

Today

Review for Midterm.

First there was logic...

A statement is a true or false.

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A statement is a true or false.

Don't worry about Gödel.

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Statements?

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Statements?

$$3 = 4 - 1 \text{ ?}$$

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Statements?

$3 = 4 - 1$? Statement!

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Statements?

$3 = 4 - 1$? Statement!

$3 = 5$?

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$3 = 4 - 1$? Statement!

$3 = 5$? Statement!

3 ? Not a statement!

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$n = 3$? Not a statement...

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$3 = 4 - 1$? Statement!

$3 = 5$? Statement!

3 ? Not a statement!

$n = 3$? Not a statement...but a predicate.

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$3 = 4 - 1$? Statement!

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Predicate: Statement with free variable(s).

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Example: $x = 3$

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Predicate: Statement with free variable(s).

Example: $x = 3$ Given a value for x , becomes a statement.

Predicate?

$n > 3$? Predicate: $P(n)$!

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Example: $x = 3$ Given a value for x , becomes a statement.

Predicate?

$n > 3$? Predicate: $P(n)$!

$x = y$?

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Predicate?

$n > 3$? Predicate: $P(n)$!

$x = y$? Predicate: $P(x, y)$!

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$x + y$?

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$x + y$? No.

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$x + y$? No. An expression, not a statement.

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Quantifiers:

$(\forall x) P(x)$.

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Quantifiers:

$(\forall x) P(x)$. For every x , $P(x)$ is true.

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Quantifiers:

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$(\forall n \in \mathbb{N}), n^2 \geq n$:

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$(\forall n \in \mathbb{N}), n^2 \geq n$: Any free variables? No. So it's a statement.

$(\forall x \in \mathbb{R})(\exists y \in \mathbb{R})y > x$.

Connecting Statements

$$A \wedge B, A \vee B, \neg A, A \implies B.$$

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Propositional Expressions and Logical Equivalence

Connecting Statements

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Propositional Expressions and Logical Equivalence

$$(A \implies B) \equiv (\neg A \vee B)$$

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Propositional Expressions and Logical Equivalence

$$(A \implies B) \equiv (\neg A \vee B)$$

$$\neg(A \vee B) \equiv (\neg A \wedge \neg B)$$

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Propositional Expressions and Logical Equivalence

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Proofs: truth table or manipulation of known formulas.

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Proofs: truth table or manipulation of known formulas.

$$(\forall x \in \mathbb{R})(P(x) \wedge Q(x)) \equiv (\forall x \in \mathbb{R})P(x) \wedge (\forall x \in \mathbb{R})Q(x)$$

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Step 1:

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If you think it's true:

Step 1: Show that when the thing on the left is true, the thing on the right is true. No matter what P and Q are!

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If you think it's true:

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Step 2: Show that when the thing on the right is true, the thing on the left is true. No matter what P and Q are!

Or manipulate the formulas.

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Or manipulate the formulas.

If you think it's not true:

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$$(\forall x \in \mathbb{R})(P(x) \wedge Q(x)) \equiv (\forall x \in \mathbb{R})P(x) \wedge (\forall x \in \mathbb{R})Q(x)$$

If you think it's true:

Step 1: Show that when the thing on the left is true, the thing on the right is true. No matter what P and Q are!

Step 2: Show that when the thing on the right is true, the thing on the left is true. No matter what P and Q are!

Or manipulate the formulas.

If you think it's not true:

Find an example of $P(x)$ and $Q(x)$

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$$(\forall x \in \mathbb{R})(P(x) \wedge Q(x)) \equiv (\forall x \in \mathbb{R})P(x) \wedge (\forall x \in \mathbb{R})Q(x)$$

If you think it's true:

Step 1: Show that when the thing on the left is true, the thing on the right is true. No matter what P and Q are!

Step 2: Show that when the thing on the right is true, the thing on the left is true. No matter what P and Q are!

Or manipulate the formulas.

If you think it's not true:

Find an example of $P(x)$ and $Q(x)$ such that one of the above steps fails.

...and then proofs...

Direct: $P \implies Q$

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Direct: $P \implies Q$

Example: a is even $\implies a^2$ is even.

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Approach: What is even?

...and then proofs...

Direct: $P \implies Q$

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Approach: What is even? $a = 2k$

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$$a^2 = 4k^2$$

...and then proofs...

Direct: $P \implies Q$

Example: a is even $\implies a^2$ is even.

Approach: What is even? $a = 2k$

$$a^2 = 4k^2 = 2(2k^2)$$

...and then proofs...

Direct: $P \implies Q$

Example: a is even $\implies a^2$ is even.

Approach: What is even? $a = 2k$

$$a^2 = 4k^2 = 2(2k^2)$$

Integers closed under multiplication!

...and then proofs...

Direct: $P \implies Q$

Example: a is even $\implies a^2$ is even.

Approach: What is even? $a = 2k$

$$a^2 = 4k^2 = 2(2k^2)$$

Integers closed under multiplication! So $2k^2$ is even.

...and then proofs...

Direct: $P \implies Q$

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Contrapositive: $P \implies Q$

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$$a^2 = 4k^2 = 2(2k^2)$$

Integers closed under multiplication! So $2k^2$ is even.

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Contrapositive: $P \implies Q$ or $\neg Q \implies \neg P$.

...and then proofs...

Direct: $P \implies Q$

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a^2 is even.

Contrapositive: $P \implies Q$ or $\neg Q \implies \neg P$.

Example: a^2 is odd $\implies a$ is odd.

...and then proofs...

Direct: $P \implies Q$

Example: a is even $\implies a^2$ is even.

Approach: What is even? $a = 2k$

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Contrapositive: $P \implies Q$ or $\neg Q \implies \neg P$.

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...and then proofs...

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Example: rogue couple does not exist.

...jumping forward..

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Find a place where induction step doesn't hold.

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Something about the Well ordering principle.

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First day where no woman improves.

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Base: $8|3^2 - 1$.

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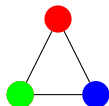
Proof Idea: Original graph connected.

Graph Coloring.

Given $G = (V, E)$, a coloring of a G assigns colors to vertices V where for each edge the endpoints have different colors.

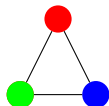
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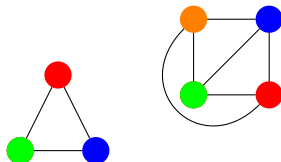
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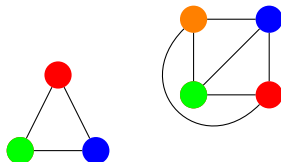
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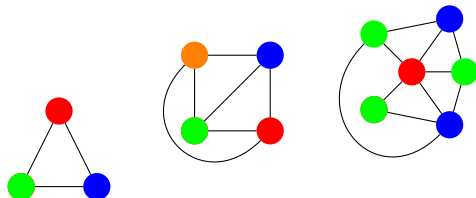
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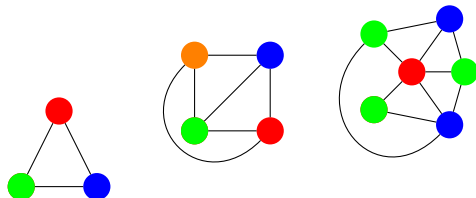
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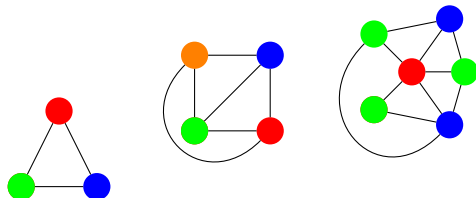
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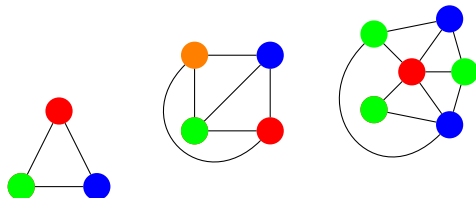
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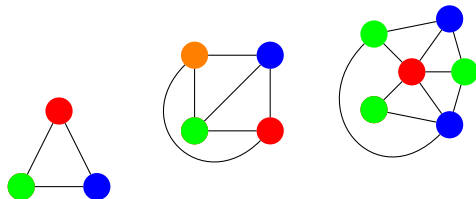
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Notice that the last one, has one three colors.

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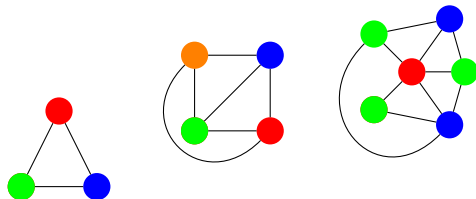
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Fewer colors than number of vertices.

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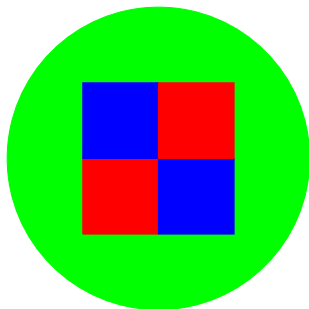
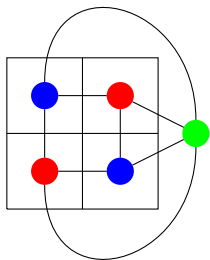
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Fewer colors than number of vertices.

Fewer colors than max degree node.

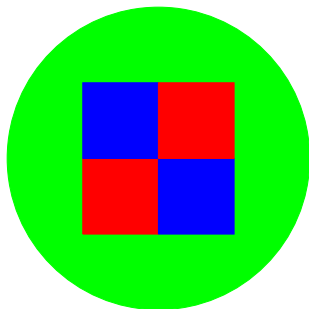
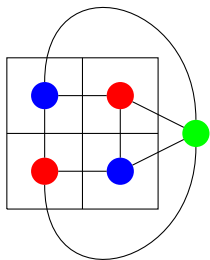
Planar graphs and maps.

Planar graph coloring \equiv map coloring.



Planar graphs and maps.

Planar graph coloring \equiv map coloring.



Four color theorem is about planar graphs!

Six color theorem.

Theorem: Every planar graph can be colored with six colors.

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Proof:

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Recall: $e \leq 3v - 6$ for any planar graph.

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From Euler's Formula:

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There exists a vertex with degree < 6 or at most 5.

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Five color theorem

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Five color theorem

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Proof:

Five color theorem

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Proof: Not Today!

Four Color Theorem

Four Color Theorem

Theorem: Any planar graph can be colored with four colors.

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Proof:

Four Color Theorem

Theorem: Any planar graph can be colored with four colors.

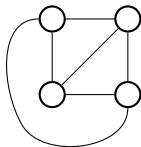
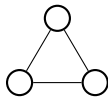
Proof: Not Today!

Four Color Theorem

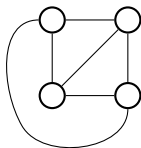
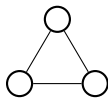
Theorem: Any planar graph can be colored with four colors.

Proof: Not Today!

Graph Types: Complete Graph.

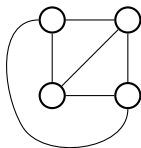
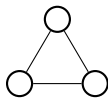


Graph Types: Complete Graph.



$$K_n, |V| = n$$

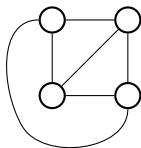
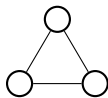
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every edge present.

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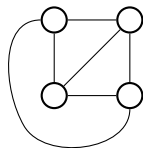
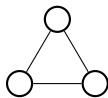


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degree of vertex?

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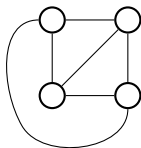
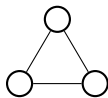


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degree of vertex? $|V| - 1$.

Graph Types: Complete Graph.



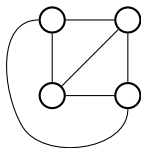
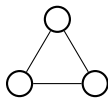
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Very connected.

Graph Types: Complete Graph.



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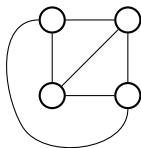
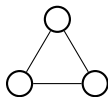
every edge present.

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Very connected.

Lots of edges:

Graph Types: Complete Graph.



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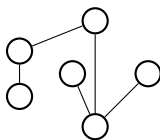
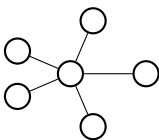
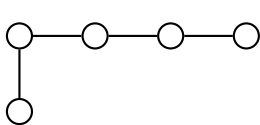
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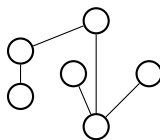
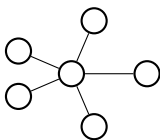
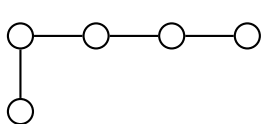
Lots of edges: $n(n-1)/2$.

Trees.



Definitions:

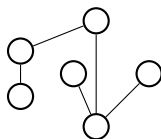
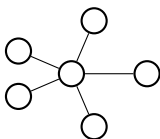
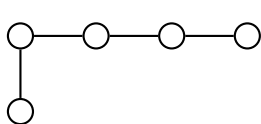
Trees.



Definitions:

A connected graph without a cycle.

Trees.

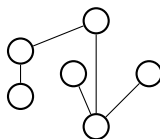
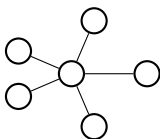
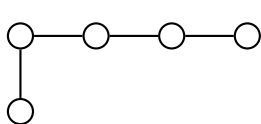


Definitions:

A connected graph without a cycle.

A connected graph with $|V| - 1$ edges.

Trees.



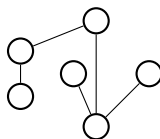
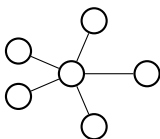
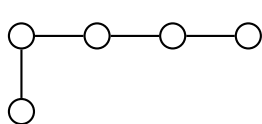
Definitions:

A connected graph without a cycle.

A connected graph with $|V| - 1$ edges.

A connected graph where any edge removal disconnects it.

Trees.



Definitions:

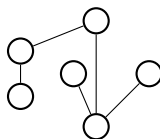
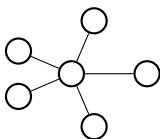
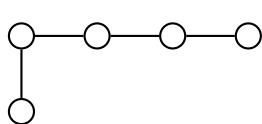
A connected graph without a cycle.

A connected graph with $|V| - 1$ edges.

A connected graph where any edge removal disconnects it.

An acyclic graph where any edge addition creates a cycle.

Trees.



Definitions:

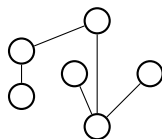
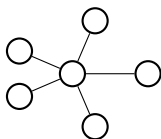
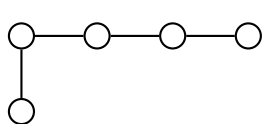
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Trees.



Definitions:

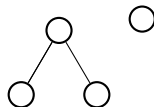
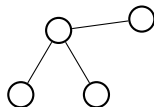
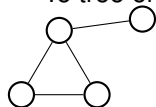
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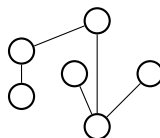
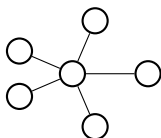
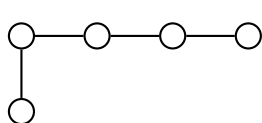
A connected graph where any edge removal disconnects it.

An acyclic graph where any edge addition creates a cycle.

To tree or not to tree!



Trees.



Definitions:

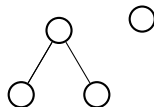
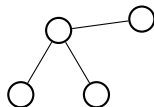
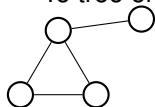
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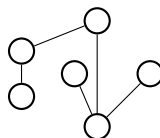
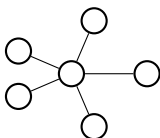
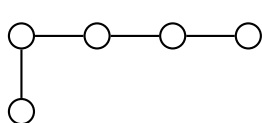
An acyclic graph where any edge addition creates a cycle.

To tree or not to tree!



Minimally connected, minimum number of edges to connect.

Trees.



Definitions:

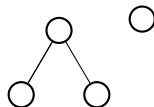
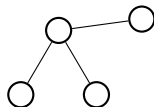
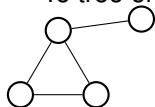
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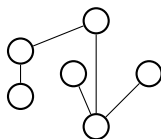
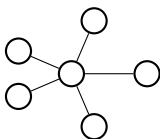
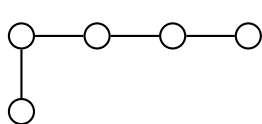
To tree or not to tree!



Minimally connected, minimum number of edges to connect.

Property:

Trees.



Definitions:

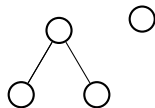
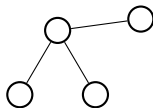
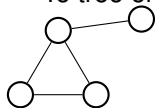
A connected graph without a cycle.

A connected graph with $|V| - 1$ edges.

A connected graph where any edge removal disconnects it.

An acyclic graph where any edge addition creates a cycle.

To tree or not to tree!



Minimally connected, minimum number of edges to connect.

Property:

Can remove a single node and break into components of size at most $|V|/2$.

Hypercube

Hypercubes.

Hypercube

Hypercubes. Really connected.

Hypercube

Hypercubes. Really connected. $|V| \log |V|$ edges!

Hypercube

Hypercubes. Really connected. $|V| \log |V|$ edges!
Wait what?

Hypercube

Hypercubes. Really connected. $|V| \log |V|$ edges!
Wait what? I thought it was $n2^n$.

Hypercube

Hypercubes. Really connected. $|V| \log |V|$ edges!

Wait what? I thought it was $n2^n$. Oh...

Hypercube

Hypercubes. Really connected. $|V| \log |V|$ edges!

Wait what? I thought it was $n2^n$. Oh... $2^n = |V|$...

Hypercube

Hypercubes. Really connected. $|V| \log |V|$ edges!

Wait what? I thought it was $n2^n$. Oh... $2^n = |V|$...

Also represents bit-strings nicely.

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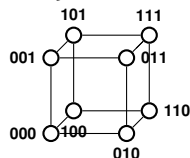
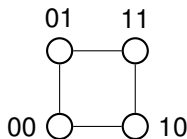
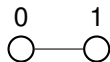
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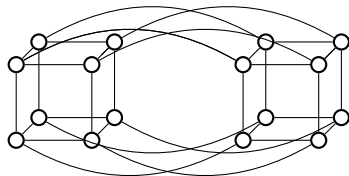
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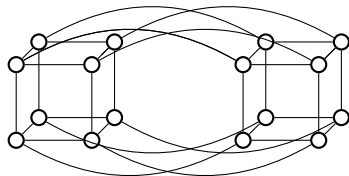
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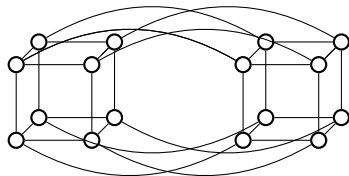
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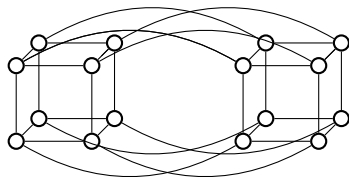
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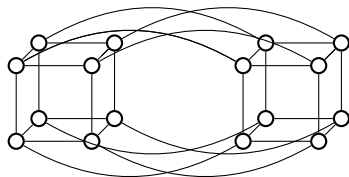
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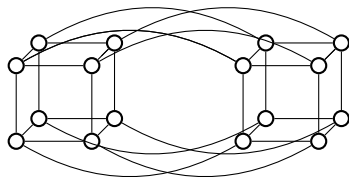
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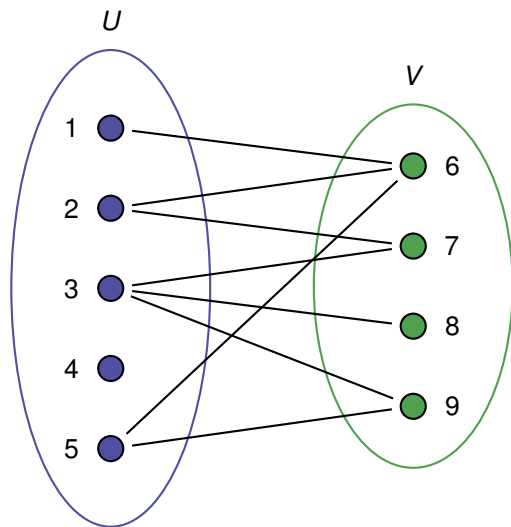
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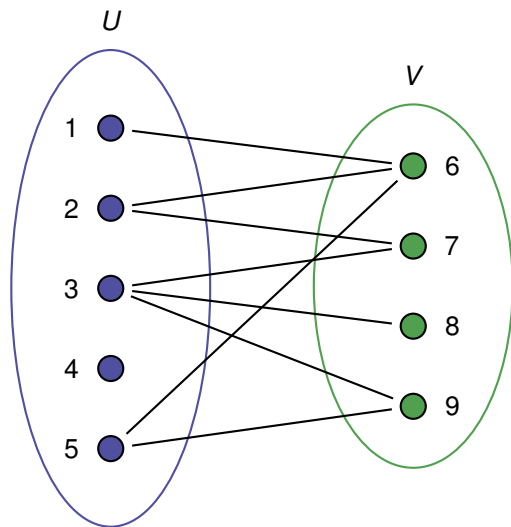
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Good communication network!

Bipartite graphs

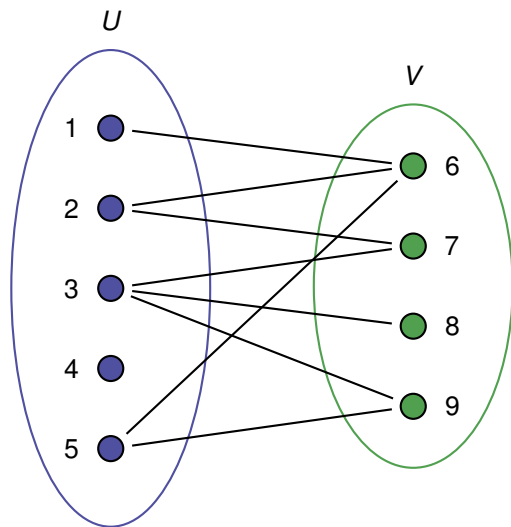


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Cycles have length 4 or more edges.

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n -men, n -women.

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Man optimal \implies Woman pessimal.

Optimality/Pessimal

Optimal partner if best partner in any **stable** pairing.

Not necessarily first in list.

Possibly no stable pairing with that partner.

Man-optimal pairing is pairing where every man gets optimal partner.

Thm: TMA produces male optimal pairing, S .

Proof by contradiction:

Let M be the first man to lose optimal partner.

Better partner W for M .

Different stable pairing T .

TMA: M asked W first!

There is M' who bumps M in TMA.

W prefers M' .

M' likes W at least as much as optimal partner.

Not first bump.

M' and W is rogue couple in T .

Thm: woman pessimal.

Man optimal \implies Woman pessimal.

Woman optimal \implies Man pessimal.

And then countability

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More than one infinities

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Some things are countable

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More than one infinities

Some things are countable , like the natural numbers

And then countability

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Some things are countable , like the natural numbers , or the rationals...

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Why? There is a list!!

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Why? **Diagonalization:**

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Why? **Diagonalization:** Well, assume there is a list.

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Why? **Diagonalization:** Well, assume there is a list. Can construct a diagonal element x . x is not in the list!

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Some things are not countable , like the reals , or the set of all subsets of the naturals...

Why? **Diagonalization:** Well, assume there is a list. Can construct a diagonal element x . x is not in the list! Contradiction.

HALTING

HALTING

The HALT problem:

HALTING

The HALT problem: Is there a program that can tell you if another (generic) program halts on an input?

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Why? Self reference!

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Who cares?

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Who cares? Using the same trick I can show that a bunch of problems are undecidable!

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Like: Will this program P even print "Hello World"?

HALTING

The HALT problem: Is there a program that can tell you if another (generic) program halts on an input?

NO!

Why? Self reference!

Who cares? Using the same trick I can show that a bunch of problems are undecidable!

Like: Will this program P even print "Hello World"?

Or "Is there an input for this program P that will give an attacker admin access?"

Counting!

Sample k items out of n .

	With Replacement	Without Replacement
Order matters	n^k	$\frac{n!}{(n-k)!}$
Order doesn't matter	$\binom{n+k-1}{n-1}$	$\binom{n}{k}$

Stars and bars!

Stars and bars!

Confusion yesterday: 10 hats.

Stars and bars!

Confusion yesterday: 10 hats. 7 days.

Stars and bars!

Confusion yesterday: 10 hats. 7 days. I can wear the same hat on different days (replacement).

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Confusion yesterday: 10 hats. 7 days. I can wear the same hat on different days (replacement). I don't care which day I wore what (order doesn't matter).

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Why is this stars and bars?

How many stars?

Stars and bars!

Confusion yesterday: 10 hats. 7 days. I can wear the same hat on different days (replacement). I don't care which day I wore what (order doesn't matter).

Why is this stars and bars?

How many stars? One for each day.

Stars and bars!

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How many stars? One for each day. So 7

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How many stars? One for each day. So 7

How many bars? One fewer than the hats.

Stars and bars!

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Why is this stars and bars?

How many stars? One for each day. So 7

How many bars? One fewer than the hats. So 9

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|| * | ** | ** || | *** ||

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`|| * | ** | ** || | *** ||`

Didn't wear hats 1 and 2. Wore hat 3 for 1 day, hat 4 for 2 days, hat 5 days. Didn't wear hats 6 and 7. Hat 8 for 3 days. Didn't wear hats 9 and 10.

Combinatorial Proofs.

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Easy ones: $\binom{n}{k} = \binom{n}{n-k}$

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What's the thing on the left? Number of subsets of size k of $\{1, 2, \dots, n+1\}$.

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What's the thing on the left? Number of subsets of size k of $\{1, 2, \dots, n+1\}$.

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Add them up. (**Sum rule**)

Midterm format

Time: 110 minutes.

Midterm format

Time: 110 minutes.

Some short answers.

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Get at ideas that you learned.

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If something is taking too long maybe there is a trick!

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Know material well:

Midterm format

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If something is taking too long maybe there is a trick!

Know material well: fast,

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Know material well: fast, correct.

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Know material medium:

Midterm format

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Know material well: fast, correct.

Know material medium: slower,

Midterm format

Time: 110 minutes.

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If something is taking too long maybe there is a trick!

Know material well: fast, correct.

Know material medium: slower, less correct.

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Know material well: fast, correct.

Know material medium: slower, less correct.

Know material not so well:

Midterm format

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If something is taking too long maybe there is a trick!

Know material well: fast, correct.

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Remember that a problem from hw and/or discussions is in the midterm! (identical or almost identical)

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Remember that a problem from hw and/or discussions is in the midterm! (identical or almost identical)

So study those!

Wrapup.

Wrapup.

If you sent us an email about Midterm conflicts

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If you sent us an email about Midterm conflicts
Other arrangements.

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If you sent us an email about Midterm conflicts
Other arrangements.
Should have received an email from us.

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Other issues....

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Wrapup.

If you sent us an email about Midterm conflicts
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