CS70: Discrete Math and Probability

Fan Ye

June 23, 2016

More inductions!

Bunch of examples

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Bunch of examples
Good ones

More inductions!

Bunch of examples
Good ones and bad ones

Suppose I start with 0 written on a piece of paper. Each time, I choose a digit written on the paper and erase it. If it was a 0, I replace it with 010. If it was a 1, I replace it with 1001. Prove that it's not possible for me to get two 1's in a row.

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Induct on number of operations I make.

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Base case:

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Induct on number of operations I make.
Base case: After the first step, we get 010, which does not have two 1's in a row.

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Case 1: If we choose a 1, its neighbor must be 0's (based on ind hyp).

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By principle of induction, ... $\square$

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Theorem: Every positive integer n can be written as a sum of distinct powers of 2.

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Base on induction hypothesis, $n+1-2^k$ can be written as a sum of distinct powers of 2.

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Done? Need to make sure 2^k is unique!
Since $n+1-2^k < 2^{k+1} - 2^k < 2^k$, all terms must have power less than k, thus 2^k must be unique in this sum for $n+1$.

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By principle of induction, ... $\square$

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Theorem: For all $n \geq 1$, $\sum_{i=1}^{n} \frac{1}{i^2} \leq 2$. ($S_n = \sum_{i=1}^{n} \frac{1}{i^2}$.)

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Base: $P(1)$. $1 \leq 2$.

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Base: $P(1)$. $1 \leq 2$.
Ind Step: $\sum_{i=1}^{k} \frac{1}{i^2} \leq 2$.

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&\quad \leq 2 + \frac{1}{(k+1)^2}
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$$= \sum_{i=1}^{k} \frac{1}{i^2} + \frac{1}{(k+1)^2}.$$
$$\leq 2 + \frac{1}{(k+1)^2}$$

Uh oh?

Hmmm... It better be that any sum is *strictly less than* 2.

Strengthening: need to...

Theorem: For all $n \geq 1$, $\sum_{i=1}^{n} \frac{1}{i^2} \leq 2$. ($S_n = \sum_{i=1}^{n} \frac{1}{i^2}$.)

Base: $P(1)$. $1 \leq 2$.
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How much less?

Strengthening: need to...

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Need to prove "$S_{k+1} \leq 2 - \frac{1}{(k+2)^2}$".

Oooops.....

Strengthening: how?

Theorem: For all $n \geq 1$, $\sum_{i=1}^{n} \frac{1}{i^2} \leq 2 - f(n)$. ($S_n = \sum_{i=1}^{n} \frac{1}{i^2}$.)

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Theorem: For all $n \geq 1$, $\sum_{i=1}^{n} \frac{1}{i^2} \leq 2 - f(n)$. ($S_n = \sum_{i=1}^{n} \frac{1}{i^2}$.)

Proof:

Strengthening: how?

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Ind hyp: $P(k)$

Strengthening: how?

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Proof:

Ind hyp: $P(k)$ — "$S_k \leq 2 - f(k)$"

Strengthening: how?

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Ind hyp: $P(k)$ — "$S_k \leq 2 - f(k)$"

Prove: $P(k+1)$

Strengthening: how?

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Ind hyp: $P(k)$ — "$S_k \leq 2 - f(k)$"

Prove: $P(k+1)$ – "$S_{k+1} \leq 2 - f(k+1)$"

$S(k+1) = S_k + \frac{1}{(k+1)^2}$

Strengthening: how?

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Proof:

Ind hyp: $P(k)$ — "$S_k \leq 2 - f(k)$"

Prove: $P(k+1)$ – "$S_{k+1} \leq 2 - f(k+1)$"

$$\begin{aligned} S(k+1) &= S_k + \tfrac{1}{(k+1)^2} \\ &\leq 2 - f(k) + \tfrac{1}{(k+1)^2} \end{aligned}$$

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Choose $f(k+1) \leq f(k) - \frac{1}{(k+1)^2}$.

Strengthening: how?

Theorem: For all $n \geq 1$, $\sum_{i=1}^{n} \frac{1}{i^2} \leq 2 - f(n)$. ($S_n = \sum_{i=1}^{n} \frac{1}{i^2}$.)

Proof:

Ind hyp: $P(k)$ — "$S_k \leq 2 - f(k)$"

Prove: $P(k+1)$ – "$S_{k+1} \leq 2 - f(k+1)$"

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$$\implies S(k+1) \leq 2 - f(k+1).$$

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Can you?

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Theorem: For all $n \geq 1$, $\sum_{i=1}^{n} \frac{1}{i^2} \leq 2 - f(n)$. ($S_n = \sum_{i=1}^{n} \frac{1}{i^2}$.)

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Choose $f(k+1) \leq f(k) - \frac{1}{(k+1)^2}$.

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Choose $f(k+1) \leq f(k) - \frac{1}{(k+1)^2}$.

$\implies S(k+1) \leq 2 - f(k+1)$.

Can you?

Subtracting off a quadratically decreasing function every time.

Strengthening: how?

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Can you?

Subtracting off a quadratically decreasing function every time.

Maybe a linearly decreasing function to keep positive?

Strengthening: how?

Theorem: For all $n \geq 1$, $\sum_{i=1}^{n} \frac{1}{i^2} \leq 2 - f(n)$. ($S_n = \sum_{i=1}^{n} \frac{1}{i^2}$.)

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Ind hyp: $P(k)$ — "$S_k \leq 2 - f(k)$"

Prove: $P(k+1)$ – "$S_{k+1} \leq 2 - f(k+1)$"

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Try $f(k) = \frac{1}{k}$

Strengthening: how?

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Ind hyp: $P(k)$ — "$S_k \leq 2 - f(k)$"

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Ind hyp: $P(k)$ — “$S_k \leq 2 - f(k)$”

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$$\frac{1}{k+1} \leq \frac{1}{k} - \frac{1}{(k+1)^2}?$$

$$1 \leq \frac{k+1}{k} - \frac{1}{k+1}$$

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Theorem: For all $n \geq 1$, $\sum_{i=1}^{n} \frac{1}{i^2} \leq 2 - f(n)$. ($S_n = \sum_{i=1}^{n} \frac{1}{i^2}$.)

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$\frac{1}{k+1} \leq \frac{1}{k} - \frac{1}{(k+1)^2}$?

$1 \leq \frac{k+1}{k} - \frac{1}{k+1}$ Multiplied by $k+1$.

$1 \leq 1 + (\frac{1}{k} - \frac{1}{k+1})$

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$1 \leq \frac{k+1}{k} - \frac{1}{k+1}$ Multiplied by $k+1$.

$1 \leq 1 + (\frac{1}{k} - \frac{1}{k+1})$ Some math.

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$\frac{1}{k+1} \leq \frac{1}{k} - \frac{1}{(k+1)^2}$?

$1 \leq \frac{k+1}{k} - \frac{1}{k+1}$ Multiplied by $k+1$.

$1 \leq 1 + (\frac{1}{k} - \frac{1}{k+1})$ Some math. So yes!

Strengthening: how?

Theorem: For all $n \geq 1$, $\sum_{i=1}^{n} \frac{1}{i^2} \leq 2 - f(n)$. ($S_n = \sum_{i=1}^{n} \frac{1}{i^2}$.)

Proof:

Ind hyp: $P(k)$ — "$S_k \leq 2 - f(k)$"

Prove: $P(k+1)$ – "$S_{k+1} \leq 2 - f(k+1)$"

$$\begin{aligned} S(k+1) &= S_k + \frac{1}{(k+1)^2} \\ &\leq 2 - f(k) + \frac{1}{(k+1)^2} \text{ By ind. hyp.} \end{aligned}$$

Choose $f(k+1) \leq f(k) - \frac{1}{(k+1)^2}$.

$\implies S(k+1) \leq 2 - f(k+1)$.

Can you?

Subtracting off a quadratically decreasing function every time.

Maybe a linearly decreasing function to keep positive?

Try $f(k) = \frac{1}{k}$

$\frac{1}{k+1} \leq \frac{1}{k} - \frac{1}{(k+1)^2}$?

$1 \leq \frac{k+1}{k} - \frac{1}{k+1}$ Multiplied by $k+1$.

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Theorem: For all $n \geq 1$, $\sum_{i=1}^{n} \frac{1}{i^2} \leq 2 - \frac{1}{n}$.

Careful!

Careful!

Horses of the same color...

Theorem: All horses have the same color.

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Base Case: $P(1)$ - trivially true.

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First k have same color by $P(k)$. $\quad 1,2,3,\ldots,k,k+1$

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(There are two horses is $\not\equiv$ For all two horses!!!)

Of course it doesn't work.

As we will see, it is more subtle to catch errors in proofs of correct theorems!!

Good? Bad? –1

Use induction to prove the follow equality:

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Use induction to prove the follow equality:

$$n = \sqrt{1+(n-1)\sqrt{1+n\sqrt{1+(n+1)\sqrt{1+(n+2)\ldots}}}}$$

for all positive integers n.

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Proof by induction:

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Proof by induction:
Base case: for $n=1$, $1 = \sqrt{1+0} = 1$, equality holds.

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Use induction to prove the follow equality:

$$n = \sqrt{1 + (n-1)\sqrt{1 + n\sqrt{1 + (n+1)\sqrt{1 + (n+2)\dots}}}}$$

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Base case: for $n=1$, $1=\sqrt{1+0}=1$, equality holds.
Induction hypothesis: Assume this equality holds for $n=k$, i.e.

$$k = \sqrt{1+(k-1)\sqrt{1+k\sqrt{1+(k+1)\sqrt{1+(k+2)\dots}}}}$$

Good? Bad? –2

Ind hyp:

$$k = \sqrt{1 + (k-1)\sqrt{1 + k\sqrt{1 + (k+1)\sqrt{1 + (k+2)\dots}}}}$$

Good? Bad? –2

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Induction step:

Good? Bad? –2

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Induction step:Need to show it holds for $n = k+1$.

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$$k = \sqrt{1+(k-1)\sqrt{1+k\sqrt{1+(k+1)\sqrt{1+(k+2)\ldots}}}}$$

Induction step:Need to show it holds for $n = k+1$.
By square both sides of the induction hypothesis we can get:

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By square both sides of the induction hypothesis we can get:

$$k^2 = 1+(k-1)\sqrt{1+k\sqrt{1+(k+1)\sqrt{1+(k+2)\ldots}}}$$

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By square both sides of the induction hypothesis we can get:

$$k^2 = 1 + (k-1)\sqrt{1 + k\sqrt{1 + (k+1)\sqrt{1 + (k+2)\ldots}}}$$

Easy to get

$$\frac{k^2 - 1}{k-1} = k + 1 = \sqrt{1 + k\sqrt{1 + (k+1)\sqrt{1 + (k+2)\ldots}}}$$

Good? Bad? –2

Ind hyp:

$$k=\sqrt{1+(k-1)\sqrt{1+k\sqrt{1+(k+1)\sqrt{1+(k+2)\dots}}}}$$

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Easy to get

$$\frac{k^2-1}{k-1}=k+1=\sqrt{1+k\sqrt{1+(k+1)\sqrt{1+(k+2)\dots}}}$$

Therefore it holds for $n=k+1$, by principle of induction, ... $\square$

Good? Bad? –2

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$$k=\sqrt{1+(k-1)\sqrt{1+k\sqrt{1+(k+1)\sqrt{1+(k+2)\dots}}}}$$

Induction step:Need to show it holds for $n=k+1$.

By square both sides of the induction hypothesis we can get:

$$k^2=1+(k-1)\sqrt{1+k\sqrt{1+(k+1)\sqrt{1+(k+2)\dots}}}$$

Easy to get

$$\frac{k^2-1}{k-1}=k+1=\sqrt{1+k\sqrt{1+(k+1)\sqrt{1+(k+2)\dots}}}$$

Therefore it holds for $n=k+1$, by principle of induction, ... ☐Good or bad?

Good? Bad? –3

Bad proof!

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Bad proof! We need $k \neq 1$ to divide both sides by $k - 1$

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Or in other words, $p(1)$ does not imply $p(2)$

Good? Bad? –3

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Or in other words, $p(1)$ does not imply $p(2)$

Be careful.

Graphs!

Graphs!

Definitions: model.

Note 4: Graph theory

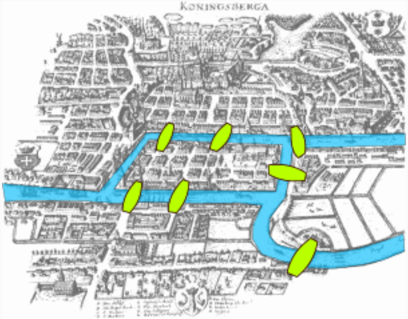
Graphs!

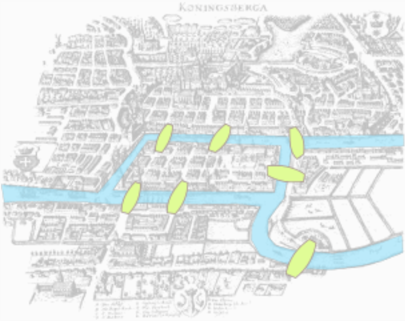
Definitions: model.

Konigsberg bridges problem.

Can you make a tour visiting each bridge exactly once?

Figure 1: "Konigsberg bridges" by Bogdan Giuşcă - License.





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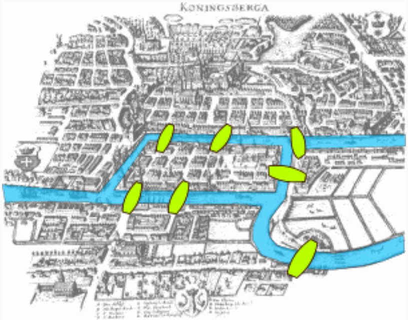

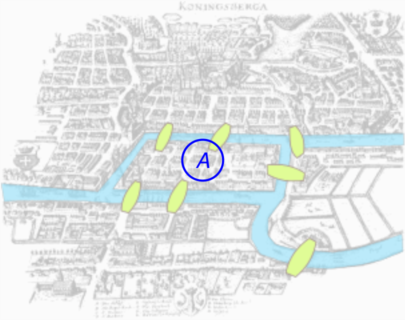

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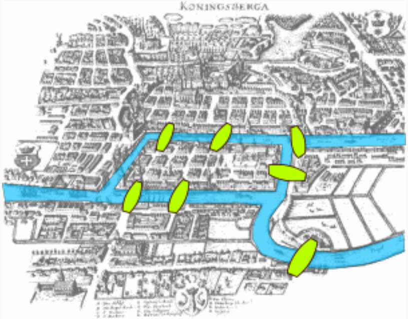

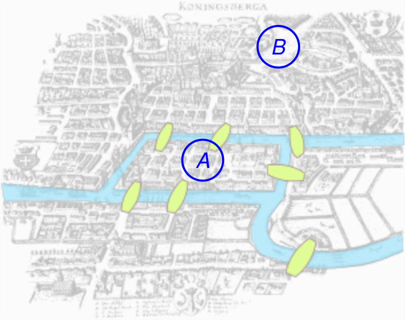

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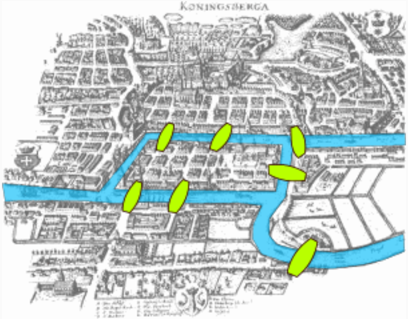

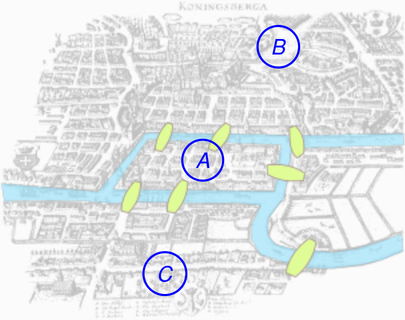

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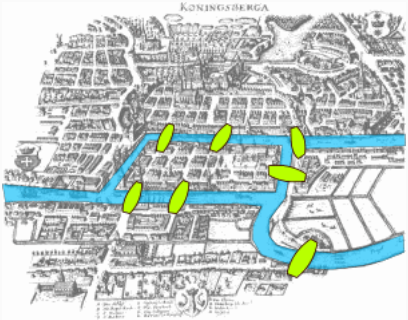

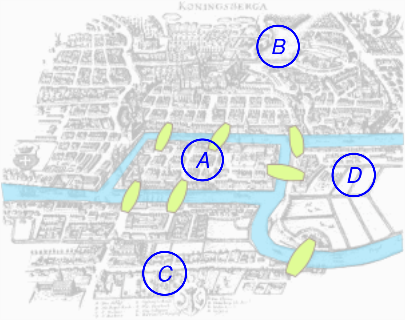

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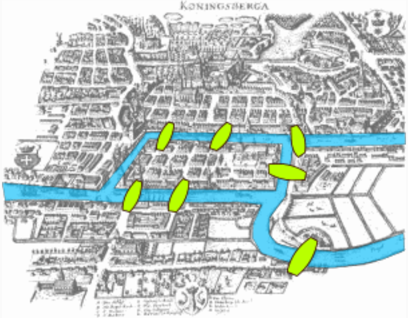

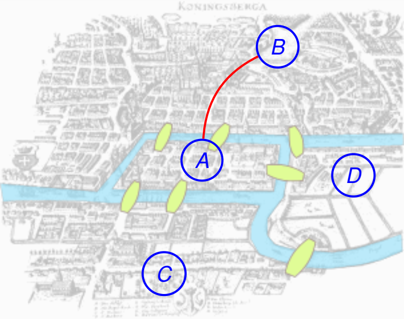

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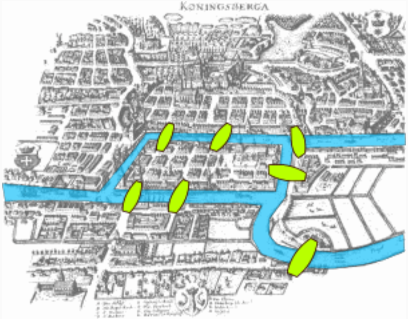

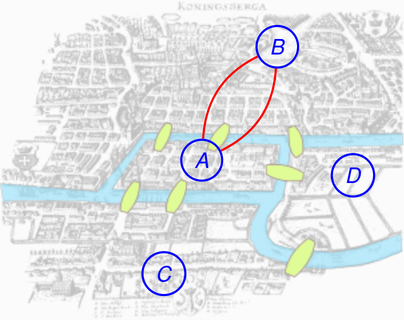

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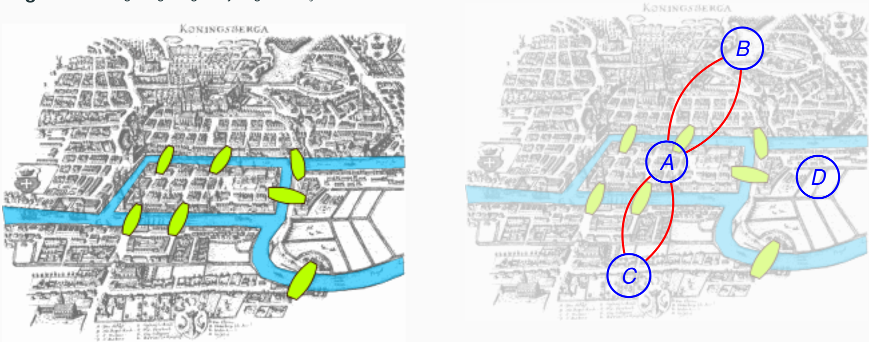

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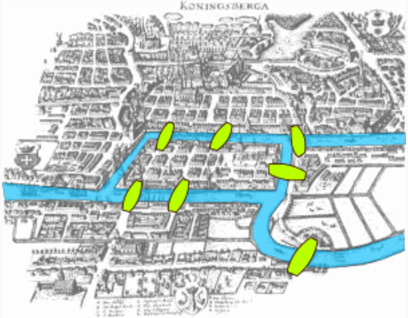

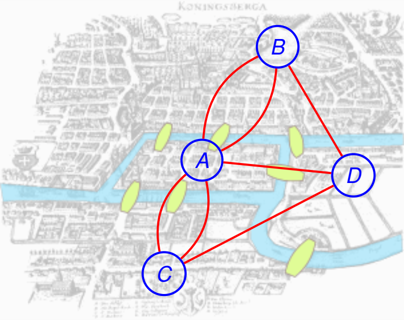

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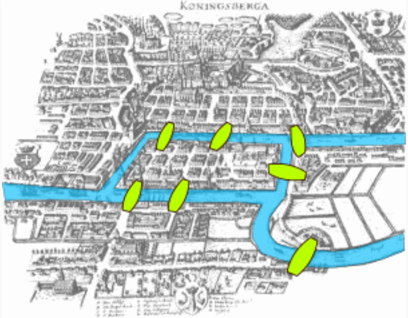
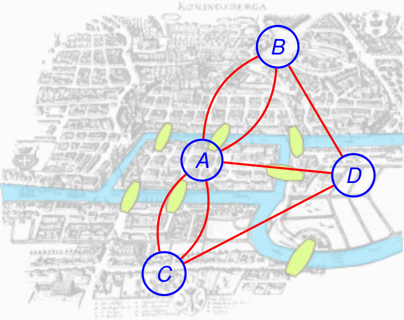


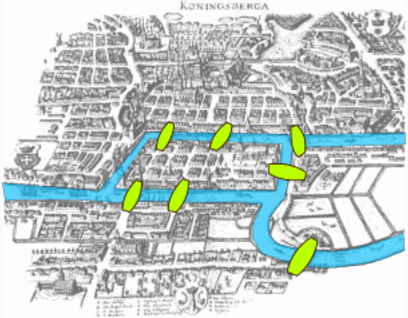


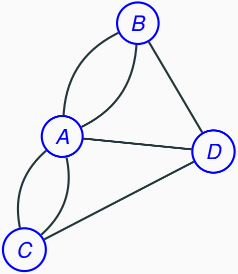
Can you draw a tour in the graph where you visit each edge once?

Konigsberg bridges problem.

Can you make a tour visiting each bridge exactly once?

Figure 1: "Konigsberg bridges" by Bogdan Giuşcă - License.





Can you draw a tour in the graph where you visit each edge once? Yes?

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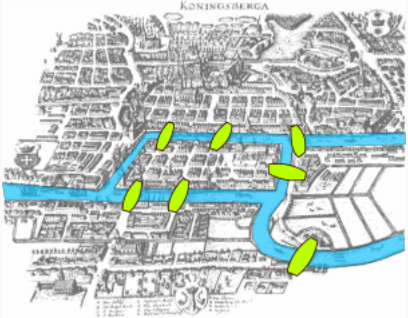

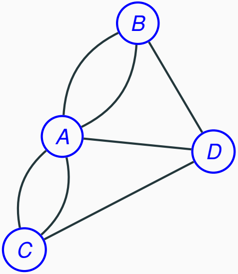

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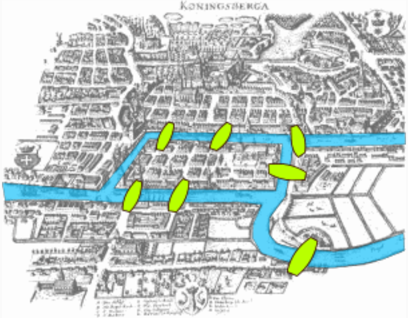
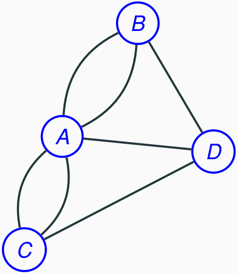
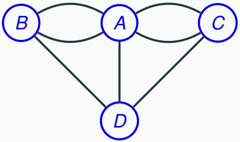




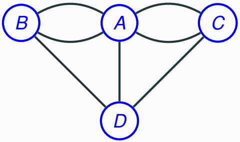
Can you draw a tour in the graph where you visit each edge once? Yes? No?
We will see!

Graphs: formally.



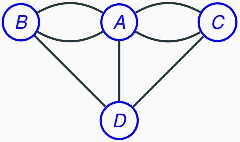
Graph:

Graphs: formally.



Graph: $G = (V, E)$.

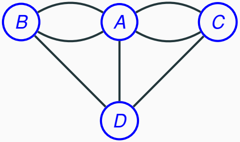
Graphs: formally.



Graph: $G = (V, E)$.

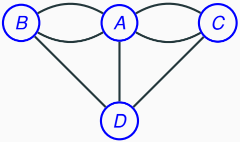
V - set of vertices.

Graphs: formally.



Graph: $G = (V, E)$.
 V - set of vertices.
 $\{A, B, C, D\}$

Graphs: formally.



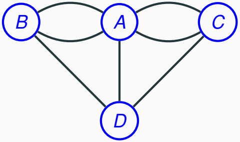
Graph: $G = (V, E)$.

V - set of vertices.

$\{A, B, C, D\}$

$E \subseteq V \times V$ -

Graphs: formally.



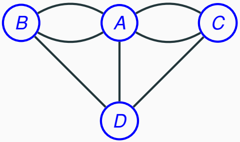
Graph: $G = (V, E)$.

V - set of vertices.

$\{A, B, C, D\}$

$E \subseteq V \times V$ - set of edges.

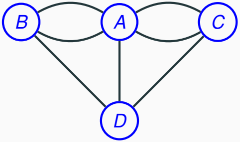
Graphs: formally.



Graph: $G = (V, E)$.

- V - set of vertices.
  - $\{A, B, C, D\}$
- $E \subseteq V \times V$ - set of edges.
  - $\{\{A, B\}$

Graphs: formally.



Graph: $G = (V, E)$.

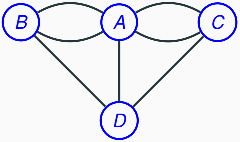
V - set of vertices.

$\{A, B, C, D\}$

$E \subseteq V \times V$ - set of edges.

$\{\{A, B\}, \{A, B\}$

Graphs: formally.



Graph: $G = (V, E)$.

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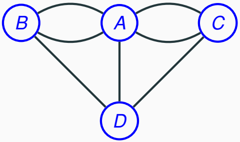
Graphs: formally.



Graph: $G = (V, E)$.

V - set of vertices.

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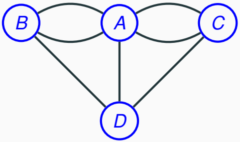
$E \subseteq V \times V$ - set of edges.

$\{\{A,B\},\{A,B\},\{A,C\},\{B,C\},\{B,D\},\{B,D\},\{C,D\}\}$.

Graphs: formally.



Graph: $G = (V, E)$.

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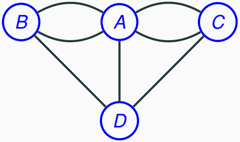
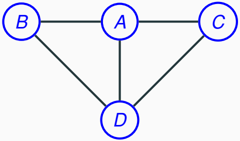
For CS 70, usually simple graphs.

Graphs: formally.





Graph: $G = (V, E)$.

V - set of vertices.

$\{A, B, C, D\}$

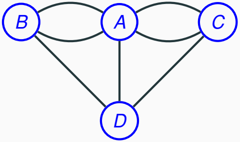
$E \subseteq V \times V$ - set of edges.

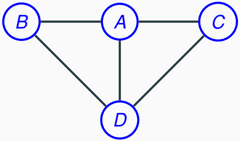
$\{\{A,B\},\{A,B\},\{A,C\},\{B,C\},\{B,D\},\{B,D\},\{C,D\}\}$.

For CS 70, usually simple graphs.

No parallel edges.

Graphs: formally.





Graph: $G = (V, E)$.

V - set of vertices.

$\{A, B, C, D\}$

$E \subseteq V \times V$ - set of edges.

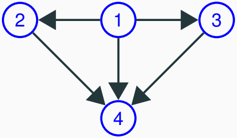
$\{\{A, B\}, \{A, B\}, \{A, C\}, \{B, C\}, \{B, D\}, \{B, D\}, \{C, D\}\}$.

For CS 70, usually simple graphs.

No parallel edges.

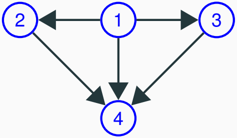
Multigraph above.

Directed Graphs



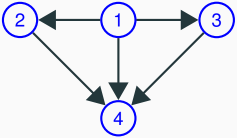
$G = (V, E)$.

Directed Graphs



$G = (V, E)$.
V - set of vertices.

Directed Graphs

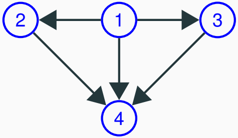


$G = (V, E)$.

V - set of vertices.

$\{1, 2, 3, 4\}$

Directed Graphs



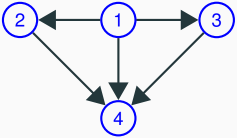
$G = (V, E)$.

V - set of vertices.

$\{1, 2, 3, 4\}$

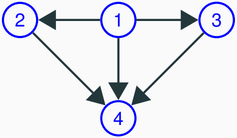
E ordered pairs of vertices.

Directed Graphs



$G = (V, E)$.
V - set of vertices.
$\{1, 2, 3, 4\}$
E ordered pairs of vertices.
$\{(1,2),$

Directed Graphs



$G = (V, E)$.

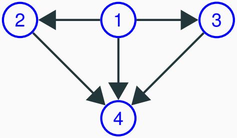
V - set of vertices.

$\{1, 2, 3, 4\}$

E ordered pairs of vertices.

$\{(1,2), (1,3),$

Directed Graphs



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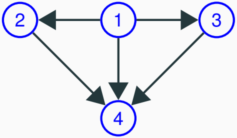
Directed Graphs



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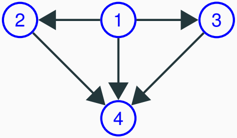
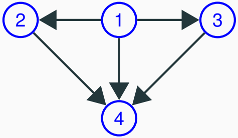
Directed Graphs



$G = (V, E)$.
V - set of vertices.
$\{1, 2, 3, 4\}$
E ordered pairs of vertices.
$\{(1,2),(1,3),(1,4),(2,4),(3,4)\}$

One way streets.

Directed Graphs



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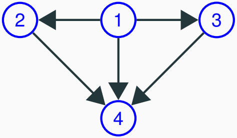
$\{(1,2),(1,3),(1,4),(2,4),(3,4)\}$

One way streets.
Tournament:

Directed Graphs



$G = (V, E)$.
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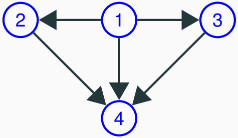
One way streets.
Tournament: 1 beats 2,

Directed Graphs



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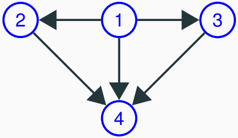
One way streets.
Tournament: 1 beats 2, ...
Precedence:

Directed Graphs



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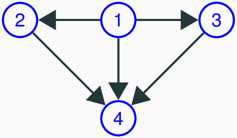
One way streets.
Tournament: 1 beats 2, ...
Precedence: 1 is before 2,

Directed Graphs



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One way streets.
Tournament: 1 beats 2, ...
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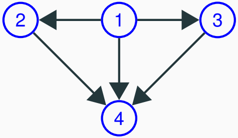
Directed Graphs



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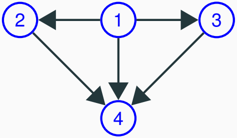
One way streets.
Tournament: 1 beats 2, ...
Precedence: 1 is before 2, ..

Social Network:

Directed Graphs



$G = (V, E)$.
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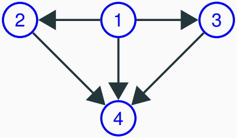
$\{(1,2),(1,3),(1,4),(2,4),(3,4)\}$

One way streets.
Tournament: 1 beats 2, ...
Precedence: 1 is before 2, ..

Social Network: Directed?

Directed Graphs



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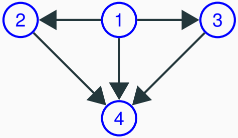
One way streets.
Tournament: 1 beats 2, ...
Precedence: 1 is before 2, ..

Social Network: Directed? Undirected?

Directed Graphs



$G = (V, E)$.
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E ordered pairs of vertices.
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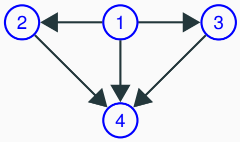
One way streets.
Tournament: 1 beats 2, ...
Precedence: 1 is before 2, ..

Social Network: Directed? Undirected?
Friends.

Directed Graphs



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E ordered pairs of vertices.

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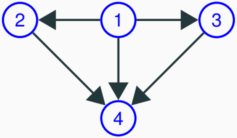
One way streets.
Tournament: 1 beats 2, ...
Precedence: 1 is before 2, ..

Social Network: Directed? Undirected?
Friends. Undirected.

Directed Graphs



$G = (V, E)$.
V - set of vertices.
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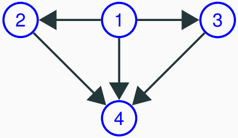
E ordered pairs of vertices.
$\{(1,2),(1,3),(1,4),(2,4),(3,4)\}$

One way streets.
Tournament: 1 beats 2, ...
Precedence: 1 is before 2, ..

Social Network: Directed? Undirected?
Friends. Undirected.
Likes.

Directed Graphs



$G = (V, E)$.
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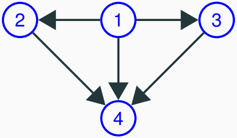
One way streets.
Tournament: 1 beats 2, ...
Precedence: 1 is before 2, ..

Social Network: Directed? Undirected?
Friends. Undirected.
Likes. Directed.

Directed Graphs



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One way streets.
Tournament: 1 beats 2, ...
Precedence: 1 is before 2, ..

Social Network: Directed? Undirected?
Friends. Undirected.
Likes. Directed.

Graph Concepts and Definitions.

Graph: $G = (V, E)$

Graph Concepts and Definitions.

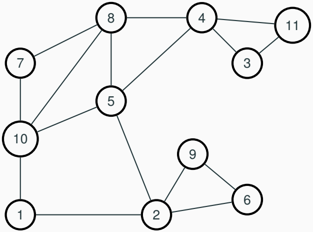
Graph: $G = (V, E)$

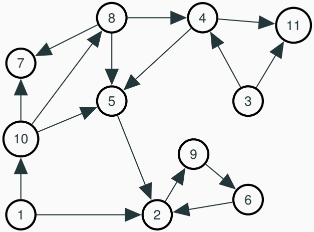
neighbors, adjacent, degree, incident, in-degree, out-degree

Graph Concepts and Definitions.

Graph: $G = (V, E)$

neighbors, adjacent, degree, incident, in-degree, out-degree



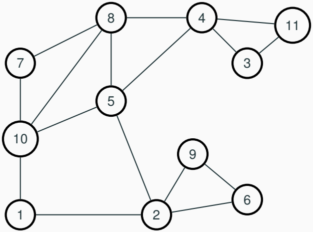


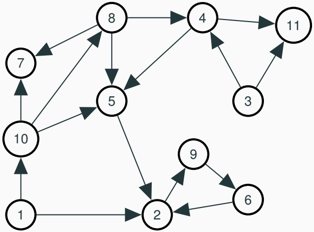
Neighbors of 10?

Graph Concepts and Definitions.

Graph: $G = (V, E)$

neighbors, adjacent, degree, incident, in-degree, out-degree

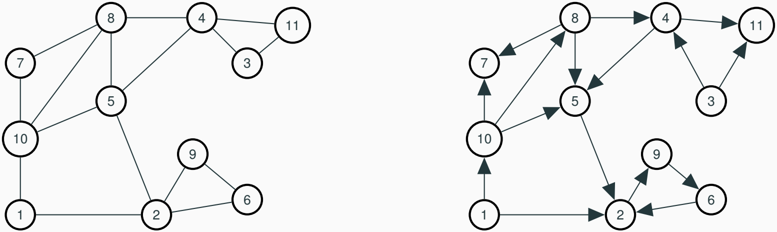




Neighbors of 10? 1,

Graph Concepts and Definitions.

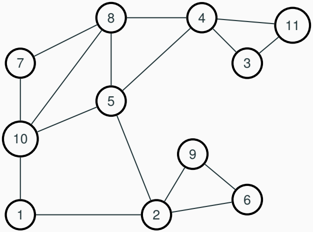
Graph: $G = (V, E)$
neighbors, adjacent, degree, incident, in-degree, out-degree

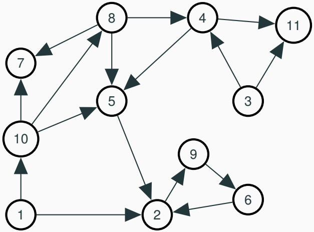


Neighbors of 10? 1,5,

Graph Concepts and Definitions.

Graph: $G = (V, E)$
neighbors, adjacent, degree, incident, in-degree, out-degree



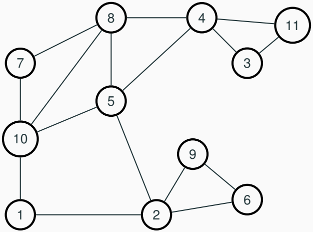


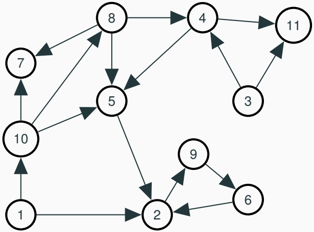
Neighbors of 10? 1,5,7,

Graph Concepts and Definitions.

Graph: $G = (V, E)$

neighbors, adjacent, degree, incident, in-degree, out-degree



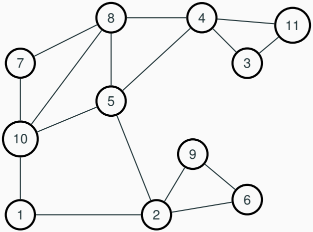


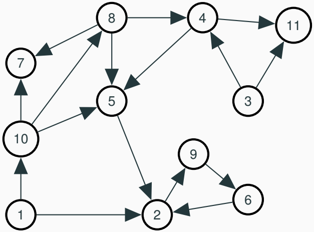
Neighbors of 10? 1,5,7, 8.

Graph Concepts and Definitions.

Graph: $G = (V, E)$

neighbors, adjacent, degree, incident, in-degree, out-degree





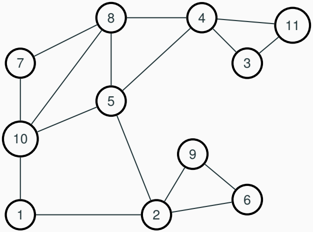
Neighbors of 10? 1,5,7, 8.

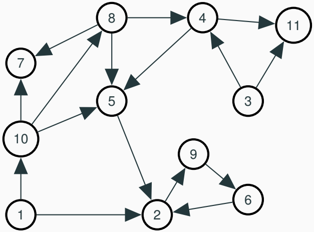
u is neighbor of v if $(u, v) \in E$.

Graph Concepts and Definitions.

Graph: $G = (V, E)$

neighbors, adjacent, degree, incident, in-degree, out-degree





Neighbors of 10? 1,5,7, 8.

u is neighbor of v if $(u, v) \in E$.

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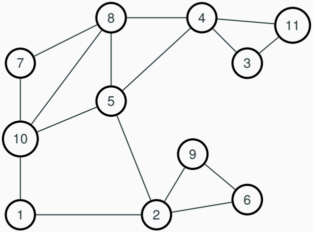
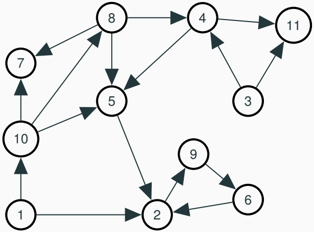
Graph Concepts and Definitions.

Graph: $G = (V, E)$
neighbors, adjacent, degree, incident, in-degree, out-degree





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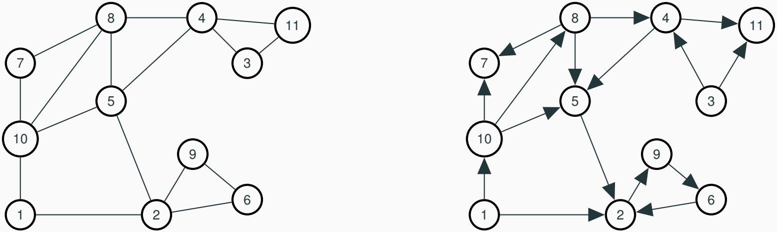

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Degree of vertex 1?

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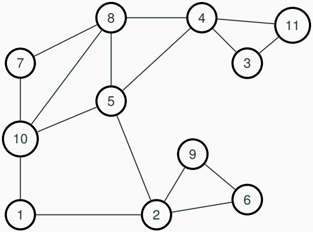

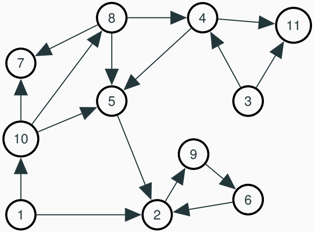

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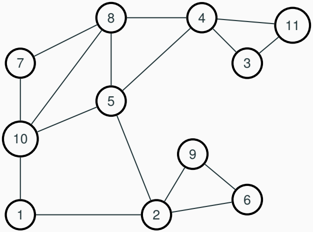

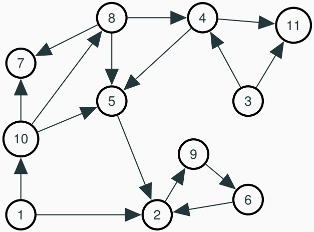

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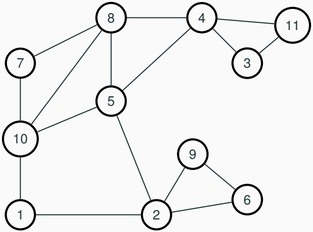

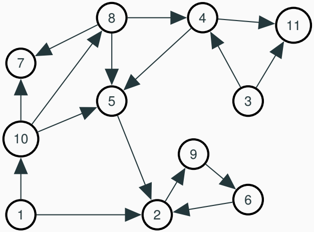

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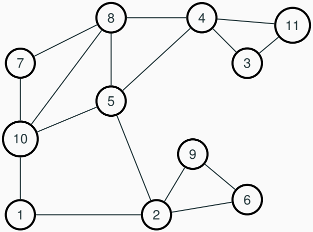

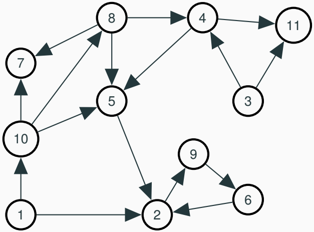

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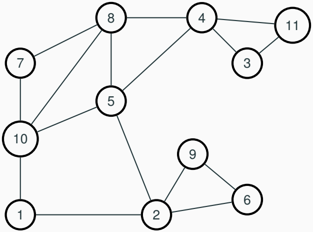

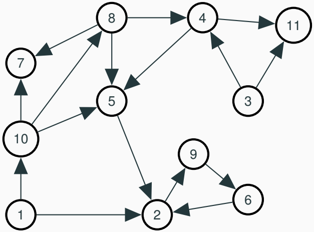

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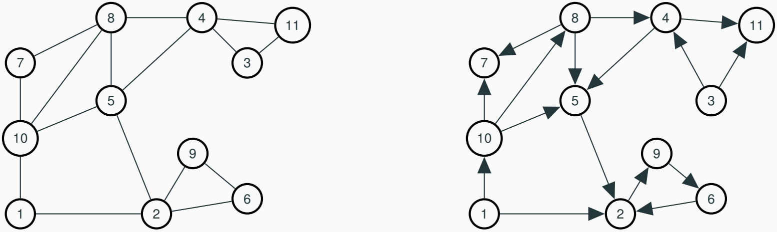
Directed graph?

Graph Concepts and Definitions.

Graph: $G = (V, E)$

neighbors, adjacent, degree, incident, in-degree, out-degree



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In-degree of 10?

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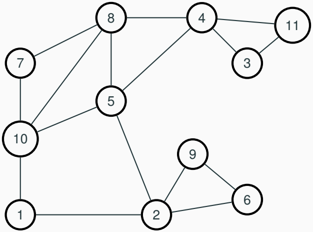

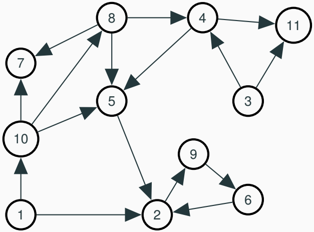

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Directed graph?

In-degree of 10? 1

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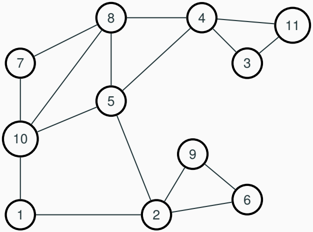

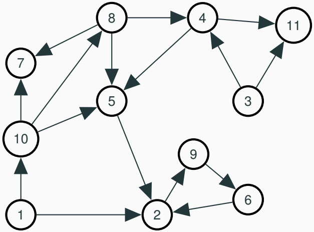

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In-degree of 10? 1 Out-degree of 10?

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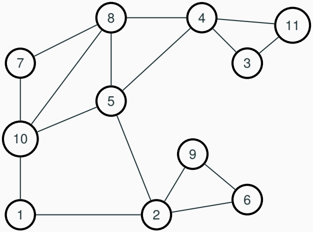

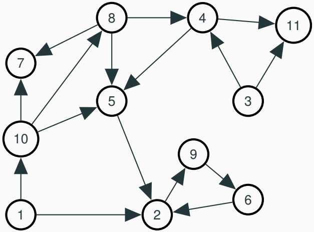

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Directed graph?

In-degree of 10? 1 Out-degree of 10? 3

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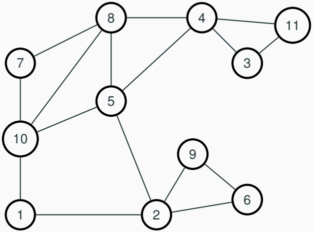

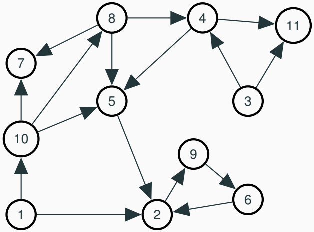

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Quick Proof.

The sum of the vertex degrees is equal to

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(C) What?

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Not (A)!

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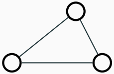

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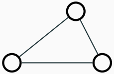

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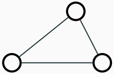

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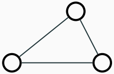

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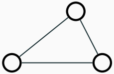

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What? For triangle number of edges is 3, the sum of degrees is 6.

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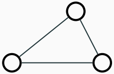

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Could it always be...

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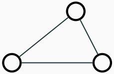

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Could it always be...$2|E|$?

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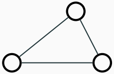

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How many incidences does each edge contribute?

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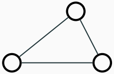

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How many incidences does each edge contribute? 2.

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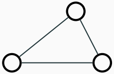

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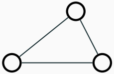

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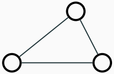

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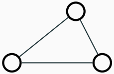

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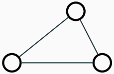

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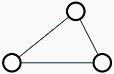

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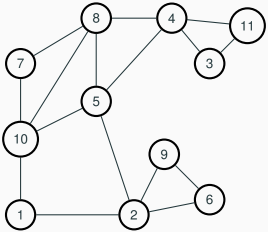
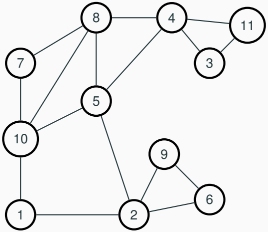
Thm: Sum of vertex degress is $2|E|$.

Paths, walks, cycles, tour.



A path in a graph is a sequence of edges.

Paths, walks, cycles, tour.



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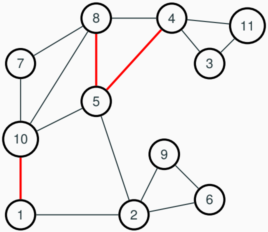
Path?

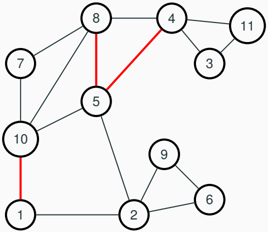
Paths, walks, cycles, tour.



A path in a graph is a sequence of edges.

Path? $\{1, 10\}$, $\{8,5\}$, $\{4,5\}$?

Paths, walks, cycles, tour.



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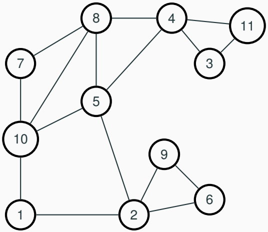

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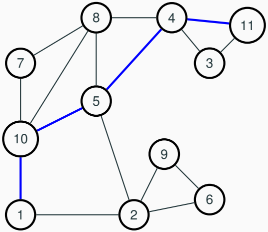

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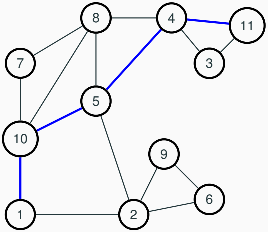
Paths, walks, cycles, tour.



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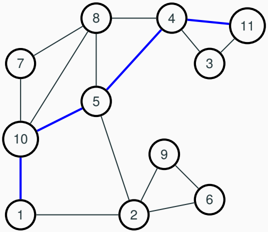

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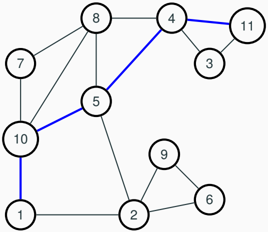
Paths, walks, cycles, tour.



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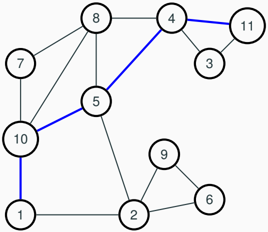
Paths, walks, cycles, tour.



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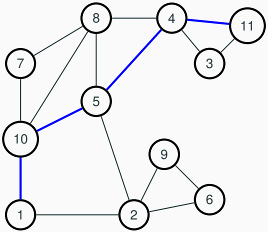
Quick Check! Length of path?

Paths, walks, cycles, tour.



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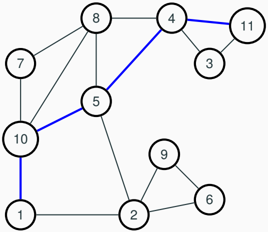
Quick Check! Length of path? k vertices

Paths, walks, cycles, tour.



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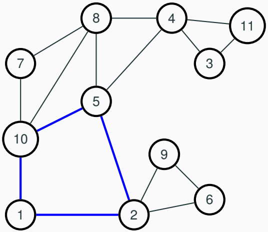
Quick Check! Length of path? k vertices or $k-1$ edges.

Paths, walks, cycles, tour.



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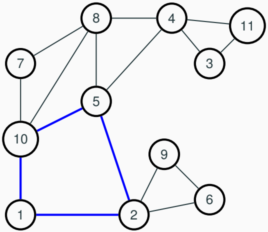
Cycle: Path with $v_1 = v_k$.

Paths, walks, cycles, tour.



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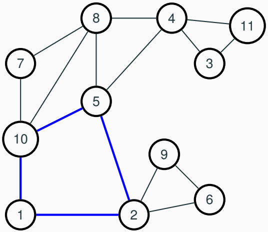

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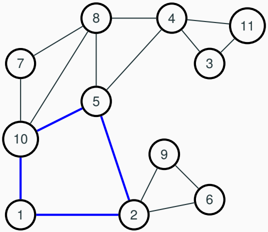
Paths, walks, cycles, tour.



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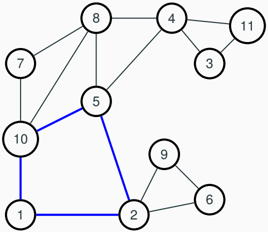
Paths, walks, cycles, tour.



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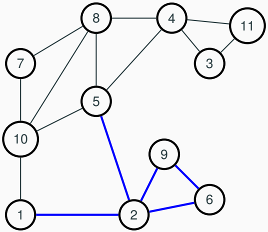

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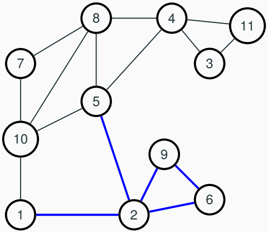
Walk is sequence of edges with possible repeated vertex or edge.

Paths, walks, cycles, tour.



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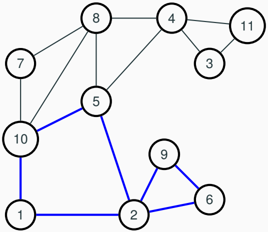

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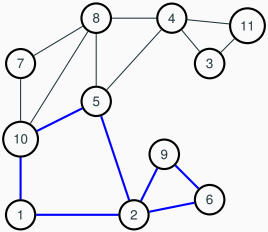

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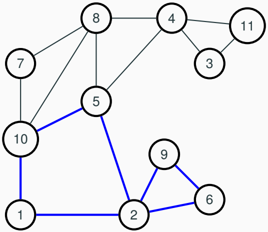

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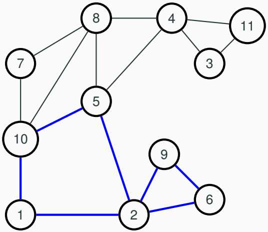

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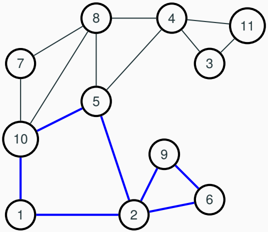
Path is to Walk as Cycle is to ??

Paths, walks, cycles, tour.



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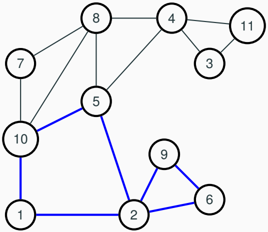

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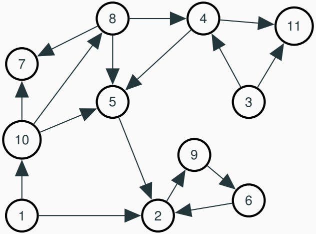

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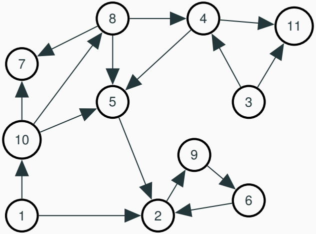

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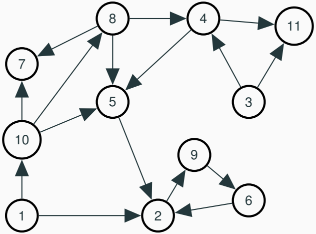

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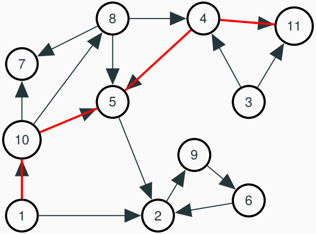

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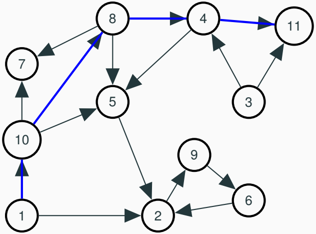

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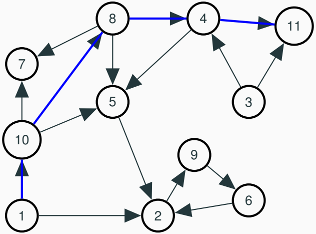

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Paths,

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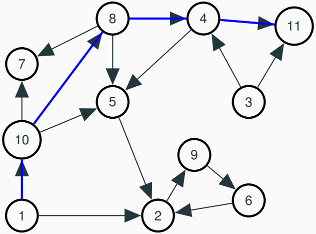

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Paths, walks,

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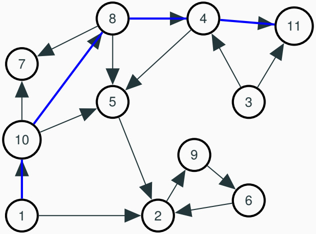

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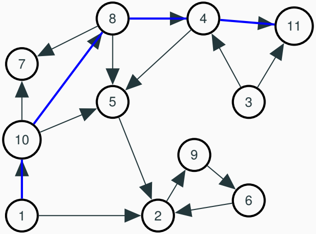

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Paths, walks, cycles, tours

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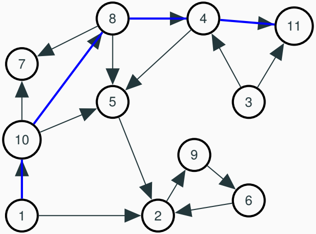

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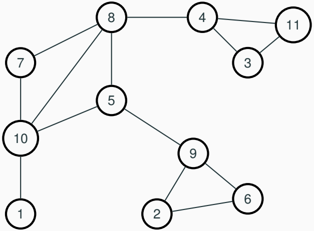
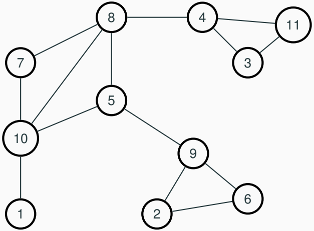
Paths, walks, cycles, tours ... are analagous to undirected now.

Connectivity



u and v are connected if there is a path between u and v.

Connectivity



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A connected graph is a graph where all pairs of vertices are connected.

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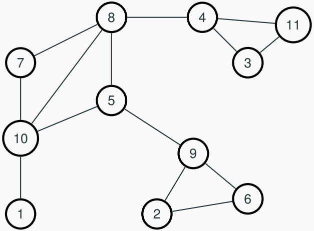

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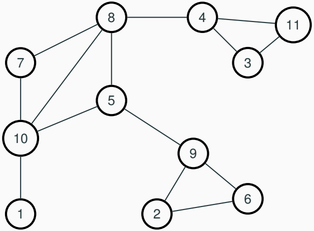

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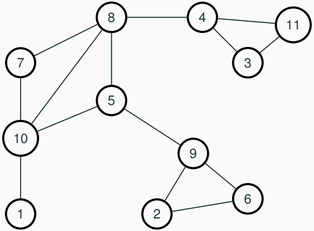
Is graph connected?

Connectivity



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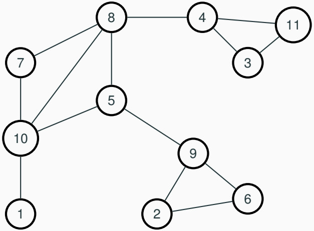
Is graph connected? Yes?

Connectivity



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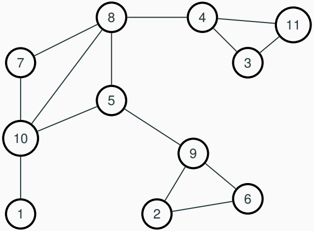

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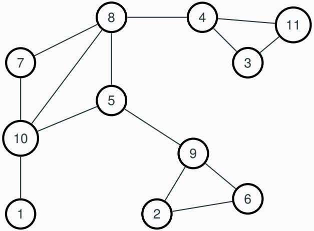
Proof:

Connectivity



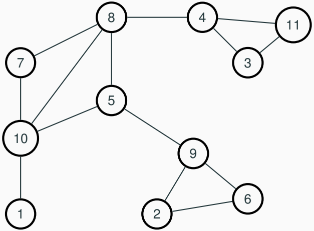
u and v are connected if there is a path between u and v.

A connected graph is a graph where all pairs of vertices are connected.

If one vertex x is connected to every other vertex.
Is graph connected? Yes? No?

Proof: Use path from u to x and then from x to v.

Connectivity



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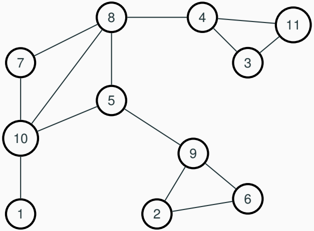

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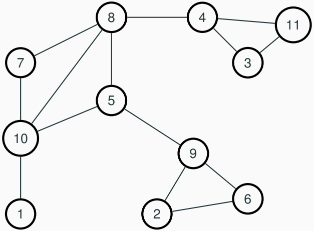

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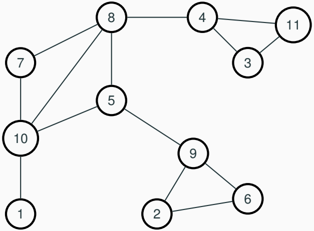

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Or cut out cycles.

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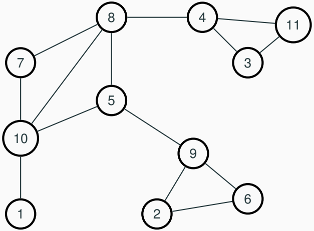

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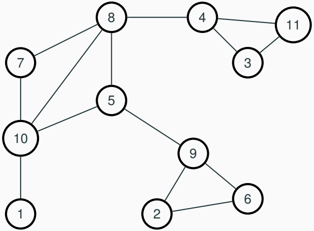

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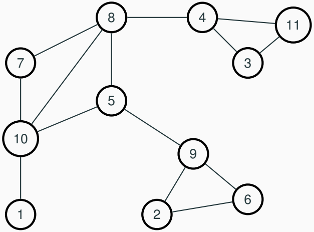
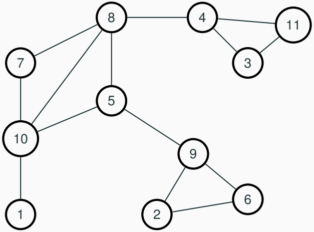
Connected component



Is graph above connected?

Connected component



Is graph above connected? Yes!

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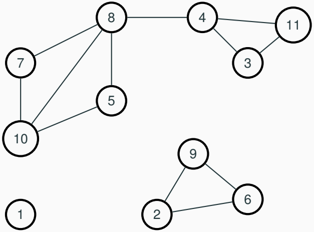
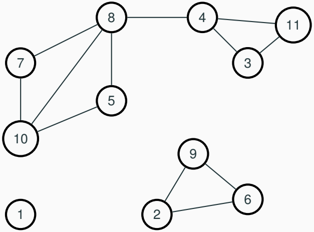


Is graph above connected? Yes!

How about now?

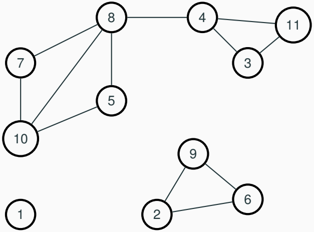
Connected component



Is graph above connected? Yes!

How about now? No!

Connected component

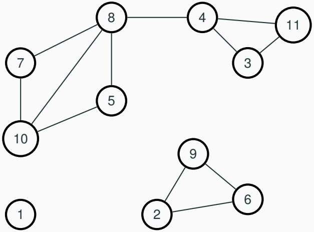


Is graph above connected? Yes!

How about now? No!

Connected Components?

Connected component

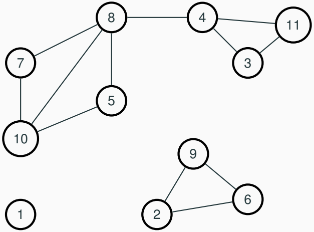


Is graph above connected? Yes!

How about now? No!

Connected Components? $\{1\},\{10,7,5,8,4,3,11\},\{2,9,6\}$.

Connected component



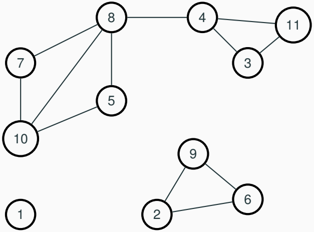
Is graph above connected? Yes!

How about now? No!

Connected Components? $\{1\}, \{10, 7, 5, 8, 4, 3, 11\}, \{2, 9, 6\}$.

Connected component - maximal set of connected vertices.

Connected component



Is graph above connected? Yes!

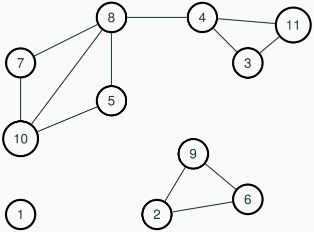
How about now? No!

Connected Components? $\{1\}, \{10, 7, 5, 8, 4, 3, 11\}, \{2, 9, 6\}$.

Connected component - maximal set of connected vertices.

Quick Check: Is $\{10, 7, 5\}$ a connected component?

Connected component



Is graph above connected? Yes!

How about now? No!

Connected Components? $\{1\},\{10,7,5,8,4,3,11\},\{2,9,6\}$.
Connected component - maximal set of connected vertices.
Quick Check: Is $\{10,7,5\}$ a connected component? No.

Thank you!

Congrats on surviving the first week!

Thank you!

Congrats on surviving the first week!Have a good weekend!

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Congrats on surviving the first week!Have a good weekend!

Don’t forget your homework, homework party tonight.