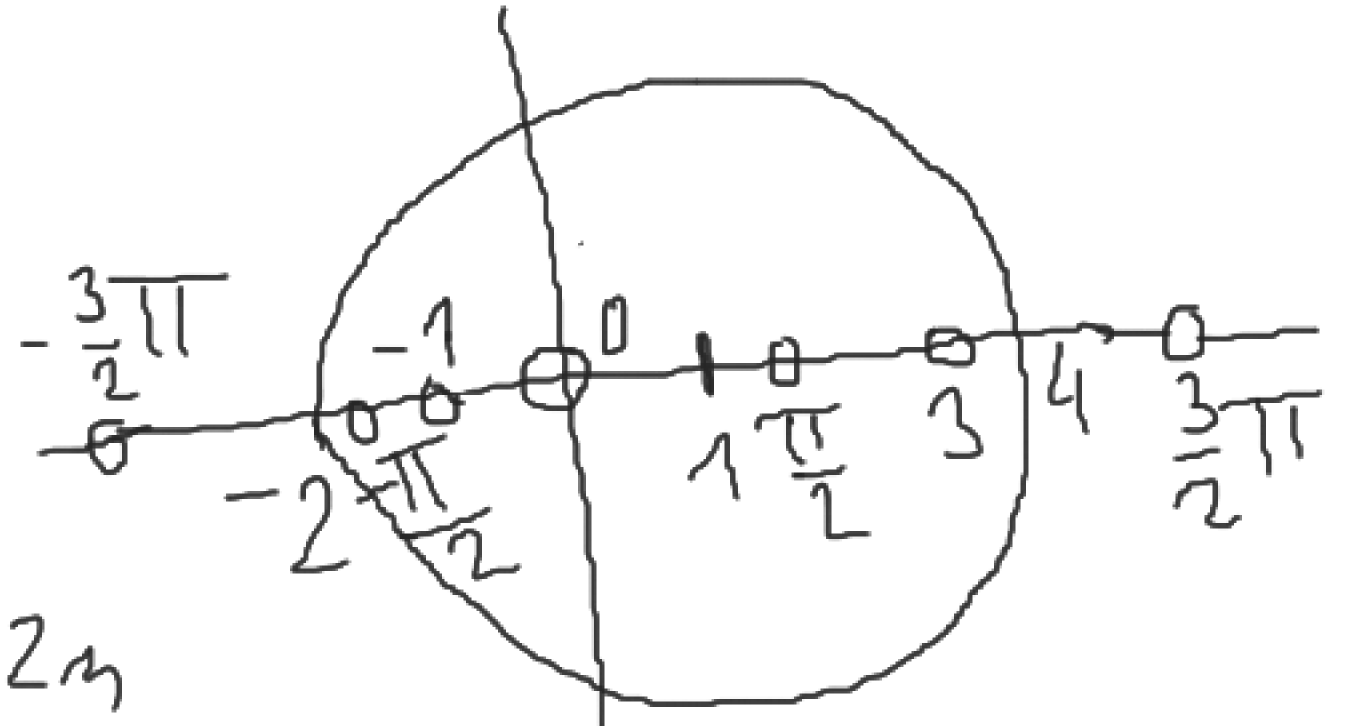
$$\int_C \left((z+1)\cos\left(\frac{4}{z+1}\right) + \frac{\operatorname{tg} z}{z} + \operatorname{tg} z + \frac{2z^3-5z^2}{(z-3)^3} \right) dz \qquad \operatorname{tg} z \to \frac{\sin z}{\cos z}$$

$C: |z-1| = 3$

$z \neq -1 \qquad z \neq 0 \qquad z \neq \frac{\pi}{2} + k\pi,\ k \in \mathbb{Z} \qquad z \neq 3$

$$\cos z = \sum_{n=0}^{\infty} (-1)^n \frac{z^{2n}}{(2n)!}$$

$$(z+1)\cos\left(\frac{4}{z+1}\right) = (z+1)\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!}\left(\frac{4}{z+1}\right)^{2n}$$



$$= (z+1)^1 \sum_{n=0}^{\infty} \frac{(-1)^n 4^{2n}}{(2n)!}(z+1)^{-2n} = \sum_{n=0}^{\infty} \frac{(-1)^n 4^{2n}}{(2n)!}(z+1)^{-2n+1}$$

$$\sum_{n=0}^{\infty} \frac{(-1)^n 4^{2n}}{(2n)!} (z+1)^{-2n+1}$$

c_{-2n+1}

Hlavná časť Laurentovho radu má nekonečne veľa členov

$\Downarrow$

$\boxed{z = -1 \text{ je PODSTATNE singulárny bod}}$

$$-2n+1 = -1$$

$$-2n = -2$$

$$n = 1$$

$$\operatorname{res}_{z=-1} (z+1)\cos\left(\frac{4}{z+1}\right) = c_{-1} = \frac{(-1)^1 4^{2\cdot 1}}{(2\cdot 1)!} = \frac{-1\cdot 16}{2} = -8$$

$$\operatorname{res}_{z=-1} (z+1)\cos\left(\frac{4}{z+1}\right) = c_{-1} = \boxed{-8}$$

$$\lim_{z \to 0} \frac{\operatorname{tg} z}{z} = \lim_{z \to 0} \frac{\frac{\sin z}{\cos z}}{z} = \lim_{z \to 0} \underbrace{\frac{\sin z}{z}}_{1} \cdot \underbrace{\frac{1}{\cos z}}_{1} = 1$$

$\Downarrow$

konečné číslo

$\boxed{z = 0 \text{ je ODSTRÁNITEĽNÝ singulárny bod}}$

$\Downarrow$

Hlavná časť Laurentovho radu má 0 členov

$\Downarrow$

$$\operatorname{res}_{z=0} \frac{\operatorname{tg} z}{z} = c_{-1} = \boxed{0}$$

$$\operatorname{tg} z = \frac{\sin z = h(z)}{\cos z = g(z)} \quad \cos z = 0 \text{ pre } z = \frac{\pi}{2} + k\pi,\ k \in \mathbb{Z}$$

iba $z = \pm\frac{\pi}{2}$ patria oblasti

$$h\left(\frac{\pi}{2}\right) = \sin\frac{\pi}{2} = 1 \neq 0$$

$$g\left(\frac{\pi}{2}\right) = \cos\frac{\pi}{2} = 0$$

$$g'\left(\frac{\pi}{2}\right) = \cos'\frac{\pi}{2} = -\sin\frac{\pi}{2} = -1 \neq 0$$

Platia tieto 3 podmienky

$\Downarrow$

$\boxed{z = \frac{\pi}{2} \text{ je pól}}$

$$\operatorname*{res}_{z=\frac{\pi}{2}} \operatorname{tg} z = \frac{h\left(\frac{\pi}{2}\right)}{g'\left(\frac{\pi}{2}\right)} = \frac{1}{-1} = \boxed{-1}$$

$$h\left(-\frac{\pi}{2}\right) = \sin\left(-\frac{\pi}{2}\right) = -1$$

$$g\left(-\frac{\pi}{2}\right) = \cos\left(-\frac{\pi}{2}\right) = 0$$

$$g'\left(-\frac{\pi}{2}\right) = -\sin\left(-\frac{\pi}{2}\right) = 1$$

Platia tieto 3 podmienky

$\Downarrow$

$\boxed{z = -\frac{\pi}{2} \text{ je pól}}$

$$\operatorname*{res}_{z=-\frac{\pi}{2}} \operatorname{tg} z = \frac{-1}{1} = \boxed{-1}$$

$$\frac{2z^3-5z^2}{(z-3)^3}$$ z = 3 je nulový bod a pól 3. rádu

$$\operatorname*{res}_{z=3}\frac{2z^3-5z^2}{(z-3)^3}=\frac{1}{(3-1)!}\lim_{z\to3}\frac{d^{3-1}}{dz^{3-1}}\left[\cancel{(z-3)^3}\frac{(2z^3-5z^2)}{\cancel{(z-3)^3}}\right]$$

$$=\frac{1}{2!}\lim_{z\to3}\frac{d^2}{dz^2}\left[2z^3-5z^2\right]=\frac{1}{2}\lim_{z\to3}\frac{d}{dz}\left[6z^2-10z\right]=$$

$$=\frac{1}{2}\lim_{z\to3}(12z-10)=\frac{1}{2}(12\cdot3-10)=\frac{36-10}{2}=\frac{26}{2}=\boxed{13}$$

pre pól m-tého rádu

$$\operatorname*{res}_{z=a}f(z)=\frac{1}{(m-1)!}\lim_{z\to a}\frac{d^{m-1}}{dz^{m-1}}\left[(z-a)^m f(z)\right]$$

$$\int_c \left((z+1)\cos\left(\frac{4}{z+1}\right) + \frac{\operatorname{tg} z}{z} + \operatorname{tg} z + \frac{2z^3 - 5z^2}{(z-3)^3} \right) dz =$$

$= 2\pi i \cdot$ (súčet rezídui pre všetky singulárne body ležiace vnútri oblasti)

$$= 2\pi i \left(-8 + 0 + (-1) + (-1) + 13 \right) = 2\pi i \cdot 3$$

$$= 6\pi i$$