

Econometrics III

Assignment Part I

Disclaimer: This assignment was originally prepared for the 2022/23 version of the course in teams of two, but Thao Le (523716) has already passed this course and hence the assignment is resubmitted by David Gyaraki (582340). The assignment has been slightly changed to align with the 2024/25 version.

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```
# load packages
if(!require(pacman)){install.packages("pacman")}

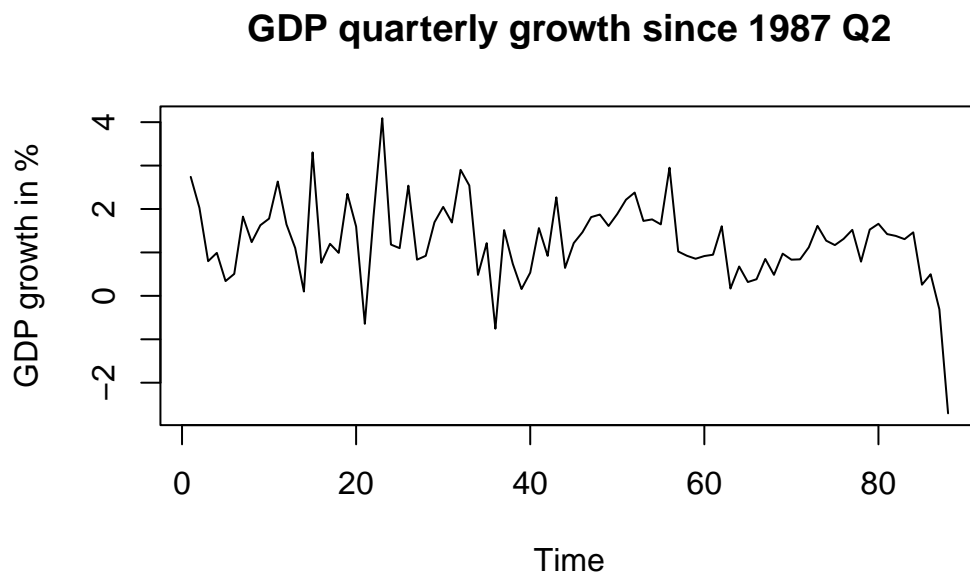
p_load(devtools,tidyverse,dplyr,ggplot2,latex2exp,stargazer, fixest,
  ↳ modelsummary, knitr, readr, tseries, lmtest, forecast, dynlm, stats)

dfAssign_p1 <- as.data.frame(read_csv("data/data_assign_p1.csv"))

# Encode quarters
dfAssign_p1 <-
  ↳ cbind(dfAssign_p1,c(seq(1,nrow(dfAssign_p1),length.out=nrow(dfAssign_p1))))
colnames(dfAssign_p1) <- c("obs", "GDP_QGR", "ind")
```

1 Question 1

```
plot.ts(dfAssign_p1$GDP_QGR, main="GDP quarterly growth since 1987 Q2",
  ↳ ylab="GDP growth in %")
```



The Dutch quarterly growth rates seems to be fluctuating around 1 % every quarter until it

experiences a sharp drop at Quarter 1 of 2009. This growth rate reflect the influence of the economic recession.

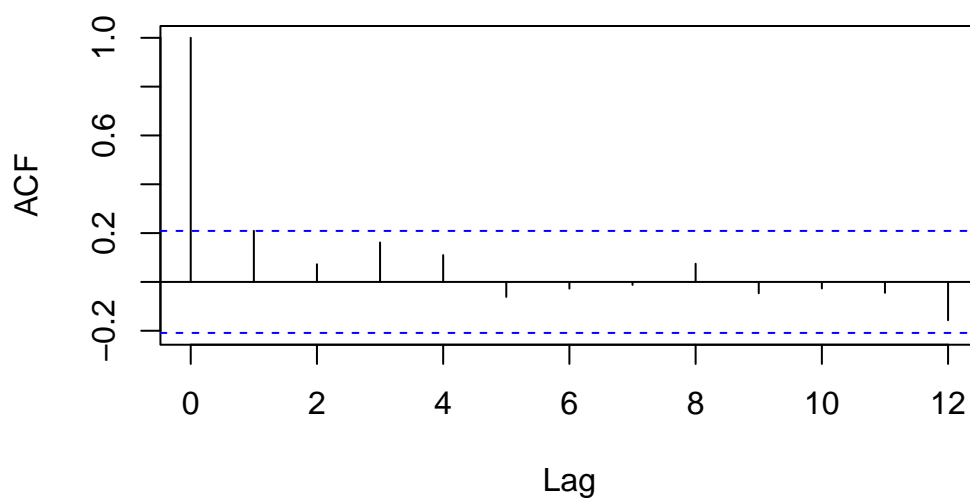
```
acf(dfAssign_p1$GDP_QGR,12,pl=F)
```

Autocorrelations of series 'dfAssign_p1\$GDP_QGR', by lag

| | | | | | | | | | | |
|--------|--------|-------|-------|-------|--------|--------|--------|-------|--------|--------|
| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 1.000 | 0.209 | 0.072 | 0.162 | 0.110 | -0.061 | -0.027 | -0.012 | 0.075 | -0.046 | -0.027 |
| 11 | 12 | | | | | | | | | |
| -0.045 | -0.157 | | | | | | | | | |

```
acf(dfAssign_p1$GDP_QGR,12,pl=T)
```

Series dfAssign_p1\$GDP_QGR



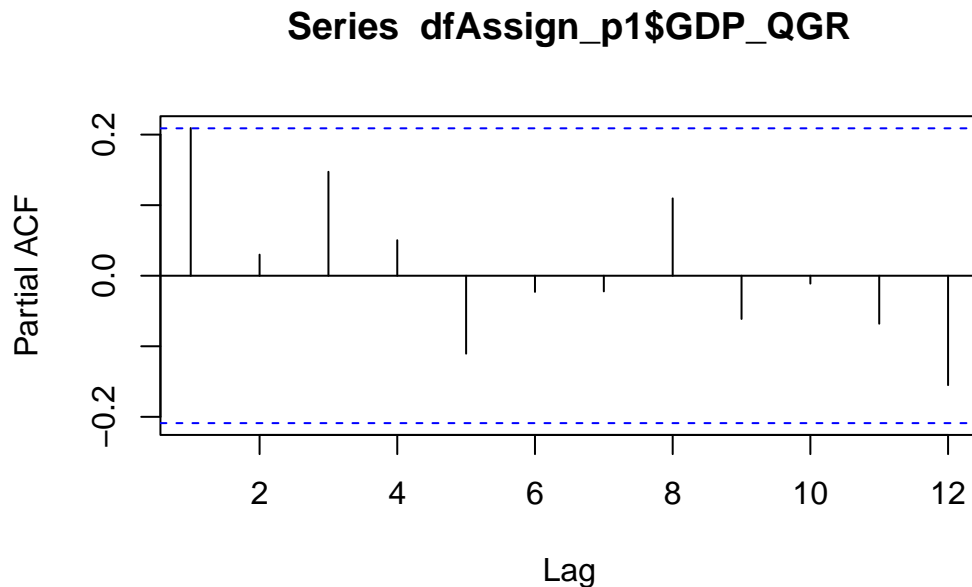
```
pacf(dfAssign_p1$GDP_QGR,12,pl=F)
```

Partial autocorrelations of series 'dfAssign_p1\$GDP_QGR', by lag

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|----|----|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
|---|---|---|---|---|---|---|---|---|----|----|

```
0.209 0.030 0.147 0.050 -0.110 -0.023 -0.022 0.110 -0.061 -0.011 -0.068
12
-0.155
```

```
pacf(dfAssign_p1$GDP_QGR,12,pl=T)
```



From the sample ACF and partial ACF plots, we can observe the autocorrelation of the data with each defined lag up to 12. From the sample ACF plot (former), we can observe that there is quite a bit of positive autocorrelation with the first, second, third and fourth lags (although only the first reaches the white-noise threshold). After these, there is little correlations, with changing signs, only the twelfth shows a relatively larger negative autocorrelation. From the partial plot, we can observe more or less the same picture, except that the first and third lags are quite enhanced while the second and fourth are less pronounced now. Furthermore we can see that the fifth lag is more pronounced negative now along with the twelfth, and interestingly the eighth shows quite a large positive correlation too.

Using the sample ACF, the dynamics seems to be that GDP quarterly growth rate seems to be correlated to the first 4 lags.

2 Question 2

```
ar4 <- arima(dfAssign_p1$GDP_QGR, order=c(4,0,0))
#coeftest(ar4)

ar3 <- arima(dfAssign_p1$GDP_QGR, order=c(3,0,0))
#coeftest(ar3)

ar2 <- arima(dfAssign_p1$GDP_QGR, order=c(2,0,0))
#coeftest(ar2)

ar1 <- arima(dfAssign_p1$GDP_QGR, order=c(1,0,0))
#coeftest(ar1)
```

We start by estimating an AR(4) model and we attempt to investigate the coefficients in the model. Table 1 shows us the estimated model parameters in the AR(4) model and the subsequent restricted models. We find that the fourth lag coefficient (along with second and third) is not significant and therefore we start an elimination of lags from the model one-by-one. Column (2) contains the AR(3) model's estimated parameters then, where we can see that by removing the fourth lag, the third one became more enhanced, but this is not enough for our criteria of 5% significance level. This implies that the third lag might be important to us later, but for now we will remove it from the model. Then the second lag should also be removed due to insignificance, and then we are left with only one, the AR(1) model with first-order autocorrelation, which is significant in predicting this quarter's GDP growth. Hence we can say that the first lag of GDP growth seems to be a significant predictor of the GDP growth in any quarter shown by (4) column in Table 1, in a positive relationship, i.e. if the GDP growth was positive last period, we predict 0.272 times that growth in this period (plus the intercept, which is significant positive). This also implies that GDP tends to exhibit similar patterns in one lag with some moderated coefficient, so if GDP growth was positive in a period, this is likely to repeat in the next period with some moderated effect.

```
stargazer::stargazer(ar4,ar3,ar2,ar1, title="Estimating the AR(4) to
  ↪ AR(1) models on GDP data", align=TRUE, label = "tab_ar4",
  ↪ table.placement="H", out = "tab_ar4.tex")
```

Table 1: Estimating the AR(4) to AR(1) models on GDP data

| | <i>Dependent variable:</i> | | | |
|-------------------|----------------------------|---------------------|-----------------------------|---------------------|
| | (1) | (2) | (3) | (4) |
| ar1 | 0.242** (0.118) | 0.256** (0.117) | 0.263** (0.119) | 0.272** (0.118) |
| ar2 | 0.030 (0.120) | 0.030 (0.120) | 0.058 (0.121) | |
| ar3 | 0.189 (0.119) | 0.200* (0.119) | | |
| ar4 | 0.086 (0.120) | | | |
| intercept | 1.214*** (0.204) | 1.228*** (0.180) | 1.249*** (0.140) | 1.253*** (0.130) |
| Observations | 88 | 88 | 88 | 88 |
| Log Likelihood | -113.211 | -113.468 | -114.849 | -114.963 |
| σ^2 | 0.765 | 0.770 | 0.796 | 0.798 |
| Akaike Inf. Crit. | 238.421 | 236.936 | 237.698 | 235.925 |
| <i>Note:</i> | | | *p<0.1; **p<0.05; ***p<0.01 | |

3 Question 3

The final model we chose is the AR(1) model.

```
ar1_res <- as.numeric(ar1[["residuals"]]) #estimated residual ACF
↪ function
acf(ar1_res,12,pl=F)
```

Autocorrelations of series 'ar1_res', by lag

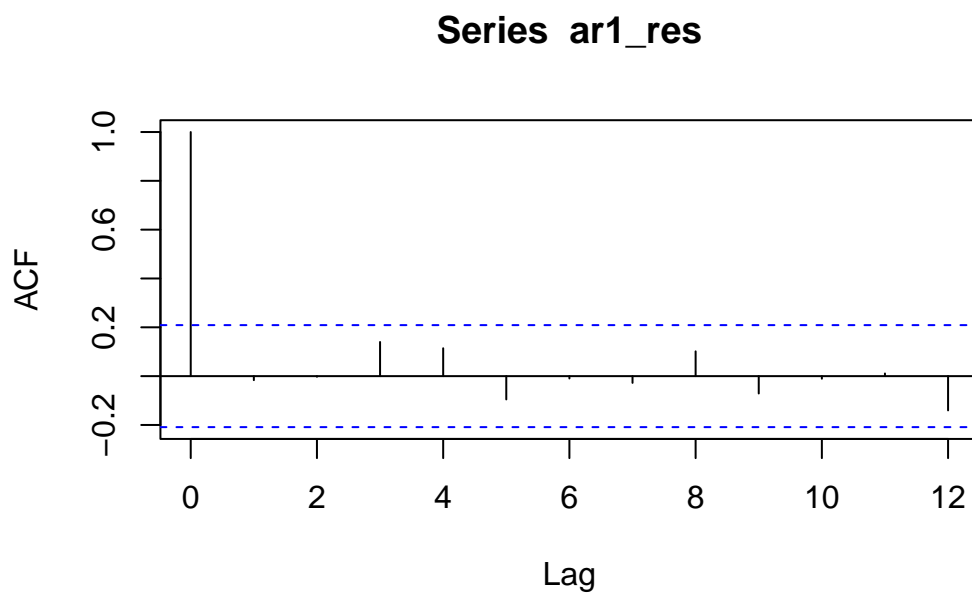
0 1 2 3 4 5 6 7 8 9 10

```

1.000 -0.017 -0.003  0.140  0.114 -0.096 -0.010 -0.028  0.102 -0.071 -0.011
    11    12
0.011 -0.140

```

```
acf(ar1_res,12,pl=T)
```



```
resar4 <- arima(ar1_res, order=c(4,0,0))
coeftest(resar4)
```

z test of coefficients:

| | Estimate | Std. Error | z value | Pr(> z) |
|-----------|-----------|------------|---------|-----------|
| ar1 | -0.036058 | 0.116826 | -0.3086 | 0.75759 |
| ar2 | 0.025976 | 0.117600 | 0.2209 | 0.82518 |
| ar3 | 0.205153 | 0.118412 | 1.7325 | 0.08318 . |
| ar4 | 0.180039 | 0.118957 | 1.5135 | 0.13016 |
| intercept | -0.037084 | 0.147513 | -0.2514 | 0.80151 |

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

In this case, we can investigate the autocorrelation of the residuals in the AR(1) model. The plot above investigates the autocorrelation in the residuals up to 12 lags, and immediately we can see that the previously large first order autocorrelation has been removed thanks to our AR(1) model estimation. However, there are quite some correlations appearing in the third and fourth lags, along with some levels of correlation in the fifth, eighth and twelfth as previously seen. This would imply that the autoregressive model is not defined as best, since we still leave some autocorrelation behind. However, if we estimate an AR(4) model on the residuals, we still find that the third lag is only significant with 10% level so this could provide us with some justification even if our model is not so well specified.

4 Question 4

```
forecast8 <- forecast(ar1, 8, level = c(95))
predlbl <- c("2009Q2", "2009Q3", "2009Q4", "2010Q1", "2010Q2", "2010Q3",
  ↪ "2010Q4", "2011Q1")
forecast_q4 <- cbind(predlbl, as.data.frame(forecast8))

pred <- predict(ar1, 8)

kable(forecast_q4[,1:2], caption="The predicted quarterly GDP growth
  ↪ rates using ARIMA(1,0,0)", col.names = c("Quarter", "Predicted
  ↪ growth %"), digits = 3, row.names = F)
```

Table 2: The predicted quarterly GDP growth rates using ARIMA(1,0,0)

| Quarter | Predicted growth % |
|---------|--------------------|
| 2009Q2 | 0.177 |
| 2009Q3 | 0.960 |
| 2009Q4 | 1.173 |
| 2010Q1 | 1.231 |
| 2010Q2 | 1.247 |
| 2010Q3 | 1.251 |
| 2010Q4 | 1.252 |
| 2011Q1 | 1.253 |

```
mean(dfAssign_p1$GDP_QGR)
```

```
[1] 1.263169
```


Since we estimated the AR(1) model with nonzero mean, our prediction for the first period follows the calculation:

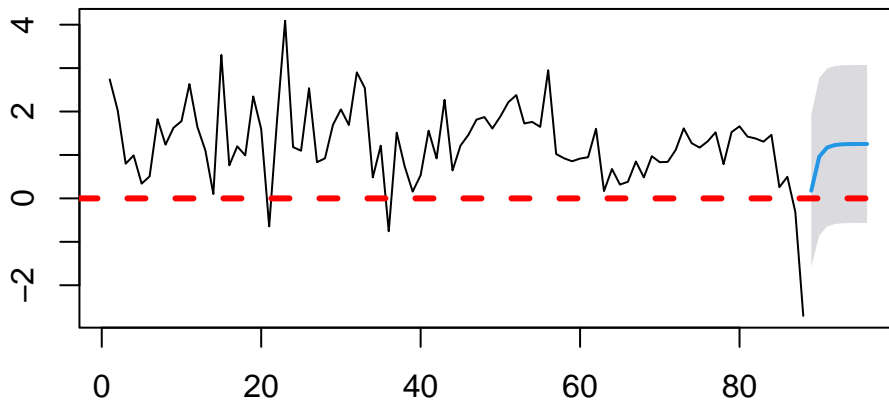
$$\hat{Y}_{T+1} = \delta + \phi(Y_T - \mu), \quad (1)$$

where δ is the estimated intercept, ϕ is the coefficient of the AR(1) lag term and $\mu = 1.26317$ is the mean of the GDP growth in the estimation period. Therefore, we can calculate the value for the Q2 of 2009 by $(-2.7051 - 1.26317) * 0.272 + 1.253 = 0.174$, which closely corresponds to the function estimates of 0.177. For all further values in time period t , we simply use the predicted values for period $t - 1$. We can then manually confirm that indeed the values predicted by the function are correct.

5 Question 5

```
plot(forecast8) %>%
  abline(h=0, col="red", lwd=3, lty=2)
```

Forecasts from ARIMA(1,0,0) with non-zero mean



If we assume that the innovations in our AR model are IID Gaussian, we can use the estimated model and its coefficients for forecasting the values and their confidence interval with

precision. In this case, we assume that our estimates obtained in Section 4 are not influenced by any remaining correlation in the residuals (potentially from the second, third or fourth lags) and the predictions produced will not be tainted by autocorrelation. From the plot in Section 5 we can see that the prediction interval is quite wide, with 0 continuously within the interval as well as some negative values. Therefore, even if our prediction is positive (expecting growth in GDP in 2009 Q2), there is indeed a really good probability that the growth in Q2 will stay negative, since almost half of the confidence interval lies within the negative territory.

6 Question 6

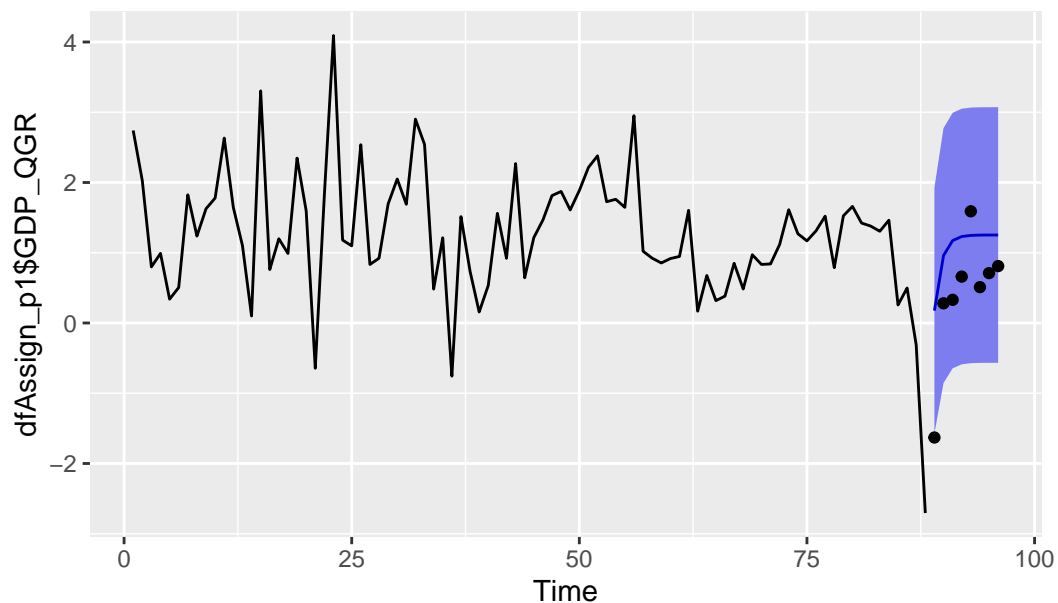
The assumption that the residuals are iid Gaussian is not reasonable. As we have seen before, even though not at 5% level, but the second, third and fourth lag coefficients came close to being significant, implying that there might be some positive autocorrelation up to the fourth lag. Since we have removed them from the model, we will likely introduce autocorrelated errors in the estimated model, which might explain the strong uncertainty in the confidence interval around our predictions. Therefore, as the graph also implies, we may have an overestimation bias by not including the second-to-fourth lags in the model and the CI of the estimates will implode because of the reduced efficiency of the model (although the coefficient estimates will be consistent and unbiased).

7 Question 7

```
dfActuals <- data.frame(x=c(89, 90, 91, 92, 93, 94, 95, 96),
                        y=c(-1.63, 0.28, 0.33, 0.66, 1.59, 0.51, 0.71, 0.81))

autoplot(forecast8, showgap = F) + geom_point(data = dfActuals, aes(x=x,
↪ y=y))
```

Forecasts from ARIMA(1,0,0) with non-zero mean



From the plot we can read off the predicted quarterly GDP growth for two years along with the confidence intervals at a 95% level. Then we can compare these predicted values and the confidence interval to the actual observed growth rates per period. While we can claim that most actual values are within our confidence intervals (apart from the first predicted period 2009 Q2, where the actual growth is slightly below the lower bound), this has more to do with the uncertainty of our predictions rather than the accuracy. The CI intervals are quite large, generally spanning 3-3.5% GDP growth, which makes our predictions quite useless (predicting that GDP growth will be between 0 and 3.5% next quarter offers relatively no insight). Also our AR(1) model seems to consistently overestimate the actual values with the predictions, apart from one (2010 Q2), all observations are below the predicted line. Therefore our accuracy is quite weak and it seems that our model consistently overestimates the actual GDP growth in the prediction window.