Math 0413 Quiz 8 Talk Veith p1

1. Prove that $\sum_{n=1}^{\infty} \frac{7^n}{2^{3n}}$ is a convergent series & find its sum.

$\forall n \in \mathbb{N}$, $\frac{7^n}{2^{3n}} = \left(\frac{7}{2^3}\right)^n = \left(\frac{7}{8}\right)^n$.

Since $\left|\frac{7}{8}\right| = \frac{7}{8} < \frac{8}{8} = 1$, by the Geometric Series Test,

$\sum_{n=1}^{\infty} \frac{7^n}{2^{3n}}$ is a convergent series.

Thus, we can find the sum of the series:

for $a=1$, $r=7/8$:

$$\frac{ar}{1-r} = \frac{1}{1-7/8}(7/8) = \frac{7}{8-7} = \frac{7}{1} = 7.$$

Thus $\sum_{n=1}^{\infty} \frac{7^n}{2^{3n}}$ is a convergent series whose sum is 7.

QED.