Nájdite analytickú funkciu: f: $A(\subset\mathbb{C})\to\mathbb{C}$, f(z) = f (x + i y) = u(x,y) + i v(x,y), ak je daná funkcia v: $R \to R^2$, v(x,y) = 2xy + 3x

$$\frac{\partial v}{\partial x} = 2y+3 = -\frac{\partial u}{\partial y} \Rightarrow \frac{\partial u}{\partial y} = -2y-3$$

$$\frac{\partial v}{\partial y} = 2x = \frac{\partial u}{\partial x}$$

Príklad sa ďalej dá riešiť 2 rovnocennými spôsobmi.

1. možnosť

$$\frac{\partial u}{\partial x} = 2x \Rightarrow u(x,y) = \int 2x\,dx = \frac{2x^2}{2} + \phi(y) = x^2 + \phi(y)$$

rovnaké premenné

rôzne premenné

rovnaké premenné

$$\frac{\partial u}{\partial y} = \phi'(y)$$

$$u(x,y) = x^2 - y^2 - 3y + C$$

$\phi(y)$

$$\frac{\partial u}{\partial y} = \frac{\partial u}{\partial y}$$

$$\phi'(y) = -2y-3$$

rovnaké premenné

$$\phi(y) = \int(-2y-3)\,dy = -\frac{2y^2}{2} - 3y + C = -y^2 - 3y + C$$

$$f(z) = u(x,y) + i\,v(x,y) = x^2 - y^2 - 3y + C + i(2xy + 3x)$$

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2. možnosť

rôzne premenné

$$\frac{\partial u}{\partial y} = -2y-3 \Rightarrow u(x,y) = \int(-2y-3)\,dy = -\frac{2y^2}{2} - 3y + \phi(x) = -y^2 - 3y + \phi(x)$$

rovnaké premenné

$$\frac{\partial u}{\partial x} = \phi'(x)$$

$$u(x,y) = -y^2 - 3y + x^2 + C$$

$\phi(x)$

$$\frac{\partial u}{\partial x} = \frac{\partial u}{\partial x}$$

$$\phi'(x) = 2x$$

$$\phi(x) = \int 2x\,dx = \frac{2x^2}{2} + C = x^2 + C$$

rovnaké premenné

$$f(z) = u(x,y) + i\,v(x,y) = -y^2 - 3y + x^2 + C + i(2xy + 3x)$$

Nech $u: R^2 \to R$, $u(x,y) = x^2 - y^2$

Nájdite harmonicky združenú funkciu $v: R^2 \to R$, **takú, že v(0,0) = 0**

Najskôr overíme, že u(x,y) je harmonická funkcia.

$$\frac{\partial u}{\partial x} = 2x = \frac{\partial v}{\partial y} \qquad \frac{\partial u}{\partial y} = -2y = -\frac{\partial v}{\partial x} \qquad \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 2 + (-2) = 0$$

$$\frac{\partial^2 u}{\partial x^2} = 2 \qquad \frac{\partial^2 u}{\partial y^2} = -2$$

u(x,y) je harmonická funkcia

$$\frac{\partial v}{\partial x} = 2y$$

Dá sa to riešiť 2 rovnocennými spôsobmi

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1. možnosť

$$\frac{\partial v}{\partial x} = 2y \Rightarrow v(x,y) = \int 2y\,dx = 2yx + \phi(y)$$

$$\frac{\partial v}{\partial y} = 2x + \phi'(y) \qquad v(x,y) = 2yx + C$$

$$\frac{\partial v}{\partial y} = \frac{\partial v}{\partial y}$$

$$\cancel{2x} + \phi'(y) = \cancel{2x}$$

$$\phi'(y) = 0$$

$$\phi(y) = C$$

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2. možnosť

$$\frac{\partial v}{\partial y} = 2x \Rightarrow v(x,y) = \int 2x\,dy = 2xy + \phi(x)$$

$$v(x,y) = 2xy + C$$

$$\frac{\partial v}{\partial x} = 2y + \phi'(x)$$

$$\frac{\partial v}{\partial x} = \frac{\partial v}{\partial x}$$

$$\cancel{2y} + \phi'(x) = \cancel{2y}$$

$$\phi'(x) = 0$$

$$\phi(x) = C$$

$$v(0,0) = 0$$

$$v(x,y) = 2xy + C$$

$$v(0,0) = 2 \cdot 0 \cdot 0 + C = 0$$

$$C = 0 \qquad \Rightarrow v(x,y) = 2xy$$