Chapter 28

Magnetic Fields

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28-1 Magnetic Fields and the Definition of *B*

Learning Objectives

28.01 Distinguish an electromagnet from a permanent magnet.

28.02 Identify that a magnetic field is a vector quantity and thus has both magnitude and direction.

28.03 Explain how a magnetic field can be defined in terms of what happens to a charged particle moving through the field.

28.04 For a charged particle moving through a uniform magnetic field, apply the relationship between the force on the charge F_B, charge q, speed v, field magnitude B, and the angle Φ between the directions of the velocity vector v and the magnetic field vector B.

28.05 For a charged particle sent through a uniform magnetic field, find the direction of the magnetic force F_B by (1) applying the right-hand rule to find the direction of the cross product $\boldsymbol{v}\times\boldsymbol{B}$ and (2) determining what effect the charge q has on the direction.

28-1 Magnetic Fields and the Definition of *B*

Learning Objectives (Contd.)

28.06 Find the magnetic force F_B acting on a moving charged particle by evaluating the cross product q ($\boldsymbol{v}\times\boldsymbol{B}$) in unit-vector notation and magnitude-angle notation.

28.07 Identify that the magnetic force vector F_B must always be perpendicular to both the velocity vector ***v*** and the magnetic field vector ***B***.

28.08 Identify the effect of the magnetic force on the particle's speed and kinetic energy.

28.09 Identify a magnet as being a magnetic dipole.

28.10 Identify that opposite magnetic poles attract each other and like magnetic poles repel each other.

28.11 Explain magnetic field lines, including where they originate and terminate and what their spacing represents.

28-1 Magnetic Fields and the Definition of *B*

The Definition of *B*

The Field. We can define a magnetic field ***B*** to be a vector quantity that exists when it exerts a force $\boldsymbol{F_B}$ on a charge moving with velocity ***v***. We can next measure the magnitude of $\boldsymbol{F_B}$ when ***v*** is directed perpendicular to that force and then define the magnitude of ***B*** in terms of that force magnitude:

$$B = \frac{F_B}{|q|v},$$

where q is the charge of the particle. We can summarize all these results with the following vector equation:

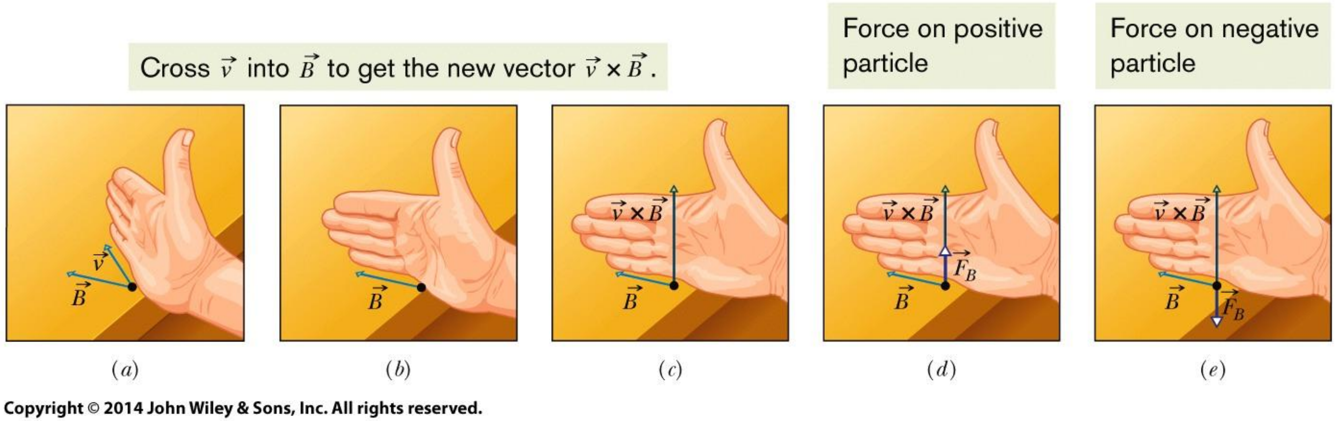
$$\vec{F}_B = q\vec{v} \times \vec{B};$$

that is, the force $\boldsymbol{F_B}$ on the particle by the field ***B*** is equal to the charge q times the cross product of its velocity ***v*** and the field ***B*** (all measured in the same reference frame). We can write the magnitude of $\boldsymbol{F_B}$ as

$$F_B = |q|vB \sin \phi,$$

where ϕ is the angle between the directions of velocity ***v*** and magnetic field ***B***.

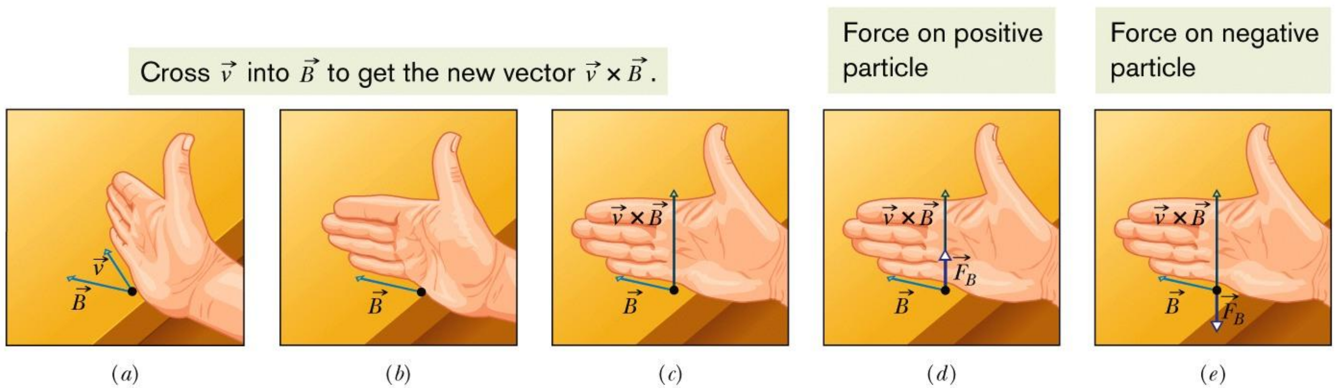
Finding the Magnetic Force on a Particle



$$\vec{F}_B = q\vec{v} \times \vec{B};$$

This equation tells us the direction of ***F***. We know the cross product of ***v*** and ***B*** is a vector that is perpendicular to these two vectors. The right-hand rule (Figs. a-c) tells us that the thumb of the right hand points in the direction of ***v*** × ***B*** when the fingers sweep ***v*** into ***B***. If *q* is positive, then (by the above Eq.) the force $\boldsymbol{F_B}$ has the same sign as ***v*** × ***B*** and thus must be in the same direction; that is, for positive *q*, $\boldsymbol{F_B}$ is directed along the thumb (Fig. d). If *q* is negative, then the force $\boldsymbol{F_B}$ and cross product ***v*** × ***B*** have opposite signs and thus must be in opposite directions. For negative *q*, ***F*** is directed opposite the thumb (Fig. e).

Finding the Magnetic Force on a Particle

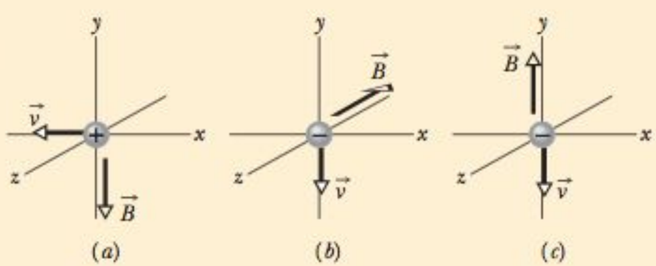


$$\vec{F}_B = q\vec{v}\times\vec{B};$$

The force $\vec{F}_B$ acting on a charged particle moving with velocity $\vec{v}$ through a magnetic field $\vec{B}$ is *always* perpendicular to $\vec{v}$ and $\vec{B}$.

Checkpoint 1

The figure shows three situations in which a charged particle with velocity $\vec{v}$ travels through a uniform magnetic field $\vec{B}$. In each situation, what is the direction of the magnetic force $\vec{F}_B$ on the particle?



Answer:

(a) towards the positive z-axis

(b) towards the negative x-axis

(c) none (cross product is zero)

Magnetic Field Lines

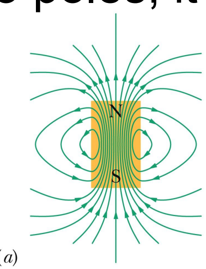
We can represent magnetic fields with field lines, as we did for electric fields. Similar rules apply:

(1) the direction of the tangent to a magnetic field line at any point gives the direction of ***B*** at that point

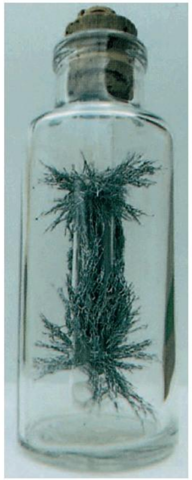
(2) the spacing of the lines represents the magnitude of ***B*** —the magnetic field is stronger where the lines are closer together, and conversely.

Two Poles. The (closed) field lines enter one end of a magnet and exit the other end. The end of a magnet from which the field lines emerge is called the north pole of the magnet; the other end, where field lines enter the magnet, is called the south pole. Because a magnet has two poles, it is said to be a **magnetic dipole**.

> Opposite magnetic poles attract each other, and like magnetic poles repel each other.



(a)


(b)

Courtesy Dr. Richard Cannon, Southeast Missouri State University, Cape Girardeau

(a) The magnetic field lines for a bar magnet.

(b) A “cow magnet” — a bar magnet that is intended to be slipped down into the rumen of a cow to recover accidentally ingested bits of scrap iron and to prevent them from reaching the cow’s intestines. The iron filings at its ends reveal the magnetic field lines.

28-2 Crossed Fields: Discovery of The Electron

Learning Objectives

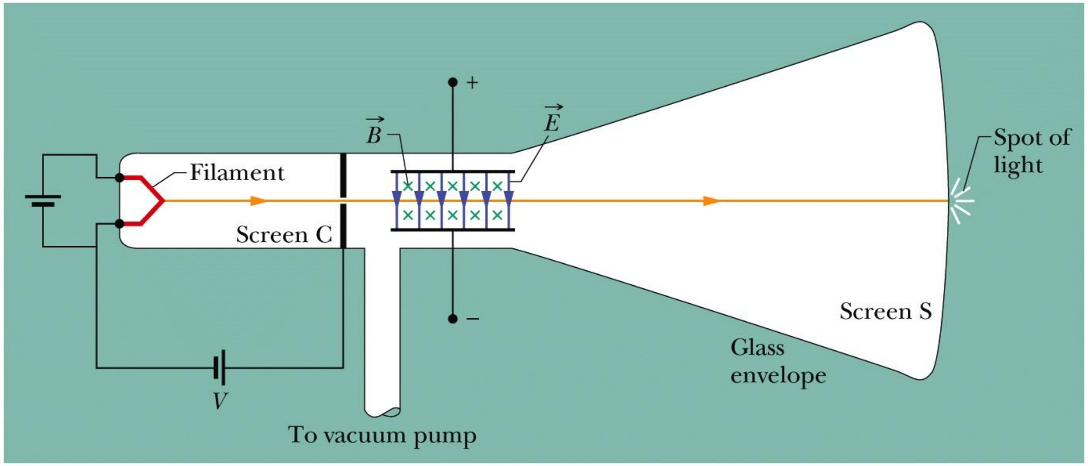
28.12 Describe the experiment of J. J. Thomson.

28.13 For a charged particle moving through a magnetic field and an electric field, determine the net force on the particle in both magnitude-angle notation and unit-vector notation.

28.14 In situations where the magnetic force and electric force on a particle are in opposite directions, determine the speed at which these forces cancel, the speeds at which the magnetic force dominates, and the speeds at which the electric force dominates.

28-2 Crossed Fields: Discovery of The Electron

A modern version of J.J. Thomson's apparatus for measuring the ratio of mass to charge for the electron. An electric field ***E*** is established by connecting a battery across the deflecting-plate terminals. The magnetic field ***B*** is set up by means of a current in a system of coils (not shown). The magnetic field shown is into the plane of the figure, as represented by the array of Xs (which resemble the feathered ends of arrows).



If a charged particle moves through a region containing both an electric field and a magnetic field, it can be affected by both an electric force and a magnetic force.

When the two fields are perpendicular to each other, they are said to be **crossed fields**.

If the forces are in opposite directions, one particular speed will result in no deflection of the particle.

28-3 Crossed Fields: The Hall Effect

Learning Objectives

28.15 Describe the Hall effect for a metal strip carrying current, explaining how the electric field is set up and what limits its magnitude.

28.16 For a conducting strip in a Hall-effect situation, draw the vectors for the magnetic field and electric field. For the conduction electrons, draw the vectors for the velocity, magnetic force, and electric force.

28.17 Apply the relationship between the Hall potential difference V, the electric field magnitude E, and the width of the strip d.

28.18 Apply the relationship between charge-carrier number density n, magnetic field magnitude B, current i, and Hall-effect potential difference V.

28.19 Apply the Hall-effect results to a conducting object moving through a uniform magnetic field, identifying the width across which a Hall-effect potential difference V is set up and calculating V.

28-3 Crossed Fields: The Hall Effect

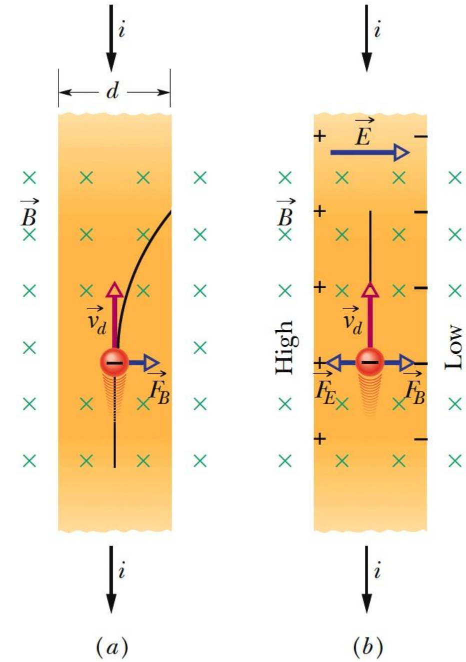
As we just discussed, a beam of electrons in a vacuum can be deflected by ad magnetic field. Can the drifting conduction electrons in a copper wire also be deflected by a magnetic field?

In 1879, Edwin H. Hall, then a 24-year-old graduate student at the Johns Hopkins University, showed that they can.

This **Hall effect** allows us to find out whether the charge carriers in a conductor are positively or negatively charged. Beyond that, we can measure the number of such carriers per unit volume of the conductor.

Figure (a) shows a copper strip of width *d*, carrying a current *i* whose conventional direction is from the top of the figure to the bottom. The charge carriers are electrons and, as we know, they drift (with drift speed v_d) in the opposite direction, from bottom to top.

As time goes on, electrons move to the right, mostly piling up on the right edge of the strip, leaving uncompensated positive charges in fixed positions at the left edge as shown in figure (b).



28-3 Crossed Fields: The Hall Effect

When a uniform magnetic field B is applied to a conducting strip carrying current i, with the field perpendicular to the direction of the current, a Hall-effect potential difference *V* is set up across the strip.
The electric force $\boldsymbol{F}_E$ on the charge carriers is then balanced by the magnetic force $\boldsymbol{F}_B$ on them.
The number density *n* of the charge carriers can then be determined from

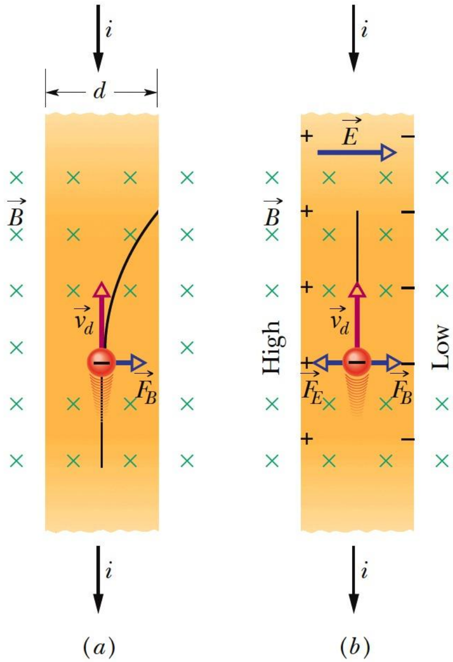
$$n = \frac{Bi}{Vle},$$

in which *l* (= *A/d*) is the thickness of the strip. With this equation we can find *n* from measurable quantities.

When a conductor moves through a uniform magnetic field ***B*** at speed *v*, the Hall-effect potential difference *V* across it is

$$V = vBd,$$

Where *d* is the width perpendicular to both velocity ***v*** and field ***B***.



28-4 A Circulating Charged Particle

Learning Objectives

28.20 For a charged particle moving through a uniform magnetic field, identify under what conditions it will travel in a straight line, in a circular path, and in a helical path.

28.21 For a charged particle in uniform circular motion due to a magnetic force, start with Newton's second law and derive an expression for the orbital radius *r* in terms of the field magnitude *B* and the particle's mass *m*, charge magnitude *q*, and speed *v*.

28.22 For a charged particle moving along a circular path in a magnetic field, calculate and relate speed, centripetal force, centripetal acceleration, radius, period, frequency, and angular frequency, and identify which of the quantities do not depend on speed.

28.23 For a positive particle and a negative particle moving along a circular path in a uniform magnetic field, sketch the path and indicate the magnetic field vector, the velocity vector, the result of the cross product of the velocity and field vectors, and the magnetic force vector.

28-4 A Circulating Charged Particle

Learning Objectives (Contd.)

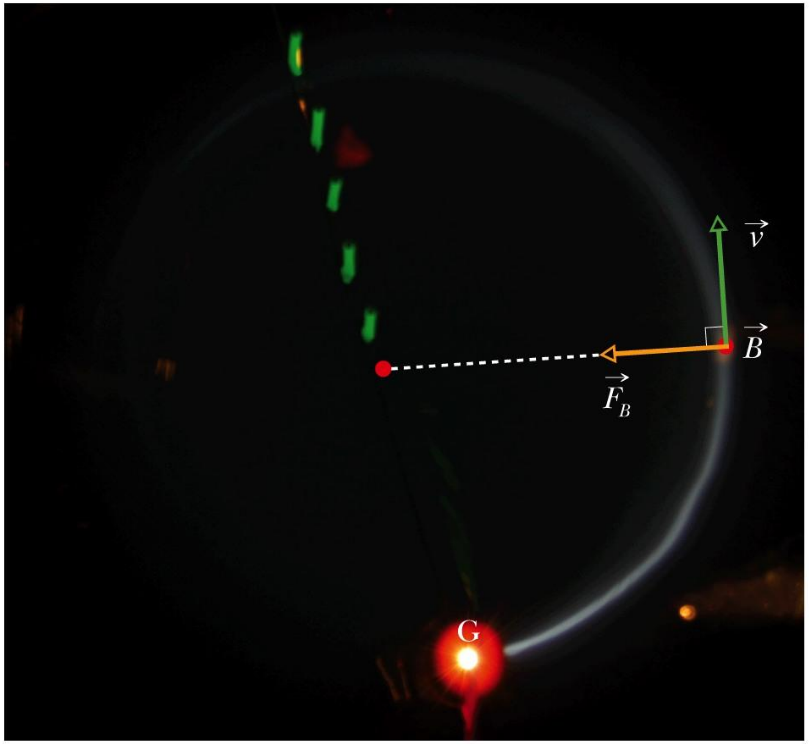
28.24 For a charged particle moving in a helical path in a magnetic field, sketch the path and indicate the magnetic field, the pitch, the radius of curvature, the velocity component parallel to the field, and the velocity component perpendicular to the field

28.25 For helical motion in a magnetic field, apply the relationship between the radius of curvature and one of the velocity components.

28.26 For helical motion in a magnetic field, identify pitch *p* and relate it to one of the velocity components.

28-4 A Circulating Charged Particle

A beam of electrons is projected into a chamber by an electron gun G. The electrons enter in the plane of the page with speed *v* and then move in a region of uniform magnetic field ***B*** directed out of that plane. As a result, a magnetic force $F_B = q\,(\mathbf{v}\times\mathbf{B})$ continuously deflects the electrons, and because ***v*** and B are always perpendicular to each other, this deflection causes the electrons to follow a circular path. The path is visible in the photo because atoms of gas in the chamber emit light when some of the circulating electrons collide with them.



Applying Newton's second law to the circular motion yields

$$|q|vB = \frac{mv^2}{r}.$$

Therefore the radius *r* of the circle is

$$r = \frac{mv}{|q|B}.$$

28-5 Cyclotrons and Synchrotrons

Learning Objectives

28.27 Describe how a cyclotron works, and in a sketch, indicate a particle's path and the regions where the kinetic energy is increased.

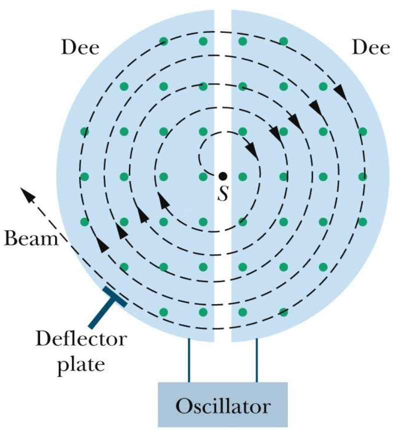
28.28 Identify the resonance condition

28.29 For a cyclotron, apply the relationship between the particle's mass and charge, the magnetic field, and the frequency of circling.

28.30 Distinguish between a cyclotron and a synchrotron.

28-5 Cyclotrons and Synchrotrons

The protons spiral outward in a cyclotron, picking up energy in the gap.



The Cyclotron: The figure is a top view of the region of a cyclotron in which the particles (protons, say) circulate. The two hollow D-shaped objects (each open on its straight edge) are made of sheet copper. These dees, as they are called, are part of an electrical oscillator that alternates the electric potential difference across the gap between the dees. The electrical signs of the dees are alternated so that the electric field in the gap alternates in direction, first toward one dee and then toward the other dee, back and forth. The dees are immersed in a large magnetic field directed out of the plane of the page. The magnitude *B* of this field is set via a control on the electromagnet producing the field.

The key to the operation of the cyclotron is that the frequency *f* at which the proton circulates in the magnetic field (and that does not depend on its speed) must be equal to the fixed frequency f_{osc} of the electrical oscillator, or

$$f = f_{\text{osc}} \quad \text{(resonance condition).}$$

This resonance condition says that, if the energy of the circulating proton is to increase, energy must be fed to it at a frequency f_{osc} that is equal to the natural frequency *f* at which the proton circulates in the magnetic field.

28-5 Cyclotrons and Synchrotrons

Proton Synchrotron: The magnetic field ***B*** and the oscillator frequency f_{osc}, instead of having fixed values as in the conventional cyclotron, are made to vary with time during the accelerating cycle. When this is done properly, (1) the frequency of the circulating protons remains in step with the oscillator at all times, and (2) the protons follow a circular — not a spiral — path. Thus, the magnet need extend only along that circular path, not over some 4×10^6 m^2. The circular path, however, still must be large if high energies are to be achieved.

28-6 Magnetic Force on Current-Carrying Wire

Learning Objectives

28.31 For the situation where a current is perpendicular to a magnetic field, sketch the current, the direction of the magnetic field, and the direction of the magnetic force on the current (or wire carrying the current).

28.32 For a current in a magnetic field, apply the relationship between the magnetic force magnitude F_B, the current i, the length of the wire L, and the angle f between the length vector $\boldsymbol{L}$ and the field vector $\boldsymbol{B}$.

28.33 Apply the right-hand rule for cross products to find the direction of the magnetic force on a current in a magnetic field.

28.34 For a current in a magnetic field, calculate the magnetic force $\boldsymbol{F}_B$ with a cross product of the length vector $\boldsymbol{L}$ and the field vector $\boldsymbol{B}$, in magnitude-angle and unit-vector notations.

28.35 Describe the procedure for calculating the force on a current-carrying wire in a magnetic field if the wire is not straight or if the field is not uniform.

28-6 Magnetic Force on a Current-Carrying Wire

A straight wire carrying a current *i* in a uniform magnetic field experiences a sideways force

$$\vec{F}_B = i\vec{L} \times \vec{B} \quad \text{(force on a current).}$$

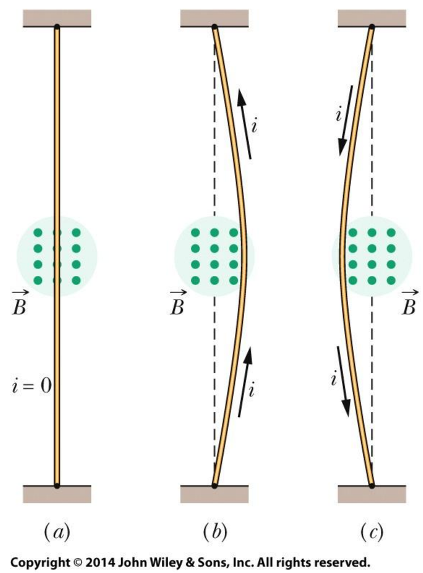
Here ***L*** is a length vector that has magnitude *L* and is directed along the wire segment in the direction of the (conventional) current.

Crooked Wire. If a wire is not straight or the field is not uniform, we can imagine the wire broken up into small straight segments. The force on the wire as a whole is then the vector sum of all the forces on the segments that make it up. In the differential limit, we can write

$$d\vec{F}_B = i\,d\vec{L} \times \vec{B}.$$

and the direction of length vector ***L*** or *d**L*** is in the direction of *i.*

A force acts on a current through a *B* field.



A flexible wire passes between the pole faces of a magnet (only the farther pole face is shown). (a) Without current in the wire, the wire is straight. (b) With upward current, the wire is deflected rightward. (c) With downward current, the deflection is leftward.

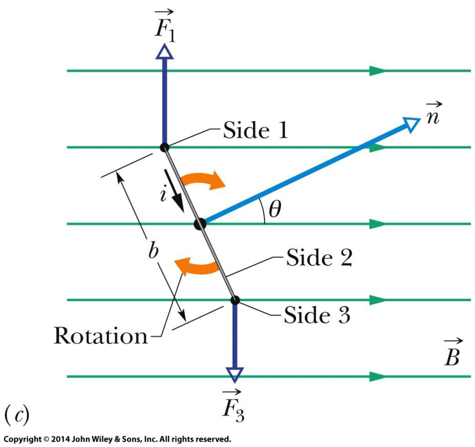
28-7 Torque on a Current Loop

Learning Objectives

28.36 Sketch a rectangular loop of current in a magnetic field, indicating the magnetic forces on the four sides, the direction of the current, the normal vector ***n***, and the direction in which a torque from the forces tends to rotate the loop.

28.37 For a current-carrying coil in a magnetic field, apply the relationship between the torque magnitude τ, the number of turns N, the area of each turn A, the current i, the magnetic field magnitude B, and the angle θ between the normal vector ***n*** and the magnetic field vector ***B***.

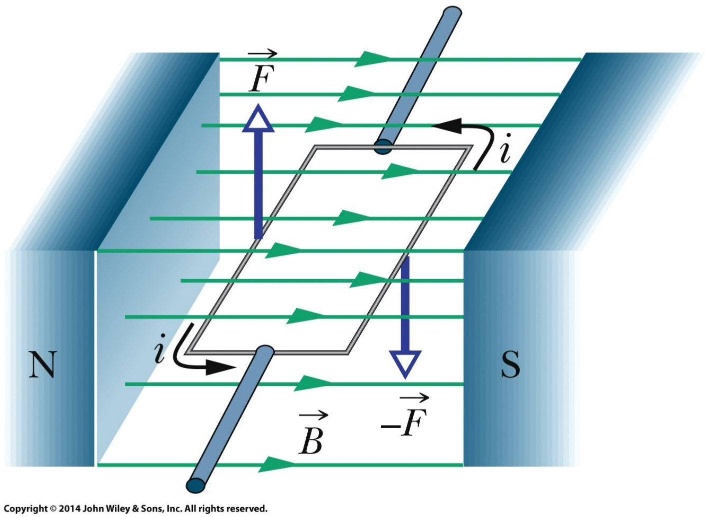
28-7 Torque on a Current Loop



As shown in the figure (right) the net force on the loop is the vector sum of the forces acting on its four sides and comes out to be zero. The net torque acting on the coil has a magnitude given by

$$\tau = NiAB \sin \theta,$$

where *N* is the number of turns in the coil, *A* is the area of each turn, *i* is the current, *B* is the field magnitude, and *θ* is the angle between the magnetic field ***B*** and the normal vector to the coil ***n***.



The elements of an electric motor: A rectangular loop of wire, carrying a current and free to rotate about a fixed axis, is placed in a magnetic field. Magnetic forces on the wire produce a torque that rotates it. A commutator (not shown) reverses the direction of the current every half-revolution so that the torque always acts in the same direction.

28-8 The Magnetic Dipole Moment

Learning Objectives

28.38 Identify that a current-carrying coil is a magnetic dipole with a magnetic dipole moment $\boldsymbol{\mu}$ that has the direction of the normal vector $\boldsymbol{n}$, as given by a right-hand rule.

28.39 For a current-carrying coil, apply the relationship between the magnitude μ of the magnetic dipole moment, the number of turns N, the area A of each turn, and the current i.

28.40 On a sketch of a current-carrying coil, draw the direction of the current, and then use a right-hand rule to determine the direction of the magnetic dipole moment vector $\boldsymbol{\mu}$.

28.41 For a magnetic dipole in an external magnetic field, apply the relationship between the torque magnitude τ, the dipole moment magnitude μ, the magnetic field magnitude $\boldsymbol{B}$, and the angle θ between the dipole moment vector $\boldsymbol{\mu}$ and the magnetic field vector $\boldsymbol{B}$.

28.42 Identify the convention of assigning a plus or minus sign to a torque according to the direction of rotation.

28-8 The Magnetic Dipole Moment

Learning Objectives (Contd.)

28.43 Calculate the torque on a magnetic dipole by evaluating a cross product of the dipole moment vector ***μ*** and the external magnetic field vector ***B***, in magnitude-angle notation and unit-vector notation.

28.44 For a magnetic dipole in an external magnetic field, identify the dipole orientations at which the torque magnitude is minimum and maximum.

28.45 For a magnetic dipole in an external magnetic field, apply the relationship between the orientation energy *U*, the dipole moment magnitude *μ*, the external magnetic field magnitude ***B***, and the angle *θ* between the dipole moment vector *μ* and the magnetic field vector ***B***.

28.46 Calculate the orientation energy *U* by taking a dot product of the dipole moment vector ***μ*** and the external magnetic field vector ***B***, in magnitude-angle and unit-vector notations.

28-8 The Magnetic Dipole Moment

Learning Objectives (Contd.)

28.47 Identify the orientations of a magnetic dipole in an external magnetic field that give the minimum and maximum orientation energies.

28.48 For a magnetic dipole in a magnetic field, relate the orientation energy U to the work W_a done by an external torque as the dipole rotates in the magnetic field.

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28-8 The Magnetic Dipole Moment

A coil (of area A and N turns, carrying current i) in a uniform magnetic field $\boldsymbol{B}$ will experience a torque $\boldsymbol{\tau}$ given by

$$\vec{\tau} = \vec{\mu} \times \vec{B},$$

where $\boldsymbol{\mu}$ is the **magnetic dipole moment** of the coil, with magnitude $\mu = NiA$ and direction given by the right- hand rule.

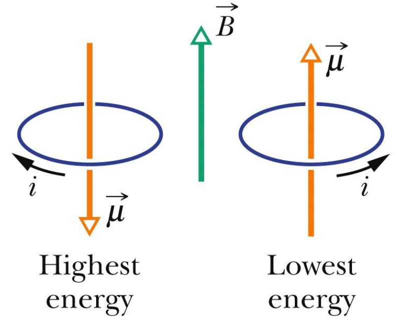
The orientation energy of a magnetic dipole in a magnetic field is

$$U(\theta) = -\vec{\mu} \cdot \vec{B}.$$

If an external agent rotates a magnetic dipole from an initial orientation θ_i to some other orientation θ_f and the dipole is stationary both initially and finally, the work W_a done on the dipole by the agent is

$$W_a = \Delta U = U_f - U_i.$$

The magnetic moment vector attempts to align with the magnetic field.



28 Summary

The Magnetic Field *B*

- Defined in terms of the force $\boldsymbol{F_B}$ acting on a test particle with charge *q* moving through the field with velocity ***v***

$$\vec{F}_B = q\vec{v} \times \vec{B}. \qquad \text{Eq. 28-2}$$

A Charge Particle Circulating in a Magnetic Field

- Applying Newton's second law to the circular motion yields

$$|q|vB = \frac{mv^2}{r} \qquad \text{Eq. 28-15}$$

- from which we find the radius r of the orbit circle to be

$$r = \frac{mv}{|q|B}. \qquad \text{Eq. 28-16}$$

Magnetic Force on a Current Carrying wire

- A straight wire carrying a current *i* in a uniform magnetic field experiences a sideways force

$$\vec{F}_B = i\vec{L} \times \vec{B}. \qquad \text{Eq. 28-26}$$

- The force acting on a current element i dL in a magnetic field is

$$d\vec{F}_B = i\,d\vec{L} \times \vec{B}. \qquad \text{Eq. 28-28}$$

Torque on a Current Carrying Coil

- A coil (of area *A* and *N* turns, carrying current *i*) in a uniform magnetic field ***B*** will experience a torque ***τ*** given by

$$\vec{\tau} = \vec{\mu} \times \vec{B}. \qquad \text{Eq. 28-37}$$

28 Summary

The Hall Effect

- When a conducting strip carrying a current *i* is placed in a uniform magnetic field ***B***, some charge carriers (with charge *e*) build up on one side of the conductor, creating a potential difference *V* across the strip. The polarities of the sides indicate the sign of the charge carriers.

Orientation Energy of a Magnetic Dipole

- The orientation energy of a magnetic dipole in a magnetic field is

$$U(\theta) = -\vec{\mu} \cdot \vec{B}. \qquad \textbf{Eq. 28-38}$$

- If an external agent rotates a magnetic dipole from an initial orientation θ_i to some other orientation θ_f and the dipole is stationary both initially and finally, the work W_a done on the dipole by the agent is

$$W_a = \Delta U = U_f - U_i. \qquad \textbf{Eq. 28-39}$$