Université Grenoble Alpes
MAP350 – L3 MI

TP5 : Interpolation paramétrique

Given a sequence of points $M_i = (x_i, y_i)$, $0 \leq i \leq n$, we want to construct a polynomial parametric curve of degree n (because of the $n+1$ constraints) which interpolates these data points, that is which goes through each point (x_i, y_i) for some parameter t_i. The main question consists in the choice of the interpolation parameters t_i, which are also called the *interpolation nodes* or simply the *nodes*.

Precisely, we look for a n-degree polynomial parametric curve

$$m : \begin{array}{ccc} [a,b] \in \mathbb{R} & \longrightarrow & \mathbb{R}^2 \\ t & \longmapsto & m(t) = \begin{pmatrix} m_x(t) \\ m_y(t) \end{pmatrix} \end{array}$$

such that

$$m(t_i) = M_i \quad \Leftrightarrow \quad \begin{cases} m_x(t_i) = x_i \\ m_y(t_i) = y_i \end{cases}, \quad 0 \leq i \leq n$$

with a sequence of interpolation nodes $a \leq t_0 < t_1 < \cdots < t_n \leq b$, and where $m_x(t)$ and $m_y(t)$ are polynomials of degree n. As a result, we are reduced to solve two separate Lagrange interpolation problems.

We will consider three choices for the interpolation parameters. The first one is the *uniform parametrization* : parameters t_i are evenly spaced in the parameters domain. The second one is the *Chebyshev parametrization* as introduced in the course on interpolation. The last one is the *chordal parametrization* : parameters are chosen in such a way that distances between successive parameters t_i are proportional to the distances between associate successive data points M_i.

- Uniform parameterization : $t_i = a + i\,\dfrac{b-a}{n}$.
- Chebyshev parameterization : $\hat{t}_i = \dfrac{a+b}{2} + \dfrac{b-a}{2}\,\cos\Big((2i+1)\,\dfrac{\pi}{2n+2}\Big)$.
- Chordal parameterization : $\tilde{t}_{i+1} - \tilde{t}_i = \dfrac{d_i}{\sum_{k=0}^{n-1} d_k}$ with $d_k = \mathrm{dist}(M_k, M_{k+1})$ over $[0,1]$.

Notice that the choice of the parameter domain $[a,b]$ does not affect the method, so that we will consider $[a,b] = [0,1]$.

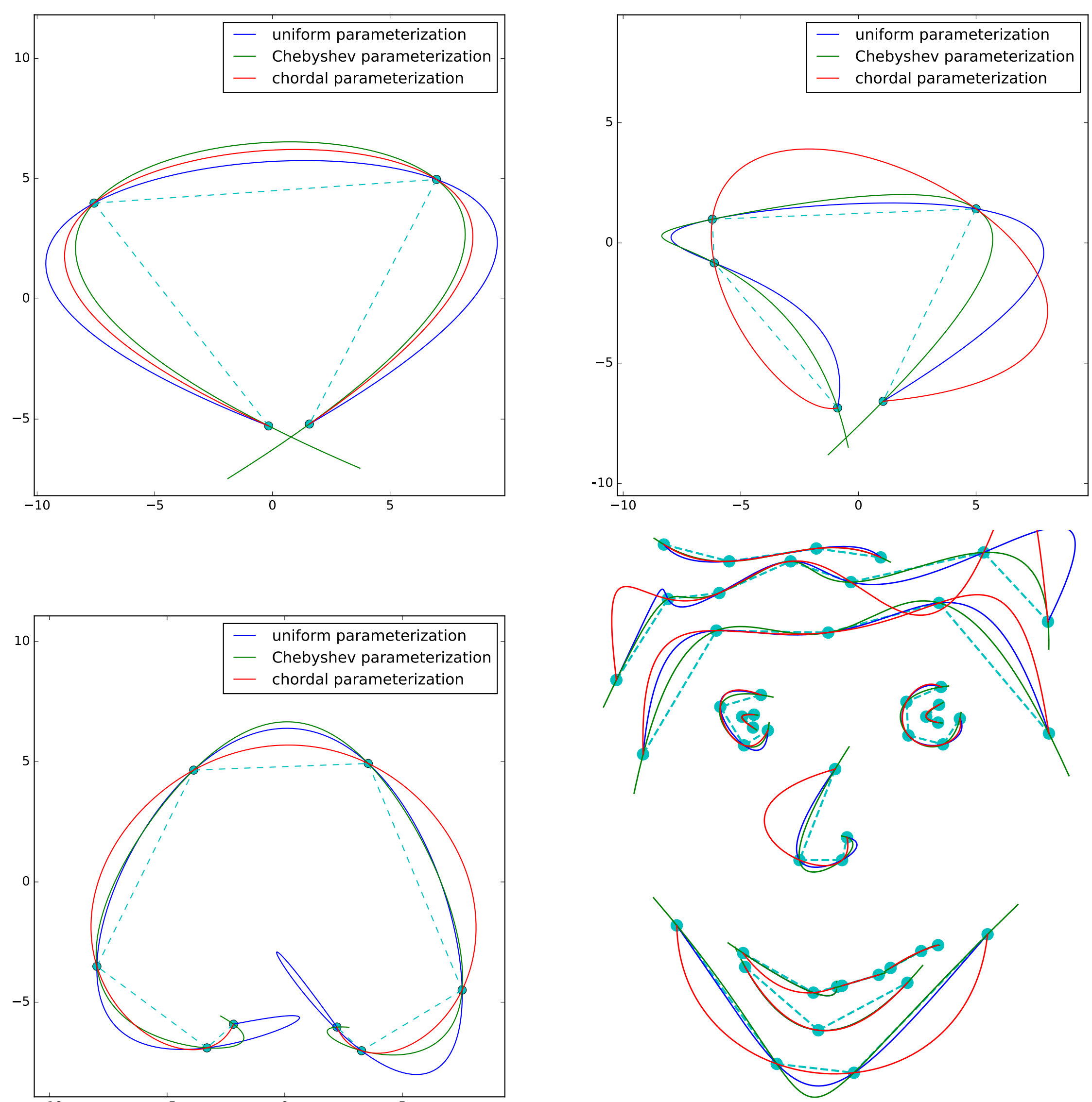
1) Récupérer le script Python “ `TP3parametricInterpolStudents.py` ” permettant d’acquérir à la souris un polygone dans une fenêtre graphique et de récupérer les coordonnées (x_i, y_i) des sommets de ce polygone dans deux vecteurs `xi` et `yi` :

ligne 83 : `xi, yi = PolygonAcquisition('oc','c--')`

Ce script fournit également les procédures permettant de réaliser l’interpolation de Lagrange dans la base de Newton à l’aide des différences divisées et de la méthode de Horner.

2) Compléter ce script afin de réaliser l’interpolation paramétrique des sommets du polygone acquis à la souris. Les 3 choix de points d’interpolation décrits ci-dessus seront implémentés.

2) Expérimeter ce script afin d’obtenir des figures semblables aux figures ci-dessous.



Il n’est pas demandé de compte rendu écrit pour ce TP — Néanmoins il sera testé en séance.