Fair Spread for Basket CDS

1. Get historical CDS spread data and associated yield curve  
   - 5 single-name par spread CDS with maturities up to 5Y.  
   - Matching term structure of discounting curve.
2. Estimate the inferred linear correlation of default probabilities  
   <<M5L8, 2nd part 7min>>  
   Correlation matrix is estimated from historical PD data, as implied by credit spreads  
   - 5Y maturity is a good reference point  
   - daily or weekly changes; one-two year period  
     
   **PD or Hazard Rates (empirical)**  
   Historical CDS spread 🡺 Historical Probability of Default 🡪 Correlate change (i.e. difference) in Probability of Default  
   or correlate Hazard rates (because they are already log difference of survival probabilities)  
   but still difference in hazard rates should be better  
     
   << experimentation to be envisaged. Diff of PD, Hazard Rate, Diff of Hazard rates, log changes>>  
   << compare with correlation in CDS spreads or CDS spreads changes>>  
     
     
     
     
   elliptical copulae require a linear correlation. Linear correlation is always estimated from Normal variables.  
   convert your date into U using normal CDF if X is normal (e.g. return) or using F\_bar if X is not normal (difference in PD)  
   a) transform data X into uniform U = F\_bar(X), U is properly scaled belong to [0,1] and F\_bar is empirical CDF  
   b) apply normal CDF Z = phi-1(U)  
   c) calculate a linear correlation matrix Sigma = Corr(Z)  
   compare with correlation of returns  
   plot histogram of the U  
     
   research on f\_bar (empirical density), kernel density  
     
   Correlation for Gaussian copula (to be detailed)  
   Correlation for student’s t copula + determine degree of freedom (Laximum Log Likelyhood)  
   Kernel smooting + formula of student’s t copula density. If MLE too complex, assume degree of freedom between 7 and 13 (shortcut, see Q&A document)  
   inverse cumulative from cumulative density function (cf. Peter Jaeckel lecture)  
     
   Getting daily historical quotes for CDS and discounting a year or two back can be a challenge. Common substitutes are  
   - historical returns data, i.e., bonds or equity  
   - base correlation available from traded correlation instruments, often the same among all reference names.
3. Get CDS spread data for one particular date  
   Bootstrap curve (survival probability, hazard rate) for each reference name  
   How to convert random uniform variable into Tau (default time) – iterative procedure  
   First between which 2 tenors, and then approx. delta\_t. Mode details on Q&A document
4. Discounting can be approximated   
   See if discounting curve can be retrieved
5. Bootstrap implied default probabilities from quotes CDS and convert then to hazard rates (empirical marginal distribution)  
   Use constant recover rate = 40%
6. Calculation of the Spread from Copula sampling  
   Compare CDS basket spreads in Gausian Copula and Student t copula  
     
   around 100,000 simulation min required  
   not accepting default time < 0.25 (throwing the simulations, increase stability)  
   always justify number of simulations. Provide convergence diagram  
   logarithmic base 2 convergence sequence. Value at 32000, 16000, 8000… don’t report below 128  
   plot running average as a function of the number iterations N with plot points for N being powers of 2  
     
   Not advisable to use cholesky decomposition with sobol number  
   Ideally we want the the first column of A (C = A\*t(A)) to be the representing the first Principal Component (i.e. eigen vector corresponding to the largest eigen value)  
   Perform PCA (spectral decomposition)  
     
   path construction using Brownian bridge (over time, i.e. large correlation matrix)  
   between asset spectral decomposition  
   incremental (cholesky is too slow)