Today

Review for Midterm.

First there was logic...

A statement is a true or false.

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Don’t worry about Gödel.

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Statements?

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Statements?
$3 = 4 - 1$?

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Statements?

$3 = 4 - 1$? Statement!

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Statements?

$3 = 4 - 1$? Statement!

$3 = 5$?

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3 ?

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Statements?

$3 = 4 - 1$? Statement!

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$n = 3$? Not a statement...

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$3 = 4 - 1$? Statement!
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$n = 3$? Not a statement...but a predicate.

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Predicate: Statement with free variable(s).

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- $3 = 4 - 1$? Statement!
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Predicate: Statement with free variable(s).

- Example: $x = 3$

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Predicate?

$n > 3$?

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$n = 3$? Not a statement...but a predicate.

Predicate: Statement with free variable(s).

Example: $x = 3$ Given a value for x, becomes a statement.

Predicate?

$n > 3$? Predicate: $P(n)$!

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Example: $x = 3$ Given a value for x, becomes a statement.
Predicate?
$n > 3$? Predicate: $P(n)$!
$x = y$?

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Predicate: Statement with free variable(s).

Example: $x = 3$ Given a value for x, becomes a statement.

Predicate?

$n > 3$? Predicate: $P(n)$!

$x = y$? Predicate: $P(x, y)$!

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Predicate: Statement with free variable(s).

- Example: $x = 3$ Given a value for x, becomes a statement.

Predicate?

- $n > 3$? Predicate: $P(n)$!
- $x = y$? Predicate: $P(x, y)$!
- $x + y$?

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$x + y$? No.

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$x + y$? No. An expression, not a statement.

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Quantifiers:

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Quantifiers:

$(\forall x)\ P(x)$.

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  $(\forall x)\ P(x)$.   For every x, $P(x)$ is true.

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$(\forall n \in N), n^2 \geq n$:

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$(\forall n \in N), n^2 \geq n$: Any free variables? No.

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$(\forall x)\ P(x)$. For every x, $P(x)$ is true.
$(\exists x)\ P(x)$. There exists an x, where $P(x)$ is true.

$(\forall n \in N), n^2 \geq n$: Any free variables? No. So it's a statement.
$(\forall x \in R)(\exists y \in R) y > x$.

Connecting Statements

$A \wedge B$, $A \vee B$, $\neg A$, $A \implies B$.

Connecting Statements

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Propositional Expressions and Logical Equivalence

Connecting Statements

$A \wedge B$, $A \vee B$, $\neg A$, $A \implies B$.

Propositional Expressions and Logical Equivalence

$(A \implies B) \equiv (\neg A \vee B)$

Connecting Statements

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Propositional Expressions and Logical Equivalence

$(A \implies B) \equiv (\neg A \vee B)$
$\neg(A \vee B) \equiv (\neg A \wedge \neg B)$

Connecting Statements

$A \wedge B$, $A \vee B$, $\neg A$, $A \implies B$.

Propositional Expressions and Logical Equivalence

$(A \implies B) \equiv (\neg A \vee B)$
$\neg(A \vee B) \equiv (\neg A \wedge \neg B)$

Proofs: truth table or manipulation of known formulas.

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$(\forall x \in \mathbb{R})(P(x) \wedge Q(x)) \equiv (\forall x \in \mathbb{R})P(x) \wedge (\forall x \in \mathbb{R})Q(x)$

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If you think it's true:

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If you think it's true:

Step 1:

Connecting Statements

$A \land B$, $A \lor B$, $\neg A$, $A \implies B$.

Propositional Expressions and Logical Equivalence

$$(A \implies B) \equiv (\neg A \lor B)$$
$$\neg(A \lor B) \equiv (\neg A \land \neg B)$$

Proofs: truth table or manipulation of known formulas.

$$(\forall x \in \mathbb{R})(P(x) \land Q(x)) \equiv (\forall x \in \mathbb{R})P(x) \land (\forall x \in \mathbb{R})Q(x)$$

If you think it's true:

Step 1: Show that when the thing on the left is true, the thing on the right is true.

Connecting Statements

$A \wedge B$, $A \vee B$, $\neg A$, $A \implies B$.

Propositional Expressions and Logical Equivalence

$$(A \implies B) \equiv (\neg A \vee B)$$
$$\neg(A \vee B) \equiv (\neg A \wedge \neg B)$$

Proofs: truth table or manipulation of known formulas.

$$(\forall x \in \mathbb{R})(P(x) \wedge Q(x)) \equiv (\forall x \in \mathbb{R})P(x) \wedge (\forall x \in \mathbb{R})Q(x)$$

If you think it's true:

Step 1: Show that when the thing on the left is true, the thing on the right is true. No matter what P and Q are!

Connecting Statements

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$$(A \implies B) \equiv (\neg A \vee B)$$
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If you think it's true:

Step 1: Show that when the thing on the left is true, the thing on the right is true. No matter what P and Q are!

Step 2: Show that when the thing on the right is true, the thing on the left is true. No matter what P and Q are!

Or manipulate the formulas.

Connecting Statements

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If you think it's true:

Step 1: Show that when the thing on the left is true, the thing on the right is true. No matter what P and Q are!

Step 2: Show that when the thing on the right is true, the thing on the left is true. No matter what P and Q are!

Or manipulate the formulas.

If you think it's not true:

Connecting Statements

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Propositional Expressions and Logical Equivalence

$$(A \implies B) \equiv (\neg A \vee B)$$
$$\neg(A \vee B) \equiv (\neg A \wedge \neg B)$$

Proofs: truth table or manipulation of known formulas.

$$(\forall x \in \mathbb{R})(P(x) \wedge Q(x)) \equiv (\forall x \in \mathbb{R})P(x) \wedge (\forall x \in \mathbb{R})Q(x)$$

If you think it's true:

Step 1: Show that when the thing on the left is true, the thing on the right is true. No matter what P and Q are!

Step 2: Show that when the thing on the right is true, the thing on the left is true. No matter what P and Q are!

Or manipulate the formulas.

If you think it's not true:

Find an example of $P(x)$ and $Q(x)$

Connecting Statements

$A \wedge B$, $A \vee B$, $\neg A$, $A \implies B$.

Propositional Expressions and Logical Equivalence

$$(A \implies B) \equiv (\neg A \vee B)$$
$$\neg(A \vee B) \equiv (\neg A \wedge \neg B)$$

Proofs: truth table or manipulation of known formulas.

$$(\forall x \in \mathbb{R})(P(x) \wedge Q(x)) \equiv (\forall x \in \mathbb{R})P(x) \wedge (\forall x \in \mathbb{R})Q(x)$$

If you think it's true:

Step 1: Show that when the thing on the left is true, the thing on the right is true. No matter what P and Q are!

Step 2: Show that when the thing on the right is true, the thing on the left is true. No matter what P and Q are!

Or manipulate the formulas.

If you think it's not true:

Find an example of $P(x)$ and $Q(x)$ such that one of the above steps fails.

...and then proofs...

Direct: $P \implies Q$

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Direct: $P \implies Q$
Example: a is even $\implies$ a^2 is even.

...and then proofs...

Direct: $P \implies Q$

Example: a is even $\implies a^2$ is even.

Approach: What is even?

...and then proofs...

Direct: $P \implies Q$

Example: a is even $\implies$ a^2 is even.

Approach: What is even? $a = 2k$

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$a^2 = 4k^2$

...and then proofs...

Direct: $P \implies Q$

Example: a is even $\implies a^2$ is even.

Approach: What is even? $a = 2k$

$a^2 = 4k^2 = 2(2k^2)$

...and then proofs...

Direct: $P \implies Q$

Example: a is even $\implies a^2$ is even.

Approach: What is even? $a = 2k$

$a^2 = 4k^2 = 2(2k^2)$

Integers closed under multiplication!

...and then proofs...

Direct: $P \implies Q$

Example: a is even $\implies$ a^2 is even.

Approach: What is even? $a = 2k$

$a^2 = 4k^2 = 2(2k^2)$

Integers closed under multiplication! So $2k^2$ is even.

...and then proofs...

Direct: $P \implies Q$

Example: a is even $\implies$ a^2 is even.

Approach: What is even? $a = 2k$

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Integers closed under multiplication! So $2k^2$ is even.

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Integers closed under multiplication! So $2k^2$ is even.

a^2 is even.

Contrapositive: $P \implies Q$

...and then proofs...

Direct: $P \implies Q$

Example: a is even $\implies a^2$ is even.

Approach: What is even? $a = 2k$

$a^2 = 4k^2 = 2(2k^2)$

Integers closed under multiplication! So $2k^2$ is even.

a^2 is even.

Contrapositive: $P \implies Q$ or $\neg Q \implies \neg P$.

...and then proofs...

Direct: $P \implies Q$

Example: a is even $\implies a^2$ is even.

Approach: What is even? $a = 2k$

$a^2 = 4k^2 = 2(2k^2)$

Integers closed under multiplication! So $2k^2$ is even.

a^2 is even.

Contrapositive: $P \implies Q$ or $\neg Q \implies \neg P$.

Example: a^2 is odd $\implies a$ is odd.

...and then proofs...

Direct: $P \implies Q$

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Approach: What is even? $a = 2k$

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Contrapositive: a is even $\implies a^2$ is even.

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Integers closed under multiplication! So $2k^2$ is even.
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Contrapositive: $P \implies Q$ or $\neg Q \implies \neg P$.
Example: a^2 is odd $\implies a$ is odd.
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Example: rogue couple does not exist.

...jumping forward..

Contradiction in induction:

...jumping forward..

Contradiction in induction:

Find a place where induction step doesn't hold.

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$$3^{2n+2}-1 = 9(3^{2n})-1$$

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$(3^{2n} - 1 = 8d)$

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$3^{2n+2} - 1 = 9(3^{2n}) - 1$ (by induction hypothesis)

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\begin{aligned}
3^{2n+2} - 1 &= 9(3^{2n}) - 1 \quad \text{(by induction hypothesis)} \\
&= 9(8d + 1) - 1 \\
&= 72d + 8 \\
&= 8(9d + 1)
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$\square$

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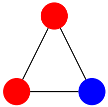

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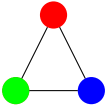

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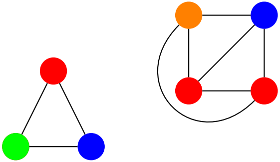

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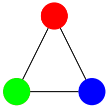

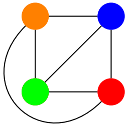

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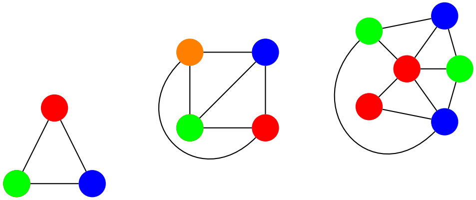

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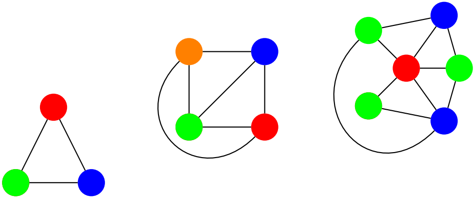

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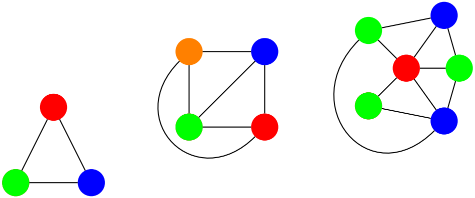

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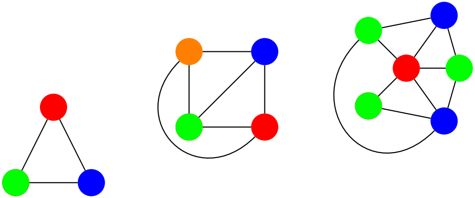

Notice that the last one, has one three colors.

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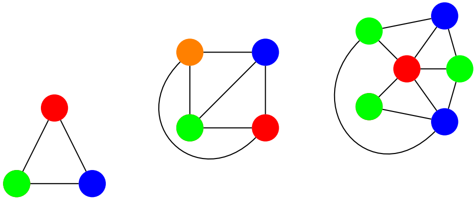

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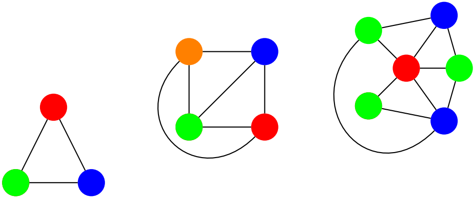
Fewer colors than number of vertices.

Graph Coloring.

Given $G = (V, E)$, a coloring of a G assigns colors to vertices V where for each edge the endpoints have different colors.



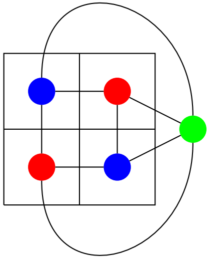
Notice that the last one, has one three colors.

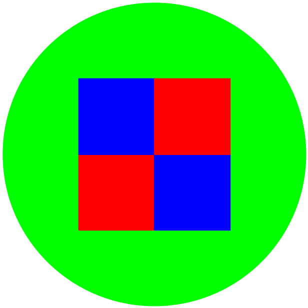
Fewer colors than number of vertices.

Fewer colors than max degree node.

Planar graphs and maps.

Planar graph coloring $\equiv$ map coloring.





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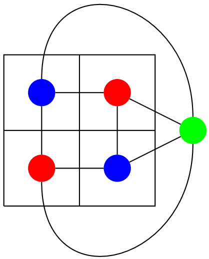

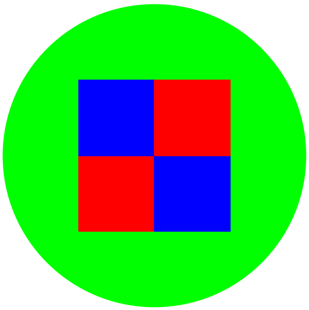

Four color theorem is about planar graphs!

Six color theorem.

Theorem: Every planar graph can be colored with six colors.

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Proof:

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Proof: Not Today!

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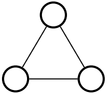
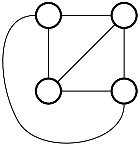
Graph Types: Complete Graph.





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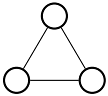

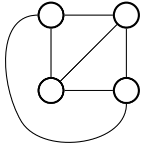

K_n, $|V| = n$

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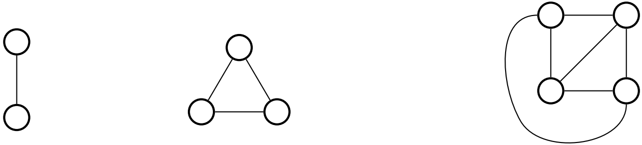

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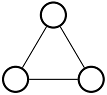

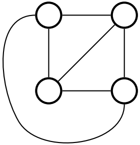

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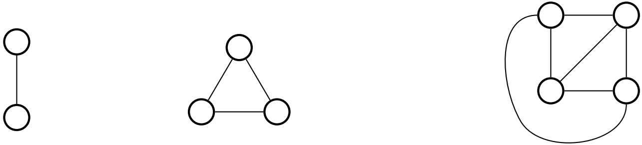

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degree of vertex? $|V| - 1$.

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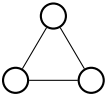

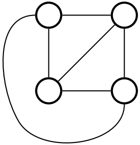

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Very connected.

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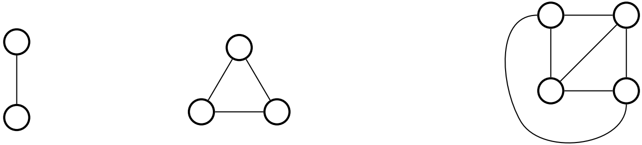

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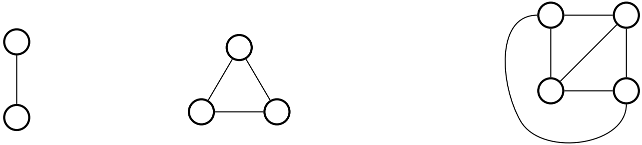

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Very connected.
Lots of edges: $n(n-1)/2$.

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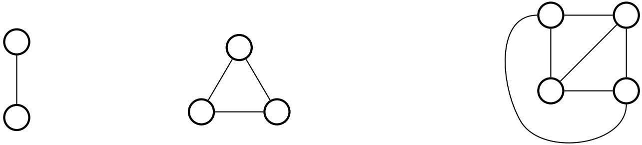

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every edge present.
degree of vertex? $|V|-1$.

Very connected.
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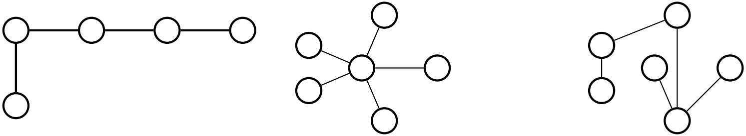
Wow.

Trees.



Definitions:

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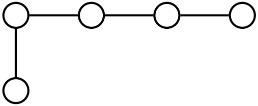
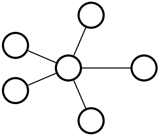
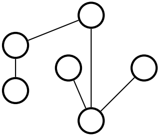

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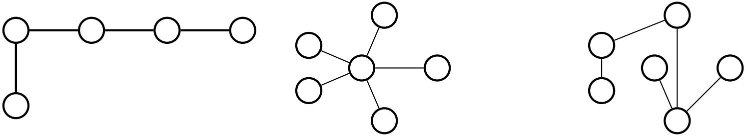
A connected graph without a cycle.

Trees.



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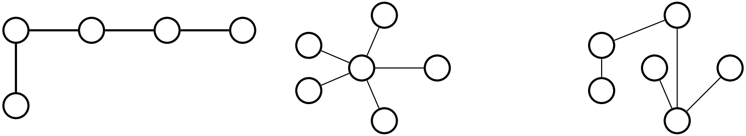

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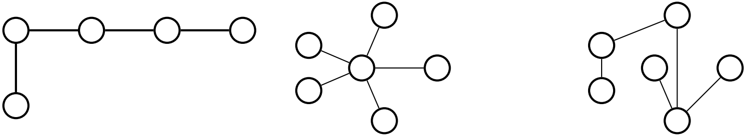

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An acyclic graph where any edge addition creates a cycle.

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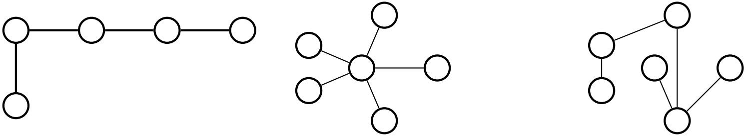

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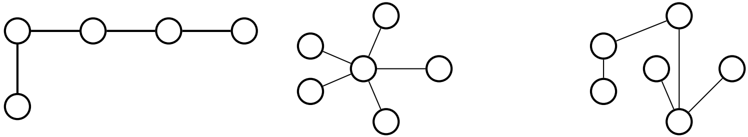

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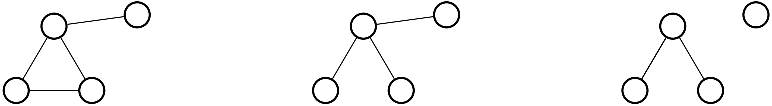
To tree or not to tree!



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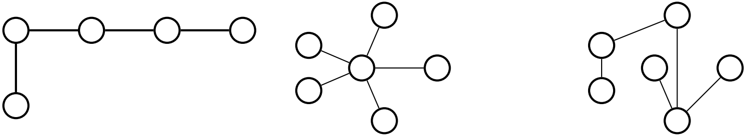

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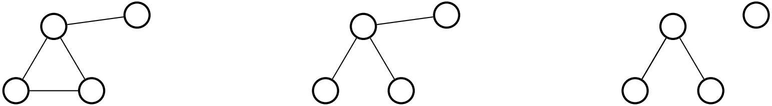
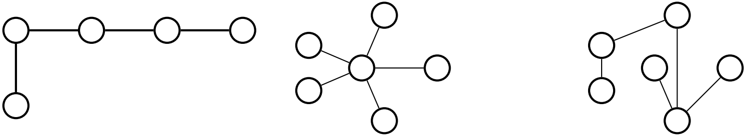


Minimally connected, minimum number of edges to connect.

Trees.



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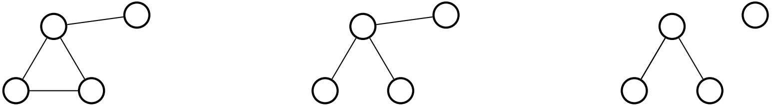

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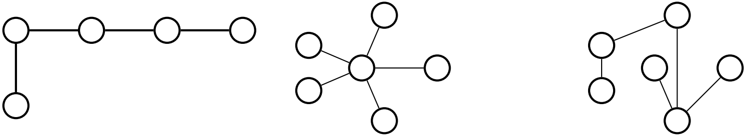
Property:

Trees.



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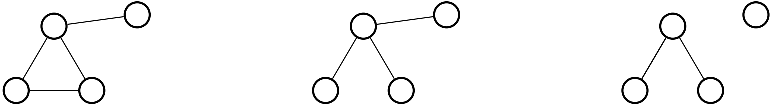

Minimally connected, minimum number of edges to connect.

Property:
Can remove a single node and break into components of size at most $|V|/2$.

Hypercube

Hypercubes.

Hypercube

Hypercubes. Really connected.

Hypercube

Hypercubes. Really connected. $O(|V| \log |V|)$ edges!

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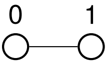

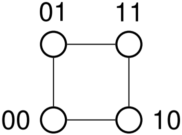

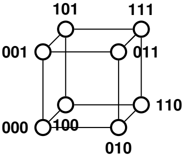

Recursive Definition.

A 0-dimensional hypercube is a node labelled with the empty string of bits.

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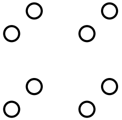

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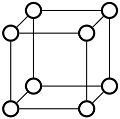

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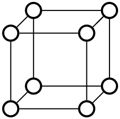

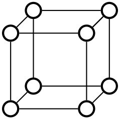

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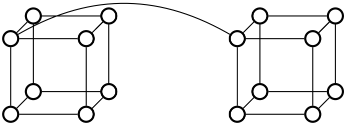

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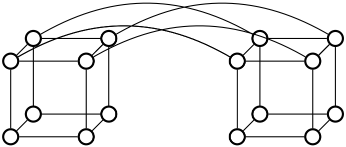

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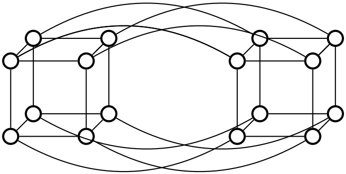

Hypercube:properties

Dense cuts: Cutting off k nodes needs $\geq k$ edges.

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Get from 000100 to 101000.

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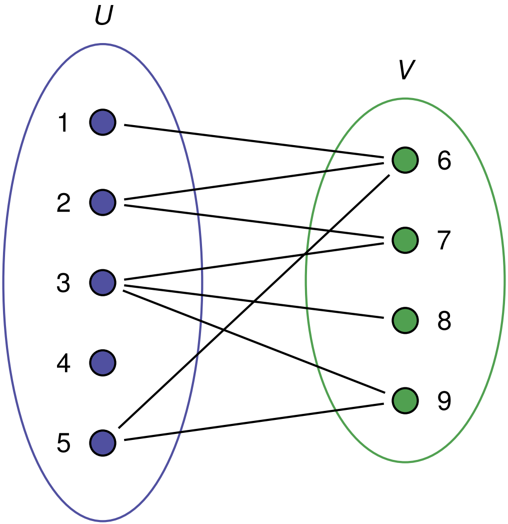
Good communication network!

Bipartite graphs



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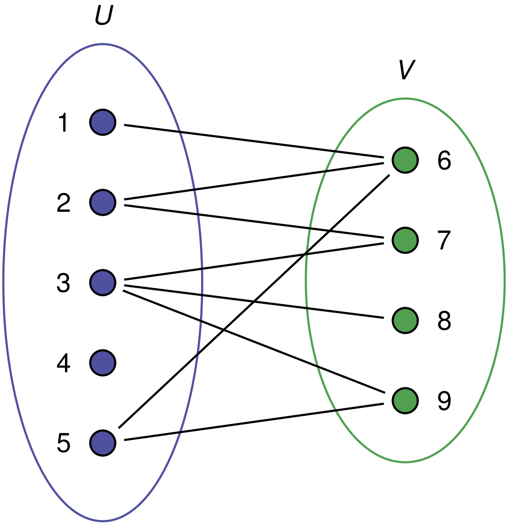

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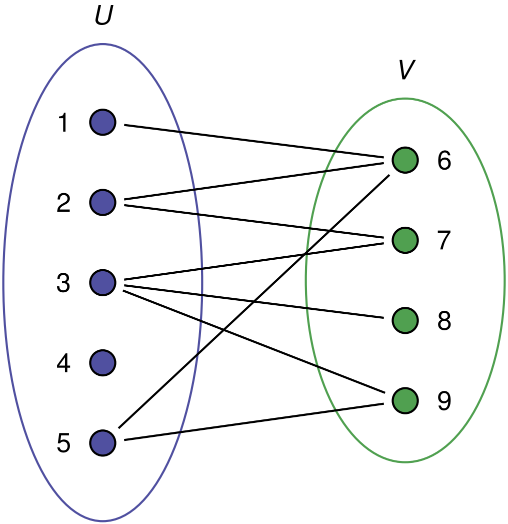

There is a cut with all the edges.

Cycles have length 4 or more edges.

Stable Marriage: a study in definitions and WOP.

n-men, n-women.

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Does stable pairing exist?

Yes for matching.

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Pairing with no rogue couples.

Does stable pairing exist?

Yes for matching.
No, for roommates problem.

TMA.

Traditional Marriage Algorithm:

TMA.

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Every man proposes to his favorite woman from the ones that haven't already rejected him.

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Not rogue couple!

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Man optimal $\implies$ Woman pessimal.

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Man optimal $\Longrightarrow$ Woman pessimal.
Woman optimal $\Longrightarrow$ Man pessimal.

And then countability

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More than one infinities

And then countability

More than one infinities

Some things are countable

And then countability

More than one infinities

Some things are countable , like the natural numbers

And then countability

More than one infinities

Some things are countable , like the natural numbers , or the rationals...

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Why?

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Why? There is a list!!

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Why? **Diagonalization:** Well, assume there is a list.

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Why? **Diagonalization:** Well, assume there is a list. Can construct a diagonal element x. x is not in the list! Contradiction.

HALTING

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Like: Will this program *P* even print ”Hello World”?

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The HALT problem: Is there a program that can tell you if another (generic) program halts on an input?

NO!

Why? Self reference!

Who cares? Using the same trick I can show that a bunch of problems are undecidable!
Like: Will this program *P* even print ”Hello World”?
Or ”Is there an input for this program *P* that will give an attacker admin access?

Counting!

Sample k items out of n.

	With Replacement	Without Replacement
Order matters	n^k	$\frac{n!}{(n-k)!}$
Order doesn't matter	$\binom{n+k-1}{n-1}$	$\binom{n}{k}$

Stars and bars!

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Confusion yesterday: 10 hats.

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Confusion yesterday: 10 hats. 7 days.

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Confusion yesterday: 10 hats. 7 days. I can wear the same hat on different days (replacement).

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How many stars?

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How many stars? One for each day.

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Didn't wear hats 1 and 2. Wore hat 3 for 1 day, hat 4 for 2 days, hat 5 days. Didn't wear hats 6 and 7. Hat 8 for 3 days. Didn't wear hats 9 and 10.

Combinatorial Proofs.

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Add them up. (**Sum rule**)

Midterm format

Time: 110 minutes.

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Some short answers.

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  Get at ideas that you learned.

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Midterm format

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Know material well:	fast, correct.
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Know material not so well:	

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Remember that a problem from hw and/or discussions is in the midterm! (identical or almost identical)

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So study those!

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  Also, big proofs are usually a bunch of little proofs put together. And every proof is a new trick. And we like tricks!

Wrapup.

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- Should have received an email from us.

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