$$\int \frac{1-\sin x}{1+\cos x}\,dx \quad \left|\; \begin{aligned} t &= tg\frac{x}{2} \\ arctg\, t &= arctg\left(tg\frac{x}{2}\right) \\ arctg\, t &= \frac{x}{2} \\ x &= 2\cdot arctg\, t \\ \boxed{dx} &\boxed{= \frac{2}{1+t^2}\,dt} \end{aligned} \qquad \begin{aligned} \sin x &= \frac{2t}{t^2+1} \\ \cos x &= \frac{1-t^2}{1+t^2} \end{aligned} \;\right| =$$

nevyužijeme to

$(t-1)^2$

$$= \int \frac{1-\frac{2t}{t^2+1}}{1+\frac{1-t^2}{t^2+1}}\,\frac{2\,dt}{1+t^2} = 2\int \frac{\frac{t^2+1-2t}{\cancel{t^2+1}}}{\frac{t^2+1+1-\cancel{t^2}}{\cancel{t^2+1}}}\,\frac{1\,dt}{1+t^2} = \cancel{2}\int \frac{t^2-2t+1}{\cancel{2}\cdot(1+t^2)}\,dt =$$

$$\frac{f'(t)}{f(t)}$$

$$= \int \frac{t^2-2t+1}{t^2+1}\,dt = \int\left(\frac{t^2+1}{t^2+1}+\frac{-2t}{t^2+1}\right)dt = \int\left(1-\frac{2t}{t^2+1}\right)dt = t-\ln|t^2+1|+C$$

$$= tg\frac{x}{2}-\ln\left|tg^2\frac{x}{2}+1\right|+C$$

$$\int \frac{dx}{\sin^2 x+3\cos^2 x+2} \quad \left|\; \begin{aligned} t &= tg\, x \\ arctg\, t &= arctg(tg\, x) \\ arctg\, t &= x \\ \boxed{\frac{dt}{1+t^2}} &\boxed{= dx} \end{aligned} \qquad \begin{aligned} \sin^2 x &= \frac{t^2}{t^2+1} \\ \cos^2 x &= \frac{1}{t^2+1} \end{aligned} \;\right| =$$

$$= \int \frac{\frac{dt}{1+t^2}}{\frac{t^2}{t^2+1}+\frac{3}{t^2+1}+2} = \int \frac{\frac{dt}{\cancel{1+t^2}}}{\frac{t^2+3+2t^2+2}{\cancel{t^2+1}}} = \int \frac{dt}{3t^2+5} =$$

$$= \int \frac{dt}{3\left(t^2+\frac{5}{3}\right)} = \frac{1}{3}\int \frac{dt}{t^2+\frac{5}{3}} = \frac{1}{3}\,\frac{1}{\frac{\sqrt5}{\sqrt3}}\,arctg\,\frac{t}{\frac{\sqrt5}{\sqrt3}}+C$$

$$a^2=\frac{5}{3} \Rightarrow a=\frac{\sqrt5}{\sqrt3}$$

$$\frac{1}{3}=\frac{1}{\sqrt3\sqrt3}$$

$$\frac{1}{\sqrt3\cancel{\sqrt3}}\,\frac{1}{\frac{\sqrt5}{\cancel{\sqrt3}}}\,arctg\,\frac{\sqrt3\,t}{\sqrt5}+C = \frac{1}{\sqrt{15}}\,arctg\,\frac{\sqrt3\,tg\,x}{\sqrt5}+C$$

$$\int \frac{1+\operatorname{tg} x}{1-\operatorname{tg} x}\,dx \quad \left| \begin{array}{l} t = \operatorname{tg} x \\ \operatorname{arctg} t = \operatorname{arctg}(\operatorname{tg} x) \\ \operatorname{arctg} t = x \\ \boxed{\frac{dt}{1+t^2} = dx} \end{array} \right| =$$

$$\int \frac{1+t}{1-t}\,\frac{dt}{1+t^2}$$

znamienko mínus sme presunuli do čitateľa

$$\frac{t+1}{(1-t)(t^2+1)} = \frac{-t-1}{(t-1)(t^2+1)} =$$

$$= \frac{A}{t-1} + \frac{Bt+C}{t^2+1} = \frac{(t-1)(Bt+C)+A(t^2+1)}{(t-1)(t^2+1)}$$

$$= \frac{Bt^2 - Bt + Ct - C + At^2 + A}{(t-1)(t^2+1)}$$

t^2: $0 = A + B$ I.

t^1: $-1 = -B + C$ II.

t^0: $-1 = A - C$ III.

$-2 = A - B$ II. + III.

$$\begin{aligned} 0 &= A + B \\ -2 &= A - B \\ \hline -2 &= 2A \end{aligned}$$

$$\boxed{A = -1}$$

$B = -A$

$$\boxed{B = 1}$$

$C = A + 1$

$C = -1 + 1$

$$\boxed{C = 0}$$

$$= \int \left(\frac{-1}{t-1} + \frac{2t}{2(t^2+1)} \right) dt$$

$$= -\ln|t-1| + \frac{1}{2}\ln|t^2+1| + C$$

$$= -\ln|\operatorname{tg} x - 1| + \frac{1}{2}\ln|\operatorname{tg}^2 x + 1| + C$$