Chapter 31

Electromagnetic Oscillations and Alternating Current

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31-1 Electromagnetic Oscillations

Learning Objectives

31.01 Sketch an LC oscillator and explain which quantities oscillate and what constitutes one period of the oscillation.

31.02 For an LC oscillator, sketch graphs of the potential difference across the capacitor and the current through the inductor as functions of time, and indicate the period T on each graph.

31.03 Explain the analogy between a block–spring oscillator and an LC oscillator.

31.04 For an LC oscillator, apply the relationships between the angular frequency ω (and the related frequency f and period T) and the values of the inductance and capacitance.

31.05 Starting with the energy of a block–spring system, explain the derivation of the differential equation for charge q in an LC oscillator and then identify the solution for q(t).

31.06 For an LC oscillator, calculate the charge q on the capacitor for any given time and identify the amplitude Q of the charge oscillations.

31-1 Electromagnetic Oscillations

Learning Objectives

31.07 Starting from the equation giving the charge *q(t)* on the capacitor in an LC oscillator, find the current *i(t)* in the inductor as a function of time.

31.08 For an LC oscillator, calculate the current *i* in the inductor for any given time and identify the amplitude *I* of the current oscillations.

31.09 For an LC oscillator, apply the relationship between the charge amplitude *Q*, the current amplitude *I*, and the angular frequency *ω*.

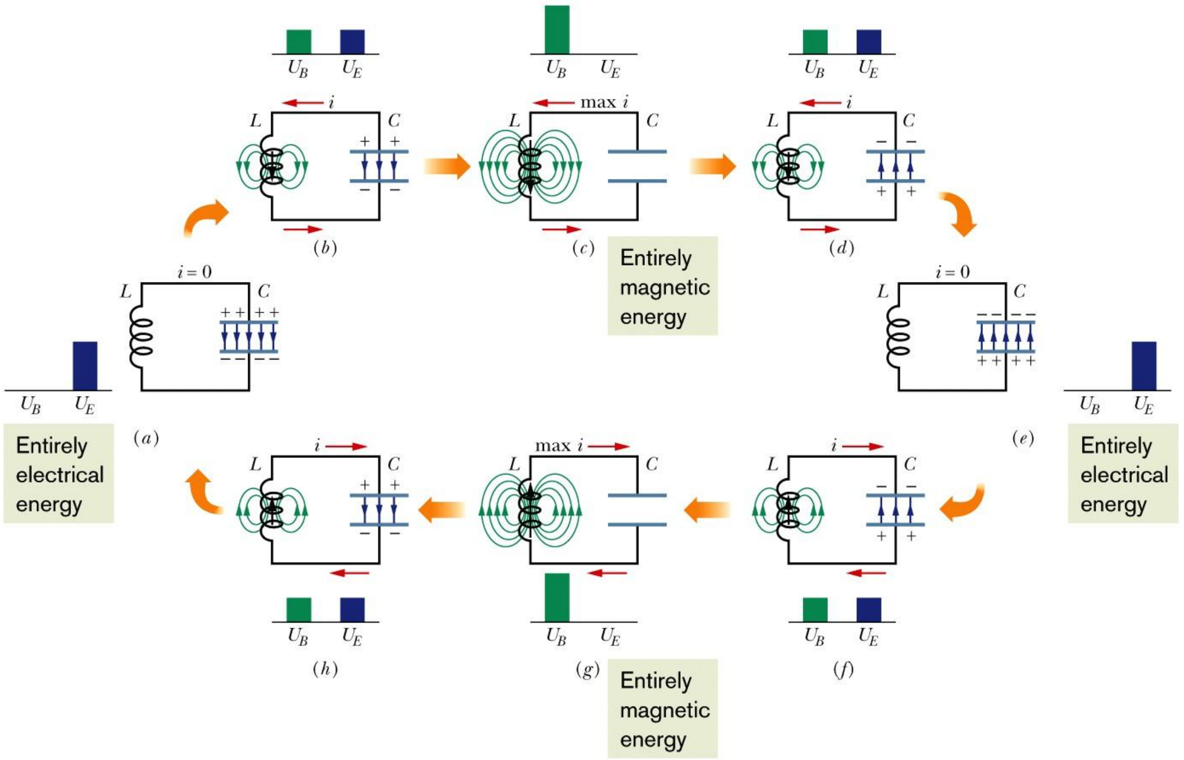
31.10 From the expressions for the charge *q* and the current *i* in an LC oscillator, find the magnetic field energy U_B(t) and the electric field energy U_E(t) and the total energy.

31.11 For an LC oscillator, sketch graphs of the magnetic field energy U_B(t), the electric field energy U_E(t), and the total energy, all as functions of time

31.12 Calculate the maximum values of the magnetic field energy U_B and the electric field energy U_E and also calculate the total energy..

31-1 Electromagnetic Oscillations

Eight stages in a single cycle of oscillation of a resistance less LC circuit. The bar graphs by each figure show the stored magnetic and electrical energies. The magnetic field lines of the inductor and the electric field lines of the capacitor are shown. (a) Capacitor with maximum charge, no current. (b) Capacitor discharging, current increasing. (c) Capacitor fully discharged, current maximum. (d) Capacitor charging but with polarity opposite that in (a), current decreasing.



(e) Capacitor with maximum charge having polarity opposite that in (a), no current. (f) Capacitor discharging, current increasing with direction opposite that in (b). (g) Capacitor fully discharged, current maximum. (h) Capacitor charging, current decreasing.

31-1 Electromagnetic Oscillations

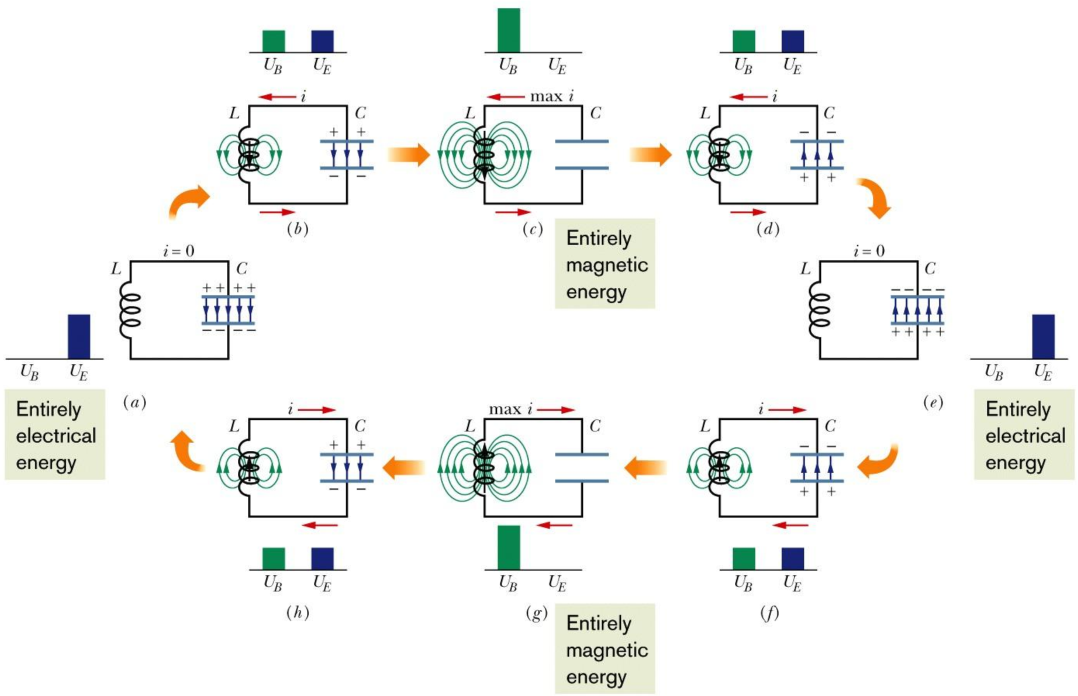
Parts (a) through (h) of the Figure show succeeding stages of the oscillations in a simple LC circuit. The energy stored in the electric field of the **capacitor** at any time is

$$U_E = \frac{q^2}{2C}$$

where q is the charge on the capacitor at that time. The energy stored in the magnetic field of the **inductor** at any time is

$$U_B = \frac{Li^2}{2}$$

where i is the current through the inductor at that time.



The resulting oscillations of the capacitor’s electric field and the inductor’s magnetic field are said to be **electromagnetic oscillations**.

31-1 Electromagnetic Oscillations

Table 31-1 Comparison of the Energy in Two Oscillating Systems

Block–Spring System		*LC* Oscillator	
Element	Energy	Element	Energy
Spring	Potential, $\frac{1}{2}kx^2$	Capacitor	Electrical, $\frac{1}{2}(1/C)q^2$
Block	Kinetic, $\frac{1}{2}mv^2$	Inductor	Magnetic, $\frac{1}{2}Li^2$
	$v = dx/dt$		$i = dq/dt$

From the table we can deduce the correspondence between these systems. Thus,

q corresponds to x, $1/C$ corresponds to k,
i corresponds to v, and L corresponds to m.

The correspondences listed above suggest that to find the angular frequency of oscillation for an ideal (resistanceless) LC circuit, k should be replaced by $1/C$ and m by L, yielding

$$\omega = \frac{1}{\sqrt{LC}} \quad (LC \text{ circuit}).$$

LC Oscillator

The total energy U present at any instant in an oscillating LC circuit is given by

$$U = U_B + U_E = \frac{Li^2}{2} + \frac{q^2}{2C}$$

in which U_B is the energy stored in the magnetic field of the inductor and U_E is the energy stored in the electric field of the capacitor. Since we have assumed the circuit resistance to be zero, no energy is transferred to thermal energy and U remains constant with time. In more formal language, dU/dt must be zero. This leads to

$$\frac{dU}{dt} = \frac{d}{dt}\left(\frac{Li^2}{2} + \frac{q^2}{2C}\right) = Li\frac{di}{dt} + \frac{q}{C}\frac{dq}{dt} = 0.$$

However, $i = dq/dt$ and $di/dt = d^2q/dt^2$. With these substitutions, we get

$$L\frac{d^2q}{dt^2} + \frac{1}{C}q = 0$$

This is the **differential equation** that describes the oscillations of a resistanceless LC circuit.

Charge and Current Oscillation

The solution for the differential equation equation that describes the oscillations of a resistanceless LC circuit is

$$q = Q\cos(\omega t + \phi)$$

where Q is the amplitude of the charge variations, ω is the angular frequency of the electromagnetic oscillations, and ϕ is the phase constant. Taking the first derivative of the above Eq. with respect to time gives us the current:

$$i = \frac{dq}{dt} = -\omega Q\sin(\omega t + \phi)$$

Checkpoint 2

A capacitor in an LC oscillator has a maximum potential difference of 17 V and a maximum energy of 160 μJ. When the capacitor has a potential difference of 5 V and an energy of 10 μJ, what are (a) the emf across the inductor and (b) the energy stored in the magnetic field?

Answer: (a) ε_L= 12 V
(b) U_B=150 μJ

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31-1 Electromagnetic Oscillations

Electrical and Magnetic Energy Oscillations

The electrical energy stored in the LC circuit at time *t* is,

$$U_E = \frac{q^2}{2C} = \frac{Q^2}{2C}\cos^2(\omega t + \phi).$$

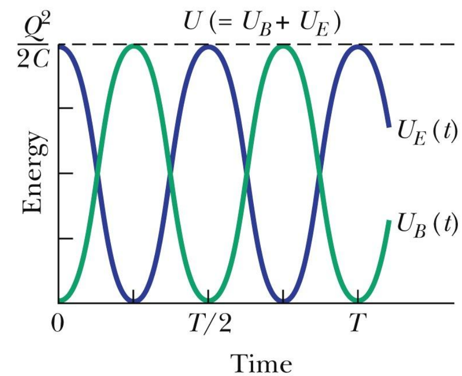
The magnetic energy is,

$$U_B = \frac{Q^2}{2C}\sin^2(\omega t + \phi).$$

Figure shows plots of U_E (t) and U_B (t) for the case of $\phi=0$. Note that

1. The maximum values of U_E and U_B are both $Q^2/2C$.
2. At any instant the sum of U_E and U_B is equal to $Q^2/2C$, a constant.
3. When U_E is maximum, U_B is zero, and conversely.

The electrical and magnetic energies vary but the total is constant.



The stored magnetic energy and electrical energy in the RL circuit as a function of time.

31-2 Damped Oscillation in an RLC circuit

Learning Objectives

31.13 Draw the schematic of a damped RLC circuit and explain why the oscillations are damped.

31.14 Starting with the expressions for the field energies and the rate of energy loss in a damped RLC circuit, write the differential equation for the charge q on the capacitor.

31.15 For a damped RLC circuit, apply the expression for charge q(t).

31.16 Identify that in a damped RLC circuit, the charge amplitude and the amplitude of the electric field energy decrease exponentially with time.

31.17 Apply the relationship between the angular frequency ω' of a given damped RLC oscillator and the angular frequency ω of the circuit if R is removed.

31.18 For a damped RLC circuit, apply the expression for the electric field energy U_E as a function of time.

31-2 Damped Oscillation in an RLC circuit

To analyze the oscillations of this circuit, we write an equation for the total electromagnetic energy U in the circuit at any instant. Because the resistance does not store electromagnetic energy, we can write

$$U = U_B + U_E = \frac{Li^2}{2} + \frac{q^2}{2C}.$$

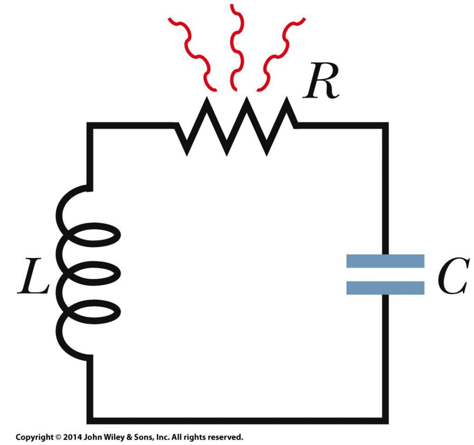
Now, however, this total energy decreases as energy is transferred to thermal energy. The rate of that transfer is,

$$\frac{dU}{dt} = -i^2R,$$

where the minus sign indicates that *U* decreases. By differentiating *U* with respect to time and then substituting the result we eventually get,

$$L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{1}{C}q = 0$$

which is the differential equation for **damped oscillations** in an RLC circuit.



A series RLC circuit. As the charge contained in the circuit oscillates back and forth through the resistance, electromagnetic energy is dissipated as thermal energy, damping (decreasing the amplitude of) the oscillations.

Charge Decay. The solution to above Eq. is $q = Qe^{-Rt/2L}\cos(\omega' t + \phi)$

in which $\omega' = \sqrt{\omega^2 - (R/2L)^2}$ and $\omega = 1/\sqrt{LC}$.

31-3 Forced Oscillations of Three Simple Circuits

Learning Objectives

31.19 Distinguish alternating current from direct current.

31.20 For an ac generator, write the *emf* as a function of time, identifying the *emf* amplitude and driving angular frequency.

31.21 For an ac generator, write the current as a function of time, identifying its amplitude and its phase constant with respect to the *emf*.

31.22 Draw a schematic diagram of a (series) RLC circuit that is driven by a generator.

31.23 Distinguish driving angular frequency ω_d from natural angular frequency ω.

31.24 In a driven (series) RLC circuit, identify the conditions for resonance and the effect of resonance on the current amplitude.

31.25 For each of the three basic circuits (purely resistive load, purely capacitive load, and purely inductive load), draw the circuit and sketch graphs and **phasor diagrams** for voltage *v(t)* and current *i(t)*.

31-3 Forced Oscillations of Three Simple Circuits

Learning Objectives

31.26 For the three basic circuits, apply equations for voltage *v(t)* and current *i(t)*.

31.27 On a phasor diagram for each of the basic circuits, identify angular speed, amplitude, projection on the vertical axis, and rotation angle.

31.28 For each basic circuit, identify the phase constant, and interpret it in terms of the relative orientations of the current phasor and voltage phasor and also in terms of leading and lagging.

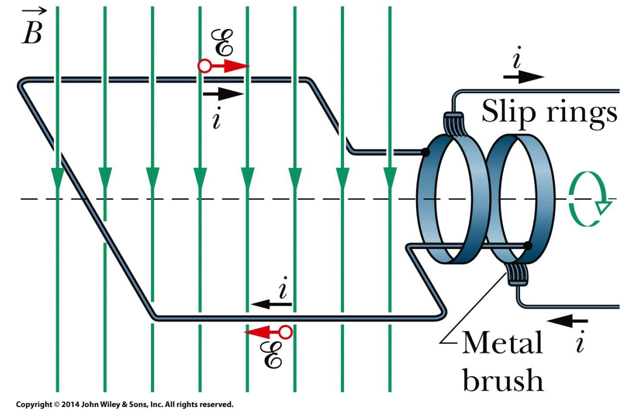
31.29 Apply the mnemonic "ELI positively is the ICE man."

31.30 For each basic circuit, apply the relationships between the voltage amplitude V and the current amplitude I.

31.31 Calculate capacitive reactance X_C and inductive reactance X_L.

31-3 Forced Oscillations of Three Simple Circuits

Why ac? The basic advantage of alternating current is this: As the current alternates, so does the magnetic field that surrounds the conductor. This makes possible the use of Faraday's law of induction, which, among other things, means that we can step up (increase) or step down (decrease) the magnitude of an alternating potential difference at will, using a device called a transformer, as we shall discuss later. Moreover, alternating current is more readily adaptable to rotating machinery such as generators and motors than is (nonalternating) direct current.



The basic mechanism of an alternating-current generator is a conducting loop rotated in an external magnetic field. In practice, the alternating emf induced in a coil of many turns of wire is made accessible by means of slip rings attached to the rotating loop. Each ring is connected to one end of the loop wire and is electrically connected to the rest of the generator circuit by a conducting brush against which the ring slips as the loop (and ring) rotates.

Forced Oscillations

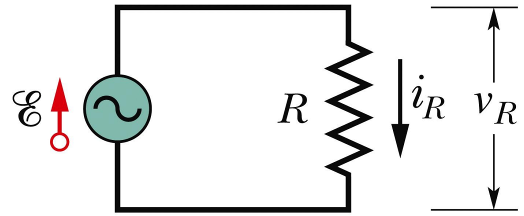
> Whatever the natural angular frequency ω of a circuit may be, forced oscillations of charge, current, and potential difference in the circuit always occur at the driving angular frequency ω_d.

31-3 Forced Oscillations of Three Simple Circuits

Resistive Load

The alternating potential difference across a resistor has amplitude

$$V_R = I_R R \quad \text{(resistor)}.$$



where V_R and I_R are the amplitudes of alternating current i_R and alternating potential difference v_r across the resistance in the circuit.

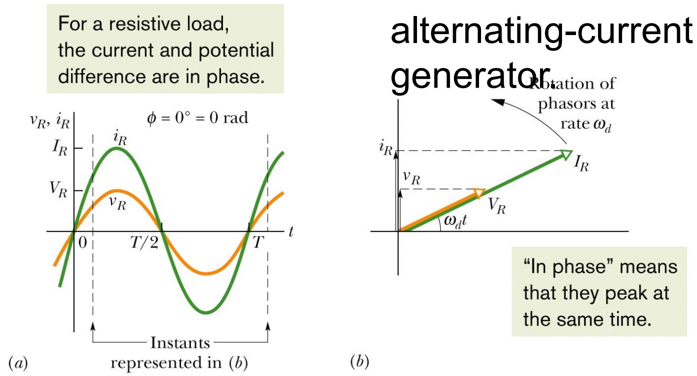
A resistor is connected across an alternating-current generator.

Angular speed: Both current and potential difference phasors rotate counterclockwise about the origin with an angular speed equal to the angular frequency ω_d of v_R and i_R.

Length: The length of each phasor represents the amplitude of the alternating quantity: V_R for the voltage and I_R for the current.

Projection: The projection of each phasor on the vertical axis represents the value of the alternating quantity at time t: v_R for the voltage and i_R for the current.

Rotation angle: The rotation angle of each phasor is equal to the phase of the alternating quantity at time t.



(a) The current i_R and the potential difference v_R across the resistor are plotted on the same graph, both versus time t. They are in phase and complete one cycle in one period T. (b) A phasor diagram shows the same thing as (a).

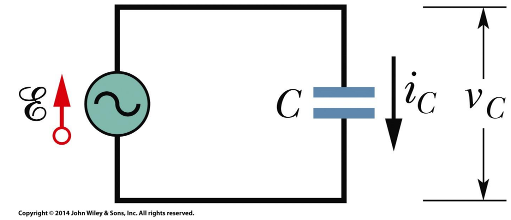
Inductive Load

The **inductive reactance** of an inductor is defined as

$$X_L = \omega_d L$$

Its value depends not only on the inductance but also on the driving angular frequency ω_d.

The voltage amplitude and current amplitude are related by



A capacitor is connected across an alternating-current generator.

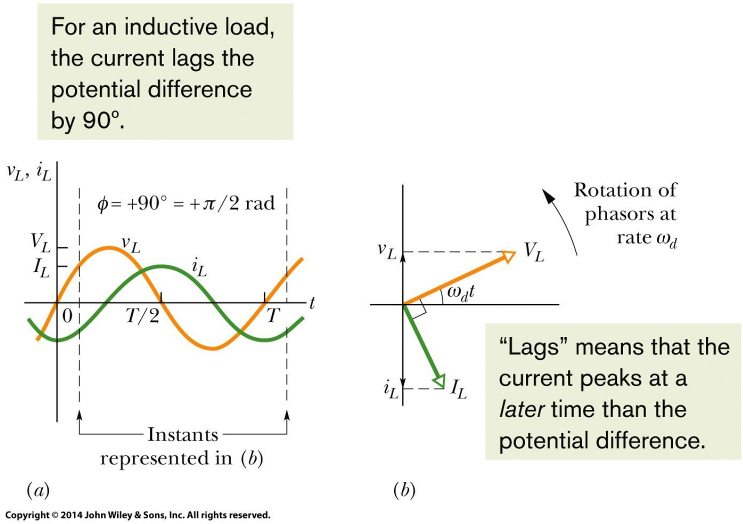


Fig. (left), shows that the quantities i_L and v_L are 90° out of phase. In this case, however, i_L lags v_L; that is, monitoring the current i_L and the potential difference v_L in the circuit of Fig. (top) shows that i_L reaches its maximum value after v_L does, by one-quarter cycle.

(a)The current in the capacitor leads the voltage by 90° (= π/2 rad). (b) A phasor diagram shows the same thing.

31-3 Forced Oscillations of Three Simple Circuits

Capacitive Load

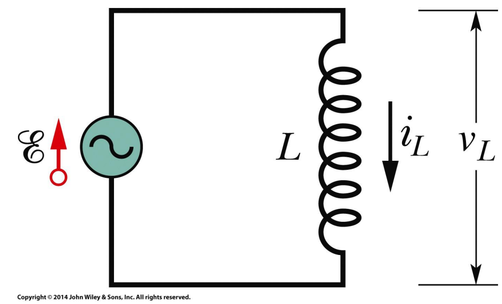
The **capacitive reactance** of a capacitor, defined as

$$X_C = \frac{1}{\omega_d C}$$

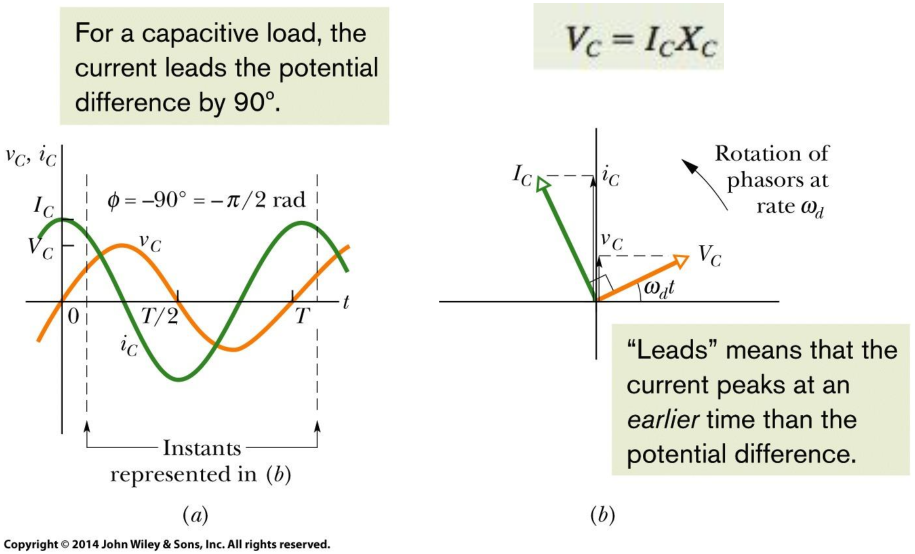
Its value depends not only on the capacitance but also on the driving angular frequency ω_d.

The voltage amplitude and current amplitude are related by

$$V_C = I_C X_C$$



An inductor is connected across an alternating-current generator.



(a)The current in the capacitor lags the voltage by 90° (= π/2 rad). (b) A phasor diagram shows the same thing.

In the phasor diagram we see that i_C leads v_C, which means that, if you monitored the current i_C and the potential difference v_C in the circuit above, you would find that i_C reaches its maximum before v_C does, by one-quarter cycle.

31-4 The Series RLC Circuits

Learning Objectives

31.32 Draw the schematic diagram of a series RLC circuit.

31.33 Identify the conditions for a mainly inductive circuit, a mainly capacitive circuit, and a resonant circuit.

31.34 For a mainly inductive circuit, a mainly capacitive circuit, and a resonant circuit, sketch graphs for voltage *v(t)* and current *i(t)* and sketch phasor diagrams, indicating leading, lagging, or resonance.

31.35 Calculate impedance Z.

31.36 Apply the relationship between current amplitude I, impedance Z, and emf amplitude.

31.37 Apply the relationships between phase constant ϕ and voltage amplitudes V_L and V_C, and also between phase constant ϕ, resistance R, and reactances X_L and X_C.

31.38 Identify the values of the phase constant ϕ corresponding to a mainly inductive circuit, a mainly capacitive circuit, and a resonant circuit.

31-4 The Series RLC Circuits

Learning Objectives

31.39 For resonance, apply the relationship between the driving angular frequency ω_d, the natural angular frequency ω, the inductance L, and the capacitance C.

31.40 Sketch a graph of current amplitude versus the ratio ω_d/ω, identifying the portions corresponding to a mainly inductive circuit, a mainly capacitive circuit, and a resonant circuit and indicating what happens to the curve for an increase in the resistance.

31-4 The Series RLC Circuit

For a series RLC circuit with an external emf given by

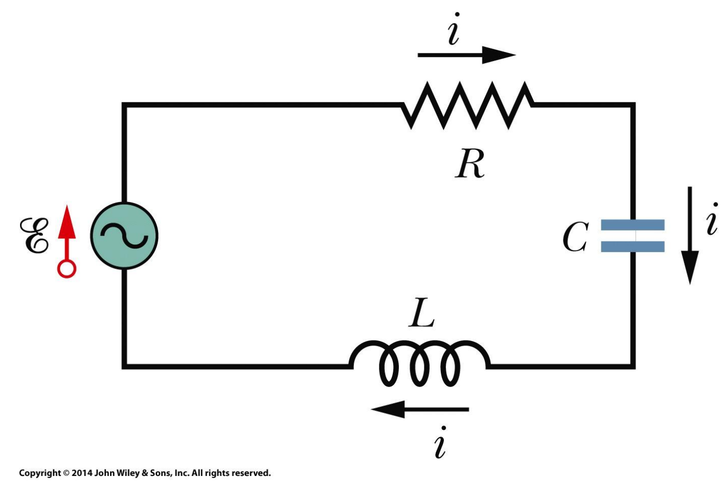
$$\mathcal{E} = \mathcal{E}_m \sin \omega_d t$$

The current is given by

$$i = I \sin(\omega_d t - \phi)$$

the current amplitude is given by

$$I = \frac{\mathcal{E}_m}{\sqrt{R^2 + (X_L - X_C)^2}}.$$



Series RLC circuit with an external emf

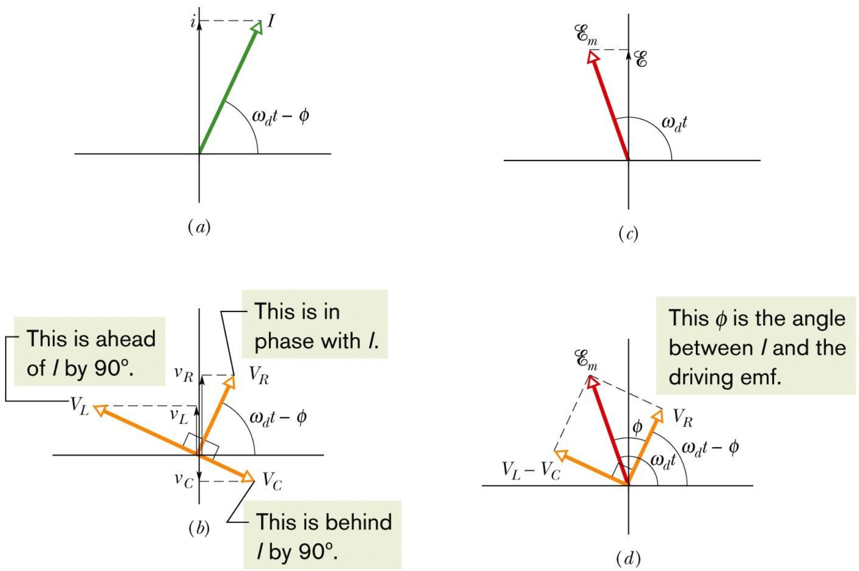
The denominator in the above equation is called the impedance *Z* of the circuit for the driving angular frequency ω_d.

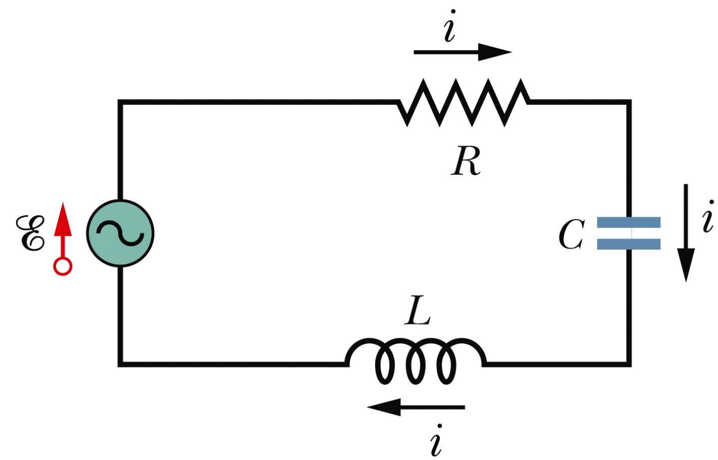
$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

If we substitute the value of X_L and X_C in the equation for current (I), the equation becomes:

$$I = \frac{\mathcal{E}_m}{\sqrt{R^2 + (\omega_d L - 1/\omega_d C)^2}}$$

31-4 The Series RLC Circuits





Series RLC circuit with an external emf

From the right-hand phasor triangle in Fig.(d) we can write

$$\tan\phi = \frac{V_L - V_C}{V_R} = \frac{IX_L - IX_C}{IR}, \quad \Rightarrow \quad \tan\phi = \frac{X_L - X_C}{R}$$

Phase Constant

The current amplitude I is maximum when the driving angular frequency ω_d equals the natural angular frequency ω of the circuit, a condition known as **resonance**. Then $X_C = X_L$, $\phi = 0$, and the current is in phase with the emf.

$$\omega_d = \omega = \frac{1}{\sqrt{LC}} \quad \text{(resonance)}.$$

31-5 Power in Alternating-Current Circuits

Learning Objectives

31.41 For the current, voltage, and emf in an ac circuit, apply the relationship between the rms values and the amplitudes.

31.42 For an alternating emf connected across a capacitor, an inductor, or a resistor, sketch graphs of the sinusoidal variation of the current and voltage and indicate the peak and rms values.

31.43 Apply the relationship between average power P_{avg}, rms current I_{rms}, and resistance R.

31.44 In a driven RLC circuit, calculate the power dissipated by each element.

31.45 For a driven RLC circuit in steady state, explain what happens to (a) the value of the average stored energy with time and (b) the energy that the generator puts into the circuit.

31.46 Apply the relationship between the power factor $\cos\phi$, the resistance R, and the impedance Z.

31-5 Power in Alternating-Current Circuits

The instantaneous rate at which energy is dissipated in the resistor can be written as

$$P = i^2R = [I\sin(\omega_d t - \phi)]^2 R = I^2 R \sin^2(\omega_d t - \phi).$$

Over one complete cycle, the average value of sinθ, where θ is any variable, is zero (Fig.a) but the average value of sin$^2\theta$ is 1/2(Fig.b). Thus the power

$$P_{avg} = \frac{I^2R}{2} = \left(\frac{I}{\sqrt{2}}\right)^2 R.$$

The quantity I/ √2 is called the **root-mean-square**, or rms, value of the current *i*:

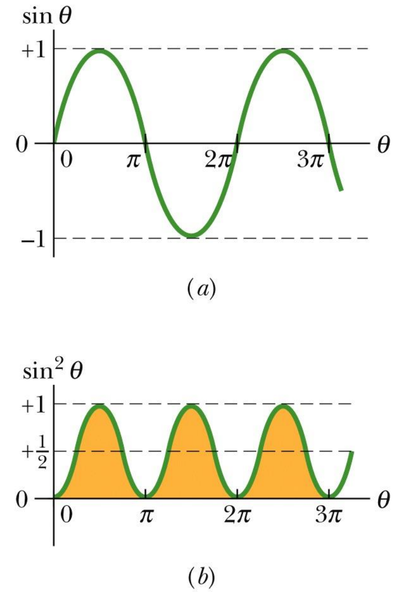
$$I_{rms} = \frac{I}{\sqrt{2}} \Rightarrow P_{avg} = I_{rms}^2 R$$

We can also define rms values of voltages and emfs for alternating-current circuits:

$$V_{rms} = \frac{V}{\sqrt{2}} \quad \text{and} \quad \mathscr{E}_{rms} = \frac{\mathscr{E}_m}{\sqrt{2}}$$

In a series RLC circuit, the average power P_{avg} of the generator is equal to the production rate of thermal energy in the resistor:

$$P_{avg} = \mathscr{E}_{rms} I_{rms} \cos\phi$$



(a) A plot of sinθ versus θ. The average value over one cycle is zero.
(b) A plot of sin$^2\theta$ versus θ . The average value over one cycle is 1/2.

31-6 Transformers

Learning Objectives

31.49 For power transmission lines, identify why the transmission should be at low current and high voltage.

31.50 Identify the role of transformers at the two ends of a transmission line.

31.51 Calculate the energy dissipation in a transmission line.

31.52 Identify a transformer's primary and secondary.

31.53 Apply the relationship between the voltage and number of turns on the two sides of a transformer.

31.54 Distinguish between a step-down transformer and a step-up transformer.

31.55 Apply the relationship between the current and number of turns on the two sides of a transformer.

31.56 Apply the relationship between the power into and out of an ideal transformer.

31-6 Transformers

Learning Objectives

31.57 Identify the equivalent resistance as seen from the primary side of a transformer.

31.58 Apply the relationship between the equivalent resistance and the actual resistance.

31.59 Explain the role of a transformer in impedance matching.

31-6 Transformers

A transformer (assumed to be ideal) is an iron core on which are wound a primary coil of N_p turns and a secondary coil of N_s turns. If the primary coil is connected across an alternating-current generator, the primary and secondary voltages are related by

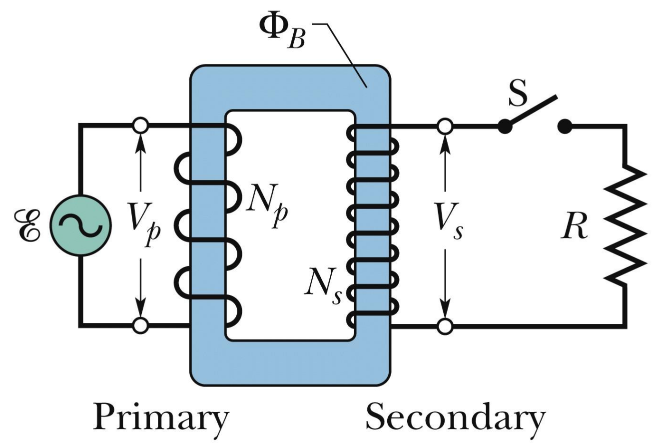
$$V_s = V_p \frac{N_s}{N_p}$$

Energy Transfers. The rate at which the generator transfers energy to the primary is equal to I_pV_p. The rate at which the primary then transfers energy to the secondary (via the alternating magnetic field linking the two coils) is I_sV_s. Because we assume that no energy is lost along the way, conservation of energy requires that

$$I_pV_p = I_sV_s. \Rightarrow I_s = I_p \frac{N_p}{N_s}$$

The equivalent resistance of the secondary circuit, as seen by the generator, is

$$R_{eq} = \left(\frac{N_p}{N_s}\right)^2 R.$$



An ideal transformer (two coils wound on an iron core) in a basic trans- former circuit. An ac generator produces current in the coil at the left (the primary). The coil at the right (the secondary) is connected to the resistive load R when switch S is closed.

31 Summary

LC Energy Transfer

- In an oscillating LC circuit, instantaneous values of the two forms of energy are

$$U_E = \frac{q^2}{2C} \quad \text{and} \quad U_B = \frac{Li^2}{2}$$

Eq. 31-1&2

LC Charge and Current Oscillations

- The principle of conservation of energy leads to

$$L\frac{d^2q}{dt^2} + \frac{1}{C}q = 0$$

Eq. 31-11

- The solution of Eq. 31-11 is

$$q = Q\cos(\omega t + \phi)$$

Eq. 31-12

- the angular frequency v of the oscillations is

$$\omega = \frac{1}{\sqrt{LC}}.$$

Eq. 31-4

Damped Oscillations

- Oscillations in an LC circuit are damped when a dissipative element R is also present in the circuit. Then

$$L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{1}{C}q = 0$$

Eq. 31-24

- The solution of this differential equation is

$$q = Qe^{-Rt/2L}\cos(\omega' t + \phi),$$

Eq. 31-25

Alternating Currents; Forced Oscillations

- A series RLC circuit may be set into forced oscillation at a driving angular frequency by an external alternating emf

$$\mathscr{E} = \mathscr{E}_m \sin \omega_d t.$$

Eq. 31-28

- The current driven in the circuit is

$$i = I\sin(\omega_d t - \phi)$$

Eq. 31-29

31 Summary

Series RLC Circuits

- For a series RLC circuit with an alternating external emf and a resulting alternating current,

$$I = \frac{\mathscr{E}_m}{\sqrt{R^2 + (X_L - X_C)^2}} = \frac{\mathscr{E}_m}{\sqrt{R^2 + (\omega_d L - 1/\omega_d C)^2}}$$

Eq. 31-60&63

- and the phase constant is,

$$\tan \phi = \frac{X_L - X_C}{R}$$

Eq. 31-65

- The impedance is

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

Eq. 31-61

Power

- In a series RLC circuit, the average power of the generator is,

$$P_{avg} = I_{rms}^2 R = \mathscr{E}_{rms} I_{rms} \cos \phi.$$

Eq. 31-71&76

Transformers

- Primary and secondary voltage in a transformer is related by

$$V_s = V_p \frac{N_s}{N_p}$$

Eq. 31-79

- The currents through the coils,

$$I_s = I_p \frac{N_p}{N_s}$$

Eq. 31-80

- The equivalent resistance of the secondary circuit, as seen by the generator, is

$$R_{eq} = \left(\frac{N_p}{N_s}\right)^2 R,$$

Eq. 31-82