GATE CSE NOTES

by

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EXTRAS

* DFA for $L = \{w \in \{a,b\}^* \mid n_a\%3 > n_b\%3\}$

→

	$n_a\%3$	$n_b\%3$	Cond^n satisfied?
POSSIBILITIES	0	0	x
	0	1	x
	0	2	x
	1	0	✓
	1	1	x
	1	2	x
	2	0	✓
	2	1	✓
	2	2	x

Make product DFA

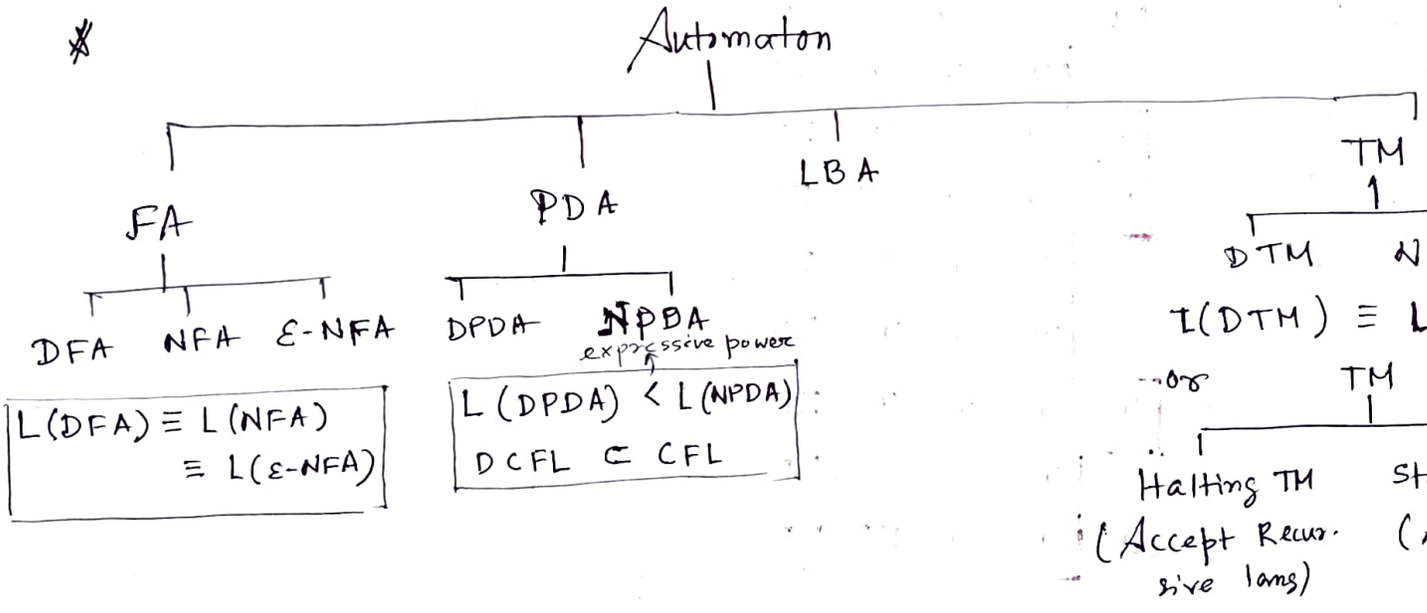
$$\left[\begin{array}{l} n_a\%3 = 0 \\ n_b\%3 = 0 \end{array}\right]$$

Final states are –

q_{10}, q_{20}, q_{21}.

* Formal language is the abstraction of generalized characteristic of programming languages.

*



HTM < TM

* Expressive power of automata

(FA < DPDA < NPDA or PDA < LBA < HTM < TM)

(FA ≡ TM with read only tape ≡ TM with unidirectional tape ≡ TM with finite tape ≡ PDA with finite stack)

(PDA ≡ FA with stack)

(TM ≡ PDA with additional stack ≡ FA with 2 stacks)

* ME 136 Dgm

TOC	(related to) →	Compiler
FA	RL	Lexical Analysis
PDA	CFG	Syntax "
LBA	CSL	Semantic "
TM	REL	Logic (Whole compiler)

TOC	Algo / Program
FA	Algo w/o using any memory
PDA	Algo using 1 stack (Palindrome)
LBA	Bounded memory
TM	Any algo

* (DFA = NFA) < DPDA < NPDA < LBA < (NTM = DTM)

* FA with 1 stack = PDA
FA with 2 stacks = TM.

* Automata with a queue ≈ TM

* TM with 3 states ≈ TM ≈ Multitape TM with 'stay' & at most 2 states. ≈ NPDA with 2 independent stacks.

* NDTM with only stack = PDA

TM with finite tape = FA

TM with part of tape only where ip is present = LBA (use to check CSL)

* Grammar - decidable / undecidable

Decision Problem	RL	DCFL	CFL	CSL	RECL	REL
Membership $w \in L$?	D	D	D	D	D	UD
Emptiness $L = \phi$?	D	D	D	UD	UD	UD
Finiteness	D	D	D	UD	UD	UD
Equivalence $L_1 = L_2$?	D	D	UD	UD	UD	UD
Intersⁿ empty $L_1 \cap L_2 = \phi$? (Disjoint)	D	UD[I]	UD	UD	UD	UD
Totality $L = \Sigma^*$?	D	D	UD	UD	UD	UD
Subset $L_1 \subseteq L_2$? (Containment)	D	UD[S]	UD	UD	UD	UD
Intersⁿ finiteness ($L_1 \cap L_2$ finite?)	D	UD[I]	UD	UD	UD	UD
Cofiniteness ($\bar{L}$ finite?)	D	D	UD	UD	UD	UD
Regularity (L = reg?)	D	D	UD	UD	UD	UD
Ambiguity	D	UD[A]	UD	UD	UD	UD
C L same type?	D	D	UD	D	D	UD
I $L_1 \cap L_2$ is same type?	D	UD[I]	UD	UD	UD	UD
Halting	D	D	D	D	D	UD.

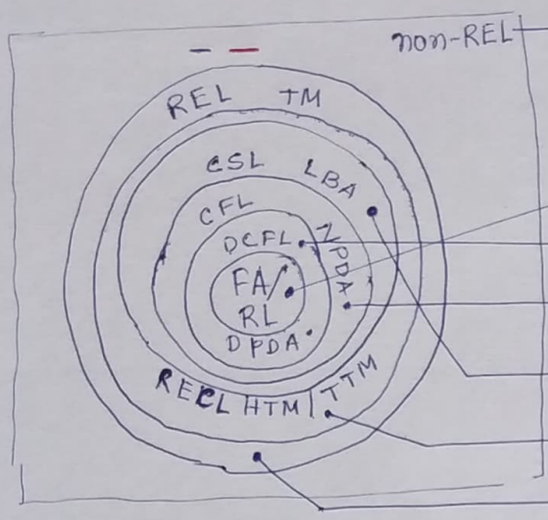
- Arbitrary CFGs G, G_1, G_2 & arbitrary Regex R,
undecidable – whether $L(R) \subseteq L(G)$, whether $L(G)$ is DCFL, whether $L(G) = L(R)$.
decidable – Whether $L(G) \subseteq L(R)$ [test $L(G) \cap \overline{L(R)} = \phi$ or not]

- Arbitrary DCFGs G, G_1, G_2 & arbitrary regex R, decidable – whether $L(G) = L(R)$, whether $L(G) \subseteq L(R)$, Whether $L(R) \subseteq L(G)$, whether $L(G)$ is CFL. (trivial)

* L accepted by LBA

$\{a^n b^n c^n \mid n \geq 1\}$, $\{a^{n!} \mid n \geq 0\}$, $\{a^n \mid n$ prime$\}$, $\{a^n, n = m^2, m \geq 1\}$, $\{a^n \mid n$ not prime$\}$, $\{ww \mid w \in (a,b)^+\}$, $\{w^n \mid w \in (a,b)^+, n \geq 1\}$, $\{www^r \mid w \in (a,b)^+\}$

Closure property



(undecidable) (no TM exists)
eg. Set of real numbers, $2^{\mathbb{N}}$ (power set of $\mathbb{N}$), 2^{Σ^*}.

$a^n b^m$

$a^n b^n$

ww^r

$a^n b^n c^n$.

Turing decidable (Decidable)

Turing recognizable (undecidable)

* Closure properties of formal lang.

	RL	DCFL	CFL	CSL	RECL (Turing Decidable)	REL (Turing recognizable / Enumerable / semi)
✓ Union (finite)	✓	✗	✓	✓	✓	✓
✓ Intersection (finite)	✓	✗	✗	✓	✓	✓
✓ Set difference (finite)	✓	✗	✗	✓	✓	✗
✓ Complementation	✓	✓	✗	✓	✓	✗
✓ Intersection with a RL	✓	✓	✓	✓	✓	✓
✓ Concatenation (finite)	✓	✗	✓	✓	✓	✓
Kleene closure	✓	✗	✓	✓	✓	✓
Positive closure	✓	✗	✓	✓	✓	✓
✓ Reversal	✓	✗	✓	✓	✓	✓
Homomorphism	✓	✗	✓	✗	✗	✓
ϵ-free homomorphism	✓	✗	✓	✓	✓	✓
Inverse homomorphism	✓	✓	✓	✓	✓	✓
Substitution	✓	✗	✓	✗	✗	✓
ϵ-free substitution	✓	✗	✓	✓	✓	✓
Right quotient with a RL	✓	✓	✓	✗		✓
Left quotient with a RL	✓	✓	✓	✗		✓
✓ Union with RL	✓	✓	✓			
Left difference with RL		✓	✗			
Right difference with RL		✓	✓			
✓ Infinite union	✗	✗	✗	✗	✗	✗
✓ Infinite intersection	✗	✗	✗	✓	✓	✓
Infinite difference	✗					
Infinite concatenation	✗					
✓ Prefix	✓	✓	✓			
Suffix						
Cycle	✓		✓			
Min		✓				
Max		✓				
XOR / Symmetric Difference	✓		✗			
NAND			✗			
NOR	✓		✗			
COR	✓					

	RL	DCFL	CFL	CSL	RECL	REL
Square root of L, $\sqrt{L}$	✓					
Square of L	✗					
Shuffle (L_1, L_2)	✓					
One-third of L	✓					
Half of L	✓					
Subsequence	✓					
Subword	✓					
✓ Subset	✗		✗			
✓ Superset			✗			

• Decᵈ

	RL	DCFL	CFL	RECL	REL
Membership	D	D	D	D	UD (Semidec) ~~but~~ und.
Halting	D	D	D	D.	UD (Semidec.) ~~but decl.~~
Emptiness	D	D	D	UD (non-re)	UD (non-re)
Finiteness	D	D	D	UD (non-re)	UD (non-re)
Totality	D	D	UD (non-re)	UD (non-re)	UD (non-re)
Equivalence	D	D	UD (non-re)	UD (non-re)	UD (non-re)
Disjoint	D	UD (non-re)	UD (non-re)	UD (non-re)	UD (non-re)
Set containment	D	UD (non-re)	UD (non-re)	UD (non-re)	UD (non-re)
Ambiguity	D	UD	UD	UD	UD

Marvelous humble employee failed to equate dogs & cats. & ants.

Regularity	D	D	UD ? ~~Non-re~~	UD ? ~~Non-re~~	UD non-re

Non-membership
Decidable upto RECL.
For REL, undecidable (non-RE).

Non-emptiness
$\{\langle M \rangle \mid L(M) \neq \phi\}$
For TM, it is semidecidable.

Non-equivalence
non-re for TM
SD for PDA, HTM.

Decision problem	RL	DCFL	CFL	RECL	REL
Membership	D	D	D	D	UD RE, not REC
Halting	D	D	D	D	UD RE, not REC
Emptiness	D	D	D	UD non-RE	UD non-RE
Finiteness	D	D	D	UD non RE (nr)	UD (nr)
Totality	D	D	UD	UD nr	UD nr
Equivalence	D	D	UD nr	UD nr	UD nr
Disjoint	D	UD nr	UD nr	UD nr	UD nr
Set containment	D	UD nr	UD nr	UD nr	UD nr
Ambiguity	D	UD nr	UD nr	UD nr	UD nr
Regularity	D	D	UD	UD	UD non-re

- Non-membership : Decidable upto RECL for REL, non-RE.
- Non emptiness : $\{\langle M\rangle \mid L(M) \neq \phi\}$ for RE, semidecidable
- Non-equivalence : for CFL, RECL semidecidable for REL, non-RE.

Marvelous humble employee failed to equate dogs, cats, ants, rats.