卢卡斯定理

$$\text{设}p\text{为素数，}a, b \in N\text{，并且}$$
$$a = a_k p^k + a_{k-1} p^{k-1} + \ldots + a_1 p + a_0,$$
$$b = b_k p^k + b_{k-1} p^{k-1} + \ldots + b_1 p + b_0,$$
$$0 \le a_i, b_i \le p-1\text{都是整数，则}C_a^b \equiv C_{a_k}^{b_k} C_{a_{k-1}}^{b_{k-1}} \ldots C_{a_0}^{b_0}\ (mod\ p)$$

$$\text{引入多项式同余记号，}f(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_0, g(x) = b_n x^n + b_{n-1} x^{n-1} + \ldots + b_0$$
$$\text{如果对}0 \le i \le n,\text{都有}a_i \equiv b_i (mod\ m)\text{，那么称}f(x)\text{与}g(x)\text{对模}m\text{同余，记作}f(x) \equiv g(x)(mod\ m)$$
$$\text{由于}p\text{是质数，所以对}1 \le j \le p-1\text{，有}C_p^j = \frac{p}{j} C_{p-1}^{j-1} \equiv 0 (mod\ p)$$
$$\text{于是，}(1+x)^p = 1 + C_p^1 x + \ldots + C_p^{p-1} x^{p-1} + x^p \equiv 1 + x^p (mod\ p)$$
$$\begin{aligned}(1+x)^a &= (1+x)^{a_0} ((1+x)^p)^{a_1} \ldots ((1+x)^{p^k})^{a^k} \\ &\equiv (1+x)^{a_0} ((1+x)^p)^{a_1} \ldots ((1+x)^{p^k})^{a^k} (mod\ p)\end{aligned}$$
$$\text{对比上式中}x^b\text{的系数，可得}C_a^b \equiv C_{a_k}^{b_k} C_{a_{k-1}}^{b_{k-1}} \ldots C_{a_0}^{b_0}\ (mod\ p)$$

原题链接 Acwing 887 求组合数III

```
#include <iostream>

using namespace std;

typedef long long LL;

const int N = 10010;
int fact[N], infact[N];

int qmi(LL a, LL p, int mod)
{
    int res = 1;
    while (p)
    {
        if (p & 1) res = (LL) res * a % mod;
        a = (LL) a * a % mod;
        p >>= 1;
    }
    return res;
}

int C(LL a, LL b, int p)
{
    if (b > a) return 0;
    int res = 1;
    for (int i = 1, j = a; i <= b; i ++, j --)
    {
        res = (LL) res * j % p;
        res = (LL) res * qmi(i, p - 2, p) % p;
    }
    return res;
}
```

```
int lucas(LL a, LL b, int p)
{
    if (a < p && b < p) return C(a, b, p);
    return (LL) C(a % p, b % p, p) * lucas(a / p, b / p, p) % p;
}

int main()
{
    int n;
    cin >> n;
    for (int i = 0; i < n; i ++)
    {
        LL a, b;
        int p;
        cin >> a >> b >> p;
        cout << lucas(a, b, p) << endl;
    }
    return 0;
}
```