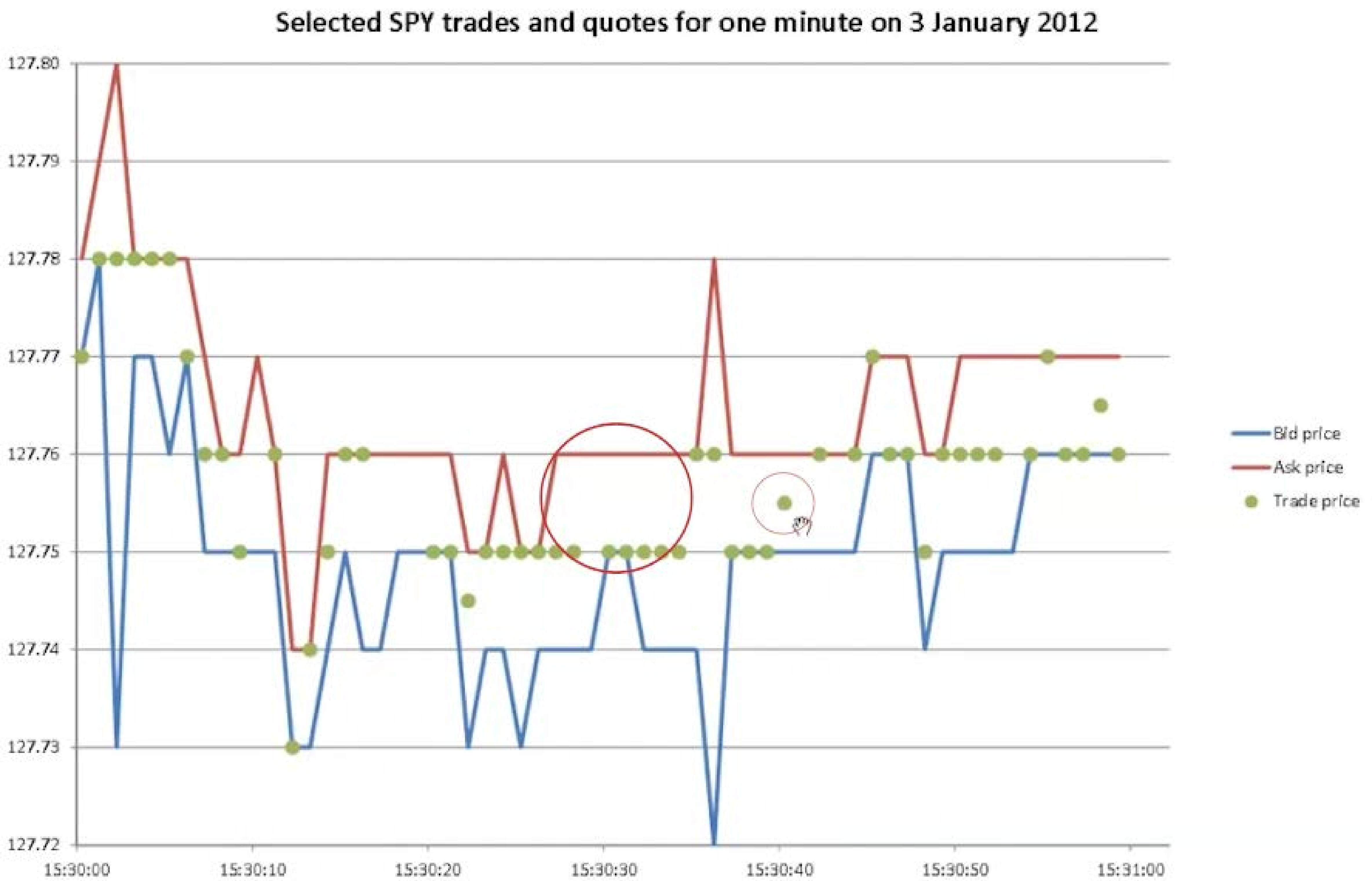
- There are 16,498 trades and 313,617 pairs of bid and ask quotes.
- The chart shows the first pair of quotes for each second.
- These quotes are not always the most competitive.



Selected SPY quotes for 15 minutes on 3 January 2012
127.94
127.93
127.92
127.91
127.90
127.89
127.88
127.87
127.86
127.85
127.84
127.83
127.82
127.81
127.80
127.79
127.78
127.77
127.76
127.75
127.74
127.73
127.72
127.71
127.70
127.69
127.68
127.67
15:30:00
15:33:00
15:36:00
15:39:00
15:42:00
15:45:00
Bid price
Ask price

From 15:30:00 to 15:30:59,

- The chart shows the first trade and the first pair of quotes for each second.
- A few seconds contain no trades.
- Trades times are *not* exactly equal to quote times.
- Some recorded trade prices include fractions of a cent; these are 'dark pool' prices.


Selected SPY trades and quotes for one minute on 3 January 2012
127.80
127.79
127.78
127.77
127.76
127.75
127.74
127.73
127.72
15:30:00
15:30:10
15:30:20
15:30:30
15:30:40
15:30:50
15:31:00
Bid price
Ask price
Trade price

1. Stylized facts for intraday returns

- Intraday returns have a fat-tailed distribution, whose kurtosis increases as the frequency of price observations increases.
- Intraday returns from **traded** assets are almost uncorrelated. Any important dependence is usually negative and between consecutive returns.
- There is substantial positive dependence among intraday absolute returns. It occurs at many low lags. It **also occurs** when the intraday returns are separated by an integer number of days.

Figure 12.4 Autocorrelations for DM/$ 30-minute absolute returns

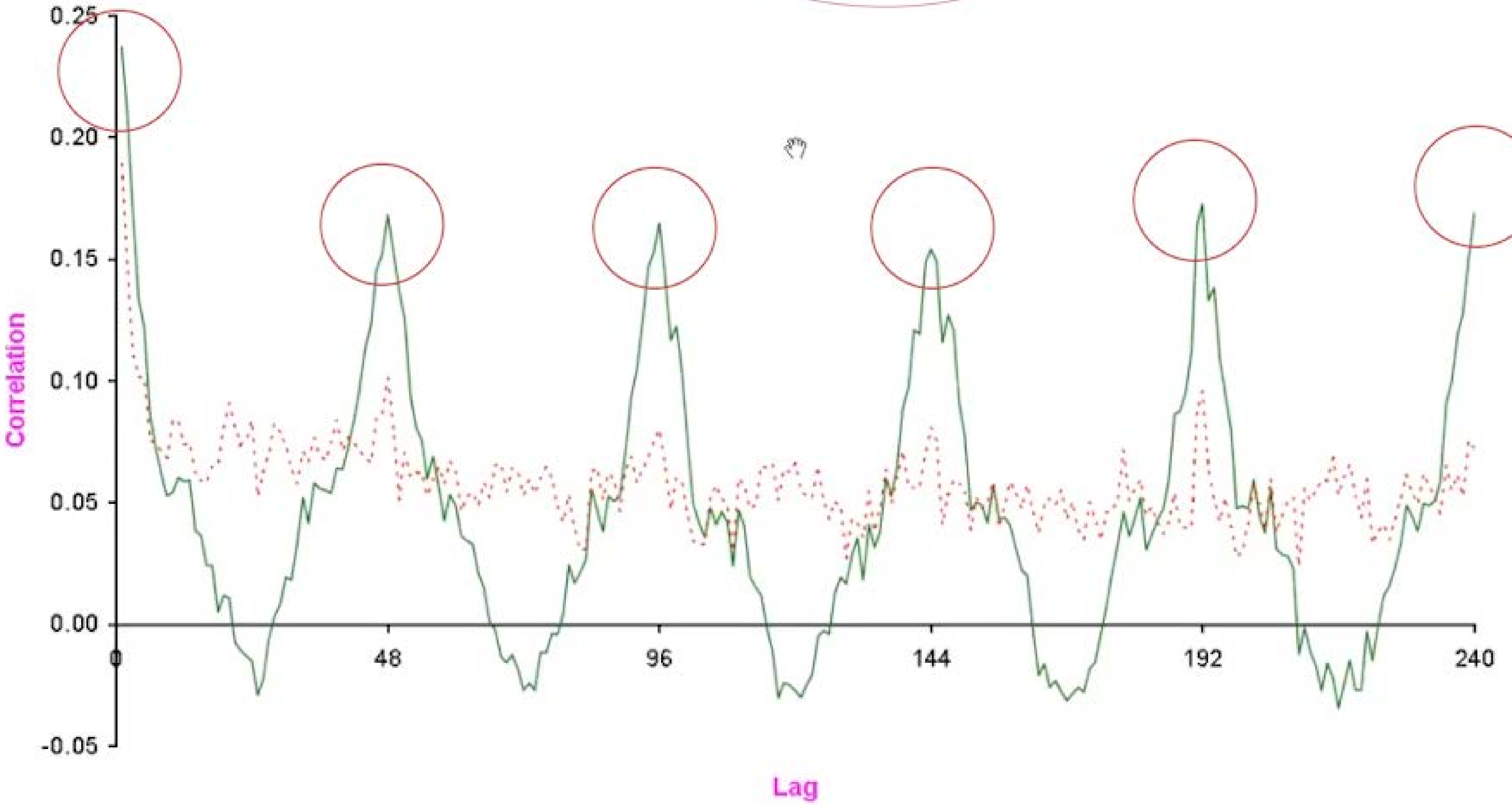
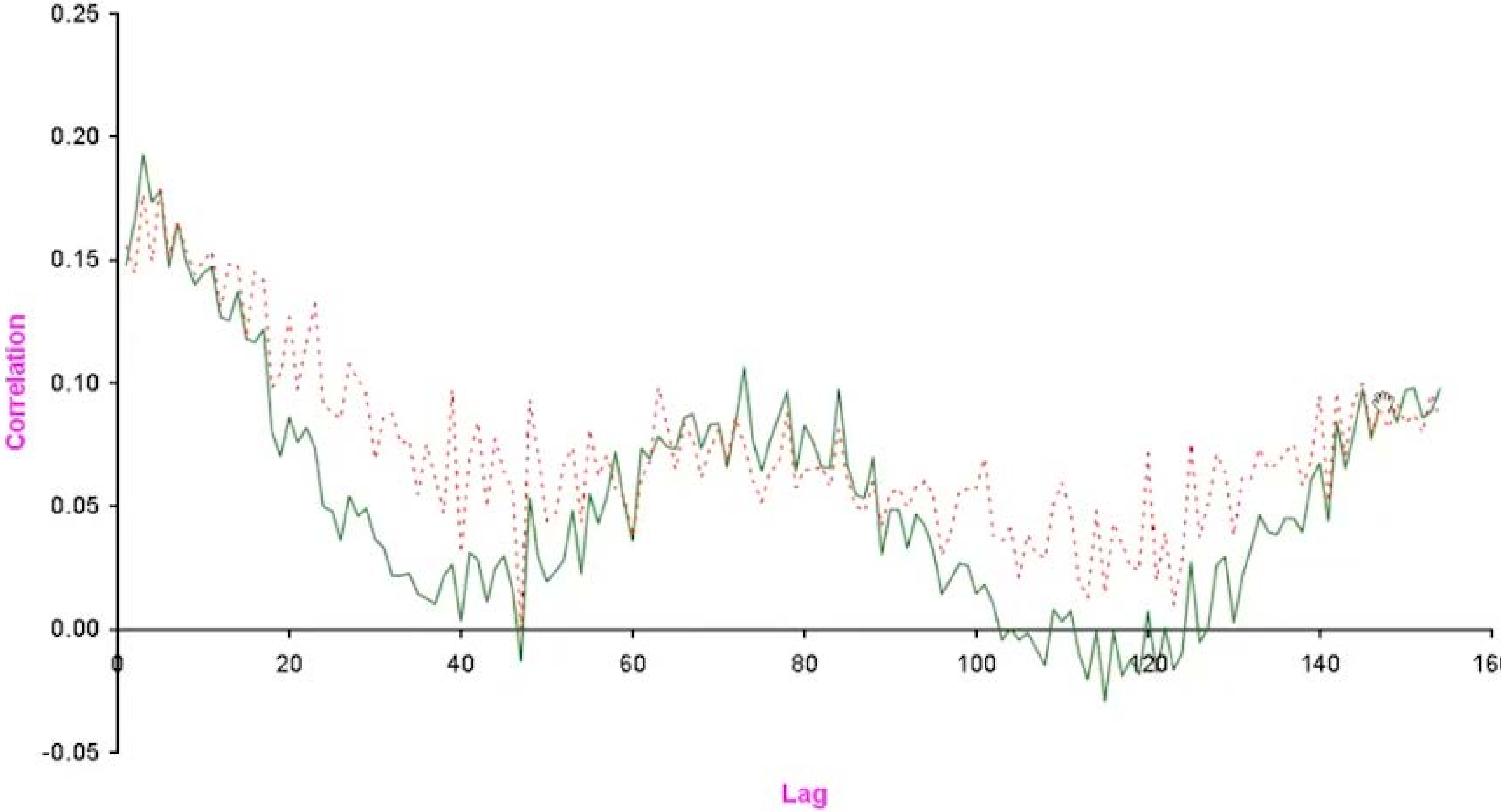


Figure 12.5 Autocorrelations for intraday absolute S & P returns



- Figure 12.4, FX pattern repeats after 48 thirty-minute periods (one calendar day).
- Figure 12.5, equity pattern repeats after 77 five-minute periods (one trading day).

- The average level of volatility depends on the time of day, with a significant intraday variation.
- There are short bursts of high volatility in intraday prices that follow major macroeconomic announcements.

Recommended reading: Section 12.4.

2. Intraday volatility patterns

There are distinctive volatility patterns that depend on:

- The time during the day,
- The day of the week,
- Scheduled macroeconomic news announcements about unemployment, trade balances, GNP, inflation, money supply, etc.

Figure 12.7 summarises the intraday pattern for the London stock futures market, across **all days** of the week, from Nov. 1993 to July 1998.

It shows variance *proportions* (factors) for 91 five-minute intervals when the market was open - from 08:35 to 16:10 local time. These,

- Sum to 1 across the day.
- Are proportional to the sample variances of five-minute returns for the intervals.
- Are highest when the market opens at 08:35.
- Increase slightly when U.K. macro. news is announced at 09:30 local time.
- Decline from 09:35 to 13:30.

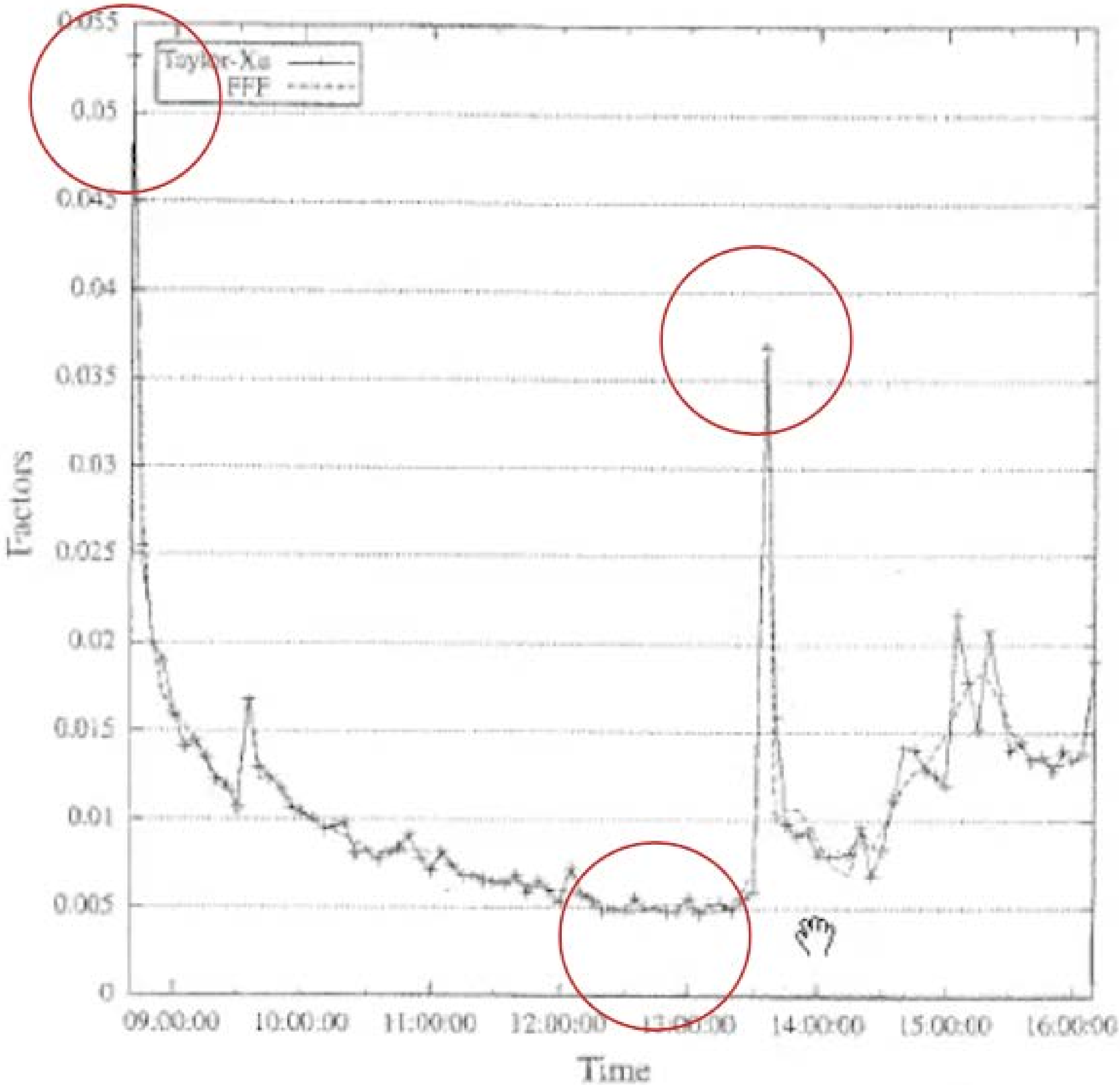


FIGURE 2

Five-minute fitted open-market variance proportions for the FTSE-100 futures index, using all days of the week, for the period from 18/11/1993 to 17/07/1998.

- Rise sharply at 13:30 in London (08:30 in New York) when U.S. macro. news is announced on ***some*** days, particularly some ***Fridays***.
- Are higher when U.S. stock markets are also open, from 14:30 onwards.
- Only refer to the open-market period. About 30% of all price variation occurs when the market is closed.

Figure 12.8 shows estimates of the London pattern by day of the week.

- Monday has the highest volatility peak at the open, presumably reflecting the longer closed market period.
- Friday has the highest mid-day peak, because the most important U.S. macro. news is released then.
- Otherwise, the pattern is similar for all five days.

However, U.S. macro. news is only released on a half of all Fridays. Ederington and Lee show that the volatility spike only occurs on the announcement days.

3. Realized volatility

Volatility can be estimated more precisely from high-frequency returns than from daily returns, because more observations can be used.

Let $r_{t,j}$, $j = 1, 2, ..., N$ represent the high frequency (e.g. five-minute) returns on day t.

Then the *realized volatility*, defined by

$$\sigma_t = \sqrt{\sum_{j=1}^{N} r_{t,j}^2},$$

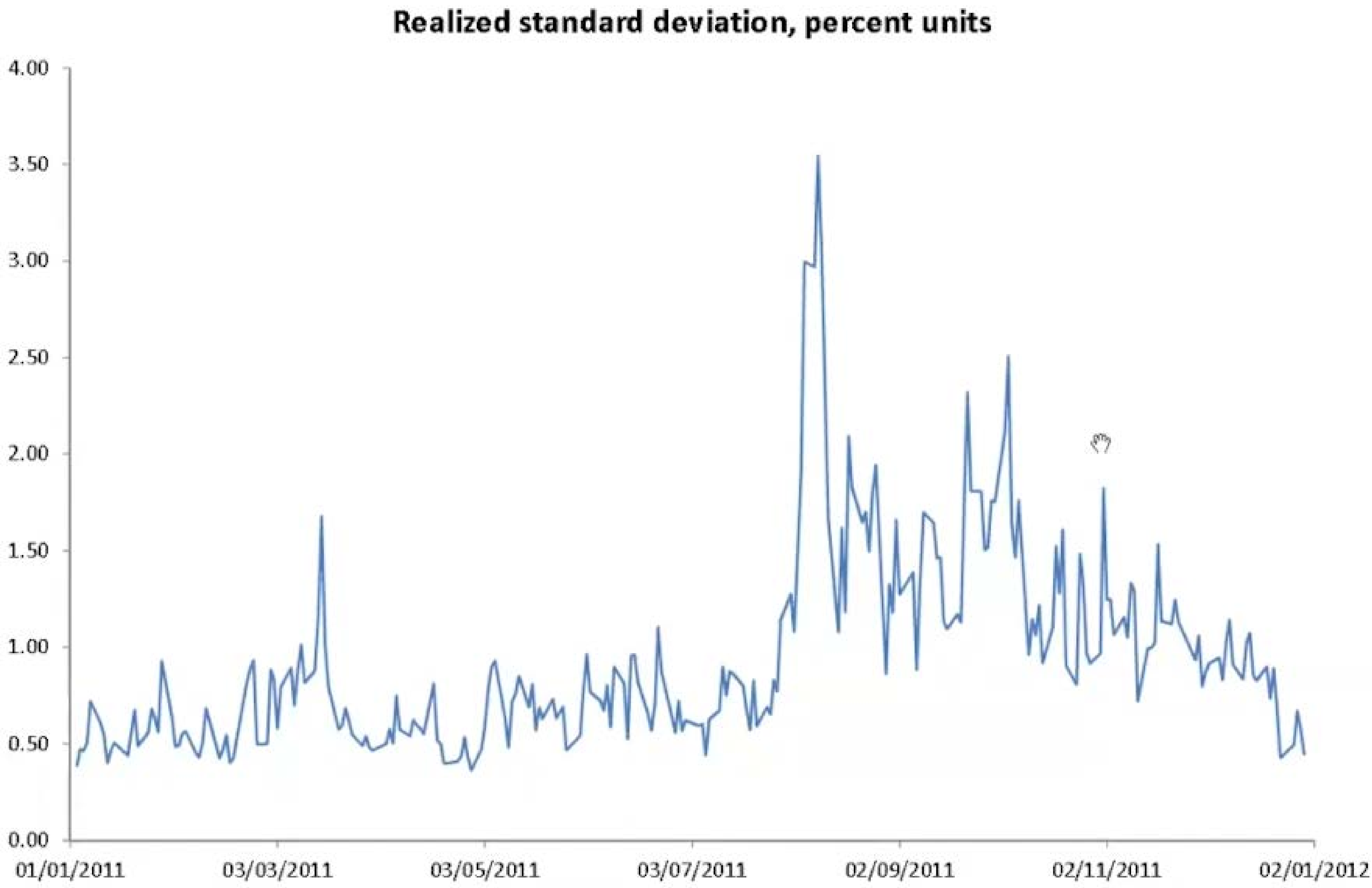
is a fairly accurate measure of the volatility during trading hours on day t.

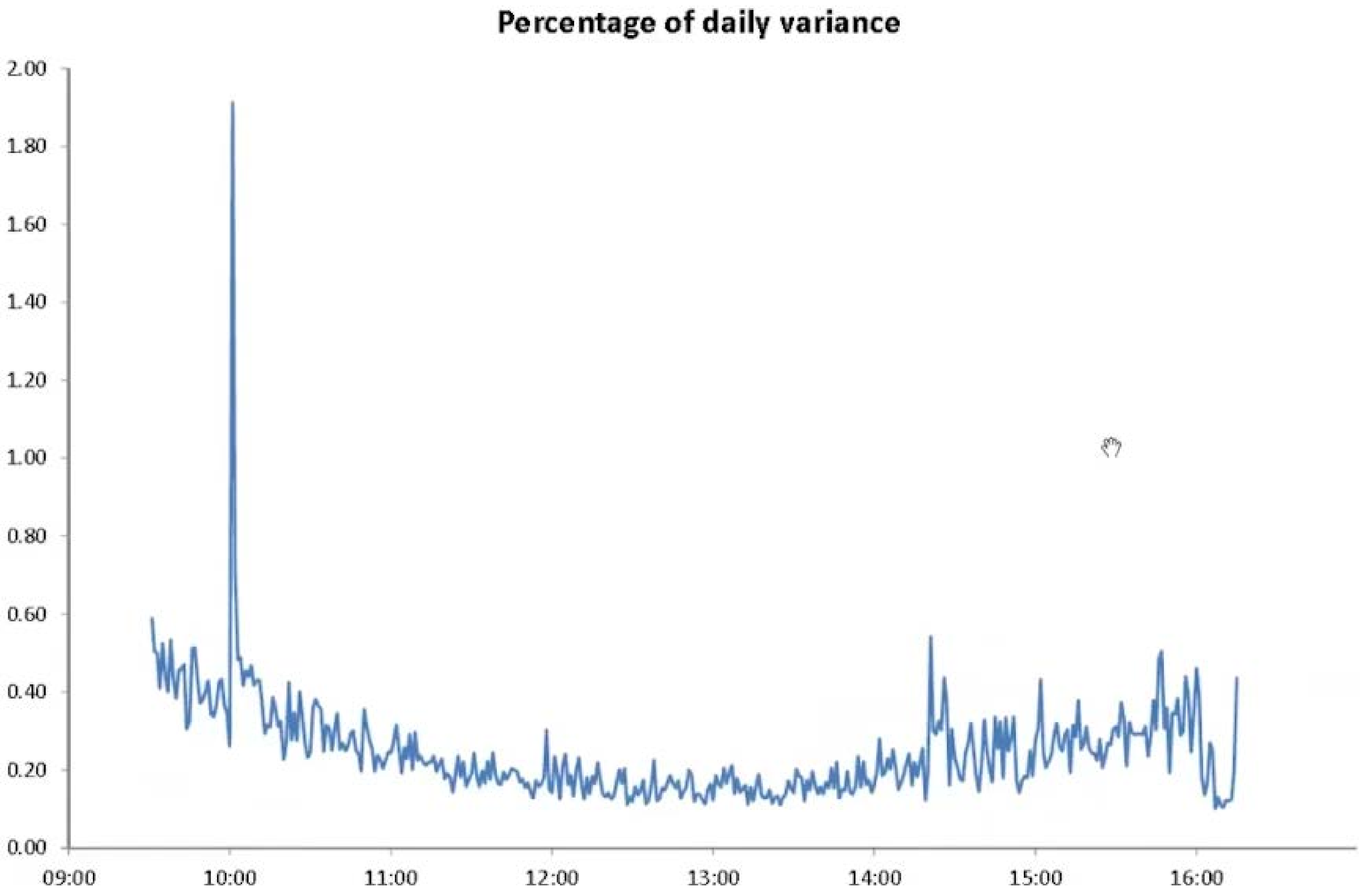
File SPY_2011_by_minute for Fitch students.xlsm:

- Contains SPY trade prices for each full trading day in 2011.
- There are 406 prices for each day, one a minute from 09:30 to 16:15 EST inclusive.

- Realized variance shoots up in August.
- Is visibly higher during Aug/Sep/Oct than during Apr/May/Jun.
- A realized s.d. of 1% is the same as an annualized open-market volatility of 16%.

- Percentages of daily variance sum to 100 across 405 one-minute intervals.
- Clear peak at 10:00 EST, explained by macroeconomic news.
- More volatility around open and close, than mid-day.


Realized standard deviation, percent units
4.00
3.50
3.00
2.50
2.00
1.50
1.00
0.50
0.00
01/01/2011
03/03/2011
03/05/2011
03/07/2011
02/09/2011
02/11/2011
02/01/2012


Percentage of daily variance
2.00
1.80
1.60
1.40
1.20
1.00
0.80
0.60
0.40
0.20
0.00
09:00
10:00
11:00
12:00
13:00
14:00
15:00
16:00

For many markets it is found that:

- The distribution of daily returns divided by realised volatility, i.e. of r_t / σ_t, is **approximately normal**, unlike the distribution of returns. See Figure 12.12: the dotted curve (normal) is near the solid curve (empirical data).
- The distribution, through time, of σ_t is **approximately lognormal**. See Figure 12.13: again the two curves are similar.
- The autocorrelations of volatility (and its logarithm) decay slowly and resemble those of a "*long memory*" process. See Figure 12.14.

Further reading: Sections 12.8 and 12.9. Both sections contain technical material, particularly 12.8.

Figure 12.12

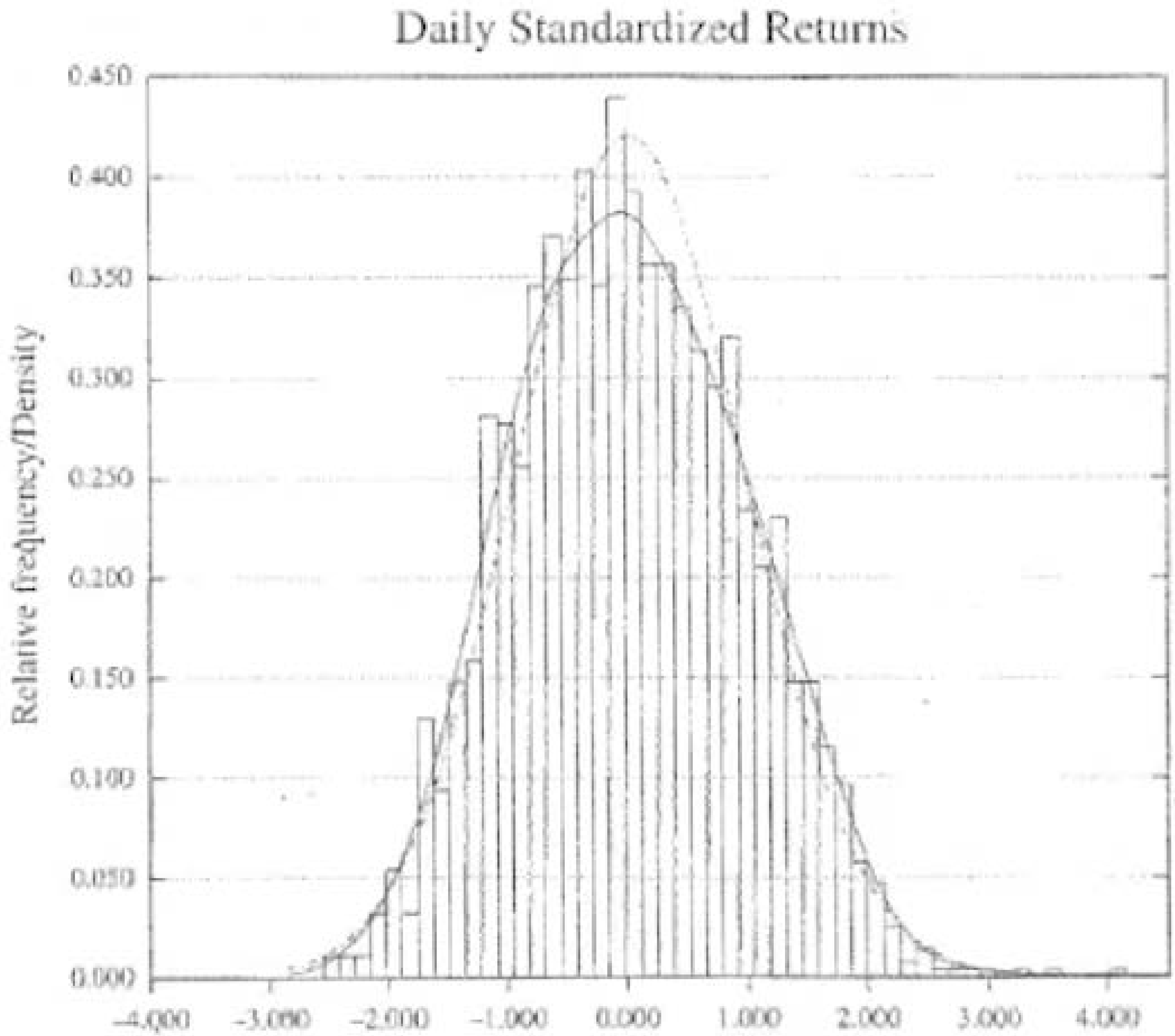


FIGURE 10
The distribution of the daily standardized returns of FTSE-100 index futures from March 1990 to July 1998.

FIGURE 12.13

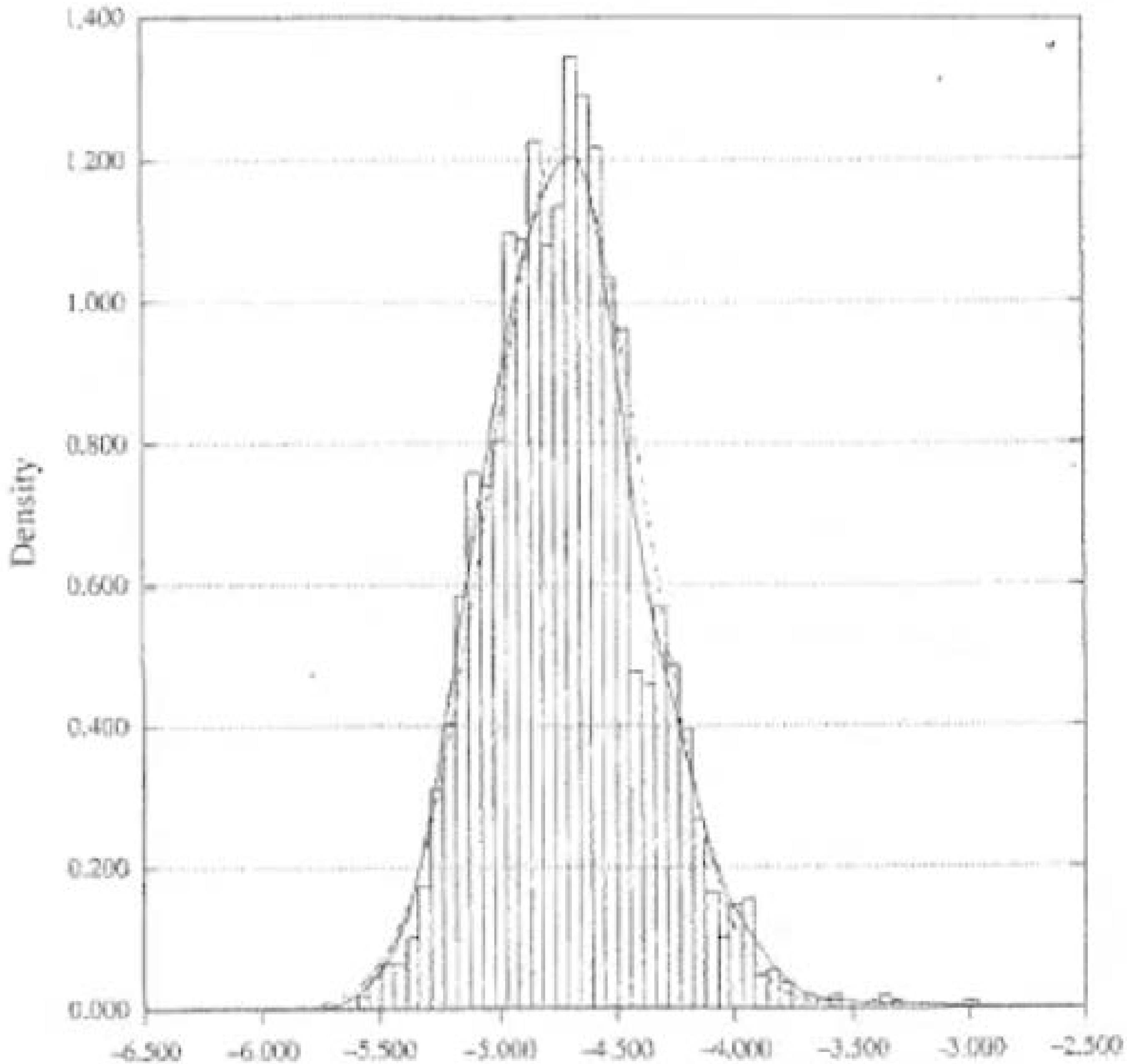


FIGURE 5
The distribution of the logarithm of realized volatility for the FTSE-100 index from 1990 to 1998.

FIGURE 12.13

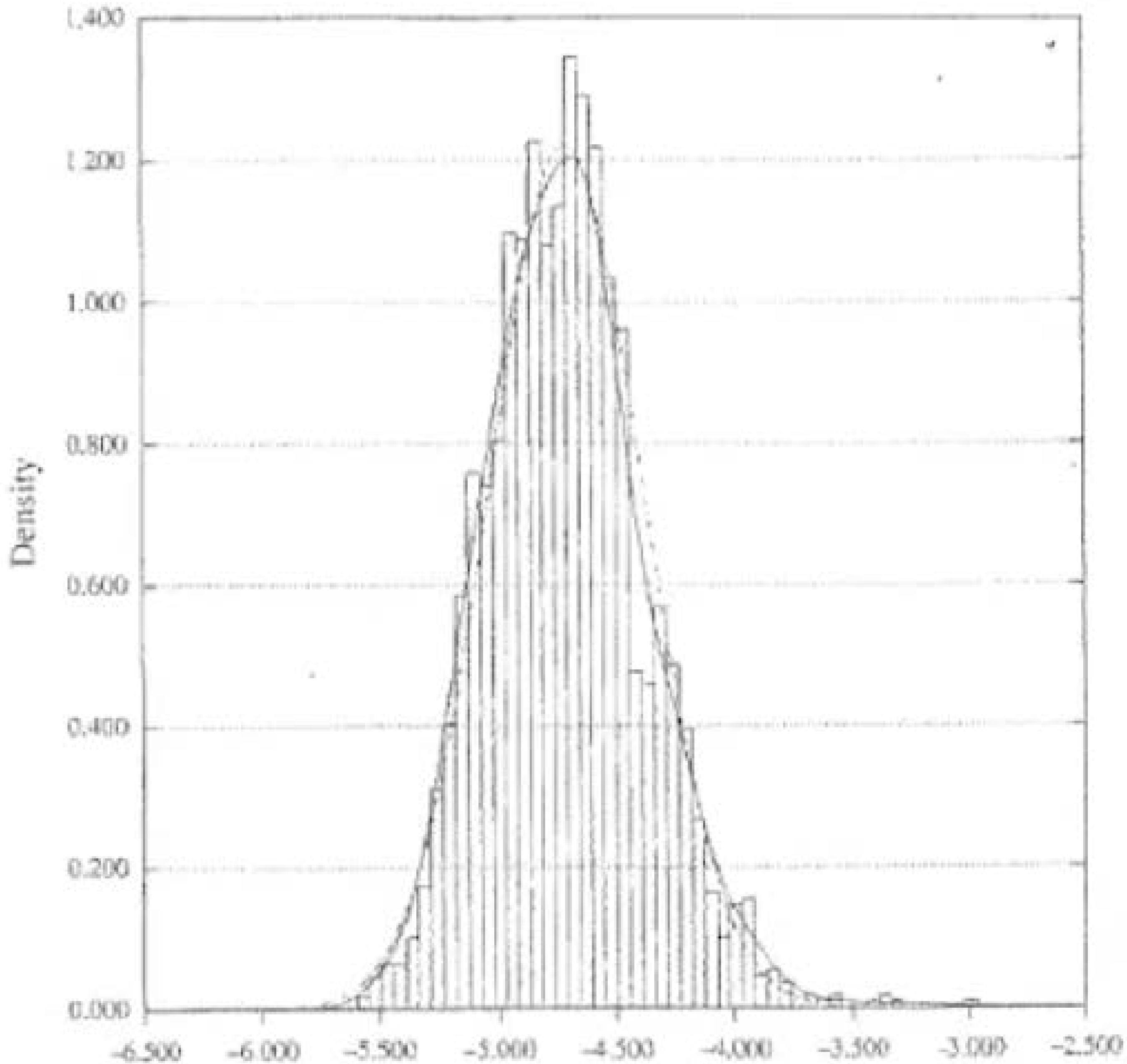


FIGURE 5
The distribution of the logarithm of realized volatility for the FTSE-100 index from 1990 to 1998.

FIGURE 12.14

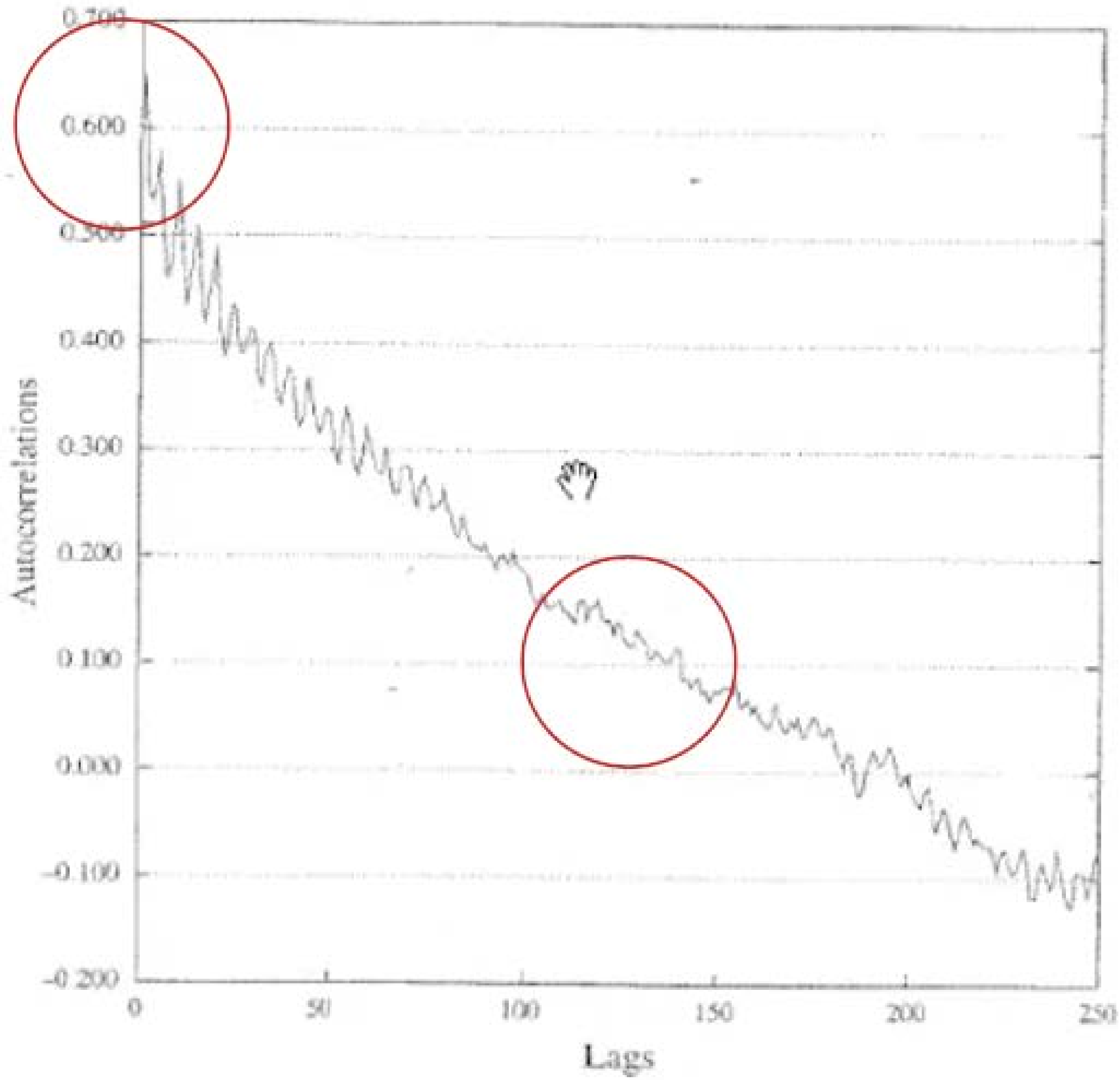


FIGURE 6

Autocorrelations of the logarithm of realized volatility for the FTSE-100 index from March 1990 to July 1998.

4. Further insights from high-frequency prices

Information about future volatility

- There is additional information in five-minute returns, compared with that in daily returns, which can be used to forecast future volatility more accurately. This is true for both equity and FX markets.
- There is sometimes incremental information in five-minute returns, when compared with that of implied volatilities.

News and volatility

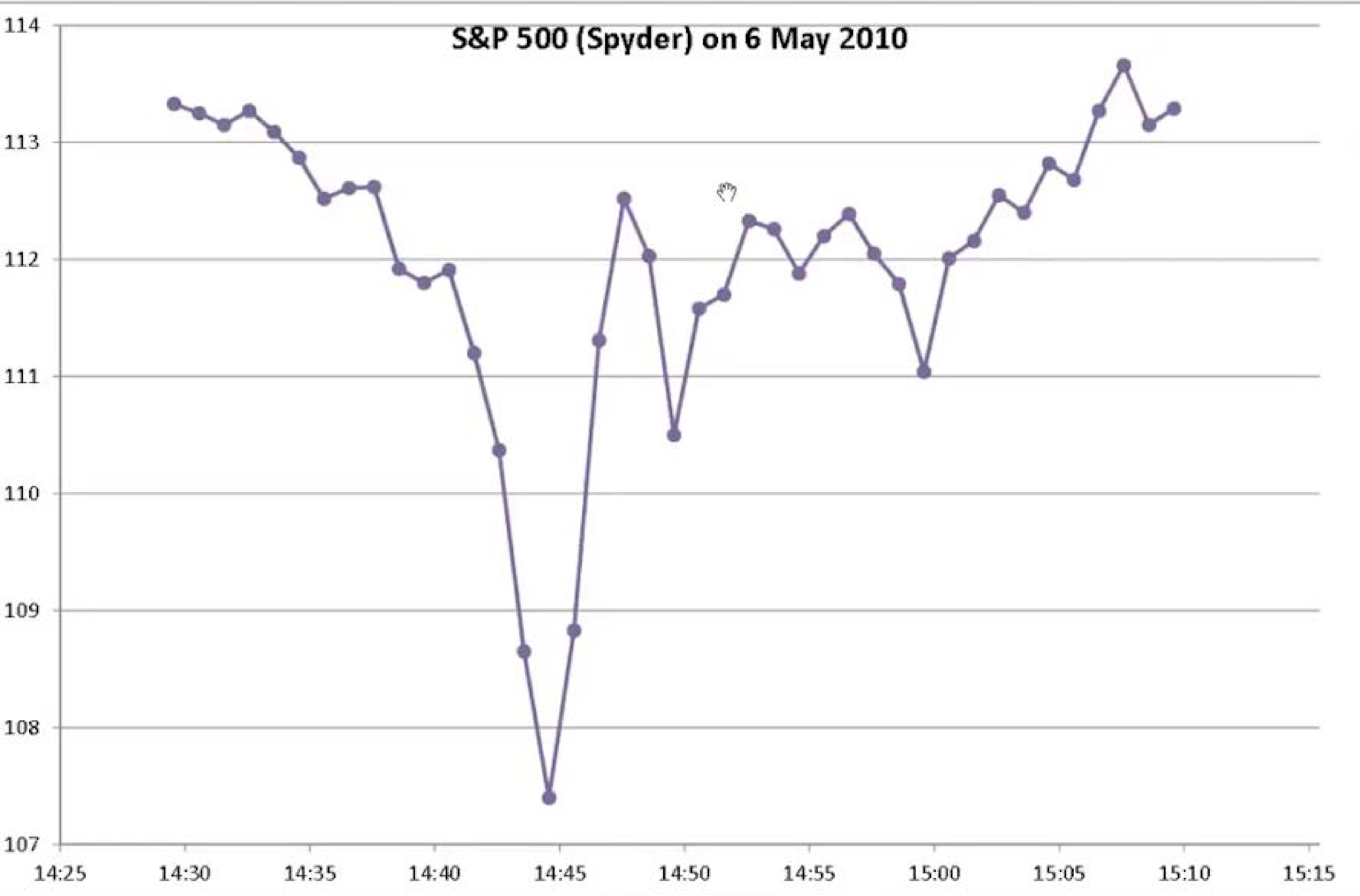
- Specific items of news, particularly macro., create short bursts of higher volatility as prices respond to the unexpected component in the news.
- News in general is only weakly associated with the level of volatility. For example, the quantity of news headlines displayed by Reuters seems to be only weakly correlated with volatility.

Exceptional price movements

The "flash crash" phenomenon on 6 May 2010 was exceptional and it is only clearly visible in high-frequency data. Explanations of this event are controversial!?

U.S. stock indices collapsed around 14:45 but rapidly recovered.

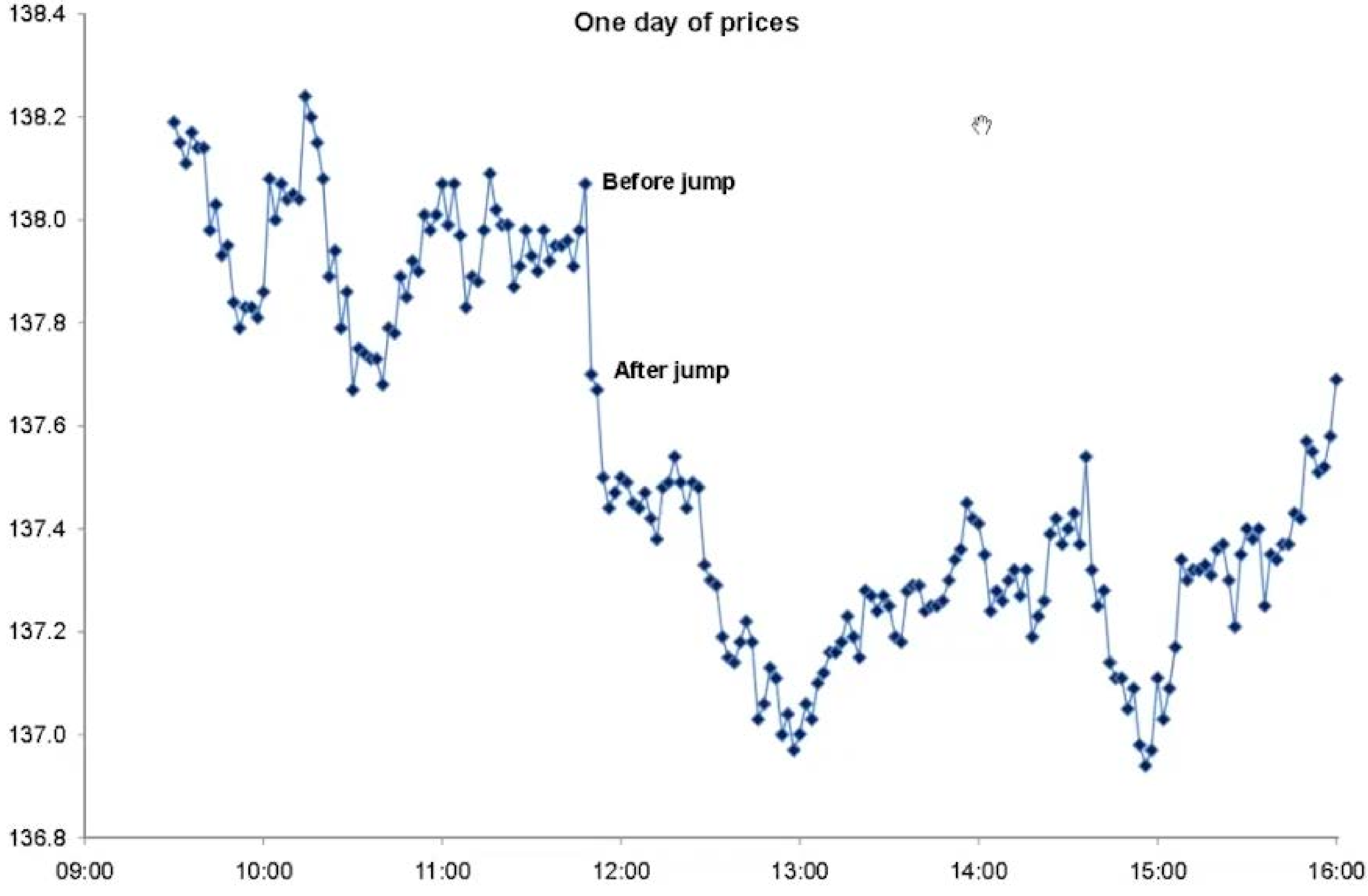
The realized variance on the S&P 500 ETF (SPY) during the half-hour around this event was equivalent to an annualized volatility (standard deviation) of 240%.

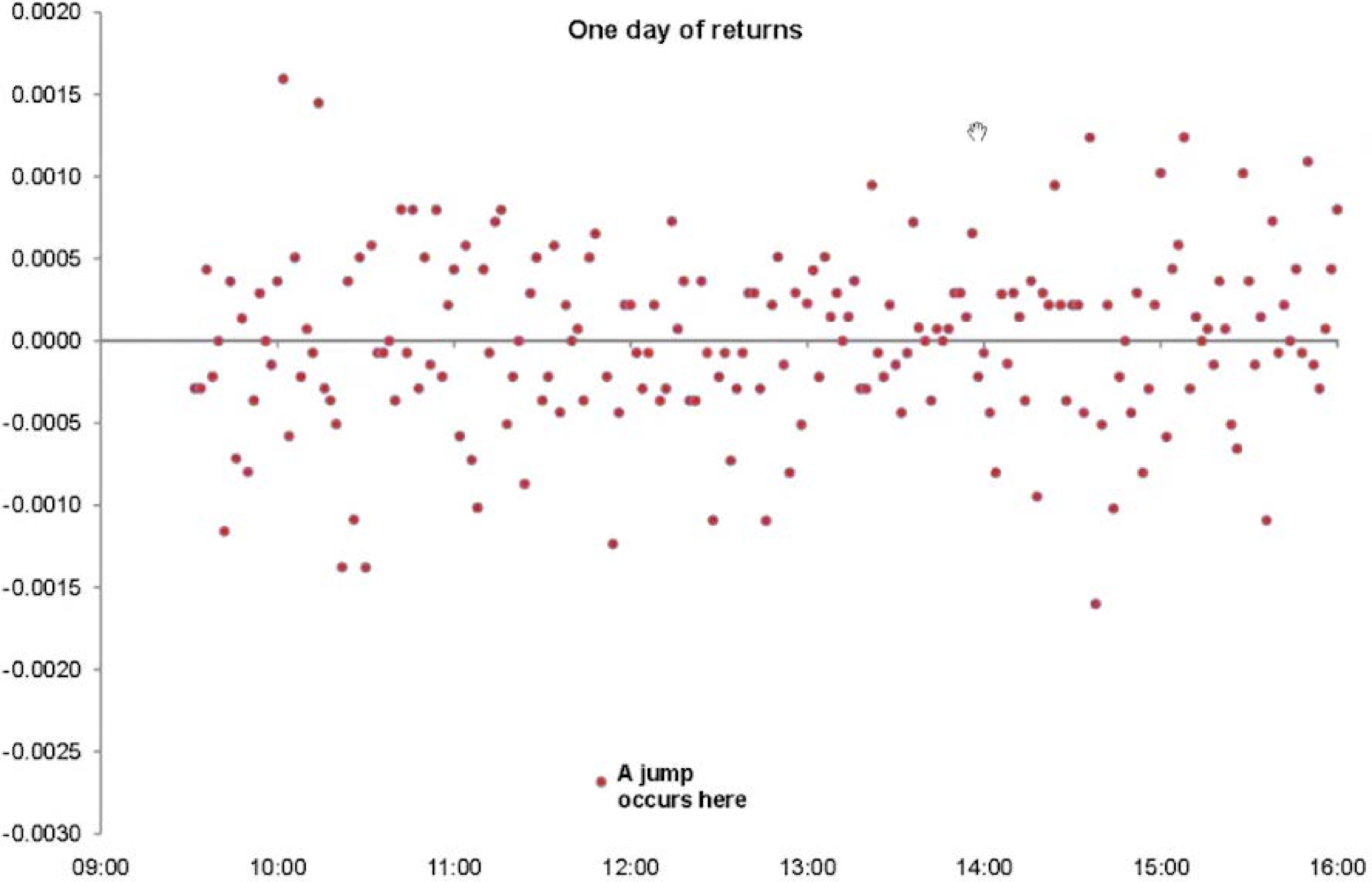

S&P 500 (Spyder) on 6 May 2010
114
113
112
111
110
109
108
107
14:25
14:30
14:35
14:40
14:45
14:50
14:55
15:00
15:05
15:10
15:15

Price jumps

High-frequency prices provide conclusive evidence that there are jumps in asset prices. It would be difficult to show this by using daily prices.

For example, consider SPY on 22 April 2008, with prices recorded every two minutes. A test motivated by Andersen, Bollerslev & Dobrev (J Econometrics, 2007) finds a jump between 11:48 and 11:50 EST. See the prices and returns on the two following pages.


One day of prices
Before jump
After jump
138.4
138.2
138.0
137.8
137.6
137.4
137.2
137.0
136.8
09:00
10:00
11:00
12:00
13:00
14:00
15:00
16:00



One day of returns
0.0020
0.0015
0.0010
0.0005
0.0000
-0.0005
-0.0010
-0.0015
-0.0020
-0.0025
-0.0030
09:00
10:00
11:00
12:00
13:00
14:00
15:00
16:00
A jump occurs here