Nájdite deriváciu funkcie: $f: \mathbb{C}\setminus\{\frac{4}{3}i\} \to \mathbb{C},\ f(z)=\dfrac{z-2i}{3iz+4}$

$$\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$$

menovateľ $\neq 0$

$$3iz+4 \neq 0$$
$$3iz \neq -4$$
$$z \neq \frac{-4}{3i}\cdot\frac{i}{i} = \frac{-4i}{3i^2} = \frac{-4i}{3(-1)} = \frac{4}{3}i$$

$$D_f = \mathbb{C}\setminus\{\tfrac{4}{3}i\}$$

$$D_{f'} = \mathbb{C}\setminus\{\tfrac{4}{3}i\}$$

$$f'(z) = \frac{(z-2i)'(3iz+4)-(z-2i)(3iz+4)'}{(3iz+4)^2}$$
$$= \frac{1\cdot(3iz+4)-(z-2i)3i}{(3iz+4)^2}$$
$$= \frac{\cancel{3iz}+4-\cancel{3iz}+6i^2}{(3iz+4)^2} \quad (-1)$$
$$= \frac{-2}{(3iz+4)^2}$$

$$f': \mathbb{C}\setminus\{\tfrac{4}{3}i\} \to \mathbb{C},\ f'(z) = \frac{-2}{(3iz+4)^2}$$

Nájdite deriváciu funkcie: $f: \mathbb{C}\setminus\{-\frac{2+i}{3}\} \to \mathbb{C},\ f(z)=\ln(2+i+3z)$

$\ln(A)$

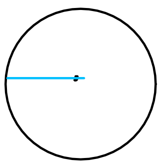
$A \neq 0$

$$2+i+3z \neq 0$$
$$3z \neq -2-i$$
$$z \neq \frac{-2-i}{3}$$
$$z \neq -\frac{2+i}{3}$$

$$(\ln(A))' = \frac{1}{A}\cdot A'$$

$$f'(z) = \frac{1}{2+i+3z}\cdot 3 = \frac{3}{2+i+3z}$$

Logaritmická funkcia v komplexnej analýze nie je spojitá a teda ani diferencovateľná (analytická) pre reálne záporné čísla.



$z = x + iy$ (x – Re z, y – Im z)

$$A = 2+i+3z = 2+i+3(x+iy)$$
$$= 2+i+3x+3iy = \underbrace{(2+3x)}_{\text{Re } A} + i\underbrace{(1+3y)}_{\text{Im } A}$$

Re $A<0$: $2+3x<0 \Rightarrow 3x<-2 \Rightarrow x<-\frac{2}{3}$

Im $=0$: $1+3y=0 \Rightarrow 3y=-1 \Rightarrow y=-\frac{1}{3}$

(x – Re z, y – Im z)

$$f': \mathbb{C}\setminus\left(\{-\tfrac{2+i}{3}\} \cup \{z\in\mathbb{C}: \text{Re } z < -\tfrac{2}{3} \wedge \text{Im } z = -\tfrac{1}{3}\}\right) \to \mathbb{C},$$
$$f'(z) = \frac{3}{2+i+3z}$$

Keby bolo zadanie:
Na akej množine je funkcia analytická?

Odpoveď by bola:

f(z) je analytická na $\mathbb{C}\setminus\left(\{-\frac{2+i}{3}\} \cup \{z\in\mathbb{C}: \text{Re } z<-\frac{2}{3} \wedge \text{Im } z=-\frac{1}{3}\}\right)$