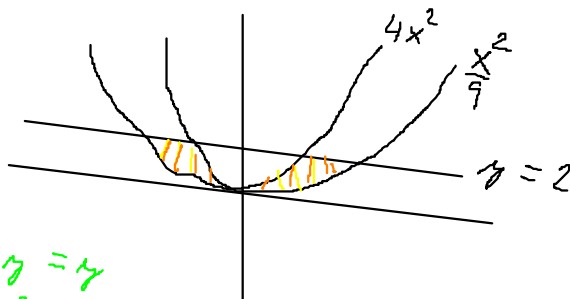


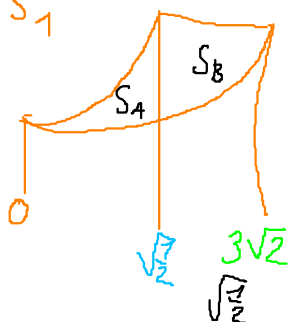
$S = ?$ obsah oblasti ohraničenej: $y = 4x^2$

$$y = \frac{x^2}{9}$$

$$y = 2$$



$$S = 2S_1$$



$$\begin{aligned} y &= 4 \\ 4x^2 &= 2 \\ x^2 &= \frac{2}{4} = \frac{1}{2} \\ x &= \pm \sqrt{\frac{1}{2}} \end{aligned}$$

$$\begin{aligned} y &= 4 \\ \frac{x^2}{9} &= 2 \\ x^2 &= 18 \\ x &= \pm \sqrt{18} = \pm \sqrt{2 \cdot 9} = \pm 3\sqrt{2} \end{aligned}$$

$$S_1 = S_A + S_B = \int_0^{\sqrt{\frac{1}{2}}} (4x^2 - \frac{x^2}{9}) dx + \int_{\sqrt{\frac{1}{2}}}^{3\sqrt{2}} (2 - \frac{x^2}{9}) dx$$

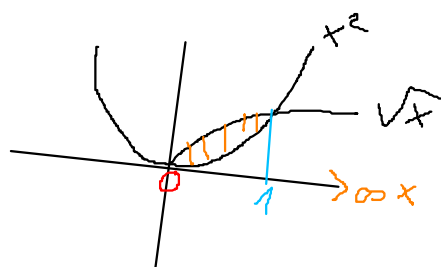
$$= \int_0^{\sqrt{\frac{1}{2}}} \frac{35}{9} x^2 dx + \left[2x - \frac{x^3}{27} \right]_{\sqrt{\frac{1}{2}}}^{3\sqrt{2}} = \frac{35}{9} \left[\frac{x^3}{3} \right]_0^{\sqrt{\frac{1}{2}}} + \left[2x - \frac{x^3}{27} \right]_{\sqrt{\frac{1}{2}}}^{3\sqrt{2}} =$$

$$= \frac{35}{27} \left(\left(\sqrt{\frac{1}{2}} \right)^3 - 0^3 \right) + \left[6\sqrt{2} - \frac{(3\sqrt{2})^3}{27} - \left(2\sqrt{\frac{1}{2}} - \frac{(\sqrt{\frac{1}{2}})^3}{27} \right) \right] =$$

$$\frac{35}{27 \cdot 2\sqrt{2}} + \left(6\sqrt{2} - \frac{27 \cdot 2\sqrt{2}}{27} - \left(\frac{2}{\sqrt{2}} + \frac{1}{27 \cdot 2\sqrt{2}} \right) \right) = \frac{36}{54\sqrt{2}} + 5\sqrt{2} - 2\sqrt{2} = \frac{\sqrt{2}}{3} + 3\sqrt{2}$$

$$S = 2 \cdot S_1 = \frac{2\sqrt{2}}{3} + 6\sqrt{2} = \frac{\sqrt{2}(2+18)}{3} = \frac{20\sqrt{2}}{3} \approx 9.2$$

$V = ?$ rotácia oblasti ohraničenej: $y = x^2$
 $y = \sqrt{x}$



$$V = \pi \cdot \int_a^b (\text{horná}(x)^2 - \text{dolná}(x)^2) dx$$

$$V = \pi \int_0^1 ((\sqrt{x})^2 - (x^2)^2) dx = \pi \int_0^1 (x - x^4) dx = \pi \left[\frac{x^2}{2} - \frac{x^5}{5} \right]_0^1 =$$

$$= \pi \left(\frac{1}{2} - \frac{1}{5} \right) - \left(\frac{0}{2} - \frac{0}{5} \right) = \pi \frac{5-2}{10} = \frac{3}{10} \pi \cdot 3$$

$$\begin{aligned} y &= 3 \\ x^2 &= \sqrt{x} \\ x^4 &= x \\ x^4 - x &= 0 \\ x(x^3 - 1) &= 0 \\ x &= 0 \quad \downarrow \quad x = 1 \end{aligned}$$