CS70: Discrete Math and Probability

Slides adopted from Satish Rao, CS70 Spring 2016

June 20, 2016

Introduction

Programming Computers

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Programming Computers $\equiv$ Superpower!

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What are your super powerful programs doing?

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Logic and Proofs!

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Induction $\equiv$ Recursion.

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What can computers do?
  Work with discrete objects.

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Discrete Math $\implies$ immense application.

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E.g. machine learning, data analysis.
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See note 1, for more discussion.

Admin.

Course Webpage: **www.eecs70.org**

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Explains policies, has homework/discussion worksheets, slides, exam dates, etc.

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Assessment:
- Homework: 20%
- Midterm 1 (07/08): 20%
- Midterm 2 (07/29): 20%
- Final (08/12): 35%
- Quiz: 4%
- Sundry: 1%

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Conflicts? Piazza pinned post.

Learning tips

Take homework seriously!

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Questions?

Staffs

3 Co-Instructors

Fan Ye

Just graduated,

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Just graduated, from Berkeley

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Been TA for CS70 for two semesters

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David Dinh

Email: `dinh@cs.berkeley.edu`
Office Hours: M/W 3:30-5:00 (right after lecture) in 606 Soda
I just finished my first year of grad school. My research interests are numerical algorithms and complexity theory - essentially, I work on making faster algorithms for doing things like solving equations, factoring matrices, etc. (and proving that they run fast!), as well as showing that there are limits on how fast we can make these algorithms.
Also did my undergrad here at Cal - CS70 was by far my favorite lower-div.
Fun fact: I like to make ice cream.

Alex Psomas

Not here today.

Alex Psomas

Not here today. Tomorrow lecture

Staffs

3 Co-Instructors
12 awesome and talented TAs.

Wason's experiment:1

Suppose we have four cards on a table:

- 1st about Alice, 2nd about Bob, 3rd Charlie, 4th Donna.

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Suppose we have four cards on a table:

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Suppose we have four cards on a table:

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- Consider the theory:
  "If a person travels to Chicago, he/she flies."

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- Consider the theory: "If a person travels to Chicago, he/she flies."
- Suppose you see that Alice went to Baltimore, Bob drove, Charlie went to Chicago, and Donna flew.

Alice	Bob	Charlie	Donna
Baltimore	drove	Chicago	flew

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Answer:

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Answer: Later.

CS70: Lecture 1. Outline.

Today: Note 1.

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The language of proofs!

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The language of proofs!

1. Propositions.
2. Propositional Forms.
3. Implication.
4. Truth Tables
5. Quantifiers
6. More De Morgan's Laws

Propositions: Statements that are true or false.

$\sqrt{2}$ is irrational

2+2 = 4

2+2 = 3

826th digit of pi is 4

Stephen Curry is a good player.

All evens > 2 are sums of 2 primes

$4 + 5$

$x + x$

Alice travelled to Chicago

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Propositional Forms.

Put propositions together to make another...

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Propositional Forms: quick check!

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$P =$ "$\sqrt{2}$ is rational"
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$\neg P$ ... True

Put them together..

Propositions:

C_1 - Take class 1

Put them together..

Propositions:

C_1 - Take class 1

C_2 - Take class 2

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C_1 - Take class 1
C_2 - Take class 2
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Put them together..

Propositions:

C_1 - Take class 1
C_2 - Take class 2
....

You can only take one of class 1, class 2 and one of class 3, class 4, or take both class 2, class 3 and take either class 5 or class 6, as long as you are not taking class 4 at the same time.

Put them together..

Propositions:

C_1 - Take class 1
C_2 - Take class 2
....

You can only take one of class 1, class 2 and one of class 3, class 4, or take both class 2, class 3 and take either class 5 or class 6, as long as you are not taking class 4 at the same time.

Propositional Form:

Put them together..

Propositions:

C_1 - Take class 1
C_2 - Take class 2
....

You can only take one of class 1, class 2 and one of class 3, class 4, or take both class 2, class 3 and take either class 5 or class 6, as long as you are not taking class 4 at the same time.

Propositional Form:

$$((C_1 \vee C_2) \wedge (C_3 \vee C_4)) \vee ((C_2 \wedge C_3) \wedge (C_5 \vee C_6) \wedge (\neg C_4))$$

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Can you take class 1?

Put them together..

Propositions:

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Can you take class 1?
Can you take class 1 and class 5 together?

Put them together..

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....

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Can you take class 1?
Can you take class 1 and class 5 together?

This seems ...

Put them together..

Propositions:

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C_2 - Take class 2
....

You can only take one of class 1, class 2 and one of class 3, class 4, or take both class 2, class 3 and take either class 5 or class 6, as long as you are not taking class 4 at the same time.

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$$((C_1 \vee C_2) \wedge (C_3 \vee C_4)) \vee ((C_2 \wedge C_3) \wedge (C_5 \vee C_6) \wedge (\neg C_4))$$

Can you take class 1?
Can you take class 1 and class 5 together?

This seems ...complicated.

Put them together..

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Propositional Form:

$$((C_1 \lor C_2) \land (C_3 \lor C_4)) \lor ((C_2 \land C_3) \land (C_5 \lor C_6) \land (\neg C_4))$$

Can you take class 1?
Can you take class 1 and class 5 together?

This seems ...complicated.

We can program!!!!

Put them together..

Propositions:

C_1 - Take class 1
C_2 - Take class 2
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Can you take class 1?
Can you take class 1 and class 5 together?

This seems ...complicated.

We can program!!!!We need a way to keep track!

Truth Tables for Propositional Forms.

P	Q	$P \wedge Q$
T	T	T
T	F	
F	T	
F	F	

Notice: $\wedge$ and $\vee$ are commutative.

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One use for truth tables: Logical Equivalence of propositional forms!

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Example: $\neg(P \wedge Q)$ logically equivalent to $\neg P \vee \neg Q$

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P	Q	$P \land Q$
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T	F	F
F	T	F
F	F	F

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T	T	T
T	F	T
F	T	T
F	F	F

Notice: $\land$ and $\lor$ are commutative.

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T	T	F	F
T	F	F	F
F	T	F	F
F	F	T	

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DeMorgan's Law's for Negation: distribute and flip!
$\neg(P \wedge Q)$

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DeMorgan's Law's for Negation: distribute and flip!

$\neg(P \wedge Q) \quad \equiv \quad \neg P \vee \neg Q$ $\qquad\qquad$ $\neg(P \vee Q) \quad \equiv \quad \neg P \wedge \neg Q$

Distributive?

$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$?

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$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$?

Simplify: $(T \wedge Q) \equiv Q$,

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$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$?

Simplify: $(T \wedge Q) \equiv Q$, $(F \wedge Q) \equiv F$.

Distributive?

$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$?

Simplify: $(T \wedge Q) \equiv Q$, $(F \wedge Q) \equiv F$.

Cases:

P is True .

LHS: $T \wedge (Q \vee R)$

Distributive?

$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$?

Simplify: $(T \wedge Q) \equiv Q$, $(F \wedge Q) \equiv F$.

Cases:

P is True .

LHS: $T \wedge (Q \vee R) \equiv (Q \vee R)$.

Distributive?

$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$?

Simplify: $(T \wedge Q) \equiv Q$, $(F \wedge Q) \equiv F$.

Cases:

P is True .

LHS: $T \wedge (Q \vee R) \equiv (Q \vee R)$.

RHS: $(T \wedge Q) \vee (T \wedge R)$

Distributive?

$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$?

Simplify: $(T \wedge Q) \equiv Q$, $(F \wedge Q) \equiv F$.

Cases:

P is True .

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Distributive?

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Simplify: $(T \wedge Q) \equiv Q$, $(F \wedge Q) \equiv F$.

Cases:

P is True .

LHS: $T \wedge (Q \vee R) \equiv (Q \vee R)$.

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P is False .

Distributive?

$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$?

Simplify: $(T \wedge Q) \equiv Q$, $(F \wedge Q) \equiv F$.

Cases:

P is True .

LHS: $T \wedge (Q \vee R) \equiv (Q \vee R)$.

RHS: $(T \wedge Q) \vee (T \wedge R) \equiv (Q \vee R)$.

P is False .

LHS: $F \wedge (Q \vee R)$

Distributive?

$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$?

Simplify: $(T \wedge Q) \equiv Q$, $(F \wedge Q) \equiv F$.

Cases:

P is True .

LHS: $T \wedge (Q \vee R) \equiv (Q \vee R)$.

RHS: $(T \wedge Q) \vee (T \wedge R) \equiv (Q \vee R)$.

P is False .

LHS: $F \wedge (Q \vee R) \equiv F$.

Distributive?

$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$?

Simplify: $(T \wedge Q) \equiv Q$, $(F \wedge Q) \equiv F$.

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P is True .

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Cases:

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P is False .

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RHS: $(F \wedge Q) \vee (F \wedge R) \equiv (F \vee F) \equiv F$.

Distributive?

$P \land (Q \lor R) \equiv (P \land Q) \lor (P \land R)$?

Simplify: $(T \land Q) \equiv Q$, $(F \land Q) \equiv F$.

Cases:
P is True .
LHS: $T \land (Q \lor R) \equiv (Q \lor R)$.
RHS: $(T \land Q) \lor (T \land R) \equiv (Q \lor R)$.
P is False .
LHS: $F \land (Q \lor R) \equiv F$.
RHS: $(F \land Q) \lor (F \land R) \equiv (F \lor F) \equiv F$.

$P \lor (Q \land R) \equiv (P \lor Q) \land (P \lor R)$?

Distributive?

$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$?

Simplify: $(T \wedge Q) \equiv Q$, $(F \wedge Q) \equiv F$.

Cases:

P is True .

LHS: $T \wedge (Q \vee R) \equiv (Q \vee R)$.

RHS: $(T \wedge Q) \vee (T \wedge R) \equiv (Q \vee R)$.

P is False .

LHS: $F \wedge (Q \vee R) \equiv F$.

RHS: $(F \wedge Q) \vee (F \wedge R) \equiv (F \vee F) \equiv F$.

$P \vee (Q \wedge R) \equiv (P \vee Q) \wedge (P \vee R)$?

Simplify: $T \vee Q \equiv T$,

Distributive?

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$P \vee (Q \wedge R) \equiv (P \vee Q) \wedge (P \vee R)$?

Simplify: $T \vee Q \equiv T$, $F \vee Q \equiv Q$.

Distributive?

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Foil 1:

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$P \vee (Q \wedge R) \equiv (P \vee Q) \wedge (P \vee R)$?

Simplify: $T \vee Q \equiv T$, $F \vee Q \equiv Q$.

Foil 1:

$(A \vee B) \wedge (C \vee D) \equiv (A \wedge C) \vee (A \wedge D) \vee (B \wedge C) \vee (B \wedge D)$?

Distributive?

$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$?

Simplify: $(T \wedge Q) \equiv Q$, $(F \wedge Q) \equiv F$.

Cases:

P is True .

LHS: $T \wedge (Q \vee R) \equiv (Q \vee R)$.

RHS: $(T \wedge Q) \vee (T \wedge R) \equiv (Q \vee R)$.

P is False .

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Foil 2:

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Simplify: $(T \wedge Q) \equiv Q$, $(F \wedge Q) \equiv F$.

Cases:

P is True .

LHS: $T \wedge (Q \vee R) \equiv (Q \vee R)$.

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Implication.

$P \implies Q$ interpreted as

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If P, then Q.

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Statement: If you stand in the rain, then you'll get wet.

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Non-Consequences/consequences of Implication

The statement "$P \implies Q$"

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Some Fun: use propositional formulas to describe implication?
$((P \implies Q) \wedge P) \implies Q$.

Implication and English.

$P \implies Q$

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  Just reversing the order.

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  Remember if P is true then Q must be true.
    this suggests that P can only be true if Q is true.
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  since if Q is false P must have been false.
- P is sufficient for Q.
  This means that proving P allows you
  to conclude that Q is true.
- Q is necessary for P.
  For P to be true it is necessary that Q is true.
  Or if Q is false then we know that P is false.

Truth Table: implication.

P	Q	$P \implies Q$
T	T	T
T	F	
F	T	
F	F	

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T	F	
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$\neg P \vee Q \quad \equiv P \implies Q.$

These two propositional forms are logically equivalent!

Contrapositive, Converse

- Contrapositive of $P \implies Q$ is $\neg Q \implies \neg P$.

Contrapositive, Converse

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    - If the plant pollutes, fish die.

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- **Definition:** If $P \implies Q$ and $Q \implies P$ is P if and only if Q or $P \iff Q$.
  (Logically Equivalent: $\iff$.)

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Propositions?

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Next: Statements about boolean valued functions!!

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Wait! What is $\mathbb{N}$?

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Proposition: "For all natural numbers n**,** $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$**."**

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- See note 0 for more!

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Yes. $P(B) \implies Q(B) \equiv \neg Q(B) \implies \neg P(B)$.
So $P(Bob)$ must be False .

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Back to: Wason's experiment:1

Theory:
"If a person travels to Chicago, he/she flies."

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Only have to turn over cards for Bob and Charlie.

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Idea alert: Restrict domain using implication.

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Note that we may omit universe if clear from context.

Quantifiers..not commutative.

- In English: “there is a natural number that is the square of every natural number”.

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Talk logically

Theorem: No three positive integers a, b, c satisfy the equation $a^n + b^n = c^b$ for any integer n strictly larger than two.

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Contrapositive: $\neg Q \implies \neg P$

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Now can state theorems!

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Now can state theorems! And disprove false ones!

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Next Time: proofs!