Econometrics III
Assignment Part II

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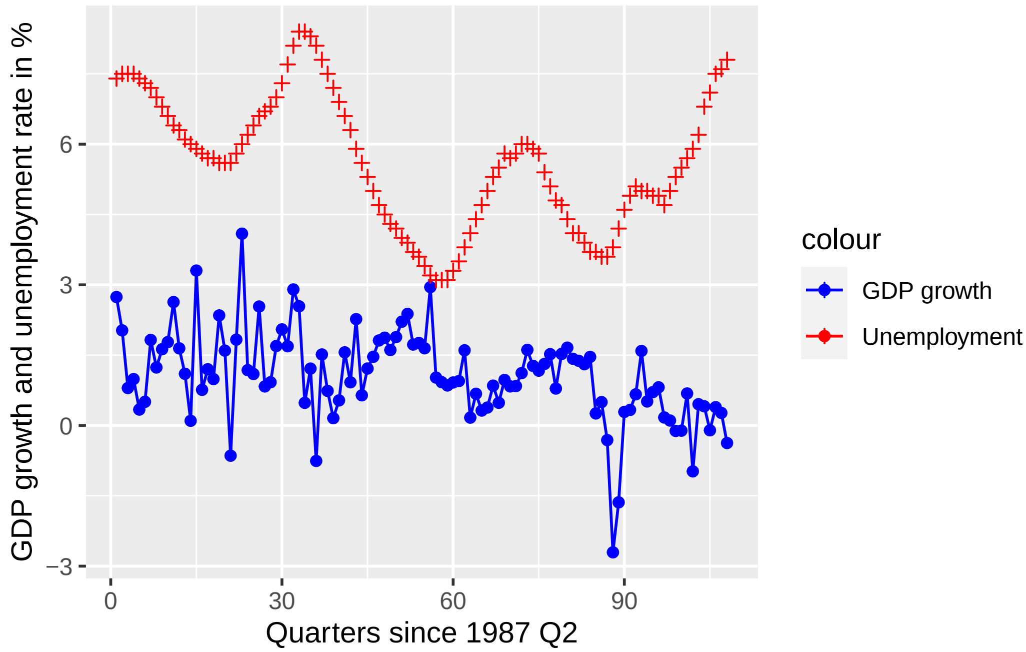
Contents

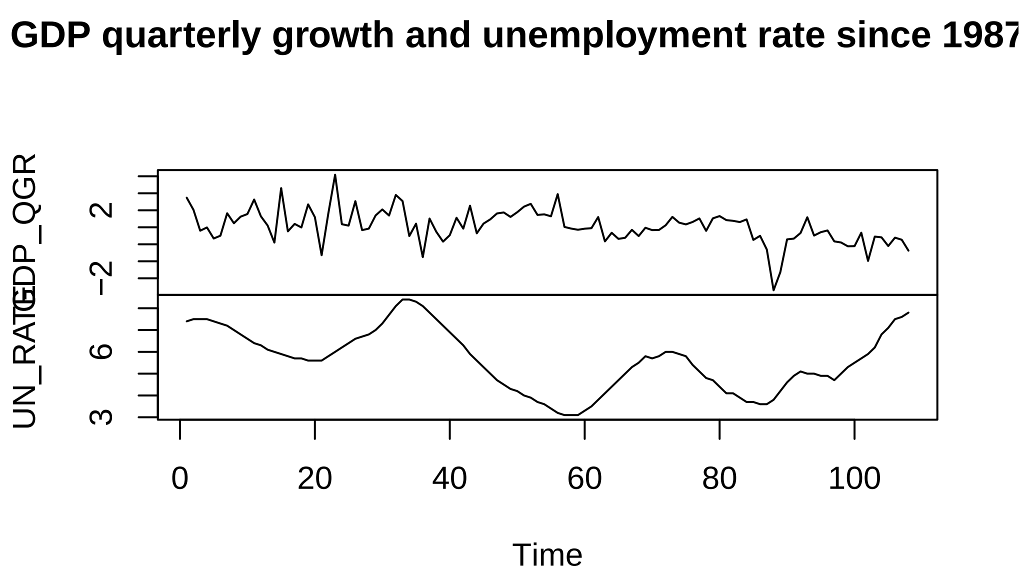
```r
# load packages
if(!require(pacman)){install.packages("pacman")}

p_load(devtools,tidyverse,dplyr,ggplot2,latex2exp,stargazer, fixest,
↪ modelsummary, knitr, readr, tseries, lmtest, forecast, dynlm, vars,
↪ gridExtra)

dfAssign_p2 <- as.data.frame(read_csv("data/data_assign_p2.csv"))

# Encode quarters
dfAssign_p2 <-
↪ cbind(dfAssign_p2,c(seq(1,nrow(dfAssign_p2),length.out=nrow(dfAssign_p2))))
colnames(dfAssign_p2) <- c("obs", "GDP_QGR", "UN_RATE", "ind")

dfData <- dfAssign_p2[,2:3]
```

1 Question 1

```r
ggplot(dfAssign_p2) +
  geom_point(aes(x = ind, y = GDP_QGR, color="GDP growth")) +
  geom_line(aes(x = ind, y = GDP_QGR, color="GDP growth")) +
  geom_point(data=dfAssign_p2, aes(x=ind, y=UN_RATE,
  ↪ color="Unemployment"), shape=3) +
  ylab("GDP growth and unemployment rate in %") +
  xlab("Quarters since 1987 Q2") +
  scale_color_manual(values = c(
    "Unemployment" = "red",
    "GDP growth" = "blue"))
```




```
plot.ts(dfAssign_p2[,2:3], plot.type = c("multiple"), main="GDP
↪ quarterly growth and unemployment rate since 1987 Q2")
```



The Dutch quarterly growth rates seems to be fluctuating around 1 % every quarter until it experiences a sharp drop to almost -3% at Quarter 1 of 2009. This growth rate reflect the influence of the economic recession. Afterwards, it rises again and fluctuate around 0%, indicating that the economy have recovered, but not back to the full swing before the recession.

Regarding the unemployment rate, it resembles a wave pattern. As of now, it is unclear how to relate this pattern to the GDP growth rate or the eco nomic recession. Thus, we proceed to model it in the following steps.

```
# Creating the autoregression of GDP growth
ar4_gdp <- arima(dfAssign_p2$GDP_QGR, order=c(4,0,0))
coeftest(ar4_gdp) #Remove the least significant lag: the second lag
```

```
z test of coefficients:

          Estimate Std. Error z value  Pr(>|z|)
ar1       0.332074   0.097217  3.4158  0.000636 ***
ar2       0.056504   0.100260  0.5636  0.573047
ar3       0.207863   0.100008  2.0785  0.037667 *
ar4       0.045652   0.097498  0.4682  0.639614
intercept 1.064601   0.220502  4.8281 1.379e-06 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
ar1_3_4_gdp <- arima(dfAssign_p2$GDP_QGR, order=c(4,0,0), fixed =
 ↪  c(NA,0,NA,NA,NA))
coeftest(ar1_3_4_gdp) #Remove the least significant lag: the forth lag
```

```
z test of coefficients:

          Estimate Std. Error z value  Pr(>|z|)
ar1       0.349605   0.092313  3.7872 0.0001524 ***
ar3       0.224838   0.095511  2.3541 0.0185694 *
ar4       0.049150   0.097448  0.5044 0.6139976
intercept 1.063916   0.210443  5.0556  4.29e-07 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
ar1_3_gdp <- arima(dfAssign_p2$GDP_QGR, order=c(3,0,0), fixed =
 ↪  c(NA,0,NA,NA))
coeftest(ar1_3_gdp) #Stop here, 1st and 3rd lag coefficients are
 ↪  significant
```

```
z test of coefficients:

           Estimate Std. Error z value  Pr(>|z|)
ar1        0.361536   0.089338  4.0468 5.192e-05 ***
ar3        0.241553   0.089670  2.6938  0.007064 **
intercept 1.065869   0.200668  5.3116 1.087e-07 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

The final AR model is one with the first and third lag, we call it the AR(1-3) model.

$$X_t = 1.066 + 0.362 * X_{t-1} + 0.242 * X_{t-3} + u_t \quad (1)$$

Looking at the estimated coefficients, we can say that the quarterly GDP growth rate is influenced by the growth of the previous quarter and the third previous quarters.

```
dfAssign_p2$GDP_QGR <- ts(dfAssign_p2$GDP_QGR, start = c(1987, 2),
 ↪  frequency = 4)
dfAssign_p2$UN_RATE <- ts(dfAssign_p2$UN_RATE, start = c(1987, 2),
 ↪  frequency = 4)

adl_4_4 <- dynlm(UN_RATE ~ L(UN_RATE, 1) + L(UN_RATE, 2) + L(UN_RATE, 3)
 ↪  + L(UN_RATE, 4) + GDP_QGR + L(GDP_QGR, 1) + L(GDP_QGR, 2) +
 ↪  L(GDP_QGR, 3) + L(GDP_QGR, 4), data = dfAssign_p2)
coeftest(adl_4_4)
```

```
t test of coefficients:

                Estimate Std. Error t value  Pr(>|t|)
(Intercept)    0.1664281  0.0573297  2.9030  0.004605 **
L(UN_RATE, 1)  1.6044535  0.1030613 15.5680 < 2.2e-16 ***
L(UN_RATE, 2) -0.3044370  0.1908063 -1.5955  0.113951
L(UN_RATE, 3) -0.4354095  0.1942187 -2.2419  0.027323 *
L(UN_RATE, 4)  0.1111869  0.1055289  1.0536  0.294760
```

```
GDP_QGR       -0.0138627  0.0147644 -0.9389  0.350174
L(GDP_QGR, 1) -0.0201461  0.0151944 -1.3259  0.188088
L(GDP_QGR, 2) -0.0084318  0.0150829 -0.5590  0.577470
L(GDP_QGR, 3)  0.0205418  0.0153721  1.3363  0.184678
L(GDP_QGR, 4) -0.0069244  0.0146330 -0.4732  0.637162
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```r
adl_4_3 <- dynlm(UN_RATE ~ L(UN_RATE, 1) + L(UN_RATE, 2) + L(UN_RATE, 3)
 ↪  + L(UN_RATE, 4) + GDP_QGR + L(GDP_QGR, 1) + L(GDP_QGR, 2) +
 ↪  L(GDP_QGR, 3), data = dfAssign_p2)
coeftest(adl_4_3)
```

```
t test of coefficients:

                Estimate Std. Error t value  Pr(>|t|)    
(Intercept)    0.1644335  0.0569405  2.8878  0.004804 ** 
L(UN_RATE, 1)  1.5984568  0.1018606 15.6926 < 2.2e-16 ***
L(UN_RATE, 2) -0.2929561  0.1884828 -1.5543  0.123442    
L(UN_RATE, 3) -0.4361600  0.1934173 -2.2550  0.026428 *  
L(UN_RATE, 4)  0.1061712  0.1045656  1.0154  0.312515    
GDP_QGR       -0.0139329  0.0147032 -0.9476  0.345733    
L(GDP_QGR, 1) -0.0215160  0.0148550 -1.4484  0.150797    
L(GDP_QGR, 2) -0.0087843  0.0150028 -0.5855  0.559592    
L(GDP_QGR, 3)  0.0185162  0.0147037  1.2593  0.211012    
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```r
# Remove the 2nd exogenous lag.
adl_4_3 <- dynlm(UN_RATE ~ L(UN_RATE, 1) + L(UN_RATE, 2) + L(UN_RATE, 3)
 ↪  + L(UN_RATE, 4) + GDP_QGR + L(GDP_QGR, 1) + L(GDP_QGR, 3), data =
 ↪  dfAssign_p2)
coeftest(adl_4_3)
```

```
t test of coefficients:

               Estimate Std. Error t value  Pr(>|t|)    
(Intercept)    0.160028   0.056248  2.8451  0.005428 ** 
L(UN_RATE, 1)  1.607262   0.100399 16.0087 < 2.2e-16 ***
```

```
L(UN_RATE, 2) -0.301943   0.187213 -1.6128  0.110063
L(UN_RATE, 3) -0.439869   0.192651 -2.2832  0.024621 *
L(UN_RATE, 4)  0.110183   0.103983  1.0596  0.291973
GDP_QGR       -0.014597   0.014609 -0.9992  0.320223
L(GDP_QGR, 1) -0.023896   0.014239 -1.6782  0.096564 .
L(GDP_QGR, 3)  0.016282   0.014151  1.1506  0.252767
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```r
# Remove 2nd exogenous lag, and the exogenous variable with no lag.
adl_4_3 <- dynlm(UN_RATE ~ L(UN_RATE, 1) + L(UN_RATE, 2) + L(UN_RATE, 3)
↪  + L(UN_RATE, 4) + L(GDP_QGR, 1) + L(GDP_QGR, 3), data = dfAssign_p2)
coeftest(adl_4_3)
```

```
t test of coefficients:

               Estimate Std. Error t value  Pr(>|t|)
(Intercept)    0.162108   0.056209  2.8840  0.004837 **
L(UN_RATE, 1)  1.599186   0.100072 15.9803 < 2.2e-16 ***
L(UN_RATE, 2) -0.301586   0.187211 -1.6109  0.110441
L(UN_RATE, 3) -0.412020   0.190622 -2.1614  0.033122 *
L(UN_RATE, 4)  0.088545   0.101702  0.8706  0.386104
L(GDP_QGR, 1) -0.028710   0.013399 -2.1427  0.034638 *
L(GDP_QGR, 3)  0.012749   0.013702  0.9304  0.354473
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```r
# Looking at the model above, we remove the 4th lag of the unemployment
↪  rate (autolag).
adl_3_3 <- dynlm(UN_RATE ~ L(UN_RATE, 1) + L(UN_RATE, 2) + L(UN_RATE, 3)
↪  + L(GDP_QGR, 1) + L(GDP_QGR, 3), data = dfAssign_p2)
coeftest(adl_3_3)
```

```
t test of coefficients:

               Estimate Std. Error t value  Pr(>|t|)
(Intercept)    0.176507   0.053005  3.3300  0.001221 **
L(UN_RATE, 1)  1.578350   0.096533 16.3504 < 2.2e-16 ***
L(UN_RATE, 2) -0.337712   0.181214 -1.8636  0.065340 .
```

```
L(UN_RATE, 3) -0.269030   0.096080 -2.8001  0.006144 **
L(GDP_QGR, 1) -0.028172   0.013258 -2.1249  0.036085 *
L(GDP_QGR, 3)  0.012055   0.013346  0.9032  0.368596
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```r
# Remove the 3rd exogenous lag.
adl_3_1 <- dynlm(UN_RATE ~ L(UN_RATE, 1) + L(UN_RATE, 2) + L(UN_RATE, 3)
 ↪   + L(GDP_QGR, 1), data = dfAssign_p2)
coeftest(adl_3_1)
```

```
t test of coefficients:

               Estimate Std. Error t value  Pr(>|t|)
(Intercept)    0.184821   0.052152  3.5439 0.0006011 ***
L(UN_RATE, 1)  1.572280   0.096210 16.3422 < 2.2e-16 ***
L(UN_RATE, 2) -0.339772   0.181033 -1.8769 0.0634533 .
L(UN_RATE, 3) -0.260515   0.095528 -2.7271 0.0075469 **
L(GDP_QGR, 1) -0.025568   0.012929 -1.9776 0.0507287 .
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```r
# Remove the second autolag.
adl_3_1 <- dynlm(UN_RATE ~ L(UN_RATE, 1) + L(UN_RATE, 3) + L(GDP_QGR,
 ↪   1), data = dfAssign_p2)
coeftest(adl_3_1)
```

```
t test of coefficients:

               Estimate Std. Error  t value  Pr(>|t|)
(Intercept)    0.207594   0.051350   4.0427 0.0001033 ***
L(UN_RATE, 1)  1.399745   0.028735  48.7114 < 2.2e-16 ***
L(UN_RATE, 3) -0.431554   0.029004 -14.8789 < 2.2e-16 ***
L(GDP_QGR, 1) -0.027101   0.013063  -2.0746 0.0405617 *
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

The final ADL model is one with lag 1 and lag 3 of Y_t and lag 1 of the exogenous variable X_t.

$$\begin{aligned} Y_t &= 0.208 + 1.400 * Y_{t-1} - 0.432 * Y_{t-3} - 0.027 * X_{t-1} + \epsilon_t \\ X_t &= 1.066 + 0.362 * X_{t-1} + 0.242 * X_{t-3} + u_t \end{aligned} \tag{2}$$

Looking at the coefficients of the final model, we can see that the quarterly unemployment rate is related to the unemployment rates of the previous and the 3rd pervious quarter, moreover, it is related to the GDP growth rate of the previous quarter.

Moreover, it seems that if GDP growth rate and the unemployment rate of the previous quarter increase, unemployment decreases; and if the unemployment rate of the third previous quarter increases, the current quarter's unemployment rate decreases.

2 Question 2

The short-run multiplier is the impact of a unit increase in X_t on Y_t, which is the coefficient β_0 of X_t. Since X_t is not significant in this model, the short run multiplier is $\beta_0 = 0$. Interpretation: with every percent increase in the contemporaneous GDP growth rate, the quarterly unemployment rate of the same period is not affected.

The 2-step-ahead multiplier is the impact of a unit increase in X_t on Y_{t+h}, which is:

$$\phi^2\beta_0 + \phi\beta_1 = 0 + 1.400 * (-0.027) = -0.0378 \tag{3}$$

Interpretation: with every percent increase in the current GDP growth rate, the quarterly unemployment rate of the second following quarter decreases by 0.0378%.

The long-run multiplier is the total impact of a permanent unit increase in X_t on Y_t. Let $\phi(L) = 1 - \phi_1 L - ... - \phi_p L^p$ and $\beta(L) = \beta_0 + \beta_1 L + ... + \beta_q L^q$, then, the long-run multiplier is given by:

$$\begin{aligned} \Delta\bar{Y} &= \frac{\beta(1)}{\phi(1)} = \frac{\beta_1}{1 - \phi_1 - \phi_3} \\ &= \frac{-0.027}{1 - 1.400 + 0.432} = -0.84375 \end{aligned} \tag{4}$$

Interpretation: with every permanent percent increase in the current GDP growth rate, the quarterly un-employment rate decrease by 0.84375% on average.

3 Question 3

Statement: “An increase in the GDP growth rate causes a reduction in the unemployment rate.”

Looking at the short-run, 2-step ahead, and the long-run multipliers above, we still cannot confirm this statement. This is because the multipliers above only indicates association, not causation.

The multipliers above measure relationship between the dependent variable and the (lagged values) of the independent variables. It tells us the impact of a change in the independent variable on the dependent variable, holding all other factors constant. However, this change can be caused by 3 main reasons:

1. Causation: the independent variable causes the dependent variable.
2. Reserved causality: the dependent variable causes the independent variable.
3. Spurious association: it can occur when two variables are both influenced by a common cause or when one variable influences a third variable, which in turn influences the second variable.

To establish causation, we need to rely on other methods such as randomized controlled trials or natural experiments, which can help us isolate the effect of the independent variable on the dependent variable while controlling for other factors that may influence the outcome.

Moreover, the ADL model is a time series method that can only suggest a relationship between variables based on observed data. It cannot establish causality on its own. Therefore, we need to exercise caution in interpreting the results of the ADL model and use other methods to establish causation.

4 Question 4

Using the ADL model, we can calculate the predicted value of the unemployment rate of 2014Q2

$$
\begin{aligned}
Y_{2014Q2} &= 0.208 + 1.400 * Y_{2014Q1} - 0.432 * Y_{2013Q3} - 0.027 * X_{2014Q1} \\
&= 0.208 + 1.400 * 7.8 - 0.432 * 7.5 - 0.027 * -0.374 \\
&= 7.898
\end{aligned} \tag{5}
$$

Then, we can get the standard error of our model:

```
# Get the residuals
residuals = resid(adl_3_1)

# Estimate the standard errors
y_hat = 7.898
n = 108
k = 4
SE = (sum(residuals^2)/(n-k))^(1/2)

# Assuming innovations are IID Gaussian
# The probability of observing Unemployment rate below 7.8%:
cat("The probability of observing Unemployment rate below 7.8%
 ↪  is:",pnorm(7.8, mean = 7.898, sd = SE),"\n")
```

```
The probability of observing Unemployment rate below 7.8% is: 0.2099289
```

```
# The probability of observing Unemployment rate above 7.8%:
cat("The probability of observing Unemployment rate below 7.8%
 ↪  is:",1-pnorm(7.8, mean = 7.898, sd = SE))
```

```
The probability of observing Unemployment rate below 7.8% is: 0.7900711
```

We should not trust the IID Gaussian assumption because in many cases, the assumption of IID Gaussian errors may not hold. This could be due to several reasons: errors have a time-varying variance, not normally distributed, or exhibit serial correlation (i.e., the errors are correlated over time), then assuming IID Gaussian errors could result in biased parameter estimates and incorrect inference.

Therefore, it's important to assess the validity of the IID Gaussian assumption in the context of the specific data and research question at hand. This can be done through various diagnostic tests, such as examining the autocorrelation function of the residuals, testing for heteroscedasticity, and checking the normality of the residuals. If these tests suggest that the assumption of IID Gaussian errors is not appropriate, then alternative models or estimation techniques may be necessary.

5 Question 5

For this question, we can use package function *predict()* to make predictions for our AR model:

```r
pred8 <- as.vector(predict(ar1_3_gdp, 8)[["pred"]])
```

Unfortunately due to a bug, the *dynlm* dynamic linear model estimation package is not compatible with either *predict()* or *forecast()* functions. Hence we need to implement our manual forecasting function, which uses the predictions from the AR model for GDP growth (the exogenous variable in the ADL model):

```r
#' Predictor function for ADL model
#'
#' @param model The dynlm estimation object for the model to be used for
↪  predictions
#' @param dfData The dataframe used for training the model, with the
↪  exogenous variable assumed to be column 1 (and Y as column 2)
#' @param iForward integer, the number of periods forwards we wish to
↪  estimate
#' @param vExgen The vector of predicted exogenous variable values with
↪  the AR model, assumed to be the same length as the iForward
#'
#' @return vPred The vector of predictions
#'
fPredADL <- function(model, dfData, iForward, vExgen){
  cat("The model estimation equation is:",
   ↪  as.character(model[["call"]][["formula"]]))
  # Gathering regressors in the model, their coefficients and lags
  regressors <- model[["call"]][["formula"]][[3]]
  coefficients <- as.vector(model[["coefficients"]][-1])
  dfPredictors <- data.frame(coefficients=c(0,0,0), varnames=c("none",
↪  "none", "none"), lags=c(0,0,0))
  k <- length(coefficients)
  while(k > 1){
    dfPredictors[k,1] <- coefficients[k]
    dfPredictors[k,2] <- as.character(regressors[[3]][[2]])
    dfPredictors[k,3] <- regressors[[3]][[3]]
    regressors <- regressors[[2]]
    k <- k-1
  }
  dfPredictors[1,1] <- coefficients[1]
  dfPredictors[1,2] <- as.character(regressors[[2]])
  dfPredictors[1,3] <- regressors[[3]]

  vPred <- c(rep(0,iForward))
```

```r
  t <- 1
  while(t <= iForward){
    prediction <- as.vector(model[["coefficients"]][1])
    for(var in 1:nrow(dfPredictors)){
      varname <- as.character(dfPredictors[var,2])
      index = nrow(dfData)-dfPredictors[var,3]
      prediction <- prediction + dfPredictors[var,1] *
    dfData[index,varname]
    }
    dfData <- rbind(dfData, c(vExgen[t], prediction))
    vPred[t] <- prediction
    t = t+1
  }
  return(vPred)
}
```

```r
predicted_result <- fPredADL(adl_3_1, dfData, 8, pred8)
```

```
The model estimation equation is: ~ UN_RATE L(UN_RATE, 1) + L(UN_RATE, 3) + L(GDP_QGR, 1)
```

```r
predicted_result
```

```
[1] 7.774367 7.899095 7.799557 7.881228 7.754777 7.810972 7.673781 7.716494
```

```r
predlbl <- c("2014Q2", "2014Q3", "2014Q4", "2015Q1", "2015Q2", "2015Q3",
    "2015Q4", "2016Q1")
forecasted <- cbind(predlbl, data.frame(GDP=pred8,
    UN_RATE=predicted_result))

kable(forecasted, caption="The predicted quarterly GDP growth rates
    using AR(1-3) and unemployment rates using ADL(1-3,1)", col.names =
    c("Quarter", "Predicted growth %", "Predicted unemployment %"),
    digits = 3, row.names = F)
```

Table 1: The predicted quarterly GDP growth rates using AR(1-3) and unemployment rates using ADL(1-3,1)

Quarter	Predicted growth %	Predicted unemployment %
2014Q2	0.382	7.774
2014Q3	0.626	7.899
2014Q4	0.559	7.800
2015Q1	0.717	7.881
2015Q2	0.834	7.755
2015Q3	0.859	7.811
2015Q4	0.907	7.674
2016Q1	0.952	7.716

The obtained estimates indicate that the GDP growth rates increases steadily. On the other hand, the unemployment rates experiences fluctuations and there is no clear increasing trend.

6 Question 6

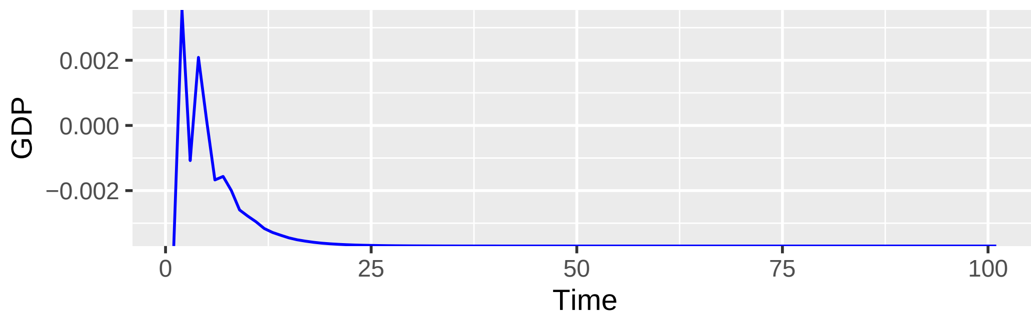
```
# IRF function
fIRF = function(n_periods, init_y, init_x, epsilon, beta_1, beta_3,
↪  phi_1, phi_3, theta_1){
  # Create vectors to store the results
  y_values <- rep(0, n_periods + 1)
  x_values <- rep(0, n_periods + 1)
  y_derivs <- rep(0, n_periods + 1)
  x_derivs <- rep(0, n_periods + 1)

  # calculate derivatives for x from t=s to t=s+2
  x_derivs[1] <- 1
  x_derivs[2] <- beta_1
  x_derivs[3] <- beta_1^2
  # calculate derivatives for y from t=s to t=s+2
  y_derivs[1] <- 0
  y_derivs[2] <- theta_1
  y_derivs[3] <- phi_1*theta_1 + theta_1*beta_1

  # calculate the rest of x and y_derives
```

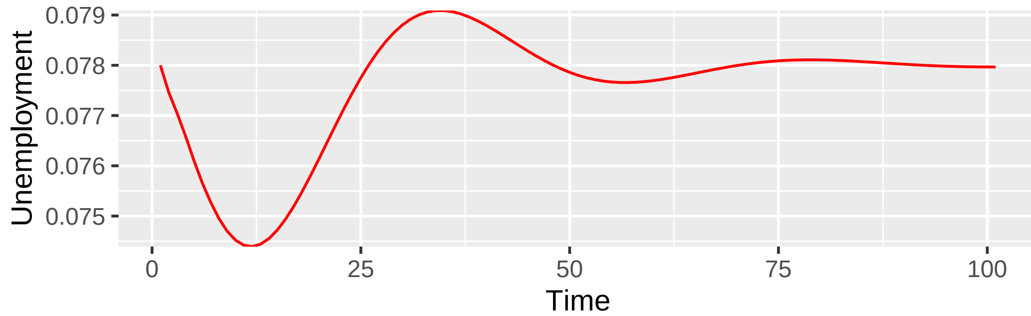
```
  for (i in 4:length(x_derivs)) {
    x_derivs[i] = beta_1*x_derivs[i-1] + beta_3*x_derivs[i-3]
    y_derivs[i] = phi_1*y_derivs[i-1] + phi_3*y_derivs[i-3] +
 ↪  theta_1*x_derivs[i-1]
  }

  # Calculate X and Y values
  y_values[1] <- init_y
  x_values[1] <- init_x
  for (i in 2:(n_periods+1)){
    x_values[i] = init_x+ x_derivs[i]*epsilon
    y_values[i] = init_y+ y_derivs[i]*epsilon
  }
  return(list(x_values=x_values,y_values=y_values))
}

# Function to plot IRF values
plotIRF = function(n_periods, x_values, y_values){
  # Create two time series vectors
  t <- 1:(n_periods+1)
  y1 <- x_values
  y2 <- y_values

  df1 <- data.frame(t, y1)
  df2 <- data.frame(t, y2)

  # Create two plots, one for each time series vector, with different
  ↪  y-limits
  p1 <- ggplot(df1, aes(x = t, y = y1)) +
    geom_line(color = "blue") +
    scale_y_continuous(limits = c(min(x_values), max(x_values)), expand
    ↪  = c(0, 0)) +
    labs(x = "Time", y = "GDP")

  p2 <- ggplot(df2, aes(x = t, y = y2)) +
    geom_line(color = "red") +
    scale_y_continuous(limits = c(min(y_values), max(y_values)), expand
    ↪  = c(0, 0)) +
    labs(x = "Time", y = "Unemployment")

  # Combine the two plots into one using the gridExtra package
```

```
  grid.arrange(p1, p2, nrow = 2)
}
```

6.1 Question 6a

```
n_periods=100
init_y = 0.078
init_x = -0.0037

# Coeficients of AR model
beta_1 = 0.362
beta_3 = 0.242

# Coefficients of ADL model
phi_1 = 1.400
phi_3 = -0.432
theta_1 = -0.027

# For the positive scenarios
epsilon = 0.02
lIRF= fIRF(n_periods, init_y, init_x, epsilon, beta_1, beta_3, phi_1,
↪   phi_3, theta_1)
x_values=lIRF$x_values
y_values=lIRF$y_values
plotIRF(n_periods, x_values, y_values)
```





In this scenario, the GDP growth rate got a boost of positive 2% at time 0, and then the impact of the shock quickly decreases, and GDP growth decreases close to the original rate at around t=12, then the effect completely dies down at around t=25.

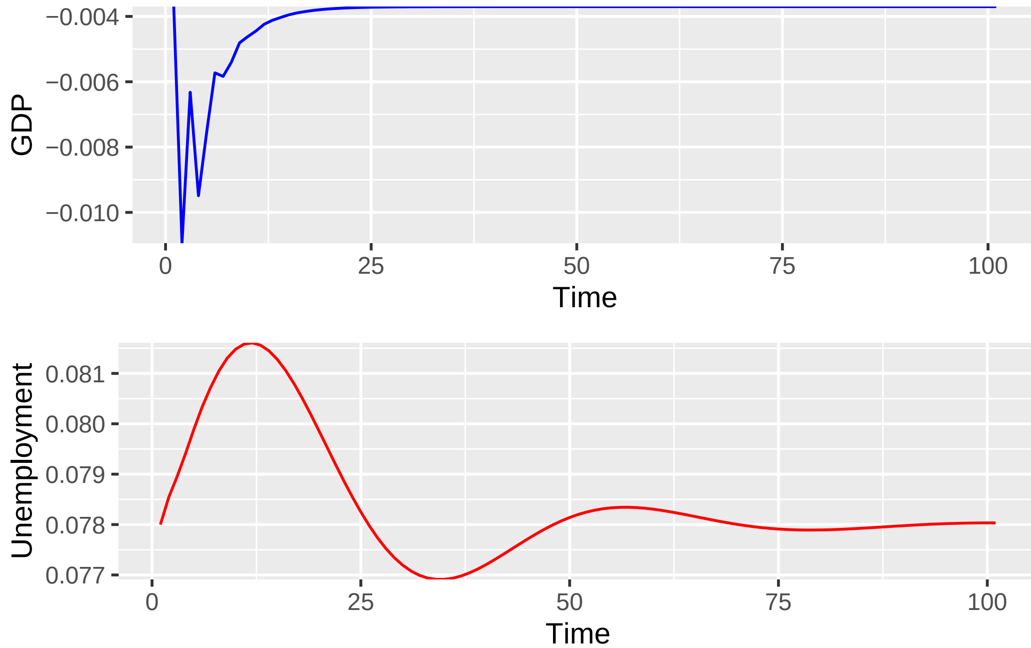
On the other hand, the effect of the shock on the Unemployment rate shows a more gradual oscillating behaviour and the effect dies down at a much later stage. When the GDP growth rate received a boost, unemployment rate quickly decrease. However, when the effect of the boost becomes smaller, and the GDP growth rate returns to a negative value, the Unemployment goes up again. Then the system reaches the steady state.

This behavior of the model makes sense, because as the GDP growth rate increases, Unemployment should decreases.

6.2 Question 6b

```
# For the negative scenarios
epsilon = -0.02
lIRF= fIRF(n_periods, init_y, init_x, epsilon, beta_1, beta_3, phi_1,
 ↪   phi_3, theta_1)
x_values=lIRF$x_values
y_values=lIRF$y_values
```

```r
plotIRF(n_periods, x_values, y_values)
```



In the bad scenario where the GDP rate experience a negative shock of 2%, the behaviour of the system is exactly the opposite of the previous scenario. Specifically, we see a decrease in GDP growth rate and increase in unemployment rate. The effect of the shock on GDP is more prononced in magnitude and short-lived in terms of convergence time. The effect on unemployment rate is more gradual.

This behavior of the model makes sense, because as the GDP growth rate decreases, Unemployment should increases.