CS70: Discrete Math and Probability

Fan Ye

June 27, 2016

Today

More graphs

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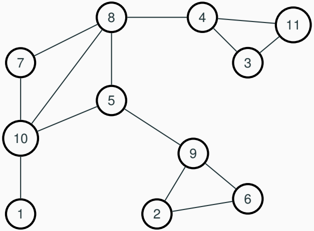
- Connectivity
- Eulerian Tour
- Planar graphs
  - 5 coloring theorem

Connectivity



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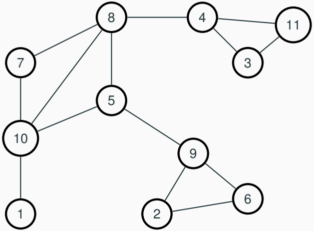

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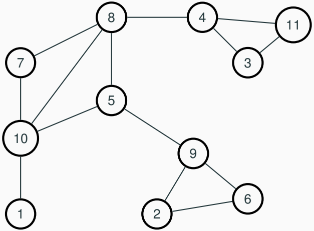

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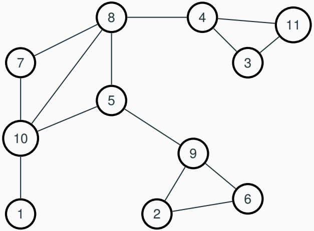

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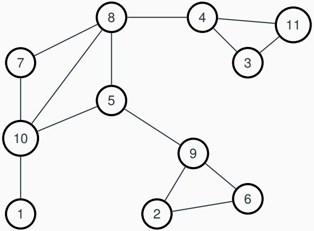

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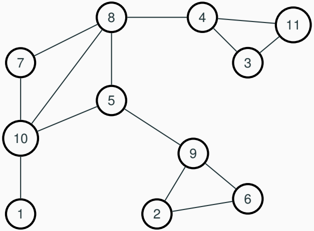

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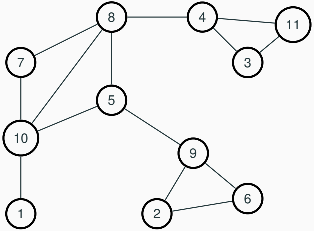

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Proof idea:

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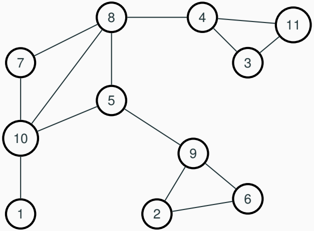

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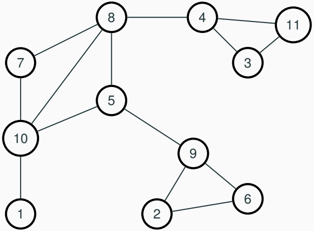

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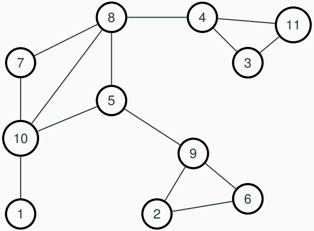

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Either modify definition to walk.

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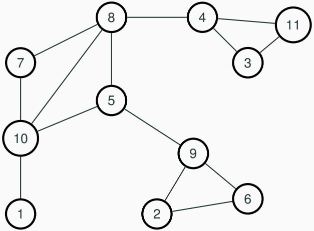

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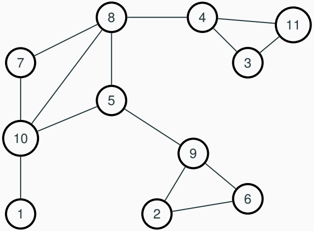

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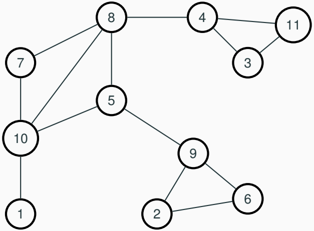

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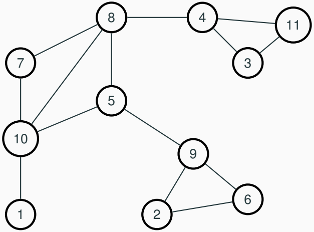
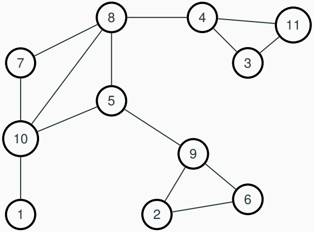


Is graph above connected?

Connected component



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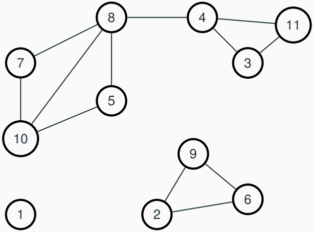
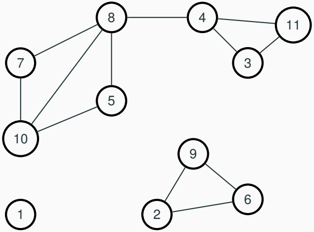


Is graph above connected? Yes!

How about now?

Connected component



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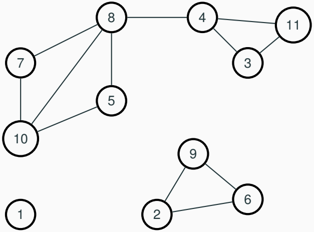

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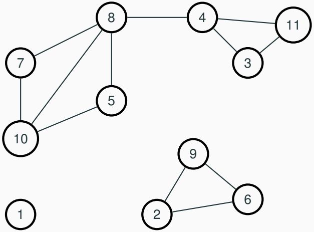
Connected Components?

Connected component

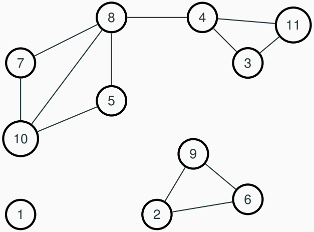


Is graph above connected? Yes!

How about now? No!

Connected Components? $\{1\},\{10,7,5,8,4,3,11\},\{2,9,6\}$.

Connected component



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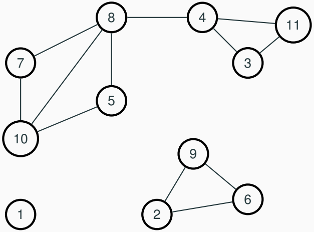
Connected Components? $\{1\}, \{10, 7, 5, 8, 4, 3, 11\}, \{2, 9, 6\}$.
Connected component - maximal set of connected vertices.

Connected component



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Quick Check: Is $\{10, 7, 5\}$ a connected component?

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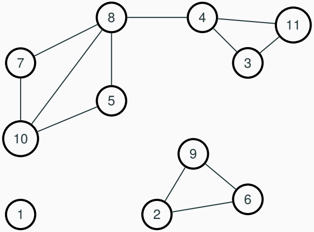

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Finally..back to bridges!

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Definition:

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Proof of only if: Eulerian $\implies$ connected and all even degree.

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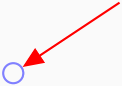

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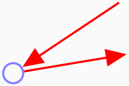

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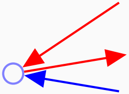

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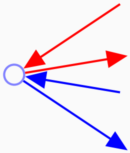

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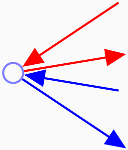

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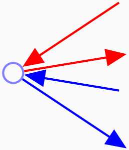

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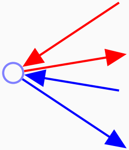

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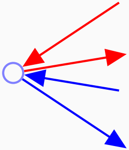

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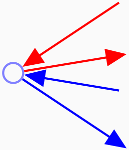

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Proof of if: Even + connected $\implies$ Eulerian Tour.
We will give an algorithm.

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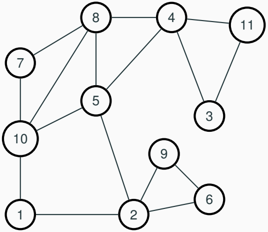

1. Take a walk starting from v (1) on "unused" edges

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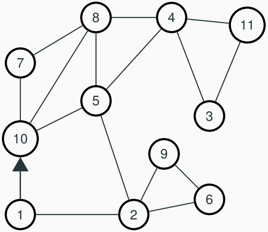

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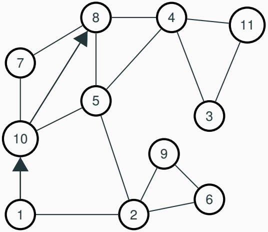

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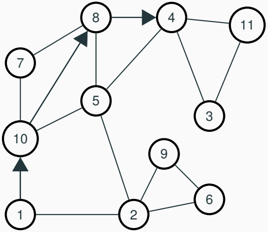

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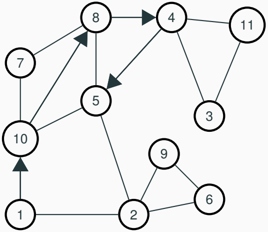

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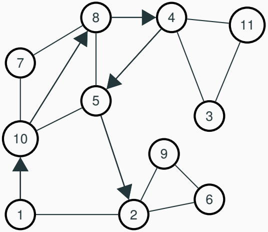

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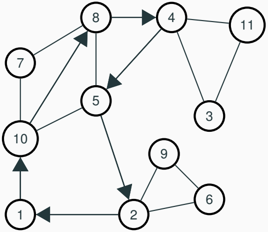

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... till you get back to v.

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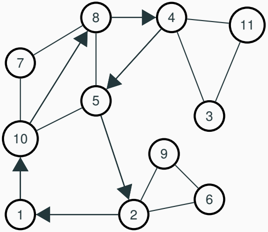

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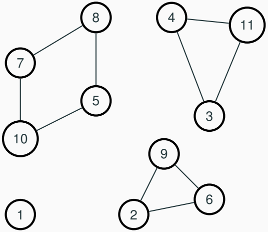
2. Remove tour, C.

Finding a tour!

Proof of if: Even + connected $\implies$ Eulerian Tour.
We will give an algorithm. First by picture.



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3. Let $G_1, \ldots, G_k$ be connected components.

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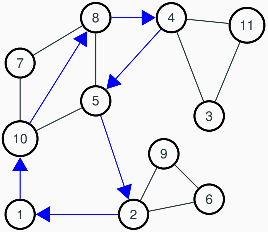

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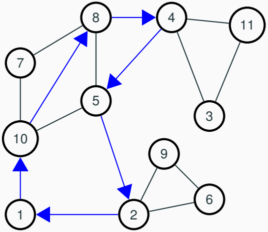

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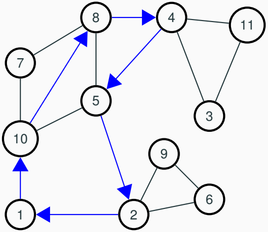

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3. Let $G_1, \ldots, G_k$ be connected components.
   Each is touched by C.
      Why? G was connected.

Finding a tour!

Proof of if: Even + connected $\implies$ Eulerian Tour.
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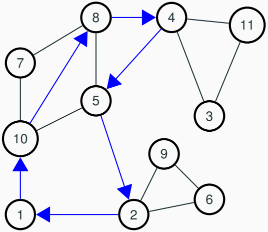

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   Why? G was connected.

   Let v_i be (first) node in G_i touched by C.

Finding a tour!

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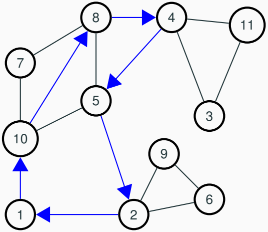

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3. Let $G_1, \ldots, G_k$ be connected components.
   Each is touched by C.
      Why? G was connected.
   Let v_i be (first) node in G_i touched by C.
    Example: $v_1 = 1$,

Finding a tour!

Proof of if: Even + connected $\implies$ Eulerian Tour.
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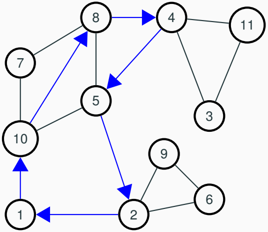

1. Take a walk starting from v (1) on "unused" edges

... till you get back to v.

2. Remove tour, C.
3. Let $G_1, \ldots, G_k$ be connected components.

   Each is touched by C.

   Why? G was connected.

   Let v_i be (first) node in G_i touched by C.

   Example: $v_1 = 1$, $v_2 = 10$,

Finding a tour!

Proof of if: Even + connected $\implies$ Eulerian Tour.

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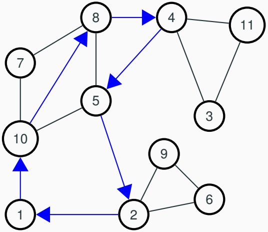

1. Take a walk starting from v (1) on "unused" edges
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3. Let $G_1, \ldots, G_k$ be connected components.
Each is touched by C.
Why? G was connected.
Let v_i be (first) node in G_i touched by C.
Example: $v_1 = 1$, $v_2 = 10$, $v_3 = 4$,

Finding a tour!

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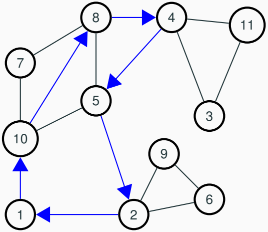

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      Why? G was connected.
   Let v_i be (first) node in G_i touched by C.
    Example: $v_1 = 1$, $v_2 = 10$, $v_3 = 4$, $v_4 = 2$.

Finding a tour!

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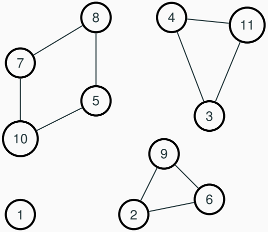

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   Let v_i be (first) node in G_i touched by C.
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4. Recurse on $G_1, \ldots, G_k$ starting from v_i

Finding a tour!

Proof of if: Even + connected $\implies$ Eulerian Tour.
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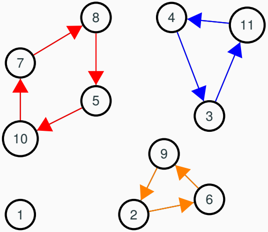

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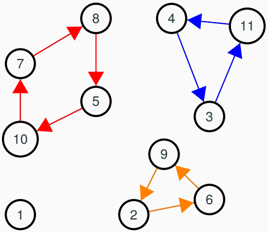

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   Let v_i be (first) node in G_i touched by C.
    Example: $v_1 = 1$, $v_2 = 10$, $v_3 = 4$, $v_4 = 2$.
4. Recurse on $G_1, \ldots, G_k$ starting from v_i
5. Splice together.

Finding a tour!

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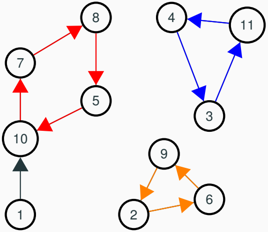

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5. Splice together.
   1,10

Finding a tour!

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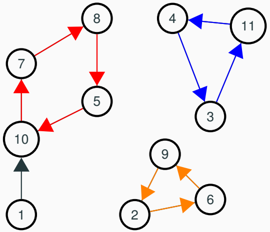

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   Example: $v_1 = 1$, $v_2 = 10$, $v_3 = 4$, $v_4 = 2$.
4. Recurse on $G_1, \ldots, G_k$ starting from v_i
5. Splice together.
   1,10,7,8,5,10

Finding a tour!

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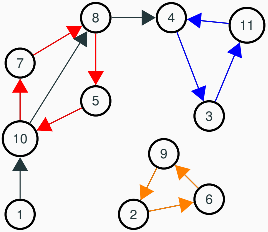

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  1,10,7,8,5,10 ,8,4

Finding a tour!

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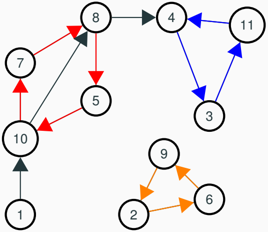

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    Example: $v_1 = 1$, $v_2 = 10$, $v_3 = 4$, $v_4 = 2$.
4. Recurse on $G_1, \ldots, G_k$ starting from v_i
5. Splice together.
   1,10,7,8,5,10 ,8,4,3,11,4

Finding a tour!

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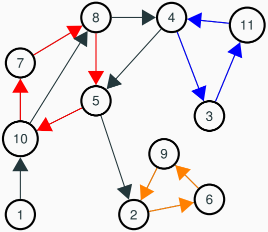

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    Example: $v_1 = 1$, $v_2 = 10$, $v_3 = 4$, $v_4 = 2$.
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5. Splice together.
   1,10,7,8,5,10 ,8,4,3,11,4 5,2

Finding a tour!

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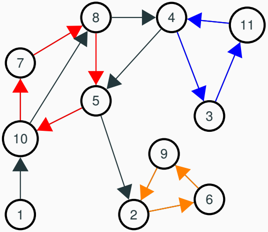

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Finding a tour!

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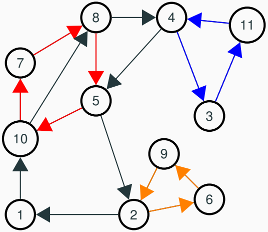

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   1,10,7,8,5,10 ,8,4,3,11,4 5,2,6,9,2 and to 1!

Finding a tour: in general.

1. Take a walk from arbitrary node v, until you get back to v.

Finding a tour: in general.

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Claim: Do get back to v!

Finding a tour: in general.

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Claim: Do get back to v!
Proof of Claim: Even degree.

Finding a tour: in general.

1. Take a walk from arbitrary node v, until you get back to v.

Claim: Do get back to v!
Proof of Claim: Even degree. If enter, can leave

Finding a tour: in general.

1. Take a walk from arbitrary node v, until you get back to v.

Claim: Do get back to v!
Proof of Claim: Even degree. If enter, can leave except for v.

Finding a tour: in general.

1. Take a walk from arbitrary node v, until you get back to v.

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Finding a tour: in general.

1. Take a walk from arbitrary node v, until you get back to v.

Claim: Do get back to v!
Proof of Claim: Even degree. If enter, can leave except for v. $\square$

2. Remove cycle, C, from G.

Finding a tour: in general.

1. Take a walk from arbitrary node v, until you get back to v.

Claim: Do get back to v!
Proof of Claim: Even degree. If enter, can leave except for v. $\square$

2. Remove cycle, C, from G.
Resulting graph may be disconnected. (Removed edges!)

Finding a tour: in general.

1. Take a walk from arbitrary node v, until you get back to v.

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Proof of Claim: Even degree. If enter, can leave except for v. □

2. Remove cycle, C, from G.
Resulting graph may be disconnected. (Removed edges!)
Let components be $G_1, \ldots, G_k$.

Finding a tour: in general.

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Claim: Do get back to v!
Proof of Claim: Even degree. If enter, can leave except for v. □

2. Remove cycle, C, from G.
Resulting graph may be disconnected. (Removed edges!)
Let components be $G_1, \ldots, G_k$.
Let v_i be first vertex of C that is in G_i.

Finding a tour: in general.

1. Take a walk from arbitrary node v, until you get back to v.

Claim: Do get back to v!
Proof of Claim: Even degree. If enter, can leave except for v. $\square$

2. Remove cycle, C, from G.

Resulting graph may be disconnected. (Removed edges!)
Let components be $G_1, \ldots, G_k$.
Let v_i be first vertex of C that is in G_i.
Why is there a v_i in C?

Finding a tour: in general.

1. Take a walk from arbitrary node v, until you get back to v.

Claim: Do get back to v!
Proof of Claim: Even degree. If enter, can leave except for v. $\square$

2. Remove cycle, C, from G.

Resulting graph may be disconnected. (Removed edges!)
Let components be $G_1, \ldots, G_k$.
Let v_i be first vertex of C that is in G_i.
Why is there a v_i in C?
G was connected $\implies$

Finding a tour: in general.

1. Take a walk from arbitrary node v, until you get back to v.

Claim: Do get back to v!
Proof of Claim: Even degree. If enter, can leave except for v. $\square$

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Let components be $G_1, \ldots, G_k$.
Let v_i be first vertex of C that is in G_i.
Why is there a v_i in C?
G was connected $\implies$
a vertex in G_i must be incident to a removed edge in C.

Finding a tour: in general.

1. Take a walk from arbitrary node v, until you get back to v.

Claim: Do get back to v!
Proof of Claim: Even degree. If enter, can leave except for v. $\square$

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Let components be $G_1, \ldots, G_k$.
Let v_i be first vertex of C that is in G_i.
Why is there a v_i in C?
G was connected $\implies$
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Finding a tour: in general.

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Proof of Claim: Even degree. If enter, can leave except for v. $\square$

2. Remove cycle, C, from G.

Resulting graph may be disconnected. (Removed edges!)
Let components be $G_1, \ldots, G_k$.
Let v_i be first vertex of C that is in G_i.
Why is there a v_i in C?
G was connected $\implies$
a vertex in G_i must be incident to a removed edge in C.

Claim: Each vertex in each G_i has even degree

Finding a tour: in general.

1. Take a walk from arbitrary node v, until you get back to v.

Claim: Do get back to v!
Proof of Claim: Even degree. If enter, can leave except for v. $\square$

2. Remove cycle, C, from G.

Resulting graph may be disconnected. (Removed edges!)
Let components be $G_1, \ldots, G_k$.
Let v_i be first vertex of C that is in G_i.
Why is there a v_i in C?
G was connected $\implies$
a vertex in G_i must be incident to a removed edge in C.

Claim: Each vertex in each G_i has even degree and is connected.

Finding a tour: in general.

1. Take a walk from arbitrary node v, until you get back to v.

Claim: Do get back to v!
Proof of Claim: Even degree. If enter, can leave except for v. $\square$

2. Remove cycle, C, from G.

Resulting graph may be disconnected. (Removed edges!)
Let components be $G_1, \ldots, G_k$.
Let v_i be first vertex of C that is in G_i.
Why is there a v_i in C?
G was connected $\implies$
a vertex in G_i must be incident to a removed edge in C.

Claim: Each vertex in each G_i has even degree and is connected.
Prf: Tour C has even incidences to any vertex v.

Finding a tour: in general.

1. Take a walk from arbitrary node v, until you get back to v.

Claim: Do get back to v!
Proof of Claim: Even degree. If enter, can leave except for v. $\square$

2. Remove cycle, C, from G.

Resulting graph may be disconnected. (Removed edges!)
Let components be $G_1, \ldots, G_k$.
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Why is there a v_i in C?
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Finding a tour: in general.

1. Take a walk from arbitrary node v, until you get back to v.

Claim: Do get back to v!
Proof of Claim: Even degree. If enter, can leave except for v. $\square$

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Resulting graph may be disconnected. (Removed edges!)
Let components be $G_1, \ldots, G_k$.
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G was connected $\implies$
a vertex in G_i must be incident to a removed edge in C.

Claim: Each vertex in each G_i has even degree and is connected.
Prf: Tour C has even incidences to any vertex v. $\square$

3. Find tour T_i of G_i

Finding a tour: in general.

1. Take a walk from arbitrary node v, until you get back to v.

Claim: Do get back to v!
Proof of Claim: Even degree. If enter, can leave except for v. $\square$

2. Remove cycle, C, from G.

Resulting graph may be disconnected. (Removed edges!)
Let components be $G_1, \ldots, G_k$.
Let v_i be first vertex of C that is in G_i.
Why is there a v_i in C?
G was connected $\implies$
a vertex in G_i must be incident to a removed edge in C.

Claim: Each vertex in each G_i has even degree and is connected.
Prf: Tour C has even incidences to any vertex v. $\square$

3. Find tour T_i of G_i starting/ending at v_i.

Finding a tour: in general.

1. Take a walk from arbitrary node v, until you get back to v.

Claim: Do get back to v!
Proof of Claim: Even degree. If enter, can leave except for v. $\square$

2. Remove cycle, C, from G.
Resulting graph may be disconnected. (Removed edges!)
Let components be $G_1, \ldots, G_k$.
Let v_i be first vertex of C that is in G_i.
Why is there a v_i in C?
G was connected $\implies$
a vertex in G_i must be incident to a removed edge in C.

Claim: Each vertex in each G_i has even degree and is connected.
Prf: Tour C has even incidences to any vertex v. $\square$

3. Find tour T_i of G_i starting/ending at v_i. Induction.

Finding a tour: in general.

1. Take a walk from arbitrary node v, until you get back to v.

Claim: Do get back to v!
Proof of Claim: Even degree. If enter, can leave except for v. $\square$

2. Remove cycle, C, from G.
Resulting graph may be disconnected. (Removed edges!)
Let components be $G_1, \ldots, G_k$.
Let v_i be first vertex of C that is in G_i.
Why is there a v_i in C?
G was connected $\implies$
a vertex in G_i must be incident to a removed edge in C.

Claim: Each vertex in each G_i has even degree and is connected.
Prf: Tour C has even incidences to any vertex v. $\square$

3. Find tour T_i of G_i starting/ending at v_i. Induction.
4. Splice T_i into C where v_i first appears in C.

Finding a tour: in general.

1. Take a walk from arbitrary node v, until you get back to v.

Claim: Do get back to v!
Proof of Claim: Even degree. If enter, can leave except for v. $\square$

2. Remove cycle, C, from G.
Resulting graph may be disconnected. (Removed edges!)
Let components be $G_1, \ldots, G_k$.
Let v_i be first vertex of C that is in G_i.
Why is there a v_i in C?
G was connected $\Longrightarrow$
a vertex in G_i must be incident to a removed edge in C.

Claim: Each vertex in each G_i has even degree and is connected.
Prf: Tour C has even incidences to any vertex v. $\square$

3. Find tour T_i of G_i starting/ending at v_i. Induction.
4. Splice T_i into C where v_i first appears in C.

Visits every edge once:
Visits edges in C

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Visits edges in C exactly once.
By induction for all edges in each G_i.

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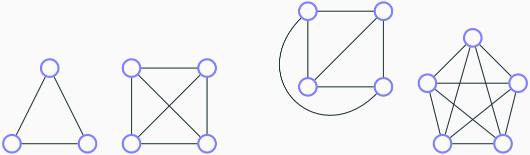
Planar graphs.

A graph that can be drawn in the plane without edge crossings.

Planar graphs.

A graph that can be drawn in the plane without edge crossings.



Planar?

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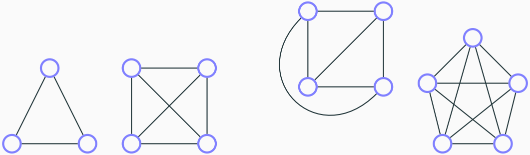

Planar? Yes for Triangle.

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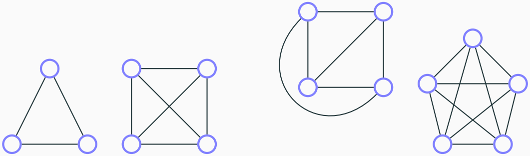

Planar? Yes for Triangle.
Four node complete?

Planar graphs.

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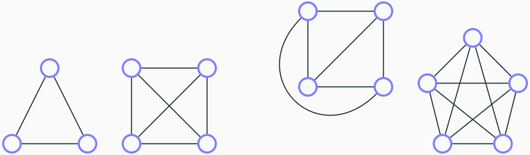

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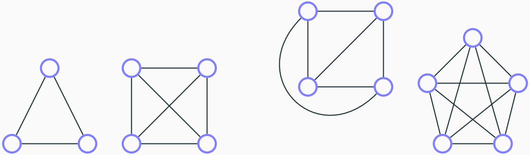

Planar? Yes for Triangle.
Four node complete? Yes.
Five node complete or K_5?

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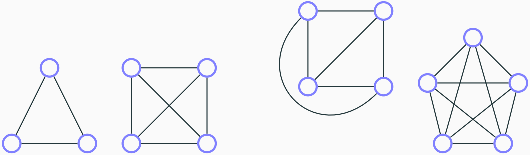

Planar? Yes for Triangle.
Four node complete? Yes.
Five node complete or K_5? No!

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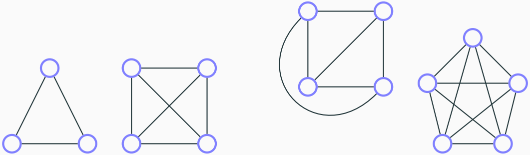

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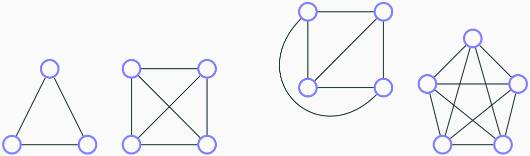

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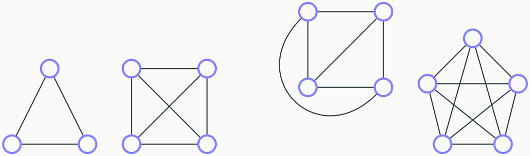

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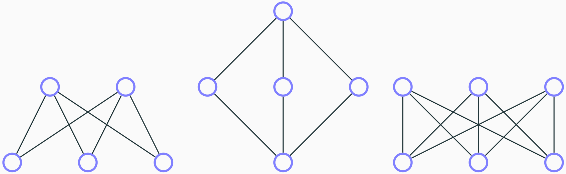

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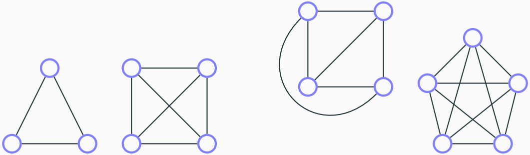

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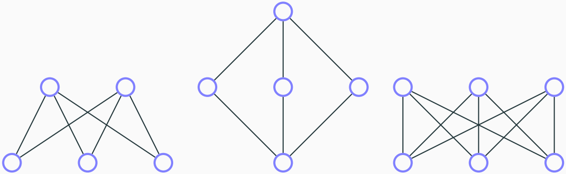

Two to three nodes, bipartite?

Planar graphs.

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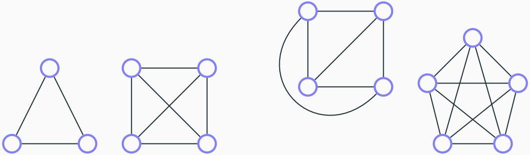

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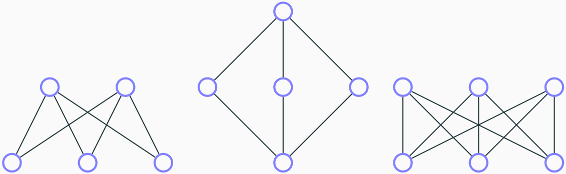

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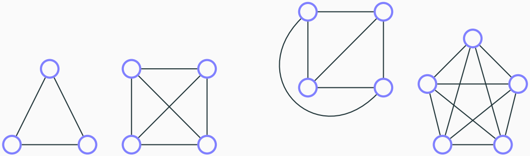

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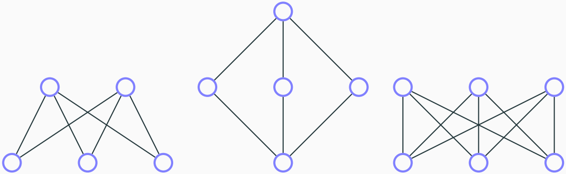

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Three to three nodes, complete/bipartite or $K_{3,3}$.

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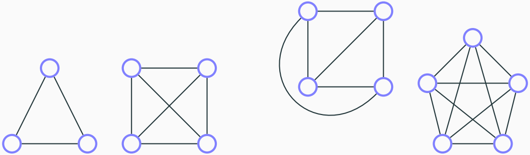

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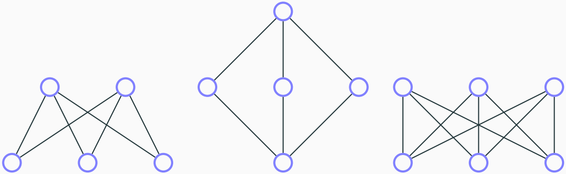

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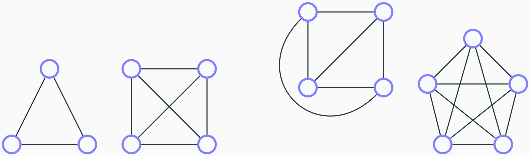

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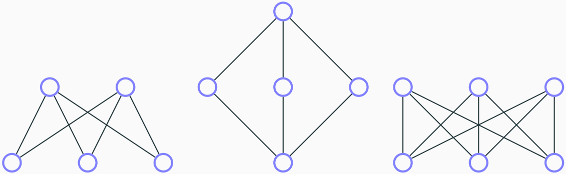

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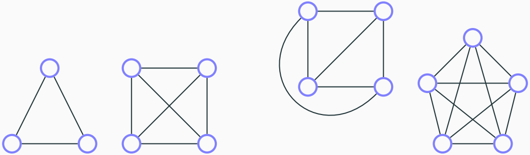

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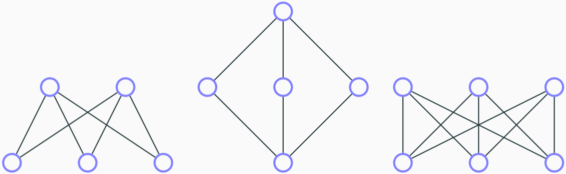

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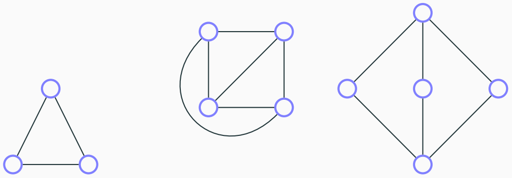
Euler's Formula.



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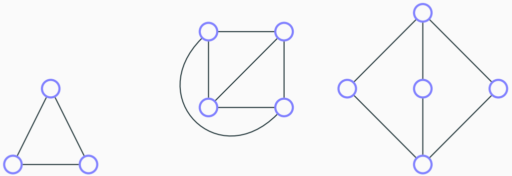
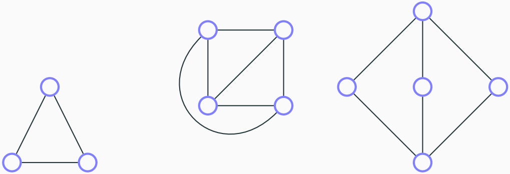


Faces: connected regions of the plane.

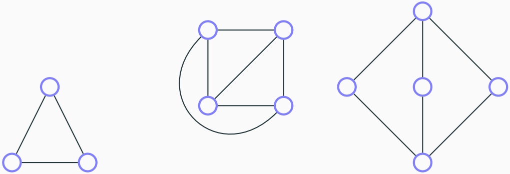
Euler's Formula.



Faces: connected regions of the plane.

How many faces for

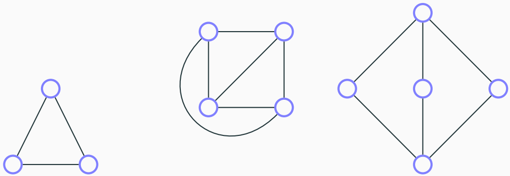
Euler’s Formula.



Faces: connected regions of the plane.

How many faces for
triangle?

Euler's Formula.

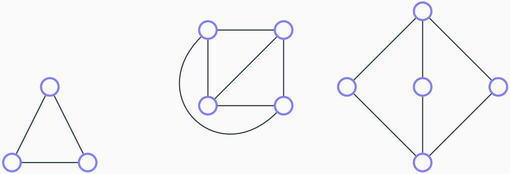


Faces: connected regions of the plane.

How many faces for

triangle? 2

Euler's Formula.



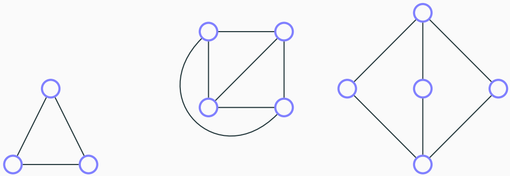
Faces: connected regions of the plane.

How many faces for

triangle? 2

complete on four vertices or K_4?

Euler's Formula.

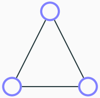
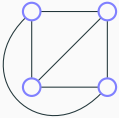
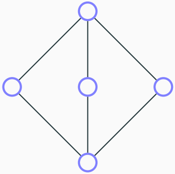


Faces: connected regions of the plane.

How many faces for

- triangle? 2
- complete on four vertices or K_4? 4

Euler's Formula.

Faces: connected regions of the plane.

How many faces for

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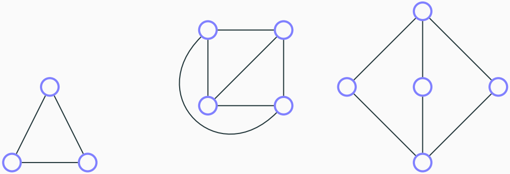
- bipartite, complete two/three or $K_{2,3}$?

Euler's Formula.



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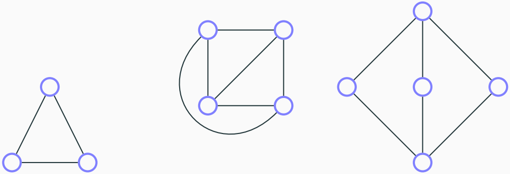

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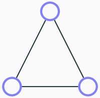
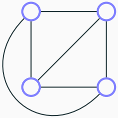
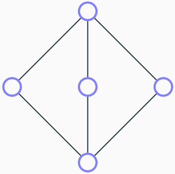

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v is number of vertices, e is number of edges, f is number of faces.

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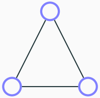 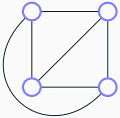 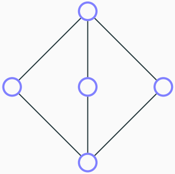

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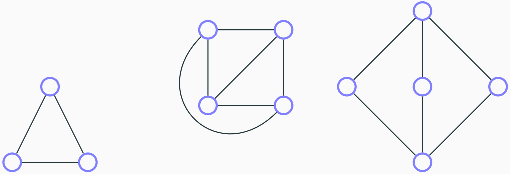
Euler's Formula: Connected planar graph has $v + f = e + 2$.

Euler's Formula.



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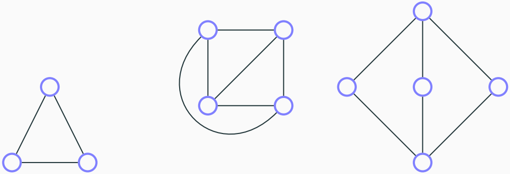

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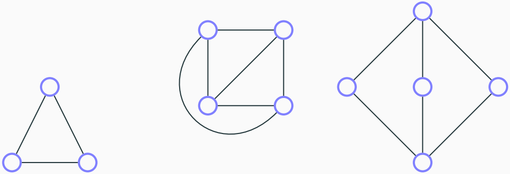

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Triangle: $3+2=3+2$!

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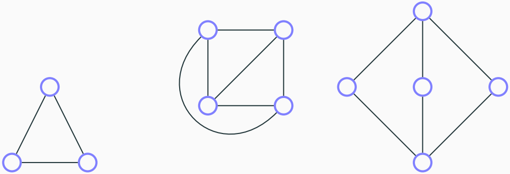

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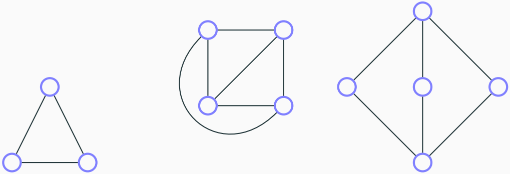
K_4:

Euler's Formula.



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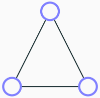
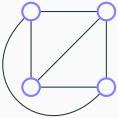
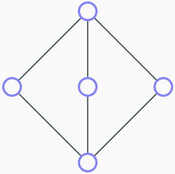
Triangle: $3+2=3+2$!
K_4: $4+4=6+2$!

Euler's Formula.

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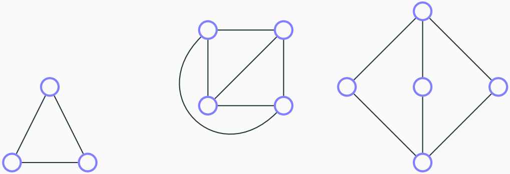

Faces: connected regions of the plane.

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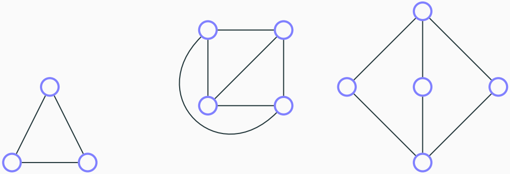

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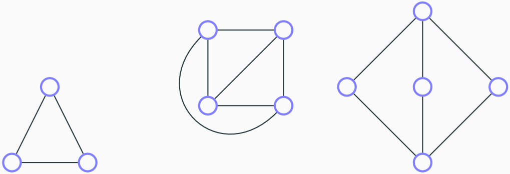

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Examples = 3!

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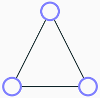

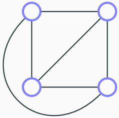

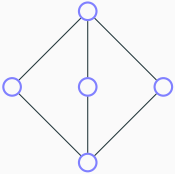

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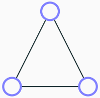

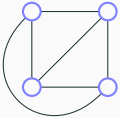

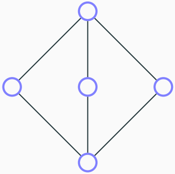

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Greeks knew formula for polyhedron.

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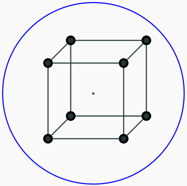

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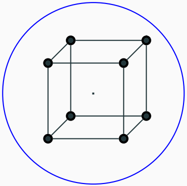

Faces?

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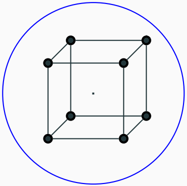

Faces? 6. Edges?

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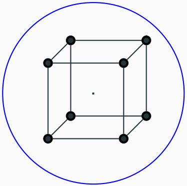

Faces? 6.   Edges? 12.

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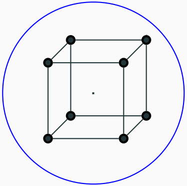

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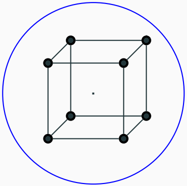

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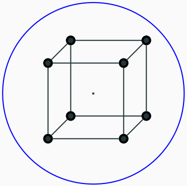

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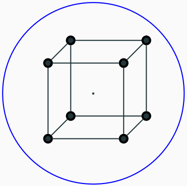

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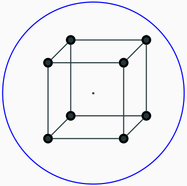

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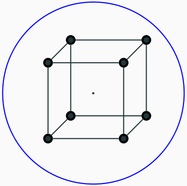

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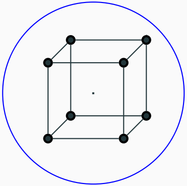

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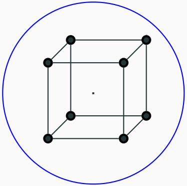

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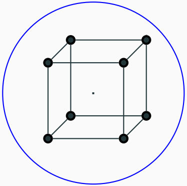

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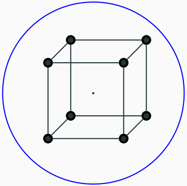

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Polyhedron without holes

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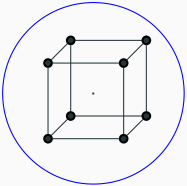

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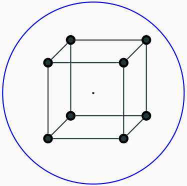

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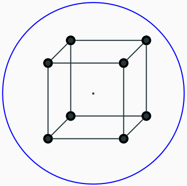

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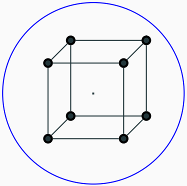
Polyhedron without holes $\equiv$ Planar graphs.

Surround by sphere.

Euler and Polyhedron.

Greeks knew formula for polyhedron.



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Polyhedron without holes $\equiv$ Planar graphs.

Surround by sphere.
Project from point inside polytope onto sphere.

Euler and Polyhedron.

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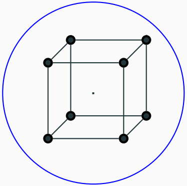

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Polyhedron without holes $\equiv$ Planar graphs.

Surround by sphere.
Project from point inside polytope onto sphere.
Sphere

Euler and Polyhedron.

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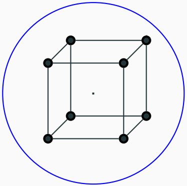

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Project from point inside polytope onto sphere.
Sphere $\equiv$ Plane!

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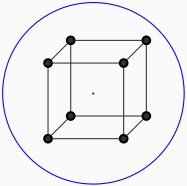

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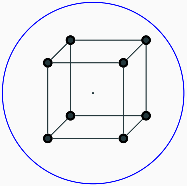

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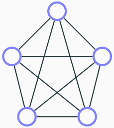
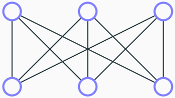
Euler proved formula thousands of years later!

Euler and planarity of K_5 and $K_{3,3}$





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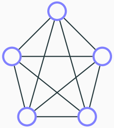

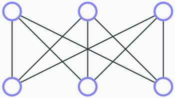

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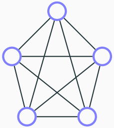

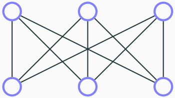

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Each face is adjacent to at least three edges.

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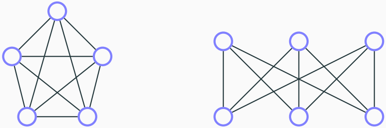
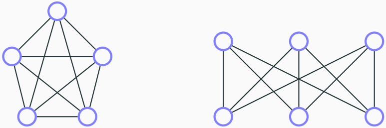


Euler: $v + f = e + 2$ for connected planar graph.

Each face is adjacent to at least three edges. face-edge adjacencies. $\geq 3f$

Euler and planarity of K_5 and $K_{3,3}$



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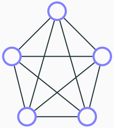

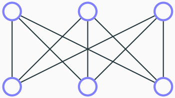

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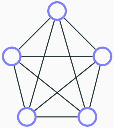

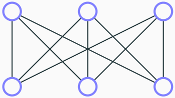

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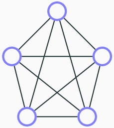
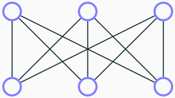

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Euler: $v + \frac{2}{3}e \geq e + 2$

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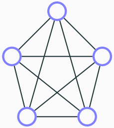

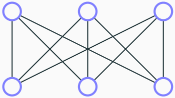

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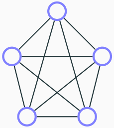
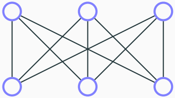
Euler and planarity of K_5 and $K_{3,3}$





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K_5

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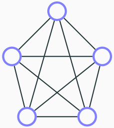

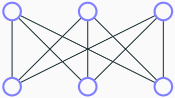

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K_5 Edges?

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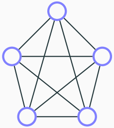

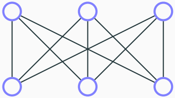

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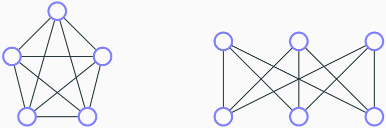

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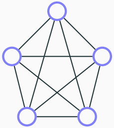

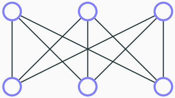

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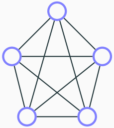
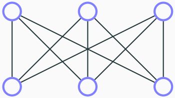

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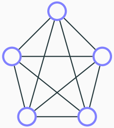

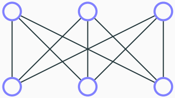

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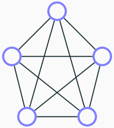

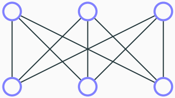

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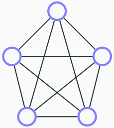

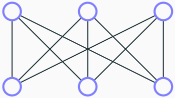

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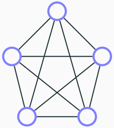

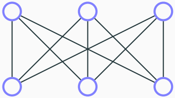

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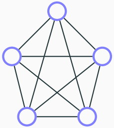

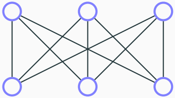

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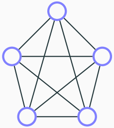

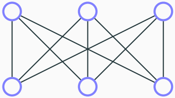

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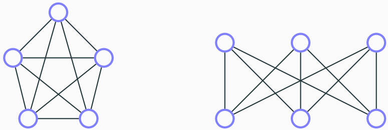

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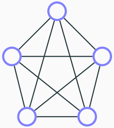

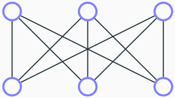

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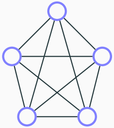

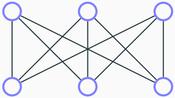

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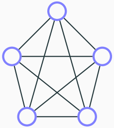

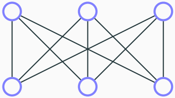

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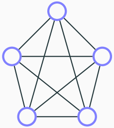

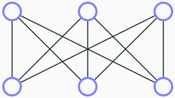

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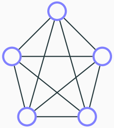

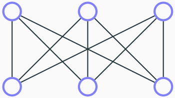

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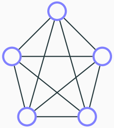

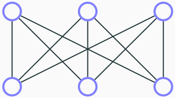

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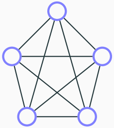

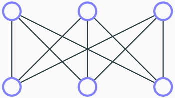

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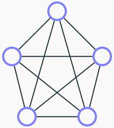

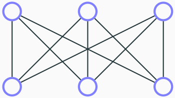

Euler: $v + f = e + 2$ for connected planar graph.

Each face is adjacent to at least three edges. face-edge adjacencies. $\geq 3f$
Each edge is adjacent to exactly two faces. face-edge adjacencies. $= 2e$
$\implies 3f \leq 2e$

Euler: $v + \frac{2}{3}e \geq e + 2 \implies e \leq 3v - 6$

K_5 Edges? $4 + 3 + 2 + 1 = 10$. Vertices? 5.
$10 \not\leq 3(5) - 6 = 9$. $\implies$ K_5 is not planar.

$K_{3,3}$? Edges? 9. Vertices. 6. $9 \leq 3(6) - 6$? Sure!
But no cycles that are triangles. Face is of length ≥ 4.
.... $4f \leq 2e$.
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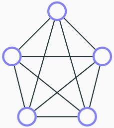
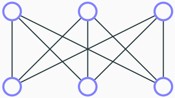
$9 \not\leq 2(6) - 4$.

Euler and planarity of K_5 and $K_{3,3}$





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$9 \not\leq 2(6) - 4$. $\implies$ $K_{3,3}$ is not planar!

Tree.

A tree is a connected acyclic graph.

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To tree or not to tree!

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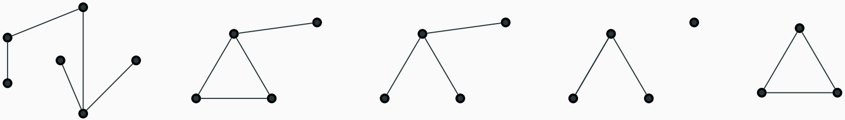

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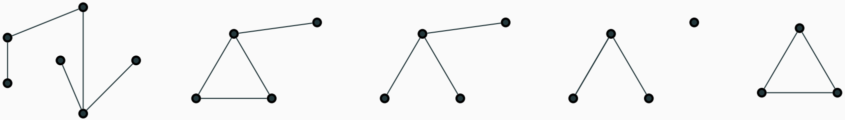

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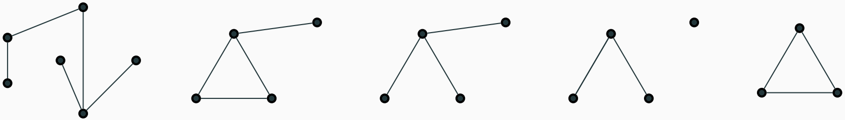

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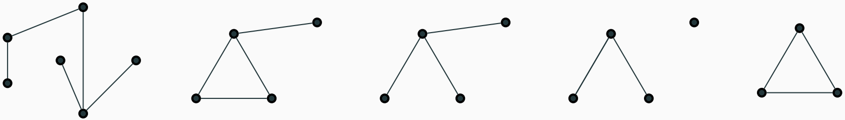

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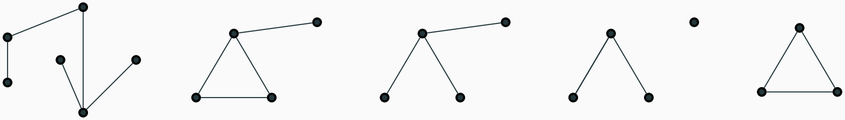

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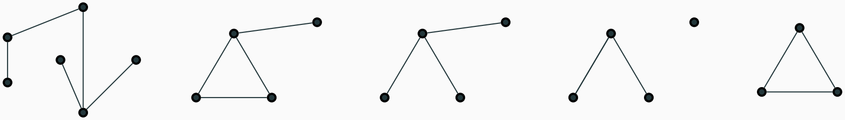

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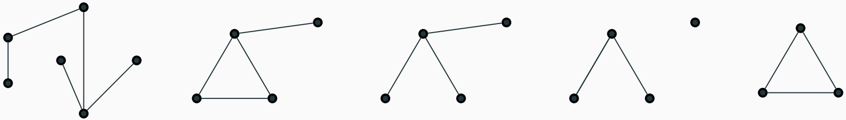

Yes. No. Yes. No. No.

Faces?

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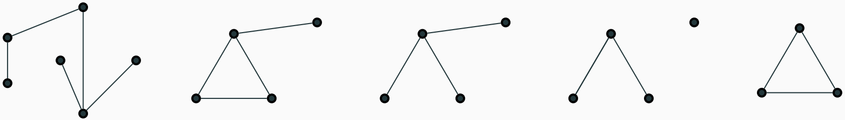

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Faces? 1.

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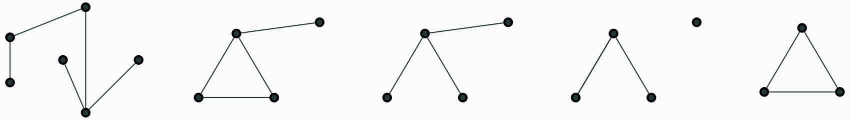

Yes. No. Yes. No. No.

Faces? 1. 2.

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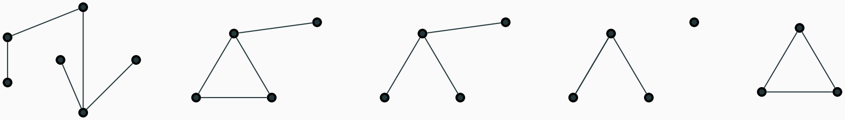

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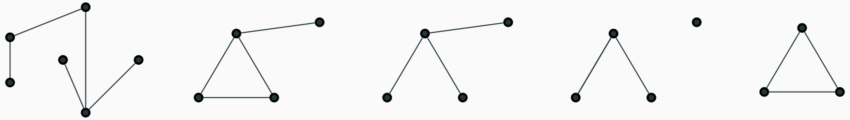

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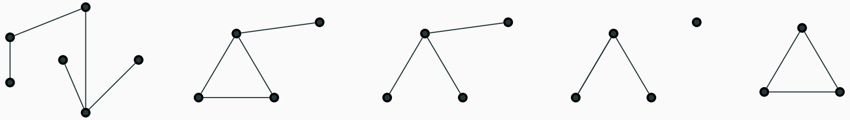

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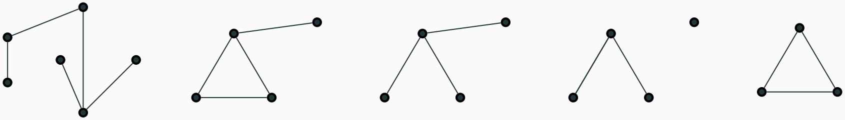

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Faces? 1. 2. 1. 1. 2.
Vertices/Edges.

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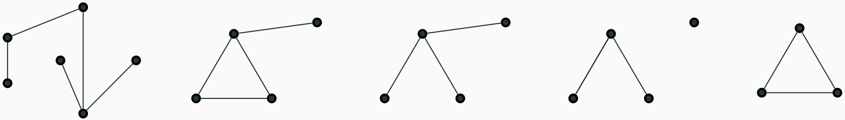

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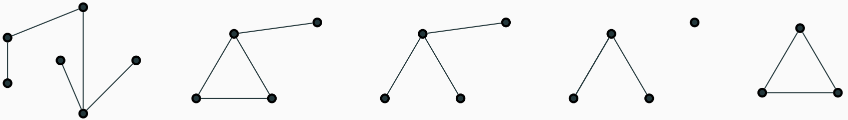

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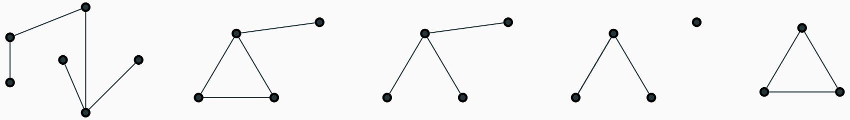

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One face for trees!

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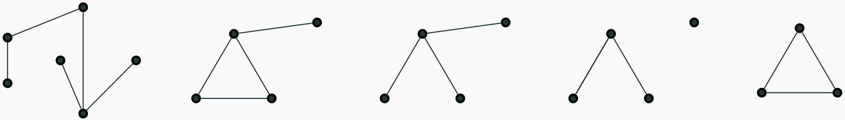

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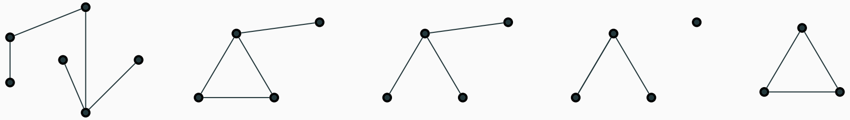

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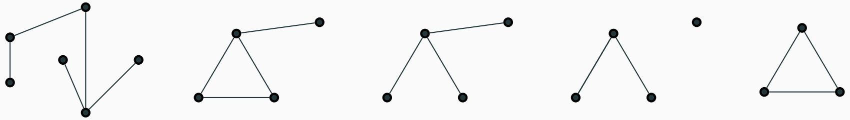

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Euler's formula.

Euler: Connected planar graph has $v + f = e + 2$.

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Proof sketch:

Euler's formula.

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Proof sketch: Induction on e.

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Proof sketch: Induction on e.
Base:

Euler's formula.

Euler: Connected planar graph has $v + f = e + 2$.

Proof sketch: Induction on e.
Base: $e = 0$,

Euler's formula.

Euler: Connected planar graph has $v+f=e+2$.

Proof sketch: Induction on e.
Base: $e=0$, $v=f=1$.

Euler's formula.

Euler: Connected planar graph has $v + f = e + 2$.

Proof sketch: Induction on e.
Base: $e = 0$, $v = f = 1$. $p(0)$ (base case) holds

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Base: $e = 0$, $v = f = 1$. $p(0)$ (base case) holds
Induction Step:

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Base: $e = 0$, $v = f = 1$. $p(0)$ (base case) holds
Induction Step:
  If it is a tree.

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  If not a tree.
    Find a cycle. Remove edge.

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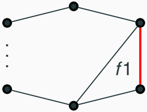

Outer face.

Joins two faces.

Euler's formula.

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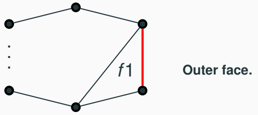

Joins two faces.
New graph: v-vertices.

Euler's formula.

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Proof sketch: Induction on e.
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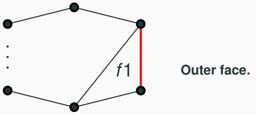

Joins two faces.
New graph: v-vertices. $e-1$ edges.

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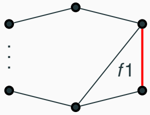

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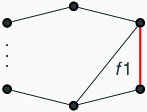

Outer face.

Joins two faces.
New graph: v-vertices. $e - 1$ edges. $f - 1$ faces. Planar.

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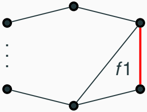

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$v+(f-1)=(e-1)+2$ by induction hypothesis for a smaller graph with $e-1$ edges.

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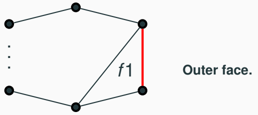

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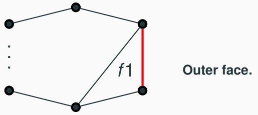

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Graph Coloring.

Given $G = (V, E)$, a coloring of a G assigns colors to vertices V where for each edge the endpoints have different colors.

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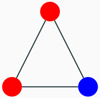

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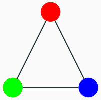

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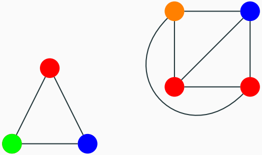

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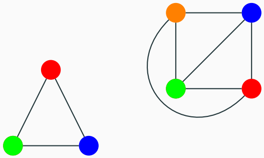

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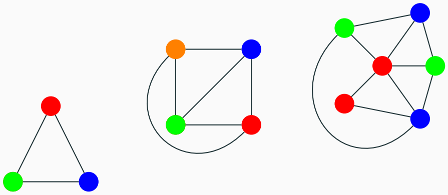

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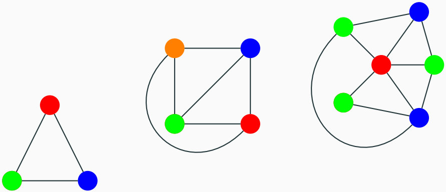

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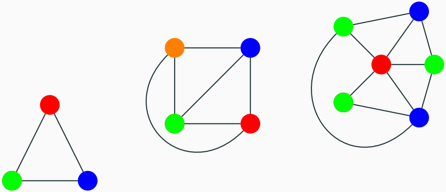

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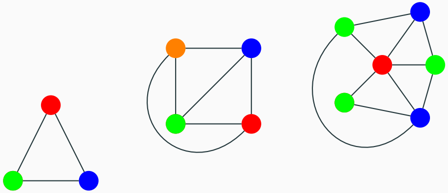

Notice that the last one, has one three colors.

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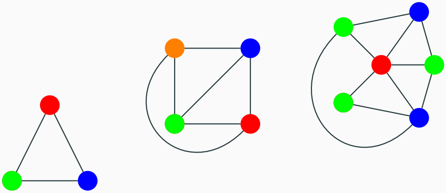

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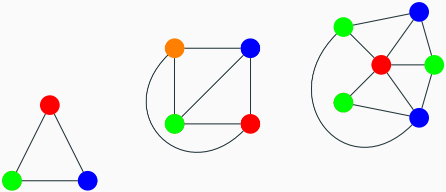

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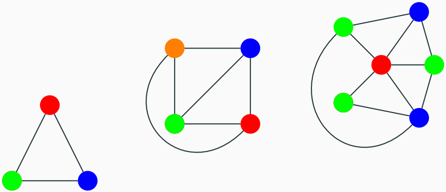

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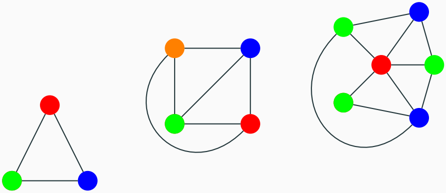

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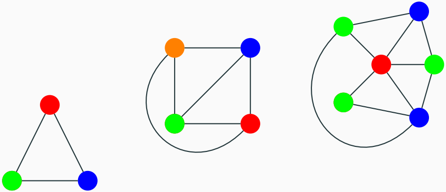

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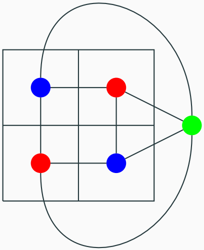
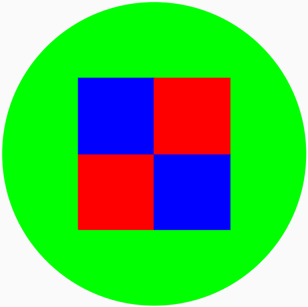
Interesting things to do. Algorithm!

Planar graphs and maps.

Planar graph coloring $\equiv$ map coloring.





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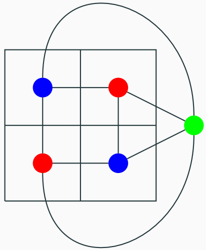

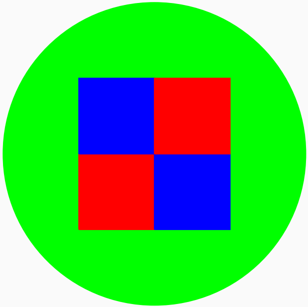

Four color theorem is about planar graphs!

Six color theorem.

Theorem: Every planar graph can be colored with six colors.

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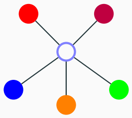

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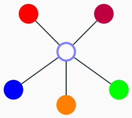

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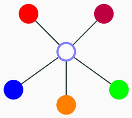

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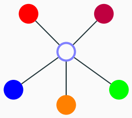

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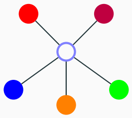

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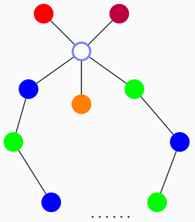

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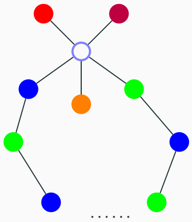

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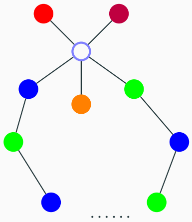

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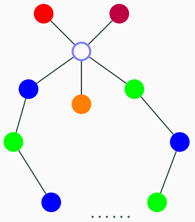

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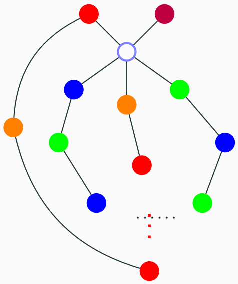

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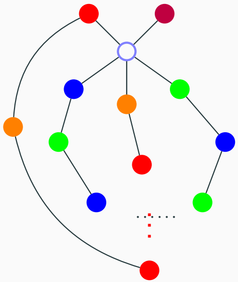

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Planar.

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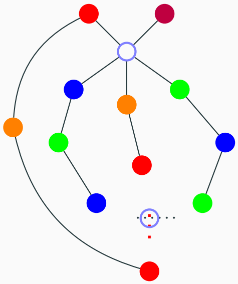

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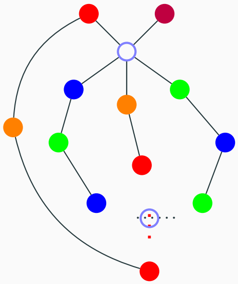

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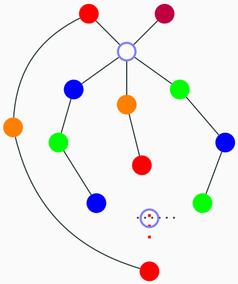

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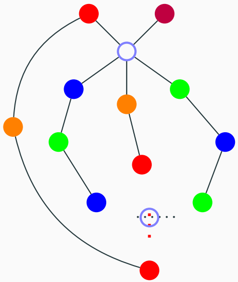

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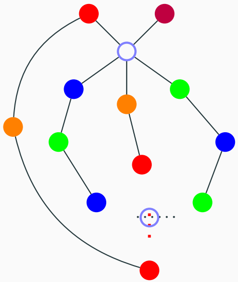

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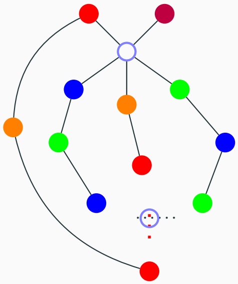

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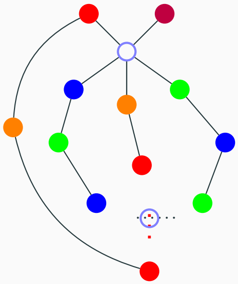

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And recolor "center" vertex.

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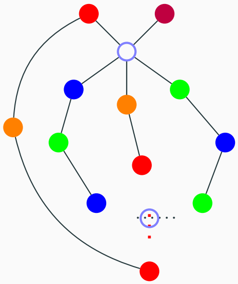

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