Assig 6 Jake Veith p1

1.

a. $L(M') = \Sigma^*$ if M accepts w. Otherwise, $L(M') = \emptyset$

TM M' = "on input x:
1. run TM M on input w
2. if M accepts w, accept.
Otherwise, reject."

b. $L(M') = \{x \mid x \text{ ends with a } 1\}$ if M acc. w. else $L(M') = \emptyset$

TM M' = "on input x:
1. if x does not end with a 1, reject.
2. else, run TM M on input w
3. if M accepts w, accept.
else reject."

c. $L(M') = \Sigma^*$ if M acc w. else $L(M') = \{x \mid x \text{ ends with a } 1\}$

TM M' = "on input x:
1. run TM M on input w:
2. if M acc w, accept.
3. else, if x ends with a 1, accept.
else, reject."

d. $L(M') = 2$ strings if M acc w, else $|L(M')| > 2$ or $|L(M')| < 2$

TM M' = "on input x:
1. Run TM M on input w.
2. if M acc. w and $x = 01$ or $x = 10$, accept.
3. else reject."

Assig 6

Jack
reith

p2

1. contd.

e. L(

M acc w: $L(M_1) = \{01\}$

$L(M_2) = \{01\}$

else $L(M_1) = \Sigma^*$

$L(M_2) = \emptyset$

TM M_1 = "on input x:
1. run TM M on input w.
2. if TM M acc w and x=01, accept.
3. if TM M acc w and x!=01, reject.
4. else accept."

TM M_2 = "on input x:
1. run TM M on input w.
2. if TM M acc w and x=01, accept.
3. if TM M acc. w and x!=01, reject.
4. else reject."

---

2. $NE_{TM} = \{\langle M \rangle \mid M$ is a TM and M acc at least 1 string$\}$

Assume that NE_{TM} is decidable. Then $\exists$ a TM R that decides NE_{TM}.

Construct TM S to decide A_{TM}.
S = "on input $\langle M, w \rangle$ where M is a TM and w is an input string:
1. Construct TM M' as follows:
M' = "on input x:
1. run M on input w.
2. if M acc. w, accept
else, reject."
2. run R on input $\langle M' \rangle$
3. if R accepts $\langle M' \rangle$, accept
else, reject."

Assig 6

Julie Veith

2 cont'd.

~~if~~ Assuming that NE_{TM} is decidable, $\exists$ TM R that decides NE_{TM}. construct TM S as described above to decide A_{TM}.

· Prove that if $\langle M,w \rangle \in A_{TM}$, S accepts $\langle M,w \rangle$. Assume that $\langle M,w \rangle \in A_{TM}$. Then M is a TM that accepts string w. Since M accepts w, from the definition of the TM M', M' accepts all strings and Thus $L(M') = \Sigma^*$. Then M' accepts at least one string, and $\langle M' \rangle \in NE_{TM}$. Thus, when R, a decider for NE_{TM}, is run on input $\langle M' \rangle$, it will accept. Then the original machine S that decides A_{TM} will accept.

· Prove that if $\langle M,w \rangle \notin A_{TM}$ S rejects $\langle M,w \rangle$. Assume that $\langle M,w \rangle \notin A_{TM}$. Then M is a TM, does not acc. string w. Since M does not w, from the definition of TM M', M' rejects all strings and thus $L(M') = \emptyset$. Then M' does not accept at least one string, and $\langle M' \rangle \notin NE_{TM}$. Thus when R, a decider for NE_{TM}, is run on input $\langle M' \rangle$, it will reject. Then the original machine S that decides A_{TM} will accept.

This shows that TM S is a decider for A_{TM}. therefore, A_{TM} is decidable. This a contradiction; thus NE_{TM} is undecidable.

Assig 6 Jack Veith p4

3. $E_\varepsilon = \{\langle M \rangle \mid M$ is a TM that accepts $\varepsilon\}$

Assume that E_ε is decidable. Then $\exists$ a TM R that decides E_ε.
Construct TM S to decide A_{TM}.
S = " on input $\langle M, w \rangle$ where M is a TM and w is a string:
1. construct TM M' as follows
   M' = "on input X:
   1. run TM M on input w
   2. if M accepts, accept.
      else reject. "
2. run R on input $\langle M' \rangle$.
3. if R accepts, accept.
   else reject. "

Pf Assuming that E_ε is decidable. Then $\exists R$ a TM that decides E_ε.
Construct TM S as above to decide A_{TM}.

· Prove that if $\langle M, w \rangle \in A_{TM}$, S accepts $\langle M, w \rangle$. Assume that $\langle M, w \rangle \in A_{TM}$.
Then M is a TM that acc. w. Then per the definition of TM M',
M' accepts all strings (including ε). Then M' accepts ε, and $\langle M' \rangle \in E_\varepsilon$.
Thus when TM R, a decider for E_ε, is run on $\langle M' \rangle$, it will accept.
Then by the definition of S, S accepts $\langle M, w \rangle$.

· Prove that if $\langle M, w \rangle \notin A_{TM}$, S rejects $\langle M, w \rangle$. Assume $\langle M, w \rangle \notin A_{TM}$.
Then M is a TM that does not acc. w. Then per the definition of TM M',
M' accepts no strings (including ε). Then M' rejects on ε, and $\langle M' \rangle \notin E_\varepsilon$.
Thus when TM R, a decider for E_ε, is run on $\langle M' \rangle$, it will reject.
Then, by the definition of S, S rejects $\langle M, w \rangle$

This shows that S is a decider for A_{TM}. Therefore A_{TM} is decidable. This is a contradiction $\rightarrow E_\varepsilon$ is undecidable. □

Assig 6

Jack Veith

4. $NEQ_{TM} = \{\langle M_1, M_2 \rangle \mid M_1 \text{ and } M_2 \text{ are TMs and } L(M_1) \neq L(M_2)\}$

Assume that NEQ_{TM} is decidable $\rightarrow \exists R$ a TM that decides NEQ_{TM}.
Construct TM S to decide A_{TM}.

S = " on input $\langle M, w \rangle$:
1. construct M_1 as follows:
   M_1 = " on input x: $L(M_1) = \Sigma^*$
   1. accept."
2. Construct M_2 as follows:
   M_2 = " on input x:
   1. run M on input w.
   2. if M accepts w, reject. $\langle M, w \rangle \in A_{TM} \rightarrow L(M_2) = \emptyset$
      if M rejects w, accept." $\langle M, w \rangle \notin A_{TM} \rightarrow L(M_2) = \Sigma^*$
3. run R on input $\langle M_1, M_2 \rangle$
4. if R accepts, accept.
   if R rejects, reject."