

Explanations

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1 AllDifferent($\{X_1, \dots, X_n\}$)

$$\begin{array}{ll} X_i = t \iff b_{it} & \forall i \in \llbracket 1, n \rrbracket \forall t \in \llbracket 1, m \rrbracket \\ \sum_{i \in \llbracket 1, n \rrbracket} b_{it} \leq 1 & \forall t \in \llbracket 1, m \rrbracket \end{array}$$

$$\frac{X_{i'} = t, \forall i', i' \neq i, i' \in \llbracket 1, n \rrbracket, i \in \llbracket 1, n \rrbracket}{X_i \neq t}$$

2 AllEqual($\{X_1, \dots, X_n\}$)

$$\begin{array}{ll} X_i \leq t \iff b_{it} & \forall i \in \llbracket 1, n \rrbracket \forall t \in \llbracket 1, m \rrbracket \\ \bigwedge_{i \in \llbracket 1, n \rrbracket} b_{it} \iff b1_t & \forall t \in \llbracket 1, m \rrbracket \\ \bigwedge_{i \in \llbracket 1, n \rrbracket} \neg b_{it} \iff b2_t & \forall t \in \llbracket 1, m \rrbracket \\ (b1_t \vee b2_t) & \forall t \in \llbracket 1, m \rrbracket \end{array}$$

$$\begin{array}{l} \frac{X_i \geq t, \exists i, i \in \llbracket 1, n \rrbracket}{X_i \geq t} \\ \frac{X_{i'} < t, \forall i', i' \neq i, i' \in \llbracket 1, n \rrbracket, i \in \llbracket 1, n \rrbracket}{X_i \geq t} \\ \frac{X_{i'} \geq t, \forall i', i' \neq i, i' \in \llbracket 1, n \rrbracket, i \in \llbracket 1, n \rrbracket}{X_i < t} \\ \frac{X_i < t, \exists i, i \in \llbracket 1, n \rrbracket}{X_i < t} \end{array}$$

3 NValue($N, \{X_1, \dots, X_n\}$)

$$\begin{array}{c}
X_i = t \iff b_{it} \qquad \forall i \in \llbracket 1, n \rrbracket \forall t \in \llbracket 1, m \rrbracket \\
\bigvee_{i \in \llbracket 1, n \rrbracket} b_{it} \iff b2_t \qquad \forall t \in \llbracket 1, m \rrbracket \\
\sum_{t \in \llbracket 1, m \rrbracket} b2_t = p \iff b3_p \qquad \forall t \in \llbracket 1, m \rrbracket \forall p \in \llbracket 1, n \rrbracket \\
N = p \iff b3_p \qquad \forall p \in \llbracket 1, n \rrbracket \\
\frac{X_i \neq t, \forall i, i \in \llbracket 1, n \rrbracket, \forall t, t \in \llbracket 1, m \rrbracket \quad N = p, \forall p, p \in \llbracket 1, n \rrbracket}{X_i = t} \\
\frac{X_{i'} \neq t, \forall i', i' \neq i, i' \in \llbracket 1, n \rrbracket, i \in \llbracket 1, n \rrbracket}{X_i = t} \\
\frac{X_i = t, \exists i, i \in \llbracket 1, n \rrbracket, \forall t, t \in \llbracket 1, m \rrbracket \quad N = p, \forall p, p \in \llbracket 1, n \rrbracket}{X_i \neq t} \\
\frac{X_i \neq t, \forall i, i \in \llbracket 1, n \rrbracket, \forall t, t \in \llbracket 1, m \rrbracket, \forall i, i \in \llbracket 1, n \rrbracket \quad X_i = t, \exists i, i \in \llbracket 1, n \rrbracket, \forall t, t \in \llbracket 1, m \rrbracket, \forall i, i \in \llbracket 1, n \rrbracket}{N = p} \\
\frac{X_i = t, \exists i, i \in \llbracket 1, n \rrbracket, \forall t, t \in \llbracket 1, m \rrbracket, \forall i, i \in \llbracket 1, n \rrbracket}{N \neq p} \\
\frac{X_i \neq t, \forall i, i \in \llbracket 1, n \rrbracket, \forall t, t \in \llbracket 1, m \rrbracket, \forall i, i \in \llbracket 1, n \rrbracket}{N \neq p}
\end{array}$$

4 AtLeastNValue($N, \{X_1, \dots, X_n\}$)

$$\begin{array}{c}
X_i = t \iff b_{it} \qquad \forall i \in \llbracket 1, n \rrbracket \forall t \in \llbracket 1, m \rrbracket \\
\bigvee_{i \in \llbracket 1, n \rrbracket} b_{it} \iff b2_t \qquad \forall t \in \llbracket 1, m \rrbracket \\
\sum_{t \in \llbracket 1, m \rrbracket} b2_t \geq p \iff b3_p \qquad \forall t \in \llbracket 1, m \rrbracket \\
N \geq p \iff b3_p \qquad \forall t \in \llbracket 1, m \rrbracket \\
\frac{X_i \neq t', \forall i, i \in \llbracket 1, n \rrbracket, \forall i, i \in \llbracket 1, n \rrbracket, \forall t', t' \neq t, t' \in \llbracket 1, m \rrbracket, t \in \llbracket 1, m \rrbracket \quad N \geq p, \forall p, p \in \llbracket 1, n \rrbracket}{X_i = t} \\
\frac{X_{i'} \neq t, \forall i', i' \neq i, i' \in \llbracket 1, n \rrbracket, i \in \llbracket 1, n \rrbracket}{X_i = t} \\
\frac{X_i = t', \exists i, i \in \llbracket 1, n \rrbracket, \forall i, i \in \llbracket 1, n \rrbracket, \forall t', t' \neq t, t' \in \llbracket 1, m \rrbracket, t \in \llbracket 1, m \rrbracket \quad N < p, \forall p, p \in \llbracket 1, n \rrbracket}{X_i \neq t} \\
\frac{X_i = t, \exists i, i \in \llbracket 1, n \rrbracket, \forall i, i \in \llbracket 1, n \rrbracket, \forall t, t \in \llbracket 1, m \rrbracket, \forall i, i \in \llbracket 1, n \rrbracket}{N \geq p} \\
\frac{X_i \neq t, \forall i, i \in \llbracket 1, n \rrbracket, \forall i, i \in \llbracket 1, n \rrbracket, \forall t, t \in \llbracket 1, m \rrbracket, \forall i, i \in \llbracket 1, n \rrbracket}{N < p}
\end{array}$$

5 AtMostNValue($N, \{X_1, \dots, X_n\}$)

$$\begin{array}{l}
X_i = t \iff b_{it} \qquad \forall i \in \llbracket 1, n \rrbracket \forall t \in \llbracket 1, m \rrbracket \\
\bigvee_{i \in \llbracket 1, n \rrbracket} b_{it} \iff b2_t \qquad \forall t \in \llbracket 1, m \rrbracket \\
\sum_{t \in \llbracket 1, m \rrbracket} b2_t < p \iff \neg b3_p \qquad \forall t \in \llbracket 1, m \rrbracket \\
N \geq p \iff b3_p \qquad \forall t \in \llbracket 1, m \rrbracket \\
\hline
X_i \neq t', \forall i, i \in \llbracket 1, n \rrbracket, \forall i, i \in \llbracket 1, n \rrbracket, \forall t', t' \neq t, t' \in \llbracket 1, m \rrbracket, t \in \llbracket 1, m \rrbracket \quad N \geq p, \forall p, p \in \llbracket 1, n \rrbracket \\
\hline
X_i = t \\
\hline
X_{i'} \neq t, \forall i', i' \neq i, i' \in \llbracket 1, n \rrbracket, i \in \llbracket 1, n \rrbracket \\
\hline
X_i = t \\
\hline
X_i = t', \exists i, i \in \llbracket 1, n \rrbracket, \forall i, i \in \llbracket 1, n \rrbracket, \forall t', t' \neq t, t' \in \llbracket 1, m \rrbracket, t \in \llbracket 1, m \rrbracket \quad N < p, \forall p, p \in \llbracket 1, n \rrbracket \\
\hline
X_i \neq t \\
\hline
X_i = t, \exists i, i \in \llbracket 1, n \rrbracket, \forall i, i \in \llbracket 1, n \rrbracket, \forall t, t \in \llbracket 1, m \rrbracket, \forall i, i \in \llbracket 1, n \rrbracket \\
\hline
N \geq p \\
\hline
X_i \neq t, \forall i, i \in \llbracket 1, n \rrbracket, \forall i, i \in \llbracket 1, n \rrbracket, \forall t, t \in \llbracket 1, m \rrbracket, \forall i, i \in \llbracket 1, n \rrbracket \\
\hline
N < p
\end{array}$$

6 Cumulative($\{X_1, \dots, X_n\}, \{d_1, \dots, d_n\}, c$)

$$\begin{array}{l}
X_i \geq t \iff b_{it} \qquad \forall i \in \llbracket 1, n \rrbracket \forall t \in \llbracket 1, m \rrbracket \\
(b_{i(t-d_i)} \wedge \neg b_{it}) \iff b2_{it} \qquad \forall i \in \llbracket 1, n \rrbracket \forall t \in \llbracket 1, m \rrbracket \\
\sum_{i \in \llbracket 1, n \rrbracket} b2_{it} \leq c \qquad \forall t \in \llbracket 1, m \rrbracket \\
\hline
X_i \geq t', t' = t - d_i \quad X_{i'} < t, \forall i', i' \neq i, i' \in \llbracket 1, n \rrbracket, i \in \llbracket 1, n \rrbracket \quad X_{i'} \geq t', \forall i', i' \neq i, i' \in \llbracket 1, n \rrbracket, i \in \llbracket 1, n \rrbracket, t' = t - d_{i'} \\
\hline
X_i \geq t \\
\hline
X_i < t', t' = t + d_i \quad X_{i'} < t', \forall i', i' \neq i, i' \in \llbracket 1, n \rrbracket, i \in \llbracket 1, n \rrbracket, t' = t + d_i \quad X_{i'} \geq t'', \forall i', i' \neq i, i' \in \llbracket 1, n \rrbracket, i \in \llbracket 1, n \rrbracket, t'' = t' - d_{i'}, t' = t + d_i \\
\hline
X_i < t
\end{array}$$

7 Element($I, \{X_1, \dots, X_n\}, V$)

$$\begin{array}{l}
X_i = t \iff b_{it}^X \qquad \forall i \in \llbracket 1, n \rrbracket \forall t \in \llbracket 1, m \rrbracket \\
I = i \iff b_i^I \qquad \forall i \in \llbracket 1, n \rrbracket \\
V = t \iff b_t^V \qquad \forall t \in \llbracket 1, m \rrbracket \\
\neg b_t^V \wedge \neg b_i^I \wedge b_{it}^X \qquad \forall i \in \llbracket 1, n \rrbracket \forall t \in \llbracket 1, m \rrbracket \\
b_t^V \wedge \neg b_i^I \wedge \neg b_{it}^X \qquad \forall i \in \llbracket 1, n \rrbracket \forall t \in \llbracket 1, m \rrbracket
\end{array}$$

$$\begin{array}{c}
\frac{I = i \quad V = t}{X_i = t} \\
\frac{I = i \quad V \neq t}{X_i \neq t} \\
\frac{X_i \neq t, \forall t, t \in \llbracket 1, m \rrbracket \quad V = t, \forall t, t \in \llbracket 1, m \rrbracket}{I \neq i} \\
\frac{X_i = t, \forall t, t \in \llbracket 1, m \rrbracket \quad V \neq t, \forall t, t \in \llbracket 1, m \rrbracket}{I \neq i} \\
\frac{X_i = t, \forall i, i \in \llbracket 1, n \rrbracket \quad I = i, \forall i, i \in \llbracket 1, n \rrbracket}{V = t} \\
\frac{X_i \neq t, \forall i, i \in \llbracket 1, n \rrbracket \quad I = i, \forall i, i \in \llbracket 1, n \rrbracket}{V \neq t}
\end{array}$$

8 $\mathbf{Gcc}(\{X_1, \dots, X_n\}, V, O)$

$$\begin{array}{c}
X_i = t \iff b_{it} \quad \forall i \in \llbracket 1, n \rrbracket \forall t \in \llbracket 1, m \rrbracket \\
O_t = p \iff b_{2tp} \quad \forall t \in \llbracket 1, m \rrbracket \forall p \in \llbracket 1, n \rrbracket \\
\sum_{i \in \llbracket 1, n \rrbracket} b_{it} \geq p \iff b_{2tp} \quad \forall t \in \llbracket 1, m \rrbracket \forall p \in \llbracket 1, n \rrbracket \\
\frac{X_{i'} \neq t, \forall i', i' \neq i, i' \in \llbracket 1, n \rrbracket, i \in \llbracket 1, n \rrbracket \quad O_t \geq p, \forall p, p \in \llbracket 1, n \rrbracket}{X_i = t} \\
\frac{X_{i'} = t, \forall i', i' \neq i, i' \in \llbracket 1, n \rrbracket, i \in \llbracket 1, n \rrbracket \quad O_t < p, \forall p, p \in \llbracket 1, n \rrbracket}{X_i \neq t} \\
\frac{X_i = t, \forall i, i \in \llbracket 1, n \rrbracket, \forall i, i \in \llbracket 1, n \rrbracket}{O_t \geq p} \\
\frac{X_i \neq t, \forall i, i \in \llbracket 1, n \rrbracket, \forall i, i \in \llbracket 1, n \rrbracket}{O_t < p}
\end{array}$$

9 $\mathbf{Increasing}(\{X_1, \dots, X_n\})$

$$\begin{array}{c}
X_i \geq t \iff b_{it} \quad \forall i \in \llbracket 1, n \rrbracket \forall t \in \llbracket 1, m \rrbracket \\
(\neg b_{(i-1)t} \vee b_{it}) \quad \forall i \in \llbracket 2, n \rrbracket \forall t \in \llbracket 1, m \rrbracket \\
\frac{X_{i'} \geq t, i' = i - 1}{X_i \geq t} \\
\frac{X_{i'} < t, i' = i + 1}{X_i < t}
\end{array}$$

10 Decreasing($\{X_1, \dots, X_n\}$)

$$\begin{array}{ll} X_i \geq t \iff b_{it} & \forall i \in \llbracket 1, n \rrbracket \forall t \in \llbracket 1, m \rrbracket \\ (b_{(i-1)t} \vee \neg b_{it}) & \forall i \in \llbracket 2, n \rrbracket \forall t \in \llbracket 1, m \rrbracket \end{array}$$

$$\frac{X_{i'} \geq t, \ i' = i + 1}{X_i \geq t}$$

$$\frac{X_{i'} < t, \ i' = i - 1}{X_i < t}$$

11 Among($c, \{X_1, \dots, X_n\}, D_4$)

$$\begin{array}{ll} X_i = t \iff b_{it} & \forall i \in \llbracket 1, n \rrbracket \forall t \in \llbracket 1, m \rrbracket \\ \bigvee_{t \in D_4} b_{it} \iff b_{2i} & \forall i \in \llbracket 1, n \rrbracket \\ \sum_{i \in \llbracket 1, n \rrbracket} b_{2i} = c & \end{array}$$

$$\frac{X_i \neq t, \ \forall t, \ t \in D_4, \ \forall i, \ i \in \llbracket 1, n \rrbracket}{X_i = t}$$

$$\frac{X_i \neq t', \ \forall t', \ t' \neq t, \ t' \in D_4, \ t \in D_4}{X_i = t}$$

$$\frac{X_i = t, \ \exists t, \ t \in D_4, \ \forall i, \ i \in \llbracket 1, n \rrbracket}{X_i \neq t}$$

12 Roots($\{X_1, \dots, X_n\}, I, V$)

$$\begin{array}{ll} X_i = t \iff b_{it} & \forall i, t \\ \sum_{t \in V} b_{it} = 1 & \forall i \in I \\ \sum_{t \in D \setminus V} b_{it} = 1 & \forall i \in \llbracket 1, n \rrbracket \setminus I \end{array}$$

$$V = D_5 \text{ et } D \setminus V = D_6$$

$$\frac{X_i \neq t, \ \forall t, \ t \in D_5}{X_i = t}$$

$$\frac{X_i \neq t, \ \forall t, \ t \in D_6}{X_i = t}$$

$$\frac{X_i = t, \forall t, t \in D_5}{X_i \neq t}$$

$$\frac{X_i = t, \forall t, t \in D_6}{X_i \neq t}$$

13 Range($\{X_1, \dots, X_n\}, I, V$)

$$X_i = t \iff b_{it} \quad \forall it$$

$$\sum_{i \in I} b_{it} \geq 1 \quad \forall t \in V$$

$$\sum_{t \in V} b_{it} = 1 \quad \forall i \in I$$

$I = D_5$ et $V = D_6$

$$\frac{X_i \neq t', \forall t', t' \neq t, t' \in D_5, t \in D_5}{X_i = t}$$

$$\frac{X_i \neq t, \forall t, t \in D_6}{X_i = t}$$

$$\frac{X_i = t, \forall t, t \in D_6}{X_i \neq t}$$

14 Xor($b1, b2, b3$)

$$b1 = (b2 \neq b3)$$

$b1, b2, b3$ intervertibles

$$\frac{b2 \neg b3}{b1}$$

$$\frac{b3 \neg b2}{b1}$$

$$\frac{b2 \ b3}{\neg b1}$$

$$\frac{\neg b2 \neg b3}{\neg b1}$$

15 BinPacking($\{X_1, \dots, X_n\}, \{w_1, \dots, w_n\}, c$)

$$X_i = p \iff b_{ip} \quad \forall i \in \llbracket 1, n \rrbracket \forall p \in \llbracket 1, n \rrbracket$$

$$\sum_{i \in \llbracket 1, n \rrbracket} w_i \times b_{ip} \leq c \quad \forall p \in \llbracket 1, n \rrbracket$$

$$\frac{X_{i'} = p, \forall i', i' \neq i, i' \in \llbracket 1, n \rrbracket, i \in \llbracket 1, n \rrbracket}{X_i \neq p}$$

16 **Knapsack**($\{X_1, \dots, X_n\}, \{w_1, \dots, w_n\}, \{p_1, \dots, p_n\}, W, P$)

$$\begin{aligned}
 X_i = t &\iff b_{it} && \forall i \in \llbracket 1, n \rrbracket \forall t \in N \\
 \sum_{t \in N} b_{it} &\leq 1 && \forall i \in \llbracket 1, n \rrbracket \\
 \sum_{t \in N} \sum_{i \in \llbracket 1, n \rrbracket} b_{it} \times t \times w_i &= W \\
 \sum_{t \in N} \sum_{i \in \llbracket 1, n \rrbracket} b_{it} \times t \times p_i &= P
 \end{aligned}$$