CS70: Discrete Math and Probability

Fan Ye

June 23, 2016

More inductions!

Bunch of examples

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Bunch of examples
Good ones

More inductions!

Bunch of examples
Good ones and bad ones

More inductions

Suppose I start with 0 written on a piece of paper. Each time, I choose a digit written on the paper and erase it. If it was a 0, I replace it with 010. If it was a 1, I replace it with 1001. Prove that it's not possible for me to get two 1's in a row.

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Induct on number of operations I make.

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Base case:

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Induct on number of operations I make.
Base case: After the first step, we get 010, which does not have two 1's in a row.

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Case 1: If we choose a 1, its neighbor must be 0's (based on ind hyp).

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By principle of induction, ... □

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Theorem: Every positive integer n can be written as a sum of distinct powers of 2.

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We can find a k such that $2^k \leq (n+1) < 2^{k+1}$

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Base on induction hypothesis, $n+1-2^k$ can be written as a sum of distinct powers of 2.

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Since $n+1-2^k < 2^{k+1} - 2^k = 2^k$, all terms must have power less than k,

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Done? Need to make sure 2^k is unique!
Since $n+1-2^k < 2^{k+1} - 2^k = 2^k$, all terms must have power less than k, thus 2^k must be unique in this sum for $n+1$.

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By principle of induction, ... $\square$

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Theorem: For all $n \geq 1$, $\sum_{i=1}^{n} \frac{1}{i^2} \leq 2$. ($S_n = \sum_{i=1}^{n} \frac{1}{i^2}$.)

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Base: $P(1)$.

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Base: $P(1)$. $1 \leq 2$.

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$$= \sum_{i=1}^{k} \frac{1}{i^2} + \frac{1}{(k+1)^2}.$$
$$\leq 2 + \frac{1}{(k+1)^2}$$

Uh oh?

Hmmm... It better be that any sum is *strictly less than* 2.

Strengthening: need to...

Theorem: For all $n \geq 1$, $\sum_{i=1}^{n} \frac{1}{i^2} \leq 2$. ($S_n = \sum_{i=1}^{n} \frac{1}{i^2}$.)

Base: $P(1)$. $1 \leq 2$.
Ind Step: $\sum_{i=1}^{k} \frac{1}{i^2} \leq 2$.

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How much less?

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How much less? At least by $\frac{1}{(k+1)^2}$ for S_k.

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Need to prove "$S_{k+1} \leq 2 - \frac{1}{(k+2)^2}$".

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Oooops.....

Strengthening: how?

Theorem: For all $n \geq 1$, $\sum_{i=1}^{n} \frac{1}{i^2} \leq 2 - f(n)$. ($S_n = \sum_{i=1}^{n} \frac{1}{i^2}$.)

Strengthening: how?

Theorem: For all $n \geq 1$, $\sum_{i=1}^{n} \frac{1}{i^2} \leq 2 - f(n)$. ($S_n = \sum_{i=1}^{n} \frac{1}{i^2}$.)

Proof:

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Ind hyp: $P(k)$

Strengthening: how?

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Proof:

Ind hyp: $P(k)$ — "$S_k \leq 2 - f(k)$"

Strengthening: how?

Theorem: For all $n \geq 1$, $\sum_{i=1}^{n} \frac{1}{i^2} \leq 2 - f(n)$. ($S_n = \sum_{i=1}^{n} \frac{1}{i^2}$.)

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Ind hyp: $P(k)$ — "$S_k \leq 2 - f(k)$"

Prove: $P(k+1)$

Strengthening: how?

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Ind hyp: $P(k)$ — "$S_k \leq 2 - f(k)$"

Prove: $P(k+1)$ – "$S_{k+1} \leq 2 - f(k+1)$"

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Ind hyp: $P(k)$ — “$S_k \leq 2 - f(k)$”

Prove: $P(k+1)$ – “$S_{k+1} \leq 2 - f(k+1)$”

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Ind hyp: $P(k)$ — "$S_k \leq 2 - f(k)$"

Prove: $P(k+1)$ – "$S_{k+1} \leq 2 - f(k+1)$"

$$S(k+1) = S_k + \frac{1}{(k+1)^2}$$

Strengthening: how?

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Prove: $P(k+1)$ – "$S_{k+1} \leq 2 - f(k+1)$"

$$\begin{aligned} S(k+1) &= S_k + \frac{1}{(k+1)^2} \\ &\leq 2 - f(k) + \frac{1}{(k+1)^2} \end{aligned}$$

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need to show: $\leq 2 - f(k+1)$

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Prove: $P(k+1)$ – “$S_{k+1} \leq 2 - f(k+1)$”

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Choose $f(k+1) \leq f(k) - \frac{1}{(k+1)^2}$.

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Can you?

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Can you?

Subtracting off a "quadratically decreasing" function every time.

Strengthening: how?

Theorem: For all $n \geq 1$, $\sum_{i=1}^{n} \frac{1}{i^2} \leq 2 - f(n)$. ($S_n = \sum_{i=1}^{n} \frac{1}{i^2}$.)

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Can you?

Subtracting off a "quadratically decreasing" function every time.

Maybe a "linearly decreasing" function to keep positive?

Strengthening: how?

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Try $f(k) = \frac{1}{k}$

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$$\frac{1}{k+1} \leq \frac{1}{k} - \frac{1}{(k+1)^2}?$$

$$1 \leq \frac{k+1}{k} - \frac{1}{k+1}$$

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Try $f(k) = \frac{1}{k}$

$\frac{1}{k+1} \leq \frac{1}{k} - \frac{1}{(k+1)^2}$?

$1 \leq \frac{k+1}{k} - \frac{1}{k+1}$ Multiplied by $k+1$.

$1 \leq 1 + (\frac{1}{k} - \frac{1}{k+1})$

Strengthening: how?

Theorem: For all $n \geq 1$, $\sum_{i=1}^{n} \frac{1}{i^2} \leq 2 - f(n)$. ($S_n = \sum_{i=1}^{n} \frac{1}{i^2}$.)

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$1 \leq 1 + (\frac{1}{k} - \frac{1}{k+1})$ Some math.

Strengthening: how?

Theorem: For all $n \geq 1$, $\sum_{i=1}^{n} \frac{1}{i^2} \leq 2 - f(n)$. ($S_n = \sum_{i=1}^{n} \frac{1}{i^2}$.)

Proof:

Ind hyp: $P(k)$ — "$S_k \leq 2 - f(k)$"

Prove: $P(k+1)$ – "$S_{k+1} \leq 2 - f(k+1)$"

$$
\begin{aligned}
S(k+1) &= S_k + \tfrac{1}{(k+1)^2} \\
&\leq 2 - f(k) + \tfrac{1}{(k+1)^2} \quad \text{By ind. hyp.} \\
&\text{need to show: } \leq 2 - f(k+1)
\end{aligned}
$$

Choose $f(k+1) \leq f(k) - \frac{1}{(k+1)^2}$.

$\implies S(k+1) \leq 2 - f(k+1)$.

Can you?

Subtracting off a "quadratically decreasing" function every time.

Maybe a "linearly decreasing" function to keep positive?

Try $f(k) = \frac{1}{k}$

$$\frac{1}{k+1} \leq \frac{1}{k} - \frac{1}{(k+1)^2}?$$

$$1 \leq \frac{k+1}{k} - \frac{1}{k+1} \quad \text{Multiplied by } k+1.$$

$$1 \leq 1 + \left(\frac{1}{k} - \frac{1}{k+1}\right) \quad \text{Some math. So yes!}$$

Strengthening: how?

Theorem: For all $n \geq 1$, $\sum_{i=1}^{n} \frac{1}{i^2} \leq 2 - f(n)$. ($S_n = \sum_{i=1}^{n} \frac{1}{i^2}$.)

Proof:

Ind hyp: $P(k)$ — "$S_k \leq 2 - f(k)$"

Prove: $P(k+1)$ – "$S_{k+1} \leq 2 - f(k+1)$"

$$S(k+1) = S_k + \frac{1}{(k+1)^2}$$
$$\leq 2 - f(k) + \frac{1}{(k+1)^2} \text{ By ind. hyp.}$$
need to show: $\leq 2 - f(k+1)$

Choose $f(k+1) \leq f(k) - \frac{1}{(k+1)^2}$.

$\implies S(k+1) \leq 2 - f(k+1)$.

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Theorem: For all $n \geq 1$, $\sum_{i=1}^{n} \frac{1}{i^2} \leq 2 - \frac{1}{n}$.

Careful!

Careful!

Horses of the same color...

Theorem: All horses have the same color.

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Base Case: $P(1)$ - trivially true.

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A horse in the middle in common!	$1, 2, 3, \ldots, k, k+1$

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New Base Case: $P(2)$: there are two horses with same color.

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Fix base case.

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(There are two horses is $\neq$ For all two horses!!!)

Of course it doesn't work.

As we will see, it is more subtle to catch errors in proofs of correct theorems!!

Good? Bad? –1

Use induction to prove the follow equality:

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$$n = \sqrt{1 + (n-1)\sqrt{1 + n\sqrt{1 + (n+1)\sqrt{1 + (n+2)\ldots}}}}$$

for all positive integers n.

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Proof by induction:

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Proof by induction:
Base case: for $n = 1$, $1 = \sqrt{1+0} = 1$, equality holds.

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Base case: for $n = 1$, $1 = \sqrt{1+0} = 1$, equality holds.
Induction hypothesis: Assume this equality holds for $n = k$, i.e.

$$k = \sqrt{1+(k-1)\sqrt{1+k\sqrt{1+(k+1)\sqrt{1+(k+2)\dots}}}}$$

Good? Bad? –2

Ind hyp:

$$k = \sqrt{1 + (k-1)\sqrt{1 + k\sqrt{1 + (k+1)\sqrt{1 + (k+2)\ldots}}}}$$

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Induction step:

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Induction step:Need to show it holds for $n = k+1$.

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Ind hyp:

$$k = \sqrt{1+(k-1)\sqrt{1+k\sqrt{1+(k+1)\sqrt{1+(k+2)\ldots}}}}$$

Induction step:Need to show it holds for $n = k+1$.
By square both sides of the induction hypothesis we can get:

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By square both sides of the induction hypothesis we can get:

$$k^2 = 1 + (k-1)\sqrt{1 + k\sqrt{1 + (k+1)\sqrt{1 + (k+2)\ldots}}}$$

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Easy to get

$$\frac{k^2-1}{k-1} = k+1 = \sqrt{1+k\sqrt{1+(k+1)\sqrt{1+(k+2)\dots}}}$$

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Ind hyp:

$$k = \sqrt{1+(k-1)\sqrt{1+k\sqrt{1+(k+1)\sqrt{1+(k+2)\ldots}}}}$$

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$$\frac{k^2-1}{k-1} = k+1 = \sqrt{1+k\sqrt{1+(k+1)\sqrt{1+(k+2)\ldots}}}$$

Therefore it holds for $n = k+1$, by principle of induction, ... $\square$

Good? Bad? –2

Ind hyp:

$$k = \sqrt{1+(k-1)\sqrt{1+k\sqrt{1+(k+1)\sqrt{1+(k+2)\ldots}}}}$$

Induction step:Need to show it holds for $n = k+1$.
By square both sides of the induction hypothesis we can get:

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Therefore it holds for $n = k+1$, by principle of induction, ... □Good or bad?

Good? Bad? –3

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Bad proof! We need $k \neq 1$ to divide both sides by $k - 1$

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Good? Bad? –3

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Or in other words, $p(1)$ does not imply $p(2)$

Be careful.

Note 4: Graph theory

Graphs!

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Definitions: model.

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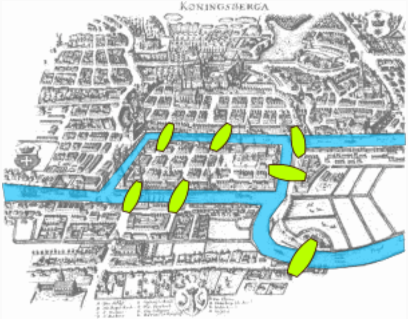
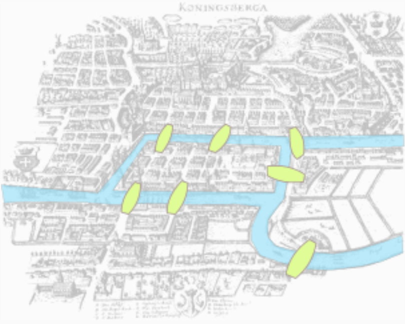
Definitions: model.

Konigsberg bridges problem.

Can you make a tour visiting each bridge exactly once?

Figure 1: “Konigsberg bridges” by Bogdan Giuşcă - License.





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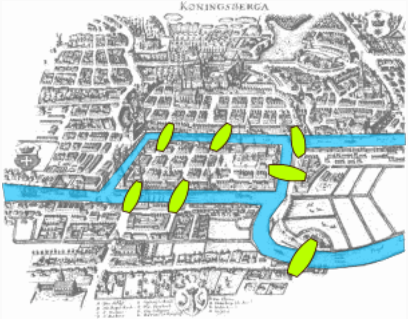

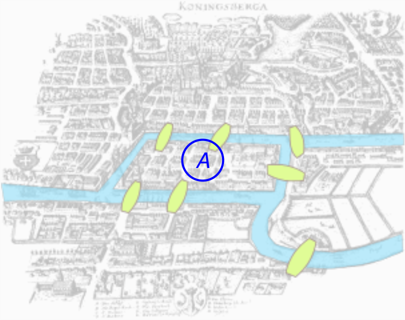

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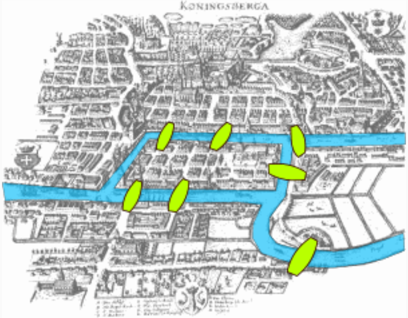

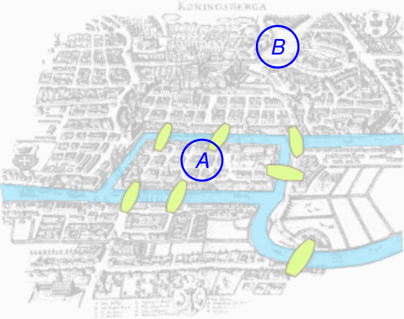

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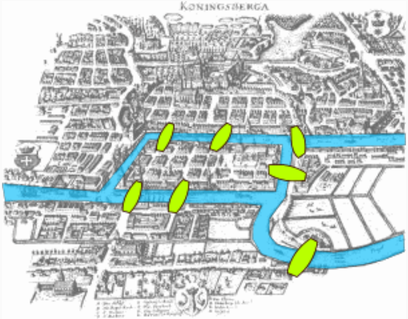

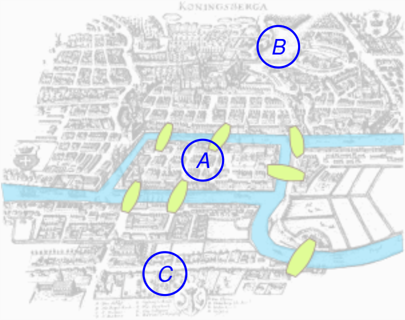

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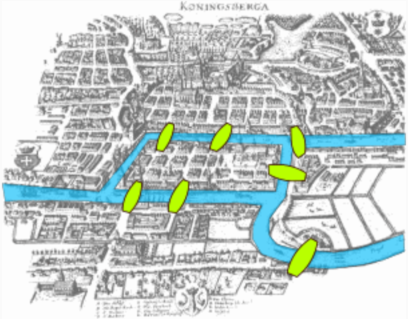

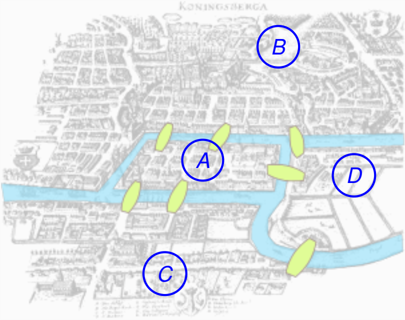

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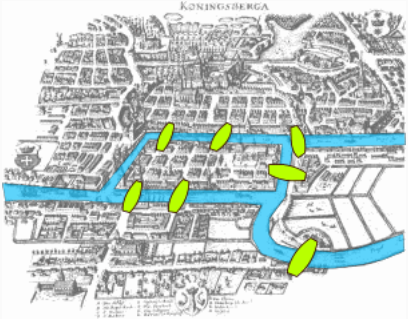

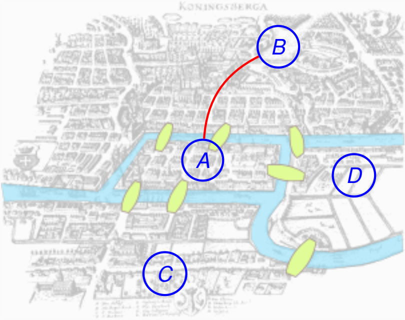

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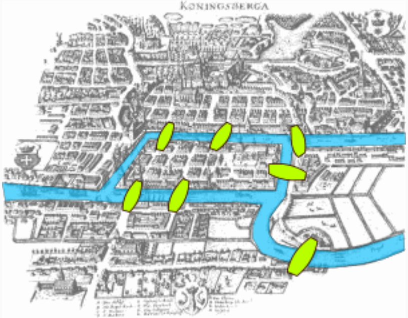

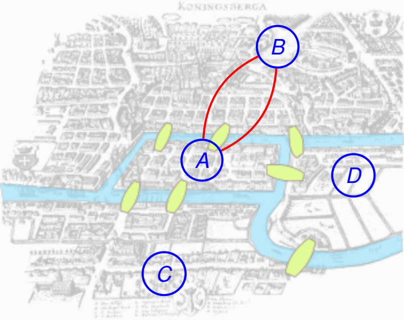

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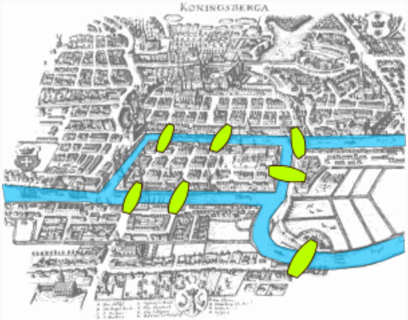

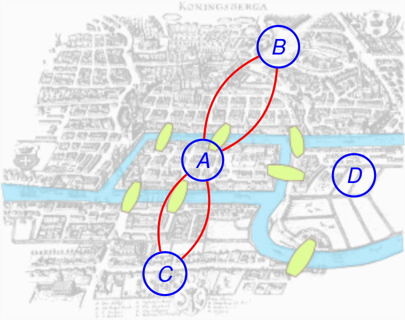

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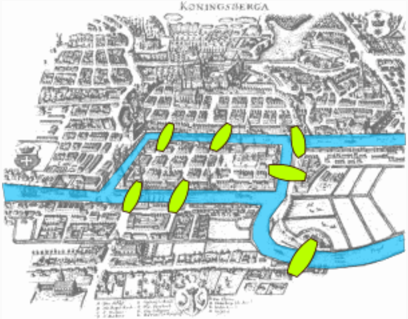

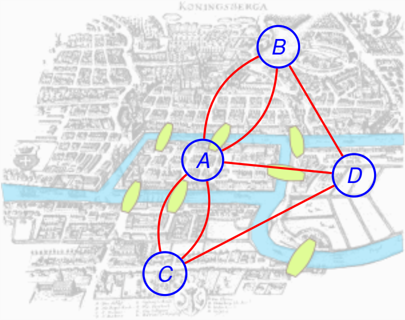

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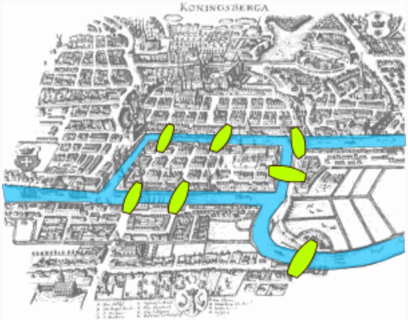

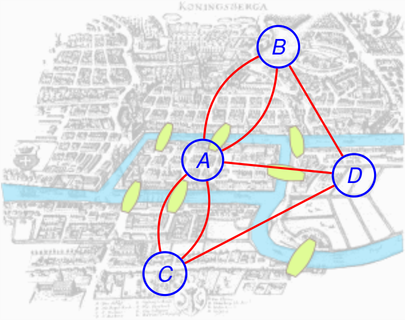

Can you draw a tour in the graph where you visit each edge once?

Konigsberg bridges problem.

Can you make a tour visiting each bridge exactly once?

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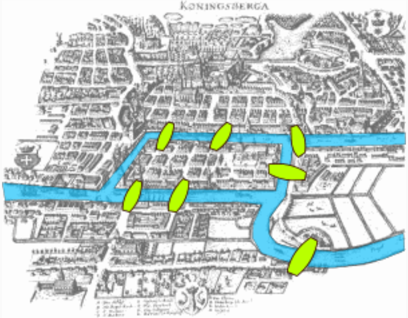

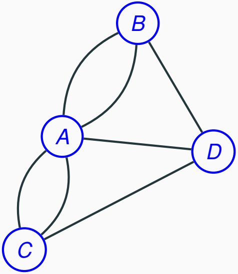

Can you draw a tour in the graph where you visit each edge once? Yes?

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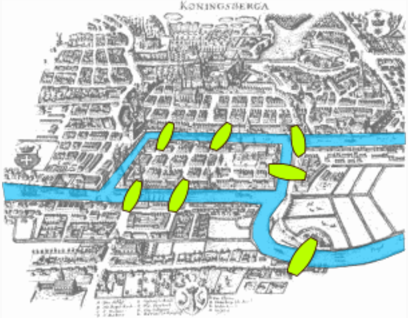
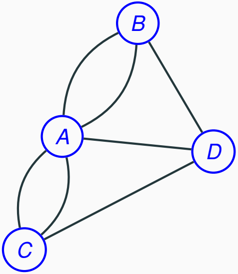
Can you make a tour visiting each bridge exactly once?

Figure 1: "Konigsberg bridges" by Bogdan Giuşcă - License.



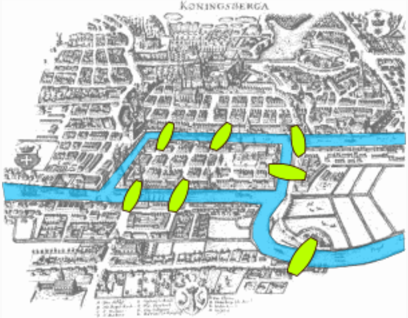


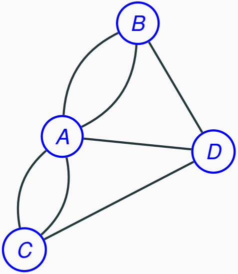
Can you draw a tour in the graph where you visit each edge once? Yes? No?

Konigsberg bridges problem.

Can you make a tour visiting each bridge exactly once?

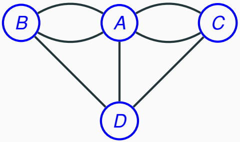
Figure 1: "Konigsberg bridges" by Bogdan Giuşcă - License.





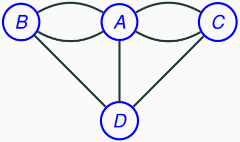
Can you draw a tour in the graph where you visit each edge once? Yes? No?
We will see!

Graphs: formally.



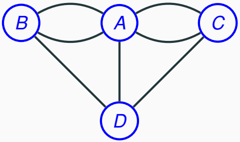
Graph:

Graphs: formally.



Graph: $G = (V, E)$.

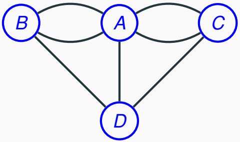
Graphs: formally.



Graph: $G = (V, E)$.

V - set of vertices.

Graphs: formally.

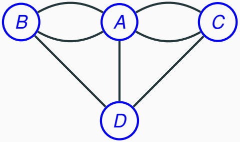


Graph: $G = (V, E)$.

V - set of vertices.

$\{A, B, C, D\}$

Graphs: formally.



Graph: $G = (V, E)$.

V - set of vertices.

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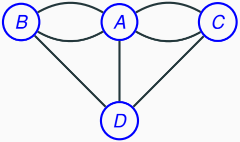
$E \subseteq V \times V$ -

Graphs: formally.



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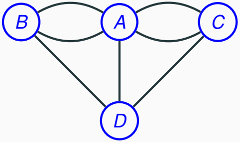

Graph: $G = (V, E)$.

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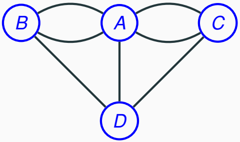
$E \subseteq V \times V$ - set of edges.

$\{\{A, B\}$

Graphs: formally.



Graph: $G = (V, E)$.

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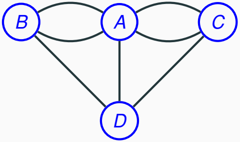
$E \subseteq V \times V$ - set of edges.

$\{\{A, B\}, \{A, B\}$

Graphs: formally.



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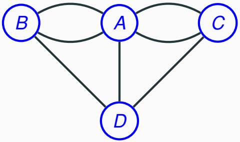

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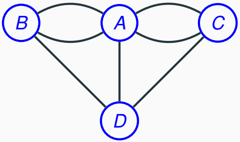
Graphs: formally.



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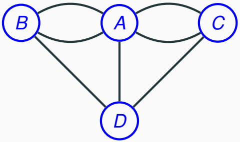
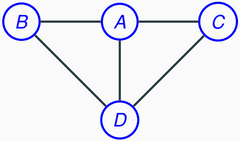
$\{\{A, B\}, \{A, B\}, \{A, C\}, \{B, C\}, \{B, D\}, \{B, D\}, \{C, D\}\}$.

For CS 70, usually simple graphs.

Graphs: formally.





Graph: $G = (V, E)$.
  V - set of vertices.
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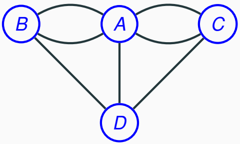
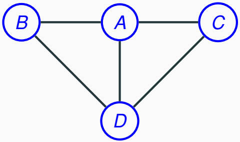
For CS 70, usually simple graphs.
  No parallel edges.

Graphs: formally.





Graph: $G = (V, E)$.

V - set of vertices.

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$E \subseteq V \times V$ - set of edges.

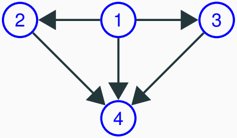
$\{\{A,B\},\{A,B\},\{A,C\},\{B,C\},\{B,D\},\{B,D\},\{C,D\}\}$.

For CS 70, usually simple graphs.

No parallel edges.

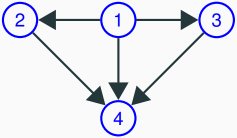
Multigraph above.

Directed Graphs



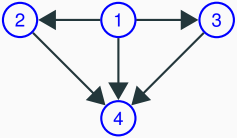
$G = (V, E)$.

Directed Graphs



$G = (V, E)$.
V - set of vertices.

Directed Graphs

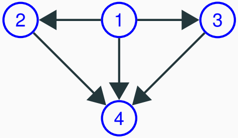


$G = (V, E)$.

V - set of vertices.

$\{1, 2, 3, 4\}$

Directed Graphs



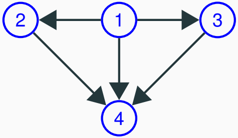
$G = (V, E)$.

V - set of vertices.

$\{1, 2, 3, 4\}$

E ordered pairs of vertices.

Directed Graphs



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V - set of vertices.

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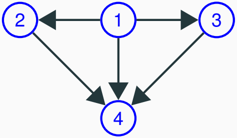
E ordered pairs of vertices.

$\{(1, 2),$

Directed Graphs



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V - set of vertices.

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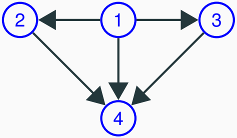
E ordered pairs of vertices.

$\{(1, 2), (1, 3),$

Directed Graphs



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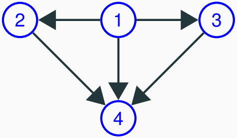
Directed Graphs



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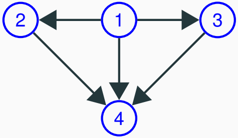
Directed Graphs



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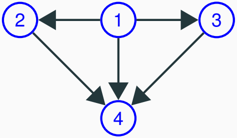
$\{(1,2),(1,3),(1,4),(2,4),(3,4)\}$

One way streets.

Directed Graphs



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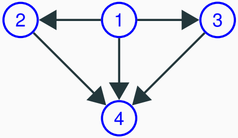
One way streets.
Tournament:

Directed Graphs



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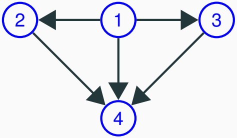
One way streets.
Tournament: 1 beats 2,

Directed Graphs



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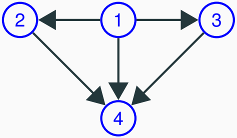
One way streets.
Tournament: 1 beats 2, ...
Precedence:

Directed Graphs



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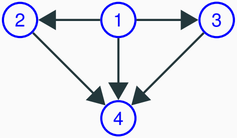
One way streets.
Tournament: 1 beats 2, ...
Precedence: 1 is before 2,

Directed Graphs



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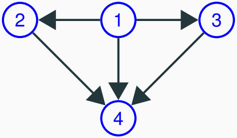
Directed Graphs



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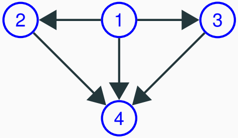
One way streets.
Tournament: 1 beats 2, ...
Precedence: 1 is before 2, ..

Social Network:

Directed Graphs



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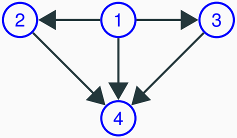
One way streets.
Tournament: 1 beats 2, ...
Precedence: 1 is before 2, ..

Social Network: Directed?

Directed Graphs



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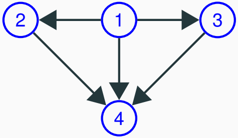
One way streets.
Tournament: 1 beats 2, ...
Precedence: 1 is before 2, ..

Social Network: Directed? Undirected?

Directed Graphs



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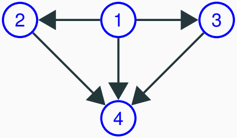
One way streets.
Tournament: 1 beats 2, ...
Precedence: 1 is before 2, ..

Social Network: Directed? Undirected?
Friends.

Directed Graphs



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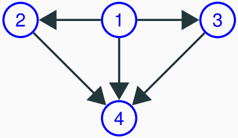
One way streets.
Tournament: 1 beats 2, ...
Precedence: 1 is before 2, ..

Social Network: Directed? Undirected?
Friends. Undirected.

Directed Graphs



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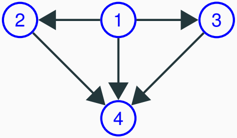
One way streets.
Tournament: 1 beats 2, ...
Precedence: 1 is before 2, ..

Social Network: Directed? Undirected?
Friends. Undirected.
Likes.

Directed Graphs



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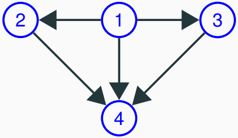
One way streets.
Tournament: 1 beats 2, ...
Precedence: 1 is before 2, ..

Social Network: Directed? Undirected?
Friends. Undirected.
Likes. Directed.

Directed Graphs



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One way streets.
Tournament: 1 beats 2, ...
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Social Network: Directed? Undirected?
Friends. Undirected.
Likes. Directed.

Graph Concepts and Definitions.

Graph: $G = (V, E)$

Graph Concepts and Definitions.

Graph: $G = (V, E)$

neighbors, adjacent, degree, incident, in-degree, out-degree

Graph Concepts and Definitions.

Graph: $G = (V, E)$

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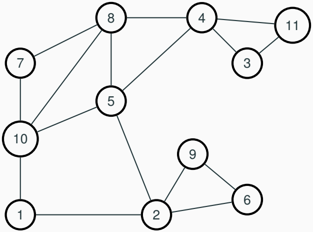
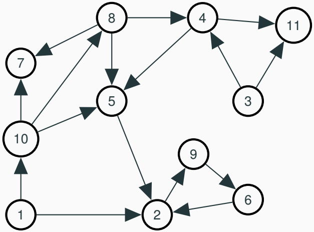
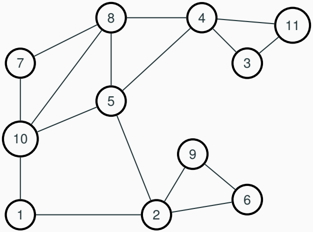


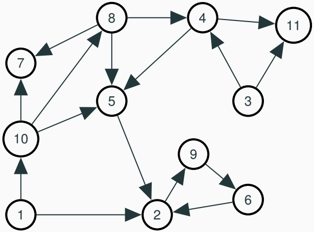


Neighbors of 10?

Graph Concepts and Definitions.

Graph: $G = (V, E)$
neighbors, adjacent, degree, incident, in-degree, out-degree



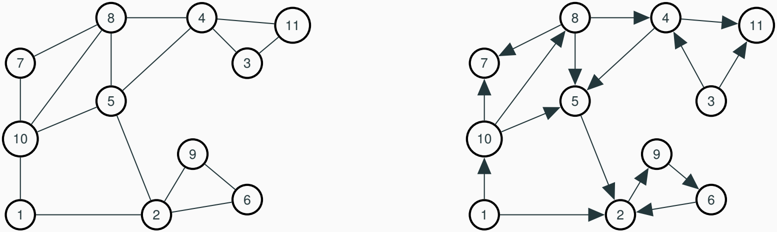


Neighbors of 10? 1,

Graph Concepts and Definitions.

Graph: $G = (V, E)$

neighbors, adjacent, degree, incident, in-degree, out-degree

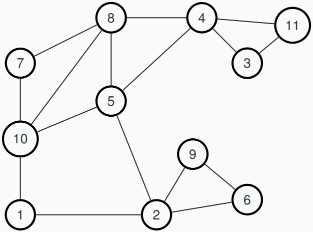


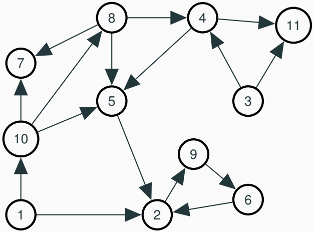
Neighbors of 10? 1,5,

Graph Concepts and Definitions.

Graph: $G = (V, E)$

neighbors, adjacent, degree, incident, in-degree, out-degree



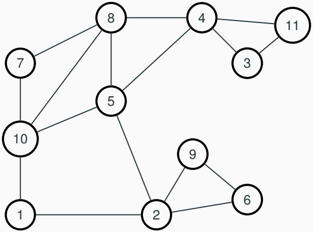


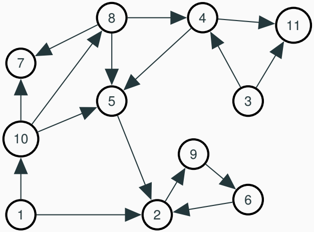
Neighbors of 10? 1,5,7,

Graph Concepts and Definitions.

Graph: $G = (V, E)$

neighbors, adjacent, degree, incident, in-degree, out-degree





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Graph Concepts and Definitions.

Graph: $G = (V, E)$

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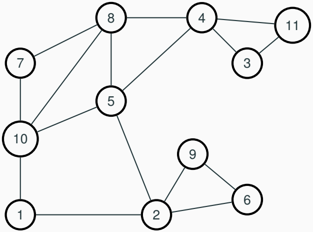

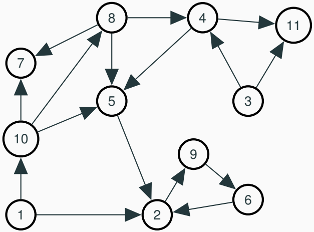

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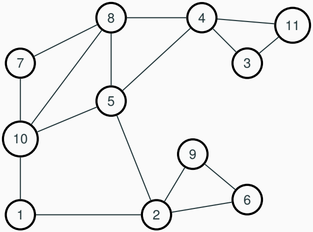

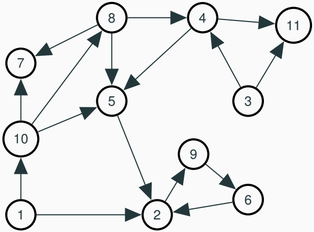

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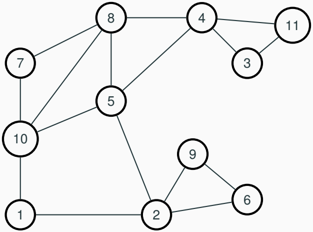

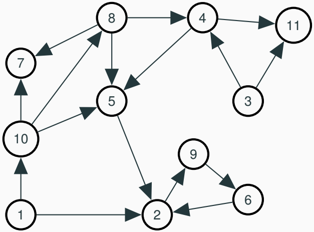

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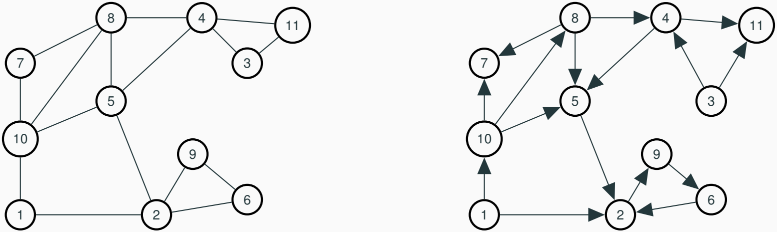

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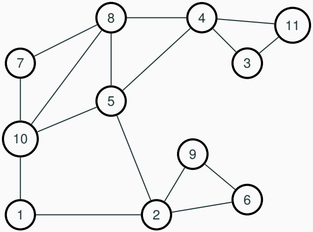

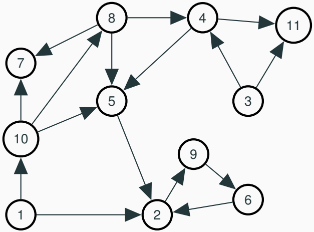

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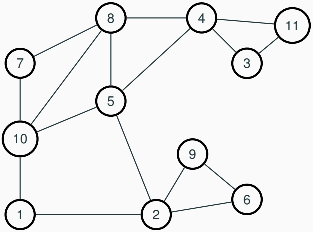

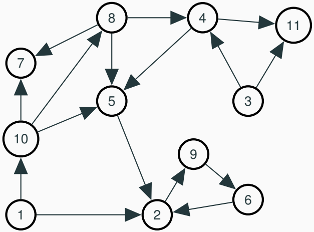

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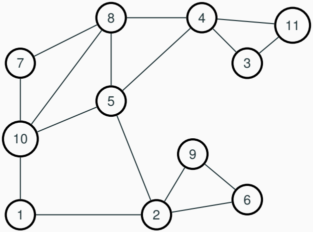

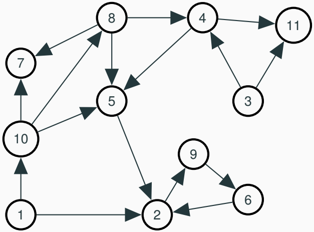

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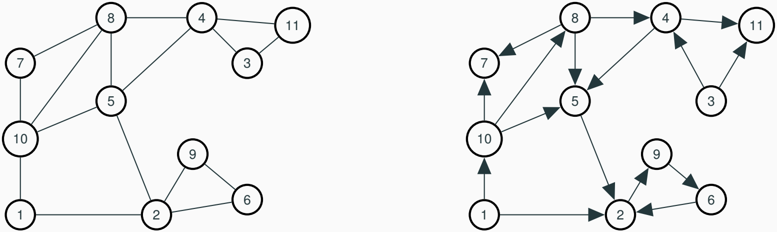

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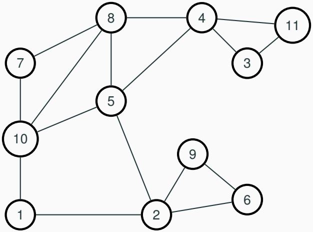

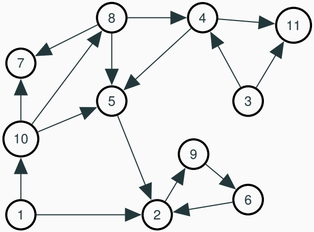

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Directed graph?

Graph Concepts and Definitions.

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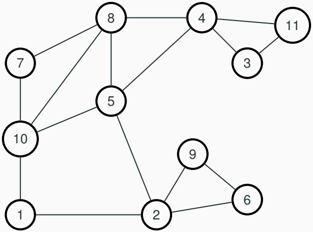

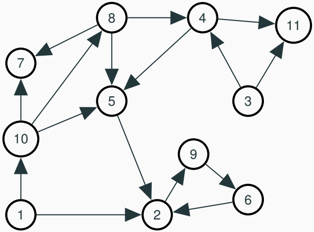

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In-degree of 10?

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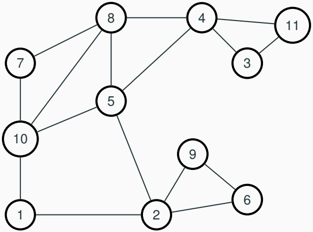

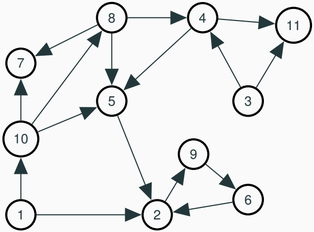

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Directed graph?

In-degree of 10? 1

Graph Concepts and Definitions.

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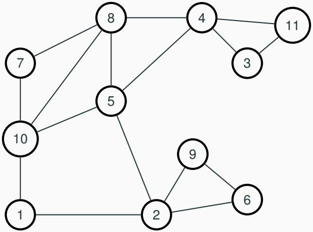
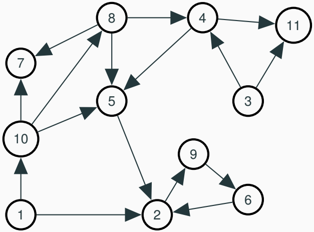
neighbors, adjacent, degree, incident, in-degree, out-degree





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In-degree of 10? 1 Out-degree of 10?

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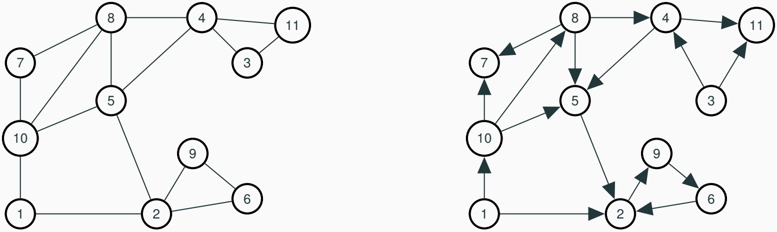

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Directed graph?

In-degree of 10? 1 Out-degree of 10? 3

Graph Concepts and Definitions.

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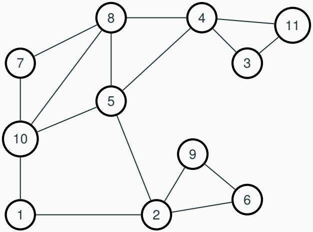

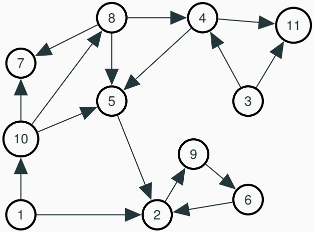

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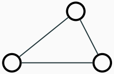

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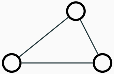

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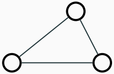

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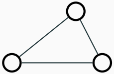

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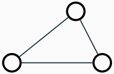

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What? For triangle number of edges is 3, the sum of degrees is 6.

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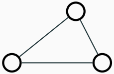

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Could it always be...

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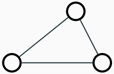

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Could it always be...$2|E|$?

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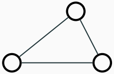

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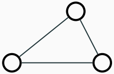

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How many incidences does each edge contribute? 2.

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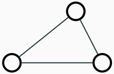

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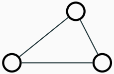

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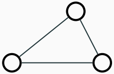

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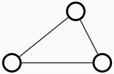

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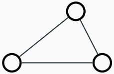

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sum of degrees is total incidences ... or $2|E|$.

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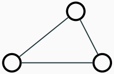


Not (B)! Triangle.

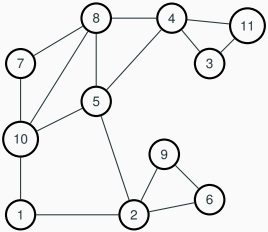
What? For triangle number of edges is 3, the sum of degrees is 6.

Could it always be...$2|E|$?

How many incidences does each edge contribute? 2.
$2|E|$ incidences are contributed in total!
What is degree v? incidences contributed to v!
sum of degrees is total incidences ... or $2|E|$.

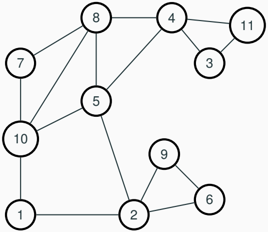
Thm: Sum of vertex degress is $2|E|$.

Paths, walks, cycles, tour.



A path in a graph is a sequence of edges.

Paths, walks, cycles, tour.



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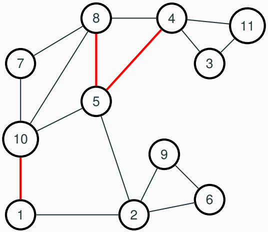
Paths, walks, cycles, tour.



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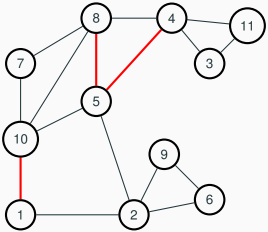
Path? $\{1,10\}$, $\{8,5\}$, $\{4,5\}$?

Paths, walks, cycles, tour.



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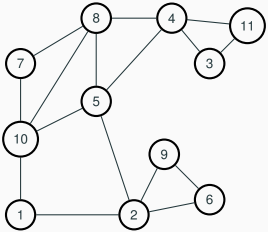

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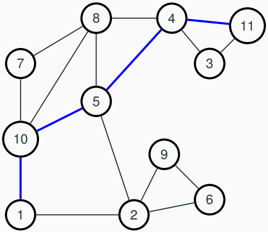

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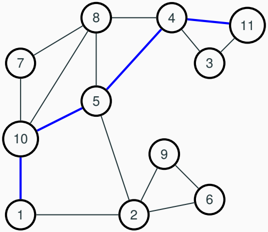
Paths, walks, cycles, tour.



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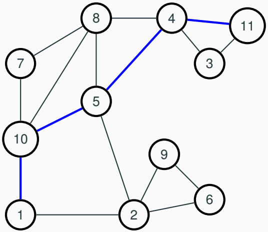

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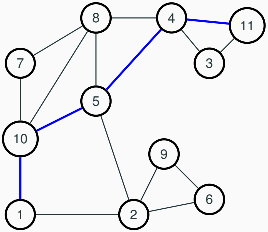
Paths, walks, cycles, tour.



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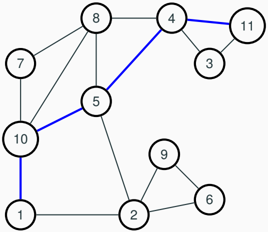
Quick Check!

Paths, walks, cycles, tour.



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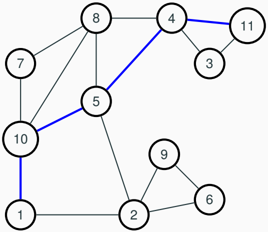
Quick Check! Length of path?

Paths, walks, cycles, tour.



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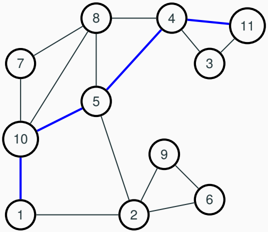
Quick Check! Length of path? k vertices

Paths, walks, cycles, tour.



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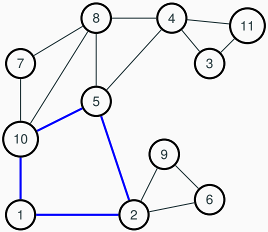
Quick Check! Length of path? k vertices or $k-1$ edges.

Paths, walks, cycles, tour.



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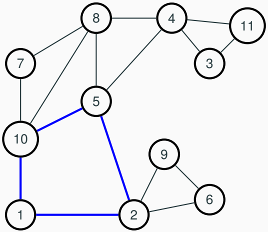
Cycle: Path with $v_1 = v_k$.



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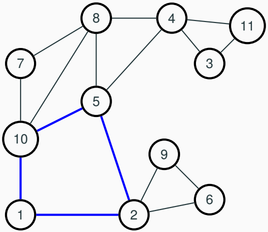
Paths, walks, cycles, tour.



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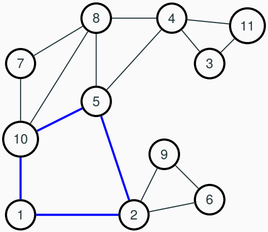
Paths, walks, cycles, tour.



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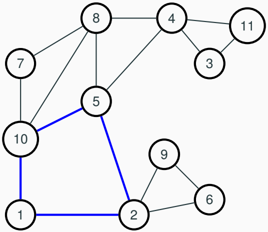
Paths, walks, cycles, tour.



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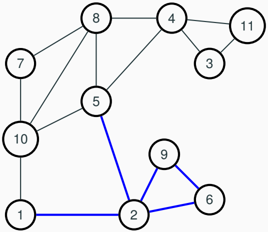
Paths, walks, cycles, tour.



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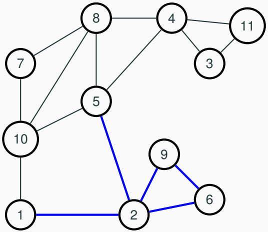
Walk is sequence of edges with possible repeated vertex or edge.

Paths, walks, cycles, tour.



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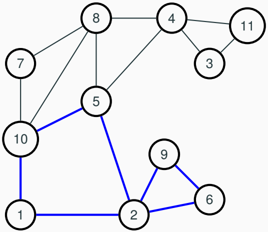

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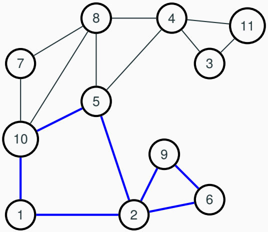

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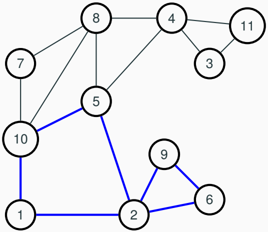

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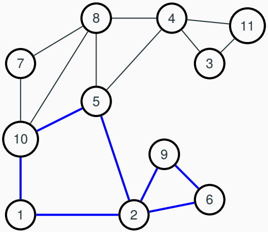

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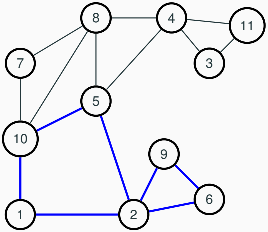
Path is to Walk as Cycle is to ??

Paths, walks, cycles, tour.



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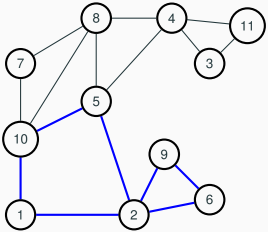

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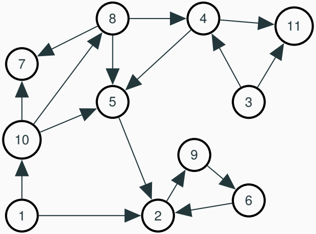

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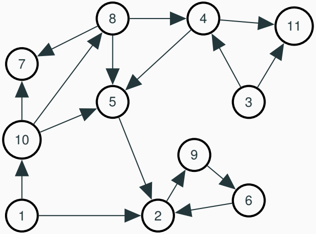

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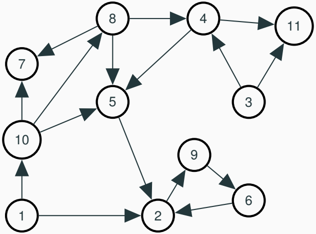

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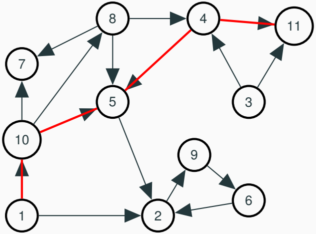

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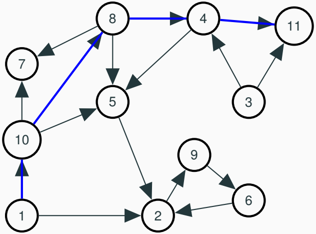

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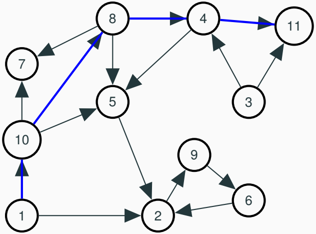

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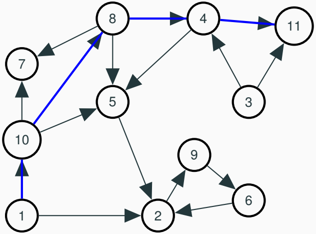

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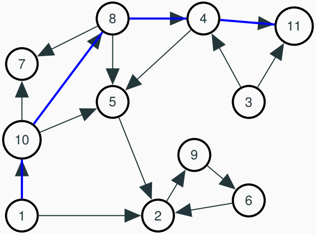

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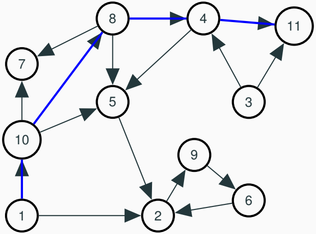

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Paths, walks, cycles, tours

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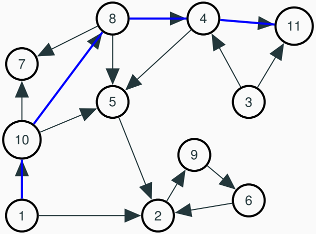

Path: $(v_1, v_2), (v_2, v_3), \ldots (v_{k-1}, v_k)$.
Paths, walks, cycles, tours ... are analagous to undirected now.

Thank you!

Congrats on surviving the first week!

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Have a good weekend!

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Don’t forget your homework, homework party tonight.