

Complex Geometric Problem: An Olympic math problem for 2D geometry

题目编号: 20191176779

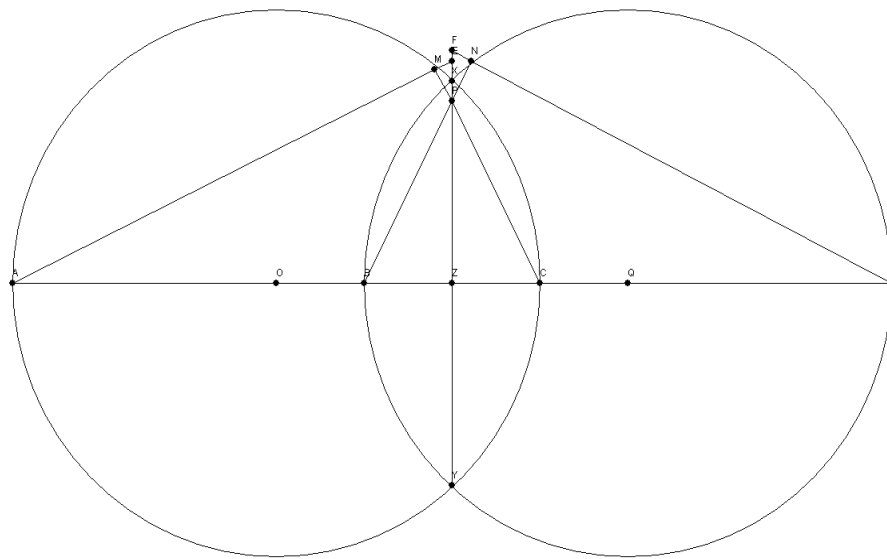
方向: Q

题干:

设A,B,C,D是一条直线上的四个点,以AC为直径的圆O与以BD为直径的圆Q相交于X,Y,直线XY交BC于点Z,若P为XY上异于点Z的一点,直线CP与以AC为直径的圆O相交于C和M,直线BP与以BD为直径的圆Q相交于B和N,试证:AM,XY和DN三线共点.

1) Problem description:

Suppose point A, B, C and D are four different points arranged in turn on a straight line, the line intersects with the circle O having a diameter AC at point X, and intersects with the circle Q having a diameter BD at point Y. Line XY intersects BC with point Z, if point P is a point different from Z on line XY, the line CP intersects with the circle O having a diameter AC at point C and M, the line BP intersects with the circle Q having a diameter BD at point B and N. Prove : AM, XY and DN three lines intersect at one point.



2) Graphic information:

```
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  "stem": {
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            "5": "F###N###D"
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            "M": "48.00,23.50",
            "N": "52.20,24.40",
            "O": "30.00,0.00",
            "Q": "70.00,0.00",
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            "Y": "50.00,-22.236",
            "Z": "50.00,0.00",
            "E": "50.00,24.455",
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}
```

3) NLP:

Common stem:

DiameterRelation{diameter=AC, circle=Circle[⊙O]{center=O, analytic=y_O=f(x_O), length=null},

DiameterRelation{diameter=BD, circle=Circle[⊙Q]{center=Q, analytic=y_Q=f(x_Q), length=null},

PointRelation:A, PointRelation:B, PointRelation:C,

LineCrossCircleRelation{line=CP, circle=⊙O, crossPoints=[C, M], crossPointNum=2},

LineCrossCircleRelation{line=BP, circle=⊙Q, crossPoints=[B, N], crossPointNum=2}

Sub stem: []

Conclusion:[ProveConclusionRelation:[MultiLineCrossRelation{lines=[DN,AM,XY]}]]

4) Strategies

Generating 3756 adding auxiliary line strategies based on Strategy Network

1	connect point M and point O
2	connect point N and point Q
3	create middle point G of segment AM
.....
1213	extended segment DN intersection segment XY at point X_107
.....
1625	extended segment AM intersection segment XY at point X_155
.....
3756	connect point X_314 and point X_352

5) Rank strategies by value network

We choose the top 10 candidates as the branching auto solving strategies.

1	create middle point G of segment DN, connect point G and point Q
2	create middle point G of segment AM, connect point G and point O
3	connect point X and point O
4	extended segment AM intersection segment XY at point E
5	extended segment DN intersection segment XY at point F
6	create vertical segment MG of segment XY through point M which the foot is point G
7	create vertical segment AG of segment DN through point A which the foot is point G
8	extended segment AM intersection segment DN at point G
9	connect point M and point N
10	connect point N and point Q

The strategies of number 4 and 5 be validated useful for problem solving.

Human-like solving processes:

6) AutoSolve:[

- (1) ∴ draw cross point E of AM and XY, draw cross point F of XY and DN
- (2) ∴ Y, Z, P, X, E, F is collinear
- (3) ∴ F, N, D is collinear
- (4) ∴ A, M, E is collinear
- (5) ∴ BD is the diameter of the circle Q
- (6) ∴ point N
- (7) ∴ by(4,5,6): $\text{Rt} \angle \text{BND}$
- (8) ∴ by(7): $\text{BN} \perp \text{DF}$, pedal point is N
- (9) ∴ by(8): $\text{Rt} \angle \text{BNF}$
- (10) ∴ by(6): $\triangle \text{FNP}$
- (11) ∴ by(9,10): $\text{Rt} \triangle \text{FNP}$ (vertex is point N)
- (12) ∴ $\odot \text{O}$
- (13) ∴ $\odot \text{Q}$
- (14) ∴ by(12,13): $\odot \text{O}$ cross with $\odot \text{Q}$
- (15) ∴ by(14): OQ is the perpendicular bisector of XY
- (16) ∴ by(15): $\text{Rt} \angle \text{AZP}$
- (17) ∴ $\triangle \text{BPZ}$
- (18) ∴ by(16,17): $\text{Rt} \triangle \text{BPZ}$ (vertex is point Z)

(19)∴ $\angle BPZ$ and $\angle NPX$ is a pair of vertical angles
 (20)∴ by(19): $\angle BPZ = \angle NPX$
 (21)∴ by(11,18,20): $BP \cdot NP = FP \cdot PZ$
 (22)∴ AC is the diameter of circle O.
 (23)∴ point M
 (24)∴ by(4,22,23): $\text{Rt} \angle AMC$
 (25)∴ by(24): $AE \perp CM$, pedal point is point M
 (26)∴ by(25): $\text{Rt} \angle CME$
 (27)∴ by(23): $\triangle EMP$
 (28)∴ by(26,27): $\text{Rt} \triangle EMP$ (vertex is point M)
 (29)∴ by(15): $\text{Rt} \angle CZP$
 (30)∴ $\triangle CPZ$
 (31)∴ by(29,30): $\text{Rt} \triangle CPZ$ (vertex is point Z)
 (32)∴ $\angle CPZ$ and $\angle MPX$ is a pair of vertical angles
 (33)∴ by(32): $\angle CPZ = \angle MPX$
 (34)∴ by(28,31,33): $MP \cdot CP = EP \cdot PZ$
 (35)∴ points B, X, N, D, Y is concyclic of $\odot Q$
 (36)∴ by(35): XY is one chord of $\odot Q$
 (37)∴ by(35): BN is one chord of $\odot Q$
 (38)∴ by (36,37): $(PX) \cdot (PY) = (BP) \cdot (NP)$
 (39)∴ points A, M, X, C, Y is concyclic of $\odot O$
 (40)∴ by(39): XY is one chord of $\odot O$
 (41)∴ by(39): CM is one chord of $\odot O$
 (42)∴ by(40,41): $(PX) \cdot (PY) = (MP) \cdot (CP)$
 (43)∴ by(21,34,38,42): point E and point F coincide
 (44)∴ by(1,2,3,43): AE, DF, FY intersected at the same point E
]

Example 2 shows the processes of automatically ranking strategies by value network.