Explanations

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1 AllDifferent$(\{X_1, \ldots, X_n\})$

$$
\begin{aligned}
X_i = t \iff b_{it} & \qquad \forall i \in [\![1, n]\!] \forall t \in [\![1, m]\!] \\
\sum_{i \in [\![1,n]\!]} b_{it} \leq 1 & \qquad \forall t \in [\![1, m]\!]
\end{aligned}
$$

$$
\frac{X_{i'} = t,\ \forall i',\ i' \neq i,\ i' \in [\![1, n]\!],\ i \in [\![1, n]\!]}{X_i \neq t}
$$

2 AllEqual$(\{X_1, \ldots, X_n\})$

$$
\begin{aligned}
X_i \geq t \iff b_{it} & \qquad \forall i \in [\![1, n]\!] \forall t \in [\![1, m]\!] \\
\bigwedge_{i \in [\![1,n]\!]} b_{it} \iff b1_t & \qquad \forall t \in [\![1, m]\!] \\
\bigwedge_{i \in [\![1,n]\!]} \neg b_{it} \iff b2_t & \qquad \forall t \in [\![1, m]\!] \\
(b1_t \vee b2_t) & \qquad \forall t \in [\![1, m]\!]
\end{aligned}
$$

$$
\frac{X_i \geq t,\ \exists i,\ i \in [\![1, n]\!]}{X_i \geq t}
$$

$$
\frac{X_{i'} < t,\ \forall i',\ i' \neq i,\ i' \in [\![1, n]\!],\ i \in [\![1, n]\!]}{X_i \geq t}
$$

$$
\frac{X_{i'} \geq t,\ \forall i',\ i' \neq i,\ i' \in [\![1, n]\!],\ i \in [\![1, n]\!]}{X_i < t}
$$

$$
\frac{X_i < t,\ \exists i,\ i \in [\![1, n]\!]}{X_i < t}
$$

3 NValue$(N, \{X_1, \ldots, X_n\})$

$$X_i = t \iff b_{it} \qquad \forall i \in [\![1,n]\!] \forall t \in [\![1,m]\!]$$

$$\bigvee_{i \in [\![1,n]\!]} b_{it} \iff b2_t \qquad \forall t \in [\![1,m]\!]$$

$$\sum_{t \in [\![1,m]\!]} b2_t = p \iff b3_p \qquad \forall t \in [\![1,m]\!] \forall p \in [\![1,n]\!]$$

$$N = p \iff b3_p \qquad \forall p \in [\![1,n]\!]$$

$$\frac{X_i \neq t,\ \forall i,\ i \in [\![1,n]\!],\ \forall t,\ t \in [\![1,m]\!] \quad N = p,\ \forall p,\ p \in [\![1,n]\!]}{X_i = t}$$

$$\frac{X_{i'} \neq t,\ \forall i',\ i' \neq i,\ i' \in [\![1,n]\!],\ i \in [\![1,n]\!]}{X_i = t}$$

$$\frac{X_i = t,\ \exists i,\ i \in [\![1,n]\!],\ \forall t,\ t \in [\![1,m]\!] \quad N = p,\ \forall p,\ p \in [\![1,n]\!]}{X_i \neq t}$$

$$\frac{X_i \neq t,\ \forall i,\ i \in [\![1,n]\!],\ \forall t,\ t \in [\![1,m]\!],\ \forall i,\ i \in [\![1,n]\!] \quad X_i = t,\ \exists i,\ i \in [\![1,n]\!],\ \forall t,\ t \in [\![1,m]\!],\ \forall i,\ i \in [\![1,n]\!]}{N = p}$$

$$\frac{X_i = t,\ \exists i,\ i \in [\![1,n]\!],\ \forall t,\ t \in [\![1,m]\!],\ \forall i,\ i \in [\![1,n]\!]}{N \neq p}$$

$$\frac{X_i \neq t,\ \forall i,\ i \in [\![1,n]\!],\ \forall t,\ t \in [\![1,m]\!],\ \forall i,\ i \in [\![1,n]\!]}{N \neq p}$$

4 AtLeastNValue$(N, \{X_1, \ldots, X_n\})$

$$X_i = t \iff b_{it} \qquad \forall i \in [\![1,n]\!] \forall t \in [\![1,m]\!]$$

$$\bigvee_{i \in [\![1,n]\!]} b_{it} \iff b2_t \qquad \forall t \in [\![1,m]\!]$$

$$\sum_{t \in [\![1,m]\!]} b2_t \geq p \iff b3_p \qquad \forall t \in [\![1,m]\!]$$

$$N \geq p \iff b3_p \qquad \forall t \in [\![1,m]\!]$$

$$\frac{X_i \neq t',\ \forall i,\ i \in [\![1,n]\!],\ \forall i,\ i \in [\![1,n]\!],\ \forall t',\ t' \neq t,\ t' \in [\![1,m]\!],\ t \in [\![1,m]\!] \quad N \geq p,\ \forall p,\ p \in [\![1,n]\!]}{X_i = t}$$

$$\frac{X_{i'} \neq t,\ \forall i',\ i' \neq i,\ i' \in [\![1,n]\!],\ i \in [\![1,n]\!]}{X_i = t}$$

$$\frac{X_i = t',\ \exists i,\ i \in [\![1,n]\!],\ \forall i,\ i \in [\![1,n]\!],\ \forall t',\ t' \neq t,\ t' \in [\![1,m]\!],\ t \in [\![1,m]\!] \quad N < p,\ \forall p,\ p \in [\![1,n]\!]}{X_i \neq t}$$

$$\frac{X_i = t,\ \exists i,\ i \in [\![1,n]\!],\ \forall i,\ i \in [\![1,n]\!],\ \forall t,\ t \in [\![1,m]\!],\ \forall i,\ i \in [\![1,n]\!]}{N \geq p}$$

$$\frac{X_i \neq t,\ \forall i,\ i \in [\![1,n]\!],\ \forall i,\ i \in [\![1,n]\!],\ \forall t,\ t \in [\![1,m]\!],\ \forall i,\ i \in [\![1,n]\!]}{N < p}$$

5 AtMostNValue$(N, \{X_1, \ldots, X_n\})$

$$
\begin{aligned}
X_i = t &\iff b_{it} && \forall i \in \llbracket 1,n \rrbracket \forall t \in \llbracket 1,m \rrbracket \\
\bigvee_{i \in \llbracket 1,n \rrbracket} b_{it} &\iff b2_t && \forall t \in \llbracket 1,m \rrbracket \\
\sum_{t \in \llbracket 1,m \rrbracket} b2_t < p &\iff \neg b3_p && \forall t \in \llbracket 1,m \rrbracket \\
N \geq p &\iff b3_p && \forall t \in \llbracket 1,m \rrbracket
\end{aligned}
$$

$$\frac{X_i \neq t',\ \forall i,\ i \in \llbracket 1,n \rrbracket,\ \forall i,\ i \in \llbracket 1,n \rrbracket,\ \forall t',\ t' \neq t,\ t' \in \llbracket 1,m \rrbracket,\ t \in \llbracket 1,m \rrbracket \quad N \geq p,\ \forall p,\ p \in \llbracket 1,n \rrbracket}{X_i = t}$$

$$\frac{X_{i'} \neq t,\ \forall i',\ i' \neq i,\ i' \in \llbracket 1,n \rrbracket,\ i \in \llbracket 1,n \rrbracket}{X_i = t}$$

$$\frac{X_i = t',\ \exists i,\ i \in \llbracket 1,n \rrbracket,\ \forall i,\ i \in \llbracket 1,n \rrbracket,\ \forall t',\ t' \neq t,\ t' \in \llbracket 1,m \rrbracket,\ t \in \llbracket 1,m \rrbracket \quad N < p,\ \forall p,\ p \in \llbracket 1,n \rrbracket}{X_i \neq t}$$

$$\frac{X_i = t,\ \exists i,\ i \in \llbracket 1,n \rrbracket,\ \forall i,\ i \in \llbracket 1,n \rrbracket,\ \forall t,\ t \in \llbracket 1,m \rrbracket,\ \forall i,\ i \in \llbracket 1,n \rrbracket}{N \geq p}$$

$$\frac{X_i \neq t,\ \forall i,\ i \in \llbracket 1,n \rrbracket,\ \forall i,\ i \in \llbracket 1,n \rrbracket,\ \forall t,\ t \in \llbracket 1,m \rrbracket,\ \forall i,\ i \in \llbracket 1,n \rrbracket}{N < p}$$

6 Cumulative$(\{X_1, \ldots, X_n\}, \{d_1, \ldots, d_n\}, c)$

$$
\begin{aligned}
X_i \geq t &\iff b_{it} && \forall i \in \llbracket 1,n \rrbracket \forall t \in \llbracket 1,m \rrbracket \\
(b_{i(t-d_i)} \wedge \neg b_{it}) &\iff b2_{it} && \forall i \in \llbracket 1,n \rrbracket \forall t \in \llbracket 1,m \rrbracket \\
\sum_{i \in \llbracket 1,n \rrbracket} b2_{it} \leq c & && \forall t \in \llbracket 1,m \rrbracket
\end{aligned}
$$

$$\frac{X_i \geq t',\ t' = t - d_i \quad X_{i'} < t,\ \forall i',\ i' \neq i,\ i' \in \llbracket 1,n \rrbracket,\ i \in \llbracket 1,n \rrbracket \quad X_{i'} \geq t',\ \forall i',\ i' \neq i,\ i' \in \llbracket 1,n \rrbracket,\ i \in \llbracket 1,n \rrbracket,\ t' = t - d_{i'}}{X_i \geq t}$$

$$\frac{X_i < t',\ t' = t + d_i \quad X_{i'} < t',\ \forall i',\ i' \neq i,\ i' \in \llbracket 1,n \rrbracket,\ i \in \llbracket 1,n \rrbracket,\ t' = t + d_i \quad X_{i'} \geq t'',\ \forall i',\ i' \neq i,\ i' \in \llbracket 1,n \rrbracket,\ i \in \llbracket 1,n \rrbracket,\ t'' = t' - d_{i'},\ t' = t + d}{X_i < t}$$

7 Element$(I, \{X_1, \ldots., X_n\}, V)$

$$
\begin{aligned}
X_i = t &\iff b^X_{it} && \forall i \in \llbracket 1,n \rrbracket \forall t \in \llbracket 1,m \rrbracket \\
I = i &\iff b^I_i && \forall i \in \llbracket 1,n \rrbracket \\
V = t &\iff b^V_t && \forall t \in \llbracket 1,m \rrbracket \\
\neg b^V_t \wedge \neg b^I_i \wedge b^X_{it} & && \forall i \in \llbracket 1,n \rrbracket \forall t \in \llbracket 1,m \rrbracket \\
b^V_t \wedge \neg b^I_i \wedge \neg b^X_{it} & && \forall i \in \llbracket 1,n \rrbracket \forall t \in \llbracket 1,m \rrbracket
\end{aligned}
$$

$$\frac{I = i \quad V = t}{X_i = t}$$

$$\frac{I = i \quad V \neq t}{X_i \neq t}$$

$$\frac{X_i \neq t,\ \forall t,\ t \in [\![1, m]\!] \quad V = t,\ \forall t,\ t \in [\![1, m]\!]}{I \neq i}$$

$$\frac{X_i = t,\ \forall t,\ t \in [\![1, m]\!] \quad V \neq t,\ \forall t,\ t \in [\![1, m]\!]}{I \neq i}$$

$$\frac{X_i = t,\ \forall i,\ i \in [\![1, n]\!] \quad I = i,\ \forall i,\ i \in [\![1, n]\!]}{V = t}$$

$$\frac{X_i \neq t,\ \forall i,\ i \in [\![1, n]\!] \quad I = i,\ \forall i,\ i \in [\![1, n]\!]}{V \neq t}$$

8 $\textbf{Gcc}(\{X_1, \ldots, X_n\}, V, O)$

$$\begin{aligned}
X_i = t &\iff b_{it} && \forall i \in [\![1, n]\!] \forall t \in [\![1, m]\!] \\
O_t = p &\iff b2_{tp} && \forall t \in [\![1, m]\!] \forall p \in [\![1, n]\!] \\
\sum_{i \in [\![1, n]\!]} b_{it} \geq p &\iff b2_{tp} && \forall t \in [\![1, m]\!] \forall p \in [\![1, n]\!]
\end{aligned}$$

$$\frac{X_{i'} \neq t,\ \forall i',\ i' \neq i,\ i' \in [\![1, n]\!],\ i \in [\![1, n]\!] \quad O_t \geq p,\ \forall p,\ p \in [\![1, n]\!]}{X_i = t}$$

$$\frac{X_{i'} = t,\ \forall i',\ i' \neq i,\ i' \in [\![1, n]\!],\ i \in [\![1, n]\!] \quad O_t < p,\ \forall p,\ p \in [\![1, n]\!]}{X_i \neq t}$$

$$\frac{X_i = t,\ \forall i,\ i \in [\![1, n]\!],\ \forall i,\ i \in [\![1, n]\!]}{O_t \geq p}$$

$$\frac{X_i \neq t,\ \forall i,\ i \in [\![1, n]\!],\ \forall i,\ i \in [\![1, n]\!]}{O_t < p}$$

9 $\textbf{Increasing}(\{X_1, \ldots, X_n\})$

$$\begin{aligned}
X_i \geq t &\iff b_{it} && \forall i \in [\![1, n]\!] \forall t \in [\![1, m]\!] \\
(\neg b_{(i-1)t} &\lor b_{it}) && \forall i \in [\![2, n]\!] \forall t \in [\![1, m]\!]
\end{aligned}$$

$$\frac{X_{i'} \geq t,\ i' = i - 1}{X_i \geq t}$$

$$\frac{X_{i'} < t,\ i' = i + 1}{X_i < t}$$

10 Decreasing($\{X_1, \dots, X_n\}$)

$$X_i \geq t \iff b_{it} \qquad \forall i \in [\![1,n]\!] \forall t \in [\![1,m]\!]$$
$$(b_{(i-1)t} \vee \neg b_{it}) \qquad \forall i \in [\![2,n]\!] \forall t \in [\![1,m]\!]$$

$$\frac{X_{i'} \geq t,\ i' = i+1}{X_i \geq t}$$
$$\frac{X_{i'} < t,\ i' = i-1}{X_i < t}$$

11 Among($c, \{X_1, \dots., X_n\}, D_4$)

$$X_i = t \iff b_{it} \qquad \forall i \in [\![1,n]\!] \forall t \in [\![1,m]\!]$$
$$\bigvee_{t \in D_4} b_{it} \iff b2_i \qquad \forall i \in [\![1,n]\!]$$
$$\sum_{i \in [\![1,n]\!]} b2_i = c$$

$$\frac{X_i \neq t,\ \forall t,\ t \in D_4,\ \forall i,\ i \in [\![1,n]\!]}{X_i = t}$$
$$\frac{X_i \neq t',\ \forall t',\ t' \neq t,\ t' \in D_4,\ t \in D_4}{X_i = t}$$
$$\frac{X_i = t,\ \exists t,\ t \in D_4,\ \forall i,\ i \in [\![1,n]\!]}{X_i \neq t}$$

12 Roots($\{X_1, \dots, X_n\}, I, V$)

$$X_i = t \iff b_{it} \qquad \forall it$$
$$\sum_{t \in V} b_{it} = 1 \qquad \forall i \in I$$
$$\sum_{t \in D \setminus V} b_{it} = 1 \qquad \forall i \in [\![1,n]\!] \setminus I$$

$V = D_5$ et $D \setminus V = D_6$

$$\frac{X_i \neq t,\ \forall t,\ t \in D_5}{X_i = t}$$
$$\frac{X_i \neq t,\ \forall t,\ t \in D_6}{X_i = t}$$

$$\frac{X_i = t,\ \forall t,\ t \in D_5}{X_i \neq t}$$

$$\frac{X_i = t,\ \forall t,\ t \in D_6}{X_i \neq t}$$

13 Range$(\{X_1, \dots, X_n\}, I, V)$

$$\begin{aligned}
&X_i = t \iff b_{it} && \forall it \\
&\sum_{i \in I} b_{it} \geq 1 && \forall t \in V \\
&\sum_{t \in V} b_{it} = 1 && \forall i \in I
\end{aligned}$$

$I = D_5$ et $V = D_6$

$$\frac{X_i \neq t',\ \forall t',\ t' \neq t,\ t' \in D_5,\ t \in D_5}{X_i = t}$$

$$\frac{X_i \neq t,\ \forall t,\ t \in D_6}{X_i = t}$$

$$\frac{X_i = t,\ \forall t,\ t \in D_6}{X_i \neq t}$$

14 Xor$(b1, b2, b3)$

$$b1 = (b2 \neq b3)$$

b1,b2,b3 intervertibles

$$\frac{b2\ \neg b3}{b1}$$

$$\frac{b3\ \neg b2}{b1}$$

$$\frac{b2\ b3}{\neg b1}$$

$$\frac{\neg b2\ \neg b3}{\neg b1}$$