

# **Econometrics III**

## **Assignment Part IV**

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```

# load packages
if(!require(pacman)){install.packages("pacman")}

p_load(devtools,tidyverse,dplyr,ggplot2,latex2exp,stargazer, fixest,
  ↪ modelsummary, knitr, readr, tseries, lmtest, forecast, dynlm, vars,
  ↪ xtable)

dfAssign_p4 <- as.data.frame(read_csv("data/data_assign_p4.csv"))

# Encode quarters
dfAssign_p4 <-
  ↪ cbind(dfAssign_p4,c(seq(1,nrow(dfAssign_p4),length.out=nrow(dfAssign_p4))))
colnames(dfAssign_p4) <- c("obs", "CONS", "INC", "TIME")

```

We are always more interested in estimating ADL(1,1) instead of 1st diff ADL(1,1) because we are not able to say anything about the long run equilibrium in case the 1st diff are used

## 1 Question 1

```

# Set the parameters
gamma <- 0.5
sigma_u <- 1
sigma_v <- 1
phi <- 0.9
init_step = 200
max_T = 2000

# Function to simulate all data
sim_data <- function(T) {
  vt <- rnorm(T, mean = 0, sd = sqrt(sigma_v))
  ut <- rnorm(T, mean = 0, sd = sqrt(sigma_u))

  #X_t and Y_t in the cointegrated model
  xt_cointegrated <- cumsum(vt)
  yt_cointegrated <- gamma * xt_cointegrated + ut

  #X_t and Y_t in the stationary model
  xt_stationary <- numeric(T)
  xt_stationary[1] <- vt[1]

```

```

for (t in 2:T) {
  xt_staionary[t] <- phi * xt_staionary[t-1] + vt[t]
}
yt_staionary <- gamma * xt_staionary + ut

return(list(xt_cointegrated = xt_cointegrated, yt_cointegrated =
  ↪ yt_cointegrated,
           xt_staionary = xt_staionary, yt_staionary = yt_staionary))
}

# Initialize dataframe to store results
sim_results <- data.frame(
  T = numeric(),
  beta_hat_1 = numeric(),
  t_stat_1 = numeric(),
  R_squared_1 = numeric(),
  beta_hat_2 = numeric(),
  t_stat_2 = numeric(),
  R_squared_2 = numeric(),
  stringsAsFactors = FALSE
)

# For each sample size T from T = 200, 400, ..., 1000
for (T in seq(init_step, max_T, 200)) {
  lData = sim_data(T)

  # Cointegrated series
  X1_t = lData$xt_cointegrated
  Y1_t = lData$yt_cointegrated

  # Stationary series
  X2_t = lData$xt_staionary
  Y2_t = lData$yt_staionary

  # Perform linear regression and get summary stats on both models
  model1 <- lm(Y1_t ~ X1_t)
  summary1 <- tidy(model1)
  beta_hat1 <- summary1$estimate[2]
  t_stat1 <- summary1$statistic[2]
  R_squared1 <- summary(model1)$r.squared

```

```

model2 <- lm(Y2_t ~ X2_t)
summary2 <- tidy(model2)
beta_hat2 <- summary2$estimate[2]
t_stat2 <- summary2$statistic[2]
R_squared2 <- summary(model2)$r.squared

# Store results
sim_results <- sim_results %>%
  add_row(T = T,
          beta_hat_1 = beta_hat1,
          t_stat_1 = t_stat1,
          R_squared_1 = R_squared1,
          beta_hat_2 = beta_hat2,
          t_stat_2 = t_stat2,
          R_squared_2 = R_squared2,
          )
}

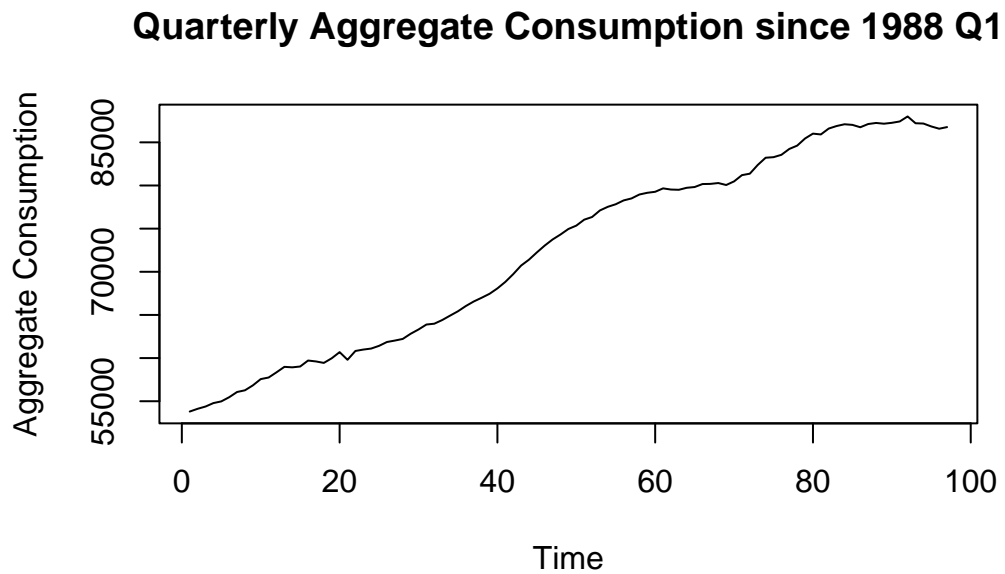
```

	T	beta_hat_1	t_stat_1	R_squared_1	beta_hat_2	t_stat_2	R_squared_2
1	200	0.54	24.62	0.75	0.51	13.08	0.46
2	400	0.50	75.90	0.94	0.49	21.06	0.53
3	600	0.50	85.59	0.92	0.49	25.48	0.52
4	800	0.50	124.18	0.95	0.51	33.60	0.59
5	1000	0.50	137.93	0.95	0.49	32.81	0.52
6	1200	0.50	211.06	0.97	0.50	40.04	0.57
7	1400	0.50	553.08	1.00	0.50	39.90	0.53
8	1600	0.50	244.21	0.97	0.49	43.78	0.55
9	1800	0.50	255.31	0.97	0.51	48.00	0.56
10	2000	0.50	415.59	0.99	0.51	51.27	0.57

Looking at the results of model 1 (using non-stationary cointegrated variables), we can see that the  $\hat{\beta}$  estimates are quite reliable and consistent, with higher T-statistics and higher R-squared scores. This indicates that we did not run into the spurious regression problems. On the other hand, the result of model 2 suggests that the model perform poorly, with lower T-statistics and low R-squared scores suggest that the model perform poorly and the results are not reliable. This suggests that we run into the spurious regression problems.

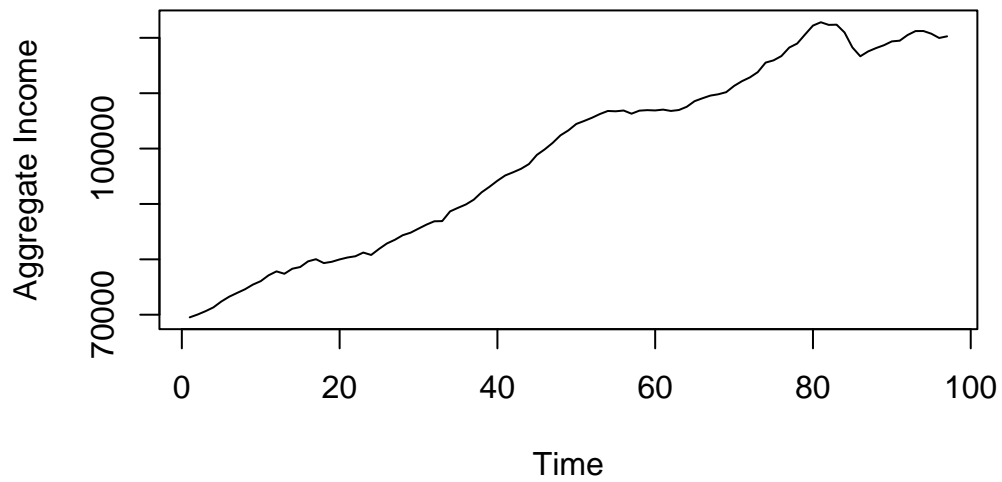
## 2 Question 2

```
plot.ts(dfAssign_p4$CONS, main="Quarterly Aggregate Consumption since  
↪ 1988 Q1", ylab="Aggregate Consumption")
```



```
plot.ts(dfAssign_p4$INC, main="Quarterly Aggregate Income since 1988  
↪ Q1", ylab="Aggregate Income")
```

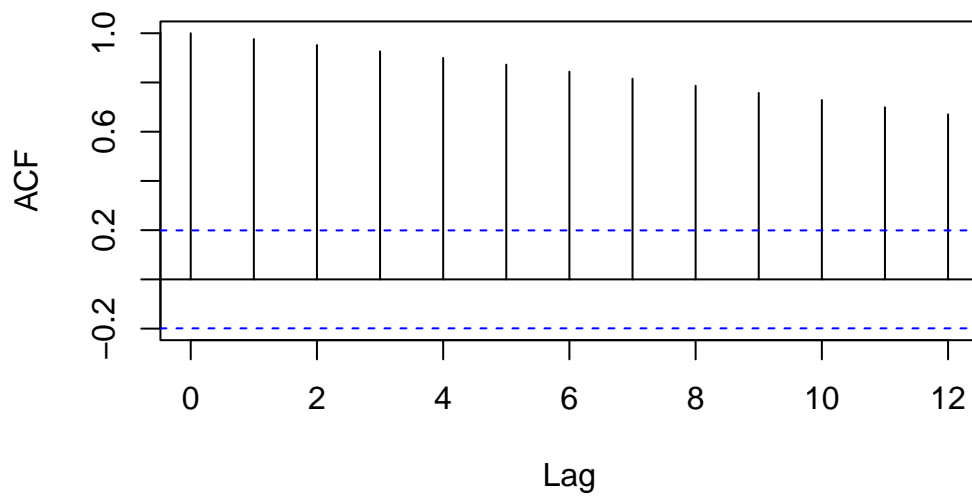
### Quarterly Aggregate Income since 1988 Q1



The two plots above show the shape of the quarterly aggregate consumption and quarterly aggregate income in the Netherlands. From this we can see that due to the aggregate nature of the two series, the series are not likely to be stationary at first glance.

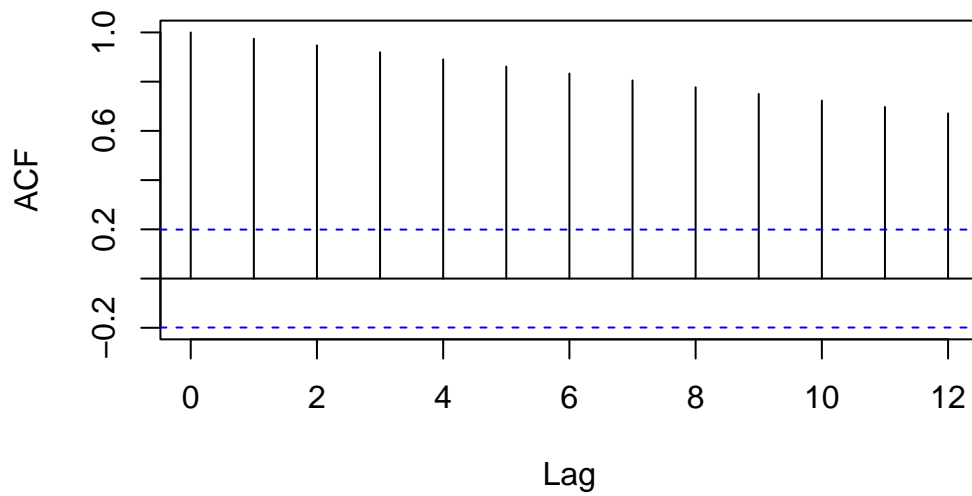
```
acf(dfAssign_p4$CONS,12,pl=T, main="ACF of the Aggregate Consumption")
```

### ACF of the Aggregate Consumption

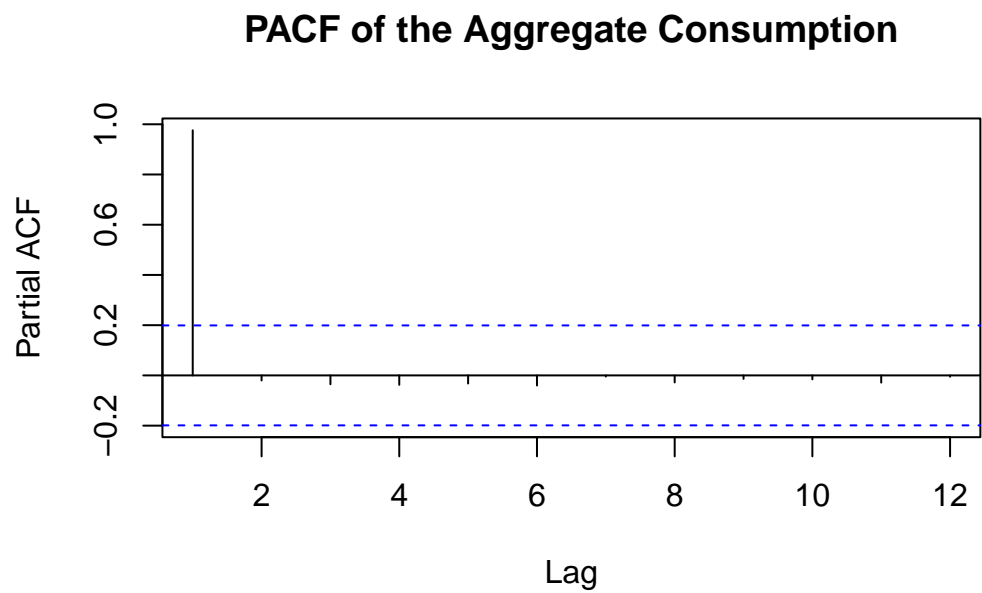


```
acf(dfAssign_p4$INC,12,pl=T, main="ACF of the Aggregate Income")
```

### ACF of the Aggregate Income



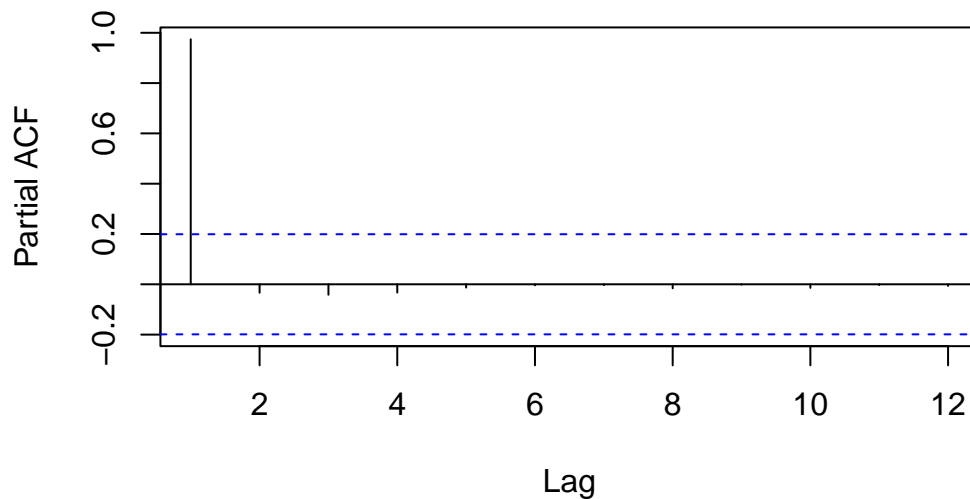
```
pacf(dfAssign_p4$CONS,12,pl=T, main="PACF of the Aggregate Consumption")
```



```
pacf(dfAssign_p4$INC,12,pl=T, main="PACF of the Aggregate Income")
```



### PACF of the Aggregate Income



From the ACF and PACF plots for the consumption and income components of the GDP, we can see that due to the aggregate nature of the series, both the consumption and income autocorrelations show significant correlation (above the white-noise threshold) across all lags all the way to the 12th period lag. However, when we investigate the partial autocorrelation function results, both variables have insignificant correlations for all lags (1st to 12th). While some correlations are more pronounced for consumption than for income, no values reach the white-noise threshold and therefore are not very significant.

### 3 Question 3

```
adf.test(dfAssign_p4$CONS)
```

Augmented Dickey-Fuller Test

```
data: dfAssign_p4$CONS
Dickey-Fuller = -1.214, Lag order = 4, p-value = 0.9006
alternative hypothesis: stationary
```

```
adf.test(dfAssign_p4$INC)
```

Augmented Dickey-Fuller Test

```
data: dfAssign_p4$INC
Dickey-Fuller = -1.4134, Lag order = 4, p-value = 0.819
alternative hypothesis: stationary
```

Based on the test results, we cannot reject in either case that the series would be non-stationary. Since we do not find sufficient evidence for the stationarity, we can stay at the null-hypothesis of the series being non-stationary.

## 4 Question 4

```
d1_CONS <- diff(dfAssign_p4$CONS, differences = 1)

d1_INC <- diff(dfAssign_p4$INC, differences = 1)

adf.test(d1_CONS)
```

Augmented Dickey-Fuller Test

```
data: d1_CONS
Dickey-Fuller = -2.1092, Lag order = 4, p-value = 0.5313
alternative hypothesis: stationary
```

```
adf.test(d1_INC)
```

Augmented Dickey-Fuller Test

```
data: d1_INC
Dickey-Fuller = -3.7139, Lag order = 4, p-value = 0.02705
alternative hypothesis: stationary
```

In the case of the first-difference series, we still cannot reject the null hypothesis on the first case (consumption), however the second Augmented Dickey-Fuller test has a p-value of  $p = 2.71\%$  and hence can be rejected at a 5% significance level. This means that the first difference of the income component of the GDP might be stationary. In terms of the order of integration, these results imply that the income series is most likely an  $I(1)$  series (integrated of order 1), while for consumption, this may be  $I(2)$  or higher order, but the ADF test implies that even the first difference of the series is not stationary.

## 5 Question 5

```
model5 <- lm(CONS ~ INC, data=dfAssign_p4)

summary(model5)
```

Call:

```
lm(formula = CONS ~ INC, data = dfAssign_p4)
```

Residuals:

Min	1Q	Median	3Q	Max
-2559.4	-524.3	-128.0	561.2	2364.8

Coefficients:

	Estimate	Std. Error	t value	Pr(> t )
(Intercept)	6.783e+03	5.550e+02	12.22	<2e-16 ***
INC	6.651e-01	5.534e-03	120.19	<2e-16 ***

---

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 911.2 on 95 degrees of freedom

Multiple R-squared: 0.9935, Adjusted R-squared: 0.9934

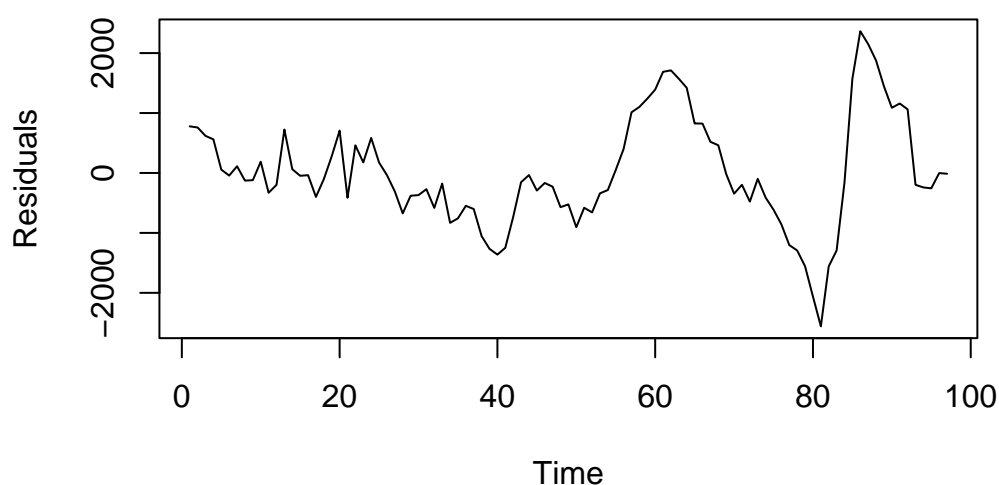
F-statistic: 1.445e+04 on 1 and 95 DF, p-value: < 2.2e-16

```
residuals <- model5$residuals

dfAssign_p4 <- cbind(dfAssign_p4, residuals)

plot.ts(dfAssign_p4$residuals, main="Residuals of Consumption regressed
↪ on Income", ylab="Residuals")
```

## Residuals of Consumption regressed on Income



From the residuals plot, one can already see that we would suspect stationarity in the residuals, which would imply cointegration between the two variables.

```
adf.test(residuals)
```

Augmented Dickey-Fuller Test

```
data: residuals
```

```
Dickey-Fuller = -3.5848, Lag order = 4, p-value = 0.03839
```

```
alternative hypothesis: stationary
```

```
adf_res = ur.df(dfAssign_p4$residuals, type = "none", lags = 10,
                selectlags = "BIC")

summary(adf_res)
```

```
#####
# Augmented Dickey-Fuller Test Unit Root Test #
#####
```

Test regression none

Call:

```
lm(formula = z.diff ~ z.lag.1 - 1 + z.diff.lag)
```

Residuals:

Min	1Q	Median	3Q	Max
-1179.90	-270.52	-7.59	274.20	1411.39

Coefficients:

	Estimate	Std. Error	t value	Pr(> t )
z.lag.1	-0.17739	0.05162	-3.436	0.000924 ***
z.diff.lag1	0.19325	0.10272	1.881	0.063428 .
z.diff.lag2	0.26739	0.10461	2.556	0.012410 *

---

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 421.2 on 83 degrees of freedom

Multiple R-squared: 0.1657, Adjusted R-squared: 0.1356

F-statistic: 5.497 on 3 and 83 DF, p-value: 0.001709

Value of test-statistic is: -3.4363

Critical values for test statistics:

	1pct	5pct	10pct
tau1	-2.6	-1.95	-1.61

The ADF test is conducted by starting with 10 number of lags and a general-to-specific approach, where we remove lags based on the Schwartz Information Criteria at each step (without considering drift or trend). Based on the ADF test on the residuals, we can observe that the non-stationarity hypothesis is rejected. This implies that the residuals series is probably stationary, which implies existing cointegration between consumption and income. Then we can observe, that the model with the best SIC is the one:

$$\Delta Z_t = \gamma Z_{t-1} + \delta_1 \Delta Z_{t-1} + \delta_2 \Delta Z_{t-2} \quad (1)$$

**6 Question 6**

**7 Question 7**

**8 Question 8**