GATE CSE NOTES

by

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Stirling no. of 2nd kind.

$$S(m,n) = S(m-1, n-1) + n\,S(m-1, n)$$

→ # ways to partition set of m elems into n subsets

m\n	0	1	2	3
0	1			
1	0	1		
2	0	1	1	
3	0	1	3	1

S(3, 2)

a, b, c

3 $\begin{cases} a, & bc \\ b, & ac \\ c, & ab \end{cases}$

$$\text{Bell}(n) = \sum_{k=1}^{n} S(n,k)$$

→ # partitions possible for a set into non empty subsets

n	
1	1
2	1 2
3	2 3 5
4	5 7 10 15
	⋮

Bell(n)

Bell(3)

a, b, c

abc   a bc   b ac   c ab   a b c

→ 5.

Partition number (unique) $p_k(n)$ n into k parts

$$p(n,k) \text{ or } p_k(n) = p(n-1,k-1) + p(n-k,k)$$

m\k	1	2	3	4	5	6
1	1					
2	1	1				
3	1	1	1			
4	1	2	1	1		
5	1	2	2	1	1	
6	1	3	3	2	1	1

→ 6 = 1+5, 2+4, 3+3

$$p(6,2) = p(5,1) + p(4,2)$$
$$= 1 + 2 = 3$$

- **Derangements.** (D_n) $\quad D_n/n! = \sum_{0}^{n} \frac{(-1)^k}{k!}$

$$D_n = (n-1)(D_{n-1} + D_{n-2})$$

$D_0 = 1$, $D_0 = 0$. $\Rightarrow$ 1, 0, 1, 2, 9, 44, ...

- **Lucas Number** (L_n) $\quad L_0 = 2$

$$L_n = L_{n-1} + L_{n-2} \qquad L_1 = 1$$

↳ Counts ~~the~~ # ways to tile a circular strip of length n using squares & dominoes.

- **Partition number of integer.** $p(n)$

eg. $p(4) = 5$

4, 31, 22, 1111, 112

Cayley's Formula

n^{n-2} trees

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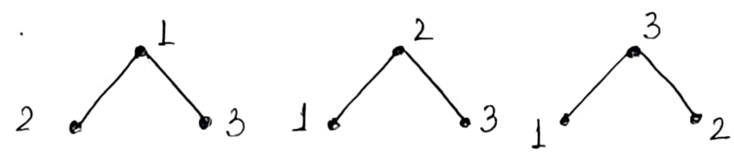
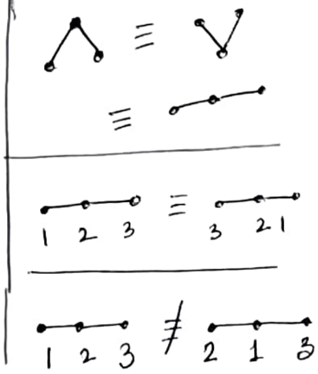
$n = 3.$



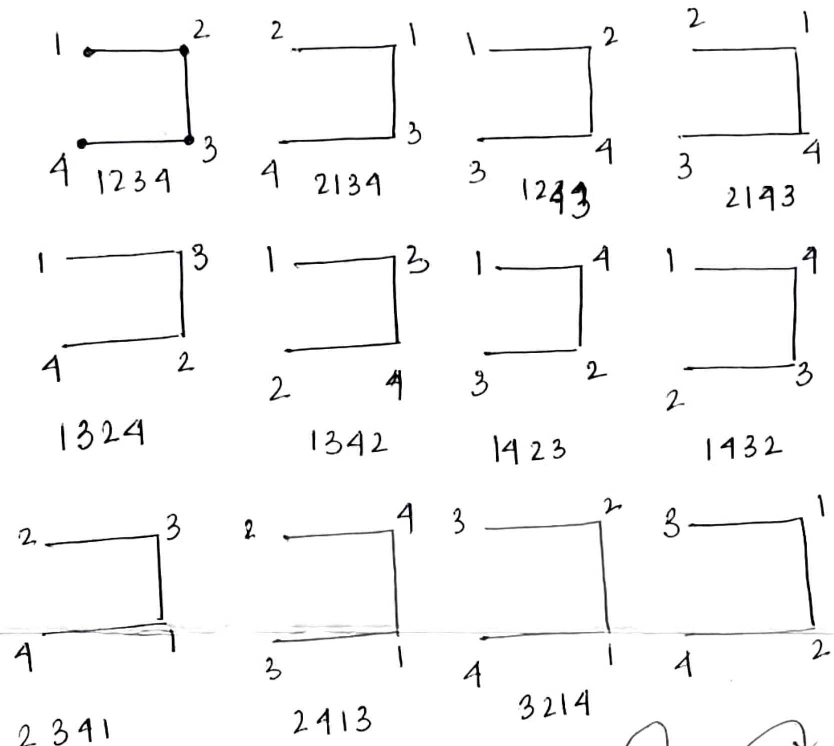
non-isomorphic trees = 1



$3^{3-2} = 3$

123
132
213
231
312
321

$n = 4.$



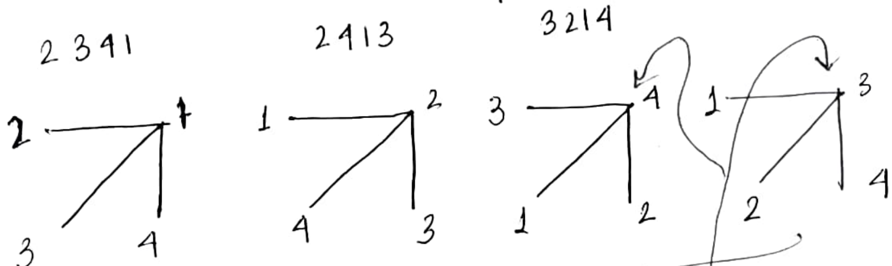
$^4C_3$

4

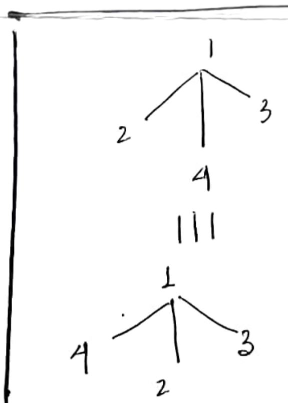
$^4P_3$

$\frac{4!}{1!}$

graph, not ds.



Choosing the centre of star.

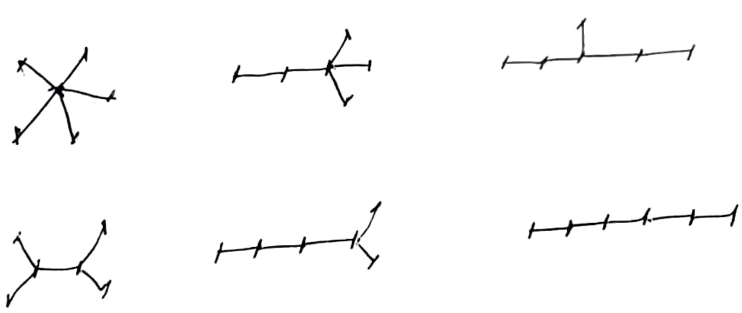


6^{6-2}

$= 6^4$

non-isomorphic trees = 2.

$n = 6$ non-isomorphic trees' structures -



Considered same in the calculation n^{n-2}.

No formula for # non-isomorphic distinct structures.

Differents roots & different left, right child are not considered (as it is not DS, it's a graph!) to count n^{n-2}