Chapter 43

Energy From The Nucleus

43.1: What is Physics?

Table 43-1

Energy Released by 1 kg of Matter

Form of Matter	Process	Time[a]
Water	A 50 m waterfall	5 s
Coal	Burning	8 h
Enriched UO_2	Fission in a reactor	690 y
^{235}U	Complete fission	3×10^4 y
Hot deuterium gas	Complete fusion	3×10^4 y
Matter and antimatter	Complete annihilation	3×10^7 y

[a]This column shows the time interval for which the generated energy could power a 100 W lightbulb.

In both atomic and nuclear burning, the release of energy is accompanied by a decrease in mass, according to the equation $Q=-\Delta m\, c^2$. The central difference between burning uranium and burning coal is that, in the former case, a much larger fraction of the available mass (by a factor of a few million) is consumed.

43.2: Nuclear Fission: The Basic Process:

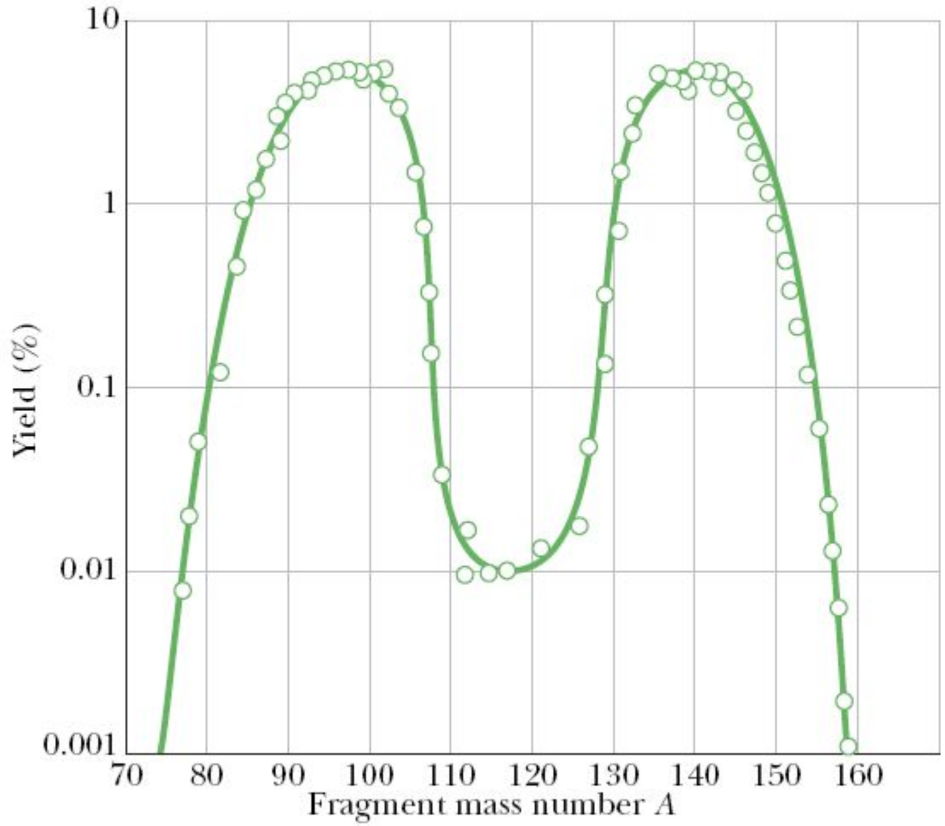


Fig. 43-1 The distribution by mass number of the fragments that are found when many fission events of ^{235}U are examined. Note that the vertical scale is logarithmic.

The most probable mass numbers, occurring in about 7% of the fission events, are centered around A ~95 and A~140.

43.2: Nuclear Fission: The Basic Process:

An example: $^{235}\text{U} + \text{n} \rightarrow {}^{236}\text{U} \rightarrow {}^{140}\text{Xe} + {}^{94}\text{Sr} + 2\text{n}.$

Since the products are not stable, they undergo further fissions, such as:

$$^{140}\text{Xe} \rightarrow {}^{140}\text{Cs} \rightarrow {}^{140}\text{Ba} \rightarrow {}^{140}\text{La} \rightarrow {}^{140}\text{Ce}$$

$T_{1/2}$	14 s	64 s	13 d	40 h	Stable
Z	54	55	56	57	58

$$^{94}\text{Sr} \rightarrow {}^{94}\text{Y} \rightarrow {}^{94}\text{Zr}$$

$T_{1/2}$	75 s	19 min	Stable
Z	38	39	40

43.2: Nuclear Fission: The Basic Process:

The energy released by the fission, Q, is:

$$Q = \begin{pmatrix} \text{total final} \\ \text{binding energy} \end{pmatrix} - \begin{pmatrix} \text{initial} \\ \text{binding energy} \end{pmatrix}.$$

$$Q = \begin{pmatrix} \text{final} \\ \Delta E_{\text{ben}} \end{pmatrix} \begin{pmatrix} \text{final number} \\ \text{of nucleons} \end{pmatrix} - \begin{pmatrix} \text{initial} \\ \Delta E_{\text{ben}} \end{pmatrix} \begin{pmatrix} \text{initial number} \\ \text{of nucleons} \end{pmatrix}.$$

For a high-mass nuclide ($A \sim 240$), the binding energy per nucleon is about 7.6 MeV/nucleon.
For middle-mass nuclides ($A \sim 120$), it is about 8.5 MeV/nucleon.
Thus, the energy released by fission of a high-mass nuclide to two middle-mass nuclides is

$$Q = \left(8.5 \frac{\text{MeV}}{\text{nucleon}}\right)(2 \text{ nuclei})\left(120 \frac{\text{nucleons}}{\text{nucleus}}\right) - \left(7.6 \frac{\text{MeV}}{\text{nucleon}}\right)(240 \text{ nucleons}) \approx 200 \text{ MeV}.$$

Example, Q-value in a fission of U-235:

Find the disintegration energy Q for the fission event of Eq. 43-1, taking into account the decay of the fission fragments as displayed in Eqs. 43-2 and 43-3. Some needed atomic and particle masses are

^{235}U	235.0439 u	^{140}Ce	139.9054 u
n	1.008 66 u	^{94}Zr	93.9063 u

KEY IDEAS

(1) The disintegration energy Q is the energy transferred from mass energy to kinetic energy of the decay products. (2) $Q = -\Delta m\, c^2$, where Δm is the change in mass.

Calculations: Because we are to include the decay of the fission fragments, we combine Eqs. 43-1, 43-2, and 43-3 to write the overall transformation as

$$^{235}\mathrm{U} \rightarrow {}^{140}\mathrm{Ce} + {}^{94}\mathrm{Zr} + \mathrm{n}. \qquad (43\text{-}7)$$

Only the single neutron appears here because the initiating neutron on the left side of Eq. 43-1 cancels one of the two neutrons on the right of that equation. The mass difference for the reaction of Eq. 43-7 is

$$\begin{aligned}\Delta m &= (139.9054\ \mathrm{u} + 93.9063\ \mathrm{u} + 1.008\,66\ \mathrm{u}) \\ &\quad - (235.0439\ \mathrm{u}) \\ &= -0.223\,54\ \mathrm{u},\end{aligned}$$

and the corresponding disintegration energy is

$$\begin{aligned}Q &= -\Delta m\, c^2 = -(-0.223\,54\ \mathrm{u})(931.494\,013\ \mathrm{MeV/u}) \\ &= 208\ \mathrm{MeV}, \qquad \text{(Answer)}\end{aligned}$$

which is in good agreement with our estimate of Eq. 43-6.

If the fission event takes place in a bulk solid, most of this disintegration energy, which first goes into kinetic energy of the decay products, appears eventually as an increase in the internal energy of that body, revealing itself as a rise in temperature. Five or six percent or so of the disintegration energy, however, is associated with neutrinos that are emitted during the beta decay of the primary fission fragments. This energy is carried out of the system and is lost.

43.3: A Model for Nuclear Fission:

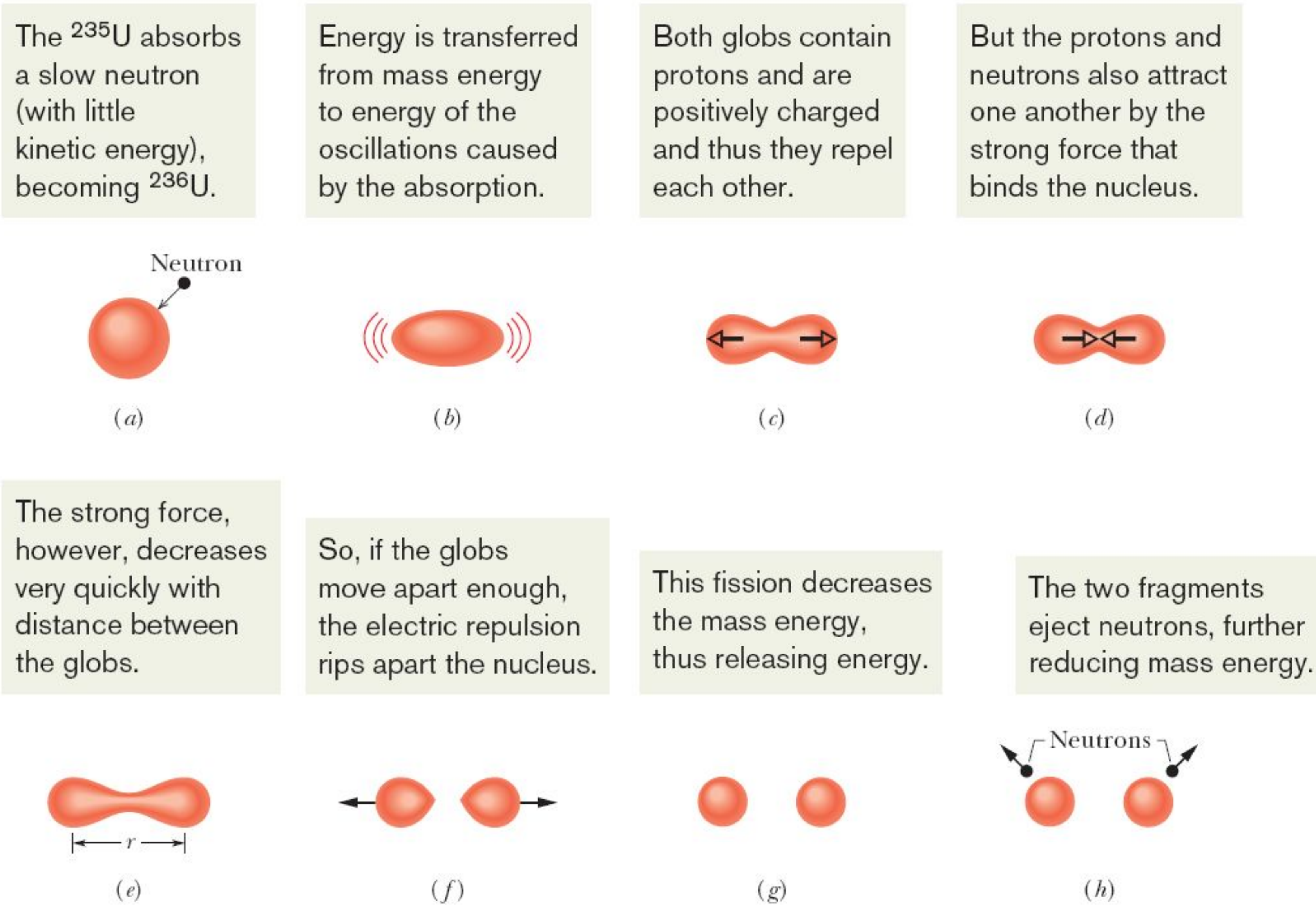
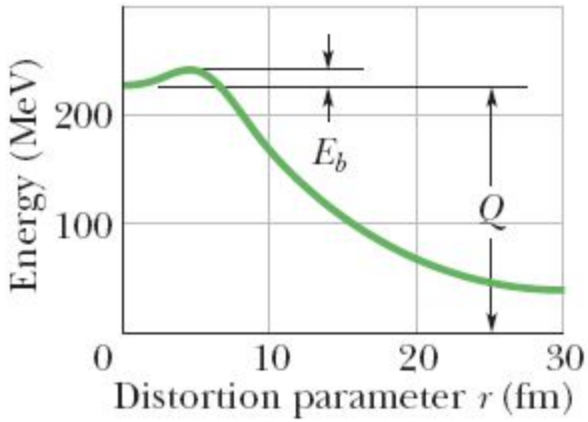


Fig. 43-2 The stages of a typical fission process, according to the collective model of Bohr and Wheeler.

43.3: A Model for Nuclear Fission:



E_b is an energy barrier that must be overcome.

Q is the energy that would then be released.

Fig. 43-3 The potential energy at various stages in the fission process, as predicted from the collective model of Bohr and Wheeler. The Q of the reaction (about 200 MeV) and the fission barrier height E_b are both indicated.

In the figure, the potential energy is plotted against the *distortion parameter r,* which is a rough measure of the extent to which the oscillating nucleus departs from a spherical shape. When the fragments are far apart, this parameter is simply the distance between their centers.

43.3: A Model for Nuclear Fission:

Table 43-2

Test of the Fissionability of Four Nuclides

Target Nuclide	Nuclide Being Fissioned	E_n (MeV)	E_b (MeV)	Fission by Thermal Neutrons?
^{235}U	^{236}U	6.5	5.2	Yes
^{238}U	^{239}U	4.8	5.7	No
^{239}Pu	^{240}Pu	6.4	4.8	Yes
^{243}Am	^{244}Am	5.5	5.8	No

Table 43-2 shows, for four high-mass nuclides, this test of whether capture of a thermal neutron can cause fissioning. For each nuclide, the table shows both the barrier height E_b of the nucleus that is formed by the neutron capture and the excitation energy E_n due to the capture.

43.3: A Model for Nuclear Fission:

Courtesy U.S. Department of Energy

Fig. 43-4 This image has transfixed the world since World War II. When Robert Oppenheimer, the head of the scientific team that developed the atomic bomb, witnessed the first atomic explosion, he quoted from a sacred Hindu text: "Now I am become Death, the destroyer of worlds."

43.4: The Nuclear Reactor:

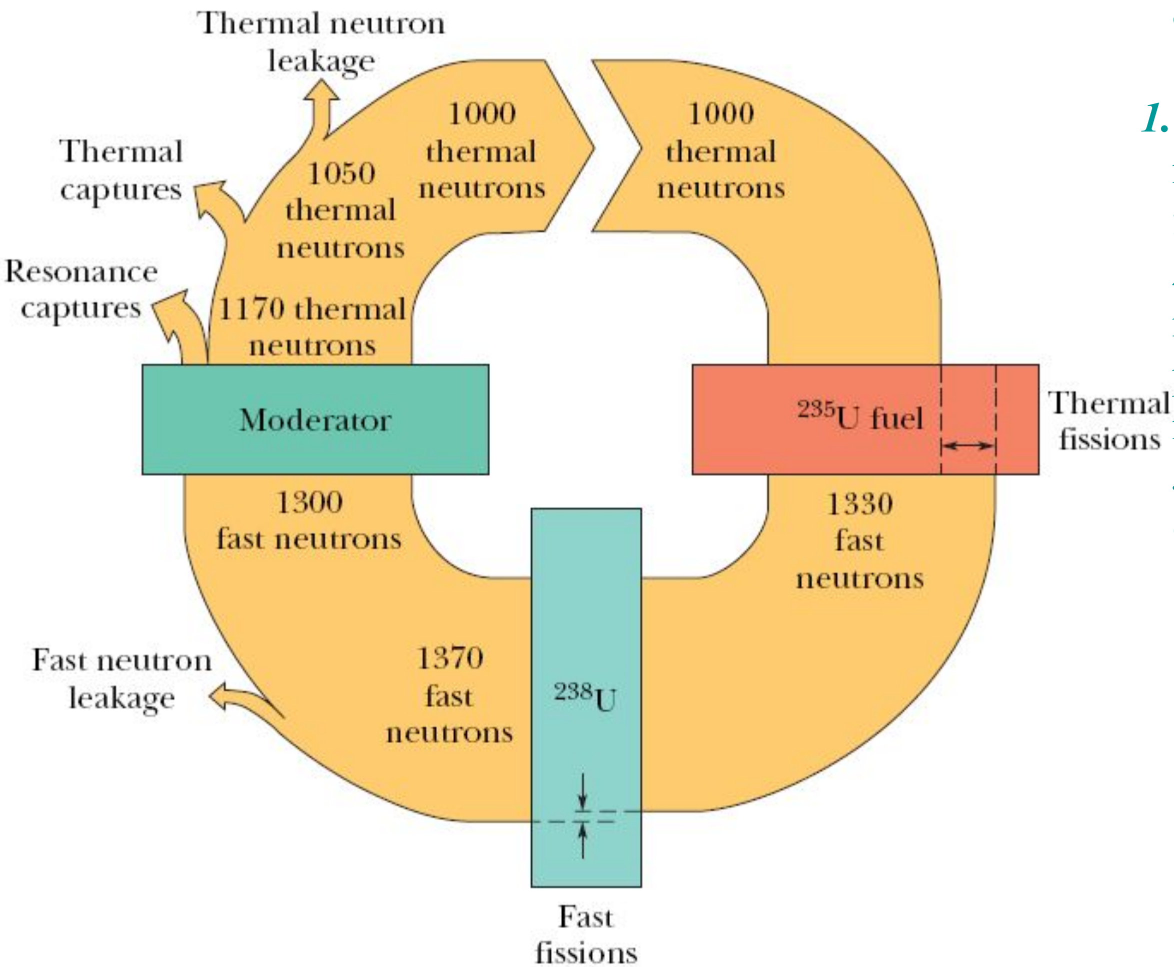


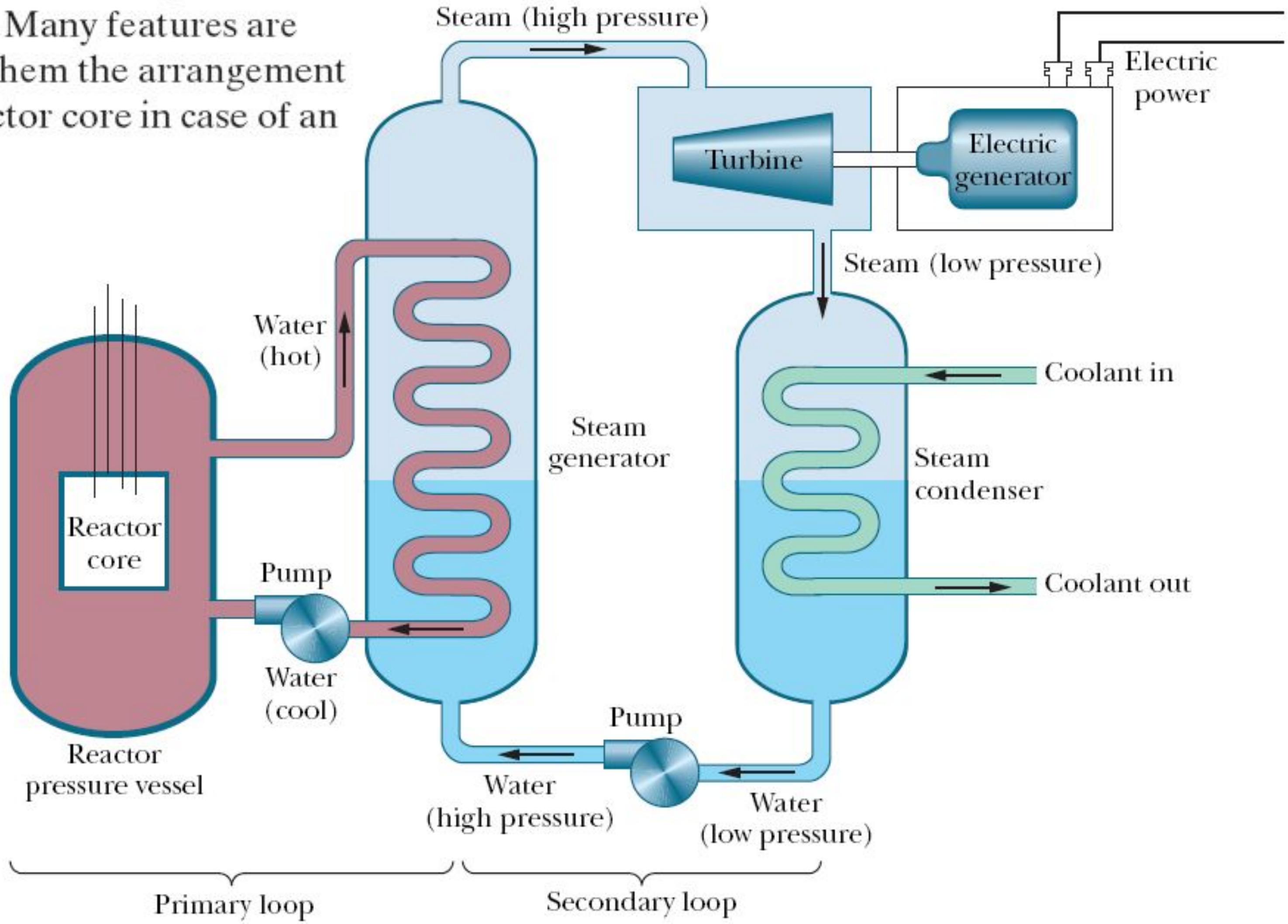
Fig. 43-5 Neutron bookkeeping in a reactor. A generation of 1000 thermal neutrons interacts with the ^{235}U fuel, the ^{238}U matrix, and the moderator. They produce 1370 neutrons by fission, but 370 of these are lost by nonfission capture or by leakage, meaning that 1000 thermal neutrons are left to form the next generation. The figure is drawn for a reactor running at a steady power level.

Three main difficulties stand in the way of a working reactor:

1. ***The Neutron Leakage Problem.*** (Some neutrons produced by fission leak out from the reactor).
2. ***The Neutron Energy Problem.*** (Fast neutrons are not as effective in producing fission as slower thermal neutrons).
3. ***The Neutron Capture Problem.*** (non-fission capture of neutrons)

43.4: The Nuclear Reactor:

Fig. 43-6 A simplified layout of a nuclear power plant, based on a pressurized-water reactor. Many features are omitted—among them the arrangement for cooling the reactor core in case of an emergency.



Example, Nuclear reactor:

A large electric generating station is powered by a pressurized-water nuclear reactor. The thermal power produced in the reactor core is 3400 MW, and 1100 MW of electricity is generated by the station. The *fuel charge* is 8.60×10^4 kg of uranium, in the form of uranium oxide, distributed among 5.70×10^4 fuel rods. The uranium is enriched to 3.0% ^{235}U.

(a) What is the station's efficiency?

Calculation: Here the efficiency (eff) is

$$\text{eff} = \frac{\text{useful output}}{\text{energy input}} = \frac{1100 \text{ MW (electric)}}{3400 \text{ MW (thermal)}}$$
$$= 0.32, \text{ or } 32\%. \qquad \text{(Answer)}$$

The efficiency—as for all power plants—is controlled by the second law of thermodynamics. To run this plant, energy at the rate of 3400 MW − 1100 MW, or 2300 MW, must be discharged as thermal energy to the environment.

(b) At what rate R do fission events occur in the reactor core?

Calculation: For steady-state operation (P is constant), we find

$$R = \frac{P}{Q} = \left(\frac{3.4 \times 10^9 \text{ J/s}}{200 \text{ MeV/fission}}\right)\left(\frac{1 \text{ MeV}}{1.60 \times 10^{-13} \text{ J}}\right)$$
$$= 1.06 \times 10^{20} \text{ fissions/s}$$
$$\approx 1.1 \times 10^{20} \text{ fissions/s}. \qquad \text{(Answer)}$$

(c) At what rate (in kilograms per day) is the ^{235}U fuel disappearing? Assume conditions at start-up.

Calculations: The total rate at which the number of atoms of ^{235}U decreases is

$$(1 + 0.25)(1.06 \times 10^{20} \text{ atoms/s}) = 1.33 \times 10^{20} \text{ atoms/s}.$$
$$\frac{dM}{dt} = (1.33 \times 10^{20} \text{ atoms/s})(3.90 \times 10^{-25} \text{ kg/atom})$$
$$= 5.19 \times 10^{-5} \text{ kg/s} \approx 4.5 \text{ kg/d}. \qquad \text{(Answer)}$$

(d) At this rate of fuel consumption, how long would the fuel supply of ^{235}U last?

Calculation: At start-up, we know that the total mass of ^{235}U is 3.0% of the 8.60×10^4 kg of uranium oxide. So, the time T required to consume this total mass of ^{235}U at the steady rate of 4.5 kg/d is

$$T = \frac{(0.030)(8.60 \times 10^4 \text{ kg})}{4.5 \text{ kg/d}} \approx 570 \text{ d}. \qquad \text{(Answer)}$$

In practice, the fuel rods must be replaced (usually in batches) before their ^{235}U content is entirely consumed.

(e) At what rate is mass being converted to other forms of energy by the fission of ^{235}U in the reactor core?

Calculation: From Einstein's relation $E = mc^2$, we can write

$$\frac{dm}{dt} = \frac{dE/dt}{c^2} = \frac{3.4 \times 10^9 \text{ W}}{(3.00 \times 10^8 \text{ m/s})^2} \qquad (43\text{-}8)$$
$$= 3.8 \times 10^{-8} \text{ kg/s} = 3.3 \text{ g/d}. \qquad \text{(Answer)}$$

43.5: A Natural Nuclear Reactor:

Some two billion years ago, in a uranium deposit recently mined in Gabon, West Africa, a natural fission reactor apparently went into operation and ran for perhaps several hundred thousand years before shutting down.

Both ^{235}U and ^{238}U are radioactive, with half-lives of 7.04×10^8 y and 44.7×10^8 y, respectively.
Thus, the half-life of the readily fissionable ^{235}U is about 6.4 times shorter than that of ^{238}U. Because ^{235}U decays faster, there was more of it, relative to ^{238}U, in the past.

Two billion years ago, in fact, this abundance of ^{235}U was not 0.72%, as it is now, but 3.8%. This abundance happens to be just about the abundance to which natural uranium is artificially enriched to serve as fuel in modern power reactors.

Of the 30 or so elements whose stable isotopes are produced in a reactor, some must still remain. Study of their isotopic abundances could provide the evidence we need. Of the several elements investigated, the case of neodymium is spectacularly convincing. The next figure explains this fact.

43.5: A Natural Nuclear Reactor:

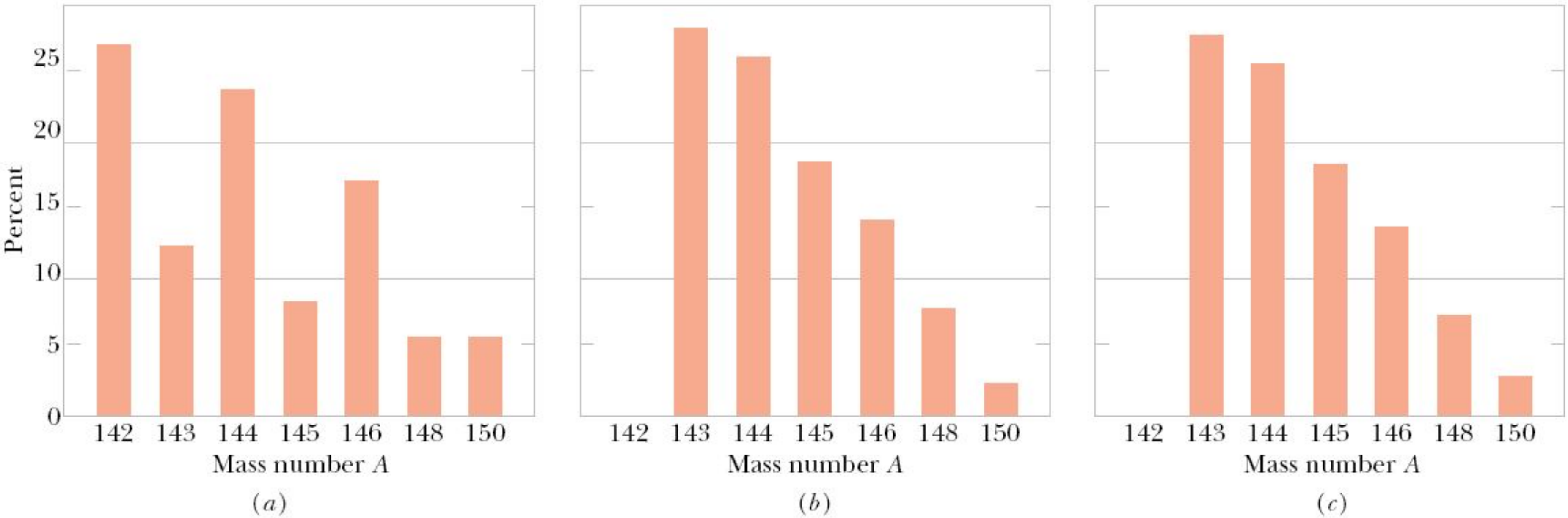


Fig. 43-9 The distribution by mass number of the isotopes of neodymium as they occur in (*a*) natural terrestrial deposits of the ores of this element and (*b*) the spent fuel of a power reactor. (*c*) The distribution (after several corrections) found for neodymium from the uranium mine in Gabon, West Africa. Note that (*b*) and (*c*) are virtually identical and are quite different from (*a*).

43.5: A Natural Nuclear Reactor:

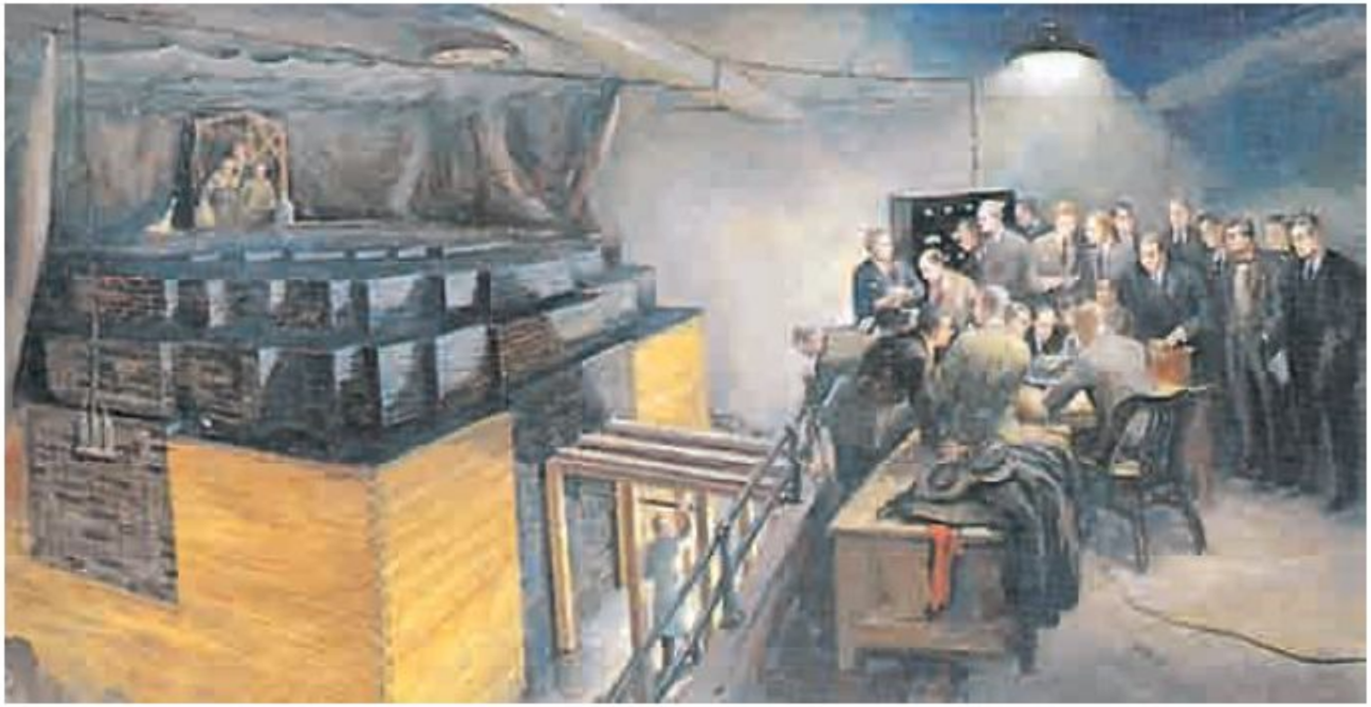


Fig. 43-8 A painting of the first nuclear reactor, assembled during World War II on a squash court at the University of Chicago by a team headed by Enrico Fermi. This reactor was built of lumps of uranium embedded in blocks of graphite.
(Gary Sheehan, Birth of the Atomic Age, *1957. Reproduced courtesy Chicago Historical Society)*

43.6: Thermonuclear Fusion, The Basic Process:

To generate useful amounts of energy, nuclear fusion must occur in bulk matter.
The best method is to raise the temperature of the material until the particles have enough energy—due to their thermal motions alone—to penetrate the Coulomb barrier. This process is called **thermonuclear fusion.**

In thermonuclear studies, temperatures are reported in terms of the kinetic energy K of interacting particles via the relation $K = kT$, in which K is the kinetic energy corresponding to the most probable speed of the interacting particles, k is the Boltzmann constant, and the temperature T is in kelvins.

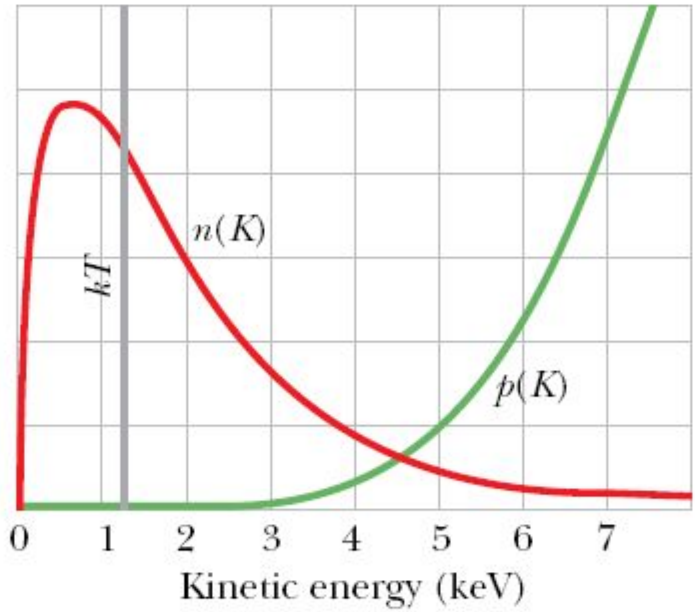


Fig. 43-10 The curve marked $n(K)$ gives the number density per unit energy for protons at the center of the Sun. The curve marked $p(K)$ gives the probability of barrier penetration (and hence fusion) for proton–proton collisions at the Sun's core temperature. The vertical line marks the value of kT at this temperature. Note that the two curves are drawn to (separate) arbitrary vertical scales.

Example, Fusion in a gas of protons, and the required temperature :

Assume a proton is a sphere of radius $R \approx 1$ fm. Two protons are fired at each other with the same kinetic energy K.

(a) What must K be if the particles are brought to rest by their mutual Coulomb repulsion when they are just "touching" each other? We can take this value of K as a representative measure of the height of the Coulomb barrier.

KEY IDEAS

The mechanical energy E of the two-proton system is conserved as the protons move toward each other and momentarily stop. In particular, the initial mechanical energy E_i is equal to the mechanical energy E_f when they stop. The initial energy E_i consists only of the total kinetic energy $2K$ of the two protons. When the protons stop, energy E_f consists only of the electric potential energy U of the system, as given by Eq. 24-43 ($U = q_1 q_2 / 4\pi\varepsilon_0 r$).

Calculations: Here the distance r between the protons when they stop is their center-to-center distance $2R$, and their charges q_1 and q_2 are both e. Then we can write the conservation of energy $E_i = E_f$ as

$$2K = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{2R}.$$

This yields, with known values,

$$\begin{aligned} K &= \frac{e^2}{16\pi\varepsilon_0 R} \\ &= \frac{(1.60 \times 10^{-19}\ \text{C})^2}{(16\pi)(8.85 \times 10^{-12}\ \text{F/m})(1 \times 10^{-15}\ \text{m})} \\ &= 5.75 \times 10^{-14}\ \text{J} = 360\ \text{keV} \approx 400\ \text{keV}. \qquad \text{(Answer)} \end{aligned}$$

(b) At what temperature would a proton in a gas of protons have the average kinetic energy calculated in (a) and thus have energy equal to the height of the Coulomb barrier?

KEY IDEA

If we treat the proton gas as an ideal gas, then from Eq. 19-24, the average energy of the protons is $K_{avg} = \frac{3}{2}kT$, where k is the Boltzmann constant.

Calculation: Solving that equation for T and using the result of (a) yield

$$\begin{aligned} T &= \frac{2K_{avg}}{3k} = \frac{(2)(5.75 \times 10^{-14}\ \text{J})}{(3)(1.38 \times 10^{-23}\ \text{J/K})} \\ &\approx 3 \times 10^9\ \text{K}. \qquad \text{(Answer)} \end{aligned}$$

The temperature of the core of the Sun is only about 1.5×10^7 K; thus fusion in the Sun's core must involve protons whose energies are *far* above the average energy.

43.7: Thermonuclear Fusion in the Sun and Other Stars:

$$^{1}H + {}^{1}H \rightarrow {}^{2}H + e^{+} + \nu \quad (Q = 0.42\ \text{MeV})$$
$$e^{+} + e^{-} \rightarrow \gamma + \gamma \quad (Q = 1.02\ \text{MeV})$$

$$^{1}H + {}^{1}H \rightarrow {}^{2}H + e^{+} + \nu \quad (Q = 0.42\ \text{MeV})$$
$$e^{+} + e^{-} \rightarrow \gamma + \gamma \quad (Q = 1.02\ \text{MeV})$$

$$^{2}H + {}^{1}H \rightarrow {}^{3}He + \gamma \quad (Q = 5.49\ \text{MeV})$$

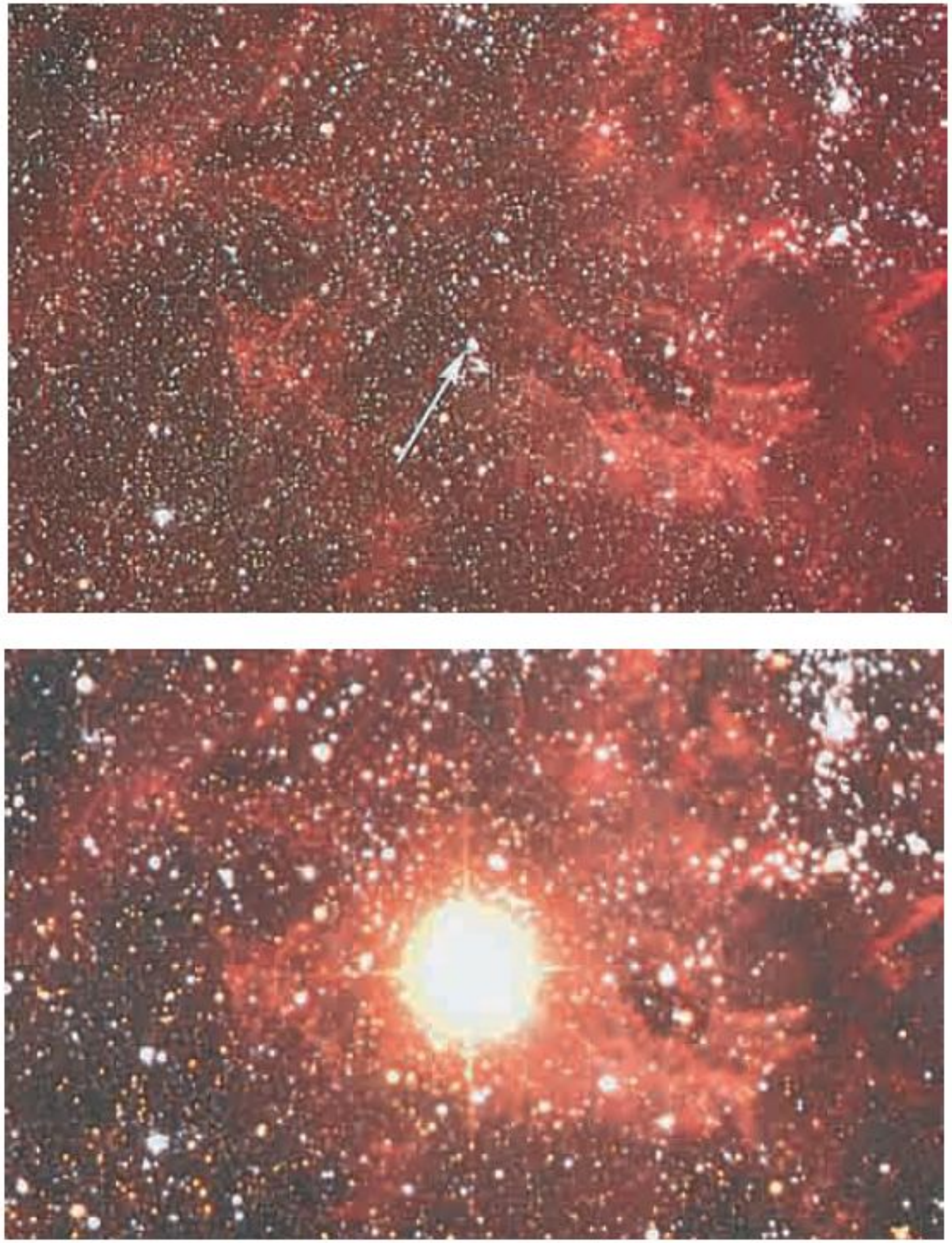
$$^{2}H + {}^{1}H \rightarrow {}^{3}He + \gamma \quad (Q = 5.49\ \text{MeV})$$

$$^{3}He + {}^{3}He \rightarrow {}^{4}He + {}^{1}H + {}^{1}H \quad (Q = 12.86\ \text{MeV})$$

$$\begin{aligned} Q &= (2)(0.42\ \text{MeV}) + (2)(1.02\ \text{MeV}) + (2)(5.49\ \text{MeV}) + 12.86\ \text{MeV} \\ &= 26.7\ \text{MeV}. \end{aligned}$$

Fig. 43-11 The proton–proton mechanism that accounts for energy production in the Sun. In this process, protons fuse to form an alpha particle (^{4}He), with a net energy release of 26.7 MeV for each event.

43.7: Thermonuclear Fusion in the Sun and Other Stars:



(a)

(b)

Fig. 43-12 (*a*) The star known as Sanduleak, as it appeared until 1987. (*b*) We then began to intercept light from the star's supernova, designated SN1987a; the explosion was 100 million times brighter than our Sun and could be seen with the unaided eye. *(Courtesy Anglo Australian Telescope Board)*

Example, Consumption rate of hydrogen in the sun:

At what rate dm/dt is hydrogen being consumed in the core of the Sun by the p-p cycle of Fig. 43-11?

KEY IDEA

The rate dE/dt at which energy is produced by hydrogen (proton) consumption within the Sun is equal to the rate P at which energy is radiated by the Sun:

$$P = \frac{dE}{dt}.$$

Calculations: To bring the mass consumption rate dm/dt into the power equation, we can rewrite it as

$$P = \frac{dE}{dt} = \frac{dE}{dm}\frac{dm}{dt} \approx \frac{\Delta E}{\Delta m}\frac{dm}{dt}, \qquad (43\text{-}12)$$

where ΔE is the energy produced when protons of mass Δm are consumed. From our discussion in this section, we know that 26.2 MeV ($= 4.20 \times 10^{-12}$ J) of thermal energy is produced when four protons are consumed. That is, $\Delta E = 4.20 \times 10^{-12}$ J for a mass consumption of $\Delta m = 4(1.67 \times 10^{-27}\text{ kg})$. Substituting these data into Eq. 43-12 and using the power P of the Sun given in Appendix C, we find that

$$\frac{dm}{dt} = \frac{\Delta m}{\Delta E}P = \frac{4(1.67 \times 10^{-27}\text{ kg})}{4.20 \times 10^{-12}\text{ J}}(3.90 \times 10^{26}\text{ W})$$

$$= 6.2 \times 10^{11}\text{ kg/s.} \qquad \text{(Answer)}$$

Thus, a huge amount of hydrogen is consumed by the Sun every second. However, you need not worry too much about the Sun running out of hydrogen, because its mass of 2×10^{30} kg will keep it burning for a long, long time.

43.8: Controlled Thermonuclear Fusion:

For controlled terrestrial use one could consider two deuteron–deuteron (d-d), and one deuteron-tritium reactions:

$$^{2}\mathrm{H} + {}^{2}\mathrm{H} \rightarrow {}^{3}\mathrm{He} + \mathrm{n} \qquad (Q = +3.27\ \mathrm{MeV}),$$

$$^{2}\mathrm{H} + {}^{2}\mathrm{H} \rightarrow {}^{3}\mathrm{H} + {}^{1}\mathrm{H} \qquad (Q = +4.03\ \mathrm{MeV}),$$

$$^{2}\mathrm{H} + {}^{3}\mathrm{H} \rightarrow {}^{4}\mathrm{He} + \mathrm{n} \qquad (Q = +17.59\ \mathrm{MeV}).$$

Three requirements for a successful thermonuclear reactor can be considered:

1.A High Particle Density n.

2. A High Plasma Temperature T.

3. A Long Confinement Time .

For the successful operation of a thermonuclear reactor using the d-t reaction, it is necessary to have ***Lawson's Criterion***: $n\tau > 10^{20}\ \mathrm{s/m^3}.$

43.8: Controlled Thermonuclear Fusion:

Magnetic Confinement

A suitably shaped magnetic field is used to confine the hot plasma in an evacuated doughnut-shaped chamber called a ***tokamak***. The magnetic forces acting on the charged particles that make up the hot plasma keep the plasma from touching the walls of the chamber.

The plasma is heated by inducing a current in it and by bombarding it with an externally accelerated beam of particles. The first goal of this approach is to achieve ***breakeven***, which occurs when the Lawson criterion is met or exceeded.

The ultimate goal is ***ignition*****,** which corresponds to a self-sustaining thermonuclear reaction and a net generation of energy.

Inertial Confinement

A second approach, involves ***"zapping"*** a solid fuel pellet from all sides with intense laser beams, evaporating some material from the surface of the pellet. This boiled-off material causes an inward-moving shock wave that compresses the core of the pellet, increasing both its particle density and its temperature. The fuel is ***confined*** to the pellet and the particles do not escape from the heated pellet during the very short zapping interval because of their inertia .

Laser fusion, using the inertial confinement approach, is being investigated in many laboratories in the United States and elsewhere.

43.8: Controlled Thermonuclear Fusion:

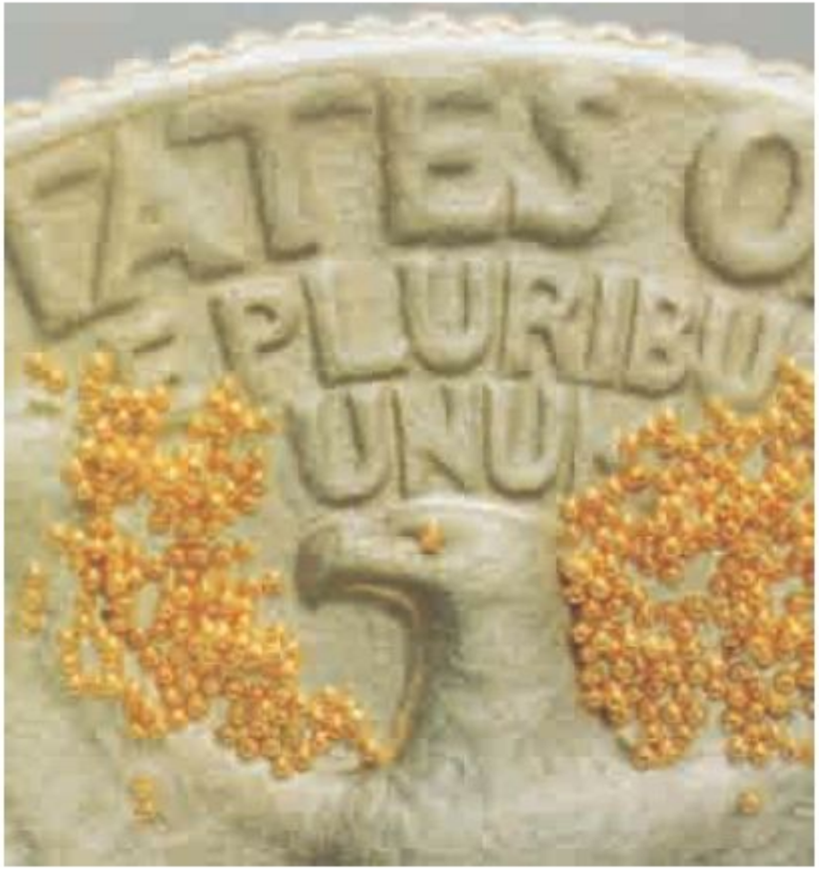


Fig. 43-13 The small spheres on the quarter are deuterium–tritium fuel pellets, designed to be used in a laser fusion chamber. *(Courtesy Los Alamos National Laboratory, New Mexico)*

Example, Laser fusion: number of particles and Lawson's criterion :

Suppose a fuel pellet in a laser fusion device contains equal numbers of deuterium and tritium atoms (and no other material). The density $d = 200\ \text{kg/m}^3$ of the pellet is increased by a factor of 10^3 by the action of the laser pulses.

(a) How many particles per unit volume (both deuterons and tritons) does the pellet contain in its compressed state? The molar mass M_d of deuterium atoms is 2.0×10^{-3} kg/mol, and the molar mass M_t of tritium atoms is 3.0×10^{-3} kg/mol.

Calculations: We can extend Eq. 43-17 to the system consisting of the two types of particles by writing the density d^* of the compressed pellet as the sum of the individual densities:

$$d^* = \frac{n}{2} m_d + \frac{n}{2} m_t, \tag{43-18}$$

where m_d and m_t are the masses of a deuterium atom and a tritium atom, respectively. We can replace those masses with the given molar masses by substituting

$$m_d = \frac{M_d}{N_A} \quad \text{and} \quad m_t = \frac{M_t}{N_A},$$

where N_A is Avogadro's number. After making those replacements and substituting $1000d$ for the compressed density d^*, we solve Eq. 43-18 for n to obtain

$$n = \frac{2000 d N_A}{M_d + M_t},$$

which gives us

$$n = \frac{(2000)(200\ \text{kg/m}^3)(6.02 \times 10^{23}\ \text{mol}^{-1})}{2.0 \times 10^{-3}\ \text{kg/mol} + 3.0 \times 10^{-3}\ \text{kg/mol}}$$

$$= 4.8 \times 10^{31}\ \text{m}^{-3}. \qquad \text{(Answer)}$$

(b) According to Lawson's criterion, how long must the pellet maintain this particle density if breakeven operation is to take place?

KEY IDEA

If breakeven operation is to occur, the compressed density must be maintained for a time period τ given by Eq. 43-16 ($n\tau > 10^{20}\ \text{s/m}^3$).

Calculation: We can now write

$$\tau > \frac{10^{20}\ \text{s/m}^3}{4.8 \times 10^{31}\ \text{m}^{-3}} \approx 10^{-12}\ \text{s}. \qquad \text{(Answer)}$$

(The plasma temperature must also be suitably high.)