CQF Exercises – Binomial Model

The following questions require you to set up a binomial tree and a Delta-hedged portfolio.

Note: When the question refers to continuous compounding you must use the quoted rate to calculate a discount factor that is applicable over the relevant period(s).

1. A share price is currently £80. At the end of three months, it will be either £84 or £76. Ignoring interest rates, calculate the value of a three-month European call option with exercise price £79.

2. A share price is currently £92. At the end of one year, it will be either £86 or £98. Calculate the value of a one-year European call option with exercise price £90 using a single-step binomial tree. The risk-free interest rate is 2% p.a. with continuous compounding.

3. A share price is currently £15. At the end of three months, it will be either £13 or £17. Interest rates are zero. Calculate the value of a three-month European option with payoff $\max(S^2 - 159, 0)$ where S is the share price at the end of three months.

4. Like question number 6 but with zero interest rate.

5. Same data as question 4 but now use the risk-neutral probabilities that you find to value an option.

6. A share price is currently £75. At the end of three months, it will be either £59 or £92. What are the risk-neutral probabilities that the share price rises or falls? The risk-free interest rate is 4% p.a. with continuous compounding.

7. A binomial model which assumes that an equity which initially has value S can during a time-step δt either rise to uS with probability p or fall to vS with probability $(1-p)$ where $0 < v < 1 < u$ and $uv = 1$. u and v are chosen so that

$$pu + (1-p)\,v = e^{\mu\ \delta t}$$

and

$$pu^{\,2} + (1-p)\,v^{\,2} = e^{\left(2\mu + \sigma^2\right)\ \delta t}$$

Show that these equations imply that

$$u + v = e^{-\mu\ \delta t} + e^{\left(\mu + \sigma^2\right)\ \delta t}$$

and hence that

$$u = \frac{1}{2}\left(e^{-\mu\ \delta t} + e^{\left(\mu + \sigma^2\right)\ \delta t}\right) + \frac{1}{2}\sqrt{\left(e^{-\mu\ \delta t} + e^{\left(\mu + \sigma^2\right)\ \delta t}\right)^2 - 4}$$

8. A stock is valued at 100. It can either rise to 103 or fall to 98. Interest rates are zero. Find the delta, hedged portfolio value, option value and risk neutral probabilities, if the strike is also 100. Fill in the blanks in the following diagram.
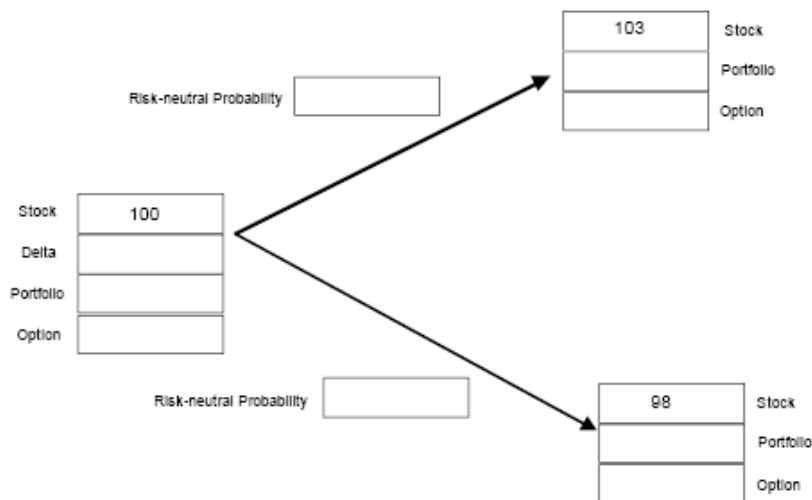
If interest rates are non zero, and there is a discount factor of 0.99, how does this affect the results? Fill in the blanks in the following diagram.

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                                                        ┌──────────────┐
                                                        │     103      │  Stock
                                                        ├──────────────┤
                                                        │              │  Portfolio
                                                        ├──────────────┤
                                    ┌──────────┐        │              │  Option
     Risk-neutral Probability      │          │        └──────────────┘
                                    └──────────┘      ↗
         ┌──────────────┐
  Stock  │     100      │
         ├──────────────┤
  Delta  │              │
         ├──────────────┤
Portfolio│              │
         ├──────────────┤
  Option │              │
         └──────────────┘
                                                      ↘
                                    ┌──────────┐        ┌──────────────┐
     Risk-neutral Probability       │          │        │      98      │  Stock
                                    └──────────┘        ├──────────────┤
                                                        │              │  Portfolio
                                                        ├──────────────┤
                                                        │              │  Option
                                                        └──────────────┘
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