Example 1: 2018121490624 A math problem for analytic geometry

题目编号: 2018121490624 查询

题干：

设F_1是椭圆C:$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1|(a>b>0)$的左焦点，M是C上一点，且$MF_1$与x轴垂直，若$|MF_1|=\frac{3}{2}$，椭圆的离心率为$\frac{1}{2}$.

（1）求椭圆C的方程；

（2）以椭圆C的左顶点A为$Rt\triangle ABD$的直角顶点，边AB,AD与椭圆C交于B,D两点，求$\triangle ABD$面积的最大值.

Let F_1 be the left focus of ellipse C:x^2/a^2+y^2/b^2=1(a>b>0), Point M lies on C, and MF_1 is perpendicular to the X-axis, if |MF_1|=3/2, the eccentricity of ellipse C is 1/2.

(1) Find the standard equation of ellipse C.

(2) If the left vertex of ellipse C is point A, and the right vertex of △ABD is point A, AB intersects ellipse C at B, and AD intersects ellipse C at D. Find the maximum value of area of △ABD.

Human-like solving processes:

Question (1):

(1)∵ line F_1M⊥line X:y=0

(2)∴ by(1): analytic of function F_1M is x=x_F_1M

(3)∴ by(1): analytic of function X is y=0

(4)∴ by(1,2,3): x_F_1M=0

(5)∵ the focus of C is F_1.

(6)∵ analytic of ellipse C is ((x^2)/(a^2))+((y^2)/(b^2))=1

(7)∴ by(5): point F_1

(8)∴ by(5,6,7): point F_1(-(a^2-b^2)^(1/2),0)

(9)∴ by(3,8): point F_1 (-(a^2-b^2)^(1/2), 0) is on line X: y = 0

(10)∴ by(2): point F_1 is on line F_1M: x = x_F_1M

(11)∴ by(9,10): line X:y=0 and line F_1M:x=x_F_1M crossing at point F_1(-(a^2-b^2)^(1/2),0)

(12)∴ by(11): point F_1(-(a^2-b^2)^(1/2), 0) is on line F_1M:x=x_F_1M

(13)∴ by(4,8,12): analytic of function F_1M is x=-(a^2-b^2)^(1/2)

(14)∴ by(13): point M is on line F_1M:x=-(a^2-b^2)^(1/2)

(15)∵ point M is on ellipse C

(16)∴ by(15): point M

(17)∴ by(15,16): point M(s_M, t_M)

(18)∴ by(13,14,17): s_M+(a^2-b^2)^(1/2)=0

(19)∴ by(6): a>b

(20)∴ by(6,15,17): s_M≥-a

(21)∴ by(6,15,17): t_M≥-b

(22)∵ F_1M=(3/2)

(23)∴ by(22): |vector F_1M| is (3/2)

(24)∴ by(23): vector F_1M is (s_M+(a^2-b^2)^(1/2), t_M)

(25)∴ by(24): vector F_1M

(26)∴ by(8,17,25): vector F_1M=(s_M+(a^2-b^2)^(1/2), t_M)

(27)∴ by(23,26): (a^2-b^2+2*s_M*(a^2-b^2)^(1/2)+s_M^2+t_M^2)^(1/2)=3/2

(28)∴ by(6): a^2≠b^2

(29)∴ by(5,6): Point F_1 is on line X:y=0

(30)∴ by(5,29): The focus of C is on X axis.

(31)∴ by(6,30): a^2>b^2

(32)∴ by(6): a>0

(33)∴ by(6,15,17): t_M≤b

(34)∴ by(6,15,17): s_M^2/a^2+t_M^2/b^2-1=0

(35)∴ by(6): b>0

(36)∴ by(6): focal length of conic C is 2*C_3

(37)∴ by(6,36): C_3>0

(38)∴ by(4,14,17): analytic of function F_1M is x=s_M

(39)∴ by(38): point F_1 is on line F_1M:x=s_M

(40)∴ by(8,38,39): -(a^2-b^2)^(1/2)-s_M=0

(41)∴ by(6,15,17): s_M≤a

(42)∴ by(18,19,20,21,27,28,31,32,33,34,35,37,40,41): s_M=-1,a=2,b=3^(1/2),t_M=3/2 or s_M=-1,a=2,b=3^(1/2),t_M=-3/2

Discussions in different conditions:

Condition 1

when [${s}_{M}=(-1)$, $a=2$, $b=\sqrt{3}$, ${t}_{M}=\frac{3}{2}$]:

(1)∵ b=3^(1/2)

(2)∵ a=2

(3)∵analytic of ellipse C is ((x^2)/(a^2))+((y^2)/(b^2))=1

(4)∴ by(2,3): analytic of ellipse C is 1/4*(b^2*x^2+4*y^2)/b^2=1

(5)∴ by(1,4): analytic of ellipse C is 1/4*x^2+1/3*y^2=1

when [${s}_{M}=(-1)$, $a=2$, $b=\sqrt{3}$, ${t}_{M}=(-\frac{3}{2})$]:

Condition 2

The same as Condition 1

To sum up, [the standard equation of ellipse C is x^2/4+y^2/3=1]

Question (2):

(1)∵ the standard equation of ellipse C is x^2/4+y^2/3=1

(2)∵ the left vertex of ellipse C is point A

(3)∴ by (1,2): A(-2,0)

(4)∵ line AB intersects ellipse C at B

(5)∴ by (3,4): the equation of function AB is y=k_AB*(x+2)

(6)∵ line AD intersects ellipse C at D

(7)∴ by (3,6): the equation of function AD is y=k_AD*(x+2)

(8)∵ let B(x_B, y_B)

(9)∴ by (1,4,8): x_B^2/4+y_B^2/3=1

(10)∴ by (3,5,8): k_AB=(y_B-0)/(x_B+2)

(11)∵ let D(x_D, y_D)

(12)∴ by (1,6,11): x_D^2/4+y_D^2/3=1

(13)∴ by (3,7,11): k_AD=(y_D-0)/(x_D+2)

(14)∵ segment AB

(15)∴ by (3,8,14): AB=((x_B+2)^2+(y_B-0)^2)^(1/2)

(16)∵ segment AD

(17)∴ by (3,11,16): AD=((x_D+2)^2+(y_D-0)^2)^(1/2)

(18)∵ Rt△ABD(vertex is point A)

(19).∴ by (18): Rt∠ BAD

(20).∴ by (19): AD⊥AB, foot point is A

(21).∴ by (20): segment AB is the height of △ABD

(22).∴ by (21): S_△ABD=((1/2)*AD)*AB

(23)∵ S_△ABD=v_0

(24).∴ by (15,17,22,23): v_0=1/2*((x_D+2)^2+(y_D-0)^2)^(1/2)*((x_B+2)^2+(y_B-0)^2)^(1/2)

(25).∴ by (20): k_AB*k_AD=-1

(26).∴ by (9,10,12,13,15,17,22,23,24,25): the maximum value of S_△ABD is 144/49

Example 1 shows the solving processes of different strategies.

Example 2: An Olympic math problem for 2D geometry

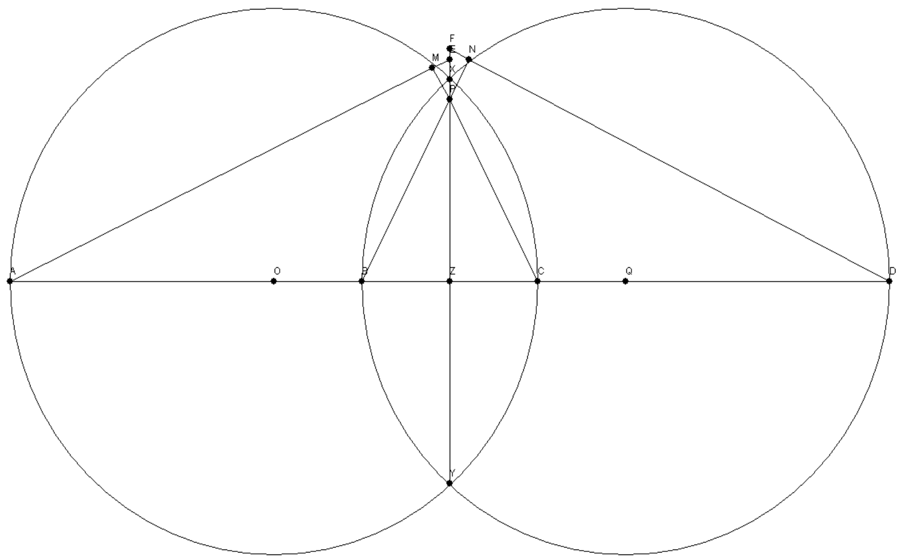
题目编号: 20191176779 查询

题干：

设A,B,C,D是一条直线上的四个点，以AC为直径的圆O与以BD为直径的圆Q相交于X,Y，直线XY交BC于点Z，若P为XY上异于点Z的一点，直线CP与以AC为直径的圆O相交于C和M，直线BP与以BD为直径的圆Q相交于B和N，试证:AM, XY 和DN三线共点.

1) Problem description:

Suppose point A, B, C and D are four different points arranged in turn on a straight line, the line intersects with the circle O having a diameter AC at point X, and intersects with the circle Q having a diameter BD at point Y. Line XY intersects BC with point Z, if point P is a point different from Z on line XY, the line CP intersects with the circle O having a diameter AC at point C and M, the line BP intersects with the circle Q having a diameter BD at point B and N. Prove : AM, XY and DN three lines intersect at one point.



2) Graphic information:

{"substems":[],"stem":{"pictures":[{"variable-equals":{},"picturename":"","circles":[{"center":"O","pointincircle":"A###M###X###C###Y"},{"center":"Q","pointincircle":"B###X###N###D###Y"}],"collineations":{"0":"A###O###B###Z###C###Q###D","1":"F###E###X###P###Z###Y","2":"M###P###C","3":"N###P###B","4":"A###M###E","5":"F###N###D"},"coordinates":{"A":"0.00,0.00","B":"40.00,0.00","C":"60.00,0.00","D":"100.00,0.00","M":"48.00,23.50","N":"52.20,24.40","O":"30.00,0.00","Q":"70.00,0.00","P":"50.00,20.00","X":"50.00,22.236","Y":"50.00,-22.236","Z":"50.00,0.00","E":"50.00,24.455","F":"50.00,25.555"}}]},"threeviews":{},"flowChart":{},"function":{}}

3) NLP:

Common stem:[

DiameterRelation{diameter=AC, circle=Circle[⊙O]{center=O, analytic=y_O=f(x_O), length=null},

DiameterRelation{diameter=BD, circle=Circle[⊙Q]{center=Q, analytic=y_Q=f(x_Q), length=null},

PointRelation:A, PointRelation:B, PointRelation:C,

LineCrossCircleRelation{line=CP, circle=⊙O, crossPoints=[C, M], crossPointNum=2},

LineCrossCircleRelation{line=BP, circle=⊙Q, crossPoints=[B, N], crossPointNum=2}

Sub stem: []

Conclusion:[ProveConclusionRelation:[MultiLineCrossRelation{lines=[DN,AM,XY] }]]]

4) Strategies

Generating 3756 adding auxiliary line strategies based on Strategy Network

1	connect point M and point O
2	connect point N and point Q
3	create middle point G of segment AM
……	……
1213	extended segment DN intersection segment XY at point X_107
……	……
1625	extended segment AM intersection segment XY at point X_155
……	……
3756	connect point X_314 and point X_352

5) Rank strategies by value network

We choose the top 10 candidates as the branching auto solving strategies.

1	create middle point G of segment DN, connect point G and point Q
2	create middle point G of segment AM, connect point G and point O
3	connect point X and point O
4	extended segment AM intersection segment XY at point E
5	extended segment DN intersection segment XY at point F
6	create vertical segment MG of segment XY through point M which the foot is point G
7	create vertical segment AG of segment DN through point A which the foot is point G
8	extended segment AM intersection segment DN at point G
9	connect point M and point N
10	connect point N and point Q

The strategies of number 4 and 5 be validated useful for problem solving.

Human-like solving processes:

6) AutoSolve:[

(1)∵ draw cross point E of AM and XY, draw cross point F of XY and DN

(2)∵ Y, Z, P, X, E, F is collinear

(3)∵ F, N, D is collinear

(4)∵ A, M, E is collinear

(5)∵ BD is the diameter of the circle Q

(6)∵ point N

(7)∴ by(4,5,6): Rt∠BND

(8)∴ by(7): BN⊥DF, pedal point is N

(9)∴ by(8): Rt∠BNF

(10)∵ by(6): △FNP

(11)∴ by(9,10): Rt△FNP(vertex is point N)

(12)∵ ⊙O

(13)∵ ⊙Q

(14)∴ by(12,13): ⊙O cross with ⊙Q

(15)∴ by(14): OQ is the perpendicular bisector of XY

(16)∴ by(15): Rt∠AZP

(17)∵ △BPZ

(18)∴ by(16,17): Rt△BPZ(vertex is point Z)

(19)∵ ∠BPZ and ∠NPX is a pair of vertical angles

(20)∴ by(19): ∠BPZ=∠NPX

(21)∴ by(11,18,20): BP*NP=FP*PZ

(22)∵ AC is the diameter of circle O.

(23)∵ point M

(24)∴ by(4,22,23): Rt∠AMC

(25)∴ by(24): AE⊥CM, pedal point is point M

(26)∴ by(25): Rt∠CME

(27)∴ by(23): △EMP

(28)∴ by(26,27): Rt△EMP(vertex is point M)

(29)∴ by(15): Rt∠CZP

(30)∵ △CPZ

(31)∴ by(29,30): Rt△CPZ(vertex is point Z)

(32)∵ ∠CPZ and ∠MPX is a pair of vertical angles

(33)∴ by(32): ∠CPZ=∠MPX

(34)∴ by(28,31,33): MP*CP=EP*PZ

(35)∵ points B, X, N, D, Y is concyclic of ⊙Q

(36)∴ by(35): XY is one chord of ⊙Q

(37)∴ by(35): BN is one chord of ⊙Q

(38)∴ by (36,37): (PX)*(PY)=(BP)*(NP)

(39)∵ points A, M, X, C, Y is concyclic of ⊙O

(40)∴ by(39): XY is one chord of ⊙O

(41)∴ by(39): CM is one chord of ⊙O

(42)∴ by(40,41): (PX)*(PY)=(MP)*(CP)

(43)∴ by(21,34,38,42): point E and point F coincide

(44)∴ by(1,2,3,43): AE, DF, FY intersected at the same point E

]

Example 2 shows the processes of automatically ranking strategies by value network.

Example 3: An Olympic math problem for algebra

1) Problem description:

Assume x, y, z are positive numbers and $xyz \geq 0$. Prove:$\frac{x^5-x^2}{x^5+y^2+z^2}+\frac{y^5-y^2}{x^2+y^5+z^2}+\frac{z^5-z^2}{x^2+y^2+z^5}\geq 0$

2) NLP:

Common stem: [

AtomAttributeRelation{atomAttribute=AtomAttribute{atomExpr=Express:[x],numberType=POSITIVE}},

AtomAttributeRelation{atomAttribute=AtomAttribute{atomExpr=Express:[y],numberType=POSITIVE}},

AtomAttributeRelation{atomAttribute=AtomAttribute{atomExpr=Express:[z],numberType=POSITIVE}},

InequalityRelation[dualExpressCompare=(x*y*z)≥1],

Conclusion:[ProveConclusionRelation:[InequalityRelation[dualExpressCompare=((((x^5)-(x^2)))/(((x^5)+(y^2)+(z^2))))+((((y^5)-(y^2)))/(((x^2)+(y^5)+(z^2))))+((((z^5)-(z^2)))/(((x^2)+(y^2)+(z^5))))≥0]]]]

3) AutoSolve:[

(1)∵P_e=((((x^5)-(x^2)))/(((x^5)+(y^2)+(z^2))))+((((y^5)-(y^2)))/(((x^2)+(y^5)+(z^2))))+((((z^5)-(z^2)))/(((x^2)+(y^2)+(z^5))))

(2)∵ (x*y*z)≥1

(3)∴by(1,2):(((x^2)+(y^2)+(z^2))/(((x^5)+(y^2)+(z^2))))+((((x^2)+(y^2)+(z^2)))/(((x^2)+(y^5)+(z^2))))+((((x^2)+(y^2)+(z^2)))/(((x^2)+(y^2)+(z^5))))≤3

(4)∴by(2,3): (x^5+y^2+z^2)*(y*z+y^2+z^2)≥((x^2)*(x*y*z)^(1/2)+y^2+z^2)^2

(5)∴by(4): (x^5+y^2+z^2)*(y*z+y^2+z^2)≥(x^2+y^2+z^2)^2

(6)∴by(1,2,3,5): (x^2+y^2+z^2)/(x^5+y^2+z^2)≤(y*z+y^2+z^2)/(x^2+y^2+z^2)

(7)∴by(1,2,3,5): (x^2+y^2+z^2)/(x^2+y^2+z^5)≤(x*y+x^2+y^2)/(x^2+y^2+z^2)

(8)∴by(1,2,3,5): (x^2+y^2+z^2)/(y^5+x^2+z^2)≤(z*x+z^2+x^2)/(x^2+y^2+z^2)

(9)∴by(6,7,8):(x^2+y^2+z^2)/(x^5+y^2+z^2)+(x^2+y^2+z^2)/(x^2+y^2+z^5)+(x^2+y^2+z^2)/(y^5+x^2+z^2)≤2+(x*y+y*z+z*x)/(x^2+y^2+z^2)≤3

(10)∴by(9):((((x^5)-(x^2)))/(((x^5)+(y^2)+(z^2))))+((((y^5)-(y^2)))/(((x^2)+(y^5)+(z^2))))+((((z^5)-(z^2)))/(((x^2)+(y^2)+(z^5))))≥0]

Example 3 shows the human-like solving processes generated by MCTS.