Université Grenoble Alpes
MAP350 – L3 MI

TP6 : Hermite interpolation

• Given two Hermite data **of order one** $(\alpha, y_\alpha, y'_\alpha)$ and $(\beta, y_\beta, y'_\beta)$, where α and β $(\alpha < \beta)$ are two distinct points, there exits a unique (Hermite) cubic polynomial $p(x)$ interpolating these data, i.e., satisfying

$$p(\alpha) = y_\alpha\,, \quad p'(\alpha) = y'_\alpha\,, \quad p'(\beta) = y'_\beta\,, \quad p(\beta) = y_\beta.$$

This cubic Hermite interpolating polynomial $p(x)$ can be written as follows

$$p(x) = y_\alpha\, H_0\Big(\frac{x-\alpha}{\beta-\alpha}\Big) + y'_\alpha\,(\beta-\alpha)\,H_1\Big(\frac{x-\alpha}{\beta-\alpha}\Big) + y'_\beta\,(\beta-\alpha)\,H_2\Big(\frac{x-\alpha}{\beta-\alpha}\Big) + y_\beta\,H_3\Big(\frac{x-\alpha}{\beta-\alpha}\Big) \quad (1)$$

where H_0, H_1, H_2, H_3 are four cubic polynomials forming the *standard cubic Hermite basis on* $[0,1]$ and characterized by the following table.

	H_0	H_1	H_2	H_3
$H_i(0)$	1	0	0	0
$H'_i(0)$	0	1	0	0
$H'_i(1)$	0	0	1	0
$H_i(1)$	0	0	0	1

$$H_0(t) = 1 - 3\,t^2 + 2\,t^3$$
$$H_1(t) = t - 2\,t^2 + t^3$$
$$H_2(t) = -t^2 + t^3$$
$$H_3(t) = 3\,t^2 - 2\,t^3$$

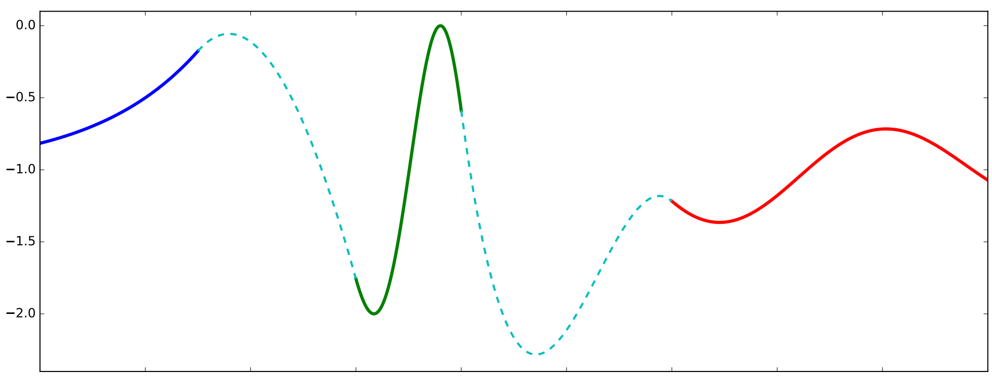
Exercise 1

Consider the three following functions which are plotted below respectively in blue, green and red.

$$f_1(x) = \frac{\exp(x)}{2} - 1, \qquad x \in [-1, 0.5],$$

$$f_2(x) = \sin(x^2) - 1, \qquad x \in [2, 3],$$

$$f_3(x) = -1 + 2\,\frac{\sin(2\,x)}{x}, \qquad x \in [5, 8].$$



Determine two functions p_1 and p_2 respectively defined on intervals $[0.5, 2]$ and $[3, 5]$, and plot all these functions, such that the concatenation of the five functions f_1, p_1, f_2, p_2, f_3 provides a C^1 function over the interval $[-1, 8]$.

• Given two Hermite data **of order two** $(\alpha, y_\alpha, y'_\alpha, y''_\alpha)$ and $(\beta, y_\beta, y'_\beta, y''_\beta)$, where α and β $(\alpha < \beta)$ are two distinct points, there exits a unique (Hermite) quintic polynomial $p(x)$ interpolating these data, i.e., satisfying

$$\begin{array}{lll} p(\alpha) = y_\alpha & p'(\alpha) = y'_\alpha & p''(\alpha) = y''_\alpha \\ p(\beta) = y_\beta & p'(\beta) = y'_\beta & p''(\beta) = y''_\beta \end{array}$$

This quintic Hermite interpolating polynomial $p(x)$ can be written as follows

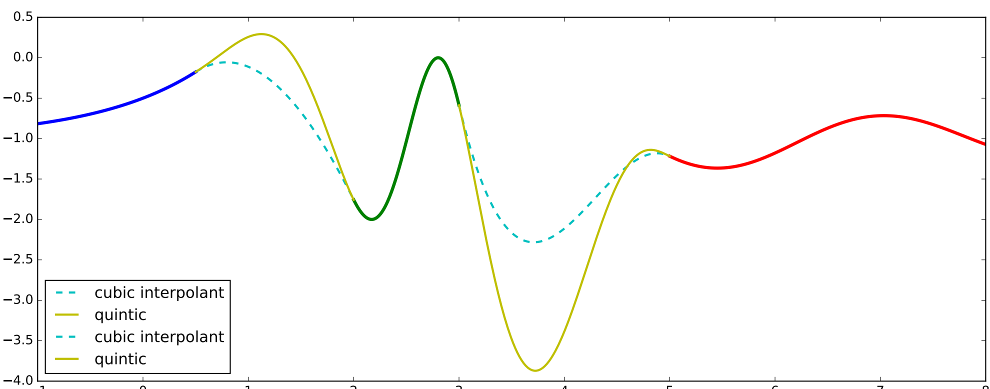
$$p(x) = \Big(Q_0(t), Q_1(t), Q_2(t), Q_3(t), Q_4(t), Q_5(t)\Big) \begin{pmatrix} y_\alpha \\ (\beta-\alpha)\, y'_\alpha \\ (\beta-\alpha)^2\, y''_\alpha \\ (\beta-\alpha)^2\, y''_\beta \\ (\beta-\alpha)\, y'_\beta \\ y_\beta \end{pmatrix} \quad \text{with} \quad t = \frac{x-\alpha}{\beta-\alpha}$$

and with the following *Quintic Hermite basis on* $[0, 1]$

$$\begin{array}{ll} Q_0(x) = -6\,x^5 + 15\,x^4 - 10\,x^3 + 1 & Q_1(x) = -3\,x^5 + 8\,x^4 - 6\,x^3 + x \\ Q_2(x) = \dfrac{1}{2}(-x^5 + 3\,x^4 - 3\,x^3 + x^2) & Q_3(x) = \dfrac{1}{2}(x^5 - 2\,x^4 + x^3) \\ Q_4(x) = -3\,x^5 + 7\,x^4 - 4\,x^3 & Q_5(x) = 6\,x^5 - 15\,x^4 + 10\,x^3 \end{array}$$

Exercise 2

Comparison with the cubic Hermite interpolation.
Consider again the three functions given in exercise 1 and determine the two quintic polynomials $q_1(x)$ *and* $q_2(x)$ *respectively defined on intervals* $[0.5, 2]$ *and* $[3, 5]$*, such that the concatenation of the five functions* f_1, q_1, f_2, q_2, f_3 *provides a* C^2 *function over the interval* $[-1, 8]$.



Comparison between cubic and quintic Hermite interpolation.

Le compte rendu de ce TP consistera en un seul fichier Python dont le nom sera `TP6_NOM1_NOM2.py`
Ce script Python contiendra en entête (et donc en commentaire) les noms NOM1 et NOM2. L'exécution de ce script devra afficher la figure donnée dans l'exercice 2 ci-dessus, et donc réaliser l'interpolation cubique et quintique des données sur un même graphique.