$$\int \frac{(x^2+1)^2}{x^3}\,dx = \int \frac{x^4+2x^2+1}{x^3}\,dx = \int\left(\frac{A}{D}+\frac{B}{D}+\frac{C}{D}\right)dx =$$

$$= \int\left(x+2x^{-1}+x^{-3}\right)dx = \frac{x^2}{2}+2\ln|x|+\frac{x^{-2}}{-2}+C \quad -\frac{1}{2x^2}$$

$$\int e^x\left(1-\frac{e^{-x}}{x^2}\right)dx = \int\left(e^x-\frac{e^0}{x^2}\right)dx = \int\left(e^x-x^{-2}\right)dx = e^x-\frac{x^{-1}}{-1}+C = e^x+\frac{1}{x}+C$$

$$\int\left(x^{\frac{7}{8}}+x^{-\frac{1}{2}}\right)dx = \frac{x^{\frac{7}{8}+\frac{8}{8}}}{\frac{15}{8}}+\frac{x^{-\frac{1}{2}+\frac{2}{2}}}{\frac{1}{2}}+C = \frac{8}{15}x^{\frac{15}{8}}+2x^{\frac{1}{2}}+C \quad \sqrt{x}$$

$$\int \frac{1}{\sin^2 x\cdot\cos^2 x}\,dx = \int\frac{\sin^2 x+\cos^2 x}{\sin^2 x\cdot\cos^2 x}\,dx = \frac{A+B}{C} = \frac{A}{C}+\frac{B}{C} =$$

$$= \int\left(\frac{\cancel{\sin^2 x}}{\cancel{\sin^2 x}\cdot\cos^2 x}+\frac{\cancel{\cos^2 x}}{\sin^2 x\cdot\cancel{\cos^2 x}}\right)dx = \int\left(\frac{1}{\cos^2 x}+\frac{1}{\sin^2 x}\right)dx =$$

$$= \operatorname{tg} x - \operatorname{cotg} x + C$$

$$\int (2x+5)(x^2+5x)^7\,dx \quad \left|\begin{array}{l} t = x^2+5x \\ dt = (2x+5)\,dx \\ dx = \frac{dt}{2x+5}\end{array}\right| \quad \int \cancel{(2x+5)}\,t^7\,\frac{dt}{\cancel{2x+5}} = \int t^7\,dt =$$

$$= \frac{t^8}{8}+C = \frac{(x^2+5x)^8}{8}+C$$

$$\int (x+3)\sqrt{x^2+6x+1}\,dx \quad \left|\begin{array}{l} t = x^2+6x+1 \\ dt = (2x+6)\,dx \\ dx = \frac{dt}{2x+6} \\ \quad 2(x+3)\end{array}\right| \quad \int \cancel{(x+3)}\sqrt{t}\,\frac{dt}{2\cancel{(x+3)}} = \frac{1}{2}\int\sqrt{t}\,dt =$$

$$= \frac{1}{2}\int t^{\frac{1}{2}}\,dt = \frac{1}{\cancel{2}}\,\frac{t^{\frac{1}{2}+\frac{2}{2}}}{\frac{3}{\cancel{2}}}+C = \frac{1}{3}t^{\frac{3}{2}}+C = \frac{1}{3}\left(x^2+6x+1\right)^{\frac{3}{2}}+C$$

$$= \frac{1}{3}\sqrt{(x^2+6x+1)^3}+C$$

$$\int\frac{\sin(\ln x)}{x}\,dx \quad \left|\begin{array}{l} t = \ln x \\ dt = \frac{1}{x}\,dx \\ dx = x\,dt\end{array}\right| = \int\frac{\sin t}{\cancel{x}}\,\cancel{x}\,dt = -\cos t + C =$$

$$= -\cos(\ln x)+C$$

$$\int x\, e^{1-x^2}\, dx \; \left|\begin{matrix} t = 1-x^2 \\ dt = -2x\, dx \\ dx = \dfrac{dt}{-2x}\end{matrix}\right| = \int \cancel{x}\, e^{t}\, \frac{dt}{-2\cancel{x}} = -\frac{1}{2}\int e^{t} + C = -\frac{1}{2e}\,t + C$$

$$= -\frac{1}{2}\, e^{1-x^2} + C$$

$$\int \frac{x}{x+16}\, dx \; \left|\begin{matrix} t = x+16 \\ dt = dx \\ x = t-16\end{matrix}\right| \int \frac{t-16}{t}\, dt \qquad \frac{A-B}{C} = \frac{A}{C} - \frac{B}{C} \qquad = \int\left(\frac{t}{t} - \frac{16}{t}\right) dt =$$

$$= \int\left(t^{0} - 16\cdot\frac{1}{t}\right) dt = t - 16\cdot \ln|t| + C = x + 16 - 16\ln|x+16| + C$$

$$\int \frac{x}{x^2+16}\, dx \; \left|\begin{matrix} t = x^2+16 \\ dt = 2x\, dx \\ dx = \dfrac{dt}{2x}\end{matrix}\right| = \int \frac{\cancel{x}}{t}\,\frac{dt}{2\cancel{x}} = \frac{1}{2}\int \frac{1}{t}\, dt = \frac{1}{2}\ln|t| + C$$

$$= \frac{1}{2}\ln|x^2+16| + C$$

$$\int \frac{x}{x^4+16}\, dx = \int \frac{x}{(x^2)^2+16}\, dx \; \left|\begin{matrix} t = x^2 \\ dt = 2x\, dx \\ dx = \dfrac{dt}{2x}\end{matrix}\right| = \int \frac{\cancel{x}}{t^2+16}\,\frac{dt}{2\cancel{x}} =$$

$$= \frac{1}{2}\int \frac{1}{t^2+16}\, dt = \frac{1}{2}\,\frac{1}{4}\,\mathrm{arctg}\,\frac{t}{4} + C = \frac{1}{8}\,\mathrm{arctg}\,\frac{x^2}{4} + C$$