Chapter 35

Interference

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35-1 Light as a Wave

Learning Objectives

35.01 Using a sketch, explain Huygens' principle.

35.02 With a few simple sketches, explain refraction in terms of the gradual change in the speed of a wavefront as it passes through an interface at an angle to the normal.

35.03 Apply the relationship between the speed of light in vacuum c, the speed of light in a material v, and the index of refraction of the material n.

35.04 Apply the relationship between a distance L in a material, the speed of light in that material, and the time required for a pulse of the light to travel through L.

35.05 Apply Snell's law of refraction.

35.06 When light refracts through an interface, identify that the frequency does not change but the wavelength and effective speed do.

35.07 Apply the relationship between the wavelength in vacuum *l*, the wavelength λ_n in a material (the internal wavelength), and the index of refraction n of the material.

35-1 Light as a Wave

Learning Objectives

35.08 For light in a certain length of a material, calculate the number of internal wavelengths that fit into the length.

35.09 If two light waves travel through different materials with different indexes of refraction and then reach a common point, determine their phase difference and interpret the resulting interference in terms of maximum brightness, intermediate brightness, and darkness.

35.10 Apply the learning objectives of Module 17-3 (sound waves there, light waves here) to find the phase difference and interference of two waves that reach a common point after traveling paths of different lengths.

35.11 Given the initial phase difference between two waves with the same wavelength, determine their phase difference after they travel through different path lengths and through different indexes of refraction.

35.12 Identify that rainbows are examples of optical interference.

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35-1 Light as a Wave

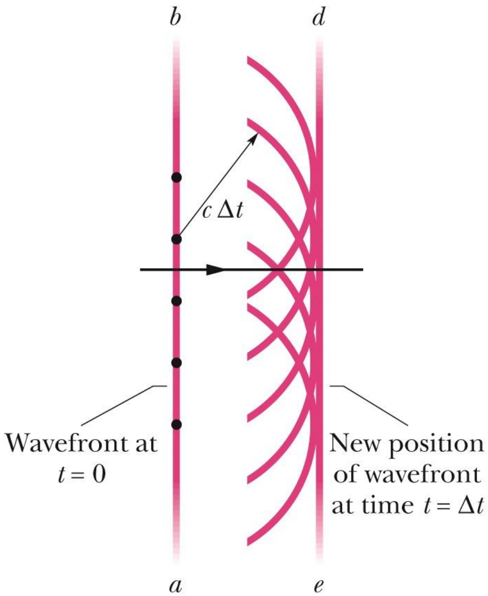
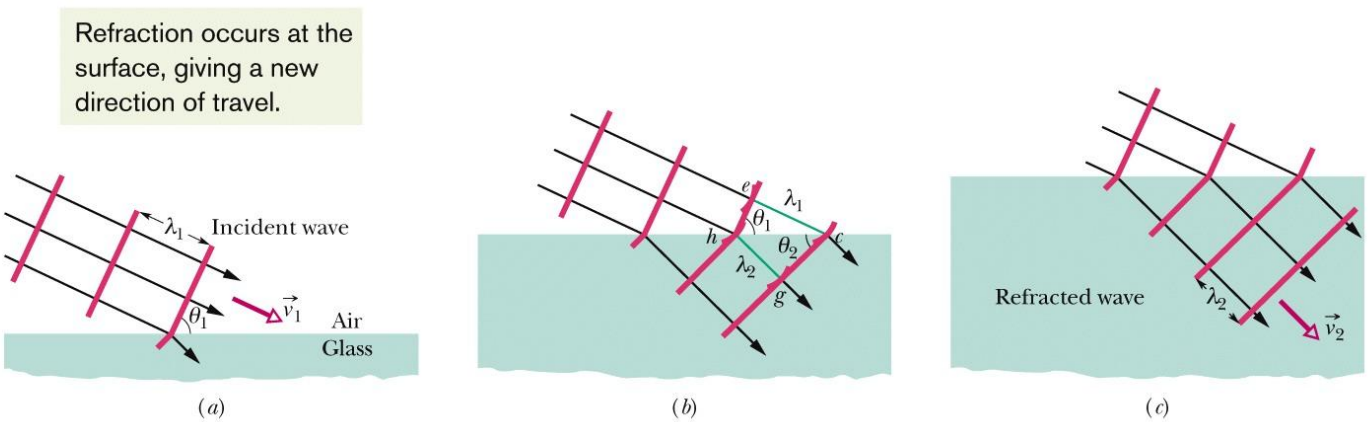


Figure 1

The three-dimensional transmission of waves, including light, may often be predicted by Huygens' principle, which states that

> All points on a wavefront serve as point sources of spherical secondary wavelets. After a time t, the new position of the wavefront will be that of a surface tangent to these secondary wavelets.

Figure 1 shows the propagation of a plane wave in vacuum, as portrayed by Huygens' principle.



The refraction of a plane wave at an air–glass interface, as portrayed by Huygens' principle. The wavelength in glass is smaller than that in air. For simplicity, the reflected wave is not shown. Parts (a) through (c) represent three successive stages of the refraction.

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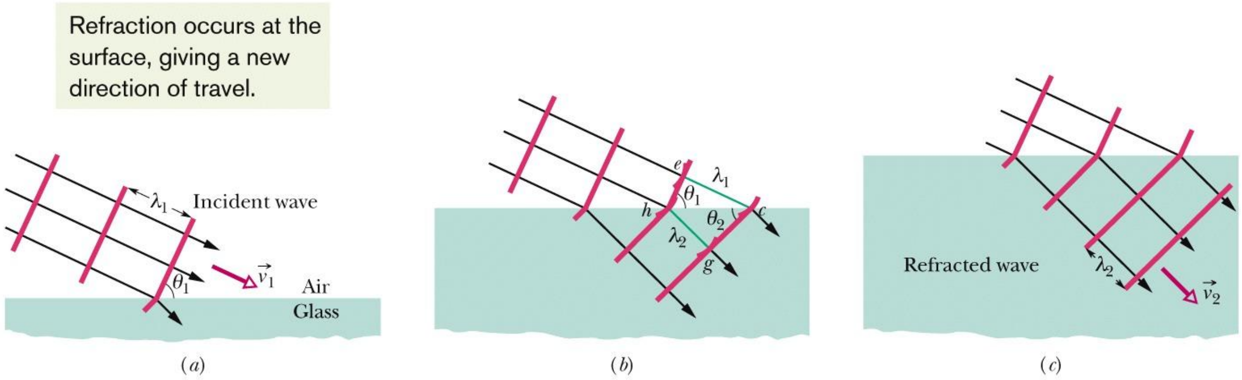
35-1 Light as a Wave



The refraction of a plane wave at an air – glass interface, as portrayed by Huygens' principle. The wavelength in glass is smaller than that in air. For simplicity, the reflected wave is not shown. Parts (a) through (c) represent three successive stages of the refraction.

The law of refraction can be derived from Huygens' principle by assuming that the index of refraction of any medium is

$$n = c/v,$$

in which v is the speed of light in the medium and c is the speed of light in vacuum. The wavelength λ_n of light in a medium depends on the index of refraction n of the medium:

$$\lambda_n = \frac{\lambda}{n},$$ where λ is the wavelength of vacuum

Because of this dependency, the phase difference between two waves can change if they pass through different materials with different indexes of refraction.

35-2 Young's Interference

Learning Objectives

35.13 Describe the diffraction of light by a narrow slit and the effect of narrowing the slit.

35.14 With sketches, describe the production of the interference pattern in a double-slit interference experiment using monochromatic light.

35.15 Identify that the phase difference between two waves can change if the waves travel along paths of different lengths, as in the case of Young's experiment.

35.16 In a double-slit experiment, apply the relationship between the path length difference ΔL and the wavelength λ, and then interpret the result in terms of interference (maximum brightness, intermediate brightness, and darkness).

35.17 For a given point in a double-slit interference pattern, express the path length difference ΔL of the rays reaching that point in terms of the slit separation d and the angle θ to that point.

35.18 In a Young's experiment, apply the relationships between the slit separation d, the light wavelength λ, and the angles θ to the minima (dark fringes) and to the maxima (bright fringes) in the interference pattern.

35-2 Young's Interference

Learning Objectives

35.19 Sketch the double-slit interference pattern, identifying what lies at the center and what the various bright and dark fringes are called (such as "first side maximum" and "third order").

35.20 Apply the relationship between the distance D between a double-slit screen and a viewing screen, the angle θ to a point in the interference pattern, and the distance y to that point from the pattern's center.

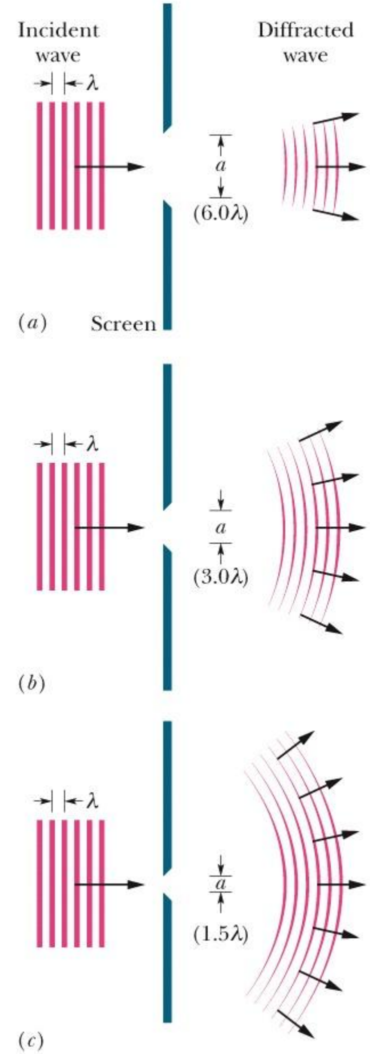
35.21 For a double-slit interference pattern, identify the effects of changing d or λ and also identify what determines the angular limit to the pattern.

35.22 For a transparent material placed over one slit in a Young's experiment, determine the thickness or index of refraction required to shift a given fringe to the center of the interference pattern.

35-2 Young's Interference

The flaring is consistent with the spreading of wavelets in the Huygens construction. **Diffraction** occurs for waves of all types, not just light waves. Figure below shows waves passing through a slit flares.

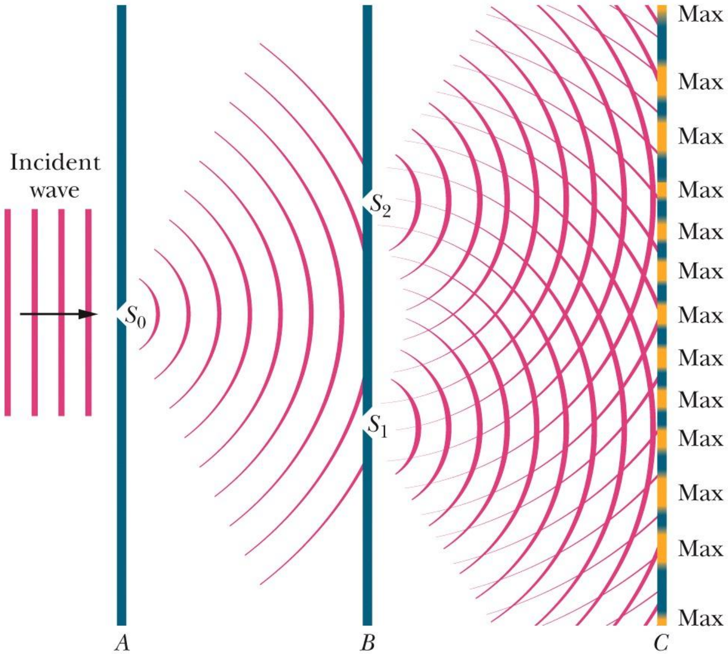
Figure (a) shows the situation schematically for an incident plane wave of wavelength λ encountering a slit that has width $a = 6.0\,\lambda$ and extends into and out of the page. The part of the wave that passes through the slit flares out on the far side. Figures (b) (with $a = 3.0\,\lambda$) and (c) ($a = 1.5\lambda$) illustrate the main feature of diffraction: the narrower the slit, the greater the diffraction.



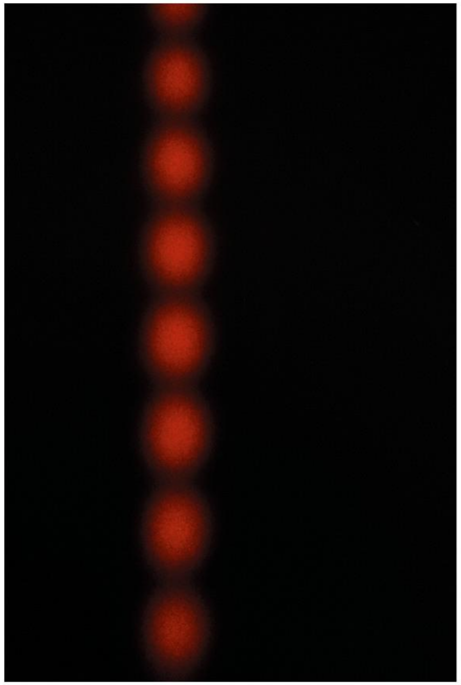
A wave passing through a slit flares (diffracts).

35-2 Young's Interference

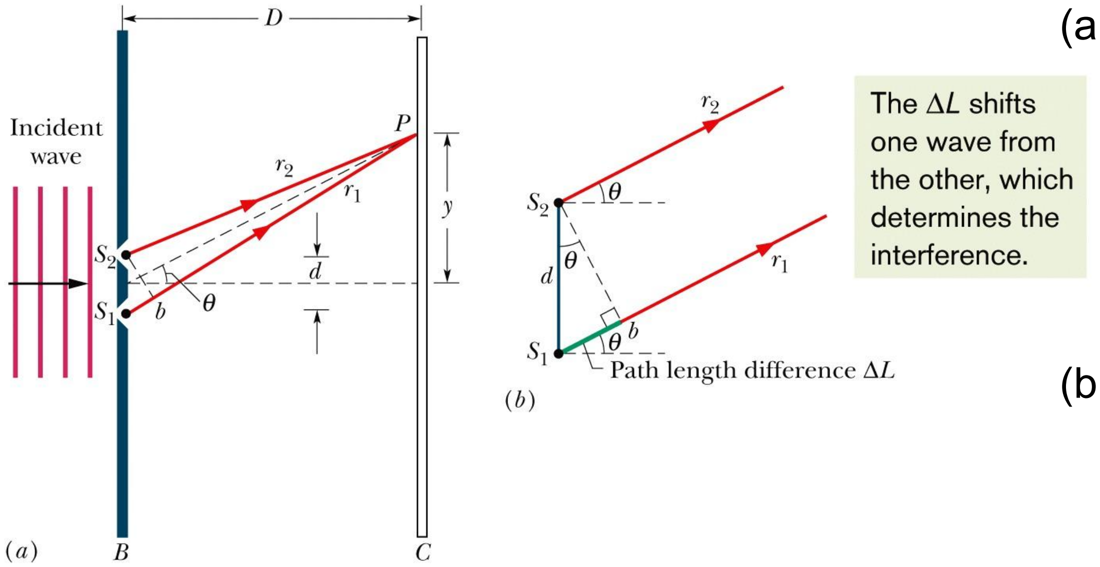
Figure gives the basic arrangement of Young's experiment. Light from a distant monochromatic source illuminates slit S_0 in screen A. The emerging light then spreads via diffraction to illuminate two slits S_1 and S_2 in screen B. Diffraction of the light by these two slits sends overlapping circular waves into the region beyond screen B, where the waves from one slit interfere with the waves from the other slit.



A photograph of the interference pattern produced by the arrangement shown in the figure(right), but with short slits. (The photograph is a front view of part of screen C of figure on left.) The alternating maxima and minima are called interference fringes (because they resemble the decorative fringe sometimes used on clothing and rugs).


Courtesy Jearl Walker

35-2 Young's Interference



(a) Waves from slits S_1 and S_2 (which extend into and out of the page) combine at P, an arbitrary point on screen C at distance *y* from the central axis. The angle θ serves as a convenient locator for *P*.

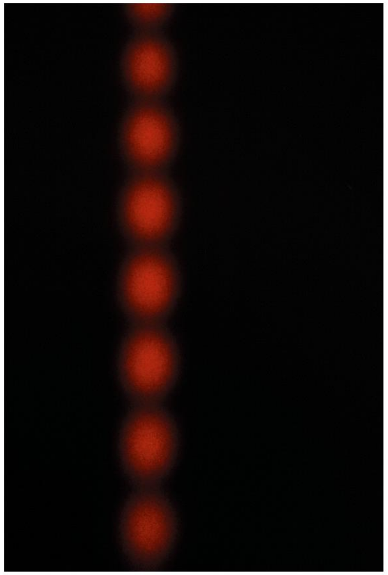
(b) For D >> d, we can approximate rays r_1 and r_2 as being parallel, at angle θ to the central axis.

The phase difference between two waves can change if the waves travel paths of different lengths.

The conditions for maximum and minimum intensity are

$$d \sin \theta = m\lambda, \qquad \text{for } m = 0, 1, 2, \ldots$$

$$d \sin \theta = (m + \tfrac{1}{2})\lambda, \qquad \text{for } m = 0, 1, 2, \ldots$$



Courtesy Jearl Walker

35-3 Interference and Double-Slit Intensity

Learning Objectives

35.23 Distinguish between coherent and incoherent light.

35.24 For two light waves arriving at a common point, write expressions for their electric field components as functions of time and a phase constant.

35.25 Identify that the phase difference between two waves determines their interference.

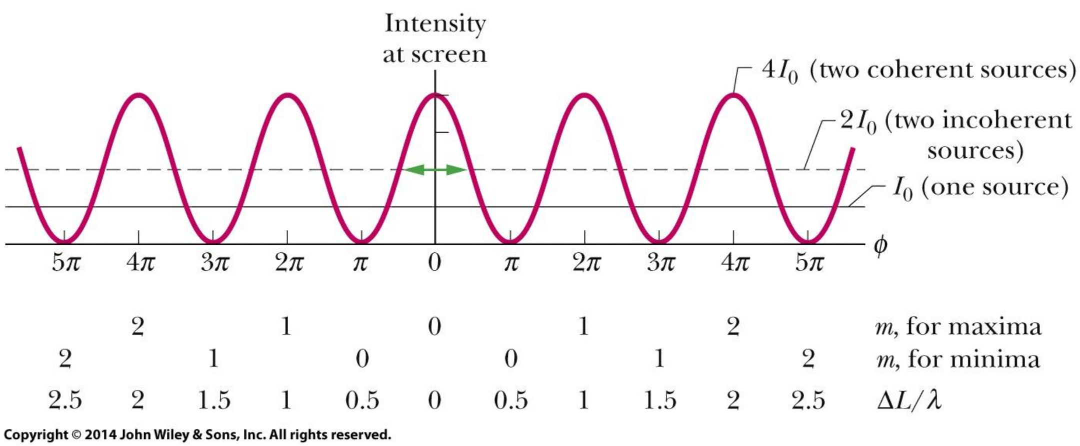
35.26 For a point in a double-slit interference pattern, calculate the intensity in terms of the phase difference of the arriving waves and relate that phase difference to the angle θ locating that point in the pattern.

35.27 Use a phasor diagram to find the resultant wave (amplitude and phase constant) of two or more light waves arriving at a common point and use that result to determine the intensity.

35.28 Apply the relationship between a light wave's angular frequency ω and the angular speed v of the phasor representing the wave.

35-3 Interference and Double-Slit Intensity

If two light waves that meet at a point are to interfere perceptibly, both must have the same wavelength and the phase difference between them must remain constant with time; that is, the waves must be coherent.



A plot of equation below, showing the intensity of a double-slit interference pattern as a function of the phase difference between the waves when they arrive from the two slits. I_0 is the (uniform) intensity that would appear on the screen if one slit were covered. The average intensity of the fringe pattern is $2I_0$, and the maximum intensity (for coherent light) is $4I_0$.

As shown in figure, in Young's interference experiment, two waves, each with intensity I_0, yield a resultant wave of intensity I at the viewing screen, with

$$I = 4I_0 \cos^2 \tfrac{1}{2}\phi,$$

where

$$\phi = \frac{2\pi d}{\lambda} \sin \theta.$$

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35-4 Interference from thin films

Learning Objectives

35.29 Sketch the setup for thin-film interference, showing the incident ray and two reflected rays (perpendicular to the film but drawn slightly slanted for clarity) and identifying the thickness of the film and the three indexes of refraction.

35.30 Identify the condition in which a reflection can result in a phase shift, and give the value of that phase shift.

35.31 Identify the three factors that determine the interference of the two reflected waves: reflection shifts, path length difference, and internal wavelength (set by the film's index of refraction).

35.32 For a thin film, use the reflection shifts and the desired result (the reflected waves are in phase or out of phase, or the transmitted waves are in phase or out of phase) to determine and then apply the necessary equation relating the thickness L, the wavelength λ (measured in air), and the index of refraction *n* of the film.

35-4 Interference from thin films

Learning Objectives

35.33 For a very thin film in air (with thickness much less than the wavelength of visible light), explain why the film is always dark.

35.34 At each end of a thin film in the form of a wedge, determine and then apply the necessary equation relating the thickness L, the wavelength λ (measured in air), and the index of refraction n of the film, and then count the number of bright bands and dark bands across the film.

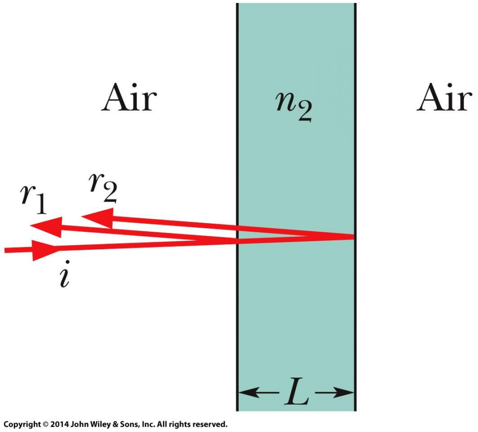
35-4 Interference from thin films

When light is incident on a thin transparent film, the light waves reflected from the front and back surfaces interfere. For near-normal incidence, the wavelength conditions for maximum and mini- mum intensity of the light reflected from a film with air on both sides are

$$2L = \left(m + \tfrac{1}{2}\right)\frac{\lambda}{n_2}, \quad \text{for } m = 0, 1, 2, \ldots \quad \text{(maxima—bright film in air).}$$

and

$$2L = m\frac{\lambda}{n_2}, \quad \text{for } m = 0, 1, 2, \ldots \quad \text{(minima—dark film in air).}$$



Reflections from a thin film in air.

where n_2 is the index of refraction of the film, L is its thickness, and λ is the wavelength of the light in air.

If a film is sandwiched between media other than air, these equations for bright and dark films may be interchanged, depending on the relative indexes of refraction.

If the light incident at an interface between media with different indexes of refraction is initially in the medium with the smaller index of refraction, the reflection causes a phase change of π *rad*, or half a wavelength, in the reflected wave. Otherwise, there is no phase change due to the reflection. Refraction causes no phase shift.

35-4 Interference from thin films

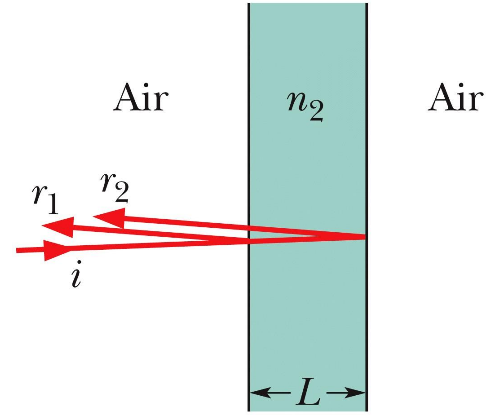
When light is incident on a thin transparent film, the light waves reflected from the front and back surfaces interfere. For near-normal incidence, the wavelength conditions for maximum and mini- mum intensity of the light reflected from a film in air are

$$2L = (m + \tfrac{1}{2})\frac{\lambda}{n_2}, \qquad \text{for } m = 0, 1, 2, \ldots \quad \text{(maxima—bright film in air).}$$

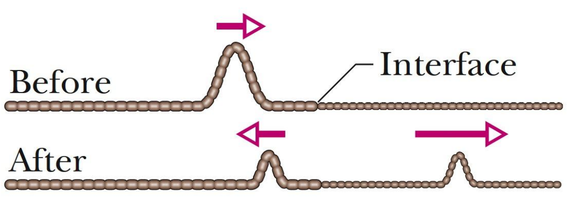
and

$$2L = m\frac{\lambda}{n_2}, \qquad \text{for } m = 0, 1, 2, \ldots \quad \text{(minima—dark film in air).}$$

where n_2 is the index of refraction of the film, L is its thickness, and λ is the wavelength of the light in air.



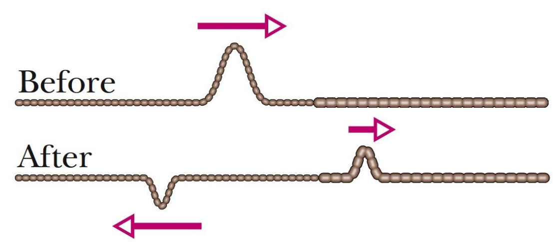
Reflections from a thin film in air.



(a)

The incident pulse is in the denser string.

Reflection	Reflection phase shift
Off lower index	0
Off higher index	0.5 wavelength



(b)

The incident pulse in the lighter string. Only here is there a phase change, and only in the reflected wave.

35-5 Michelson's Interferometer

Learning Objectives

35.35 With a sketch, explain how an interferometer works.

35.36 When a transparent material is inserted into one of the beams in an interferometer, apply the relationship between the phase change of the light (in terms of wavelength) and the material's thickness and index of refraction.

35.37 For an interferometer, apply the relationship between the distance a mirror is moved and the resulting fringe shift in the interference pattern.

35-5 Michelson's Interferometer

An interferometer is a device that can be used to measure lengths or changes in length with great accuracy by means of interference fringes.

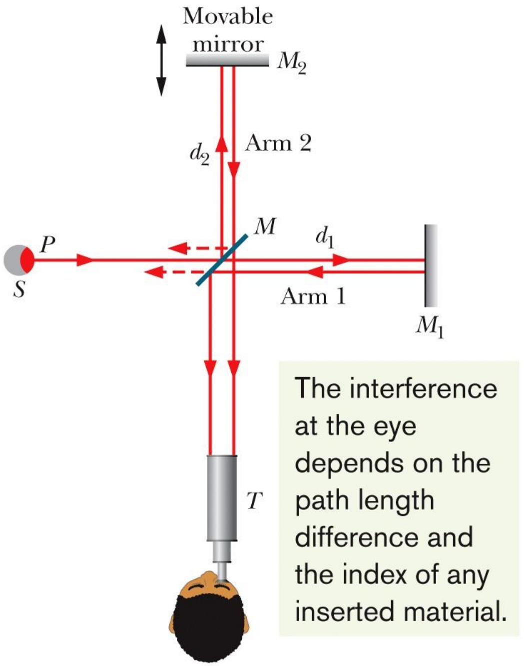
In Michelson's interferometer, a light wave is split into two beams that then recombine after traveling along different paths.

The interference pattern they produce depends on the difference in the lengths of those paths and the indexes of refraction along the paths.

If a transparent material of index n and thickness L is in one path, the phase difference (in terms of wavelength) in the recombining beams is equal to

$$\text{phase difference} = \frac{2L}{\lambda}(n-1),$$

where λ is the wavelength of the light.



Michelson's interferometer, showing the path of light originating at point P of an extended source S. Mirror M splits the light into two beams, which reflect from mirrors M_1 and M_2 back to M and then to telescope T. In the telescope an observer sees a pattern of interference fringes.

35 Summary

Huygen's Principle

- The three-dimensional transmission of waves, including light, may often be predicted by Huygens' principle, which states that all points on a wavefront serve as point sources of spherical secondary wavelets.

Wavelength and Index of Refraction

- The wavelength λ_n of light in a medium depends on the index of refraction n of the medium:

$$\lambda_n = \frac{\lambda}{n}, \qquad \textbf{Eq. 35-6}$$

in which λ is the wavelength in vacuum.

Young's Experiment

- In Young's interference experiment, light passing through a single slit falls on two slits in a screen. The light leaving these slits flares out (by diffraction), and interference occurs in the region beyond the screen. A fringe pattern, due to the interference, forms on a viewing screen.
- The conditions for maximum and minimum intensity are

$$d \sin\theta = m\lambda, \quad \text{for } m = 0, 1, 2, \ldots \text{ (maxima—bright fringes)}, \qquad \textbf{Eq. 35-14}$$

$$d \sin\theta = (m + \tfrac{1}{2})\lambda, \quad \text{for } m = 0, 1, 2, \ldots \text{ (minima—dark fringes)}, \qquad \textbf{Eq. 35-16}$$

35 Summary

Coherence

- If two light waves that meet at a point are to interfere perceptibly, both must have the same wavelength and the phase difference between them must remain constant with time; that is, the waves must be coherent.

Intensity in Two-Slit Interference

- In Young's interference experiment, two waves, each with intensity I_0, yield a resultant wave of intensity I at the viewing screen, with

$$I = 4I_0 \cos^2 \tfrac{1}{2}\phi, \qquad \text{where } \phi = \frac{2\pi d}{\lambda} \sin \theta.$$

Eqs. 35-22 & 23

Thin-Film Interference

- When light is incident on a thin transparent film, the light waves reflected from the front and back surfaces interfere. For near-normal incidence, the wavelength conditions for maximum and minimum intensity of the light reflected from a *film of index* n_2 *in air* are

$$2L = (m + \tfrac{1}{2})\frac{\lambda}{n_2}, \quad \text{for } m = 0, 1, 2, \ldots \quad \text{(maxima—bright film in air)},$$ **Eq. 35-36**

$$2L = m\frac{\lambda}{n_2}, \quad \text{for } m = 0, 1, 2, \ldots \quad \text{(minima—dark film in air)},$$ **Eq. 35-37**

Michelson's Interferometer

- In Michelson's interferometer a light wave is split into two beams that, after traversing paths of different lengths, are recombined so they interfere and form a fringe pattern.