Chapter 42

Nuclear Physics

42.2: Discovering The Nucleus:

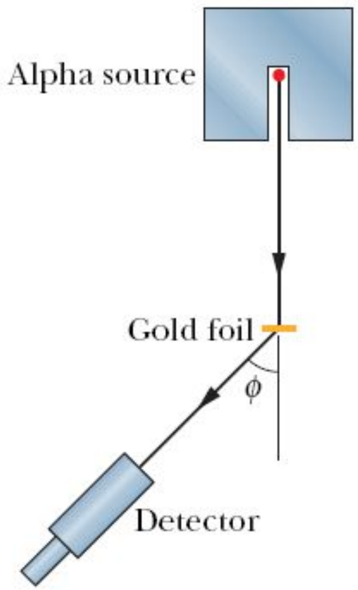
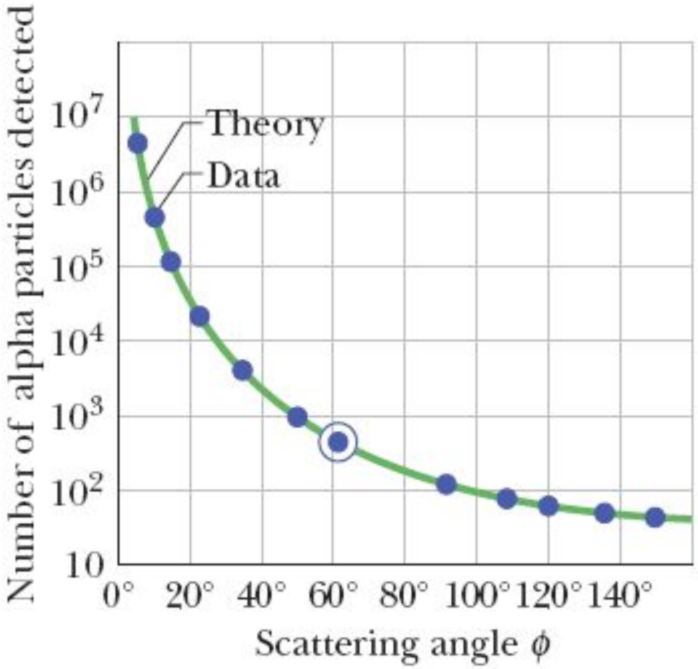


Fig. 42-1 An arrangement (top view) used in Rutherford's laboratory in 1911–1913 to study the scattering of α particles by thin metal foils. The detector can be rotated to various values of the scattering angle ϕ. The alpha source was radon gas, a decay product of radium. With this simple "tabletop" apparatus, the atomic nucleus was discovered.

In 1911 Ernest Rutherford proposed that the positive charge of the atom is densely concentrated at the center of the atom, forming its **nucleus,** and that, furthermore, the nucleus is responsible for most of the mass of the atom.

42.2: Discovering The Nucleus:

Fig. 42-2 The dots are alpha-particle scattering data for a gold foil, obtained by Geiger and Marsden using the apparatus of Fig. 42-1. The solid curve is the theoretical prediction, based on the assumption that the atom has a small, massive, positively charged nucleus. The data have been adjusted to fit the theoretical curve at the experimental point that is enclosed in a circle.



We see that most of the particles are scattered through rather small angles, but a very small fraction of them are scattered through very large angles, approaching 180°.

In Rutherford's words: "It was quite the most incredible event that ever happened to me in my life. It was almost as incredible as if you had fired a 15-inch shell at a piece of tissue paper and it [the shell] came back and hit you."

42.2: Discovering The Nucleus:

Rutherford saw that, to deflect the alpha particle backward, there must be a large force; this force could be provided if the positive charge, instead of being spread throughout the atom, were concentrated tightly at its center. Then the incoming alpha particle could get very close to the positive charge without penetrating it; such a close encounter would result in a large deflecting force.

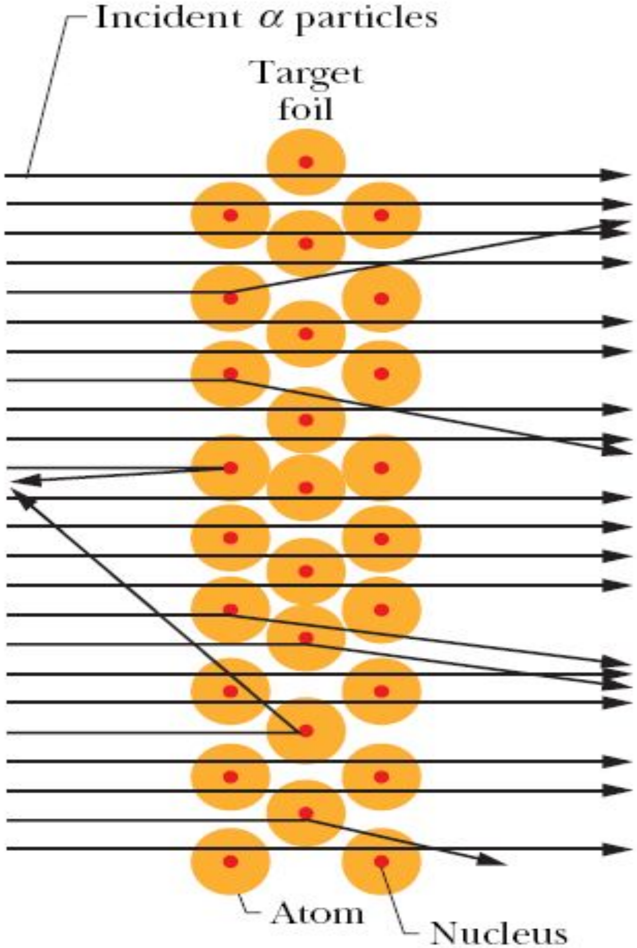
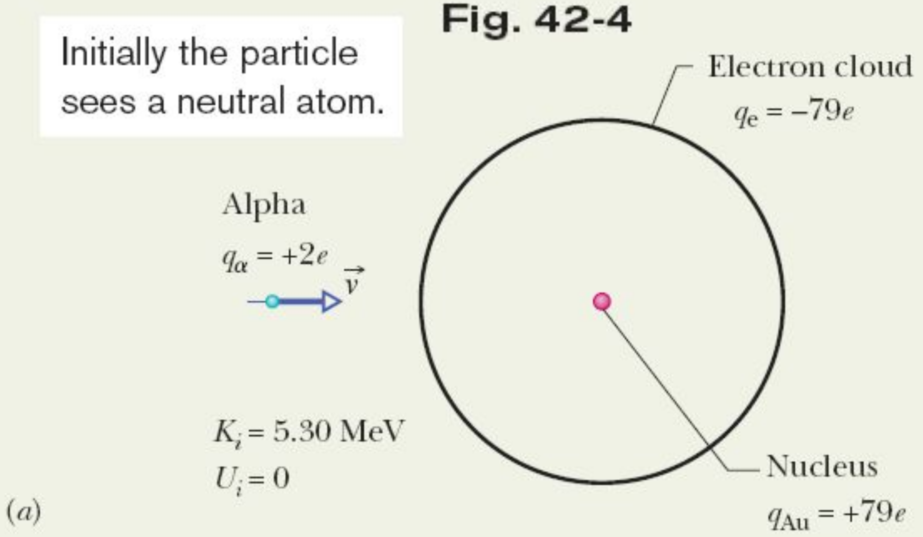


Fig. 42-3 The angle through which an incident alpha particle is scattered depends on how close the particle's path lies to an atomic nucleus. Large deflections result only from very close encounters.

Example, Rutherford scattering of an alpha particle by a gold nucleus:

An alpha particle with kinetic energy K_i = 5.30 MeV happens, by chance, to be headed directly toward the nucleus of a neutral gold atom (Fig. 42-4*a*). What is its *distance of closest approach d* (least center-to-center separation) to the nucleus? Assume that the atom remains stationary.

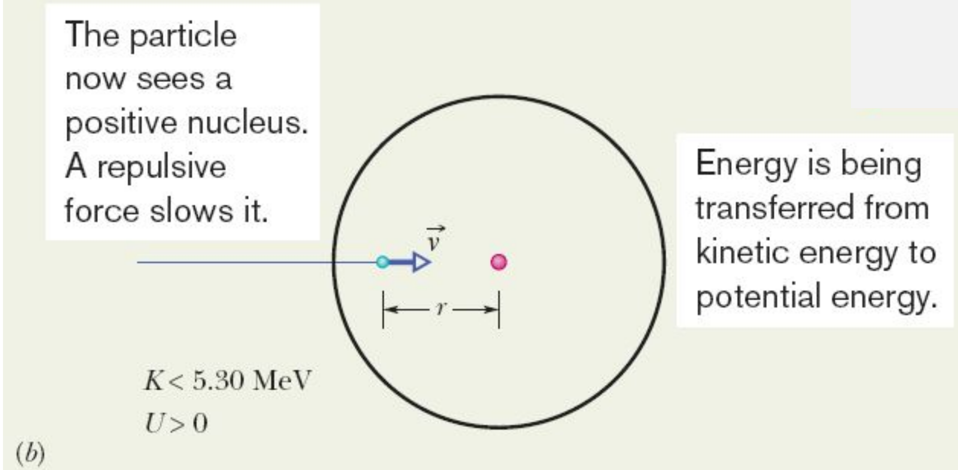
Fig. 42-4



Calculations: The alpha particle has a charge of $+2e$ because it contains two protons. The target nucleus has a charge of $q_{Au} = +79e$ because it contains 79 protons. However, that nuclear charge is surrounded by an electron "cloud" with a charge of $q_e = -79e$, and thus the alpha particle initially "sees" a neutral atom with a net charge of $q_{atom} = 0$. The electric force on the particle and the initial electric potential energy of the particle–atom system is $U_i = 0$.

Once the alpha particle enters the atom, we say that it passes through the electron cloud surrounding the nucleus. That cloud then acts as a closed conducting spherical shell and, by Gauss' law, has no effect on the (now internal) charged alpha particle. Then the alpha particle "sees" only the nuclear charge q_{Au}. Because q_α and q_{Au} are both positively charged, a repulsive electric force acts on the alpha particle, slowing it, and the particle–atom system has a potential energy

$$U = \frac{1}{4\pi\varepsilon_0}\frac{q_\alpha q_{Au}}{r}$$

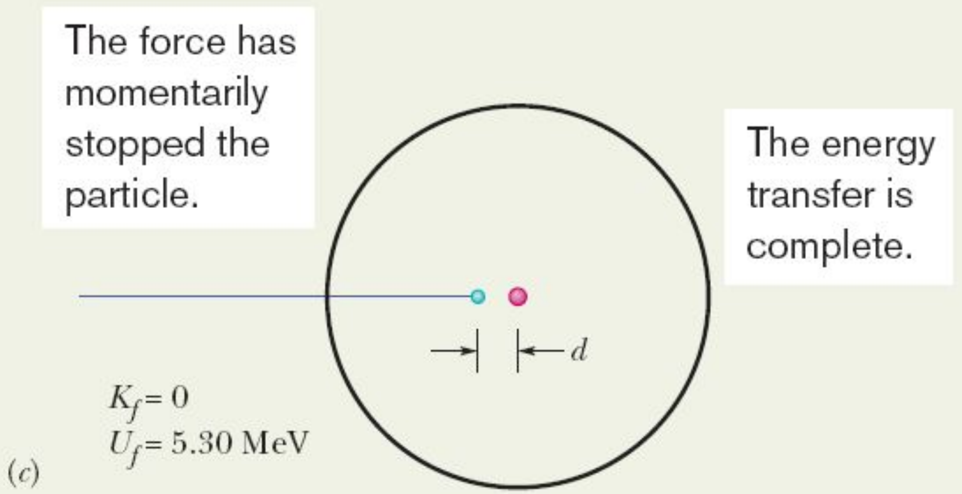


Example, Rutherford scattering of an alpha particle by a gold nucleus:

As the repulsive force slows the alpha particle, energy is transferred from kinetic energy to electric potential energy. The transfer is complete when the alpha particle momentarily stops at the distance of closest approach d to the target nucleus (Fig. 42-4c). Just then the kinetic energy is $K_f = 0$ and the particle–atom system has the electric potential energy

$$U_f = \frac{1}{4\pi\varepsilon_0}\frac{q_\alpha q_{Au}}{d}.$$

Fig. 42-4



To find d, we conserve the total mechanical energy between the initial state i and this later state f, writing

$$K_i + U_i = K_f + U_f$$

and

$$K_i + 0 = 0 + \frac{1}{4\pi\varepsilon_0}\frac{q_\alpha q_{Au}}{d}.$$

$$d = \frac{(2e)(79e)}{4\pi\varepsilon_0 K_\alpha}$$

$$= \frac{(2 \times 79)(1.60 \times 10^{-19}\ \text{C})^2}{4\pi\varepsilon_0(5.30\ \text{MeV})(1.60 \times 10^{-13}\ \text{J/MeV})}$$

$$= 4.29 \times 10^{-14}\ \text{m}. \qquad \text{(Answer)}$$

42.3: Some Nuclear Properties:

Nuclei are made up of protons and neutrons. The number of protons in a nucleus is called the **atomic number of the nucleus**, and is represented by the symbol ***Z***; the number of neutrons is the **neutron number**, and is represented by the symbol *N*.

The total number of neutrons and protons in a nucleus is called its **mass number *A*.** Neutrons and protons, when considered collectively, are called **nucleons.**

Table 42-1

Some Properties of Selected Nuclides

Nuclide	Z	N	A	Stability[a]	Mass[b] (u)	Spin[c]	Binding Energy (MeV/nucleon)
^{1}H	1	0	1	99.985%	1.007 825	$\frac{1}{2}$	—
^{7}Li	3	4	7	92.5%	7.016 004	$\frac{3}{2}$	5.60
^{31}P	15	16	31	100%	30.973 762	$\frac{1}{2}$	8.48
^{84}Kr	36	48	84	57.0%	83.911 507	0	8.72
^{120}Sn	50	70	120	32.4%	119.902 197	0	8.51
^{157}Gd	64	93	157	15.7%	156.923 957	$\frac{3}{2}$	8.21
^{197}Au	79	118	197	100%	196.966 552	$\frac{3}{2}$	7.91
^{227}Ac	89	138	227	21.8 y	227.027 747	$\frac{3}{2}$	7.65
^{239}Pu	94	145	239	24 100 y	239.052 157	$\frac{1}{2}$	7.56

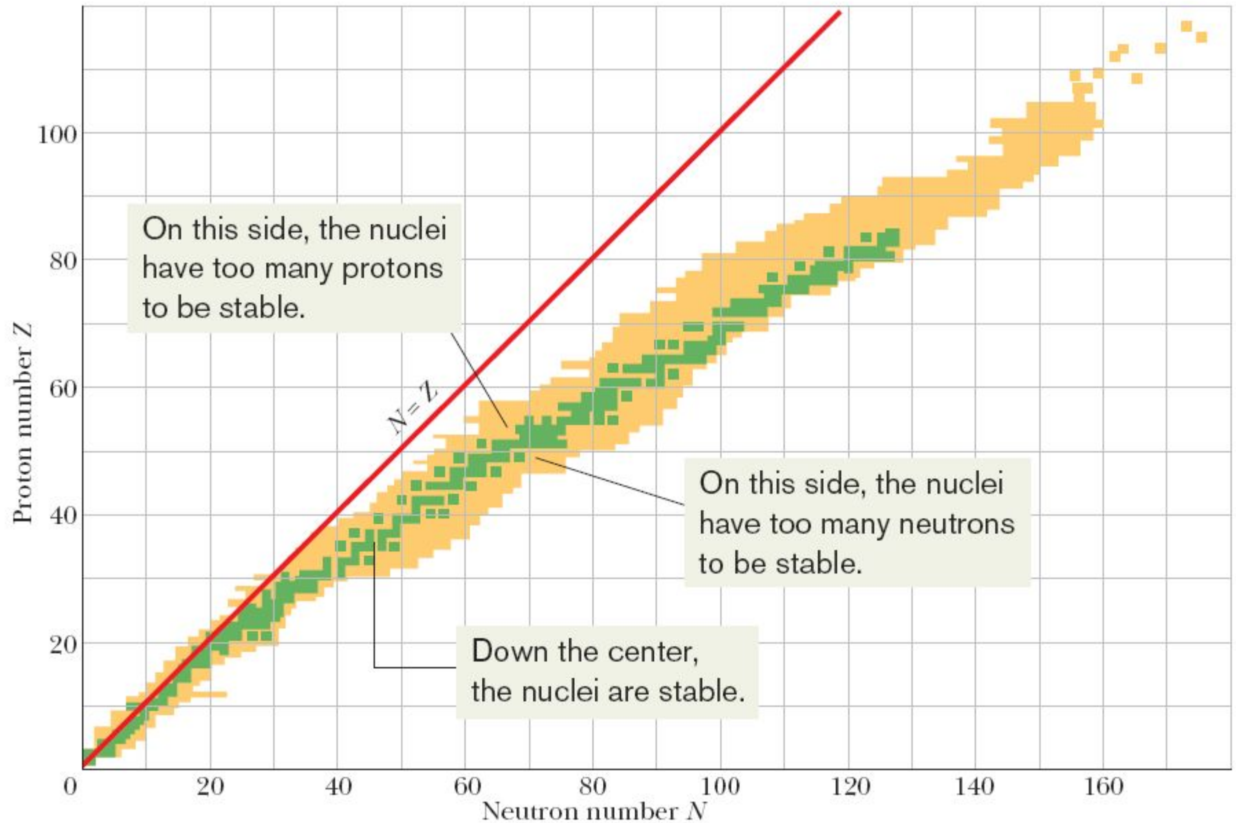
[a]For stable nuclides, the **isotopic abundance** is given; this is the fraction of atoms of this type found in a typical sample of the element. For radioactive nuclides, the half-life is given.

[b]Following standard practice, the reported mass is that of the neutral atom, not that of the bare nucleus.

[c]Spin angular momentum in units of $\hbar$.

42.3: Some Nuclear Properties:

Fig. 42-5 A plot of the known nuclides. The green shading identifies the band of stable nuclides, the beige shading the radionuclides. Low-mass, stable nuclides have essentially equal numbers of neutrons and protons, but more massive nuclides have an increasing excess of neutrons. The figure shows that there are no stable nuclides with $Z > 83$ (bismuth).



Nuclides with the same atomic number Z but different neutron numbers N are called **isotopes** of one another. The element gold has 32 isotopes, ranging from ^{173}Au to ^{204}Au. Only one of them (^{197}Au) is stable; the remaining 31 are radioactive. Such **radionuclides** undergo decay (or disintegration) by emitting a particle and thereby transforming to a different nuclide.

42.3: Some Nuclear Properties: Organizing the Nuclides

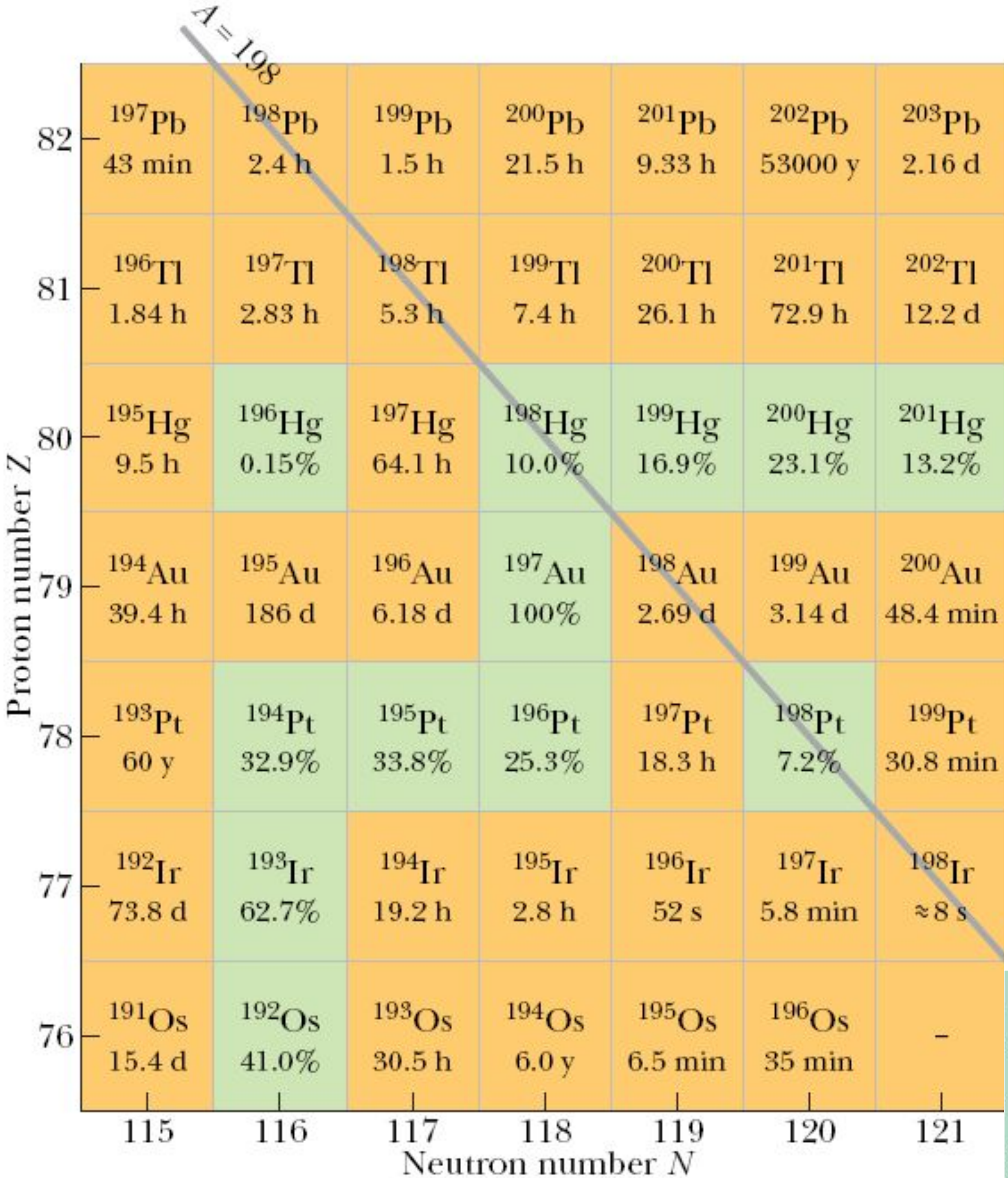


Fig. 42-6 An enlarged and detailed section of the nuclidic chart of Fig. 42-5, centered on ^{197}Au. Green squares represent stable nuclides, for which relative isotopic abundances are given. Beige squares represent radionuclides, for which half-lives are given. Isobaric lines of constant mass number A slope as shown by the example line for A = 198.

42.3: Some Nuclear Properties: Nuclear Radii

- The nucleus, like the atom, is not a solid object with a well-defined surface.

- Although most nuclides are spherical, some are notably ellipsoidal.

- Electron-scattering experiments (as well as experiments of other kinds) allow us to assign to each nuclide an effective radius given by

$$r = r_0 A^{1/3},$$

in which *A* is the mass number and $r_0 = 1.2\ fm$.

(1 femtometer =1 fermi =1 fm=10^{-15} m.

The above equation does not apply to *halo nuclides,* which are neutron-rich Nuclides, first produced in laboratories in the 1980s. These nuclides are larger than predicted by this equation, because some of the neutrons form a *halo* around a spherical core of the protons and the rest of the neutrons. Lithium isotopes are examples of this.

42.3: Some Nuclear Properties: Atomic Masses

- Atomic masses are often reported in *atomic mass units,* a system in which the atomic mass of neutral ^{12}C is defined to be exactly 12 u.

  1 u =1.660 538 86 x10^{-27} kg.

- The mass number A of a nuclide gives such an approximate mass in atomic mass units. For example, the approximate mass of both the nucleus and the neutral atom for ^{197}Au is 197 u, which is close to the actual atomic mass of 196.966 552 u.

- If the total mass of the participants in a nuclear reaction changes by an amount Δm, there is an energy release or absorption given by $Q=mc^2$.

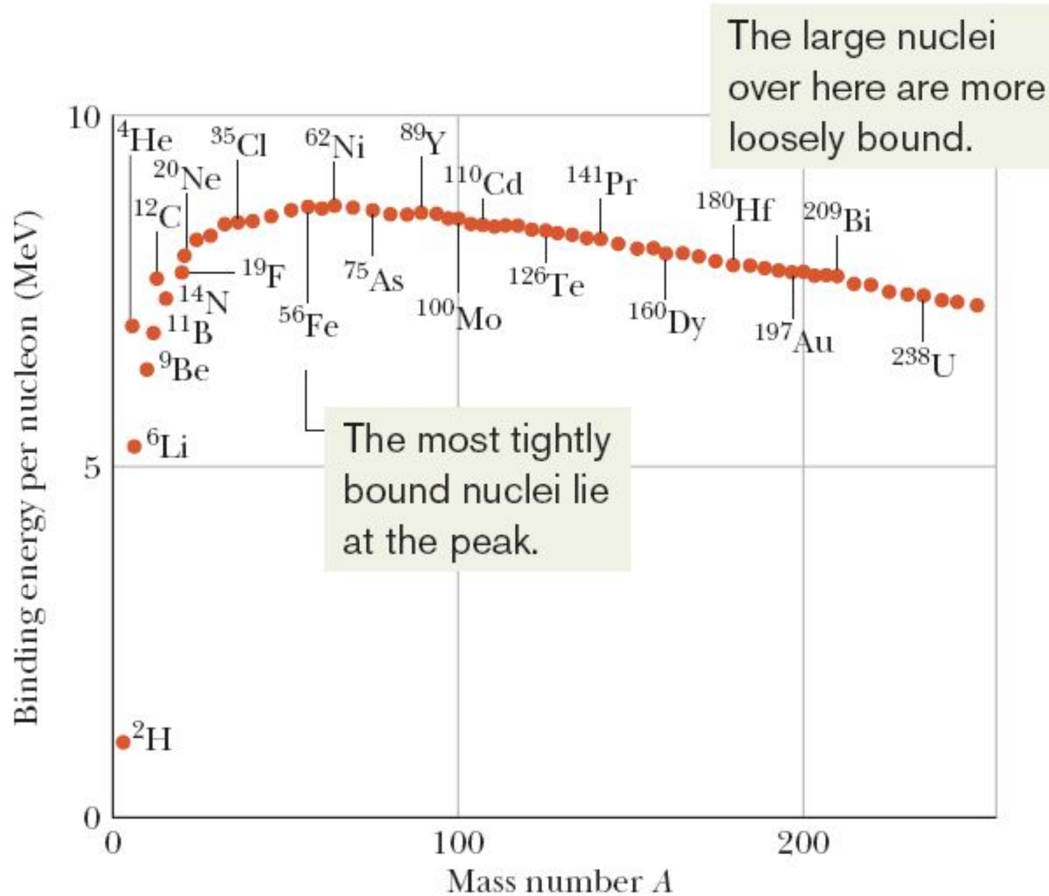
- The atom's *mass excess,* Δ, is defined as $\Delta = M - A$

  Here, M is the actual mass of the atom in atomic units, and A is the mass number for that atom's nucleus.

42.3: Some Nuclear Properties: Nuclear Binding Energies

$$\Delta E_{\text{be}} = \Sigma(mc^2) - Mc^2 \quad \text{(binding energy).}$$

Fig. 42-7 The binding energy per nucleon for some representative nuclides. The nickel nuclide ^{62}Ni has the highest binding energy per nucleon (about 8.794 60 MeV/nucleon) of any known stable nuclide. Note that the alpha particle (^{4}He) has a higher binding energy per nucleon than its neighbors in the periodic table and thus is also particularly stable.



If the nucleus splits into two nuclei, the process is called ***fission***, and occurs naturally with large (high mass number *A*).

If a pair of nuclei were to combine to form a single nucleus, the process is called ***fusion***, and occurs naturally in stars.

42.3: Some Nuclear Properties: Nuclear Energy Levels

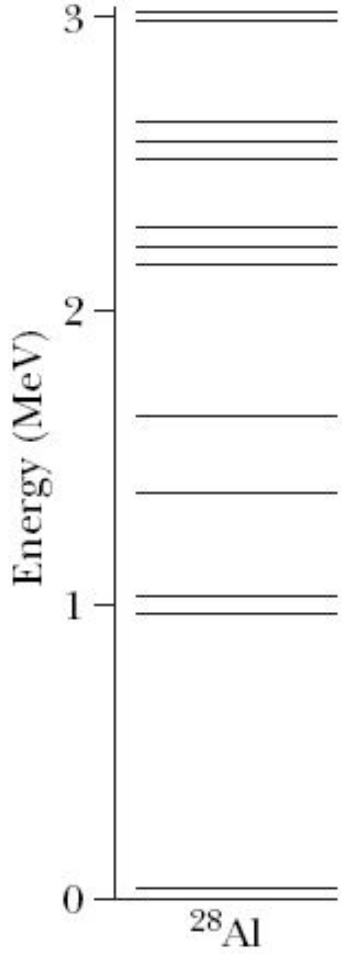


Fig. 42-8 Energy levels for the nuclide ^{28}Al, deduced from nuclear reaction experiments.

Example, Binding energy per nucleon:

What is the binding energy per nucleon for ^{120}Sn?

KEY IDEAS

1. We can find the binding energy per nucleon ΔE_{ben} if we first find the binding energy ΔE_{be} and then divide by the number of nucleons A in the nucleus, according to Eq. 42-8 ($\Delta E_{ben} = \Delta E_{be}/A$).

2. We can find ΔE_{be} by finding the difference between the mass energy Mc^2 of the nucleus and the total mass energy $\Sigma(mc^2)$ of the individual nucleons that make up the nucleus, according to Eq. 42-7 ($\Delta E_{be} = \Sigma(mc^2) - Mc^2$).

Calculations: From Table 42-1, we see that a ^{120}Sn nucleus consists of 50 protons ($Z = 50$) and 70 neutrons ($N = A - Z = 120 - 50 = 70$). Thus, we need to imagine a ^{120}Sn nucleus being separated into its 50 protons and 70 neutrons,

$$(^{120}\text{Sn nucleus}) \rightarrow 50\begin{pmatrix}\text{separate}\\\text{protons}\end{pmatrix} + 70\begin{pmatrix}\text{separate}\\\text{neutrons}\end{pmatrix}, \tag{42-9}$$

and then compute the resulting change in mass energy.

For that computation, we need the masses of a ^{120}Sn nucleus, a proton, and a neutron. However, because the mass of a neutral atom (nucleus *plus* electrons) is much easier to measure than the mass of a bare nucleus, calculations of binding energies are traditionally done with atomic masses. Thus, let's modify Eq. 42-9 so that it has a neutral ^{120}Sn atom on the left side. To do that, we include 50 electrons on the left side (to match the 50 protons in the ^{120}Sn nucleus). We must also add 50 electrons on the right side to balance Eq. 42-9. Those 50 electrons can be combined with the 50 protons, to form 50 neutral hydrogen atoms. We then have

$$(^{120}\text{Sn atom}) \rightarrow 50\begin{pmatrix}\text{separate}\\\text{H atoms}\end{pmatrix} + 70\begin{pmatrix}\text{separate}\\\text{neutrons}\end{pmatrix}. \tag{42-10}$$

From the mass column of Table 42-1, the mass M_{Sn} of a ^{120}Sn atom is 119.902 197 u and the mass m_H of a hydrogen atom is 1.007 825 u; the mass m_n of a neutron is 1.008 665 u. Thus, Eq. 42-7 yields

$$\begin{aligned}
\Delta E_{be} &= \Sigma(mc^2) - Mc^2 \\
&= 50(m_H c^2) + 70(m_n c^2) - M_{Sn}c^2 \\
&= 50(1.007\,825\text{ u})c^2 + 70(1.008\,665\text{ u})c^2 \\
&\quad - (119.902\,197\text{ u})c^2 \\
&= (1.095\,603\text{ u})c^2 \\
&= (1.095\,603\text{ u})(931.494\,013\text{ MeV/u}) \\
&= 1020.5\text{ MeV},
\end{aligned}$$

where Eq. 42-5 ($c^2 = 931.494\,013$ MeV/u) provides an easy unit conversion. Note that using atomic masses instead of nuclear masses does not affect the result because the mass of the 50 electrons in the ^{120}Sn atom subtracts out from the mass of the electrons in the 50 hydrogen atoms.

Now Eq. 42-8 gives us the binding energy per nucleon as

$$\begin{aligned}
\Delta E_{ben} &= \frac{\Delta E_{be}}{A} = \frac{1020.5\text{ MeV}}{120} \\
&= 8.50\text{ MeV/nucleon.} \qquad \text{(Answer)}
\end{aligned}$$

Example, Density of nuclear matter:

We can think of all nuclides as made up of a neutron–proton mixture that we can call *nuclear matter.* What is the density of nuclear matter?

KEY IDEA

We can find the (average) density ρ of a nucleus by dividing its total mass by its volume.

Calculations: Let m represent the mass of a nucleon (either a proton or a neutron, because those particles have about the same mass). Then the mass of a nucleus containing A nucleons is Am. Next, we assume the nucleus is spherical with radius r. Then its volume is $\frac{4}{3}\pi r^3$, and we can write the density of the nucleus as

$$\rho = \frac{Am}{\frac{4}{3}\pi r^3}.$$

The radius r is given by Eq. 42-3 ($r = r_0 A^{1/3}$), where r_0 is 1.2 fm ($= 1.2 \times 10^{-15}$ m). Substituting for r then leads to

$$\rho = \frac{Am}{\frac{4}{3}\pi r_0^3 A} = \frac{m}{\frac{4}{3}\pi r_0^3}.$$

Note that A has canceled out; thus, this equation for density ρ applies to any nucleus that can be treated as spherical with a radius given by Eq. 42-3. Using 1.67×10^{-27} kg for the mass m of a nucleon, we then have

$$\rho = \frac{1.67 \times 10^{-27}\ \text{kg}}{\frac{4}{3}\pi(1.2 \times 10^{-15}\ \text{m})^3} \approx 2 \times 10^{17}\ \text{kg/m}^3. \qquad \text{(Answer)}$$

This is about 2×10^{14} times the density of water and is the density of neutron stars, which contain only neutrons.

42.4: Radioactive Decay:

There is absolutely no way to predict whether any given nucleus in a radioactive sample will be among the small number of nuclei that decay during the next second. All have the same chance.

If a sample contains *N* radioactive nuclei, then the rate *(=dN/dt)* at which nuclei will decay is proportional to *N*:

$$-\frac{dN}{dt} = \lambda N,$$

Here λ is the disintegration or decay constant.

Therefore,

$$\frac{dN}{N} = -\lambda\, dt, \;\Rightarrow\; \int_{N_0}^{N} \frac{dN}{N} = -\lambda \int_{t_0}^{t} dt, \;\Rightarrow\; \ln\frac{N}{N_0} = -\lambda t.$$

$$\Rightarrow\; \frac{N}{N_0} = e^{-\lambda t} \;\Rightarrow\; N = N_0 e^{-\lambda t} \quad \text{(radioactive decay)},$$

Here, N_o is the number of radioactive nuclei at time t = 0.

$$1 \text{ becquerel} = 1 \text{ Bq} = 1 \text{ decay per second}.$$

$$1 \text{ curie} = 1 \text{ Ci} = 3.7 \times 10^{10} \text{ Bq}.$$

42.4: Radioactive Decay:

The decay rate:

$$R = -\frac{dN}{dt} = \lambda N_0 e^{-\lambda t}$$

Therefore,

$$R = R_0 e^{-\lambda t} \qquad \text{(radioactive decay)},$$

The half life-time ($T_{1/2}$) is the time at which both N and R have been reduced to one-half their initial values

$$\tfrac{1}{2}R_0 = R_0 e^{-\lambda T_{1/2}}.$$

Therefore,

$$T_{1/2} = \frac{\ln 2}{\lambda}.$$

And,

$$T_{1/2} = \frac{\ln 2}{\lambda} = \tau \ln 2.$$

Here τ is the ***mean life time,*** which is the time at which both N and R have been reduced to e^{-1} of their initial values.

Example, Finding the disintegration constant and the half life-time:

The table that follows shows some measurements of the decay rate of a sample of ^{128}I, a radionuclide often used medically as a tracer to measure the rate at which iodine is absorbed by the thyroid gland.

Time (min)	R (counts/s)	Time (min)	R (counts/s)
4	392.2	132	10.9
36	161.4	164	4.56
68	65.5	196	1.86
100	26.8	218	1.00

Find the disintegration constant λ and the half-life $T_{1/2}$ for this radionuclide.

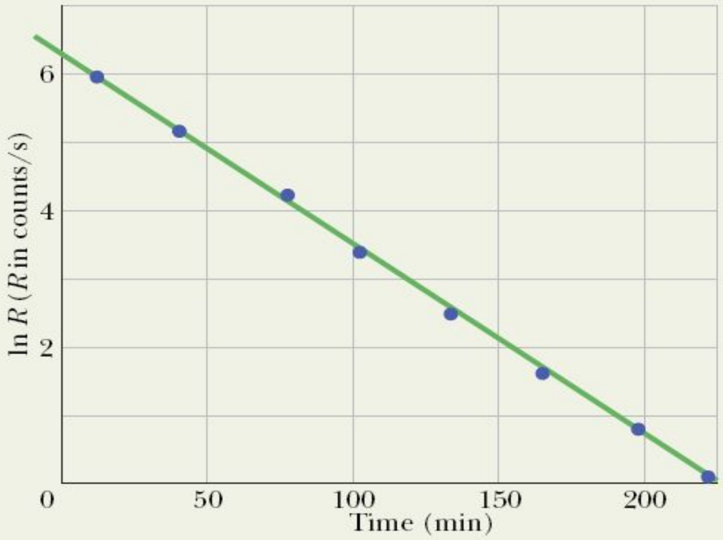


Fig. 42-9 A semilogarithmic plot of the decay of a sample of ^{128}I, based on the data in the table.

KEY IDEAS

The disintegration constant λ determines the exponential rate at which the decay rate R decreases with time t (as indicated by Eq. 42-16, $R = R_0 e^{-\lambda t}$). Therefore, we should be able to determine λ by plotting the measurements of R against the measurement times t. However, obtaining λ from a plot of R versus t is difficult because R decreases exponentially with t, according to Eq. 42-16. A neat solution is to transform Eq. 42-16 into a linear function of t, so that we can easily find λ. To do so, we take the natural logarithms of both sides of Eq. 42-16.

Calculations: We obtain

$$\ln R = \ln(R_0 e^{-\lambda t}) = \ln R_0 + \ln(e^{-\lambda t})$$
$$= \ln R_0 - \lambda t. \qquad (42\text{-}19)$$

Because Eq. 42-19 is of the form $y = b + mx$, with b and m constants, it is a linear equation giving the quantity $\ln R$ as a function of t. Thus, if we plot $\ln R$ (instead of R) versus t, we should get a straight line. Further, the slope of the line should be equal to $-\lambda$.

Figure 42-9 shows a plot of $\ln R$ versus time t for the given measurements. The slope of the straight line that fits through the plotted points is

$$\text{slope} = \frac{0 - 6.2}{225 \text{ min} - 0} = -0.0276 \text{ min}^{-1}.$$

Thus, $$-\lambda = -0.0276 \text{ min}^{-1}$$

or $$\lambda = 0.0276 \text{ min}^{-1} \approx 1.7 \text{ h}^{-1}. \qquad \text{(Answer)}$$

The time for the decay rate R to decrease by 1/2 is related to the disintegration constant λ via Eq. 42-18 ($T_{1/2} = (\ln 2)/\lambda$). From that equation, we find

$$T_{1/2} = \frac{\ln 2}{\lambda} = \frac{\ln 2}{0.0276 \text{ min}^{-1}} \approx 25 \text{ min}. \qquad \text{(Answer)}$$

Example, Finding the half life from the activity and the mass:

A 2.71 g sample of KCl from the chemistry stockroom is found to be radioactive, and it is decaying at a constant rate of 44.90 Bq. The decays are traced to the element potassium and in particular to the isotope ^{40}K, which constitutes 0.0117% of normal potassium. Calculate the half-life of this nuclide.

KEY IDEAS

1. Because the activity R of the sample is apparently constant, we cannot find the half-life $T_{1/2}$ by plotting ln R versus time t as in the preceding sample problem. (We would just get a horizontal plot.) However, we can use the following ideas.
2. We can relate the half-life $T_{1/2}$ to the disintegration constant λ via Eq. 42-18 ($T_{1/2} = (\ln 2)/\lambda$).
3. We can then relate λ to the given activity R of 44.90 Bq by means of Eq. 42-17 ($R = \lambda N$), where N is the number of ^{40}K nuclei (and thus atoms) in the sample.

Calculations: Combining Eqs. 42-18 and 42-17 yields

$$T_{1/2} = \frac{N \ln 2}{R}. \qquad (42\text{-}20)$$

We know that N in this equation is 0.0117% of the total number N_K of potassium atoms in the sample. We also know that N_K must equal the number N_{KCl} of molecules in the sample. We can obtain N_{KCl} from the molar mass M_{KCl} of KCl (the mass of one mole of KCl) and the given mass M_{sam} of the sample by combining Eqs. 19-2 ($n = N/N_A$) and 19-3 ($n = M_{sam}/M$) to write

$$N_{KCl} = \binom{\text{number of moles}}{\text{in sample}} N_A = \frac{M_{sam}}{M_{KCl}} N_A, \qquad (42\text{-}21)$$

where N_A is Avogadro's number (6.02×10^{23} mol^{-1}). From Appendix F, we see that the molar mass of potassium is 39.102 g/mol and the molar mass of chlorine is 35.453 g/mol; thus, the molar mass of KCl is 74.555 g/mol. Equation 42-21 then gives us

$$N_{KCl} = \frac{(2.71\text{ g})(6.02 \times 10^{23}\text{ mol}^{-1})}{74.555\text{ g/mol}} = 2.188 \times 10^{22}$$

as the number of KCl molecules in the sample. Thus, the total number N_K of potassium atoms is also 2.188×10^{22}, and the number of ^{40}K in the sample must be

$$N = (0.000\,117)N_K = (0.000\,117)(2.188 \times 10^{22})$$
$$= 2.560 \times 10^{18}.$$

Substituting this value for N and the given activity of 44.90 Bq (= 44.90 s^{-1}) for R into Eq. 42-20 leads to

$$T_{1/2} = \frac{(2.560 \times 10^{18}) \ln 2}{44.90\text{ s}^{-1}}$$
$$= 3.95 \times 10^{16}\text{ s} = 1.25 \times 10^{9}\text{ y}. \qquad \text{(Answer)}$$

This half-life of ^{40}K turns out to have the same order of magnitude as the age of the universe. Thus, the activity of ^{40}K in the stockroom sample decreases *very* slowly, too slowly for us to detect during a few days of observation or even an entire lifetime. A portion of the potassium in our bodies consists of this radioisotope, which means that we are all slightly radioactive.

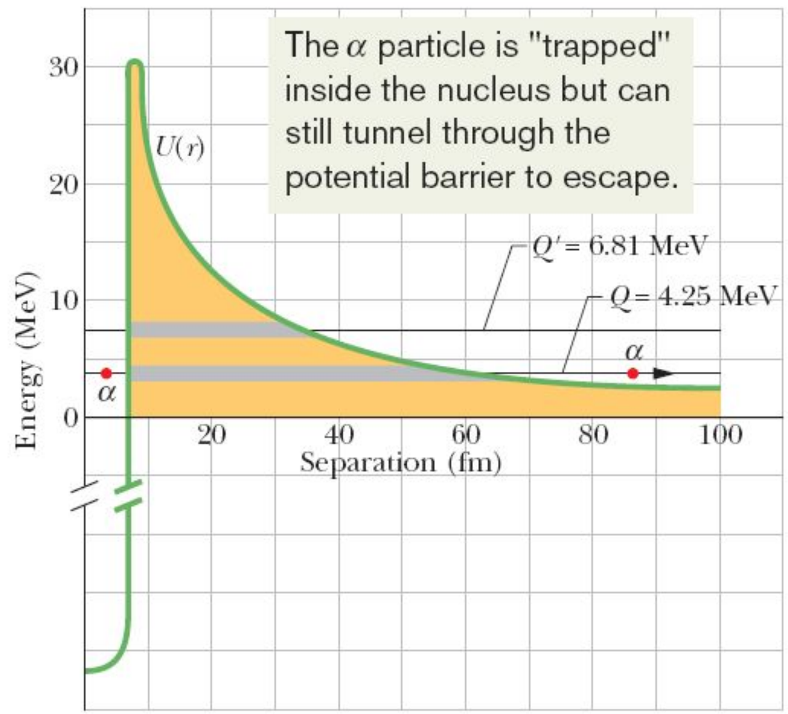
42.5: Alpha Decay:

When a nucleus undergoes **alpha decay,** it transforms to a different nuclide by emitting an alpha particle (a helium nucleus, ^{4}He). For example, when uranium ^{238}U undergoes alpha decay, it transforms to thorium ^{234}Th:

$^{238}U \rightarrow ^{234}Th + ^{4}He$. The ***disintegration energy,*** *Q*, for the decay above is 4.25.

The potential energy shown in the figure below is a combination of the potential energy associated with the (attractive) strong nuclear force that acts in the nuclear interior and a Coulomb potential associated with the (repulsive) electric force that acts between the two particles (^{234}Th and ^{4}He) before and after the decay has occurred.

Fig. 42-10 A potential energy function for the emission of an alpha particle by ^{238}U. The horizontal black line marked $Q = 4.25$ *MeV* shows the disintegration energy for the process. The thick gray portion of this line represents separations *r* that are classically forbidden to the alpha particle. The alpha particle is represented by a dot, both inside this potential energy barrier (at the left) and outside it (at the right), after the particle has tunneled through. The horizontal black line marked $Q = 6.81$ *MeV* shows the disintegration energy for the alpha decay of ^{228}U. (Both isotopes have the same potential energy function because they have the same nuclear charge.)



Example, Q value of an alpha decay using masses:

We are given the following atomic masses:

^{238}U	238.050 79 u	^{4}He	4.002 60 u
^{234}Th	234.043 63 u	^{1}H	1.007 83 u
^{237}Pa	237.051 21 u		

Here Pa is the symbol for the element protactinium ($Z = 91$).

(a) Calculate the energy released during the alpha decay of ^{238}U. The decay process is

$$^{238}\text{U} \rightarrow {}^{234}\text{Th} + {}^{4}\text{He}.$$

Note, incidentally, how nuclear charge is conserved in this equation: The atomic numbers of thorium (90) and helium (2) add up to the atomic number of uranium (92). The number of nucleons is also conserved: 238 = 234 + 4.

KEY IDEA

The energy released in the decay is the disintegration energy Q, which we can calculate from the change in mass ΔM due to the ^{238}U decay.

Calculation: To do this, we use Eq. 37-50,

$$Q = M_i c^2 - M_f c^2, \qquad (42\text{-}23)$$

where the initial mass M_i is that of ^{238}U and the final mass M_f is the sum of the ^{234}Th and ^{4}He masses. Using the atomic masses given in the problem statement, Eq. 42-23 becomes

$$\begin{aligned} Q &= (238.050\,79\text{ u})c^2 - (234.043\,63\text{ u} + 4.002\,60\text{ u})c^2 \\ &= (0.004\,56\text{ u})c^2 = (0.004\,56\text{ u})(931.494\,013\text{ MeV/u}) \\ &= 4.25\text{ MeV}. \qquad \text{(Answer)} \end{aligned}$$

Note that using atomic masses instead of nuclear masses does not affect the result because the total mass of the electrons in the products subtracts out from the mass of the nucleons + electrons in the original ^{238}U.

(b) Show that ^{238}U cannot spontaneously emit a proton; that is, protons do not leak out of the nucleus in spite of the proton–proton repulsion within the nucleus.

Solution: If this happened, the decay process would be

$$^{238}\text{U} \rightarrow {}^{237}\text{Pa} + {}^{1}\text{H}.$$

(You should verify that both nuclear charge and the number of nucleons are conserved in this process.) Using the same Key Idea as in part (a) and proceeding as we did there, we would find that the mass of the two decay products

$$237.051\,21\text{ u} + 1.007\,83\text{ u}$$

would *exceed* the mass of ^{238}U by $\Delta m = 0.008\,25$ u, with disintegration energy

$$Q = -7.68\text{ MeV}.$$

The minus sign indicates that we must *add* 7.68 MeV to a ^{238}U nucleus before it will emit a proton; it will certainly not do so spontaneously.

42.6: Beta Decay:

A nucleus that decays spontaneously by emitting an electron or a positron (a positively charged particle with the mass of an electron) is said to undergo **beta decay.** Like alpha decay, this is a spontaneous process, with a definite disintegration energy and half-life.

Examples:

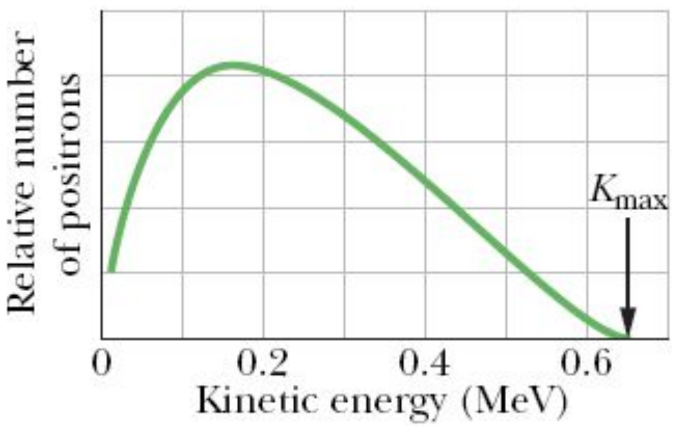
$$^{32}P \rightarrow {}^{32}S + e^- + \nu \qquad (T_{1/2} = 14.3 \text{ d}).$$

$$^{64}Cu \rightarrow {}^{64}Ni + e^+ + \nu \qquad (T_{1/2} = 12.7 \text{ h}).$$

Here, ν is a neutrino, a neutral particle which has a very small mass, that is emitted from the nucleus along with the electron or positron during the decay process.

In a beta decay the energy of the emitted electrons or positrons may range from zero up to a certain maximum K_{max}, since, unlike the alpha decay, the Q energy is shared by two components.

Fig. 42-11 The distribution of the kinetic energies of positrons emitted in the beta decay of ^{64}Cu. The maximum kinetic energy of the distribution (K_{max}) is 0.653 MeV. In all ^{64}Cu decay events, this energy is shared between the positron and the neutrino, in varying proportions. The *most probable* energy for an emitted positron is about 0.15 MeV.



42.6: Beta Decay: The Neutrino

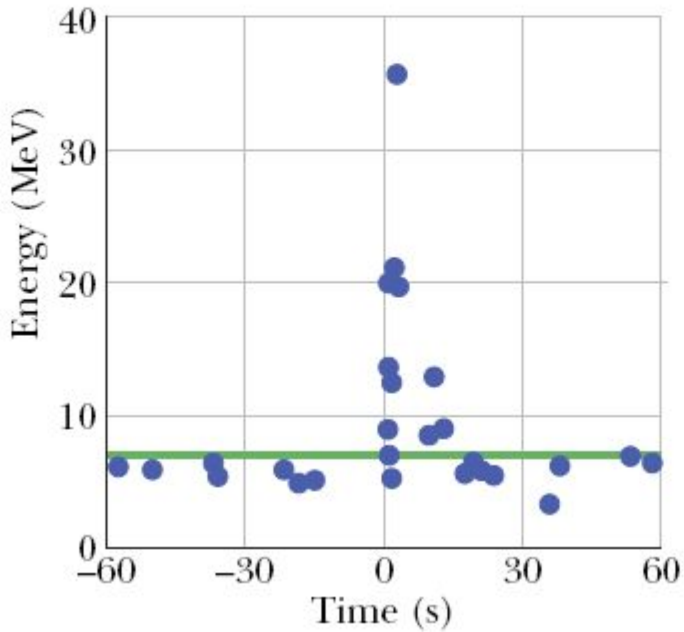


Fig. 42-12 A burst of neutrinos from the supernova SN 1987A, which occurred at (relative) time 0, stands out from the usual *background* of neutrinos. (For neutrinos, 10 is a "burst.") The particles were detected by an elaborate detector housed deep in a mine in Japan. The supernova was visible only in the Southern Hemisphere; so the neutrinos had to penetrate Earth (a trifling barrier for them) to reach the detector.

Wolfgang Pauli first suggested the existence of neutrinos in 1930.

Billions of them pass through our bodies every second, leaving no trace.

In spite of their elusive character, neutrinos have been detected in the laboratory. In spite of their elusive character, neutrinos have been detected in the laboratory.

42.6: Beta Decay: Radioactivity and the Nuclidic Chart

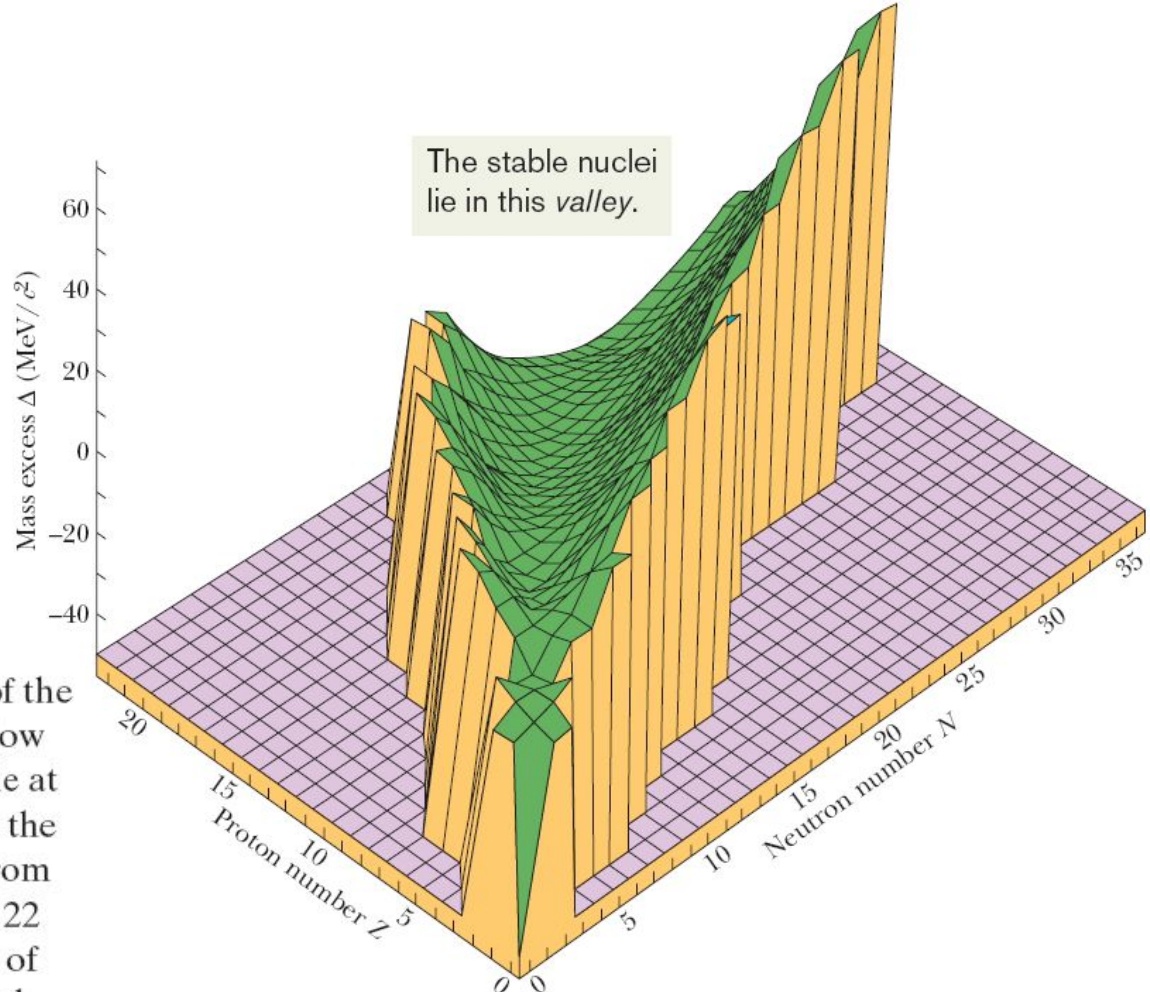


Fig. 42-13 A portion of the valley of the nuclides, showing only the nuclides of low mass. Deuterium, tritium, and helium lie at the near end of the plot, with helium at the high point. The valley stretches away from us, with the plot stopping at about $Z = 22$ and $N = 35$. Nuclides with large values of A, which would be plotted much beyond the valley, can decay into the valley by repeated alpha emissions and by fission (splitting of a nuclide).

Example, Q value of a beta decay using masses:

Calculate the disintegration energy Q for the beta decay of ^{32}P, as described by Eq. 42-24. The needed atomic masses are 31.973 91 u for ^{32}P and 31.972 07 u for ^{32}S.

KEY IDEA

The disintegration energy Q for the beta decay is the amount by which the mass energy is changed by the decay.

Calculations: Q is given by Eq. 37-50 ($Q = -\Delta M\ c^2$). However, we must be careful to distinguish between nuclear masses (which we do not know) and atomic masses (which we do know). Let the boldface symbols $\mathbf{m}_P$ and $\mathbf{m}_S$ represent the nuclear masses of ^{32}P and ^{32}S, and let the italic symbols m_P and m_S represent their atomic masses. Then we can write the change in mass for the decay of Eq. 42-24 as

$$\Delta m = (\mathbf{m}_S + m_e) - \mathbf{m}_P,$$

in which m_e is the mass of the electron. If we add and subtract $15m_e$ on the right side of this equation, we obtain

$$\Delta m = (\mathbf{m}_S + 16m_e) - (\mathbf{m}_P + 15m_e).$$

The quantities in parentheses are the atomic masses of ^{32}S and ^{32}P; so

$$\Delta m = m_S - m_P.$$

We thus see that if we subtract only the atomic masses, the mass of the emitted electron is automatically taken into account. (This procedure will not work for positron emission.)

The disintegration energy for the ^{32}P decay is then

$$\begin{aligned} Q &= -\Delta m\, c^2 \\ &= -(31.972\,07\text{ u} - 31.973\,91\text{ u})(931.494\,013\text{ MeV/u}) \\ &= 1.71\text{ MeV}. \qquad \text{(Answer)} \end{aligned}$$

Experimentally, this calculated quantity proves to be equal to K_{max}, the maximum energy the emitted electrons can have. Although 1.71 MeV is released every time a ^{32}P nucleus decays, in essentially every case the electron carries away less energy than this. The neutrino gets all the rest, carrying it stealthily out of the laboratory.

42.7: Radioactive Dating:

A fragment of the Dead Sea scrolls and the caves from which
The scrolls were recovered. (*www.BibleLandPictures.comAlamy)*

The decay of very long-lived nuclides can be used to measure the time that has elapsed since they were formed. Such measurements for rocks from Earth and the Moon, and for meteorites, yield a consistent maximum age of about 4.5×10^9 y for these bodies.

Example, Radioactive dating of a moon rock:

In a Moon rock sample, the ratio of the number of (stable) ^{40}Ar atoms present to the number of (radioactive) ^{40}K atoms is 10.3. Assume that all the argon atoms were produced by the decay of potassium atoms, with a half-life of 1.25×10^9 y. How old is the rock?

KEY IDEAS

(1) If N_0 potassium atoms were present at the time the rock was formed by solidification from a molten form, the number of potassium atoms now remaining at the time of analysis is

$$N_K = N_0 e^{-\lambda t}, \tag{42-29}$$

in which t is the age of the rock. (2) For every potassium atom that decays, an argon atom is produced. Thus, the number of argon atoms present at the time of the analysis is

$$N_{Ar} = N_0 - N_K. \tag{42-30}$$

Calculations: We cannot measure N_0; so let's eliminate it from Eqs. 42-29 and 42-30. We find, after some algebra, that

$$\lambda t = \ln\left(1 + \frac{N_{Ar}}{N_K}\right), \tag{42-31}$$

in which N_{Ar}/N_K *can* be measured. Solving for t and using Eq. 42-18 to replace λ with $(\ln 2)/T_{1/2}$ yield

$$t = \frac{T_{1/2}\ln(1 + N_{Ar}/N_K)}{\ln 2}$$

$$= \frac{(1.25 \times 10^9\ \text{y})[\ln(1 + 10.3)]}{\ln 2}$$

$$= 4.37 \times 10^9\ \text{y}. \quad \text{(Answer)}$$

Lesser ages may be found for other lunar or terrestrial rock samples, but no substantially greater ones. Thus, the oldest rocks were formed soon after the solar system formed, and the solar system must be about 4 billion years old.

42.8: Measuring Radiation Dosage:

Absorbed Dose.

This is a measure of the radiation dose (as energy per unit mass) actually absorbed by a specific object, such as a patient's hand or chest.
Its SI unit is the **gray (Gy).** An older unit, the rad (from radiation absorbed dose) is still used.
1 Gy =1 J/kg =100 rad.

Dose Equivalent.

Although different types of radiation (gamma rays and neutrons, say) may deliver the same amount of energy to the body, they do not have the same biological effect. The dose equivalent allows us to express the biological effect by multiplying the absorbed dose (in grays or rads) by a numerical ***RBE factor*** (from relative biological effectiveness).

For x rays and electrons, RBE 1; for slow neutrons, RBE 5; for alpha particles, RBE 10; and so on.

Personnel-monitoring devices such as film badges register the dose equivalent.
The SI unit of dose equivalent is the **sievert (Sv)**. An earlier unit, the rem, is still used.
1 Sv =100 rem.

42.9: Nuclear Models: The Collective Model:

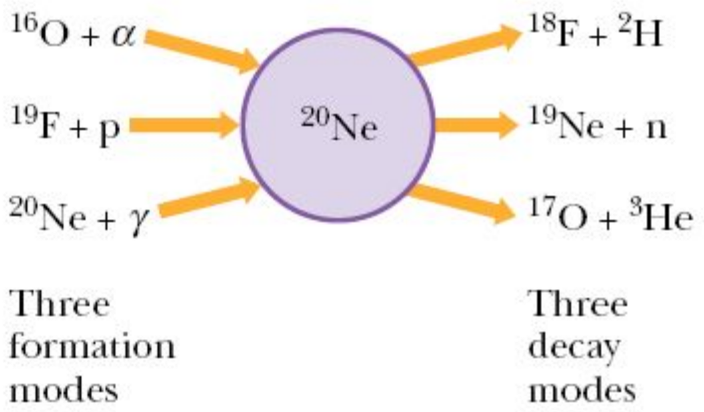


Fig. 42-14 The formation modes and the decay modes of the compound nucleus ^{20}Ne.

In the *collective model,* formulated by Niels Bohr, the nucleons, moving around within the nucleus at random, are imagined to interact strongly with each other, like the molecules in a drop of liquid.

A given nucleon collides frequently with other nucleons in the nuclear interior, its mean free path as it moves about being substantially less than the nuclear radius.

This model permits us to correlate many facts about nuclear masses and binding energies; it is useful in explaining nuclear fission. It is also useful for understanding a large class of nuclear reactions.

42.9: Nuclear Models: The Independent Particle Model:

The ***independent particle model*** is based on the assumption that each nucleon remains in a well-defined quantum state within the nucleus and makes hardly any collisions at all!

The nucleus, unlike the atom, has no fixed center of charge; we assume in this model that each nucleon moves in a potential well that is determined by the smeared-out (time-averaged) motions of all the other nucleons.

If two nucleons within the nucleus are to collide, the energy of each of them after the collision must correspond to the energy of an *unoccupied state.* If no such state is available, the collision simply cannot occur.

Some nuclei can show "closed-shell effects" such as the case of electrons in noble gases, associated with certain ***magic nucleon numbers*****:** 2, 8, 20, 28, 50, 82, 126, ...
Any nuclide whose proton number Z or neutron number N has one of these values turns out to have a special stability.

42.9: Nuclear Models: The Combined Model:

Consider a nucleus in which a small number of neutrons (or protons) exist outside a core of closed shells that contains magic numbers of neutrons or protons.

The outside nucleons occupy quantized states in a potential well established by the central core, thus preserving the central feature of the ***independent-particle model***.

These outside nucleons also interact with the core, deforming it and setting up “tidal wave” motions of rotation or vibration within it. These collective motions of the core preserve the central feature of the ***collective model***.

Such a model of nuclear structure thus succeeds in combining the irreconcilable points of view of the collective and independent- particle models. It has been remarkably successful in explaining observed nuclear properties.

Example, Lifetime of a compound nucleus made by neutron capture:

Consider the neutron capture reaction

$$^{109}\mathrm{Ag} + \mathrm{n} \rightarrow {}^{110}\mathrm{Ag} \rightarrow {}^{110}\mathrm{Ag} + \gamma, \qquad (42\text{-}35)$$

in which a compound nucleus (^{110}Ag) is formed. Figure 42-15 shows the relative rate at which such events take place, plotted against the energy of the incoming neutron. Find the mean lifetime of this compound nucleus by using the uncertainty principle in the form

$$\Delta E \cdot \Delta t \approx \hbar. \qquad (42\text{-}36)$$

Here ΔE is a measure of the uncertainty with which the energy of a state can be defined. The quantity Δt is a measure of the time available to measure this energy. In fact, here Δt is just t_{avg}, the average life of the compound nucleus before it decays to its ground state.

Reasoning: We see that the relative reaction rate peaks sharply at a neutron energy of about 5.2 eV. This suggests that we are dealing with a single excited energy level of the compound nucleus ^{110}Ag. When the available energy (of the incoming neutron) just matches the energy of this level above the ^{110}Ag ground state, we have "resonance" and the reaction of Eq. 42-35 really "goes."

However, the resonance peak is not infinitely sharp but has an approximate half-width (ΔE in the figure) of about 0.20 eV. We can account for this resonance-peak width by saying that the excited level is not sharply defined in energy but has an energy uncertainty ΔE of about 0.20 eV.

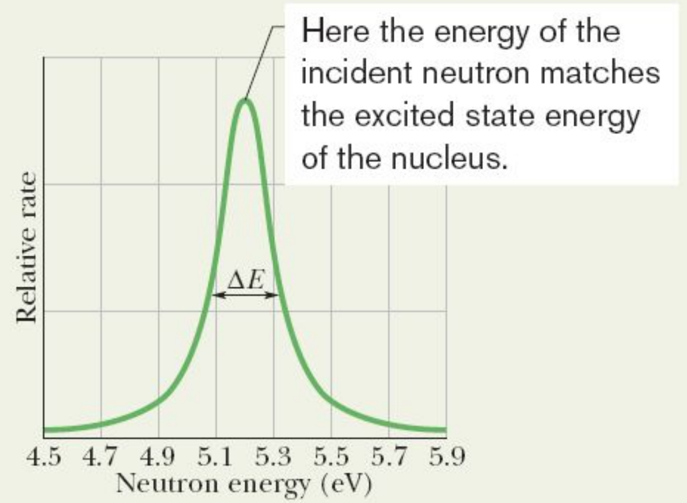


Fig. 42-15 A plot of the relative number of reaction events of the type described by Eq. 42-35 as a function of the energy of the incident neutron. The half-width ΔE of the resonance peak is about 0.20 eV.

Calculation: Substituting that uncertainty of 0.20 eV into Eq. 42-36 gives us

$$\Delta t = t_{avg} \approx \frac{\hbar}{\Delta E} \approx \frac{(4.14 \times 10^{-15}\ \mathrm{eV \cdot s})/2\pi}{0.20\ \mathrm{eV}}$$

$$\approx 3 \times 10^{-15}\ \mathrm{s}. \qquad \text{(Answer)}$$

This is several hundred times greater than the time a 0.20 eV neutron takes to cross the diameter of a ^{109}Ag nucleus. Therefore, the neutron is spending this time of 3×10^{-15} s *as part of* the nucleus.