HDSDP for Optimal Diagonal Pre-conditioning

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In this report we present the experiments results on HDSDP for the optimal diagonal pre-conditioning problem.

1 Experiment Setup

In this section we introduce the detailed experiment setup for the optimal diagonal-precondition problem.

1.1 Formulation

Given a full-rank matrix $X \in \mathbb{R}^{m\times n}$, the optimal pre-conditioning problem solves the SDP instance

$$\begin{aligned} \max_{\tau, D} \quad & \tau \\ \text{subject to} \quad & D \preceq M \\ & \tau M - D \preceq 0, \end{aligned}$$

where $M = X^\top X \in \mathbb{S}^{n\times n}_+$ and $D^{1/2}$ is applied as the pre-conditioner.

1.2 Datasets and Processing

To verify the effect of the optimal diagonal preconditioner, we test the algorithm on a extensively large collection of matrices. Currently there are threes sources for our test.

- `Tim Davis SuiteSparse Dataset` (Ready)

  https://sparse.tamu.edu

  In this dataset we are for now testing matrices X with $n \le 1000$.

- `LIBSVM Regression` (Ready)

  https://www.csie.ntu.edu.tw/~cjlin/libsvmtools/datasets/regression.html

  We take the regression ($\|X\beta - y\|^2$) datasets from `LIBSVM`.

- `OPENML Machine Learning Regression` (Preparing)

  https://www.openml.org/search?type=data

  Still in preparation

and without loss of generality, we choose the matrices whose condition number $\le 10^8$ and if a matrix does not meet the condition, we add diagonal perturbation $M \leftarrow M + \varepsilon I$ till $\kappa(M) \le 10^8$.

1.3 Experiment Environment

All the experiments in the report are carried out on Mac Mini with Apple Silicon and 16 GB memory.

1.4 Solver and Configuration

We adopt the `HDSDP` solver to solve the optimal diagonal pre-conditioning problem. To enhance the performance, we let the solver start from $(\tau, d) = (-10^\alpha, 0)$ for some $\alpha \ge 1$.

1.5 Evaluation

For each matrix X, we report the following statistics

- $\kappa(M)$ (Marked by Cbef)

  Condition number of M

- $\kappa(D^{-1/2}MD^{-1/2})$ (Marked by Caft)

  Condition number after pre-conditioning

- $\mathrm{rdc}(M)$ (Marked by Reduce)

  The relative reduction in condition number by $\mathrm{rdc}(M) = 1 - \frac{\kappa(D^{-1/2}MD^{-1/2})}{\kappa(M)}$.

  e.g., if $\mathrm{rdc}(M) = 0.99$, then pre-conditioning reduces the condition number by 99%.

Remark 1. For some problems `HDSDP` fails and in this case $\mathrm{rdc}(M)$ may be less than 0. We drop these cases from the results presented below.

A Suite-sparse Benchmark

A.1 Summary statistics

We test 391 small-medium sized Suite sparse matrices and

Reduction	Number
$\ge$80%	121
$\ge$50%	190
$\ge$20%	261

Average reduction	49.7%
Average time	1.29

Table 1. Average condition number reduction

A.2 Smaller Matrices

Mat	Size	Cbef	Caft	Reduce	Time	Mat	Size	Cbef	Caft	Reduce	Time
abb313	176	3.719000e+07	3.146000e+07	0.154092	0.374230	plskz362	362	7.743000e+05	6.933000e+05	0.104524	1.290407
ash219	85	9.150000e+00	4.194000e+00	0.541579	0.171661	pores_1	30	2.774000e+07	4.481000e+06	0.838435	0.128902
ash292	292	4.188000e+07	6.040000e+06	0.855788	1.126949	str_0	363	7.511000e+04	1.470000e+04	0.804285	0.788147
ash331	104	9.588000e+00	3.668000e+00	0.617469	0.226226	str_200	363	1.371000e+07	4.065000e+05	0.970347	2.368018
ash608	188	1.138000e+01	3.861000e+00	0.660595	0.435684	str_400	363	1.912000e+07	5.322000e+06	0.721693	0.836974
ash85	85	2.151000e+05	1.283000e+05	0.403496	0.204586	str_600	363	1.184000e+07	4.537000e+05	0.961681	2.305290
ash958	292	1.025000e+01	4.757000e+00	0.535855	0.623655	west0067	67	1.696000e+04	5.903000e+03	0.651885	0.160354
bcspwr01	39	2.756000e+03	2.164000e+03	0.214776	0.743860	west0132	132	4.772000e+07	4.199000e+02	0.999991	0.278596
bcspwr02	49	1.856000e+07	6.563000e+06	0.646473	0.134231	west0167	167	4.772000e+07	8.476000e+02	0.999982	0.396684
bcspwr03	118	2.514000e+05	1.137000e+05	0.547642	0.274450	west0381	381	2.245000e+07	2.068000e+02	0.999991	3.711164
bcspwr04	274	2.558000e+07	6.589000e+06	0.742365	1.315488	west0479	479	4.737000e+07	1.252000e+03	0.999974	2.245246
bcspwr05	443	2.681000e+07	1.200000e+07	0.552557	0.819582	west0497	497	5.540000e+07	2.286000e+03	0.999959	2.031159
bcsstk02	66	1.871000e+07	2.973000e+06	0.841086	0.162481	will199	199	1.926000e+07	1.085000e+07	0.436279	0.502647
bcsstk05	153	2.982000e+07	1.672000e+06	0.943950	0.596362	will57	57	3.781000e+07	2.533000e+07	0.330104	0.141474
bcsstk06	420	2.109000e+07	1.593000e+04	0.999245	3.996074	wm1	277	2.135000e+07	7.922000e+03	0.999629	2.268211
bcsstk07	420	2.109000e+07	1.593000e+04	0.999245	4.016814	wm2	260	1.244000e+07	8.332000e+05	0.933019	1.470060
bcsstk22	138	6.194000e+07	5.012000e+05	0.991909	0.381385	wm3	260	9.514000e+06	9.065000e+05	0.904718	1.571223
bcsstm01	48	7.782000e+07	1.000000e+00	1.000000	0.105460	bfwa398	398	8.959000e+06	1.682000e+06	0.812196	3.615892
bcsstm02	66	7.660000e+01	1.000000e+00	0.986945	0.106136	bfwa62	62	3.059000e+05	5.152000e+04	0.831580	0.174470
bcsstm03	112	5.569000e+07	1.000000e+00	1.000000	0.176999	bfwb398	398	4.465000e+02	1.565000e+02	0.649453	3.353016
bcsstm04	132	2.987000e+04	1.000000e+00	0.999967	0.134561	bfwb62	62	2.958000e+02	2.676000e+01	0.909552	0.246814
bcsstm05	153	1.612000e+02	1.000000e+00	0.993798	0.158579	bwm200	200	5.820000e+06	3.290000e+06	0.434761	0.546737
bcsstm06	420	5.512000e+07	1.000000e+00	1.000000	0.721068	ck104	104	2.987000e+07	1.143000e+06	0.961728	0.203493
bcsstm07	420	4.456000e+07	4.136000e+04	0.999072	4.501864	ck400	400	3.284000e+07	1.001000e+06	0.969527	1.006684
bcsstm22	138	8.860000e+05	1.000000e+00	0.999999	0.148473	lop163	163	1.281000e+06	5.703000e+05	0.554680	0.470454
can_144	144	2.443000e+07	2.256000e+07	0.076367	0.309114	mhda416	416	2.466000e+07	3.819000e+05	0.984513	3.291940
can_161	161	4.940000e+04	3.960000e+04	0.198256	0.440638	mhdb416	416	4.824000e+06	5.071000e+01	0.999989	1.188644
can_187	187	7.056000e+07	3.191000e+07	0.547706	0.558273	odepa400	400	2.504000e+07	1.593000e+07	0.363637	0.933260
can_229	229	1.891000e+07	1.604000e+07	0.151527	0.606216	odepb400	400	1.000000e+00	1.000000e+00	0.000000	0.330884
can_24	24	6.046000e+03	4.025000e+03	0.334338	0.127885	olm100	100	4.480000e+07	3.092000e+07	0.309857	0.243572
can_256	256	7.565000e+06	3.686000e+06	0.512763	4.624222	olm500	500	3.185000e+07	3.146000e+07	0.012234	2.274321
can_268	268	2.091000e+07	8.414000e+06	0.597536	1.479174	pde225	225	1.526000e+03	9.913000e+02	0.350364	1.132055
can_292	292	3.699000e+07	2.342000e+07	0.366954	1.222109	rbsa480	480	5.333000e+06	7.129000e+05	0.866327	9.633158
can_445	445	4.006000e+07	1.346000e+07	0.663895	2.818335	rbsb480	480	1.038000e+07	2.140000e+06	0.793806	9.278184
can_61	61	2.996000e+07	1.355000e+07	0.547665	0.163705	rw136	136	1.302000e+06	5.420000e+05	0.583550	0.342707
can_62	62	3.646000e+05	2.100000e+05	0.423995	0.164177	rw496	496	1.315000e+06	8.306000e+05	0.368444	3.604414
can_73	73	1.270000e+03	1.053000e+03	0.170424	0.191222	tub100	100	4.427000e+07	4.254000e+07	0.039051	0.226566
can_96	96	2.104000e+04	1.733000e+04	0.176483	0.198430	cavity01	317	3.947000e+07	6.467000e+04	0.998361	2.054133
curtis54	54	4.522000e+07	1.366000e+07	0.697928	0.146304	cavity02	317	1.823000e+07	3.743000e+04	0.997946	1.405985
dwt_162	162	2.204000e+07	1.579000e+07	0.283632	0.334807	cavity03	317	1.709000e+07	5.320000e+04	0.996887	2.020333
dwt_193	193	1.310000e+07	1.037000e+07	0.208211	0.597751	cavity04	317	1.966000e+07	9.305000e+04	0.995268	1.563758
dwt_198	198	3.397000e+07	1.963000e+07	0.422230	0.408961	ex1	216	1.695000e+04	7.532000e+01	0.995556	0.926692
dwt_209	209	5.449000e+07	9.330000e+06	0.828784	0.795283	ex5	27	3.457000e+07	3.443000e+07	0.004130	0.102971
dwt_221	221	3.381000e+07	1.101000e+07	0.674357	0.675951	b1_ss	7	3.896000e+04	7.115000e+01	0.998174	0.107598
dwt_234	234	1.042000e+05	4.006000e+04	0.615561	0.675201	d_dyn	87	4.941000e+07	2.718000e+07	0.449806	0.154185
dwt_245	245	3.634000e+07	1.218000e+07	0.664825	0.766070	d_dyn1	87	3.799000e+07	2.324000e+07	0.388401	0.212365
dwt_307	307	2.499000e+07	2.276000e+07	0.089230	1.154597	d_ss	53	7.596000e+07	3.611000e+07	0.524657	0.174045
dwt_310	310	1.069000e+07	5.251000e+06	0.508690	1.361014	lp_adlittle	138	2.077000e+07	1.976000e+07	0.048207	0.248500
dwt_346	346	4.967000e+07	4.011000e+06	0.919252	1.532717	lp_afiro	51	2.299000e+07	1.711000e+07	0.255975	0.129797
dwt_361	361	3.879000e+07	2.166000e+07	0.441465	1.496653	lp_bandm	472	1.055000e+07	4.584000e+06	0.565650	3.579121
dwt_419	419	2.180000e+07	1.431000e+07	0.343696	1.682823	lp_beaconfd	295	8.045000e+06	5.414000e+06	0.327030	2.481520
dwt_492	492	3.047000e+07	1.301000e+07	0.572926	2.487955	lp_blend	114	1.085000e+07	2.700000e+06	0.751234	0.229854
dwt_59	59	1.395000e+04	8.580000e+03	0.384885	0.157282	lp_bore3d	334	5.691000e+06	1.693000e+06	0.702497	1.908079
dwt_66	66	2.234000e+04	1.530000e+04	0.314936	0.146260	lp_brandy	303	1.268000e+07	2.525000e+06	0.800858	2.202008
dwt_72	72	1.220000e+07	5.763000e+06	0.527499	0.149897	lp_capri	482	1.781000e+07	3.544000e+06	0.801002	2.749435
dwt_87	87	1.024000e+04	5.077000e+03	0.504366	0.218176	lp_e226	472	7.518000e+06	2.876000e+06	0.617431	5.803117
gent113	113	3.203000e+07	8.757000e+06	0.726609	0.259552	lp_israel	316	2.213000e+07	1.540000e+07	0.303837	1.090616
gre_115	115	2.467000e+03	1.835000e+03	0.256268	0.278696	lp_kb2	68	2.378000e+07	1.584000e+07	0.333936	0.396778
gre_185	185	1.230000e+06	9.805000e+05	0.202757	0.684197	lp_lotfi	366	3.101000e+06	1.475000e+06	0.524448	3.473910
gre_216a	216	1.061000e+04	9.002000e+03	0.151722	0.655149	lp_recipe	204	2.359000e+07	9.999000e+06	0.576179	0.563452
gre_343	343	1.254000e+04	9.442000e+03	0.246972	1.478083	lp_sc105	163	1.681000e+07	1.418000e+07	0.155977	0.320327
hor_131	434	4.365000e+05	8.362000e+04	0.808423	3.846565	lp_sc205	317	1.682000e+07	1.424000e+07	0.153231	0.804767
ibm32	32	1.633000e+05	8.383000e+04	0.486670	0.131650	lp_sc50a	78	1.676000e+07	1.390000e+07	0.170654	0.150149
illc1033	320	4.540000e+06	2.175000e+06	0.520966	1.358549	lp_sc50b	78	1.505000e+07	1.227000e+07	0.184765	0.148078
impcol_a	207	2.233000e+07	1.613000e+07	0.277622	0.395068	lp_scagr7	185	1.053000e+07	1.761000e+06	0.832777	0.393990
impcol_b	59	3.727000e+07	1.754000e+06	0.952948	0.167633	lp_scorpion	466	7.188000e+06	5.136000e+06	0.285497	1.739667
impcol_c	137	4.736000e+07	3.128000e+04	0.999339	0.326395	lp_share1b	253	1.991000e+07	1.298000e+07	0.348287	1.026345
impcol_d	425	4.250000e+06	4.157000e+05	0.902187	2.347745	lp_share2b	162	2.101000e+07	1.321000e+07	0.371176	0.472567
impcol_e	225	2.384000e+07	2.028000e+01	0.999999	0.997038	lp_stocfor1	165	1.460000e+07	9.642000e+06	0.339626	0.431685
jgl009	9	3.723000e+07	2.892000e+07	0.223225	0.089136	lp_vtp_base	346	2.890000e+07	6.588000e+04	0.997720	2.345079
lshp_265	265	1.927000e+06	1.145000e+06	0.405525	1.256791	lpi_bgprtr	40	2.497000e+07	6.505000e+05	0.973943	0.147450
lshp_406	406	1.230000e+06	9.028000e+05	0.265790	2.063599	lpi_box1	261	3.616000e+07	2.108000e+07	0.416908	0.692226
lund_b	147	4.977000e+07	9.277000e+04	0.998136	0.775601	lpi_cplex2	378	1.344000e+07	8.448000e+06	0.371548	1.543225
mbeacxc	496	2.857000e+06	2.172000e+06	0.239616	5.161266	lpi_ex72a	215	8.578000e+06	5.964000e+06	0.304744	0.449019
mbeaflw	496	5.820000e+06	1.068000e+05	0.981656	8.130244	lpi_ex73a	211	8.578000e+06	5.968000e+06	0.304278	0.398350
mbeause	496	7.052000e+06	1.249000e+05	0.982292	8.919944	lpi_forest6	131	7.137000e+06	7.003000e+06	0.018793	0.217445
nnc261	261	1.652000e+07	3.333000e+06	0.798272	1.181010	lpi_galenet	14	5.895000e+06	5.324000e+06	0.096880	0.107415
nos4	100	2.492000e+06	1.096000e+06	0.559997	0.230104	lpi_itest2	13	1.786000e+07	5.946000e+06	0.667043	0.109737
nos5	468	2.975000e+07	1.447000e+06	0.951367	7.251103	lpi_itest6	17	1.124000e+07	6.547000e+06	0.417575	0.115537
plat362	362	5.995000e+05	5.944000e+05	0.008550	1.877958	lpi_klein1	108	1.431000e+07	2.869000e+06	0.799539	0.444161

Table 2. Suitesparse Matrix Collection

Mat	Size	Cbef	Caft	Reduce	Time	Mat	Size	Cbef	Caft	Reduce	Time
lpi_qual	464	2.440000e+07	1.233000e+05	0.994946	2.865512	Erdos971	472	1.551000e+07	5.380000e+05	0.965319	4.721765
lpi_refinery	464	2.440000e+07	1.154000e+05	0.995271	2.687036	Erdos981	485	8.499000e+06	3.420000e+05	0.959761	4.921555
lpi_vol1	464	2.440000e+07	1.233000e+05	0.994946	2.806833	Erdos991	492	1.629000e+07	4.908000e+05	0.969878	5.481967
lpi_woodinfe	89	8.685000e+06	8.619000e+06	0.007664	0.156514	football	35	2.949000e+07	2.832000e+05	0.990395	0.120533
lp_nug05	225	9.453000e+06	9.445000e+06	0.000832	0.484108	GD00_a	352	2.508000e+07	3.000000e+06	0.880374	0.661116
lp_nug06	486	5.863000e+06	5.862000e+06	0.000082	2.855134	GD01_a	311	1.366000e+07	3.107000e+06	0.772548	0.734467
utm300	300	5.520000e+06	2.591000e+06	0.530525	1.371771	GD01_b	18	5.560000e+06	3.200000e+06	0.424463	0.116826
pivtol	102	1.201000e+04	6.731000e+02	0.943974	0.181140	GD01_c	33	2.196000e+07	1.630000e+07	0.257784	0.123902
mesh1e1	48	2.756000e+01	1.500000e+01	0.455693	0.141812	GD02_a	23	3.395000e+07	7.171000e+06	0.788756	0.102567
mesh1em1	48	3.609000e+02	1.553000e+02	0.569767	0.187848	GD02_b	80	1.332000e+07	5.650000e+06	0.575957	0.170013
mesh1em6	48	3.731000e+01	2.348000e+01	0.370796	0.149999	GD95_a	36	1.267000e+07	5.950000e+06	0.530236	0.116576
mesh2e1	306	8.431000e+04	1.934000e+04	0.770582	1.767308	GD95_b	73	2.298000e+07	1.856000e+07	0.192322	0.141816
mesh2em5	306	6.085000e+04	2.221000e+04	0.635019	2.033301	GD95_c	62	2.651000e+03	1.348000e+03	0.491426	0.163867
mesh3e1	289	7.970000e+01	7.367000e+01	0.075652	0.948502	GD96_b	111	7.440000e+07	4.000000e+06	0.946239	0.146740
mesh3em5	289	2.466000e+01	2.383000e+01	0.033547	0.649657	GD96_c	65	1.715000e+07	1.190000e+07	0.306075	0.158284
sphere2	66	4.729000e+07	2.100000e+07	0.555919	0.143261	GD96_d	180	1.200000e+07	1.200000e+07	0.000000	0.242795
sphere3	258	2.431000e+07	2.246000e+07	0.075870	0.741558	GD97_a	84	1.574000e+07	1.431000e+07	0.090610	0.162372
cage3	5	3.552000e+02	2.324000e+02	0.345737	0.090801	GD97_b	47	3.079000e+07	8.795000e+05	0.971436	0.199645
cage4	9	3.749000e+02	2.332000e+02	0.378129	0.092323	GD98_b	121	8.121000e+06	3.000000e+06	0.630570	0.178936
cage5	37	2.377000e+02	1.446000e+02	0.391509	0.142229	GD98_c	112	9.000000e+06	8.000000e+06	0.111111	0.163736
cage6	93	1.305000e+02	5.598000e+01	0.571133	0.275322	GD99_b	64	1.586000e+07	1.582000e+07	0.002693	0.138254
cage7	340	1.709000e+02	7.342000e+01	0.570302	2.882972	GD99_c	105	7.112000e+06	5.978000e+06	0.159504	0.167981
problem1	415	4.188000e+07	2.802000e+07	0.330968	1.394374	GlossGT	72	2.923000e+07	3.664000e+01	0.999999	0.142222
oscil_dcop_01	430	5.827000e+07	5.820000e+07	0.001128	1.636190	Journals	124	4.054000e+07	2.532000e+02	0.999994	0.455923
oscil_dcop_02	430	5.827000e+07	5.820000e+07	0.001128	1.494037	Ragusa16	24	2.873000e+07	1.965000e+06	0.931590	0.129047
oscil_dcop_04	430	5.827000e+07	5.820000e+07	0.001128	1.794565	Ragusa18	23	4.879000e+07	5.236000e+06	0.892675	0.120835
oscil_dcop_05	430	5.827000e+07	5.820000e+07	0.001128	1.636854	Sandi_authors	86	1.464000e+07	8.518000e+05	0.941830	0.181722
oscil_dcop_07	430	5.827000e+07	5.820000e+07	0.001128	1.642650	Sandi_sandi	360	1.641000e+07	1.454000e+07	0.113784	0.758051
oscil_dcop_08	430	5.827000e+07	5.820000e+07	0.001128	1.559731	SmallW	396	8.277000e+06	7.182000e+06	0.132287	2.267629
oscil_dcop_09	430	5.827000e+07	5.820000e+07	0.001128	1.601949	Stranke94	10	2.676000e+03	2.339000e+03	0.125874	0.096110
oscil_dcop_10	430	5.827000e+07	5.820000e+07	0.001128	1.537608	Tina_AskCal	11	1.257000e+07	4.445000e+06	0.646435	0.108352
oscil_dcop_11	430	5.827000e+07	5.820000e+07	0.001128	1.542233	Tina_AskCog	11	3.914000e+02	2.427000e+02	0.380019	0.095339
oscil_dcop_14	430	5.827000e+07	5.820000e+07	0.001128	1.672371	Tina_DisCal	11	2.291000e+07	2.018000e+02	0.999991	0.093890
oscil_dcop_15	430	5.827000e+07	5.820000e+07	0.001128	1.729068	Tina_DisCog	11	2.707000e+07	1.267000e+07	0.532024	0.091150
oscil_dcop_17	430	5.827000e+07	5.820000e+07	0.001128	1.650611	USAir97	332	8.933000e+06	3.113000e+05	0.965153	2.799760
oscil_dcop_18	430	5.827000e+07	5.820000e+07	0.001128	1.560618	WorldCities	100	4.356000e+03	1.020000e+03	0.765829	0.320696
oscil_dcop_20	430	5.827000e+07	5.820000e+07	0.001128	1.698881	rdb200	200	1.191000e+05	6.547000e+04	0.450363	0.784539
oscil_dcop_21	430	5.827000e+07	5.820000e+07	0.001128	1.614070	rdb200l	200	1.760000e+04	1.357000e+04	0.228670	0.579784
oscil_dcop_22	430	5.827000e+07	5.820000e+07	0.001128	1.569110	rdb450	450	4.695000e+05	2.508000e+05	0.465713	3.113745
oscil_dcop_24	430	5.827000e+07	5.820000e+07	0.001128	1.664884	rdb450l	450	4.395000e+04	2.760000e+04	0.372054	2.883685
oscil_dcop_25	430	5.827000e+07	5.820000e+07	0.001128	1.571597	tols340	340	7.624000e+07	3.353000e+04	0.999560	1.283393
oscil_dcop_26	430	5.827000e+07	5.820000e+07	0.001128	1.731916	tols90	90	4.323000e+07	3.128000e+05	0.992764	0.242690
oscil_dcop_27	430	5.827000e+07	5.820000e+07	0.001128	1.561578	gams10am	171	8.804000e+06	8.724000e+06	0.009127	0.274860
oscil_dcop_28	430	5.827000e+07	5.820000e+07	0.001128	1.513176	farm	17	3.835000e+07	3.314000e+07	0.135843	0.096673
oscil_dcop_29	430	5.827000e+07	5.820000e+07	0.001128	1.681639	gams10a	171	8.804000e+06	8.722000e+06	0.009334	0.281274
oscil_dcop_30	430	5.827000e+07	5.820000e+07	0.001128	1.649562	p0033	48	3.108000e+07	3.074000e+07	0.010914	0.156589
oscil_dcop_31	430	5.827000e+07	5.820000e+07	0.001128	1.530307	p0201	334	5.041000e+06	4.148000e+06	0.177139	1.831204
oscil_dcop_33	430	5.827000e+07	5.820000e+07	0.001128	1.698440	refine	62	1.723000e+07	1.583000e+06	0.908088	0.145843
oscil_dcop_34	430	5.827000e+07	5.820000e+07	0.001128	1.518571	zed	142	2.064000e+07	8.385000e+05	0.959375	0.389350
oscil_dcop_35	430	5.827000e+07	5.820000e+07	0.001128	1.531322	Chebyshev1	261	4.901000e+07	2.200000e+05	0.995511	1.807577
oscil_dcop_36	430	5.827000e+07	5.820000e+07	0.001128	1.541858	Maragal_1	14	3.520000e+07	2.234000e+07	0.365176	0.097431
oscil_dcop_37	430	5.827000e+07	5.820000e+07	0.001128	1.746573	Maragal_2	350	1.766000e+07	9.033000e+06	0.488638	1.285429
oscil_dcop_38	430	5.827000e+07	5.820000e+07	0.001128	1.583039	photogrammetry	390	1.031000e+00	1.028000e+00	0.002058	1.937134
oscil_dcop_39	430	5.827000e+07	5.820000e+07	0.001128	1.658675	bibd_9_5	126	1.944000e+07	1.944000e+07	0.000000	0.209351
oscil_dcop_41	430	5.827000e+07	5.820000e+07	0.001128	1.543861	bibd_11_5	462	1.273000e+07	1.273000e+07	0.000000	2.089555
oscil_dcop_42	430	5.827000e+07	5.820000e+07	0.001128	1.567391	bibd_15_3	455	1.300000e+07	1.300000e+07	0.000000	1.726315
oscil_dcop_43	430	5.827000e+07	5.820000e+07	0.001128	1.526917	CAG_mat364	364	6.786000e+06	1.797000e+06	0.735207	3.660491
oscil_dcop_45	430	5.827000e+07	5.820000e+07	0.001128	1.595076	CAG_mat72	72	1.212000e+07	1.897000e+06	0.843493	0.173381
oscil_dcop_46	430	5.827000e+07	5.820000e+07	0.001128	1.632486	TF10	107	2.582000e+07	2.211000e+07	0.143588	0.247850
oscil_dcop_47	430	5.827000e+07	5.820000e+07	0.001128	1.656462	TF11	236	2.087000e+07	1.930000e+07	0.075111	0.975731
oscil_dcop_48	430	5.827000e+07	5.820000e+07	0.001128	1.532556	IG5-6	77	1.221000e+07	4.284000e+06	0.649147	0.153150
oscil_dcop_49	430	5.827000e+07	5.820000e+07	0.001128	1.836070	IG5-7	150	6.989000e+06	1.813000e+06	0.740603	0.302618
oscil_dcop_51	430	5.827000e+07	5.820000e+07	0.001128	1.569916	IG5-8	292	7.668000e+06	7.368000e+05	0.903910	1.099848
oscil_dcop_52	430	5.827000e+07	5.820000e+07	0.001128	1.580255	GL6_D_6	201	1.688000e+07	1.748000e+05	0.989642	0.641649
oscil_dcop_53	430	5.827000e+07	5.820000e+07	0.001128	1.494891	GL6_D_7	470	1.328000e+07	3.259000e+06	0.754489	4.150725
oscil_dcop_54	430	5.827000e+07	5.820000e+07	0.001128	1.566526	GL6_D_10	341	3.541000e+06	3.193000e+06	0.098214	2.102879
oscil_dcop_56	430	5.827000e+07	5.820000e+07	0.001128	1.589872	GL7d10	60	4.300000e+07	2.344000e+07	0.454839	0.113081
oscil_dcop_57	430	5.827000e+07	5.820000e+07	0.001128	1.548615	GL7d11	60	4.905000e+07	7.101000e+06	0.855245	0.160249
oscil_trans_01	430	5.827000e+07	5.562000e+07	0.045355	0.800879	robot24c1_mat5	302	2.019000e+07	2.160000e+05	0.989300	3.638249
Harvard500	500	1.830000e+07	1.673000e+07	0.085593	3.517279	robot24c1_mat5_J	404	5.047000e+06	4.816000e+06	0.045855	4.246780
lap_25	25	2.786000e+07	2.436000e+07	0.125389	0.108474	klein-b1	10	1.000000e+07	8.529000e+06	0.147110	0.092226
rajat05	301	1.466000e+07	1.316000e+06	0.910252	0.894197	n3c5-b4	210	1.000000e+07	1.000000e+07	0.000000	0.399479
rajat11	135	7.500000e+07	9.077000e+05	0.987897	0.284779	n3c5-b6	210	1.000000e+07	1.000000e+07	0.000000	0.387011
rajat14	180	5.543000e+07	1.563000e+06	0.971808	1.454099	n4c5-b11	120	1.200000e+07	1.200000e+07	0.000000	0.139600
Hamrle1	32	4.796000e+07	3.333000e+05	0.993051	0.134966	Trec3	2	1.000000e+06	1.000000e+00	0.999999	0.088600
robot	120	4.407000e+07	1.020000e+04	0.999769	0.335016	Trec4	3	1.332000e+07	3.491000e+00	1.000000	0.088323
rotor1	100	3.726000e+07	1.869000e+06	0.949828	0.305103	Trec5	7	3.019000e+07	2.920000e+07	0.032668	0.089553
LF10	18	5.170000e+07	4.958000e+04	0.999041	0.135056	Trec6	15	3.675000e+07	3.037000e+07	0.173730	0.093844
Cities	46	4.291000e+04	1.275000e+04	0.702834	0.121651	Trec7	36	1.302000e+07	1.298000e+07	0.002424	0.101188
divorce	9	3.760000e+02	1.794000e+02	0.522869	0.093020	Trec8	84	8.417000e+06	7.976000e+06	0.052324	0.169217

Table 3. Suitesparse Matrix Collection

Mat	Size	Cbef	Caft	Reduce	Time
Trec10	478	2.717000e+06	2.704000e+06	0.005090	11.140717
cat_ears_2_1	85	1.174000e+07	1.104000e+07	0.059091	0.175820
cat_ears_3_1	181	1.238000e+07	1.142000e+07	0.077634	0.368041
cat_ears_4_1	313	1.264000e+07	1.159000e+07	0.082703	0.785780
flower_4_1	129	1.245000e+07	1.222000e+07	0.018632	0.253293
flower_5_1	201	1.235000e+07	1.178000e+07	0.045937	0.454068
flower_7_1	393	1.232000e+07	1.178000e+07	0.043444	1.074669
wheel_3_1	25	1.104000e+07	1.100000e+07	0.003901	0.109810
wheel_4_1	41	1.135000e+07	1.126000e+07	0.008292	0.119916
wheel_5_1	61	1.203000e+07	1.158000e+07	0.036723	0.139474
wheel_6_1	85	1.283000e+07	1.200000e+07	0.065284	0.163871
wheel_7_1	113	1.373000e+07	1.216000e+07	0.114714	0.204933
rel3	5	3.600000e+07	3.200000e+07	0.111110	0.102103
rel4	12	2.788000e+07	2.434000e+07	0.126844	0.109623
rel5	35	2.673000e+07	2.127000e+07	0.204316	0.119203
rel6	157	1.657000e+07	1.300000e+07	0.215519	0.386222
relat3	5	4.800000e+07	4.267000e+07	0.110944	0.099093
relat4	12	3.616000e+07	3.277000e+07	0.093784	0.111163
relat5	35	2.125000e+07	1.906000e+07	0.103153	0.117217
relat6	157	1.114000e+07	8.926000e+06	0.199069	0.368550
D_5	115	1.812000e+07	2.917000e+06	0.839033	0.305395
D_6	435	1.688000e+07	2.862000e+05	0.983046	4.289448
D_11	461	2.952000e+06	2.897000e+06	0.018640	3.942477
08blocks	300	2.749000e+07	5.771000e+05	0.979006	0.672977
abtaha2	331	1.493000e+02	1.038000e+02	0.304608	4.116590
abtaha1	209	1.495000e+02	6.776000e+01	0.546865	1.368067
Trefethen_20b	19	9.212000e+02	8.697000e+00	0.990559	0.106288
Trefethen_20	20	3.980000e+03	2.859000e+01	0.992817	0.109017
Trefethen_150	150	5.928000e+05	3.893000e+01	0.999934	0.964683
Trefethen_200b	199	2.723000e+05	1.102000e+01	0.999960	1.485429
Trefethen_200	200	1.190000e+06	3.893000e+01	0.999967	1.684266
Trefethen_300	300	3.142000e+06	4.213000e+01	0.999987	3.809453
Trefethen_500	500	1.015000e+07	4.213000e+01	0.999996	12.866715
ww_36_pmec_36	66	2.185000e+07	6.710000e+06	0.692865	0.172951
adjnoun	112	1.729000e+07	8.143000e+05	0.952909	0.397979
celegansneural	297	8.250000e+06	1.025000e+05	0.987573	2.235483
dolphins	62	5.175000e+07	2.005000e+06	0.961257	0.174200
football	35	2.949000e+07	2.832000e+05	0.990395	0.000000
karate	34	2.262000e+07	9.819000e+06	0.565847	0.104732
lesmis	77	1.639000e+07	1.024000e+05	0.993754	0.220944
polbooks	105	5.187000e+05	1.738000e+05	0.664881	0.287798
jazz	198	9.052000e+06	4.181000e+06	0.538160	1.262591
celegans_metabolic	453	6.408000e+06	1.284000e+05	0.979966	13.988976
grid1	252	1.534000e+07	7.319000e+06	0.522949	0.655047
grid1_dual	224	1.519000e+07	1.393000e+07	0.082991	0.595398
chesapeake	39	4.405000e+07	5.447000e+06	0.876353	0.109485
cz148	148	6.113000e+06	5.732000e+06	0.062333	0.402147
cz308	308	5.391000e+07	5.058000e+07	0.061743	1.103020
hangGlider_1	360	2.570000e+07	8.973000e+02	0.999965	3.854804
orbitRaising_1	442	1.881000e+07	9.431000e+03	0.999499	2.991966
spaceStation_1	99	4.896000e+07	2.515000e+06	0.948639	0.278097
spaceStation_2	329	1.259000e+07	4.514000e+06	0.641467	5.425566
spaceStation_3	467	1.721000e+07	7.925000e+06	0.539570	4.392964
tumorAntiAngiogenesis_1	205	8.470000e+06	8.110000e+04	0.990425	1.249378
tumorAntiAngiogenesis_2	305	3.091000e+07	2.387000e+00	1.000000	2.574419
mycielskian2	2	1.000000e+00	1.000000e+00	0.000000	0.087407
mycielskian4	11	9.391000e+01	8.476000e+01	0.097455	0.096446
mycielskian5	23	7.641000e+02	6.110000e+02	0.200423	0.114535
mycielskian6	47	5.863000e+03	4.139000e+03	0.293979	0.119011
mycielskian7	95	4.337000e+04	2.700000e+04	0.377367	0.219110
mycielskian8	191	3.132000e+05	1.727000e+05	0.448534	0.993721
mycielskian9	383	2.227000e+06	1.072000e+06	0.518610	3.751622
breasttissue_10NN	106	4.147000e+05	4.605000e+04	0.888958	0.300496
dermatology_5NN	366	8.770000e+06	9.134000e+05	0.895847	2.231720
Ecoli_10NN	336	5.181000e+06	5.434000e+05	0.895106	2.496580
Glass_10NN	214	1.473000e+07	3.134000e+05	0.978730	1.019550
iris_dataset_30NN	150	4.617000e+05	2.363000e+05	0.488167	0.523400
Olivetti_norm_10NN	400	3.756000e+06	3.668000e+05	0.902332	6.360955
YaleA_10NN	165	2.198000e+06	1.707000e+05	0.922321	0.703458

Table 4. Suitesparse Matrix Collection

A.3 Larger Problems

Instance	Size	Cbef	Caft	Reduce	Time
m3plates	11007	2.65e+04	1.00	0.999627	492.1
EVA	8497	1.624e+07	1.620e+07	0.024330	2604.5
bcsstm38	8032	running			
plddb	5049	running			
lpi_cplex1	5224	running			

Table 5.

B LIBSVM Dataset

	Mat	Size	Cbef	Caft	Reduce
0	YearPredictionMSD	90	5233000.00	470.20	0.999910
1	YearPredictionMSD.t	90	5521000.00	359900.00	0.934816
2	abalone_scale.txt	8	2419.00	2038.00	0.157291
3	bodyfat_scale.txt	14	1281.00	669.10	0.477475
4	cadata.txt	8	8982000.00	7632.00	0.999150
5	cpusmall_scale.txt	12	20000.00	6325.00	0.683813
6	eunite2001.t	16	52450000.00	8530.00	0.999837
7	eunite2001.txt	16	67300000.00	3591.00	0.999947
8	housing_scale.txt	13	153.90	83.22	0.459371
9	mg_scale.txt	6	10.67	10.03	0.059988
10	mpg_scale.txt	7	142.50	107.20	0.247842
11	pyrim_scale.txt	27	49100000.00	3307.00	0.999933
12	space_ga_scale.txt	6	1061.00	729.60	0.312041
13	triazines_scale.txt	60	24580000.00	15460000.00	0.371034

Table 6. LIBSVM Dataset

C Randon Instances

	Mat	Size	Cbef	Caft	Reduce
0	diag-bench-100-1.000e-01	100	4261000.0	1888000.0	0.557008
1	diag-bench-500-1.000e-01	500	2152000.0	1460000.0	0.321581
2	diag-bench-1000-1.000e-02	1000	5127000.0	1713000.0	0.665939
3	diag-bench-2000-1.000e-03	2000	12510000.0	5396000.0	0.568675

Table 7. Random instances

Remark 2. Randomly generated instances are named by `diag-bench-#size#-#sparsity#`.