

## Chapter 36

# Diffraction

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## 36-1 Single-Slit Diffraction

### Learning Objectives

- 36.01** Describe the diffraction of light waves by a narrow opening and an edge, and also describe the resulting interference pattern.
- 36.02** Describe an experiment that demonstrates the Fresnel bright spot.
- 36.03** With a sketch, describe the arrangement for a single-slit diffraction experiment.
- 36.04** With a sketch, explain how splitting a slit width into equal zones leads to the equations giving the angles to the minima in the diffraction pattern.
- 36.05** Apply the relationships between width  $a$  of a thin, rectangular slit or object, the wavelength  $\lambda$ , the angle  $\theta$  to any of the minima in the diffraction pattern, the distance to a viewing screen, and the distance between a minimum and the center of the pattern.
- 36.06** Sketch the diffraction pattern for monochromatic light, identifying what lies at the center and what the various bright and dark fringes are called (such as “first minimum”).

## 36-1 Single-Slit Diffraction

### Learning Objectives

**36.07** Identify what happens to a diffraction pattern when the wavelength of the light or the width of the diffracting aperture or object is varied.

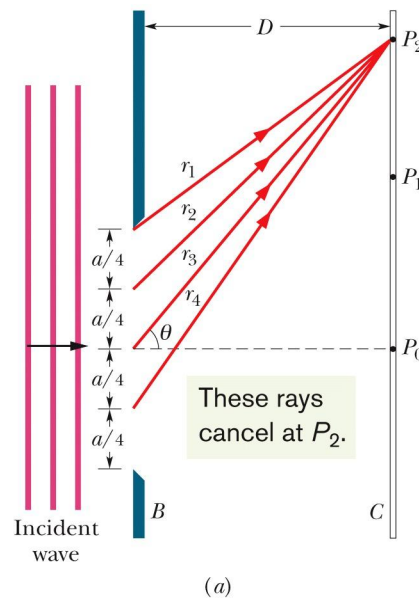
## 36-1 Single-Slit Diffraction

When waves encounter an edge, an obstacle, or an aperture the size of which is comparable to the wavelength of the waves, those waves spread out as they travel and, as a result, undergo interference. This type of interference is called diffraction.

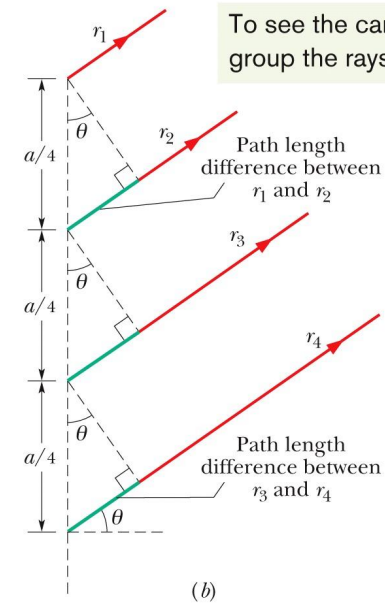
Waves passing through a long narrow slit of width  $a$  produce, on a viewing screen, a single-slit diffraction pattern that includes a central maximum (bright fringe) and other maxima. They are separated by minima that are located relative to the central axis by angles  $\theta$ :

$$a \sin \theta = m\lambda, \quad \text{for } m = 1, 2, 3, \dots$$

The maxima are located approximately halfway between minima.



(a)



(b)

- (a) Waves from the top points of four zones of width  $a/4$  undergo fully destructive interference at point  $P_2$ .  
 (b) For  $D \gg a$ , we can approximate rays  $r_1$ ,  $r_2$ ,  $r_3$ , and  $r_4$  as being parallel, at angle  $\theta$  to the central axis.

## 36-2 Intensity in Single-Slit Diffraction

### Learning Objectives

- 36.08** Divide a thin slit into multiple zones of equal width and write an expression for the phase difference of the wavelets from adjacent zones in terms of the angle  $\theta$  to a point on the viewing screen.
- 36.09** For single-slit diffraction, draw phasor diagrams for the central maximum and several of the minima and maxima off to one side, indicating the phase difference between adjacent phasors, explaining how the net electric field is calculated, and identifying the corresponding part of the diffraction pattern.
- 36.10** Describe a diffraction pattern in terms of the net electric field at points in the pattern.
- 36.11** Evaluate  $\alpha$ , the convenient connection between angle  $\theta$  to a point in a diffraction pattern and the intensity  $I$  at that point.
- 36.12** For a given point in a diffraction pattern, at a given angle, calculate the intensity  $I$  in terms of the intensity  $I_m$  at the center of the pattern.

## 36-2 Intensity in Single-Slit Diffraction

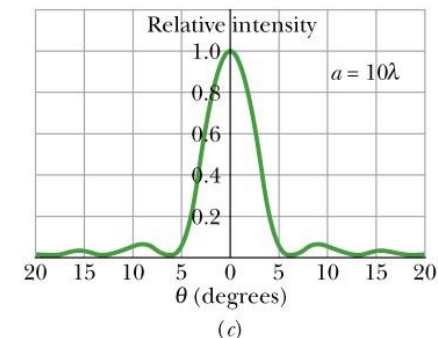
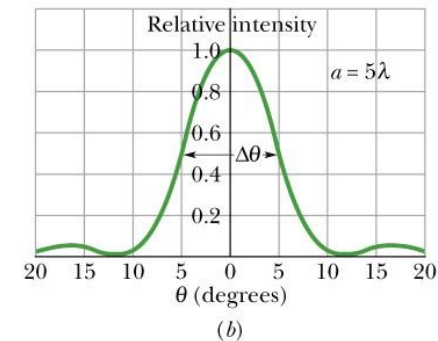
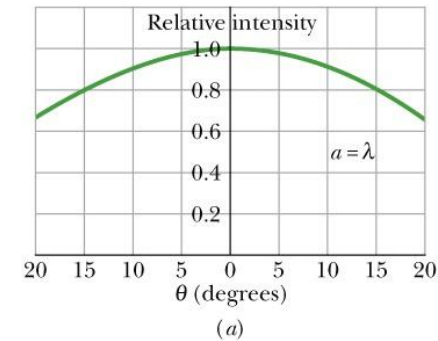
The intensity of the diffraction pattern at any given angle  $\theta$  is

$$I(\theta) = I_m \left( \frac{\sin \alpha}{\alpha} \right)^2,$$

where,  $I_m$  is the intensity at the center of the pattern and

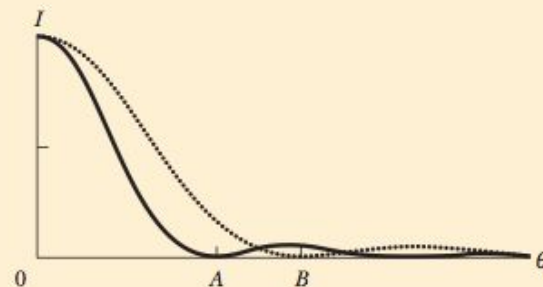
$$\alpha = \frac{1}{2}\phi = \frac{\pi a}{\lambda} \sin \theta.$$

The plots show the relative intensity in single-slit diffraction for three values of the ratio  $a/\lambda$ . The wider the slit is, the narrower is the central diffraction maximum.



### ✓ Checkpoint 3

Two wavelengths, 650 and 430 nm, are used separately in a single-slit diffraction experiment. The figure shows the results as graphs of intensity  $I$  versus angle  $\theta$  for the two diffraction patterns. If both wavelengths are then used simultaneously, what color will be seen in the combined diffraction pattern at (a) angle A and (b) angle B?



Answer  
(a) 650 nm  
(b) 430 nm

## 36-3 Diffraction by a Circular Aperture

### Learning Objectives

- 36.13** Describe and sketch the diffraction pattern from a small circular aperture or obstacle.
- 36.14** For diffraction by a small circular aperture or obstacle, apply the relationships between the angle  $\theta$  to the first minimum, the wavelength  $\lambda$  of the light, the diameter  $d$  of the aperture, the distance  $D$  to a viewing screen, and the distance  $y$  between the minimum and the center of the diffraction pattern.
- 36.15** By discussing the diffraction patterns of point objects, explain how diffraction limits visual resolution of objects.
- 36.16** Identify that Rayleigh's criterion for resolvability gives the (approximate) angle at which two point objects are just barely resolvable.
- 36.17** Apply the relationships between the angle  $\theta_R$  in Rayleigh's criterion, the wavelength  $\lambda$  of the light, the diameter  $d$  of the aperture (for example, the diameter of the pupil of an eye), the angle  $\theta$  subtended by two distant point objects, and the distance  $L$  to those objects.

## 36-3 Diffraction by a Circular Aperture

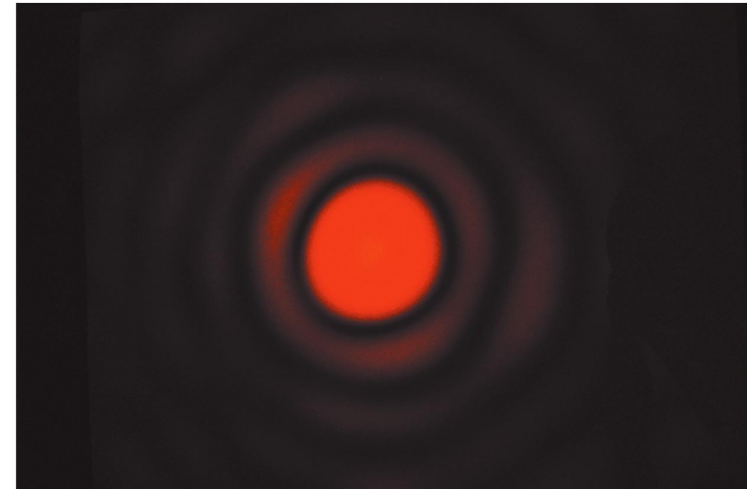
Diffraction by a circular aperture or a lens with diameter  $d$  produces a central maximum and concentric maxima and minima, given by

$$\sin \theta = 1.22 \frac{\lambda}{d} \quad (\text{first minimum—circular aperture}).$$

The angle  $\theta$  here is the angle from the central axis to any point on that (circular) minimum.

$$\sin \theta = \frac{\lambda}{a} \quad (\text{first minimum—single slit}),$$

which locates the first minimum for a long narrow slit of width  $a$ . The main difference is the factor 1.22, which enters because of the circular shape of the aperture.



Courtesy Jearl Walker

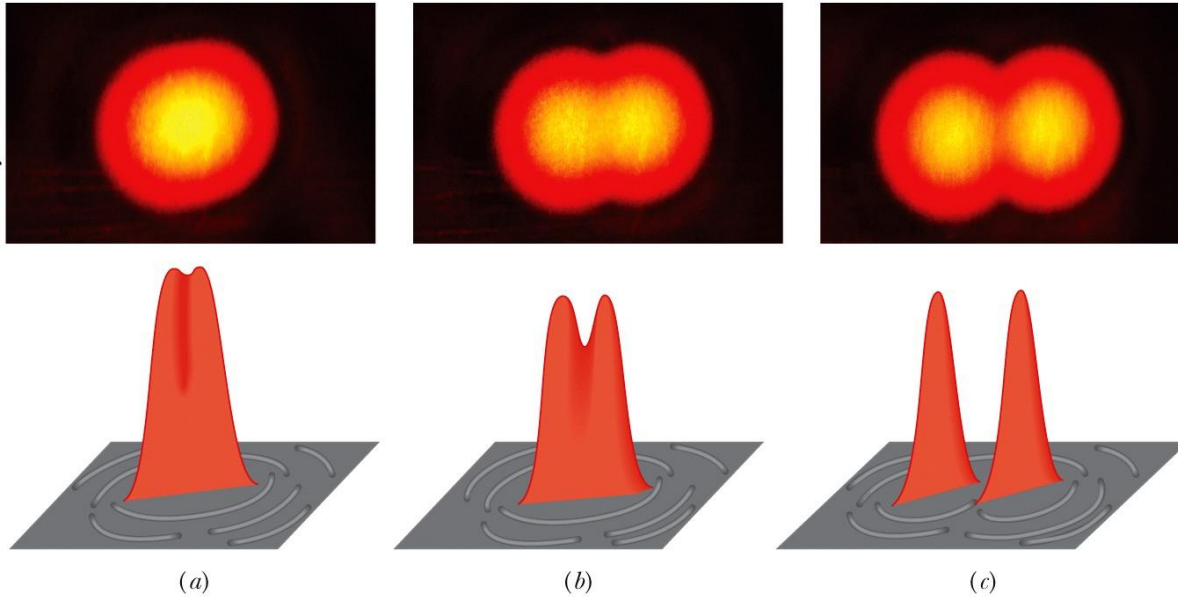
The diffraction pattern of a circular aperture. Note the central maximum and the circular secondary maxima. The figure has been overexposed to bring out these secondary maxima, which are much less intense than the central maximum.



# 36-3 Diffraction by a Circular Aperture

## Resolvability

Courtesy Jearl Walker



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The images of two point sources (stars) formed by a converging lens. At the bottom, representations of the image intensities. In (a) the angular separation of the sources is too small for them to be distinguished, in (b) they can be marginally distinguished, and in (c) they are clearly distinguished. Rayleigh's criterion is satisfied in (b), with the central maximum of one diffraction pattern coinciding with the first minimum of the other.

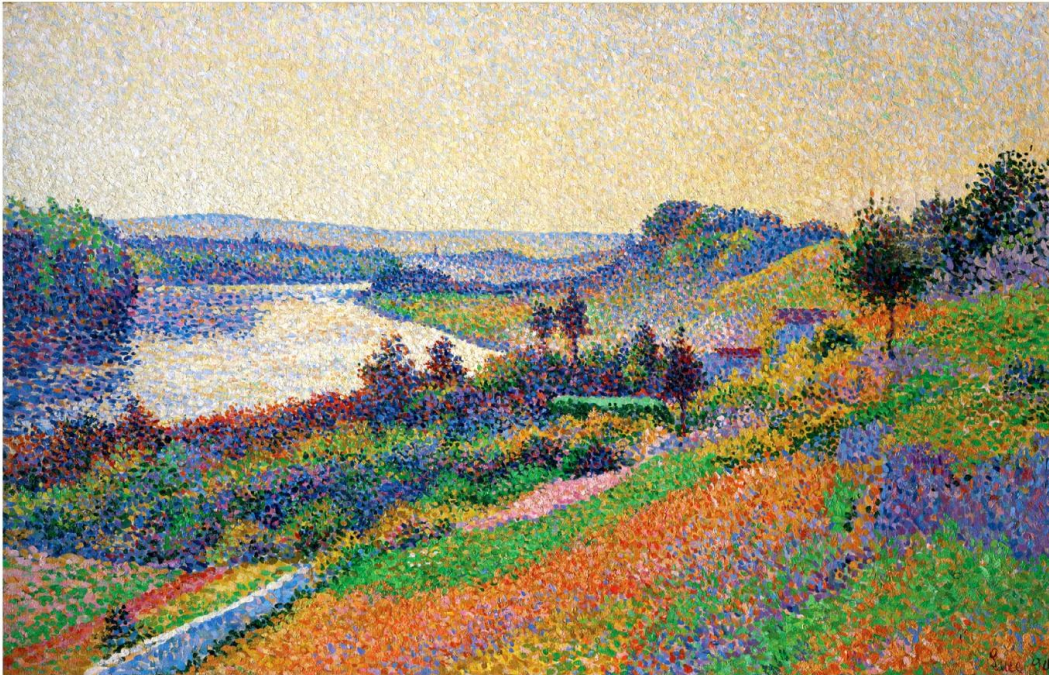
**Rayleigh's criterion** suggests that two objects are on the verge of resolvability if the central diffraction maximum of one is at the first minimum of the other. Their angular separation can then be no less than

$$\theta_R = 1.22 \frac{\lambda}{d} \quad (\text{Rayleigh's criterion}).$$

in which  $d$  is the diameter of the aperture through which the light passes.

## 36-3 Diffraction by a Circular Aperture

### Pointillism



Maximilien Luce, *The Seine at Herblay*, 1890. Musée d'Orsay, Paris, France. Photo by Erich Lessing/Art Resource

Rayleigh's criterion can explain the arresting illusions of color in the style of painting known as pointillism. In this style, a painting is made not with brush strokes in the usual sense but rather with a myriad of small colored dots. One fascinating aspect of a pointillistic painting is that when you change your distance from it, the colors shift in subtle, almost subconscious ways. This color shifting has to do with whether you can resolve the colored dots.



#### Checkpoint 4

Suppose that you can barely resolve two red dots because of diffraction by the pupil of your eye. If we increase the general illumination around you so that the pupil decreases in diameter, does the resolvability of the dots improve or diminish? Consider only diffraction. (You might experiment to check your answer.)

Answer:  
Resolvability improves.

## 36-4 Diffraction by a Double Slit

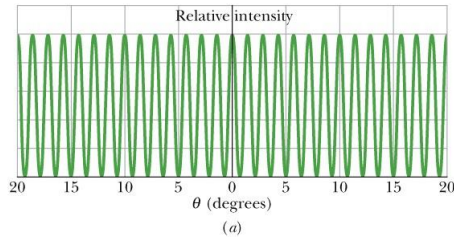
### Learning Objectives

- 36.18** In a sketch of a double-slit experiment, explain how the diffraction through each slit modifies the two-slit interference pattern, and identify the diffraction envelope, the central peak, and the side peaks of that envelope.
- 36.19** For a given point in a double-slit diffraction pattern, calculate the intensity  $I$  in terms of the intensity  $I_m$  at the center of the pattern.
- 36.20** In the intensity equation for a double-slit diffraction pattern, identify what part corresponds to the interference between the two slits and what part corresponds to the diffraction by each slit.
- 36.21** For double-slit diffraction, apply the relationship between the ratio  $d/a$  and the locations of the diffraction minima in the single-slit diffraction pattern, and then count the number of two-slit maxima that are contained in the central peak and in the side peaks of the diffraction envelope.

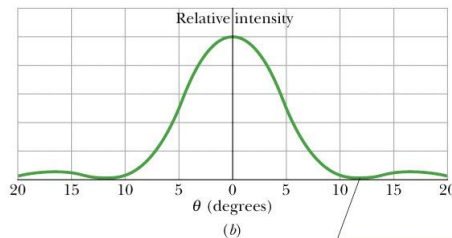
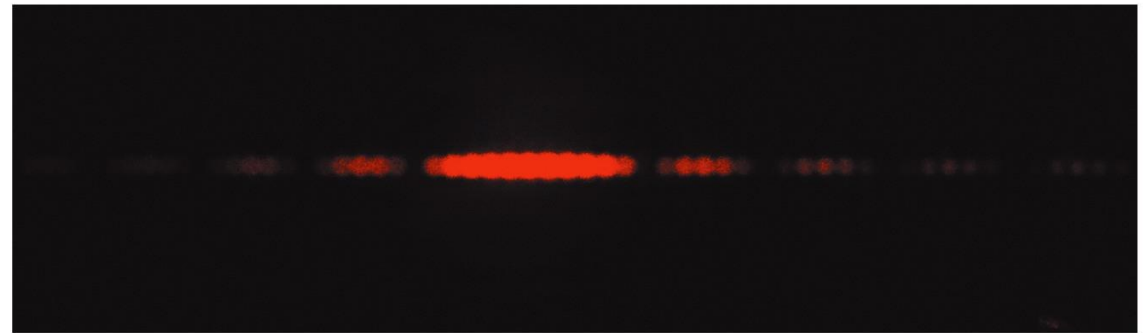


# 36-4 Diffraction by a Double Slit

Waves passing through two slits produce a combination of double-slit interference and diffraction by each slit.

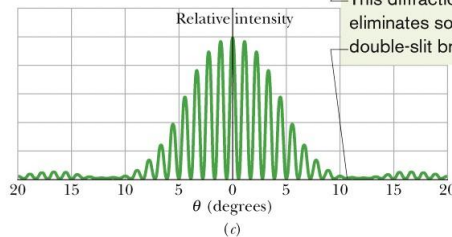
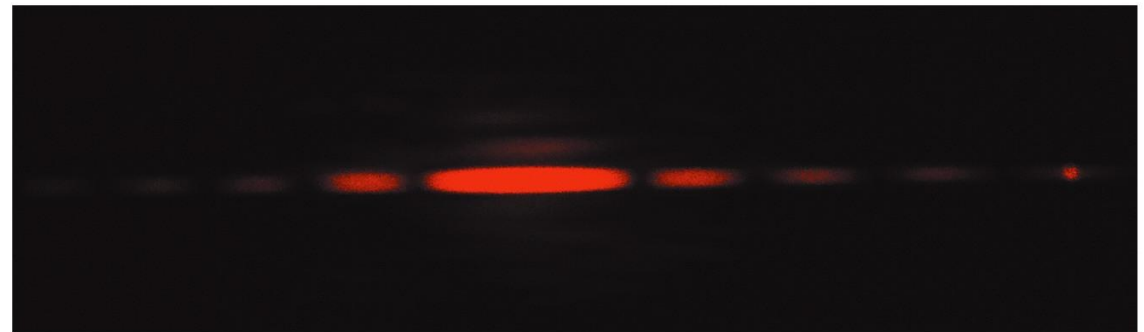


(a)



(b)

(b)



(c)

This diffraction minimum eliminates some of the double-slit bright fringes.

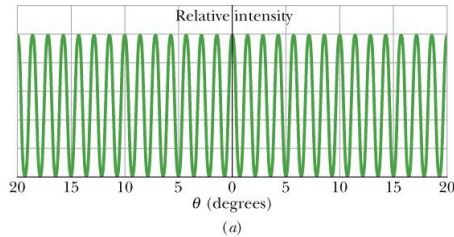
Courtesy Jearl Walker

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(a) The intensity plot to be expected in a double-slit interference experiment with vanishingly narrow slits. (b) The intensity plot for diffraction by a typical slit of width  $a$  (not vanishingly narrow). (c) The intensity plot to be expected for two slits of width  $a$ . The curve of (b) acts as an envelope, limiting the intensity of the double-slit fringes in (a). Note that the first minima of the diffraction pattern of (b) eliminate the double-slit fringes that would occur near  $12^\circ$  in (c).

## 36-4 Diffraction by a Double Slit

Waves passing through two slits produce a combination of double-slit interference and diffraction by each slit.



For identical slits with width  $a$  and center-to-center separation  $d$ , the intensity in the pattern varies with the angle  $\theta$  from the central axis as

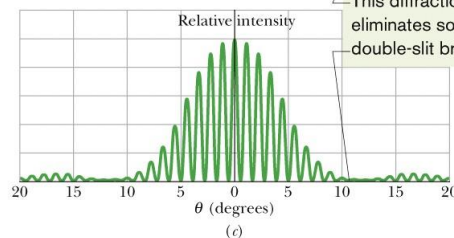
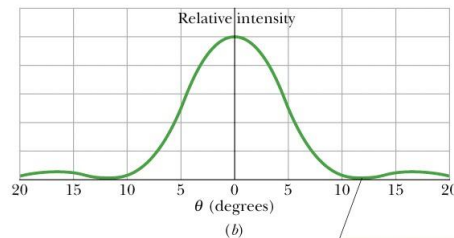
$$I(\theta) = I_m (\cos^2 \beta) \left( \frac{\sin \alpha}{\alpha} \right)^2 \quad (\text{double slit}),$$

in which

$$\beta = \frac{\pi d}{\lambda} \sin \theta$$

and

$$\alpha = \frac{\pi a}{\lambda} \sin \theta.$$



This diffraction minimum eliminates some of the double-slit bright fringes.

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Note carefully that the right side of double slit equation is the product of  $I_m$  and two factors. (1) The interference factor  $\cos^2 \beta$  is due to the interference between two slits with slit separation  $d$ . (2) The diffraction factor  $[(\sin \alpha)/\alpha]^2$  is due to diffraction by a single slit of width  $a$ .

## 36-5 Diffraction Gratings

### Learning Objectives

- 36.22** Describe a diffraction grating and sketch the interference pattern it produces in monochromatic light.
- 36.23** Distinguish the interference patterns of a diffraction grating and a double-slit arrangement.
- 36.24** Identify the terms line and order number.
- 36.25** For a diffraction grating, relate order number  $m$  to the path length difference of rays that give a bright fringe.
- 36.26** For a diffraction grating, relate the slit separation  $d$ , the angle  $\theta$  to a bright fringe in the pattern, the order number  $m$  of that fringe, and the wavelength  $\lambda$  of the light.
- 36.27** Identify the reason why there is a maximum order number for a given diffraction grating.
- 36.28** Explain the derivation of the equation for a line's half-width in a diffraction-grating pattern.

## 36-5 Diffraction Gratings

### Learning Objectives

**36.29** Calculate the half-width of a line at a given angle in a diffraction-grating pattern.

**36.30** Explain the advantage of increasing the number of slits in a diffraction grating.

**36.31** Explain how a grating spectroscope works.

## 36-5 Diffraction Gratings

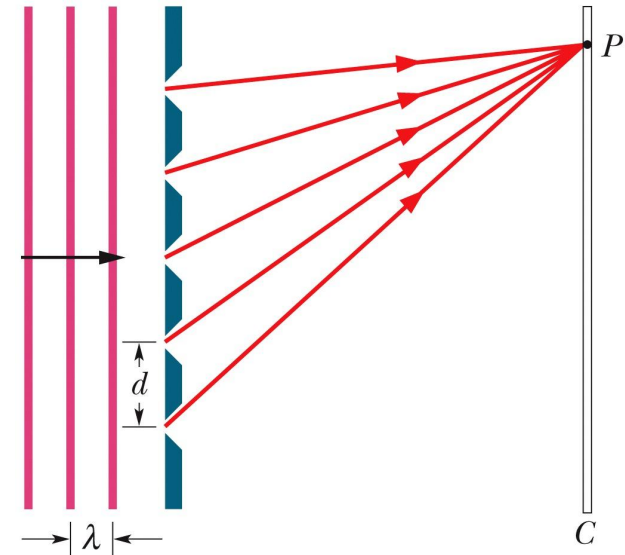
A diffraction grating is a series of “slits” used to separate an incident wave into its component wavelengths by separating and displaying their diffraction maxima. Diffraction by  $N$  (multiple) slits results in maxima (lines) at angles  $\theta$  such that

$$d \sin \theta = m\lambda, \quad \text{for } m = 0, 1, 2, \dots \quad (\text{maxima—lines}),$$

A line's **half-width** is the angle from its center to the point where it disappears into the darkness and is given by

$$\Delta\theta_{\text{hw}} = \frac{\lambda}{Nd \cos \theta} \quad (\text{half-width of line at } \theta).$$

Note that for light of a given wavelength  $\lambda$  and a given ruling separation  $d$ , the widths of the lines decrease with an increase in the number  $N$  of rulings. Thus, of two diffraction gratings, the grating with the larger value of  $N$  is better able to distinguish between wavelengths because its diffraction lines are narrower and so produce less overlap.



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An idealized diffraction grating, consisting of only five rulings, that produces an interference pattern on a distant viewing screen  $C$ .



## 36-6 Gratings: Dispersion and Resolving Power

### Learning Objectives

**36.32** Identify dispersion as the spreading apart of the diffraction lines associated with different wavelengths.

**36.33** Apply the relationships between dispersion  $D$ , wavelength difference  $\Delta\lambda$ , angular separation  $\Delta\theta$ , slit separation  $d$ , order number  $m$ , and the angle  $\theta$  corresponding to the order number.

**36.34** Identify the effect on the dispersion of a diffraction grating if the slit separation is varied.

**36.35** Identify that for us to resolve lines, a diffraction grating must make them distinguishable.

**36.36** Apply the relationship between resolving power  $R$ , wavelength difference  $\Delta\lambda$ , average wavelength  $\lambda_{avg}$ , number of rulings  $N$ , and order number  $m$ .

**36.37** Identify the effect on the resolving power  $R$  if the number of slits  $N$  is increased.

## 36-6 Gratings: Dispersion and Resolving Power

The dispersion  $D$  of a diffraction grating is a measure of the angular separation  $\Delta\theta$  of the lines it produces for two wavelengths differing by  $\Delta\lambda$ . For order number  $m$ , at angle  $\theta$ , the dispersion is given by

$$D = \frac{\Delta\theta}{\Delta\lambda} = \frac{m}{d \cos \theta} \quad (\text{dispersion}).$$

Thus, to achieve higher dispersion we must use a grating of smaller grating spacing  $d$  and work in a higher-order  $m$ . Note that the dispersion does not depend on the number of rulings  $N$  in the grating. The SI unit for  $D$  is the degree per meter or the radian per meter.



Kristen Brochmann/Fundamental Photographs

The fine rulings, each  $0.5 \mu\text{m}$  wide, on a compact disc function as a diffraction grating. When a small source of white light illuminates a disc, the diffracted light forms colored “lanes” that are the composite of the diffraction patterns from the rulings.

## 36-6 Gratings: Dispersion and Resolving Power

The resolving power  $R$  of a diffraction grating is a measure of its ability to make the emission lines of two close wavelengths distinguishable. For two wavelengths differing by  $\Delta\lambda$  and with an average value of  $\lambda_{avg}$ , the resolving power is given by

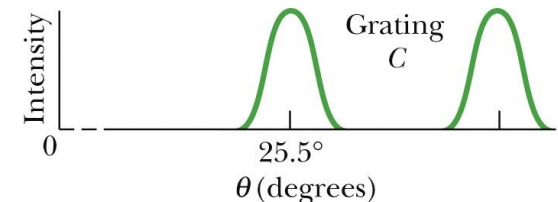
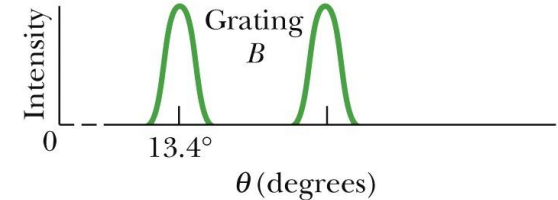
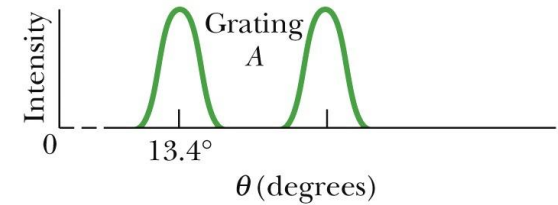
$$R = \frac{\lambda_{avg}}{\Delta\lambda} = Nm$$

**Table 36-1** Three Gratings<sup>a</sup>

Grating	$N$	$d$ (nm)	$\theta$	$D$ ( $^{\circ}/\mu\text{m}$ )	$R$
A	10 000	2540	$13.4^{\circ}$	23.2	10 000
B	20 000	2540	$13.4^{\circ}$	23.2	20 000
C	10 000	1360	$25.5^{\circ}$	46.3	10 000

<sup>a</sup>Data are for  $\lambda = 589$  nm and  $m = 1$ .

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The intensity patterns for light of two wavelengths sent through the gratings of Table 36-1. Grating B has the highest resolving power, and grating C the highest dispersion.

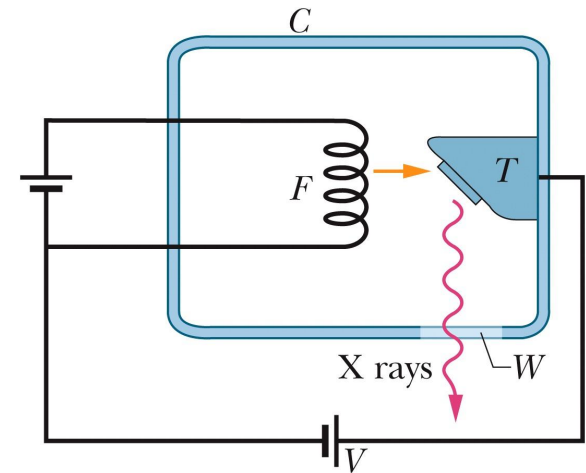
## 36-7 X-Ray Diffraction

### Learning Objectives

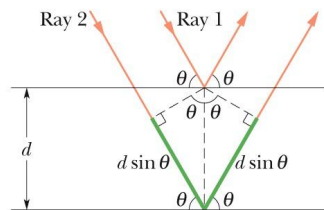
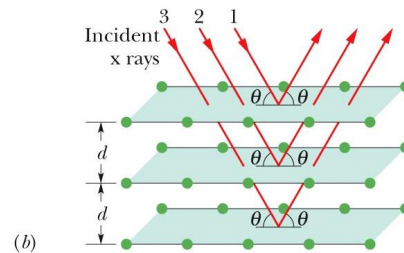
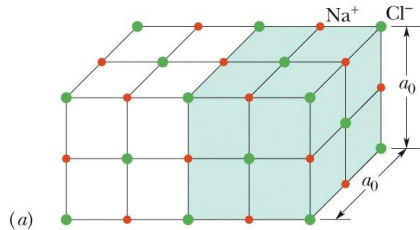
- 36.38** Identify approximately where x rays are located in the electromagnetic spectrum.
- 36.39** Define a unit cell.
- 36.40** Define reflecting planes (or crystal planes) and interplanar spacing.
- 36.41** Sketch two rays that scatter from adjacent planes.
- 36.42** For the intensity maxima in x-ray scattering by a crystal, apply the relationship between the interplanar spacing  $d$ , the angle  $\theta$  of scattering, the order number  $m$ , and the wavelength  $\lambda$  of the x rays.
- 36.43** Given a drawing of a unit cell, demonstrate how an interplanar spacing can be determined.

# 36-7 X-Ray Diffraction

X rays are electromagnetic radiation whose wavelengths are of the order of  $1 \text{ \AA} (=10^{-10} \text{ m})$ . Figure (right) shows that x rays are produced when electrons escaping from a heated filament  $F$  are accelerated by a potential difference  $V$  and strike a metal target  $T$ .



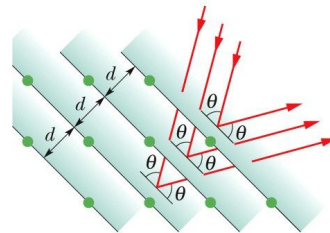
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The extra distance of ray 2 determines the interference.

(c)

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(d)

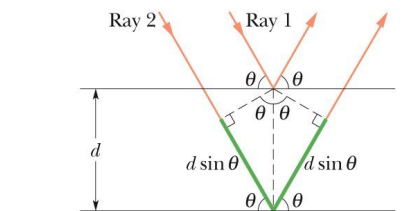
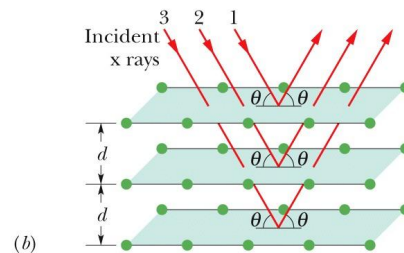
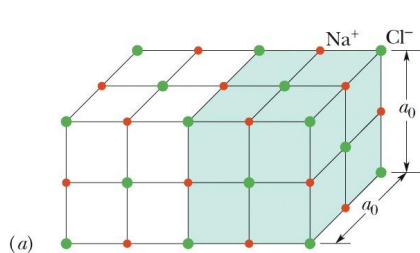
(a) The cubic structure of NaCl, showing the sodium and chlorine ions and a unit cell (shaded). (b) Incident x rays undergo diffraction by the structure of (a). The x rays are diffracted as if they were reflected by a family of parallel planes, with angles measured relative to the planes (not relative to a normal as in optics). (c) The path length difference between waves effectively reflected by two adjacent planes is  $2d \sin \theta$ . (d) A different orientation of the incident x rays relative to the structure. A different family of parallel planes now effectively reflects the x rays.

# 36-7 X-Ray Diffraction

As shown in figure below if x rays are directed toward a crystal structure, they undergo Bragg scattering, which is easiest to visualize if the crystal atoms are considered to be in parallel planes.

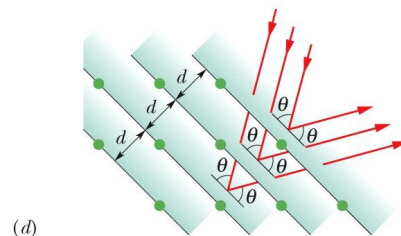
For x rays of wavelength  $\lambda$  scattering from crystal planes with separation  $d$ , the angles  $\theta$  at which the scattered intensity is maximum are given by

$$2d \sin \theta = m\lambda, \quad \text{for } m = 1, 2, 3, \dots$$



The extra distance of ray 2 determines the interference.

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(a) The cubic structure of NaCl, showing the sodium and chlorine ions and a unit cell (shaded). (b) Incident x rays undergo diffraction by the structure of (a). The x rays are diffracted as if they were reflected by a family of parallel planes, with angles measured relative to the planes (not relative to a normal as in optics). (c) The path length difference between waves effectively reflected by two adjacent planes is  $2d \sin \theta$ . (d) A different orientation of the incident x rays relative to the structure. A different family of parallel planes now effectively reflects the x rays.

## 36 Summary

### Diffraction

- When waves encounter an edge, an obstacle, or an aperture the size of which is comparable to the wavelength of the waves, those waves spread out as they travel and, as a result, undergo interference.

### Single-Slit Diffraction

- A single-slit diffraction patterns satisfy

$$a \sin \theta = m\lambda, \quad \text{for } m = 1, 2, 3, \dots \quad \text{Eq. 36-3}$$

- The intensity of the diffraction pattern at any given angle  $\theta$  is

$$I(\theta) = I_m \left( \frac{\sin \alpha}{\alpha} \right)^2, \quad \text{Eq. 36-5}$$

where  $\alpha = \frac{\pi a}{\lambda} \sin \theta$  Eq. 36-6

### Circular Aperture Diffraction

- Diffraction by a circular aperture or a lens with diameter  $d$  produces a central maximum and concentric maxima and minima, with the first minimum at an angle  $\theta$  given by

$$\sin \theta = 1.22 \frac{\lambda}{d} \quad \text{Eq. 36-12}$$

### Rayleigh's Criterion

- Rayleigh's criterion suggests that two objects are on the verge of resolvability if the central diffraction maximum of one is at the first minimum of the other. Their angular separation can then be no less than

$$\theta_R = 1.22 \frac{\lambda}{d} \quad \text{Eq. 35-14}$$



## 36 Summary

### Double-Slit Diffraction

- Waves passing through two slits, each of width  $a$ , whose centers are a distance  $d$  apart, display diffraction patterns whose intensity  $I$  at angle  $\theta$  is

$$I(\theta) = I_m(\cos^2 \beta) \left( \frac{\sin \alpha}{\alpha} \right)^2 \quad \text{Eq. 36-19}$$

### Diffraction Gratings

- Diffraction by  $N$  (multiple) slits results in maxima (lines) at angles  $\theta$  such that

$$d \sin \theta = m\lambda, \quad \text{for } m = 0, 1, 2, \dots \quad \text{Eq. 36-25}$$

with the half-widths of the lines

given by 
$$\Delta\theta_{\text{hw}} = \frac{\lambda}{Nd \cos \theta} \quad \text{Eq. 36-28}$$

and

$$D = \frac{\Delta\theta}{\Delta\lambda} = \frac{m}{d \cos \theta} \quad \text{Eq. 36-29\&30}$$

$$R = \frac{\lambda_{\text{avg}}}{\Delta\lambda} = Nm. \quad \text{Eq. 36-31\&32}$$

### X-Ray Diffraction

- Diffraction maxima (due to constructive interference) occur if the incident direction of the wave, measured from the surfaces of these planes, and the wavelength  $\lambda$  of the radiation satisfy Bragg's law:

$$2d \sin \theta = m\lambda, \quad \text{for } m = 1, 2, 3, \dots \quad \text{Eq. 36-12}$$