Enrollment No.: E22CSEU 0827

Name: MADHAV GUPTA

Department/School: SCSE T

End-Semester Examination, Even Semester 2022-23

Course Code: EMAT102L

Course Name: Linear Algebra and ODEs

Maximum Time Duration: 2 hours

Maximum Marks: 35

GENERAL INSTRUCTIONS:

1. Do not write anything on the question paper except name, enrollment number and department/school.
2. Carrying mobile phone, smart watch and any other non-permissible materials in the examination hall is an act of UFM.
3. Each question of **SECTION-A** carries **2 marks**. Do any ten questions from SECTION-A.
4. Each question of **SECTION-B** carries **5 marks**. Do any three questions from SECTION-B.

SECTION-A

1. Find all the eigenvalues of the matrix $\begin{pmatrix} \sqrt{3} & 1 \\ -1 & \sqrt{3} \end{pmatrix}$.
2. If the trace of a 2×2 **singular** matrix A is 10. Then find the value of trace $(A^2 - 10A + 2I)$. Justify your answer.
3. Find the eigenspace corresponding to the eigenvalue $\lambda = 0$ for the matrix $\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$.
4. Find the orthogonal projection vector of $v = (1, 2, 3)$ onto the vector $u = (1, -1, 1)$.
5. Let $C[0, \frac{\pi}{2}]$ be the inner product space of all continuous functions defined on an interval $[0, \frac{\pi}{2}]$ and with the inner product of functions defined as $\langle f, g \rangle = \int_0^{\frac{\pi}{2}} f(x).g(x)\ dx$. Then find $\langle \sin x, \cos x \rangle$.

6. Solve $2xydx + x^2dy = 0$.

7. Solve $\frac{dy}{dx} + y = e^{-x}$.

8. Find the value of α for which the following differential equation is exact and then find its general solution

$$\cos x.\cos y\ dx + \alpha.\sin x.\sin y\ dy = 0.$$

9. Solve the initial value problem $(2x+1)dx + (3y^2+2)dy = 0$, $y(0) = 1$.

10. Find the general solution of diffrential equation

$$\frac{d^3y}{dx^3} - \frac{d^2y}{dx^2} + \frac{dy}{dx} - y = 0.$$

11. Find the Laplace transform of $t^2 + 2t + 3$.

12. Consider the matrix $A = \begin{pmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 6 \end{pmatrix}$. Find an eigenvector of A corresponding to eigenvalue $\lambda = 4$.

SECTION-B

13. Solve the boundary value problem

$$2\frac{d^3y}{dx^3} + \frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = 0, \quad y(0) = 1,\ y'(0) = 1,\ y''(0) = 0.$$

14. Solve $(4xy^2 + 6y)dx + (5x^2y + 8x)dy = 0$ by assuming an integrating factor of the form $x^\alpha y^\beta$.

15. Given $B = \{v_1, v_2, v_3\}$ where $v_1 = (1,1,1)$, $v_2 = (1,2,1)$ and $v_3 = (2,-1,-1)$, use the Gram-Schmidt procedure to find the corresponding orthonormal basis.

16. Consider the matrix $A = \begin{pmatrix} 1 & -3 & 3 \\ 3 & -5 & 3 \\ 6 & -6 & 4 \end{pmatrix}$. Check whether the matrix is diagonalizable if so, diagonalize it. Also find the matrix which will diagonalize it.

Good Luck.

"Learn from yesterday, live for today, hope for tomorrow." —Albert Einstein