Nájdite analytickú funkciu $f: A(\subset \mathbb{C}) \to \mathbb{C}$, **$f(z) = f(x + i\,y) = u(x,y) + i\,v(x,y)$,**

ak je daná funkcia $v: \mathbb{R}^2 \to \mathbb{R}$, $v(x,y) = 2xy + 3x$

$$\frac{\partial v(x,y)}{\partial x} = 2y + 3 = -\frac{\partial u(x,y)}{\partial y}$$

$$\frac{\partial v(x,y)}{\partial y} = 2x = \frac{\partial u(x,y)}{\partial x}$$

$$\frac{\partial u}{\partial y} = -2y - 3$$

$$(c\,f)' = c \cdot f'$$

$$(x^1)' = 1 \cdot x^0 = 1 \cdot 1 = 1$$

$$(x^n)' = n \cdot x^{n-1}$$

$$(C)' = 0$$

Príklad sa dá riešiť 2 rovnocennými spôsobmi:

1. MOŽNOSŤ

rovnaké premenné

$$\frac{\partial u}{\partial x} = 2x \Rightarrow u(x,y) = \int 2x\,dx = \cancel{2}\frac{x^2}{\cancel{2}} + \phi(y)$$

opačné premenné

$\phi(y)$ **môže byť napríklad**

$\phi(y) = \cos^2 y + 7y^3 - 4$

alebo $\phi(y) = \ln y + \sqrt{y}$

Keď zderivujeme y podľa x, dostaneme 0.

$+\phi(y)$ **píšeme namiesto + C, aby sme tam zahrnuli aj časť funkcie u(x,y) obsahujúcu premennú y**

$$\boxed{u(x,y) = x^2 + \phi(y)}$$

$$\frac{\partial u}{\partial y} = \phi'(y)$$

rovnaké premenné

$$\frac{\partial u}{\partial y} = \frac{\partial u}{\partial y}$$

$$\phi'(y) = -2y - 3$$

rovnaké premenné

$$\phi(y) = \int(-2y - 3)\,dy = -\cancel{2} \cdot \frac{y^2}{\cancel{2}} - 3y + C$$

$$\boxed{\phi(y) = -y^2 - 3y + C}$$

$$u(x,y) = x^2 + \phi(y) = x^2 - y^2 - 3y + C$$

$$f(z) = u(x,y) + i \cdot v(x,y)$$

odpísané zo zadania $v(x,y)$

$$f(z) = x^2 - y^2 - 3y + C + i\,(2xy + 3x)$$

2. MOŽNOSŤ RIEŠENIA PRÍKLADU

$$\frac{\partial u}{\partial y} = -2y - 3 \Rightarrow u(x,y) = \int(-2y - 3)\,dy = -\cancel{2}\frac{y^2}{\cancel{2}} - 3y + \phi(x)$$

$$u(x,y) = -y^2 - 3y + \phi(x)$$

$$\frac{\partial u}{\partial x} = \phi'(x)$$

$$\frac{\partial u}{\partial x} = \frac{\partial u}{\partial x}$$

$$\phi'(x) = 2x \Rightarrow \phi(x) = \int 2x\,dx = \cancel{2} \cdot \frac{x^2}{\cancel{2}} + C$$

$$\phi(x) = x^2 + C$$

$$u(x,y) = -y^2 - 3y + \underbrace{x^2 + C}_{\phi(x)}$$

$$f(z) = u(x,y) + i \cdot v(x,y)$$

$$f(z) = x^2 - y^2 - 3y + C + i \cdot (2xy + 3x)$$