

Chapter 30

Induction and Inductance

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30-1 Faraday's Law and Lenz's Law

Learning Objectives

- 30.01** Identify that the amount of magnetic field piercing a surface (not skimming along the surface) is the magnetic flux Φ through the surface.
- 30.02** Identify that an area vector for a flat surface is a vector that is perpendicular to the surface and that has a magnitude equal to the area of the surface.
- 30.03** Identify that any surface can be divided into area elements (patch elements) that are each small enough and flat enough for an area vector $d\mathbf{A}$ to be assigned to it, with the vector perpendicular to the element and having a magnitude equal to the area of the element.
- 30.04** Calculate the magnetic flux Φ through a surface by integrating the dot product of the magnetic field vector \mathbf{B} and the area vector $d\mathbf{A}$ (for patch elements) over the surface, in magnitude-angle notation and unit-vector notation.
- 30.05** Identify that a current is induced in a conducting loop while the number of magnetic field lines intercepted by the loop is changing.

30-1 Faraday's Law and Lenz's Law

Learning Objectives (Contd.)

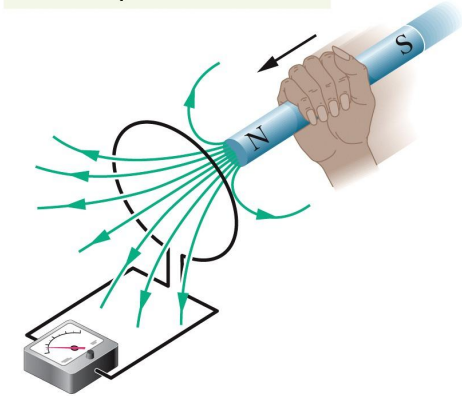
- 30.06** Identify that an induced current in a conducting loop is driven by an induced emf.
- 30.07** Apply Faraday's law, which is the relationship between an induced emf in a conducting loop and the rate at which magnetic flux through the loop changes.
- 30.08** Extend Faraday's law from a loop to a coil with multiple loops.
- 30.09** Identify the three general ways in which the magnetic flux through a coil can change.
- 30.10** Use a right-hand rule for Lenz's law to determine the direction of induced emf and induced current in a conducting loop.
- 30.11** Identify that when a magnetic flux through a loop changes, the induced current in the loop sets up a magnetic field to oppose that change.
- 30.12** If an emf is induced in a conducting loop containing a battery, determine the net emf and calculate the corresponding current in the loop.

30-1 Faraday's Law and Lenz's Law

First Experiment. Figure shows a conducting loop connected to a sensitive ammeter. Because there is no battery or other source of emf included, there is no current in the circuit. However, if we move a bar magnet toward the loop, a current suddenly appears in the circuit. The current disappears when the magnet stops moving. If we then move the magnet away, a current again suddenly appears, but now in the opposite direction. If we experimented for a while, we would discover the following:

1. A current appears only if there is relative motion between the loop and the magnet (one must move relative to the other); the current disappears when the relative motion between them ceases.
2. Faster motion of the magnet produces a greater current.
3. If moving the magnet's north pole toward the loop causes, say, clockwise current, then moving the north pole away causes counterclockwise current. Moving the south pole toward or away from the loop also causes currents, but in the reversed directions from the north pole effects.

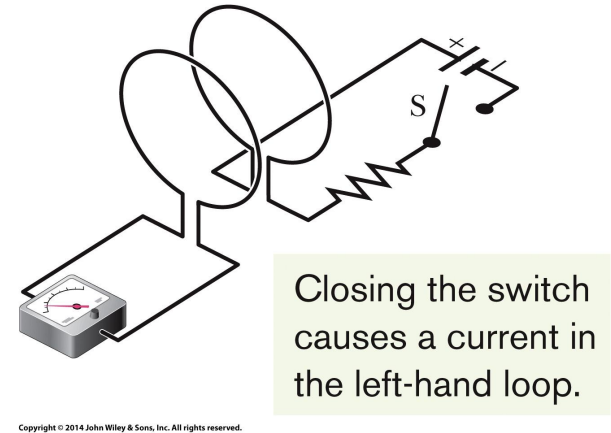
The magnet's motion creates a current in the loop.



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30-1 Faraday's Law and Lenz's Law

Second Experiment. For this experiment we use the apparatus shown in the figure, with the two conducting loops close to each other but not touching. If we close switch S to turn on a current in the right-hand loop, the meter suddenly and briefly registers a current—an induced current—in the left-hand loop. If the switch remains closed, no further current is observed. If we then open the switch, another sudden and brief induced current appears in the left-hand loop, but in the opposite direction.

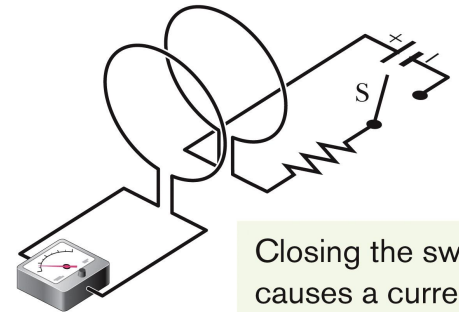


We get an induced current (from an induced emf) only when the current in the right-hand loop is changing (either turning on or turning off) and not when it is constant (even if it is large). The induced emf and induced current in these experiments are apparently caused when something changes — but what is that “something”? Faraday knew.

30-1 Faraday's Law and Lenz's Law

Faraday's Law of Induction

Faraday realized that an emf and a current can be induced in a loop, as in our two experiments, by changing the amount of magnetic field passing through the loop. He further realized that the “amount of magnetic field” can be visualized in terms of the magnetic field lines passing through the loop.



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The magnetic flux Φ_B through an area A in a magnetic field \mathbf{B} is defined as

$$\Phi_B = \int \vec{B} \cdot d\vec{A}$$

where the integral is taken over the area. The SI unit of magnetic flux is the weber, where $1 \text{ Wb} = 1 \text{ T} \cdot \text{m}^2$.

If \mathbf{B} is perpendicular to the area and uniform over it, the flux is

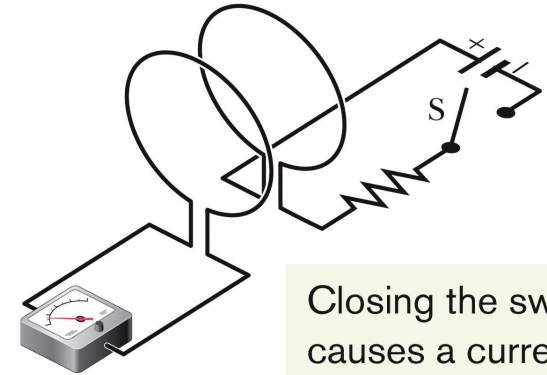
$$\Phi_B = BA \quad (\vec{B} \perp \text{area } A, \vec{B} \text{ uniform}).$$

30-1 Faraday's Law and Lenz's Law

Faraday's Law of Induction



The magnitude of the emf \mathcal{E} induced in a conducting loop is equal to the rate at which the magnetic flux Φ_B through that loop changes with time.



Closing the switch causes a current in the left-hand loop.

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Faraday's Law. With the notion of magnetic flux, we can state Faraday's law in a more quantitative and useful way:

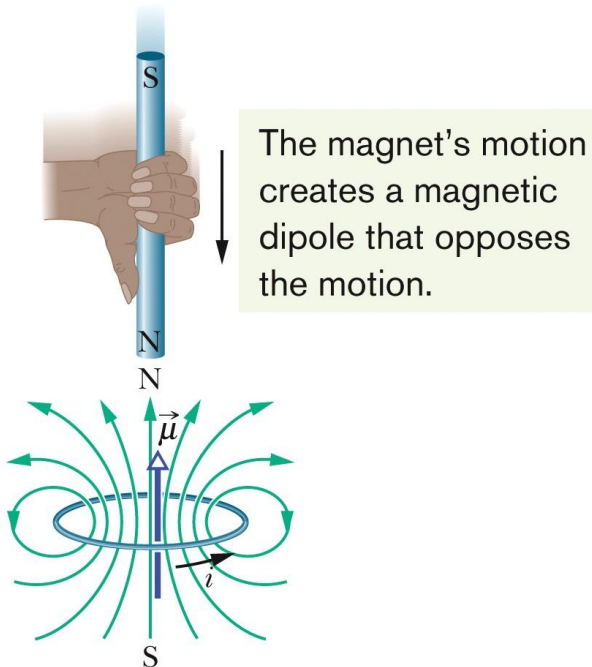
$$\mathcal{E} = -\frac{d\Phi_B}{dt}$$

the induced emf tends to oppose the flux change and the minus sign indicates this opposition. This minus sign is referred to as Lenz's Law.

30-1 Faraday's Law and Lenz's Law

Lenz's Law

An induced current has a direction such that the magnetic field due to this induced current opposes the change in the magnetic flux that induces the current. The induced emf has the same direction as the induced current.



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Lenz's law at work. As the magnet is moved toward the loop, a current is induced in the loop. The current produces its own magnetic field, with magnetic dipole moment $\vec{\mu}$ oriented so as to oppose the motion of the magnet. Thus, the induced current must be counterclockwise as shown.

30-1 Faraday's Law and Lenz's Law

Lenz's Law

Increasing the external field \vec{B} induces a current with a field \vec{B}_{ind} that *opposes the change*.

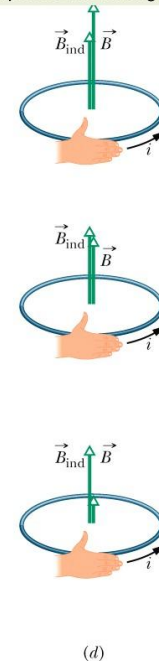
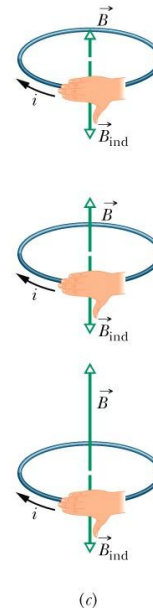
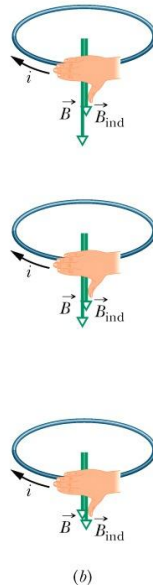
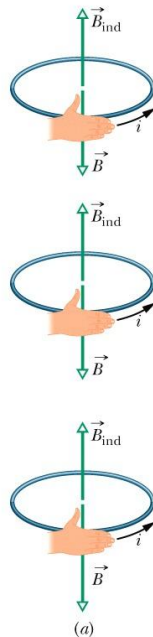
Decreasing the external field \vec{B} induces a current with a field \vec{B}_{ind} that *opposes the change*.

Increasing the external field \vec{B} induces a current with a field \vec{B}_{ind} that *opposes the change*.

Decreasing the external field \vec{B} induces a current with a field \vec{B}_{ind} that *opposes the change*.

The induced current creates this field, trying to offset the change.

The fingers are in the current's direction; the thumb is in the induced field's direction.



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The direction of the current i induced in a loop is such that the current's magnetic field \vec{B}_{ind} opposes the change in the magnetic field \vec{B} inducing i . The field \vec{B}_{ind} is always directed opposite an increasing field \vec{B} (a, c) and in the same direction as a decreasing field \vec{B} (b, d). The curled – straight right-hand rule gives the direction of the induced current based on the direction of the induced field.

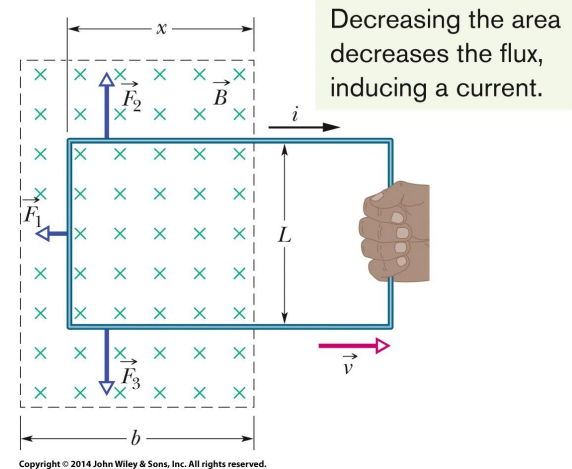
30-2 Induction and Energy Transfer

Learning Objectives

- 30.13** For a conducting loop pulled into or out of a magnetic field, calculate the rate at which energy is transferred to thermal energy.
- 30.14** Apply the relationship between an induced current and the rate at which it produces thermal energy.
- 30.15** Describe eddy currents.

30-2 Induction and Energy Transfer

In the figure, a rectangular loop of wire of width L has one end in a uniform external magnetic field that is directed perpendicularly into the plane of the loop. This field may be produced, for example, by a large electromagnet. The dashed lines in the figure show the assumed limits of the magnetic field; the fringing of the field at its edges is neglected. You are to pull this loop to the right at a constant velocity \mathbf{v} .



Flux change: Therefore, in the figure a magnetic field and a conducting loop are in relative motion at speed v and the flux of the field through the loop is changing with time (here the flux is changing as the area of the loop still in the magnetic field is changing).

Rate of Work: To pull the loop at a constant velocity \mathbf{v} , you must apply a constant force \mathbf{F} to the loop because a magnetic force of equal magnitude but opposite direction acts on the loop to oppose you. The rate at which you do work — that is, the power — is then

$$P = Fv,$$

where F is the magnitude of the force you apply to the loop.

30-2 Induction and Energy Transfer

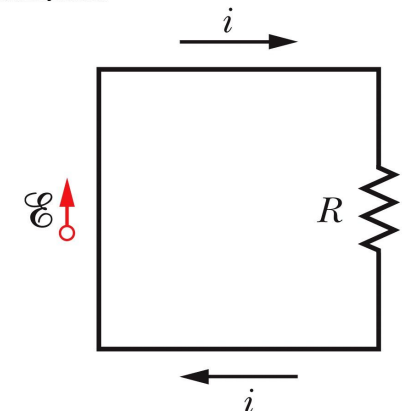
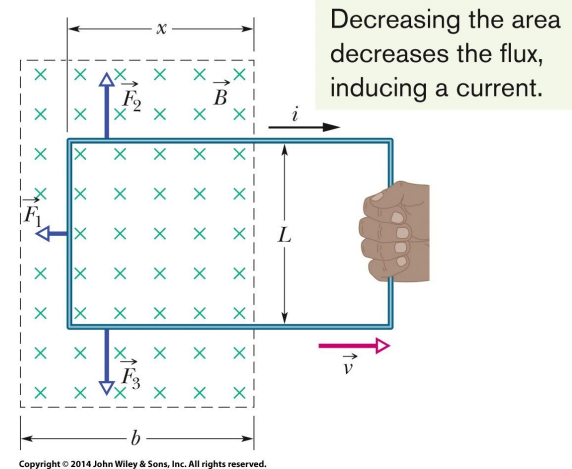
Induced emf: To find the current, we first apply Faraday's law. When x is the length of the loop still in the magnetic field, the area of the loop still in the field is Lx . Then, the magnitude of the flux through the loop is

$$\Phi_B = BA = BLx.$$

As x decreases, the flux decreases. Faraday's law tells us that with this flux i decrease, an *emf* is induced in the loop. We can write the magnitude of this emf as

$$\mathcal{E} = \frac{d\Phi_B}{dt} = \frac{d}{dt} BLx = BL \frac{dx}{dt} = BLv,$$

in which we have replaced dx/dt with v , the speed at which the loop moves.



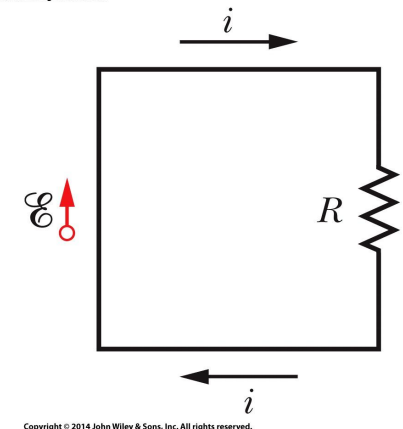
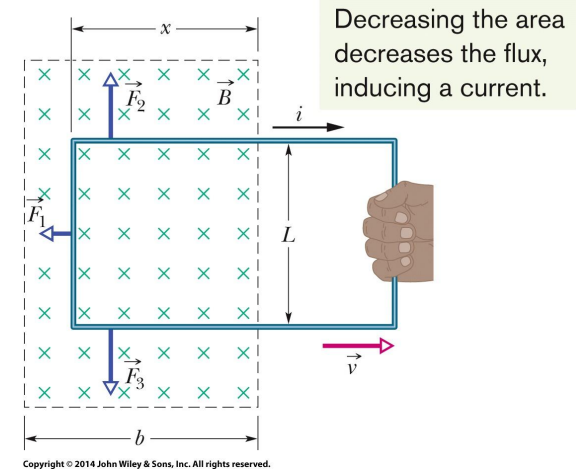
A circuit diagram for the loop of above figure while the loop is moving.

30-2 Induction and Energy Transfer

Induced Current: Figure (bottom) shows the loop as a circuit: induced *emf* is represented on the left, and the collective resistance R of the loop is represented on the right. To find the magnitude of the induced current, we can apply the equation $i = \mathcal{E}/R$, which gives

$$i = \frac{BLv}{R}.$$

In the Fig. (top), the deflecting forces acting on the three segments of the loop are marked \mathbf{F}_1 , \mathbf{F}_2 , and \mathbf{F}_3 . Note, however, that from the symmetry, forces \mathbf{F}_2 and \mathbf{F}_3 are equal in magnitude and cancel. This leaves only force \mathbf{F}_1 , which is directed opposite your force \mathbf{F} on the loop and thus is the force opposing you.



A circuit diagram for the loop of above figure while the loop is moving.

30-2 Induction and Energy Transfer

So, $\mathbf{F} = -\mathbf{F}_1$, the magnitude of \mathbf{F}_1 thus

$$F = F_1 = iLB \sin 90^\circ = iLB. \quad (\text{from } \vec{F}_d = i\vec{L} \times \vec{B}.)$$

where the angle between \mathbf{B} and the length vector \mathbf{L} for the left segment is 90° . This gives us

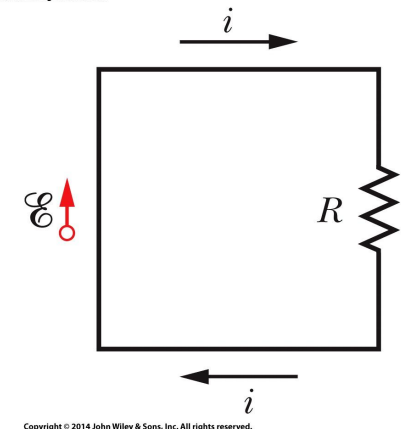
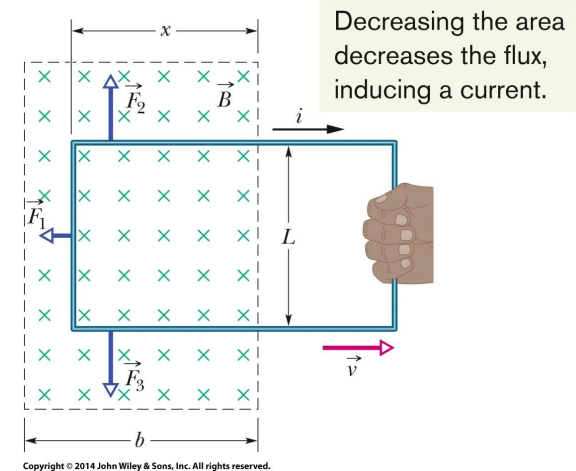
$$F = \frac{B^2 L^2 v}{R}.$$

Because B , L , and R are constants, the speed v at which you move the loop is constant if the magnitude F of the force you apply to the loop is also constant.

Rate of Work: We find the rate at which you do work on the loop as you pull it from the magnetic field:

$$P = Fv = \frac{B^2 L^2 v^2}{R}$$

NOTE: The work that you do in pulling the loop through the magnetic field appears as **thermal energy** in the loop.



A circuit diagram for the loop of above figure while the loop is moving.

30-3 Induced Electric Field

Learning Objectives

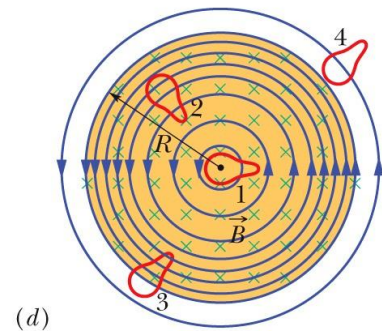
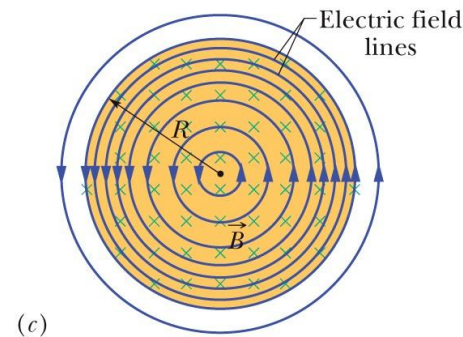
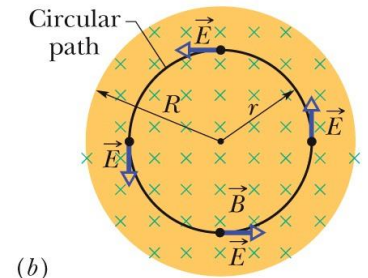
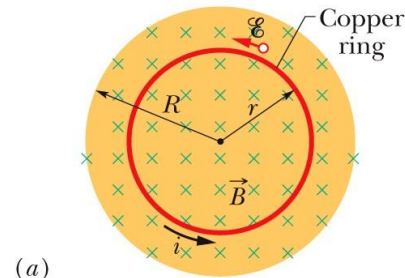
30.16 Identify that a changing magnetic field induces an electric field, regardless of whether there is a conducting loop.

30.17 Apply Faraday's law to relate the electric field \mathbf{E} induced along a closed path (whether it has conducting material or not) to the rate of change $d\Phi/dt$ of the magnetic flux encircled by the path.

30.18 Identify that an electric potential cannot be associated with an induced electric field.

30-3 Induced Electric Field

(a) If the magnetic field increases at a steady rate, a constant induced current appears, as shown, in the copper ring of radius r . (b) An induced electric field exists even when the ring is removed; the electric field is shown at four points. (c) The complete picture of the induced electric field, displayed as field lines. (d) Four similar closed paths that enclose identical areas. Equal emfs are induced around paths 1 and 2, which lie entirely within the region of changing magnetic field. A smaller emf is induced around path 3, which only partially lies in that region. No net emf is induced around path 4, which lies entirely outside the magnetic field.



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Therefore, an emf is induced by a changing magnetic flux even if the loop through which the flux is changing is not a physical conductor but an imaginary line. The changing magnetic field induces an electric field \mathbf{E} at every point of such a loop; the induced *emf* is related to \mathbf{E} by

$$\mathcal{E} = \oint \vec{E} \cdot d\vec{s}.$$

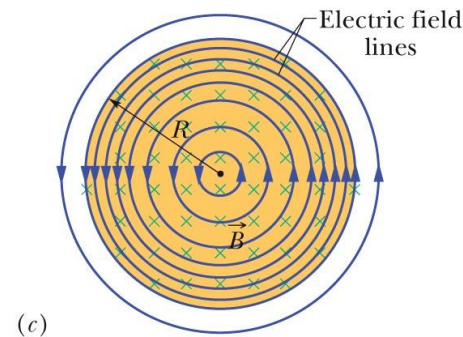
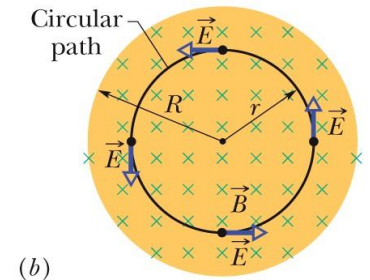
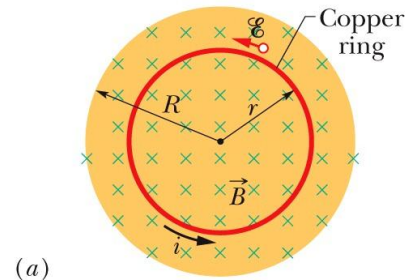
30-3 Induced Electric Field

Using the induced electric field, we can write Faraday's law in its most general form as

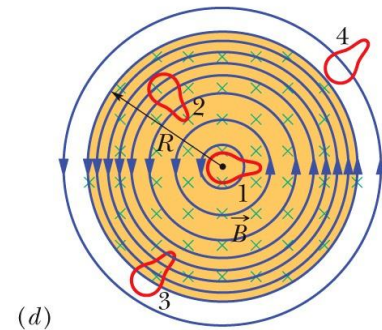
$$\oint \vec{E} \cdot d\vec{s} = - \frac{d\Phi_B}{dt}$$



A changing magnetic field produces an electric field.



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Electric Potential: Induced electric fields are produced not by static charges but by a changing magnetic flux. Therefore,



Electric potential has meaning only for electric fields that are produced by static charges; it has no meaning for electric fields that are produced by induction.

30-4 Inductors and Inductance

Learning Objectives

30.19 Identify an inductor.

30.20 For an inductor, apply the relationship between inductance L , total flux $N\Phi$, and current i .

30.21 For a solenoid, apply the relationship between the inductance per unit length L/l , the area A of each turn, and the number of turns per unit length n .

30-4 Inductors and Inductance

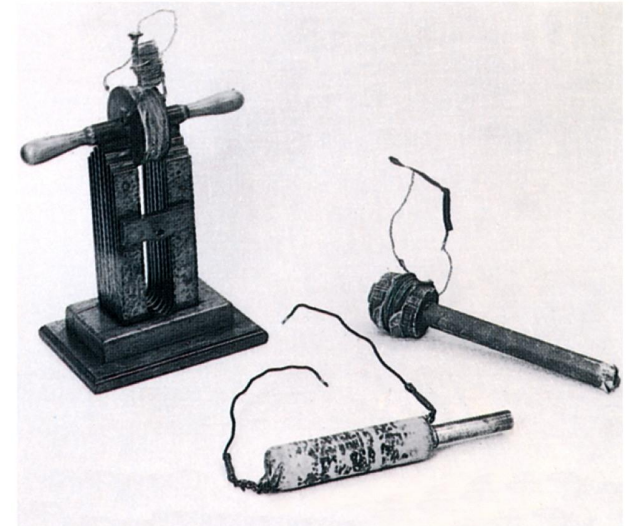
An inductor is a device that can be used to produce a known magnetic field in a specified region. If a current i is established through each of the N windings of an inductor, a magnetic flux Φ_B links those windings. The inductance L of the inductor is

$$L = \frac{N\Phi_B}{i}$$

The SI unit of inductance is the *henry* (H), where $1 \text{ henry} = 1H = 1T \cdot m^2/A$.

The inductance per unit length near the middle of a long solenoid of cross-sectional area A and n turns per unit length is

$$\frac{L}{l} = \mu_0 n^2 A$$



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The crude inductors with which Michael Faraday discovered the law of induction. In those days amenities such as insulated wire were not commercially available. It is said that Faraday insulated his wires by wrapping them with strips cut from one of his wife's petticoats.

30-5 Self-Induction

Learning Objectives

30.22 Identify that an induced *emf* appears in a coil when the current through the coil is changing.

30.23 Apply the relationship between the induced *emf* in a coil, the coil's inductance L , and the rate di/dt at which the current is changing.

30.24 When an *emf* is induced in a coil because the current in the coil is changing, determine the direction of the *emf* by using Lenz's law to show that the *emf* always opposes the change in the current, attempting to maintain the initial current.

30-5 Self-Induction

If two coils — which we can now call inductors — are near each other, a current i in one coil produces a magnetic flux Φ_B through the second coil. We have seen that if we change this flux by changing the current, an induced *emf* appears in the second coil according to Faraday's law. An induced *emf* appears in the first coil as well. This process (see Figure) is called self-induction, and the *emf* that appears is called a self-induced *emf*. It obeys Faraday's law of induction just as other induced emfs do. For any inductor,

$$N\Phi_B = Li.$$

Faraday's law tells us that

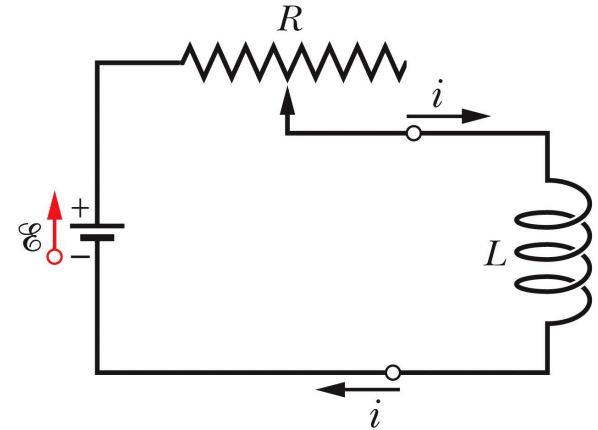
$$\mathcal{E}_L = - \frac{d(N\Phi_B)}{dt}.$$

By combining these equations, we can write

$$\mathcal{E}_L = -L \frac{di}{dt} \quad (\text{self-induced emf}).$$



An induced emf \mathcal{E}_L appears in any coil in which the current is changing.



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Note: Thus, in any inductor (such as a coil, a solenoid, or a toroid) a self-induced *emf* appears whenever the current changes with time. The magnitude of the current has no influence on the magnitude of the induced *emf*; only the rate of change of the current counts.

30-6 RL Circuits

Learning Objectives

- 30.25** Sketch a schematic diagram of an RL circuit in which the current is rising.
- 30.26** Write a loop equation (a differential equation) for an RL circuit in which the current is rising.
- 30.27** For an RL circuit in which the current is rising, apply the equation $i(t)$ for the current as a function of time.
- 30.28** For an RL circuit in which the current is rising, find equations for the potential difference V across the resistor, the rate di/dt at which the current changes, and the *emf* of the inductor, as functions of time.
- 30.29** Calculate an inductive time constant τ_L .
- 30.30** Sketch a schematic diagram of an RL circuit in which the current is decaying.

30-6 RL Circuits

Learning Objectives

- 30.31** Write a loop equation (a differential equation) for an RL circuit in which the current is decaying.
- 30.32** For an RL circuit in which the current is decaying, apply the equation $i(t)$ for the current as a function of time.
- 30.33** From an equation for decaying current in an RL circuit, find equations for the potential difference V across the resistor, the rate di/dt at which current is changing, and the *emf* of the inductor, as functions of time.
- 30.34** For an RL circuit, identify the current through the inductor and the *emf* across it just as current in the circuit begins to change (the initial condition) and a long time later when equilibrium is reached (the final condition).

30-6 RL Circuits

If a constant emf \mathcal{E} is introduced into a single-loop circuit containing a resistance R and an inductance L , the current rises to an equilibrium value of \mathcal{E}/R according to

$$i = \frac{\mathcal{E}}{R} (1 - e^{-t/\tau_L})$$

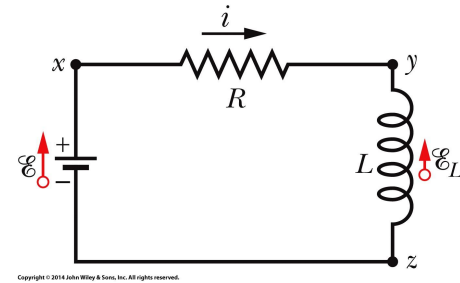
Here τ_L , the **inductive time constant**, is given by

$$\tau_L = \frac{L}{R}$$

Plot (a) and (b) shows how the potential differences $V_R (= iR)$ across the resistor and $V_L (= L di/dt)$ across the inductor vary with time for particular values of \mathcal{E} , L , and R .

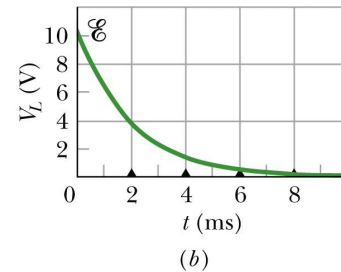
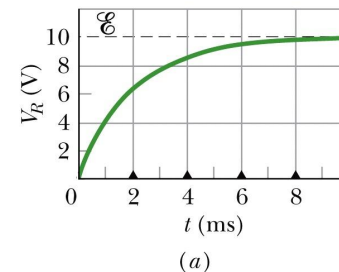
When the source of constant *emf* is removed and replaced by a conductor, the **current decays** from a value i_0 according to

$$i = \frac{\mathcal{E}}{R} e^{-t/\tau_L} = i_0 e^{-t/\tau_L}$$



An RL circuit.

The resistor's potential difference turns on.
The inductor's potential difference turns off.



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The variation with time of (a) V_R , the potential difference across the resistor in the circuit (top), and (b) V_L , the potential difference across the inductor in that circuit.

30-7 Energy Stored in a Magnetic Field

Learning Objectives

30.35 Describe the derivation of the equation for the magnetic field energy of an inductor in an RL circuit with a constant emf source.

30.36 For an inductor in an RL circuit, apply the relationship between the magnetic field energy U , the inductance L , and the current i .

30-7 Energy Stored in a Magnetic Field

If an inductor L carries a current i , the inductor's magnetic field stores an energy given by

$$U_B = \frac{1}{2} Li^2$$

A coil has an inductance of 53 mH and a resistance of $0.35 \, \Omega$.

(a) If a 12 V emf is applied across the coil, how much energy is stored in the magnetic field after the current has built up to its equilibrium value?

KEY IDEA

The energy stored in the magnetic field of a coil at any time depends on the current through the coil at that time, according to Eq. 30-49 ($U_B = \frac{1}{2} Li^2$).

Calculations: Thus, to find the energy $U_{B\infty}$ stored at equilibrium, we must first find the equilibrium current. From Eq. 30-41, the equilibrium current is

$$i_\infty = \frac{\mathcal{E}}{R} = \frac{12 \text{ V}}{0.35 \, \Omega} = 34.3 \text{ A.} \quad (30-51)$$

Then substitution yields

$$\begin{aligned} U_{B\infty} &= \frac{1}{2} Li_\infty^2 = \left(\frac{1}{2}\right)(53 \times 10^{-3} \text{ H})(34.3 \text{ A})^2 \\ &= 31 \text{ J.} \end{aligned} \quad (\text{Answer})$$

(b) After how many time constants will half this equilibrium energy be stored in the magnetic field?

Calculations: Now we are being asked: At what time t will the relation

$$U_B = \frac{1}{2} U_{B\infty}$$

be satisfied? Using Eq. 30-49 twice allows us to rewrite this energy condition as

$$\begin{aligned} \frac{1}{2} Li^2 &= \left(\frac{1}{2}\right) \frac{1}{2} Li_\infty^2 \\ \text{or} \quad i &= \left(\frac{1}{\sqrt{2}}\right) i_\infty. \end{aligned} \quad (30-52)$$

This equation tells us that, as the current increases from its initial value of 0 to its final value of i_∞ , the magnetic field will have half its final stored energy when the current has increased to this value. In general, we know that i is given by Eq. 30-41, and here i_∞ (see Eq. 30-51) is \mathcal{E}/R ; so Eq. 30-52 becomes

$$\frac{\mathcal{E}}{R} (1 - e^{-t/\tau_L}) = \frac{\mathcal{E}}{\sqrt{2}R}.$$

By canceling \mathcal{E}/R and rearranging, we can write this as

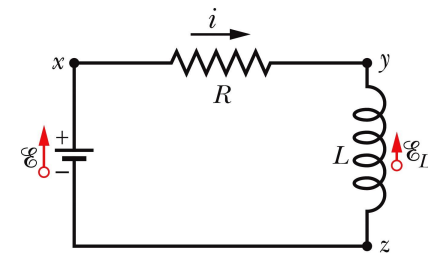
$$e^{-t/\tau_L} = 1 - \frac{1}{\sqrt{2}} = 0.293,$$

which yields

$$\frac{t}{\tau_L} = -\ln 0.293 = 1.23$$

or $t \approx 1.2\tau_L$. (Answer)

Thus, the energy stored in the magnetic field of the coil by the current will reach half its equilibrium value 1.2 time constants after the emf is applied.



An RL circuit.

30-8 Energy Density of a Magnetic Field

Learning Objectives

- 30.37** Identify that energy is associated with any magnetic field.
- 30.38** Apply the relationship between energy density u_B of a magnetic field and the magnetic field magnitude B .

30-8 Energy Density of a Magnetic Field

Consider a length l near the middle of a long solenoid of cross-sectional area A carrying current i ; the volume associated with this length is Al . The energy U_B stored by the length l of the solenoid must lie entirely within this volume because the magnetic field outside such a solenoid is approximately zero. Moreover, the stored energy must be uniformly distributed within the solenoid because the magnetic field is (approximately) uniform everywhere inside. Thus, the energy stored per unit volume of the field is

$$u_B = \frac{U_B}{Al} \longrightarrow U_B = \frac{1}{2}Li^2$$

We have,

$$u_B = \frac{Li^2}{2Al} = \frac{L}{l} \frac{i^2}{2A} \quad \text{here } L \text{ is the inductance of length } l \text{ of the solenoid}$$

Substituting for L/l we get

$$u_B = \frac{1}{2}\mu_0 n^2 i^2$$

And we can write the **energy density** as

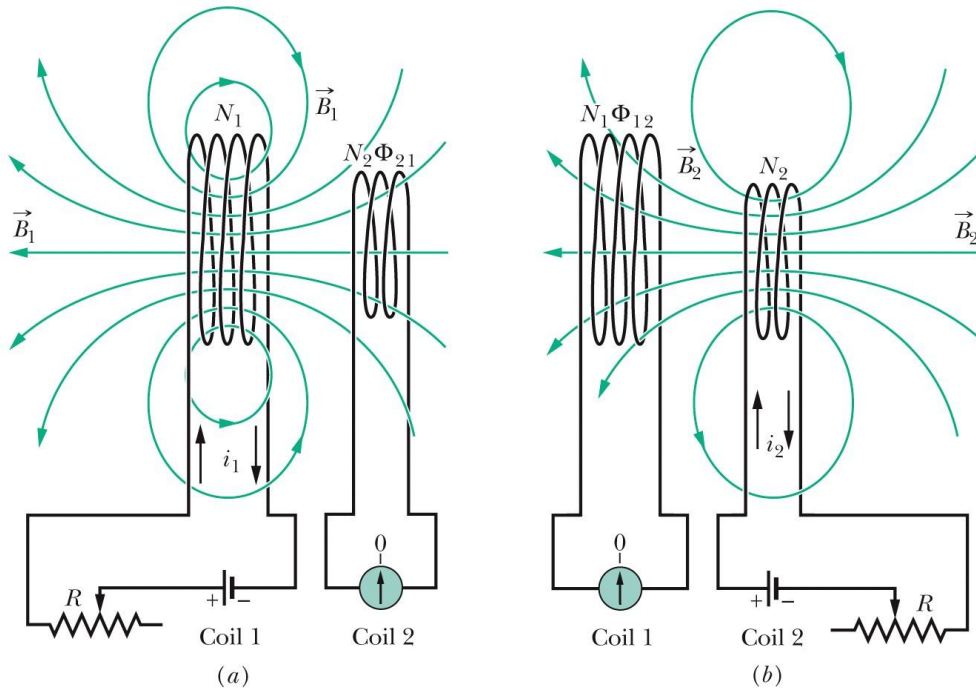
$$u_B = \frac{B^2}{2\mu_0}$$

30-8 Mutual Induction

Learning Objectives

- 30.39** Describe the mutual induction of two coils and sketch the arrangement.
- 30.40** Calculate the mutual inductance of one coil with respect to a second coil (or some second current that is changing).
- 30.41** Calculate the *emf* induced in one coil by a second coil in terms of the mutual inductance and the rate of change of the current in the second coil.

30-8 Mutual Induction



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Mutual induction. (a) The magnetic field B_1 produced by current i_1 in coil 1 extends through coil 2. If i_1 is varied (by varying resistance R), an *emf* is induced in coil 2 and current registers on the meter connected to coil 2. (b) The roles of the coils interchanged.

If coils 1 and 2 are near each other, a changing current in either coil can induce an *emf* in the other. This mutual induction is described by

and

$$\mathcal{E}_2 = -M \frac{di_1}{dt}$$

$$\mathcal{E}_1 = -M \frac{di_2}{dt}.$$

30 Summary

Magnetic Flux

- The magnetic flux through an area A in a magnetic field \mathbf{B} is defined as

$$\Phi_B = \int \vec{B} \cdot d\vec{A} \quad \text{Eq. 30-1}$$

- If \mathbf{B} is perpendicular to the area and uniform over it, Eq. 30-1 becomes

$$\Phi_B = BA \quad (\vec{B} \perp A, \vec{B} \text{ uniform}). \quad \text{Eq. 30-2}$$

Faraday's Law of Induction

- The induced *emf* is,

$$\mathcal{E} = - \frac{d\Phi_B}{dt} \quad \text{Eq. 30-4}$$

- If the loop is replaced by a closely packed coil of N turns, the induced *emf* is

$$\mathcal{E} = -N \frac{d\Phi_B}{dt}. \quad \text{Eq. 30-5}$$

Lenz's Law

- An induced current has a direction such that the magnetic field due to this induced current opposes the change in the magnetic flux that induces the current.

Emf and the Induced Magnetic Field

- The induced *emf* is related to \mathbf{E} by

$$\mathcal{E} = \oint \vec{E} \cdot d\vec{s}, \quad \text{Eq. 30-19}$$

- Faraday's law in its most general form,

$$\oint \vec{E} \cdot d\vec{s} = - \frac{d\Phi_B}{dt} \quad \text{Eq. 30-20}$$

30 Summary

Inductor

- The inductance L of the inductor is

$$L = \frac{N\Phi_B}{i} \quad \text{Eq. 30-28}$$

- The inductance per unit length near the middle of a long solenoid of cross-sectional area A and n turns per unit length is

$$\frac{L}{l} = \mu_0 n^2 A \quad \text{Eq. 30-31}$$

Self-Induction

- This self-induced *emf* is,

$$\mathcal{E}_L = -L \frac{di}{dt} \quad \text{Eq. 30-35}$$

Mutual Induction

- The mutual induction is described by,

$$\mathcal{E}_2 = -M \frac{di_1}{dt} \quad \text{Eq. 30-64}$$

$$\mathcal{E}_1 = -M \frac{di_2}{dt} \quad \text{Eq. 30-65}$$

Series *RL* Circuit

- Rise of current,

$$i = \frac{\mathcal{E}}{R} (1 - e^{-t/\tau_L}) \quad \text{Eq. 30-41}$$

- Decay of current

$$i = i_0 e^{-t/\tau_L} \quad \text{Eq. 30-45}$$

Magnetic Energy

- the inductor's magnetic field stores an energy given by

$$U_B = \frac{1}{2} Li^2 \quad \text{Eq. 30-49}$$

- The density of stored magnetic energy,

$$u_B = \frac{B^2}{2\mu_0} \quad \text{Eq. 30-55}$$