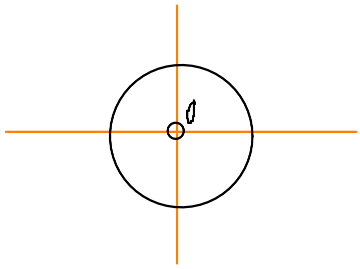
Vypočítajte: $\int_C z^2 e^{\frac{1}{z}}\,dz$, kde $C: |z| = 1$ **je kladne orientovaná.**

$|z-0| = 1$ — polomer

stred



f(z) je analytická na $\mathbb{C}\setminus\{0\}$

singulárny bod je z = 0 a ten leží vnútri krivky C

Vypočítame to pomocou Cauchyho vetyo rezíduách

$$\int_C f(z)\,dz = 2\pi i \cdot \operatorname{res}_{z=0} f(z)$$

$$z^a \cdot z^b = z^{a+b}$$

$$e^z = \sum_{n=0}^{\infty} \frac{z^n}{n!}$$

c_n – **koeficient**

$$z^2 e^{\frac{1}{z}} = z^2 e^{z^{-1}} = z^2 \cdot \sum_{n=0}^{\infty} \frac{z^{-n}}{n!} = \sum_{n=0}^{\infty} \frac{z^{2-n}}{n!} = \sum_{n=0}^{\infty} \frac{1}{n!} z^{2-n}$$

hlavná časť Laurentovho radu má nekonečne veľa členov

z = 0 je podstatne singulárny bod

$$\operatorname{res}_{z=0} f(z) = c_{-1}$$

koeficient c_{-1} **je pri člene** $(z-0)^{-1}$

$$z^{2-n} = z^{-1} \Rightarrow n = 3$$

$$c_n = \frac{1}{n!} \Longrightarrow c_{-1} = \frac{1}{3!} = \frac{1}{3\cdot 2} = \frac{1}{6}$$

$$\operatorname{res}_{z=0} f(z) = c_{-1} = \frac{1}{6}$$

$$\int_C z^2 \cdot \frac{1}{z}\,dz = 2\pi i \operatorname{res}_{z=0} f(z) = 2\pi i \cdot \frac{1}{6} = \frac{\pi i}{3}$$

$$f(z) = z^3 \sin\left(\frac{1}{z^2}\right)$$

$$\sin z = \sum_{n=0}^{\infty} (-1)^n \frac{z^{2n+1}}{(2n+1)!}$$

$$z^3 \sin\Big(\underbrace{\frac{1}{z^2}}_{z^{-2}}\Big) = z^3 \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} \left(z^{-2}\right)^{2n+1} = z^3 \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} z^{-4n-2}$$

$$z^a \cdot z^b = z^{a+b}$$

$$= \sum_{n=0}^{\infty} \underbrace{\frac{(-1)^n}{(2n+1)!}}_{c_{-1}} z^{-4n+1}$$

nekonečne veľa členov hlavnej časti Laurentovho radu

z = 0 je podstatne singulárny bod

$z^{-4n+1} = z^{-1}$ $\Rightarrow$ **neexistuje také n** $\Rightarrow c_{-1} = 0$

$$\operatorname{res}_{z=0} z^3 \sin\left(\frac{1}{z^2}\right) = c_{-1} = 0$$