Vypočítajte $\int_C (z+5)\,dz$, kde C je krivka tvorená oblúkom kružnice a úsečkou.

Oblúk kružnice $|z| = 4$, $\operatorname{Re} z \geq 0 \wedge \operatorname{Im} z \geq 0$ od bodu 4i po bod 4.

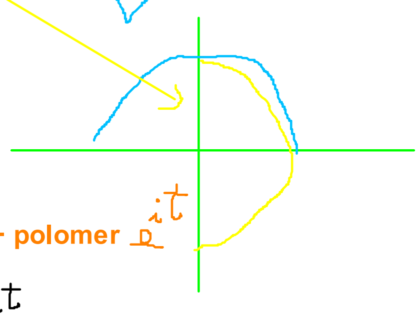
Úsečka je od bodu 4 po bod 2.

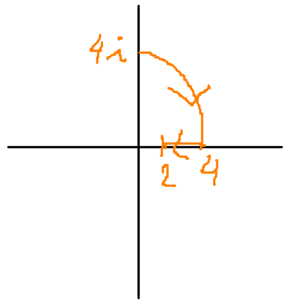
Nakreslite obrázok.

$|z - 0| = 4$

stred kružnice

polomer





$C = C_1 + C_2$

$\varphi(t) =$ stred + polomer e^{it}

$-C_1: \varphi_1: \langle 0, \frac{\pi}{2} \rangle \to C_1$ $\quad \varphi_1(t) = 4e^{it}$

$\alpha \quad \beta$ $\quad \varphi_1'(t) = 4ie^{it}$

$C_2: \varphi_2: \langle 0, 1 \rangle \to C$ $\quad \varphi(t) =$ A + t(B - A) $\quad \varphi_2(t) = 4 + t(2-4) = 4 - 2t$

$\varphi_2'(t) = -2$

$$\int_C f(z)\,dz = \int_\alpha^\beta f(\varphi(t)) \cdot \varphi'(t)\,dt$$

$$\int_C f(z)\,dz = \int_{-C_1} f(z)\,dz + \int_{C_2} f(z)\,dz$$

$$= -\int_0^{\frac{\pi}{2}} (4e^{it} + 5)\,4ie^{it}\,dt + \int_0^1 (4 - 2t + 5)(-2)\,dt =$$

$$= -\int_0^{\frac{\pi}{2}} \left(16i\,e^{2it} + 20i\,e^{it}\right) dt + \int_0^1 (-18 + 4t)\,dt$$

$$= -16i \left[\frac{e^{2it}}{2i}\right]_0^{\frac{\pi}{2}} - 20i \left[\frac{e^{it}}{i}\right]_0^{\frac{\pi}{2}} - 18\left[t\right]_0^1 + 4\left[\frac{t^2}{2}\right]_0^1 =$$

$$= -8\left(e^{i\pi} - e^0\right) - 20\left(e^{i\frac{\pi}{2}} - e^0\right) - 18(1 - 0) + 2(1^2 - 0^2) =$$

$1 \quad 1$

$$= -8e^{i\pi} + 8 - 20e^{i\frac{\pi}{2}} + 20 - 18 + 2 = 12 - 8e^{i\pi} - 20e^{i\frac{\pi}{2}} =$$

$-1 \quad 0$

$-8e^{i\pi} = -8(\cos \pi + i \sin \pi) = -8(-1) = 8$

$0 \quad 1$

$-20e^{i\frac{\pi}{2}} = -20\left(\cos \frac{\pi}{2} + i \sin \frac{\pi}{2}\right) = -20 \cdot i$

$= 12 + 8 - 20i$

$= \boxed{20 - 20i}$

2. možnosť riešenia

dá sa použiť len ak f(z) je **ANALYTICKÁ.**

$$\int_C (z+5)\,dz = \int_{4i}^{2} (z+5)\,dz = \left[\frac{z^2}{2} + 5z\right]_{4i}^{2} = \left(\frac{2^2}{2} + 5 \cdot 2\right) - \left(\frac{(4i)^2}{2} + 5 \cdot 4i\right)$$

$$= 2 + 10 - 8i^2 - 20i = 12 + 8 - 20i$$

-1

$$= \boxed{20 - 20i}$$