

Transmon Qubit 제작의 이해
(2022. Feb. Qiskit Hackathon Korea Metal Challenge 참가자)

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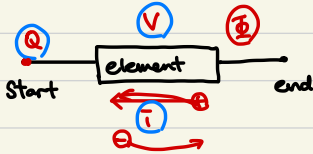
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1. 초전도회 회로의 구성요소 - 커패시터, 인덕터, 저항은 정선

1) 회로 요소의 물리량

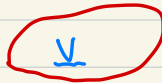


• Q, V, I, Φ

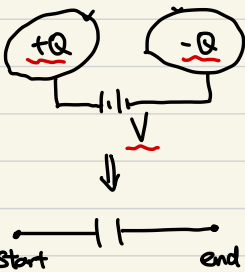
• 전하와 전하량의 관계 $\frac{dQ}{dt} = I$

2) 커패시터 "전하량"

i) 금속의 정전용량 생성



ii) 두 금속 사이의 정전용량

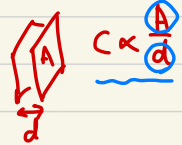


$$Q \propto V \Rightarrow Q = C \cdot V$$

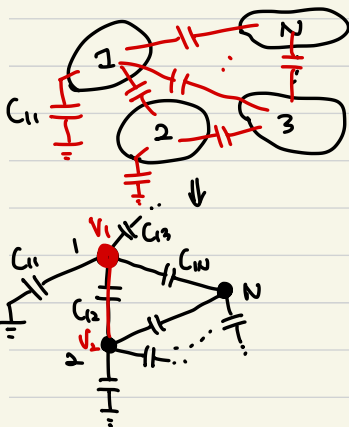
정전용량

→ 커패시터 기하구조

$$E_c = \frac{1}{2} CV^2 = \frac{Q^2}{2C} = \frac{1}{2} QV$$



iii) Maxwell Capacitance Matrix



$$Q_1 = C_{11} \cancel{V_1} + C_{12} \cancel{V_1 - V_2} + \dots + C_{1N} \cancel{V_1 - V_N}$$

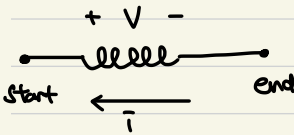
$$= (C_{11} + C_{12} + \dots + C_{1N}) V_1 - C_{12} V_2 - C_{1N} V_N$$

$$\Rightarrow \begin{pmatrix} Q_1 \\ Q_2 \\ \vdots \\ Q_N \end{pmatrix} = \begin{pmatrix} \sum_j C_{1j} & -C_{12} & -C_{13} & \dots & -C_{1N} \\ -C_{21} & \sum_j C_{2j} & -C_{23} & \dots & \vdots \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \sum_j C_{Nj} \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \\ \vdots \\ V_N \end{pmatrix}$$

Maxwell Cap. Matrix.

3) 인덕터의 유도계수

요소를 가하는 법칙.



자기선속 Φ : 어떤 면적을 수직으로 통과하는 자기장선속.

$$\Phi \propto B \propto I \Rightarrow \Phi = L \cdot I$$

유도계수.

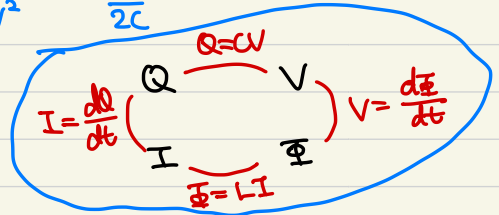
$$Q = C \cdot V$$

전기용량.

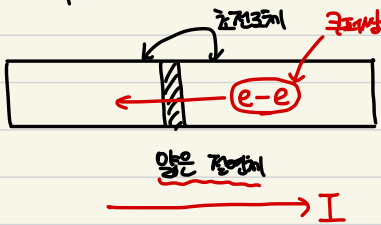
$$E_L = \frac{1}{2L} \Phi^2 = \frac{1}{2} L I^2$$

$$E_L = \frac{1}{2} C V^2 \quad \frac{Q^2}{2C}$$

* 패러데이 법칙 : $V = \frac{\partial \Phi}{\partial t}$



4) Josephson Junction



"두 초전도체 사이의 위상차"

$$I(t) = I_c \cdot \sin \phi$$

$$V(t) = \frac{\hbar}{2e} \frac{\partial \phi}{\partial t} = \frac{\partial}{\partial t} \left(\frac{\hbar}{2e} \phi \right) \Rightarrow V = \frac{\partial \Phi}{\partial t}$$

$$\Rightarrow I = I_c \cdot \sin \left(\frac{2e}{\hbar} \Phi \right), \quad \left[\frac{\hbar}{2e} \right] = \phi_0$$

flux quantum

$$= I_c \cdot \sin \left(2\pi \frac{\Phi}{\phi_0} \right)$$

$$\frac{\partial I}{\partial t} = \frac{\partial I}{\partial \Phi} \cdot \frac{\partial \Phi}{\partial t} = \frac{2\pi I_c}{\phi_0} \cos \left(2\pi \frac{\Phi}{\phi_0} \right) \frac{\partial \Phi}{\partial t} = V$$

$$\therefore L = \frac{\phi_0}{2\pi I_c} \cdot \frac{1}{\cos \left(2\pi \frac{\Phi}{\phi_0} \right)} = \frac{L_J}{\cos \left(2\pi \frac{\Phi}{\phi_0} \right)}$$

$I_c \Rightarrow J.J.$ 의 기하구조.

$\Rightarrow L_J$

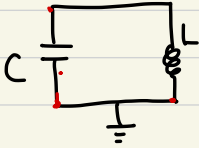
$$E = \left(\frac{\phi_0}{2\pi} \right)^2 \frac{1}{L_J} \left[1 - \cos \left(2\pi \frac{\Phi}{\phi_0} \right) \right]$$

$$I_c = \frac{\phi_0}{2\pi L_J}$$

$$\Rightarrow -E_J \cos \left(2\pi \frac{\Phi}{\phi_0} \right)$$

2 LC 회로 Josephson Junction

1) LC 회로



• 키르히호프 법칙 [전류 보존]

회로에서 전압 $\Rightarrow V_C + V_L = 0$

• 축전기에서 $Q = C V_C = C \frac{d\Phi_C}{dt}$

$\Rightarrow \Phi_C + \Phi_L = 0$

• 인덕터에서 $\Phi_L = L \cdot I$

• $I = \frac{dQ}{dt} \Rightarrow \frac{1}{L} \Phi_L = C \cdot \frac{d^2 \Phi_C}{dt^2} \Rightarrow \frac{d^2 \Phi_C}{dt^2} = -\frac{1}{LC} \Phi_C$

$\Phi = \Phi_C = -\Phi_L$

$$\Rightarrow \frac{d^2 \Phi}{dt^2} = -\frac{1}{LC} \Phi$$

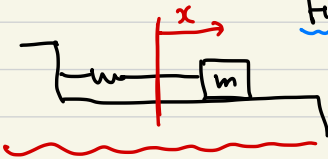
$$\Rightarrow \omega_0 = \frac{1}{\sqrt{LC}} \quad \text{진동}$$

$$\sin(\omega t) \Rightarrow \omega \cos(\omega t)$$

$$\Rightarrow -\omega^2 \sin(\omega t)$$

$$\text{저장된 에너지 } E = \frac{Q^2}{2C} + \frac{\Phi^2}{2L}$$

2) 역학계와 비교.



$$F = -kx = ma = m \frac{d^2 x}{dt^2}$$

$$\Rightarrow \frac{d^2 x}{dt^2} = -\frac{k}{m} x \quad \Leftrightarrow \quad \frac{d^2 \Phi}{dt^2} = -\frac{1}{LC} \Phi$$

$$p = m \frac{dx}{dt} \quad \Leftrightarrow \quad Q = CV = C \frac{d\Phi}{dt}$$

\Rightarrow 알므로 V, I, Q, Φ 중, Q 와 Φ 를 선택.

$\begin{pmatrix} x \leftrightarrow \Phi \\ p \leftrightarrow Q \\ m \leftrightarrow C \\ k \leftrightarrow L^{-1} \end{pmatrix}$

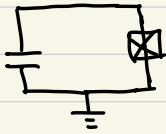
$$E = \frac{p^2}{2m} + \frac{1}{2} k x^2 \quad \Leftrightarrow \quad E = \frac{Q^2}{2C} + \frac{\Phi^2}{2L}$$

운동에너지

위치에너지



3) J.J. 가 포함된 회로.



위와 같이 $V_C + V_J = 0$

축전기에서 $Q = CV_C = C \cdot \frac{d\Phi_C}{dt} \Rightarrow I = C \frac{d^2\Phi_C}{dt^2}$

J.J.에서 $I = I_C \sin\left(2\pi \frac{\Phi}{\Phi_0}\right)$

$\therefore C \frac{d^2\Phi_C}{dt^2} = I_C \sin\left(2\pi \frac{\Phi}{\Phi_0}\right) = -I_C \sin\left(2\pi \frac{\Phi}{\Phi_0}\right)$, $\Phi = \Phi_C$

$\Rightarrow \frac{d^2\Phi}{dt^2} = -\frac{I_C}{C} \sin\left(2\pi \frac{\Phi}{\Phi_0}\right) = -\frac{\Phi_0}{2\pi L_J C} \sin\left(2\pi \frac{\Phi}{\Phi_0}\right)$

$E = \frac{Q^2}{2C} - E_J \cos\left(2\pi \frac{\Phi}{\Phi_0}\right)$

이동에너지

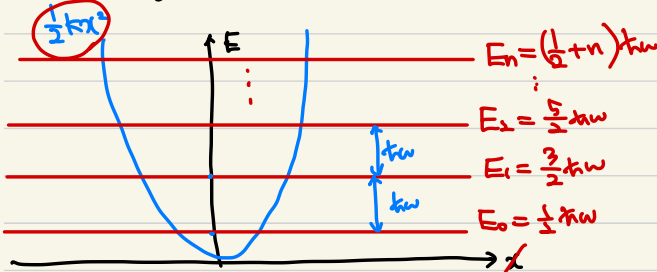
위치에너지

4) Quantum Harmonic Oscillator

* 일반적으로 물리량 Q 의 양자화를 통해 전제하는 것이 엄밀한 전개.

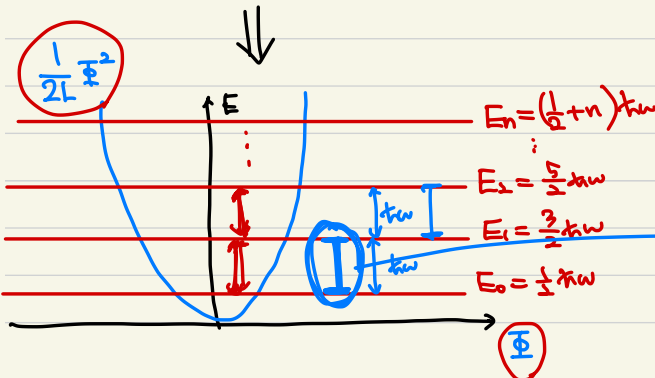
본 양자에서는 논쟁의 여지가 있음에 유의!

$-\frac{\hbar^2}{2m} \frac{d^2\psi}{dx^2} + V(x)\psi = E\psi$ (슈뢰딩거 방정식)



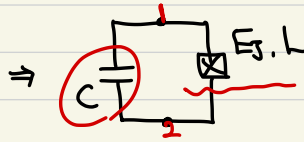
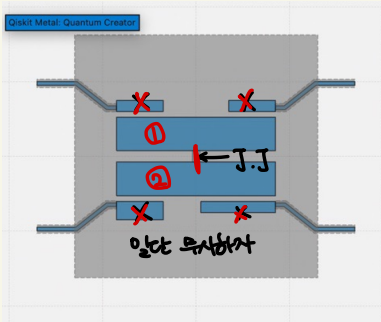
\Rightarrow LC 회로나 같은 응용방정식은
가거므로, LC 회로도
같은 방법으로 에너지가
양자화 될 것이다!

* 대신 위치에서의 파동함수가
아니라, Φ 에 대한 파동함수
임에 주의하자.



주의!

3. Transmon Qubit

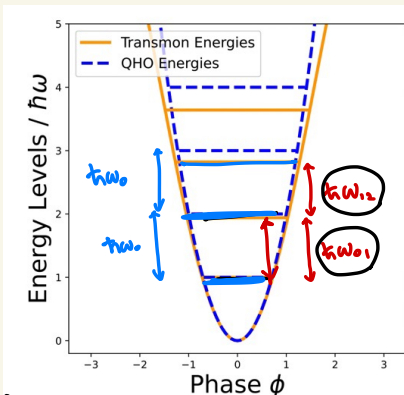
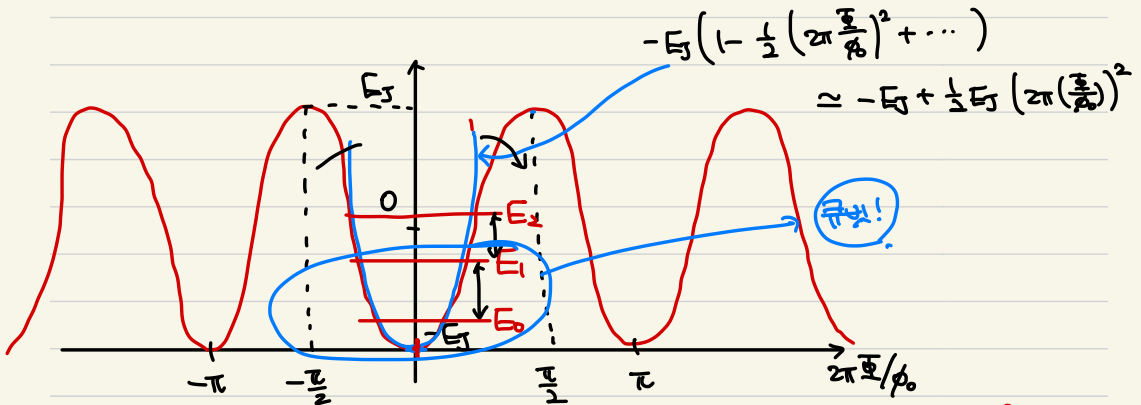


total charge
 $\tilde{Q} = 2e \cdot n$ ← number of cooper pair
 charge of cooper pair

$$Rmk. E = \frac{Q^2}{2C} - E_J \cos\left(2\pi \frac{\Phi}{\Phi_0}\right) = 4 \cdot \frac{e^2}{2C} n^2 - E_J \cos\left(2\pi \frac{\Phi}{\Phi_0}\right)$$

$\frac{e^2}{2C}$ ← E_C
 E_C

$$= 4E_C n^2 - E_J \cos\left(2\pi \frac{\Phi}{\Phi_0}\right)$$



In transmon qubit, $\omega_0 \equiv \omega_0$, $\omega_1 \equiv \omega_0 + \alpha$
 ← qubit frequency
 ← anharmonicity

근사적이지 (transmon dynamics 를 무시할 때),
 $\omega_q = \omega_0 \approx \sqrt{8 E_C E_J}$, $\alpha \approx - \frac{E_C}{E_J}$ ← $\alpha_{rel} = \frac{\alpha}{\omega_0} \sim \sqrt{\frac{E_C}{E_J}}$
 ← important!

⇒ 목표하는 qubit freq, anharmonicity 이 맞추어

E_C 를 수정해야 함. $\Rightarrow E_C = \frac{e^2}{2C}$

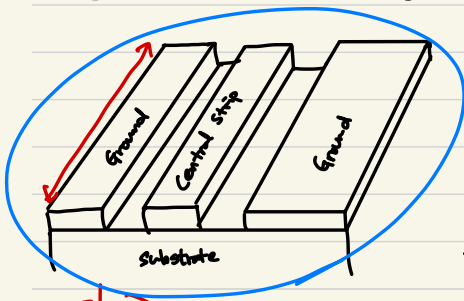
⇒ capacitance 를 바꾸는 것이 가장 효과적

(Ref: Qiskit textbook, "Introduction to Transmon physics")

4. Resonator

Quicker Metal 이서 가장 기본적으로 사용하는 resonator / transmission line 은 CO-planar waveguide (CPW) 이다.

✓ 길이 결정 \Rightarrow 공명진동수 결정



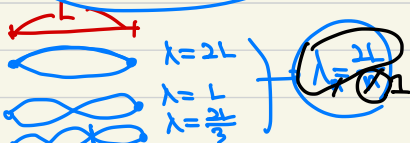
$$\Rightarrow C = \frac{1}{2} C_e \cdot l, \quad C_e = \text{단위길이당 전기용량}$$

$$L_n = \frac{2L_e}{\pi^2} l, \quad L_e = \text{단위길이당 무극계수}$$

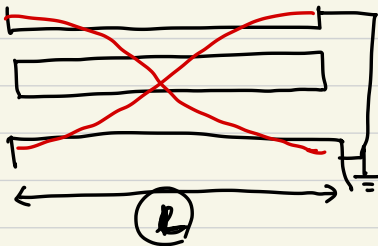
n : 정음 보정

$$\Rightarrow \omega_n = \frac{1}{\sqrt{L_n C}} = \frac{1}{\sqrt{\frac{C_e L_e}{\pi^2} l^2}}$$

$$= \frac{1}{\sqrt{L_e C_e}} \frac{\pi}{l} \propto \frac{1}{l}$$



이 결과를 더 쉽게 생각할 수 있다.



다음과 같은 CPW에서, 정상파가 생기는 것을 생각하면, $\lambda = 2L$ 이어야 한다. "위상따로 균등"

CPW에서 전자기파의 속도 (무상속도) 를 v_p 라 하면, 파동에서 $v_p = f \cdot \lambda$ 이서,

$$f = \frac{v_p}{2L} \text{ 이을 알 수 있다.}$$

$$\rightarrow v_p = \frac{\lambda}{T}$$

이를 공항하면, $\omega = 2\pi f$ 이서,

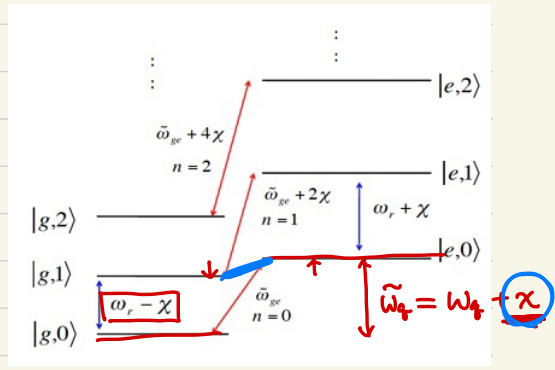
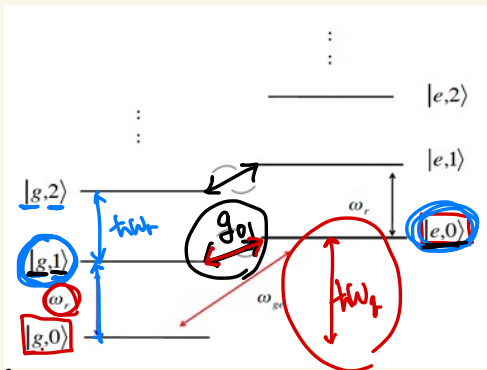
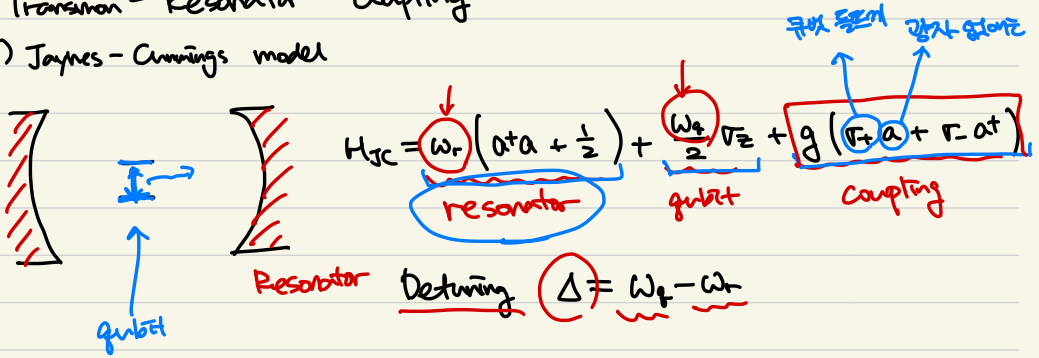
$$\omega = \frac{\pi}{\sqrt{L_e C_e}} \frac{1}{l} = \pi v_p \cdot \frac{1}{l}$$

이을 알 수 있다. 일반적으로 $v_p = \frac{1}{\sqrt{L_e C_e}}$ 은 기하적 구조에 매질의 특성이 따라 달라진다.

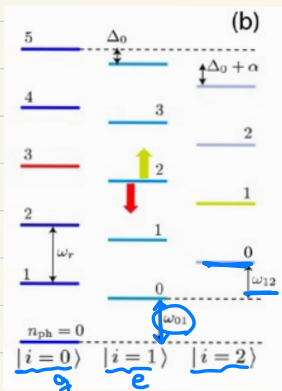
여기서 CPW의 resonant frequency를 투상하는 법을 알 수 있는데, v_p 가 상수가 하면, $\omega \propto \frac{1}{l}$ 은 이용해 CPW의 길이를 조정해서 ω 를 조절할 수 있음을 알 수 있다.

5. Transmon - Resonator Coupling

1) Jaynes - Cummings model

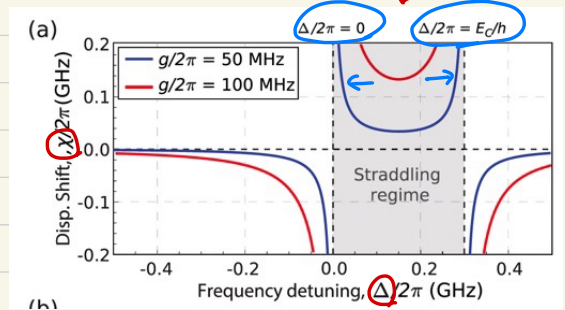


(Baladitya Suri PhD thesis)



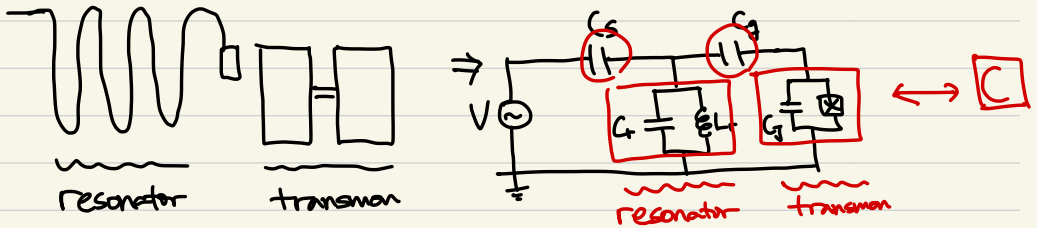
(Koch, 2007)

$$\chi = - \frac{g_{01}^2}{\Delta} \left(\frac{1}{1 + \Delta/\alpha} \right) \sim \frac{-g_{01}^2}{\Delta (1 + \Delta/\alpha)}$$



(P. Krantz, et al., 2019, Appl. Phys. Rev.)

* Capacitance Model for estimating g



$$\mathcal{H} = \underbrace{\frac{Q_r^2}{2C_{res}} + \frac{\Phi_r^2}{2L_r}}_{\text{Resonator}} + \underbrace{\frac{Q_J^2}{2C_\Sigma} - E_J \cos\left(2\pi \frac{\phi_J}{\Phi_0}\right)}_{\text{Transmon}} + \underbrace{\beta_{rJ} Q_r Q_J}_{\text{coupling}} + \underbrace{\beta_r V Q_r + \beta_J V Q_J}_{\text{drive terms}} \quad (2.125)$$

where

$$C_{res} = C_r + C_s + \frac{C_J C_g}{C_J + C_g} \quad (2.126)$$

$$C_\Sigma = C_J + \frac{C_g(C_s + C_r)}{C_g + C_s + C_r} \quad (2.127)$$

$$\beta_r = \frac{C_s(C_b + C_g)}{C_b(C_g + C_r + C_s) + C_g(C_r + C_s)} \quad (2.128)$$

$$\beta_J = \frac{C_g C_s}{C_b(C_g + C_r + C_s) + C_g(C_r + C_s)} \quad (2.129)$$

$$V \beta_{rJ} = \frac{C_g}{C_b(C_g + C_r + C_s) + C_g(C_r + C_s)} \quad (2.130)$$

(Ref: Baladitya Suri Ph.D. Thesis)

$$\omega_r = \frac{1}{\sqrt{L_r C_{res}}} \quad (2.132)$$

$$E_c = \frac{e^2}{2C_\Sigma} \quad (2.133)$$

$$\hbar \omega_J = \sqrt{8E_J E_c} - E_c \quad (2.134)$$

$$g = \frac{e \beta_{rJ}}{\sqrt{\hbar Z_r}} \left(\frac{2E_J}{E_c} \right)^{1/4} \quad (2.135)$$

← resonator-transmon coupling strength

$$\Omega_J = \frac{e}{\hbar} \beta_J V_0 \left(\frac{8E_J}{E_c} \right)^{1/4} \quad (2.136)$$

$$\Omega_r = \frac{\beta_r V_0}{\sqrt{2\hbar Z_r}} \quad (2.137)$$

Transmon n/m

$$\frac{E_J}{E_c} \sim 50$$

