Chapter 38

Photons and Matter Waves

38.2: The Photon, the Quantum of Light:

In 1905, Einstein proposed that electromagnetic radiation (or simply *light)* is quantized and exists in elementary amounts (quanta) that we now call **photons.**

According to that proposal, the quantum of a light wave of frequency f has the energy

$$E = hf \qquad \text{(photon energy).}$$

Here h is the ***Planck constant,*** which has the value

$$h = 6.63 \times 10^{-34}\ \mathrm{J \cdot s} = 4.14 \times 10^{-15}\ \mathrm{eV \cdot s}.$$

Example, Emission and absorption of light as photons:

A sodium vapor lamp is placed at the center of a large sphere that absorbs all the light reaching it. The rate at which the lamp emits energy is 100 W; assume that the emission is entirely at a wavelength of 590 nm. At what rate are photons absorbed by the sphere?

KEY IDEAS

The light is emitted and absorbed as photons. We assume that all the light emitted by the lamp reaches (and thus is absorbed by) the sphere. So, the rate R at which photons are absorbed by the sphere is equal to the rate R_{emit} at which photons are emitted by the lamp.

Calculations: That rate is

$$R_{\text{emit}} = \frac{\text{rate of energy emission}}{\text{energy per emitted photon}} = \frac{P_{\text{emit}}}{E}.$$

Into this we can substitute from Eq. 38-2 ($E = hf$), Einstein's proposal about the energy E of each quantum of light (which we here call a photon in modern language). We can then write the absorption rate as

$$R = R_{\text{emit}} = \frac{P_{\text{emit}}}{hf}.$$

Using Eq. 38-1 ($f = c/\lambda$) to substitute for f and then entering known data, we obtain

$$R = \frac{P_{\text{emit}}\lambda}{hc}$$

$$= \frac{(100\ \text{W})(590 \times 10^{-9}\ \text{m})}{(6.63 \times 10^{-34}\ \text{J}\cdot\text{s})(2.998 \times 10^{8}\ \text{m/s})}$$

$$= 2.97 \times 10^{20}\ \text{photons/s}. \qquad \text{(Answer)}$$

38.3: The Photoelectric Effect:

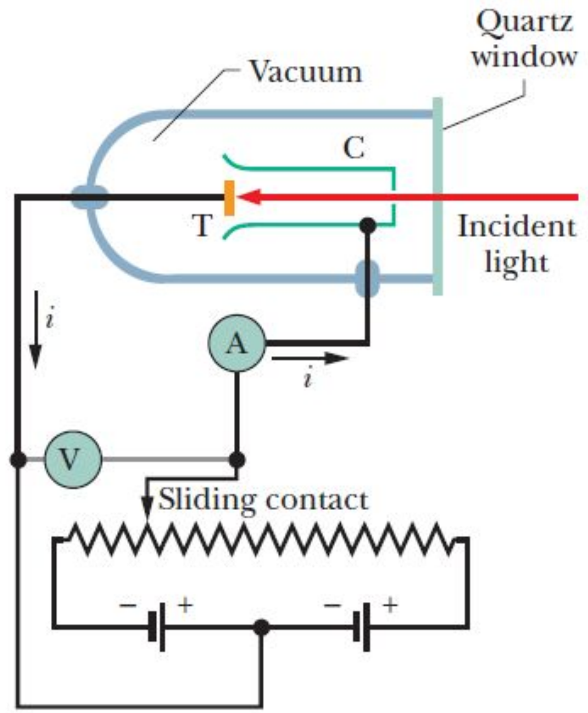


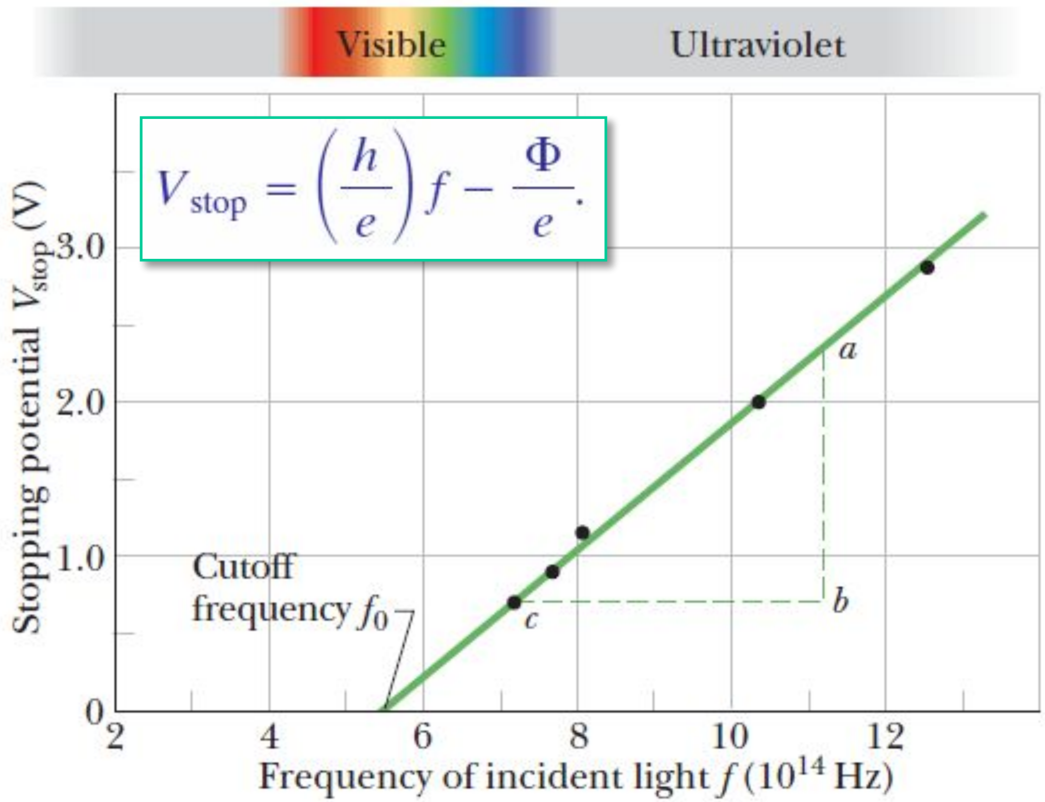
Fig. 38-1 An apparatus used to study the photoelectric effect. The incident light shines on target T, ejecting electrons, which are collected by collector cup C. The electrons move in the circuit in a direction opposite the conventional current arrows. The batteries and the variable resistor are used to produce and adjust the electric potential difference between T and C.

We then vary V until it reaches a certain value, called the **stopping potential** V_{stop}, at which point the reading of meter A has just dropped to zero. When $V = V_{stop}$, the most energetic ejected electrons are turned back just before reaching the collector. Then K_{max}, *the* kinetic energy of these most energetic electrons, is

$$K_{\max} = eV_{\text{stop}},$$

38.3: Second Photoelectric Experiment:

If the frequency f of the incident light is varied and the associated stopping potential V_{stop} is measured, then the plot of V_{stop} versus f as shown in the figure is obtained. The photoelectric effect does not occur if the frequency is below a certain **cutoff frequency** f_0 or, if the wavelength is greater than the corresponding ***cutoff* wavelength** $\lambda_0 = c/f_0$. This is so no matter how intense the incident light is.



The electrons within the target are held there by electric forces. To just escape from the target, an electron must pick up a certain minimum energy φ, where φ is a property of the target material called its **work function.** If the energy hf transferred to an electron by a photon exceeds the work function of the material (if $hf > \varphi$), the electron can escape the target.

Example, Photoelectric Effect and Work Function:

Find the work function Φ of sodium from Fig. 38-2.

KEY IDEAS

We can find the work function Φ from the cutoff frequency f_0 (which we can measure on the plot). The reasoning is this: At the cutoff frequency, the kinetic energy K_{max} in Eq. 38-5 is zero. Thus, all the energy hf that is transferred from a photon to an electron goes into the electron's escape, which requires an energy of Φ.

Calculations: From that last idea, Eq. 38-5 then gives us, with $f = f_0$,

$$hf_0 = 0 + \Phi = \Phi.$$

In Fig. 38-2, the cutoff frequency f_0 is the frequency at which the plotted line intercepts the horizontal frequency axis, about 5.5×10^{14} Hz. We then have

$$\begin{aligned}\Phi = hf_0 &= (6.63 \times 10^{-34}\ \mathrm{J \cdot s})(5.5 \times 10^{14}\ \mathrm{Hz}) \\ &= 3.6 \times 10^{-19}\ \mathrm{J} = 2.3\ \mathrm{eV}. \qquad \text{(Answer)}\end{aligned}$$

38.4: Photons Have Momentum
Compton Effect:

$$p = \frac{hf}{c} = \frac{h}{\lambda} \qquad \text{(photon momentum)}$$

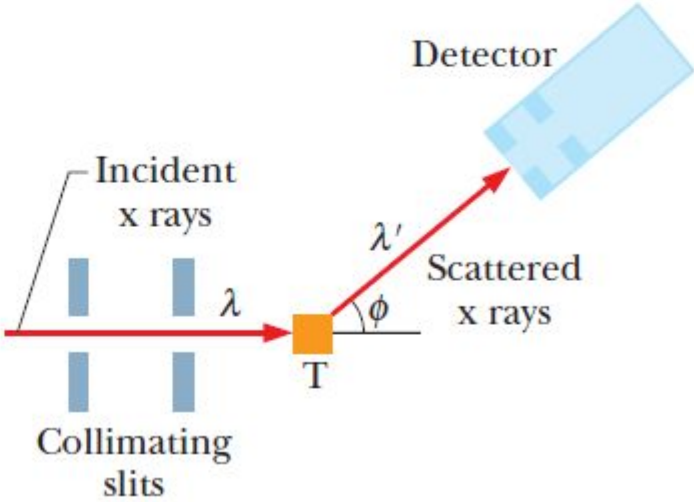


Fig. 38-3 Compton's apparatus. A beam of x rays of wavelength $\lambda = 71.1$ pm is directed onto a carbon target T. The x rays scattered from the target are observed at various angles ϕ to the direction of the incident beam. The detector measures both the intensity of the scattered x rays and their wavelength.

38.4: Photons Have Momentum, Compton Effect:

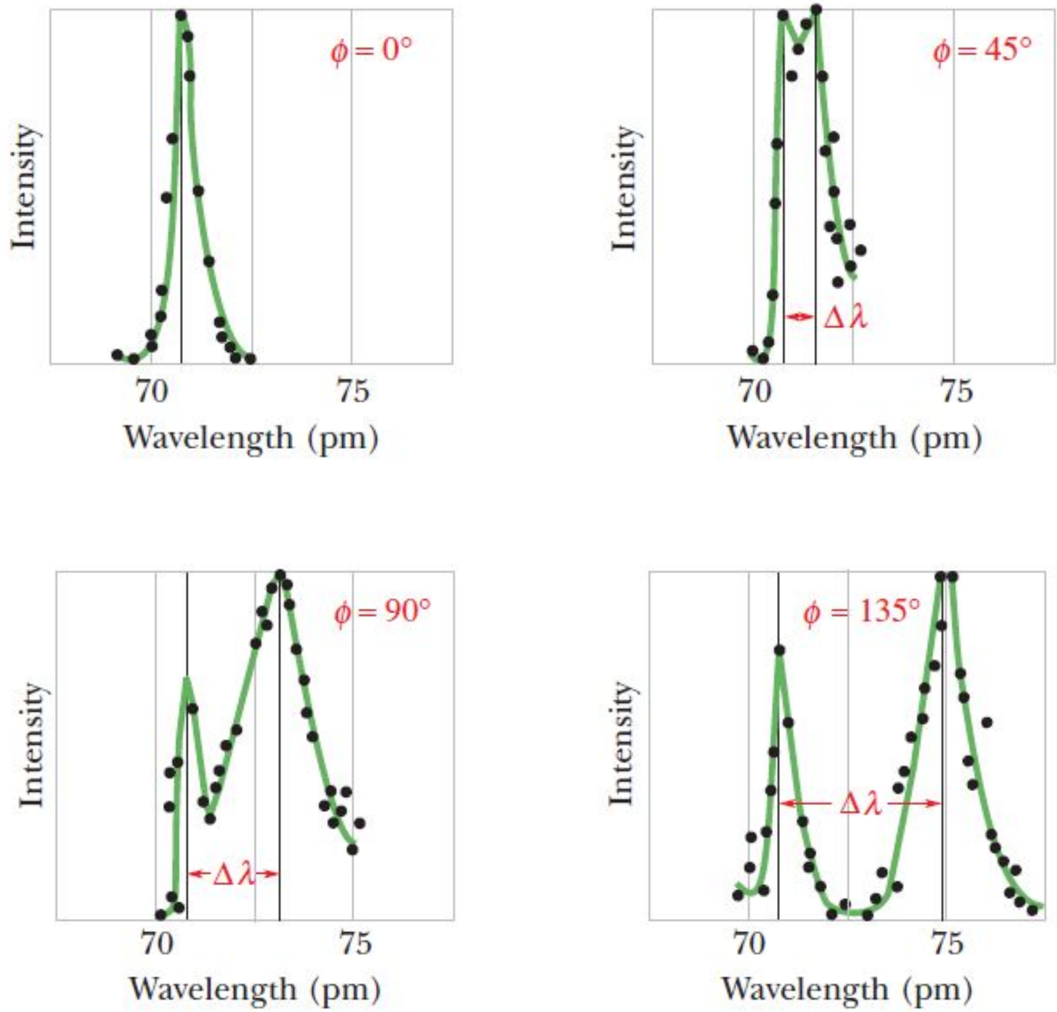


Fig. 38-4 Compton's results for four values of the scattering angle φ. Note that the Compton shift $\Delta\lambda$ increases as the scattering angle increases.

38.4: Photons Have Momentum, Compton Effect:

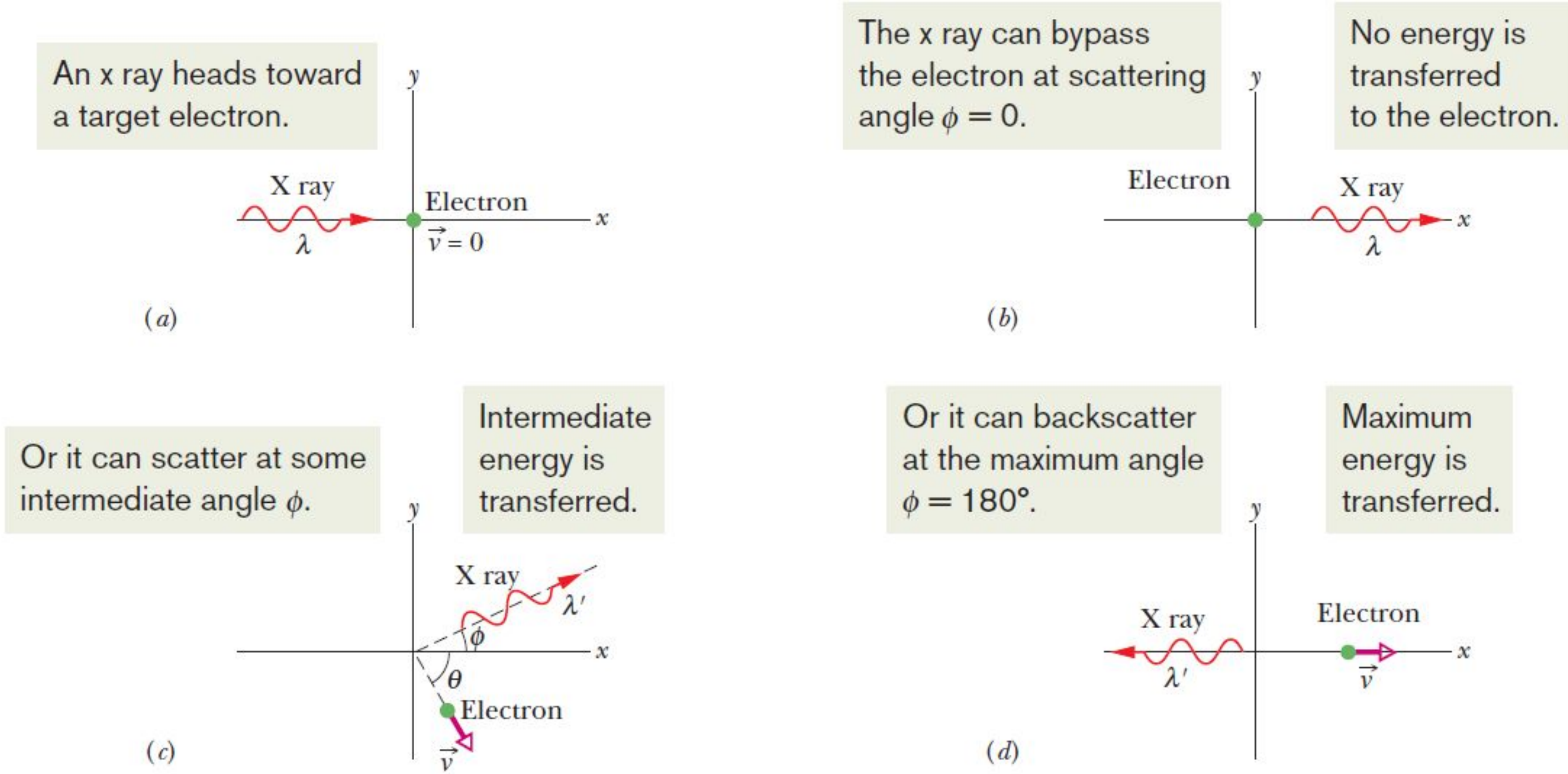


Fig. 38-5 (*a*) An x ray approaches a stationary electron. The x ray can (*b*) bypass the electron (forward scatter) with no energy or momentum transfer, (*c*) scatter at some intermediate angle with an intermediate energy and momentum transfer, or (*d*) backscatter with the maximum energy and momentum transfer.

As a result of the collision, an x ray of wavelength λ' moves off at an angle φ and the electron moves off at an angle θ, as shown. Conservation of energy then gives us

$$hf = hf' + K$$

38.4: Photons Have Momentum, Compton Effect:

$$hf = hf' + K$$

Here *hf* is the energy of the incident x-ray photon, *hf'* is the energy of the scattered x-ray photon, and *K* is the kinetic energy of the recoiling electron.
Since the electron may recoil with a speed comparable to that of light,

$$K = mc^2(\gamma - 1), \qquad \gamma = \frac{1}{\sqrt{1 - (v/c)^2}}.$$

$$\Rightarrow \quad hf = hf' + mc^2(\gamma - 1).$$

$$\Rightarrow \quad \frac{h}{\lambda} = \frac{h}{\lambda'} + mc(\gamma - 1).$$

$$\Rightarrow \quad \frac{h}{\lambda} = \frac{h}{\lambda'}\cos\phi + \gamma mv\cos\theta \qquad (x \text{ axis})$$

$$0 = \frac{h}{\lambda'}\sin\phi - \gamma mv\sin\theta \qquad (y \text{ axis}).$$

$$\Rightarrow \quad \boxed{\Delta\lambda = \frac{h}{mc}(1 - \cos\phi) \qquad (\text{Compton shift}).}$$

The quantity *h/mc* in Eq. 38-11 is a constant called the ***Compton wavelength.***

Example, Compton Scattering of Light by Electrons:

X rays of wavelength $\lambda = 22$ pm (photon energy = 56 keV) are scattered from a carbon target, and the scattered rays are detected at 85° to the incident beam.

(a) What is the Compton shift of the scattered rays?

KEY IDEA

The Compton shift is the wavelength change of the x rays due to scattering from loosely bound electrons in a target. Further, that shift depends on the angle at which the scattered x rays are detected, according to Eq. 38-11. The shift is zero for forward scattering at angle $\phi = 0°$, and it is maximum for back scattering at angle $\phi = 180°$. Here we have an intermediate situation at angle $\phi = 85°$.

Calculation: Substituting 85° for that angle and 9.11×10^{-31} kg for the electron mass (because the scattering is from electrons) in Eq. 38-11 gives us

$$\begin{aligned}\Delta\lambda &= \frac{h}{mc}(1 - \cos\phi)\\ &= \frac{(6.63 \times 10^{-34}\ \mathrm{J\cdot s})(1 - \cos 85°)}{(9.11 \times 10^{-31}\ \mathrm{kg})(3.00 \times 10^{8}\ \mathrm{m/s})}\\ &= 2.21 \times 10^{-12}\ \mathrm{m} \approx 2.2\ \mathrm{pm}. \qquad \text{(Answer)}\end{aligned}$$

(b) What percentage of the initial x-ray photon energy is transferred to an electron in such scattering?

KEY IDEA

We need to find the *fractional energy loss* (let us call it *frac*) for photons that scatter from the electrons:

$$frac = \frac{\text{energy loss}}{\text{initial energy}} = \frac{E - E'}{E}.$$

Calculations: From Eq. 38-2 ($E = hf$), we can substitute for the initial energy E and the detected energy E' of the x rays in terms of frequencies. Then, from Eq. 38-1 ($f = c/\lambda$), we can substitute for those frequencies in terms of the wavelengths. We find

$$\begin{aligned}frac &= \frac{hf - hf'}{hf} = \frac{c/\lambda - c/\lambda'}{c/\lambda} = \frac{\lambda' - \lambda}{\lambda'}\\ &= \frac{\Delta\lambda}{\lambda + \Delta\lambda}. \qquad \text{(38-12)}\end{aligned}$$

Substitution of data yields

$$frac = \frac{2.21\ \mathrm{pm}}{22\ \mathrm{pm} + 2.21\ \mathrm{pm}} = 0.091, \text{ or } 9.1\%. \qquad \text{(Answer)}$$

Although the Compton shift $\Delta\lambda$ is independent of the wavelength λ of the incident x rays (see Eq. 38-11), the *fractional* photon energy loss of the x rays does depend on λ, increasing as the wavelength of the incident radiation decreases, as indicated by Eq. 38-12.

38.5: Light as a Probability Wave:

The probability (per unit time interval) that a photon will be detected in any small volume centered on a given point in a light wave is proportional to the square of the amplitude of the wave's electric field vector at that point.

The probabilistic description of a light wave is another way to view light. It is not only an electromagnetic wave but also a **probability wave.** That is, to every point in a light wave we can attach a numerical probability (per unit time interval) that a photon can be detected in any small volume centered on that point.

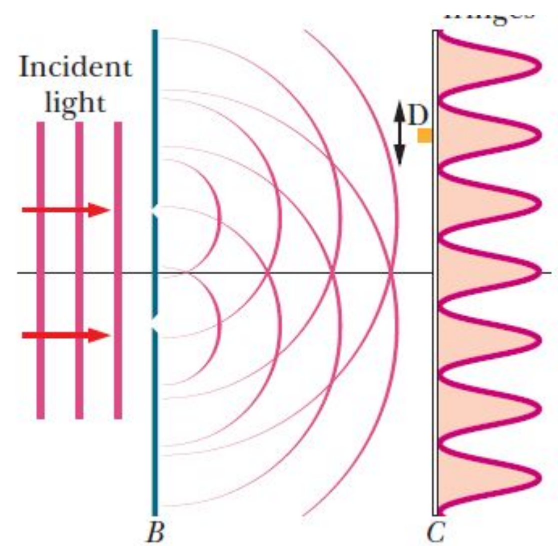


Fig. 38-6 Light is directed onto screen B, which contains two parallel slits. Light emerging from these slits spreads out by diffraction. The two diffracted waves overlap at screen C and form a pattern of interference fringes. A small photon detector D in the plane of screen C generates a sharp click for each photon that it absorbs.

38.5: Light as a Probability Wave, The Single Photon Version:

Consider the double-slit experiment again. Since an interference pattern eventually builds up on the screen, we can speculate that each photon travels from source to screen as a wave that fills up the space between source and screen and then vanishes in a photon absorption at some point on the screen, with a transfer of energy and momentum to the screen at that point.

We cannot predict where this transfer will occur (where a photon will be detected) for any given photon originating at the source.

However, we can predict the probability that a transfer will occur at any given point on the screen.

Transfers will tend to occur (and thus photons will tend to be absorbed) in the regions of the bright fringes in the interference pattern that builds up on the screen. Transfers will tend not to occur (and thus photons will tend not to be absorbed) in the regions of the dark fringes in the built-up pattern.

Thus, we can say that the wave traveling from the source to the screen is a *probability wave,* which produces a pattern of "probability fringes" on the screen.

38.5: Light as a Probability Wave, The Single Photon, Wide Angle Version:

A *single* photon can take widely different paths and still interfere with itself.

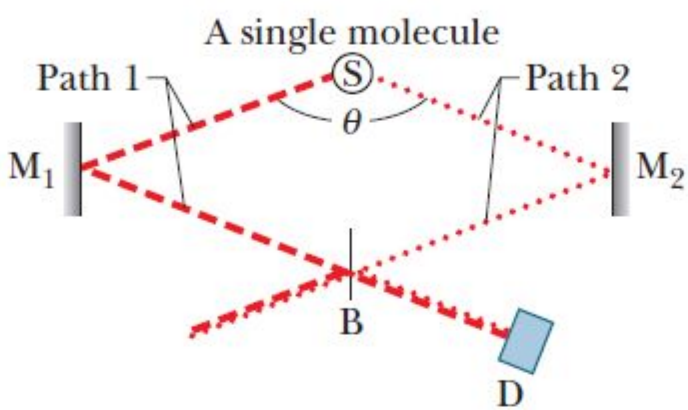


Fig. 38-7 The light from a single photon emission in source S travels over two widely separated paths and interferes with itself at detector D after being recombined by beam splitter B. (After Ming Lai and Jean-Claude Diels, *Journal of the Optical Society of America B*, **9**, 2290–2294, December 1992.)

When a molecule in the source emits a single photon, does that photon travel along path 1 or path 2 in the figure (or along any other path)? Or can it move in both directions at once?

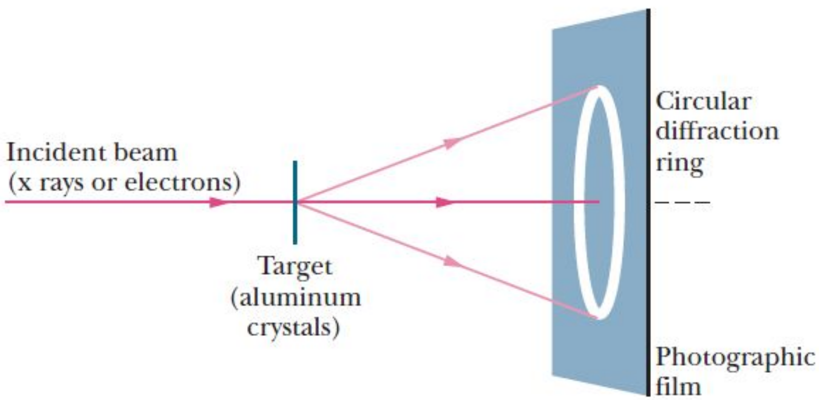
To answer, we assume that when a molecule emits a photon, a probability wave radiates in all directions from it. The experiment samples this wave in two of those directions, chosen to be nearly opposite each other.

We see that we can interpret all three versions of the double-slit experiment if we assume that (1) light is generated in the source as photons, (2) light is absorbed in the detector as photons, and (3) light travels between source and detector as a probability wave.

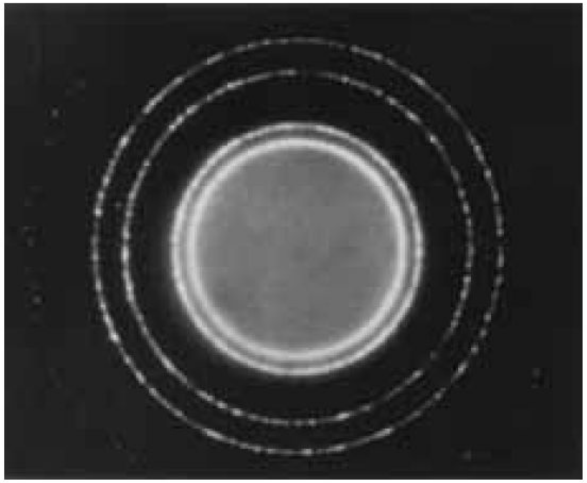
38.6: Electrons and Matter Waves:

$$\lambda = \frac{h}{p} \qquad \text{(de Broglie wavelength)}$$

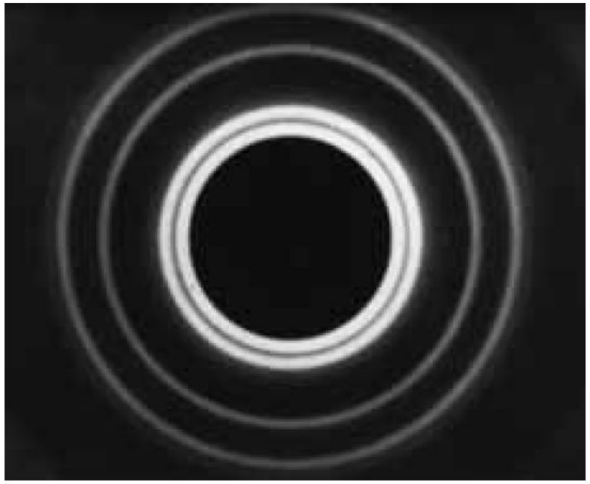
de Broglie suggested that $p = h/\lambda$ might apply not only to photons but also to electrons



(a)



(b)



(c)

Fig. 38-9 (*a*) An experimental arrangement used to demonstrate, by diffraction techniques, the wave-like character of the incident beam. Photographs of the diffraction patterns when the incident beam is (*b*) an x-ray beam (light wave) and (*c*) an electron beam (matter wave). Note that the two patterns are geometrically identical to each other. *(Photo (b) Cameca, Inc. Photo (c) from PSSC film "Matter Waves," courtesy Education Development Center, Newton, Massachusetts)*

Example, deBroglie wavelength of an electron:

What is the de Broglie wavelength of an electron with a kinetic energy of 120 eV?

KEY IDEAS

(1) We can find the electron's de Broglie wavelength λ from Eq. 38-13 ($\lambda = h/p$) if we first find the magnitude of its momentum p. (2) We find p from the given kinetic energy K of the electron. That kinetic energy is much less than the rest energy of an electron (0.511 MeV, from Table 37-3). Thus, we can get by with the classical approximations for momentum p $(= mv)$ and kinetic energy K $(= \frac{1}{2}mv^2)$.

Calculations: We are given the value of the kinetic energy. So, in order to use the de Broglie relation, we first solve the kinetic energy equation for v and then substitute into the momentum equation, finding

$$\begin{aligned} p &= \sqrt{2mK} \\ &= \sqrt{(2)(9.11 \times 10^{-31}\ \text{kg})(120\ \text{eV})(1.60 \times 10^{-19}\ \text{J/eV})} \\ &= 5.91 \times 10^{-24}\ \text{kg}\cdot\text{m/s}. \end{aligned}$$

From Eq. 38-13 then

$$\begin{aligned} \lambda &= \frac{h}{p} \\ &= \frac{6.63 \times 10^{-34}\ \text{J}\cdot\text{s}}{5.91 \times 10^{-24}\ \text{kg}\cdot\text{m/s}} \\ &= 1.12 \times 10^{-10}\ \text{m} = 112\ \text{pm}. \qquad \text{(Answer)} \end{aligned}$$

This wavelength associated with the electron is about the size of a typical atom. If we increase the electron's kinetic energy, the wavelength becomes even smaller.

38.7: Schrödinger's Equation:

If a **wave function,** $\psi(x, y, z, t)$, can be used to describe matter waves, then its space and time variables can be grouped separately and can be written in the form

$$\Psi(x, y, z, t) = \psi(x, y, z)\, e^{-i\omega t}$$

where $\omega = (2\pi f)$ is the angular frequency of the matter wave.

Suppose that a matter wave reaches a particle detector; then the probability that a particle will be detected in a specified time interval is proportional to $|\psi|^2$, where $|\psi|$ is the absolute value of the wave function at the location of the detector.

$|\psi|^2$ is always both real and positive, and it is called the **probability density,**

The probability (per unit time) of detecting a particle in a small volume centered on a given point in a matter wave is proportional to the value of $|\psi|^2$ at that point.

38.7: Schrödinger's Equation:

Matter waves are described by Schrödinger's Equation.

Suppose a particle traveling in the *x* direction through a region in which forces acting on the particle cause it to have a potential energy *U(x)*. In this special case, Schrödinger's equation can be written as:

$$\frac{d^2\psi}{dx^2} + \frac{8\pi^2 m}{h^2}[E - U(x)]\psi = 0 \qquad \text{(Schrödinger's equation, one-dimensional motion)}$$

For a free particle, *U(x)* is zero, that equation describes a free particle where a moving particle on which no net force acting on it. The particle's total energy in this case is all kinetic, and the equation becomes:

$$\frac{d^2\psi}{dx^2} + \frac{8\pi^2 m}{h^2}\left(\frac{mv^2}{2}\right)\psi = 0, \rightarrow \frac{d^2\psi}{dx^2} + \left(2\pi\frac{p}{h}\right)^2\psi = 0.$$

Using the concept of de Broglie wavelength and the definition of wave number,

$$\frac{d^2\psi}{dx^2} + k^2\psi = 0 \qquad \text{(Schrödinger's equation, free particle).}$$

The solution to this is: $\psi(x) = Ae^{ikx} + Be^{-ikx},$

Here A and B are constants.

38.7: Schrödinger's Equation, Finding the Probability Density:

$$\psi(x) = Ae^{ikx} + Be^{-ikx},$$

Consider a free particle that travels only in the positive direction of x. Let the arbitrary constant B be zero. At the same time, let us relabel the constant A as $\psi_{0.}$

$$\psi(x) = \psi_0\, e^{ikx}.$$

$$|\psi|^2 = |\psi_0\, e^{ikx}|^2 = (\psi_0^2)|e^{ikx}|^2.$$

$$|e^{ikx}|^2 = (e^{ikx})(e^{ikx})^* = e^{ikx}\, e^{-ikx} = e^{ikx-ikx} = e^0 = 1,$$

$$|\psi|^2 = (\psi_0^2)(1)^2 = \psi_0^2 \qquad \text{(a constant)}.$$

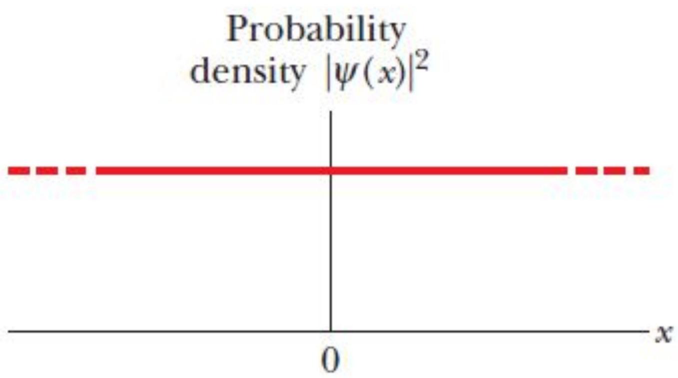


Fig. 38-12 A plot of the probability density $|\psi|^2$ for a free particle moving in the positive x direction. Since $|\psi|^2$ has the same constant value for all values of x, the particle has the same probability of detection at all points along its path.

38.8: Heisenberg's Uncertainty Principle:

$$\Delta x \cdot \Delta p_x \geq \hbar$$
$$\Delta y \cdot \Delta p_y \geq \hbar \qquad \text{(Heisenberg's uncertainty principle).}$$
$$\Delta z \cdot \Delta p_z \geq \hbar$$

Heisenberg's Uncertainty Principle states that measured values cannot be assigned to the position and the momentum of a particle simultaneously with unlimited precision.

Here Δx and Δp_x represent the intrinsic uncertainties in the measurements of the x components of $\boldsymbol{r}$ and $\boldsymbol{p}$, with parallel meanings for the y and z terms. Even with the best measuring instruments, each product of a position uncertainty and a momentum uncertainty will be greater than $\hbar$, never less.

Example, Uncertainty Principle, position and momentum:

Assume that an electron is moving along an x axis and that you measure its speed to be 2.05×10^6 m/s, which can be known with a precision of 0.50%. What is the minimum uncertainty (as allowed by the uncertainty principle in quantum theory) with which you can simultaneously measure the position of the electron along the x axis?

KEY IDEA

The minimum uncertainty allowed by quantum theory is given by Heisenberg's uncertainty principle in Eq. 38-20. We need only consider components along the x axis because we have motion only along that axis and want the uncertainty Δx in location along that axis. Since we want the minimum allowed uncertainty, we use the equality instead of the inequality in the x-axis part of Eq. 38-20, writing $\Delta x \cdot \Delta p_x = \hbar$.

Calculations: To evaluate the uncertainty Δp_x in the momentum, we must first evaluate the momentum component p_x. Because the electron's speed v_x is much less than the speed of light c, we can evaluate p_x with the classical expression for momentum instead of using a relativistic expression. We find

$$p_x = mv_x = (9.11 \times 10^{-31}\ \text{kg})(2.05 \times 10^6\ \text{m/s}) = 1.87 \times 10^{-24}\ \text{kg} \cdot \text{m/s}.$$

The uncertainty in the speed is given as 0.50% of the measured speed. Because p_x depends directly on speed, the uncertainty Δp_x in the momentum must be 0.50% of the momentum:

$$\begin{aligned}\Delta p_x &= (0.0050)p_x \\ &= (0.0050)(1.87 \times 10^{-24}\ \text{kg} \cdot \text{m/s}) \\ &= 9.35 \times 10^{-27}\ \text{kg} \cdot \text{m/s}.\end{aligned}$$

Then the uncertainty principle gives us

$$\Delta x = \frac{\hbar}{\Delta p_x} = \frac{(6.63 \times 10^{-34}\ \text{J} \cdot \text{s})/2\pi}{9.35 \times 10^{-27}\ \text{kg} \cdot \text{m/s}}$$

$$= 1.13 \times 10^{-8}\ \text{m} \approx 11\ \text{nm}, \qquad \text{(Answer)}$$

which is about 100 atomic diameters. Given your measurement of the electron's speed, it makes no sense to try to pin down the electron's position to any greater precision.

38.9: Barrier Tunneling:

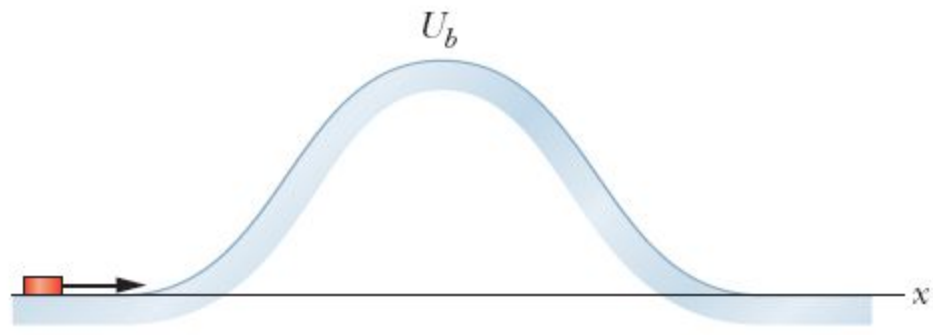


Fig. 38-13 A puck slides over frictionless ice toward a hill. The puck's gravitational potential at the top of the hill will be U_b.

As the puck climbs the hill, kinetic energy K is transformed into gravitational potential energy U. If the puck reaches the top, its potential energy is U_b. Thus, the puck can pass over the top only if its initial mechanical energy $E > U_b$.

The hill acts as a **potential energy barrier** (or, for short, **a potential barrier).**

38.9: Barrier Tunneling:

There is a potential barrier for a nonrelativistic electron traveling along an idealized wire of negligible thickness (Figure 38-14). The electron, with mechanical energy E, approaches a region (the barrier) in which the electric potential V_b is negative.

The electron, being negatively charged, will have a positive potential energy U_b $(=qV_b)$ in that region (Fig. 38-15). If $E > U_b$, we expect the electron to pass through the barrier region and come out to the right of $x = L$ in Fig. 38-14. If $E < U_b$, we expect the electron to be unable to pass through the barrier region.

Can the electron pass through the region of negative potential?

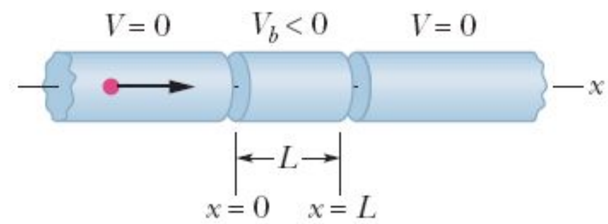


Fig. 38-14 The elements of an idealized thin wire in which an electron (the dot) approaches a negative electric potential V_b in the region $x = 0$ to $x = L$.

Classically, the electron lacks the energy to pass through the barrier region.

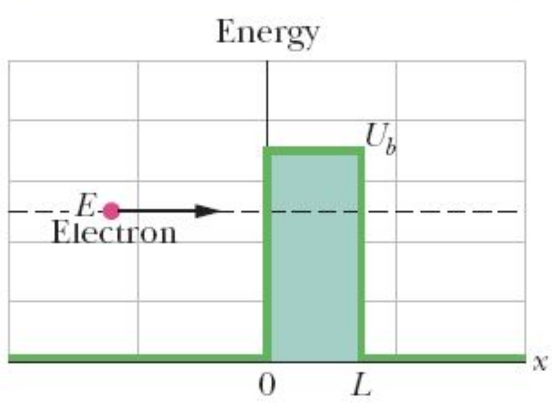


Fig. 38-15 An electron's mechanical energy E is plotted when the electron is at any coordinate $x<0$.
The electron's electric potential energy U is plotted as a function of the electron's position x, assuming that the electron can reach any value of x. The nonzero part of the plot (the potential barrier) has height U_b and thickness L.

38.9: Barrier Tunneling:

Something astounding can happen to the electron when $E < U_b$.

Since it is a matter wave, the electron has a finite probability of leaking (or, *tunneling)* through the barrier and materializing on the other side, moving rightward with energy E as though nothing had happened in the region of $0 \le x \le L$.

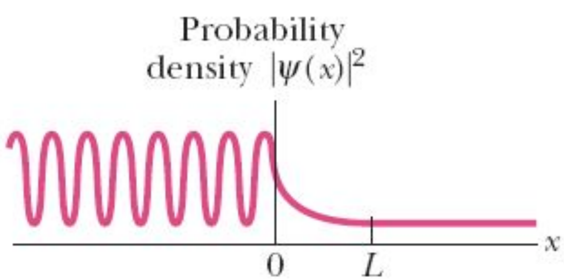


Fig. 38-16 A plot of the probability density $|\psi|^2$ of the electron matter wave for the situation of Fig. 38-15. The value of $|\psi|^2$ is nonzero to the right of the potential barrier.

The wave function $\psi(x)$ describing the electron can be found by solving Schrödinger's equation separately for the three regions: (1) to the left of the barrier, (2) within the barrier, and (3) to the right of the barrier.

The arbitrary constants that appear in the solutions can then be chosen so that the values of $\psi(x)$ and its derivative with respect to x join smoothly at $x = 0$ and at $x = L$. Squaring the absolute value of $\psi(x)$ then yields the probability density.

38.9: Barrier Tunneling:

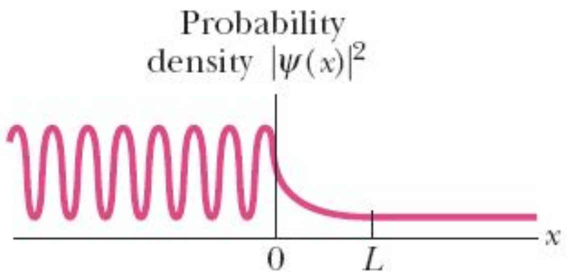


Fig. 38-16 A plot of the probability density $|\psi|^2$ of the electron matter wave for the situation of Fig. 38-15. The value of $|\psi|^2$ is nonzero to the right of the potential barrier.

- Within the barrier the probability density decreases exponentially with x.

- To the right of the barrier, the probability density plot describes a transmitted (through the barrier) wave with low but constant amplitude.

- We can assign a **transmission coefficient *T*** to the incident matter wave and the barrier. This coefficient gives the probability with which an approaching electron will be transmitted through the barrier—that is, that tunneling will occur.

Approximately,

$$T \approx e^{-2bL},$$

$$b = \sqrt{\frac{8\pi^2 m(U_b - E)}{h^2}},$$

38.9: Barrier Tunneling, The Scanning Tunneling Microscope (STM):

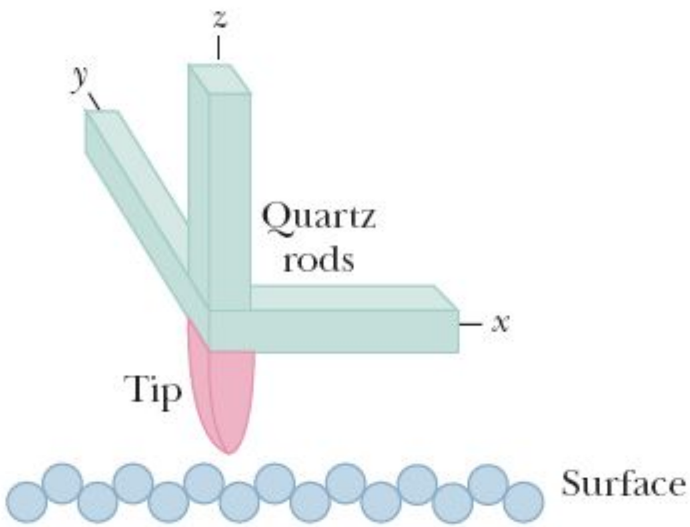


Fig. 38-17 The essence of a scanning tunneling microscope (STM). Three quartz rods are used to scan a sharply pointed conducting tip across the surface of interest and to maintain a constant separation between tip and surface. The tip thus moves up and down to match the contours of the surface, and a record of its movement provides information for a computer to create an image of the surface.

Example, Barrier tunneling by matter wave:

Suppose that the electron in Fig. 38-15, having a total energy E of 5.1 eV, approaches a barrier of height $U_b = 6.8$ eV and thickness $L = 750$ pm.

(a) What is the approximate probability that the electron will be transmitted through the barrier, to appear (and be detectable) on the other side of the barrier?

KEY IDEA

The probability we seek is the transmission coefficient T as given by Eq. 38-21 ($T \approx e^{-2bL}$), where

$$b = \sqrt{\frac{8\pi^2 m(U_b - E)}{h^2}}.$$

Calculations: The numerator of the fraction under the square-root sign is

$$(8\pi^2)(9.11 \times 10^{-31}\text{ kg})(6.8\text{ eV} - 5.1\text{ eV}) \times (1.60 \times 10^{-19}\text{ J/eV}) = 1.956 \times 10^{-47}\text{ J}\cdot\text{kg}.$$

Thus,
$$b = \sqrt{\frac{1.956 \times 10^{-47}\text{ J}\cdot\text{kg}}{(6.63 \times 10^{-34}\text{ J}\cdot\text{s})^2}} = 6.67 \times 10^9\text{ m}^{-1}.$$

The (dimensionless) quantity $2bL$ is then

$$2bL = (2)(6.67 \times 10^9\text{ m}^{-1})(750 \times 10^{-12}\text{ m}) = 10.0$$

and, from Eq. 38-21, the transmission coefficient is

$$T \approx e^{-2bL} = e^{-10.0} = 45 \times 10^{-6}. \qquad \text{(Answer)}$$

Thus, of every million electrons that strike the barrier, about 45 will tunnel through it, each appearing on the other side with its original total energy of 5.1 eV. (The transmission through the barrier does not alter an electron's energy or any other property.)

(b) What is the approximate probability that a proton with the same total energy of 5.1 eV will be transmitted through the barrier, to appear (and be detectable) on the other side of the barrier?

Reasoning: The transmission coefficient T (and thus the probability of transmission) depends on the mass of the particle. Indeed, because mass m is one of the factors in the exponent of e in the equation for T, the probability of transmission is very sensitive to the mass of the particle. This time, the mass is that of a proton (1.67×10^{-27} kg), which is significantly greater than that of the electron in (a). By substituting the proton's mass for the mass in (a) and then continuing as we did there, we find that $T \approx 10^{-186}$. Thus, although the probability that the proton will be transmitted is not exactly zero, it is barely more than zero. For even more massive particles with the same total energy of 5.1 eV, the probability of transmission is exponentially lower.