

# Chapter 41

## Conduction of Electricity in Solids

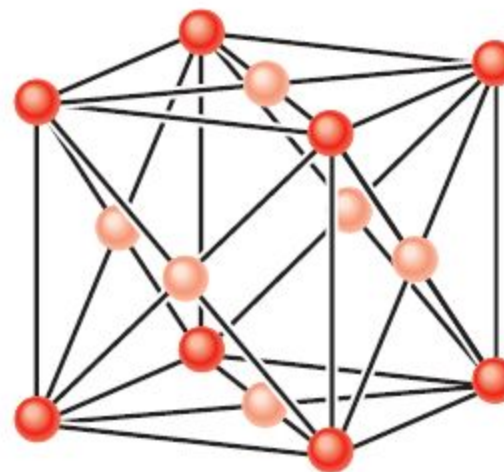
## 41.2: The Electrical Properties of Solids:

The electrical properties of solids can be categorized into following classes:

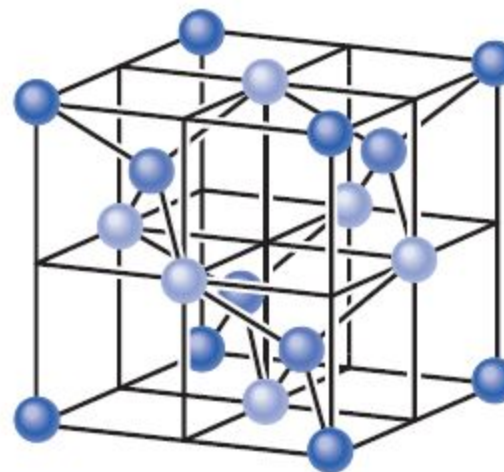
1. Their **resistivity**  $\rho$  at room temperature, with the SI unit ohm-meter ( $\Omega \cdot \text{m}$ ); resistivity is defined in Section 26-4.
2. Their **temperature coefficient of resistivity**  $\alpha$ , defined as  $\alpha = (1/\rho)(d\rho/dT)$  in Eq. 26-17 and having the SI unit inverse kelvin ( $\text{K}^{-1}$ ). We can evaluate  $\alpha$  for any solid by measuring  $\rho$  over a range of temperatures.
3. Their **number density of charge carriers**  $n$ . This quantity, the number of charge carriers per unit volume, can be found from measurements of the Hall effect, as discussed in Section 28-4, and from other measurements. It has the SI unit inverse cubic meter ( $\text{m}^{-3}$ ).

## 41.2: The Electrical Properties of Solids:

**Fig. 41-1** (a) The unit cell for copper is a cube. There is one copper atom (darker) at each corner of the cube and one copper atom (lighter) at the center of each face of the cube. The arrangement is called *face-centered cubic*. (b) The unit cell for either silicon or the carbon atoms in diamond is also a cube, the atoms being arranged in what is called a *diamond lattice*. There is one atom (darkest) at each corner of the cube and one atom (lightest) at the center of each cube face; in addition, four atoms (medium color) lie within the cube. Every atom is bonded to its four nearest neighbors by a two-electron covalent bond (only the four atoms within the cube show all four nearest neighbors).



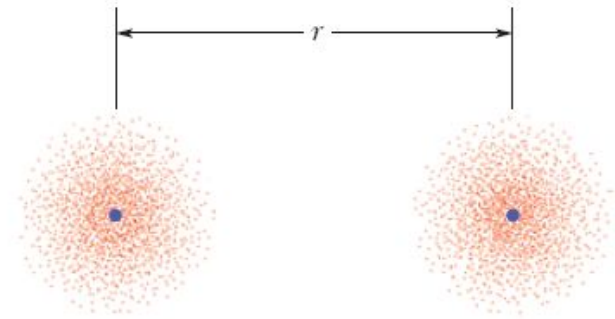
(a)



(b)

## 41.3: Energy Levels in a Crystalline Solid:

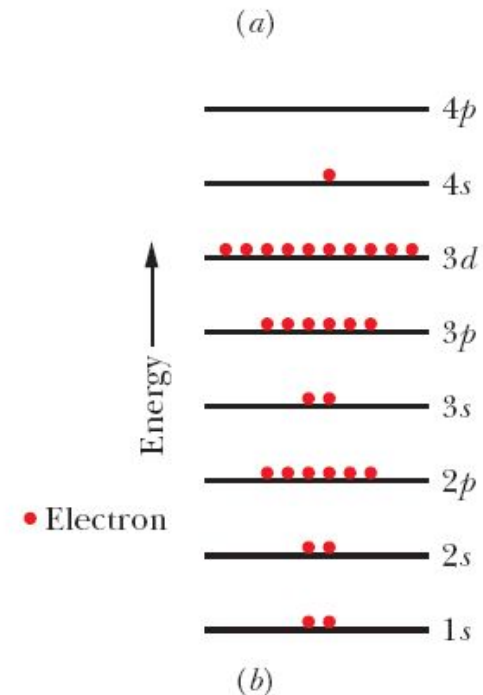
**Fig. 41-2** (a) Two copper atoms separated by a large distance; their electron distributions are represented by dot plots. (b) Each copper atom has 29 electrons distributed among a set of subshells. In the neutral atom in its ground state, all subshells up through the  $3d$  level are filled, the  $4s$  subshell contains one electron (it can hold two), and higher subshells are empty. For simplicity, the subshells are shown as being evenly spaced in energy.



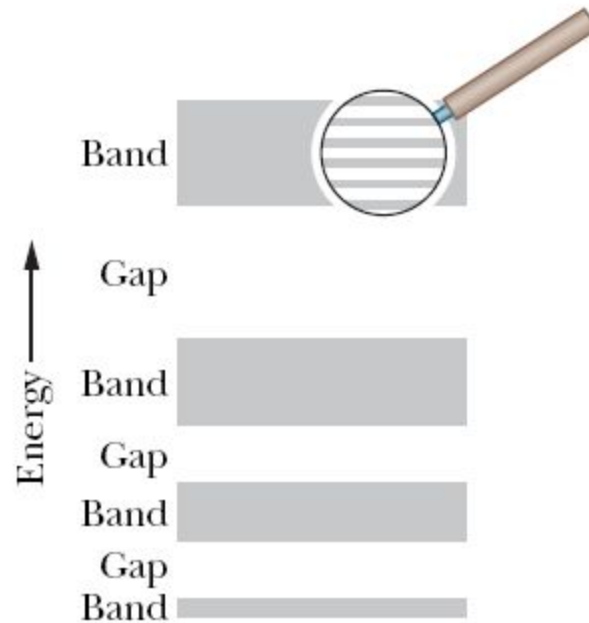
If we bring the atoms of Fig. 41-2a close together, their wave functions will overlap, beginning with those of the outermost electrons. Then we have a single two-atom system; here the system contains  $2 \times 29 = 58$  electrons.

If we bring up more atoms, we gradually assemble a lattice of solid copper. If the lattice contains  $N$  atoms, then each level of an isolated copper atom must split into  $N$  levels in the solid.

Thus, the individual energy levels of the solid form energy **bands**, adjacent bands being separated by an energy **gap**, with the gap representing a range of energies that no electron can possess.



### 41.3: Energy Levels in a Crystalline Solid:



**Fig. 41-3** The band-gap pattern of energy levels for an idealized crystalline solid. As the magnified view suggests, each band consists of a very large number of very closely spaced energy levels. (In many solids, adjacent bands may overlap; for clarity, we have not shown this condition.)

41.3: Energy Levels in a Crystalline Solid:

Table 41 -1

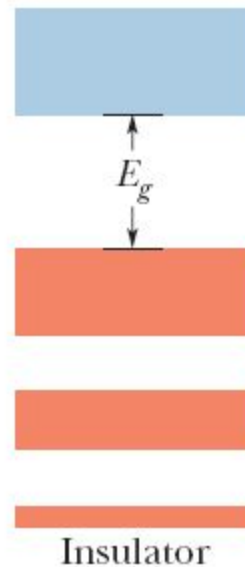
Some Electrical Properties of Two Materials<sup>a</sup>

| Property   | Unit                    | Material            |                      |
|--|-------------------------|---------------------|----------------------|
|  |                         | Copper              | Silicon              |
| Type of conductor                                |                         | Metal               | Semiconductor        |
| Resistivity, $\rho$                              | $\Omega \cdot \text{m}$ | $2 \times 10^{-8}$  | $3 \times 10^3$      |
| Temperature coefficient of resistivity, $\alpha$ | $\text{K}^{-1}$         | $+4 \times 10^{-3}$ | $-70 \times 10^{-3}$ |
| Number density of charge carriers, $n$           | $\text{m}^{-3}$         | $9 \times 10^{28}$  | $1 \times 10^{16}$   |

<sup>a</sup>All values are for room temperature.

## 41.4: Insulators:

In an insulator, electrons need a big energy jump.



**Fig. 41-4** The band-gap pattern for an insulator; filled levels are shown in red and empty levels in blue.



## Example, Probability of electron excitation in an insulator:

Approximately what is the probability that, at room temperature (300 K), an electron at the top of the highest filled band in diamond (an insulator) will jump the energy gap  $E_g$  in Fig. 41-4? For diamond,  $E_g$  is 5.5 eV.

### KEY IDEA

In Chapter 40 we used Eq. 40-29,

$$\frac{N_x}{N_0} = e^{-(E_x - E_0)/kT}, \quad (41-1)$$

to relate the population  $N_x$  of atoms at energy level  $E_x$  to the population  $N_0$  at energy level  $E_0$ , where the atoms are part of a system at temperature  $T$  (measured in kelvins);  $k$  is the Boltzmann constant ( $8.62 \times 10^{-5}$  eV/K). In this chapter we can use Eq. 41-1 to *approximate* the probability  $P$  that an electron in an insulator will jump the energy gap  $E_g$  in Fig. 41-4.

**Calculations:** We first set the energy difference  $E_x - E_0$  to  $E_g$ . Then the probability  $P$  of the jump is approximately equal to the ratio  $N_x/N_0$  of the number of electrons just above the energy gap to the number of electrons just below the gap.

For diamond, the exponent in Eq. 41-1 is

$$-\frac{E_g}{kT} = -\frac{5.5 \text{ eV}}{(8.62 \times 10^{-5} \text{ eV/K})(300 \text{ K})} = -213.$$

The required probability is then

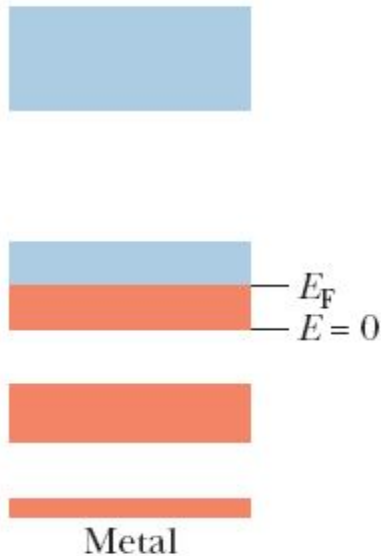
$$P = \frac{N_x}{N_0} = e^{-(E_g/kT)} = e^{-213} \approx 3 \times 10^{-93}. \quad (\text{Answer})$$

This result tells us that approximately 3 electrons out of  $10^{93}$  electrons would jump across the energy gap. Because any diamond stone has fewer than  $10^{23}$  electrons, we see that the probability of the jump is vanishingly small. No wonder diamond is such a good insulator.



## 41.5: Metals:

In a conductor, electrons need only a small energy jump.



**Fig. 41-5** The band-gap pattern for a metal. The highest filled level, called the Fermi level, lies near the middle of a band. Since vacant levels are available within that band, electrons in the band can easily change levels, and conduction can take place.

If the electric potential energy  $U$  of a conduction electron is uniform throughout the lattice, let's set  $U = 0$ , so that the mechanical energy  $E$  is entirely kinetic. Then the level at the bottom of the partially filled band of Fig. 41-5 corresponds to  $E = 0$ . The highest occupied level in this band at absolute zero ( $T = 0$  K) is called the **Fermi level**, and the energy corresponding to it is called the **Fermi energy  $E_F$** ; for copper,  $E_F = 7.0$  eV.

The electron speed corresponding to the Fermi energy is called the **Fermi speed  $v_F$** . For copper the Fermi speed is  $1.6 \times 10^6$  m/s. All motion does not cease at absolute zero; at that temperature the conduction electrons are stacked up in the partially filled band of Fig. 41-5 with energies that range from zero to the Fermi energy.

## 41.5: Metals: How Many Conduction Electrons Are There?

$$\left( \begin{array}{c} \text{number of conduction} \\ \text{electrons in sample} \end{array} \right) = \left( \begin{array}{c} \text{number of atoms} \\ \text{in sample} \end{array} \right) \left( \begin{array}{c} \text{number of valence} \\ \text{electrons per atom} \end{array} \right).$$

$$n = \frac{\text{number of conduction electrons in sample}}{\text{sample volume } V}.$$

$$\begin{aligned} \left( \begin{array}{c} \text{number of atoms} \\ \text{in sample} \end{array} \right) &= \frac{\text{sample mass } M_{\text{sam}}}{\text{atomic mass}} = \frac{\text{sample mass } M_{\text{sam}}}{(\text{molar mass } M)/N_A} \\ &= \frac{(\text{material's density})(\text{sample volume } V)}{(\text{molar mass } M)/N_A}, \end{aligned}$$

where the molar mass  $M$  is the mass of one mole of the material in the sample and  $N_A$  is Avogadro's number ( $6.02 \times 10^{23} \text{ mol}^{-1}$ ).

## Example, Number of conduction electrons in a metal:

How many conduction electrons are in a cube of magnesium of volume  $2.00 \times 10^{-6} \text{ m}^3$ ? Magnesium atoms are bivalent.

### KEY IDEAS

1. Because magnesium atoms are bivalent, each magnesium atom contributes two conduction electrons.
2. The cube's number of conduction electrons is related to its number of magnesium atoms by Eq. 41-2.
3. We can find the number of atoms with Eq. 41-4 and known data about the cube's volume and magnesium's properties.

**Calculations:** We can write Eq. 41-4 as

$$\left( \begin{array}{c} \text{number} \\ \text{of atoms} \\ \text{in sample} \end{array} \right) = \frac{(\text{density})(\text{sample volume } V)N_A}{\text{molar mass } M}.$$

Magnesium has density  $1.738 \text{ g/cm}^3$  ( $= 1.738 \times 10^3 \text{ kg/m}^3$ )

and molar mass  $24.312 \text{ g/mol}$  ( $= 24.312 \times 10^{-3} \text{ kg/mol}$ ) (see Appendix F). The numerator gives us

$$(1.738 \times 10^3 \text{ kg/m}^3)(2.00 \times 10^{-6} \text{ m}^3) \\ \times (6.02 \times 10^{23} \text{ atoms/mol}) = 2.0926 \times 10^{21} \text{ kg/mol}.$$

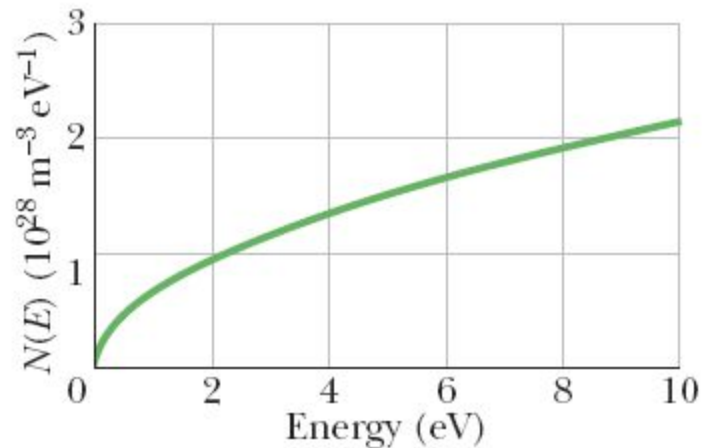
$$\text{Thus, } \left( \begin{array}{c} \text{number of atoms} \\ \text{in sample} \end{array} \right) = \frac{2.0926 \times 10^{21} \text{ kg/mol}}{24.312 \times 10^{-3} \text{ kg/mol}} \\ = 8.61 \times 10^{22}.$$

Using this result and the fact that magnesium atoms are bivalent, we find that Eq. 41-2 yields

$$\left( \begin{array}{c} \text{number of} \\ \text{conduction electrons} \\ \text{in sample} \end{array} \right) \\ = (8.61 \times 10^{22} \text{ atoms}) \left( 2 \frac{\text{electrons}}{\text{atom}} \right) \\ = 1.72 \times 10^{23} \text{ electrons.} \quad (\text{Answer})$$

## 41.5: Metals: How Many Quantum States Are There?

The density of energy levels increases upward in a band.



$$N(E) = \frac{8\sqrt{2}\pi m^{3/2}}{h^3} E^{1/2} \quad (\text{density of states, m}^{-3} \text{ J}^{-1}),$$

**Fig. 41-6** The density of states  $N(E)$ —that is, the number of electron energy levels per unit energy interval per unit volume—plotted as a function of electron energy. The density of states function simply counts the available states; it says nothing about whether these states are occupied by electrons.



## Example, Number of states per electron volt in a metal:

(a) Using the data of Fig. 41-6, determine the number of states per electron-volt at 7 eV in a metal sample with a volume  $V$  of  $2 \times 10^{-9} \text{ m}^3$ .

### KEY IDEA

We can obtain the number of states per electron-volt at a given energy by using the density of states  $N(E)$  at that energy and the sample's volume  $V$ .

**Calculations:** At an energy of 7 eV, we write

$$\left( \begin{array}{c} \text{number of states} \\ \text{per eV at 7 eV} \end{array} \right) = \left( \begin{array}{c} \text{density of states} \\ N(E) \text{ at 7 eV} \end{array} \right) \left( \begin{array}{c} \text{volume } V \\ \text{of sample} \end{array} \right).$$

From Fig. 41-6, we see that at an energy  $E$  of 7 eV, the density of states is about  $1.8 \times 10^{28} \text{ m}^{-3} \text{ eV}^{-1}$ . Thus,

$$\begin{aligned} \left( \begin{array}{c} \text{number of states} \\ \text{per eV at 7 eV} \end{array} \right) &= (1.8 \times 10^{28} \text{ m}^{-3} \text{ eV}^{-1})(2 \times 10^{-9} \text{ m}^3) \\ &= 3.6 \times 10^{19} \text{ eV}^{-1} \\ &\approx 4 \times 10^{19} \text{ eV}^{-1}. \end{aligned} \quad (\text{Answer})$$

(b) Next, determine the number of states  $N$  in the sample within a *small* energy range  $\Delta E$  of 0.003 eV centered at 7 eV.

**Calculation:** From Eq. 41-5 and Fig. 41-6, we know that the density of states is a function of energy  $E$ . However, for an energy range  $\Delta E$  that is small relative to  $E$ , we can approximate the density of states (and thus the number of states per electron-volt) to be constant. Thus, at an energy of 7 eV, we find the number of states  $N$  in the energy range  $\Delta E$  of 0.003 eV as

$$\left( \begin{array}{c} \text{number of states } N \\ \text{in range } \Delta E \text{ at 7 eV} \end{array} \right) = \left( \begin{array}{c} \text{number of states} \\ \text{per eV at 7 eV} \end{array} \right) \left( \begin{array}{c} \text{energy} \\ \text{range } \Delta E \end{array} \right)$$

$$\begin{aligned} \text{or} \quad N &= (3.6 \times 10^{19} \text{ eV}^{-1})(0.003 \text{ eV}) \\ &= 1.1 \times 10^{17} \approx 1 \times 10^{17}. \end{aligned} \quad (\text{Answer})$$

(When you are asked for the number of states in a certain energy range, first see if that range is small enough to allow this type of approximation.)

## 41.5: Metals: The Occupancy Probability $P(E)$ :

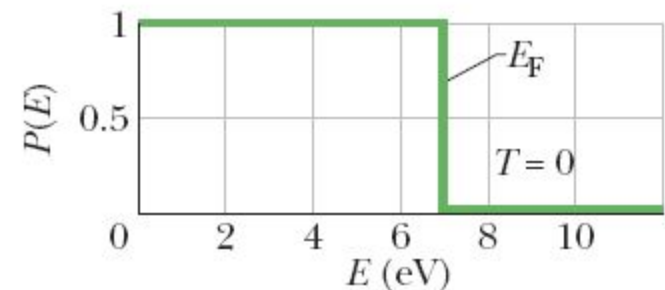


The Fermi energy of a given material is the energy of a quantum state that has the probability 0.5 of being occupied by an electron.

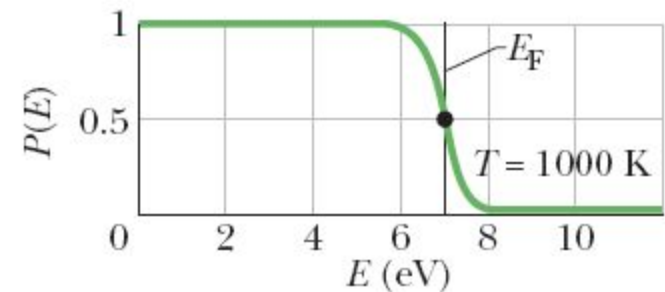
**Fig. 41-7** The occupancy probability  $P(E)$  is the probability that an energy level will be occupied by an electron. (a) At  $T = 0$  K,  $P(E)$  is unity for levels with energies  $E$  up to the Fermi energy  $E_F$  and zero for levels with higher energies. (b) At  $T = 1000$  K, a few electrons whose energies were slightly less than the Fermi energy at  $T = 0$  K move up to states with energies slightly greater than the Fermi energy. The dot on the curve shows that, for  $E = E_F$ ,  $P(E) = 0.5$ .

$$P(E) = \frac{1}{e^{(E-E_F)/kT} + 1} \quad (\text{occupancy probability}),$$

The occupancy probability is high below the Fermi level.



(a)



(b)



## Example, Probability of occupancy of an energy state in a metal :

(a) What is the probability that a quantum state whose energy is 0.10 eV above the Fermi energy will be occupied? Assume a sample temperature of 800 K.

### KEY IDEA

The occupancy probability of any state in a metal can be found from Fermi–Dirac statistics according to Eq. 41-6.

**Calculations:** Let's start with the exponent in Eq. 41-6:

$$\frac{E - E_F}{kT} = \frac{0.10 \text{ eV}}{(8.62 \times 10^{-5} \text{ eV/K})(800 \text{ K})} = 1.45.$$

Inserting this exponent into Eq. 41-6 yields

$$P(E) = \frac{1}{e^{1.45} + 1} = 0.19 \text{ or } 19\%. \quad (\text{Answer})$$

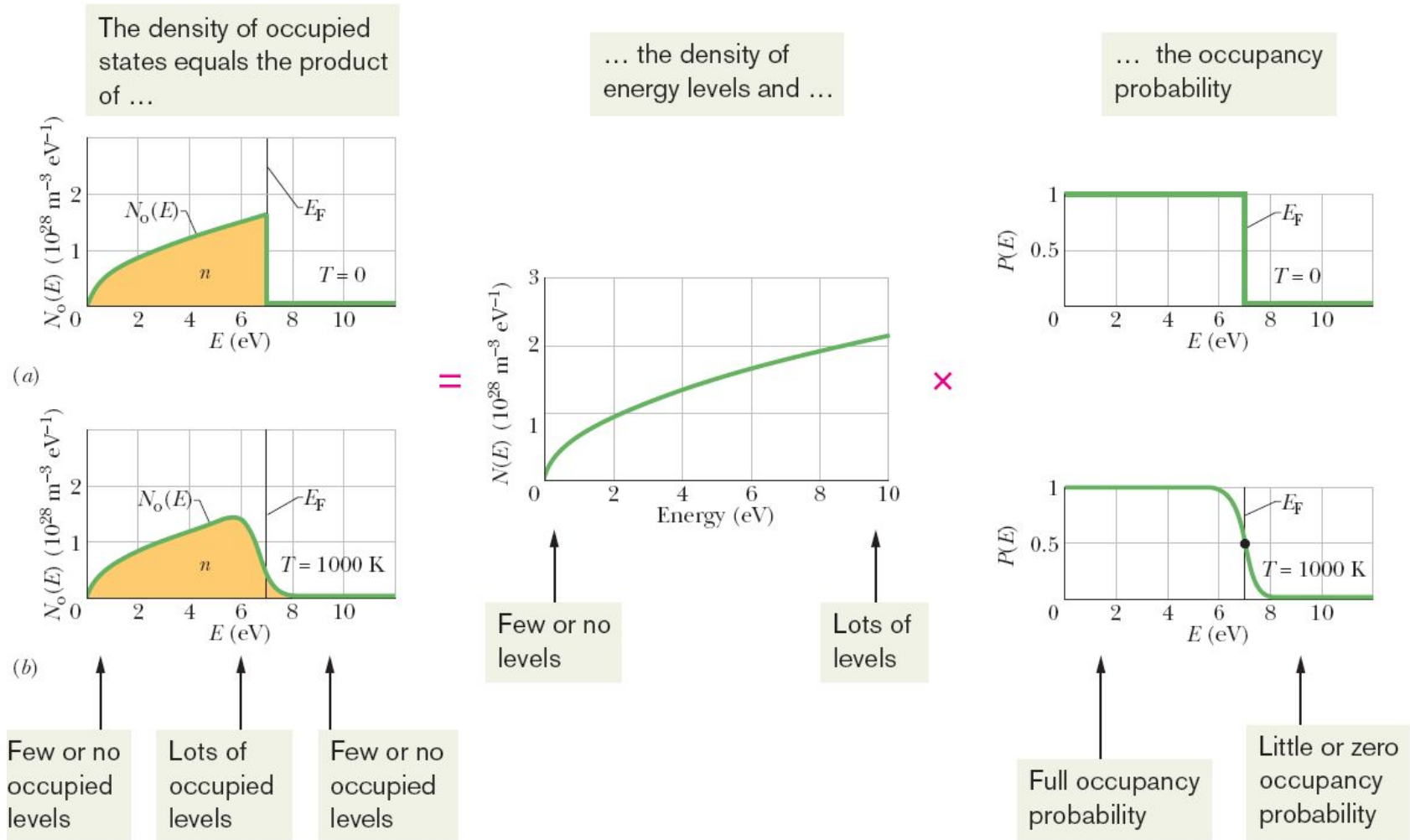
(b) What is the probability of occupancy for a state that is 0.10 eV *below* the Fermi energy?

**Calculation:** The Key Idea of part (a) applies here also except that now the state has an energy *below* the Fermi energy. Thus, the exponent in Eq. 41-6 has the same magnitude we found in part (a) but is negative, and that makes the denominator smaller. Equation 41-6 now yields

$$P(E) = \frac{1}{e^{-1.45} + 1} = 0.81 \text{ or } 81\%. \quad (\text{Answer})$$

For states below the Fermi energy, we are often more interested in the probability that the state is *not* occupied. This probability is just  $1 - P(E)$ , or 19%. Note that it is the same as the probability of occupancy in (a).

# 41.5: Metals: How Many Occupied States Are There?



**Fig. 41-8** (a) The density of occupied states  $N_o(E)$  for copper at absolute zero. The area under the curve is the number density of electrons  $n$ . Note that all states with energies up to the Fermi energy  $E_F = 7 \text{ eV}$  are occupied, and all those with energies above the Fermi energy are vacant. (b) The same for copper at  $T = 1000 \text{ K}$ . Note that only electrons with energies near the Fermi energy have been affected and redistributed.

## Example, Number of occupied states in an energy range in a metal:

A lump of copper (Fermi energy = 7.0 eV) has volume  $2 \times 10^{-9} \text{ m}^3$ . How many occupied states per eV lie in a narrow energy range around 7.0 eV?

### KEY IDEAS

(1) First we want the density of occupied states  $N_o(E)$  as given by Eq. 41-7 ( $N_o(E) = N(E) P(E)$ ). (2) Because we want to evaluate quantities for a narrow energy range around 7.0 eV (the Fermi energy for copper), the occupancy probability  $P(E)$  is 0.50.

**Calculations:** From Fig. 41-6, we see that the density of states at 7 eV is about  $1.8 \times 10^{28} \text{ m}^{-3} \text{ eV}^{-1}$ . Thus, Eq. 41-7 tells us that the density of occupied states is

$$\begin{aligned} N_o(E) &= N(E) P(E) = (1.8 \times 10^{28} \text{ m}^{-3} \text{ eV}^{-1})(0.50) \\ &= 0.9 \times 10^{28} \text{ m}^{-3} \text{ eV}^{-1}. \end{aligned}$$

Next, we write

$$\begin{aligned} \left( \begin{array}{c} \text{number of } \textit{occupied} \\ \text{states per eV at 7 eV} \end{array} \right) &= \left( \begin{array}{c} \text{density of } \textit{occupied} \\ \text{states } N_o(E) \text{ at 7 eV} \end{array} \right) \\ &\quad \times \left( \begin{array}{c} \text{volume } V \\ \text{of sample} \end{array} \right). \end{aligned}$$

Substituting for  $N_o(E)$  and  $V$  gives us

$$\begin{aligned} \left( \begin{array}{c} \text{number of occupied} \\ \text{states per eV} \\ \text{at 7 eV} \end{array} \right) &= (0.9 \times 10^{28} \text{ m}^{-3} \text{ eV}^{-1})(2 \times 10^{-9} \text{ m}^3) \\ &= 1.8 \times 10^{19} \text{ eV}^{-1} \\ &\approx 2 \times 10^{19} \text{ eV}^{-1}. \end{aligned} \quad (\text{Answer})$$

## 41.5: Metals: Calculating the Fermi Energy:

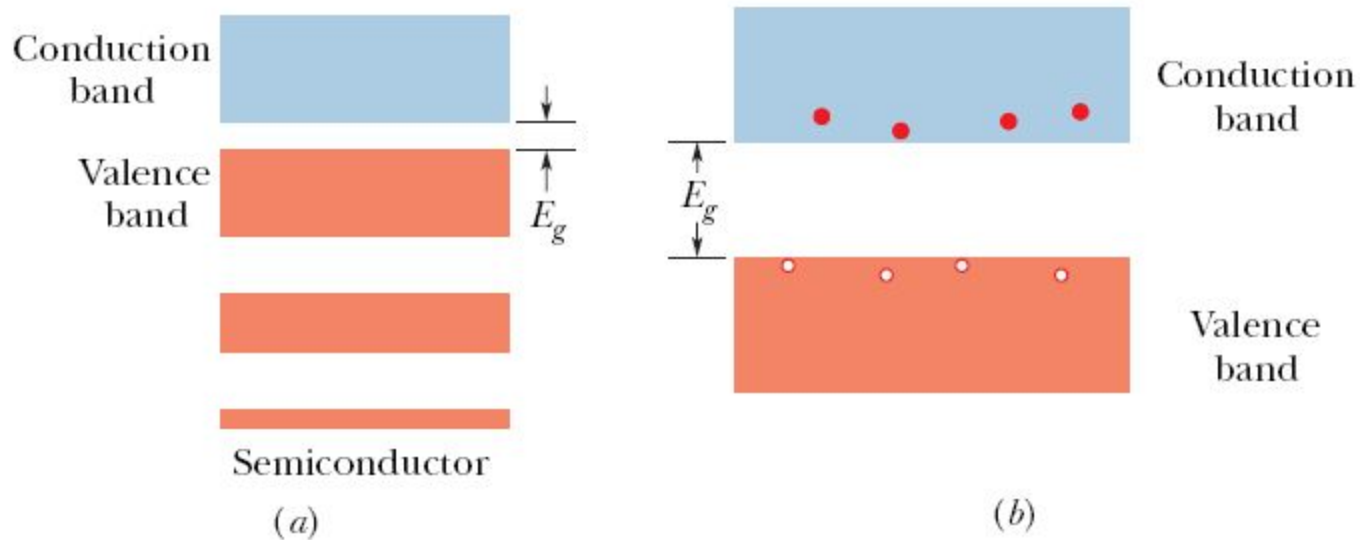
$$n = \int_0^{E_F} N_o(E) dE. \quad (\text{The number of occupied states per unit volume at } T = 0 \text{ K for all energies between } E=0 \text{ and } E=E_F).$$

$$n = \frac{8\sqrt{2}\pi m^{3/2}}{h^3} \int_0^{E_F} E^{1/2} dE \quad (\text{Here } m \text{ is the mass electron mass})$$

$$= \frac{8\sqrt{2}\pi m^{3/2}}{h^3} \frac{2E_F^{3/2}}{3},$$

$$E_F = \left( \frac{3}{16\sqrt{2}\pi} \right)^{2/3} \frac{h^2}{m} n^{2/3} = \frac{0.121h^2}{m} n^{2/3}.$$

## 41.6: Semiconductors:



**Fig. 41-9** (a) The band-gap pattern for a semiconductor. It resembles that of an insulator except that here the energy gap  $E_g$  is much smaller; thus electrons, because of their thermal agitation, have some reasonable probability of being able to jump the gap. (b) Thermal agitation has caused a few electrons to jump the gap from the valence band to the conduction band, leaving an equal number of holes in the valence band.

## 41.6: Semiconductors: Temperature Coefficient of Resistivity:

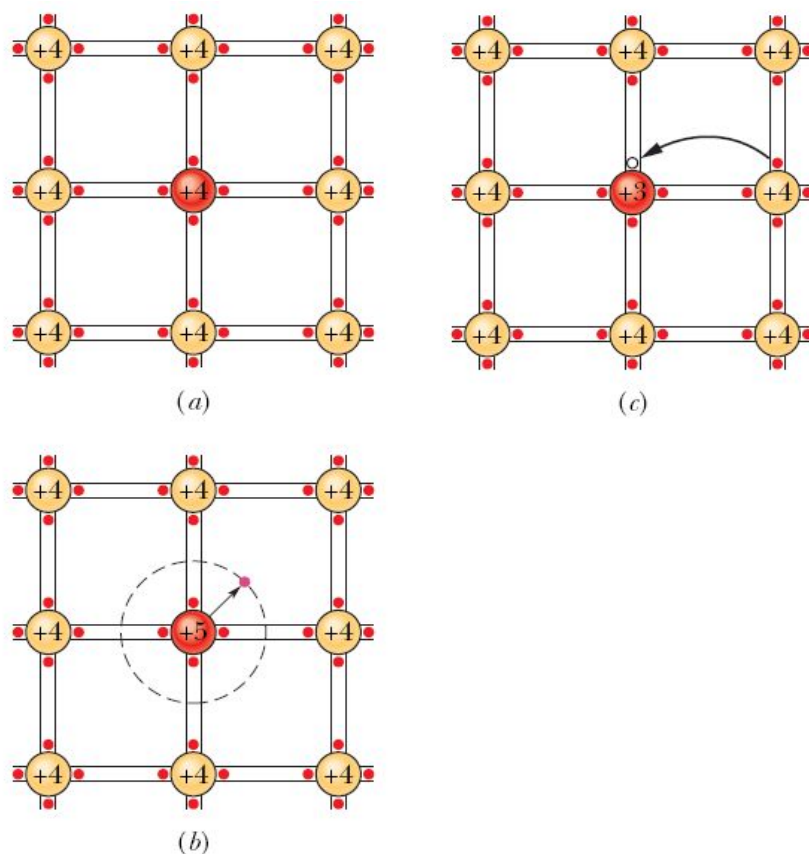
$$\alpha = \frac{1}{\rho} \frac{d\rho}{dT}. \quad (\text{Here } \rho \text{ is the resistivity})$$

The resistivity of copper increases with temperature (that is,  $d\rho/dT > 0$ ) because collisions of copper's charge carriers occur more frequently at higher temperatures. This makes  $\alpha$  positive for copper. The collision frequency also increases with temperature for silicon.

In contrast, the resistivity of silicon actually decreases with temperature ( $d\rho/dT < 0$ ) since the number of charge carriers  $n$  (electrons in the conduction band and holes in the valence band) increases so rapidly with temperature. (More electrons jump the gap from the valence band to the conduction band.) Thus, the fractional change  $\alpha$  is negative for silicon.



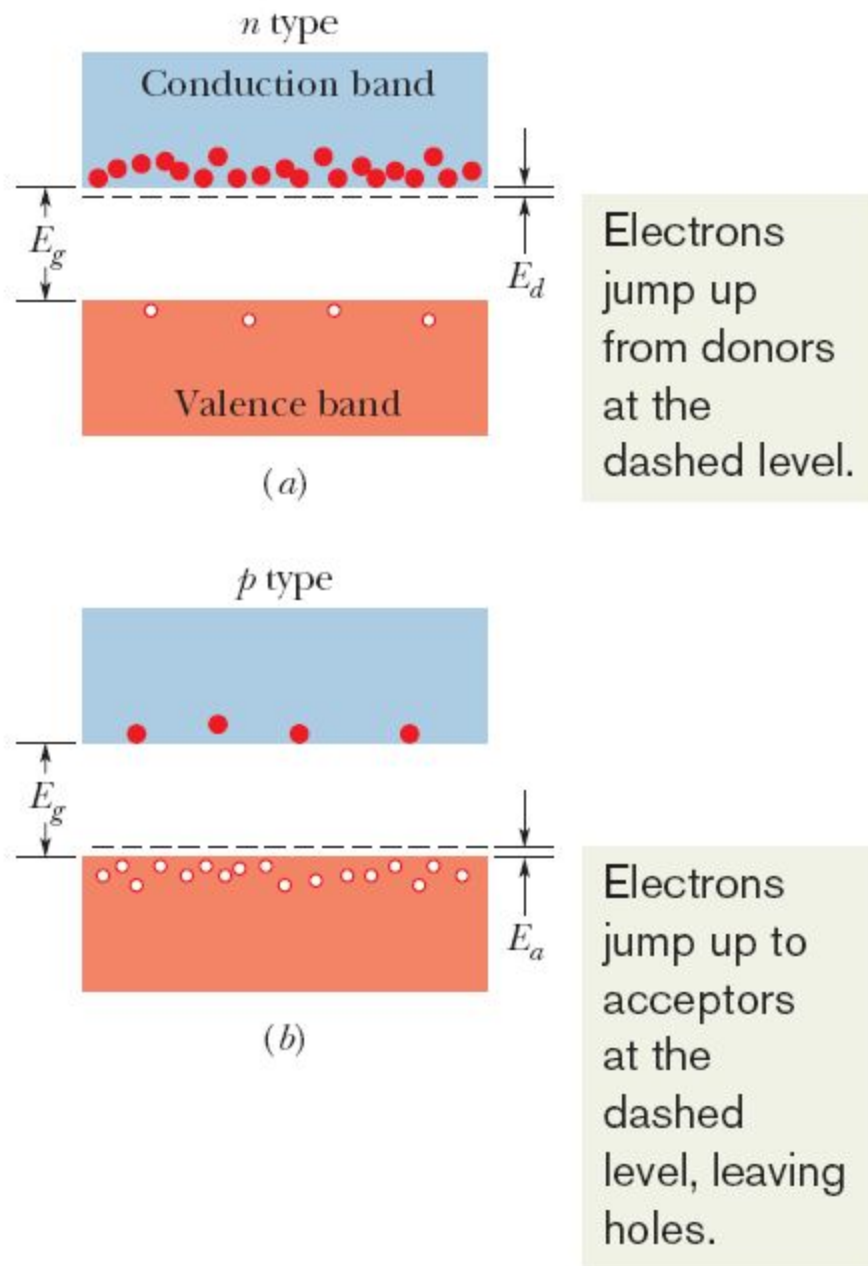
## 41.7: Doped Semiconductors:



**Fig. 41-10** (a) A flattened-out representation of the lattice structure of pure silicon. Each silicon ion is coupled to its four nearest neighbors by a two-electron covalent bond (represented by a pair of red dots between two parallel black lines). The electrons belong to the bond — not to the individual atoms — and form the valence band of the sample. (b) One silicon atom is replaced by a phosphorus atom (valence = 5). The “extra” electron is only loosely bound to its ion core and may easily be elevated to the conduction band, where it is free to wander through the volume of the lattice. (c) One silicon atom is replaced by an aluminum atom (valence = 3). There is now a hole in one of the covalent bonds and thus in the valence band of the sample. The hole can easily migrate through the lattice as electrons from neighboring bonds move in to fill it. Here the hole migrates rightward.

## 41.7: Doped Semiconductors:

**Fig. 41-11** (a) In a doped  $n$ -type semiconductor, the energy levels of donor electrons lie a small interval  $E_d$  below the bottom of the conduction band. Because donor electrons can be easily excited to the conduction band, there are now many more electrons in that band. The valence band contains the same small number of holes as before the dopant was added. (b) In a doped  $p$ -type semiconductor, the acceptor levels lie a small energy interval  $E_a$  above the top of the valence band. There are now many more holes in the valence band. The conduction band contains the same small number of electrons as before the dopant was added. The ratio of majority carriers to minority carriers in both (a) and (b) is very much greater than is suggested by these diagrams.



41.7: Doped Semiconductors:

Table 41 -2

Properties of Two Doped Semiconductors

| Property              | Type of Semiconductor    |                          |
|-----------------------|--------------------------|--------------------------|
|                       | <i>n</i>                 | <i>p</i>                 |
| Matrix material       | Silicon                  | Silicon                  |
| Matrix nuclear charge | +14 <i>e</i>             | +14 <i>e</i>             |
| Matrix energy gap     | 1.2 eV                   | 1.2 eV                   |
| Dopant                | Phosphorus               | Aluminum                 |
| Type of dopant        | Donor                    | Acceptor                 |
| Majority carriers     | Electrons                | Holes                    |
| Minority carriers     | Holes                    | Electrons                |
| Dopant energy gap     | $E_d = 0.045 \text{ eV}$ | $E_a = 0.067 \text{ eV}$ |
| Dopant valence        | 5                        | 3                        |
| Dopant nuclear charge | +15 <i>e</i>             | +13 <i>e</i>             |
| Dopant net ion charge | + <i>e</i>               | − <i>e</i>               |



## Example, Doping silicon with phosphorus:

The number density  $n_0$  of conduction electrons in pure silicon at room temperature is about  $10^{16} \text{ m}^{-3}$ . Assume that, by doping the silicon lattice with phosphorus, we want to increase this number by a factor of a million ( $10^6$ ). What fraction of silicon atoms must we replace with phosphorus atoms? (Recall that at room temperature, thermal agitation is so effective that essentially every phosphorus atom donates its “extra” electron to the conduction band.)

**Number of phosphorus atoms:** Because each phosphorus atom contributes one conduction electron and because we want the total number density of conduction electrons to be  $10^6 n_0$ , the number density of phosphorus atoms  $n_P$  must be given by

$$10^6 n_0 = n_0 + n_P.$$

Then

$$\begin{aligned} n_P &= 10^6 n_0 - n_0 \approx 10^6 n_0 \\ &= (10^6)(10^{16} \text{ m}^{-3}) = 10^{22} \text{ m}^{-3}. \end{aligned}$$

This tells us that we must add  $10^{22}$  atoms of phosphorus per cubic meter of silicon.

**Fraction of silicon atoms:** We can find the number density  $n_{\text{Si}}$  of silicon atoms in pure silicon (before the doping) from Eq. 41-4, which we can write as

$$\begin{aligned} &\left( \begin{array}{c} \text{number of atoms} \\ \text{in sample} \end{array} \right) \\ &= \frac{(\text{silicon density})(\text{sample volume } V)}{(\text{silicon molar mass } M_{\text{Si}})/N_A}. \end{aligned}$$

Dividing both sides by the sample volume  $V$  to get the number density of silicon atoms  $n_{\text{Si}}$  on the left, we then have

$$n_{\text{Si}} = \frac{(\text{silicon density})N_A}{M_{\text{Si}}}.$$

Appendix F tells us that the density of silicon is  $2.33 \text{ g/cm}^3$  ( $= 2330 \text{ kg/m}^3$ ) and the molar mass of silicon is  $28.1 \text{ g/mol}$  ( $= 0.0281 \text{ kg/mol}$ ). Thus, we have

$$\begin{aligned} n_{\text{Si}} &= \frac{(2330 \text{ kg/m}^3)(6.02 \times 10^{23} \text{ atoms/mol})}{0.0281 \text{ kg/mol}} \\ &= 5 \times 10^{28} \text{ atoms/m}^3 = 5 \times 10^{28} \text{ m}^{-3}. \end{aligned}$$

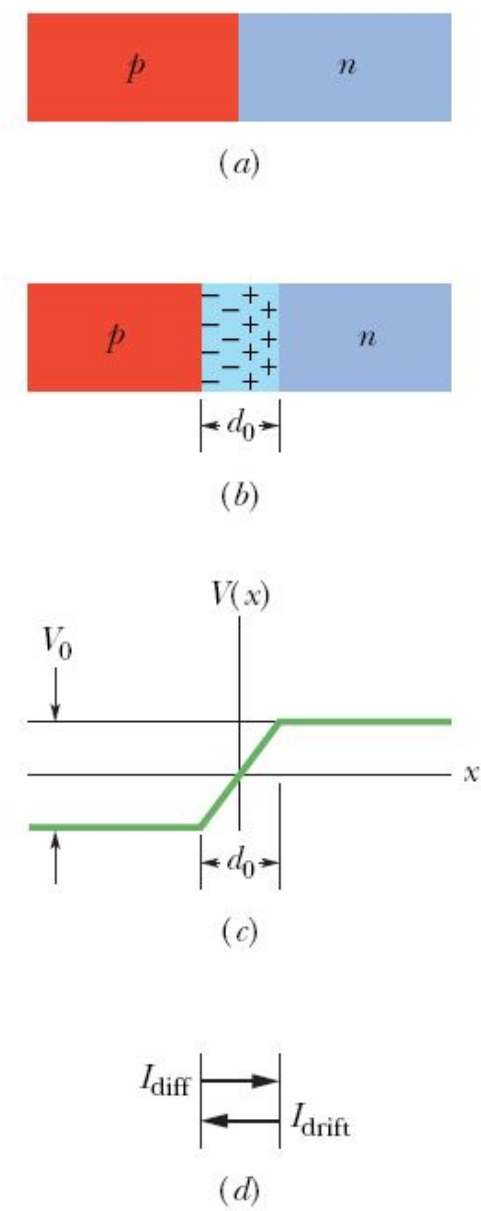
The fraction we seek is approximately

$$\frac{n_P}{n_{\text{Si}}} = \frac{10^{22} \text{ m}^{-3}}{5 \times 10^{28} \text{ m}^{-3}} = \frac{1}{5 \times 10^6}. \quad (\text{Answer})$$

If we replace only *one silicon atom in five million* with a phosphorus atom, the number of electrons in the conduction band will be increased by a factor of a million.

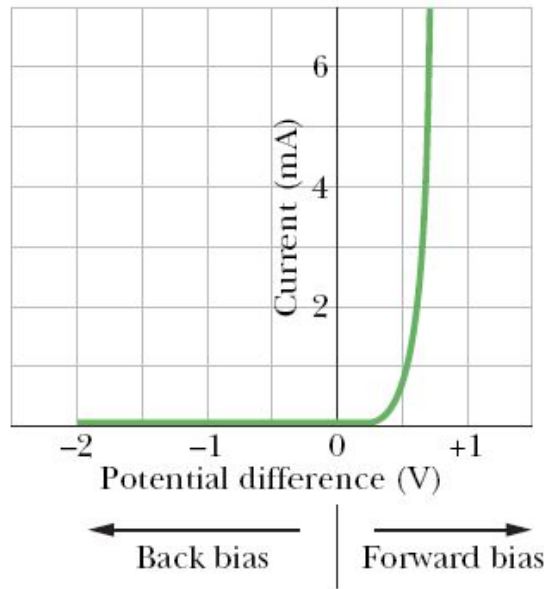
How can such a tiny admixture of phosphorus have what seems to be such a big effect? The answer is that, although the effect is very significant, it is not “big.” The number density of conduction electrons was  $10^{16} \text{ m}^{-3}$  before doping and  $10^{22} \text{ m}^{-3}$  after doping. For copper, however, the conduction-electron number density (given in Table 41-1) is about  $10^{29} \text{ m}^{-3}$ . Thus, even after doping, the number density of conduction electrons in silicon remains much less than that of a typical metal, such as copper, by a factor of about  $10^7$ .

41.8: The *p-n junction*:



**Fig. 41-12** (a) A *p-n* junction. (b) Motions of the majority charge carriers across the junction plane uncover a space charge associated with uncompensated donor ions (to the right of the plane) and acceptor ions (to the left). (c) Associated with the space charge is a contact potential difference  $V_0$  across  $d_0$ . (d) The diffusion of majority carriers (both electrons and holes) across the junction plane produces a diffusion current  $I_{\text{diff}}$ . (In a real *p-n* junction, the boundaries of the depletion zone would not be sharp, as shown here, and the contact potential curve (c) would be smooth, with no sharp corners.)

## 41.9: The Junction Rectifier:



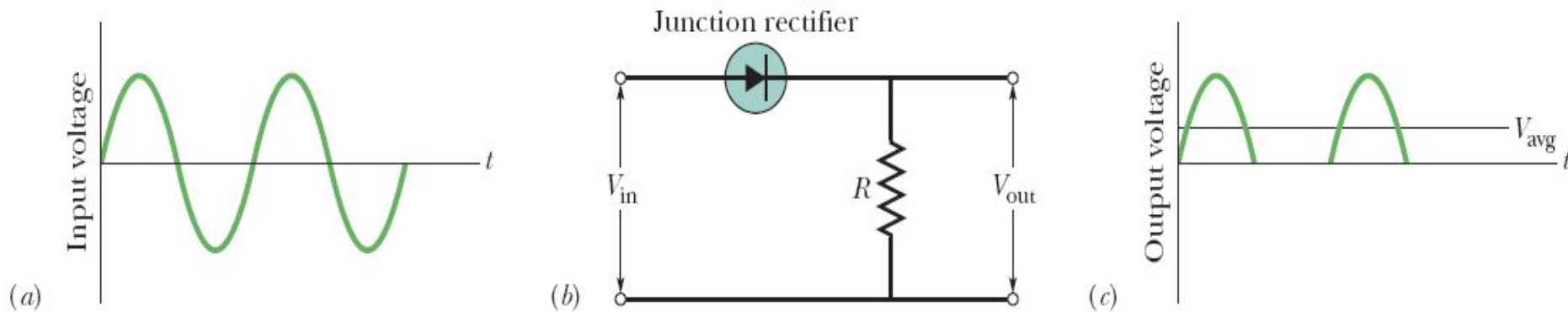
**Fig. 41-13** A current–voltage plot for a  $p$ - $n$  junction, showing that the junction is highly conducting when forward-biased and essentially nonconducting when back-biased.

If a potential difference is applied across a  $p$ - $n$  junction in one direction (here labeled and “Forward bias”), there will be a current through the junction.

However, if the direction of the potential Difference is reversed, there will be approximately zero current through the junction.

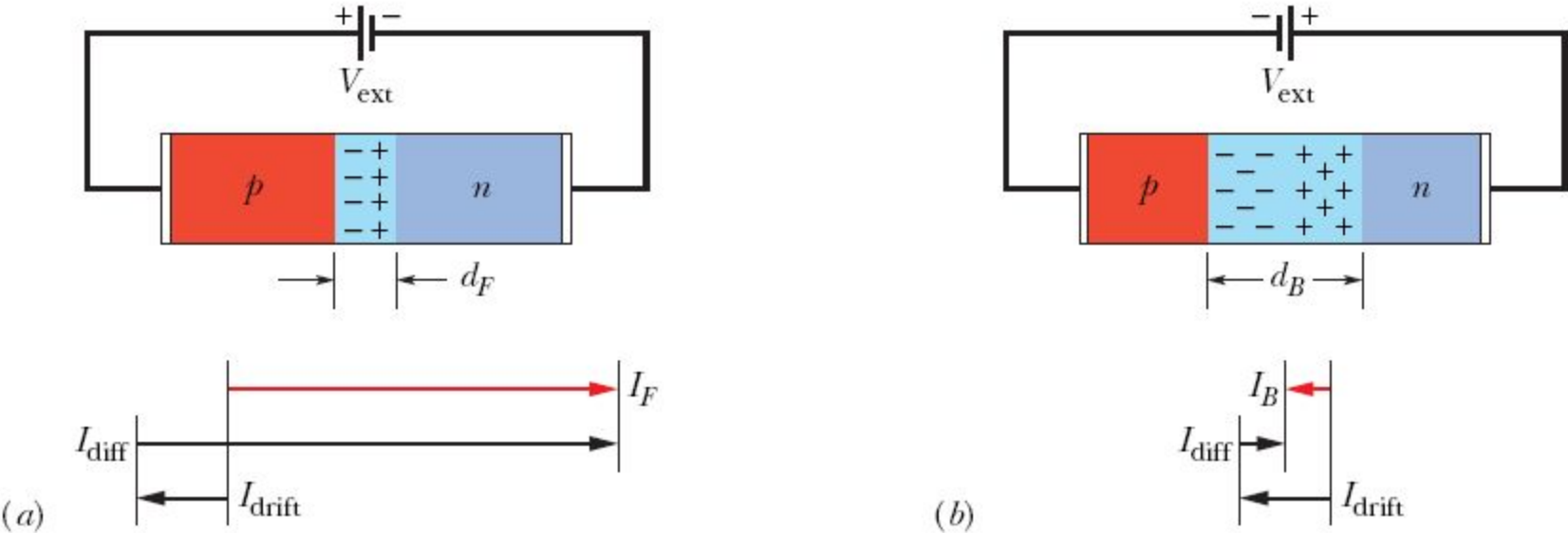


## 41.9: The Junction Rectifier, An Example:



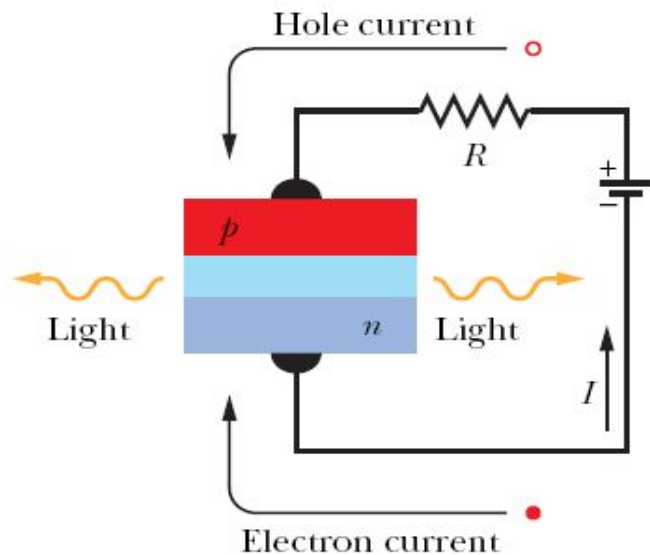
**Fig. 41-14** A  $p$ - $n$  junction connected as a junction rectifier. The action of the circuit in (b) is to pass the positive half of the input wave form in (a) but to suppress the negative half. The average potential of the input wave form is zero; that of the output wave form in (c) has a positive value  $V_{avg}$ .

41.9: The Junction Rectifier, An Example:



**Fig. 41-15** (a) The forward-bias connection of a  $p$ - $n$  junction, showing the narrowed depletion zone and the large forward current  $I_F$ . (b) The back-bias connection, showing the widened depletion zone and the small back current  $I_B$ .

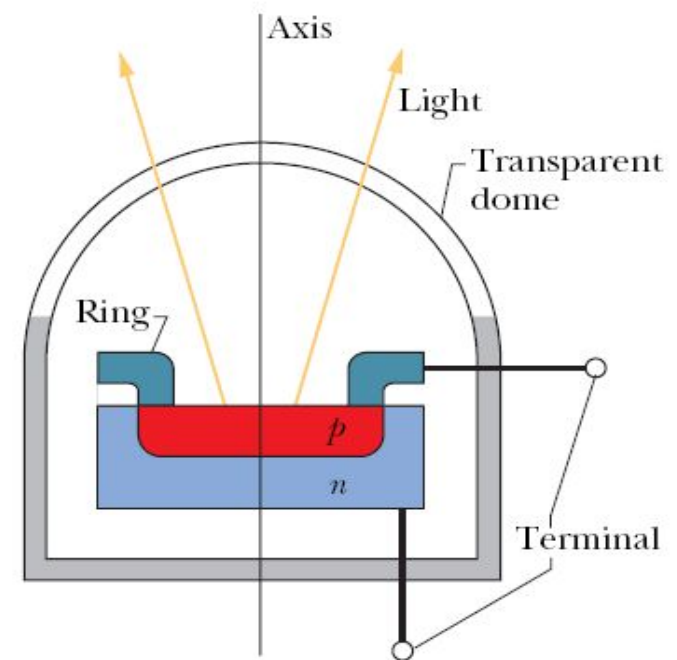
## 41.10: The Light-Emitting Diode (LED):



**Fig. 41-16** A forward-biased  $p$ - $n$  junction, showing electrons being injected into the  $n$ -type material and holes into the  $p$ -type material. (Holes move in the conventional direction of the current  $I$ , equivalent to electrons moving in the opposite direction.) Light is emitted from the narrow depletion zone each time an electron and a hole combine across that zone.

In some semiconductors, including gallium arsenide, the energy can be emitted as a photon of energy  $hf$  at wavelength

$$\lambda = \frac{c}{f} = \frac{c}{E_g/h} = \frac{hc}{E_g}.$$



**Fig. 41-17** Cross section of an LED (the device has rotational symmetry about the central axis). The  $p$ -type material, which is thin enough to transmit light, is in the form of a circular disk. A connection is made to the  $p$ -type material through a circular metal ring that touches the disk at its periphery. The depletion zone between the  $n$ -type material and the  $p$ -type material is not shown.

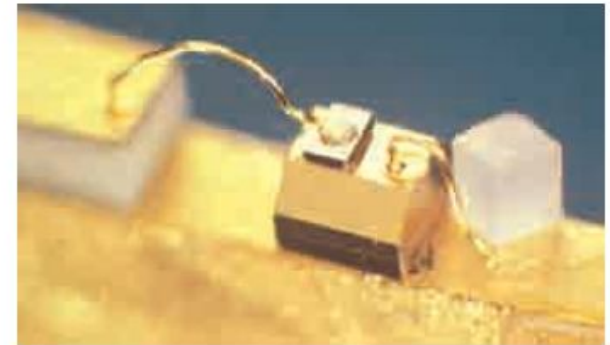
## 41.10: The Light-Emitting Diode (LED):

### Photodiode

Shining light on a suitably arranged p-n junction can produce a current in a circuit that includes the junction. This is the basis for the photodiode.

### The Junction Laser

A p-n junction can act as a junction laser, where its light output being highly coherent and much more sharply defined in wavelength than light from an LED.



**Fig. 41-18** A junction laser developed at the AT&T Bell Laboratories. The cube at the right is a grain of salt. (Courtesy AT&T Archives and History Center, Warren, NJ)

### Example, Light Emitting Diode (LED):

An LED is constructed from a  $p$ - $n$  junction based on a certain Ga-As-P semiconducting material whose energy gap is 1.9 eV. What is the wavelength of the emitted light?

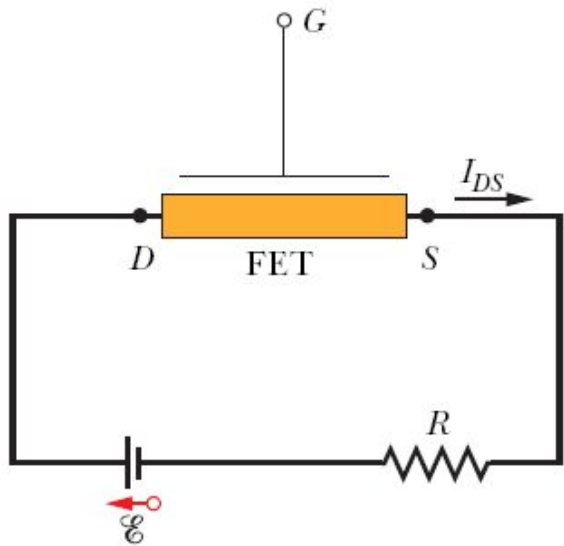
**Calculation:** For jumps from the bottom of the conduction band to the top of the valence band, Eq. 41-11 tells us

$$\begin{aligned}\lambda &= \frac{hc}{E_g} = \frac{(6.63 \times 10^{-34} \text{ J}\cdot\text{s})(3.00 \times 10^8 \text{ m/s})}{(1.9 \text{ eV})(1.60 \times 10^{-19} \text{ J/eV})} \\ &= 6.5 \times 10^{-7} \text{ m} = 650 \text{ nm.} \quad (\text{Answer})\end{aligned}$$

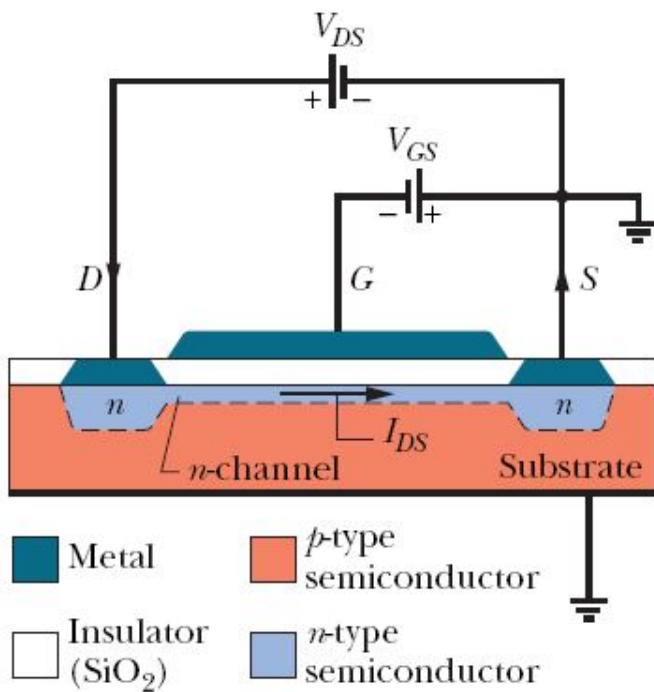
Light of this wavelength is red.

# 41.11: The Transistor:

A transistor is a three-terminal semiconducting device that can be used to amplify input signals.



**Fig. 41-19** A circuit containing a generalized field-effect transistor through which electrons flow from the source terminal  $S$  to the drain terminal  $D$ . (The conventional current  $I_{DS}$  is in the opposite direction.) The magnitude of  $I_{DS}$  is controlled by the electric field set up within the FET by a potential applied to  $G$ , the gate terminal.



**Fig. 41-20** A particular type of field-effect transistor known as a MOSFET. The magnitude of the drain-to-source conventional current  $I_{DS}$  through the  $n$  channel is controlled by the potential difference  $V_{GS}$  applied between the source  $S$  and the gate  $G$ . A depletion zone that exists between the  $n$ -type material and the  $p$ -type substrate is not shown.