Today

Review for Midterm.

First there was logic...

A statement is a true or false.

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A statement is a true or false.
Don’t worry about Gödel.

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Statements?

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Statements?
$3 = 4 - 1$?

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Statements?

$3 = 4 - 1$? Statement!

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Statements?

$3 = 4 - 1$? Statement!

$3 = 5$?

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Statements?

$3 = 4 - 1$? Statement!

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3 ?

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Statements?

$3 = 4 - 1$? Statement!

$3 = 5$? Statement!

3 ? Not a statement!

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$n = 3$? Not a statement...

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Statements?
$3 = 4 - 1$? Statement!
$3 = 5$? Statement!
3 ? Not a statement!
$n = 3$? Not a statement...but a predicate.

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Statements?

$3 = 4 - 1$? Statement!

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Predicate: Statement with free variable(s).

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Predicate: Statement with free variable(s).

Example: $x = 3$

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Predicate?

$n > 3$?

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Predicate: Statement with free variable(s).

Example: $x = 3$ Given a value for x, becomes a statement.

Predicate?

$n > 3$? Predicate: $P(n)$!

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Example: $x = 3$ Given a value for x, becomes a statement.

Predicate?

$n > 3$? Predicate: $P(n)$!

$x = y$?

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Predicate: Statement with free variable(s).

- Example: $x = 3$ Given a value for x, becomes a statement.

Predicate?

- $n > 3$? Predicate: $P(n)$!
- $x = y$? Predicate: $P(x, y)$!

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Example: $x = 3$ Given a value for x, becomes a statement.

Predicate?

$n > 3$? Predicate: $P(n)$!

$x = y$? Predicate: $P(x, y)$!

$x + y$?

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$x + y$? No.

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$x + y$? No. An expression, not a statement.

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Quantifiers:

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$n > 3$? Predicate: $P(n)$!
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Quantifiers:

$(\forall x)\ P(x)$.

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Quantifiers:
$(\forall x)\ P(x)$. For every x, $P(x)$ is true.

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Quantifiers:

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$(\forall n \in N), n^2 \geq n$:

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$(\forall n \in N), n^2 \geq n$: Any free variables?

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$(\forall n \in N), n^2 \geq n$: Any free variables? No. So it's a statement.
$(\forall x \in R)(\exists y \in R) y > x$.

Connecting Statements

$A \wedge B$, $A \vee B$, $\neg A$, $A \implies B$.

Connecting Statements

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Propositional Expressions and Logical Equivalence

Connecting Statements

$A \wedge B$, $A \vee B$, $\neg A$, $A \implies B$.

Propositional Expressions and Logical Equivalence

$(A \implies B) \equiv (\neg A \vee B)$

Connecting Statements

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Propositional Expressions and Logical Equivalence

$(A \implies B) \equiv (\neg A \vee B)$
$\neg(A \vee B) \equiv (\neg A \wedge \neg B)$

Connecting Statements

$A \wedge B$, $A \vee B$, $\neg A$, $A \implies B$.

Propositional Expressions and Logical Equivalence

$(A \implies B) \equiv (\neg A \vee B)$
$\neg(A \vee B) \equiv (\neg A \wedge \neg B)$

Proofs: truth table or manipulation of known formulas.

Connecting Statements

$A \wedge B$, $A \vee B$, $\neg A$, $A \implies B$.

Propositional Expressions and Logical Equivalence

$$(A \implies B) \equiv (\neg A \vee B)$$
$$\neg(A \vee B) \equiv (\neg A \wedge \neg B)$$

Proofs: truth table or manipulation of known formulas.

$$(\forall x \in \mathbb{R})(P(x) \wedge Q(x)) \equiv (\forall x \in \mathbb{R})P(x) \wedge (\forall x \in \mathbb{R})Q(x)$$

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If you think it's true:

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If you think it's true:

Step 1:

Connecting Statements

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If you think it's true:

Step 1: Show that when the thing on the left is true, the thing on the right is true.

Connecting Statements

$A \wedge B$, $A \vee B$, $\neg A$, $A \implies B$.

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$$(A \implies B) \equiv (\neg A \vee B)$$
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$$(\forall x \in \mathbb{R})(P(x) \wedge Q(x)) \equiv (\forall x \in \mathbb{R})P(x) \wedge (\forall x \in \mathbb{R})Q(x)$$

If you think it's true:

Step 1: Show that when the thing on the left is true, the thing on the right is true. No matter what P and Q are!

Connecting Statements

$A \wedge B$, $A \vee B$, $\neg A$, $A \implies B$.

Propositional Expressions and Logical Equivalence

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$$(\forall x \in \mathbb{R})(P(x) \wedge Q(x)) \equiv (\forall x \in \mathbb{R})P(x) \wedge (\forall x \in \mathbb{R})Q(x)$$

If you think it's true:

Step 1: Show that when the thing on the left is true, the thing on the right is true. No matter what P and Q are!

Step 2: Show that when the thing on the right is true, the thing on the left is true.

Connecting Statements

$A \wedge B$, $A \vee B$, $\neg A$, $A \implies B$.

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Proofs: truth table or manipulation of known formulas.

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If you think it's true:

Step 1: Show that when the thing on the left is true, the thing on the right is true. No matter what P and Q are!

Step 2: Show that when the thing on the right is true, the thing on the left is true. No matter what P and Q are!

Or manipulate the formulas.

Connecting Statements

$A \wedge B$, $A \vee B$, $\neg A$, $A \implies B$.

Propositional Expressions and Logical Equivalence

$$(A \implies B) \equiv (\neg A \vee B)$$
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Proofs: truth table or manipulation of known formulas.

$$(\forall x \in \mathbb{R})(P(x) \wedge Q(x)) \equiv (\forall x \in \mathbb{R})P(x) \wedge (\forall x \in \mathbb{R})Q(x)$$

If you think it's true:

Step 1: Show that when the thing on the left is true, the thing on the right is true. No matter what P and Q are!

Step 2: Show that when the thing on the right is true, the thing on the left is true. No matter what P and Q are!

Or manipulate the formulas.

If you think it's not true:

Connecting Statements

$A \land B$, $A \lor B$, $\neg A$, $A \implies B$.

Propositional Expressions and Logical Equivalence

$$(A \implies B) \equiv (\neg A \lor B)$$
$$\neg(A \lor B) \equiv (\neg A \land \neg B)$$

Proofs: truth table or manipulation of known formulas.

$$(\forall x \in \mathbb{R})(P(x) \land Q(x)) \equiv (\forall x \in \mathbb{R})P(x) \land (\forall x \in \mathbb{R})Q(x)$$

If you think it's true:

Step 1: Show that when the thing on the left is true, the thing on the right is true. No matter what P and Q are!

Step 2: Show that when the thing on the right is true, the thing on the left is true. No matter what P and Q are!

Or manipulate the formulas.

If you think it's not true:

Find an example of $P(x)$ and $Q(x)$

Connecting Statements

$A \wedge B$, $A \vee B$, $\neg A$, $A \implies B$.

Propositional Expressions and Logical Equivalence

$$(A \implies B) \equiv (\neg A \vee B)$$
$$\neg(A \vee B) \equiv (\neg A \wedge \neg B)$$

Proofs: truth table or manipulation of known formulas.

$$(\forall x \in \mathbb{R})(P(x) \wedge Q(x)) \equiv (\forall x \in \mathbb{R})P(x) \wedge (\forall x \in \mathbb{R})Q(x)$$

If you think it's true:

Step 1: Show that when the thing on the left is true, the thing on the right is true. No matter what P and Q are!

Step 2: Show that when the thing on the right is true, the thing on the left is true. No matter what P and Q are!

Or manipulate the formulas.

If you think it's not true:

Find an example of $P(x)$ and $Q(x)$ such that one of the above steps fails.

...and then proofs...

Direct: $P \implies Q$

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Direct: $P \implies Q$

Example: a is even $\implies$ a^2 is even.

...and then proofs...

Direct: $P \implies Q$

Example: a is even $\implies a^2$ is even.

Approach: What is even?

...and then proofs...

Direct: $P \implies Q$

Example: a is even $\implies a^2$ is even.

Approach: What is even? $a = 2k$

...and then proofs...

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$a^2 = 4k^2$

...and then proofs...

Direct: $P \implies Q$

Example: a is even $\implies a^2$ is even.

Approach: What is even? $a = 2k$

$a^2 = 4k^2 = 2(2k^2)$

...and then proofs...

Direct: $P \implies Q$

Example: a is even $\implies$ a^2 is even.

Approach: What is even? $a = 2k$

$a^2 = 4k^2 = 2(2k^2)$

Integers closed under multiplication!

...and then proofs...

Direct: $P \implies Q$

Example: a is even $\implies a^2$ is even.

Approach: What is even? $a = 2k$

$a^2 = 4k^2 = 2(2k^2)$

Integers closed under multiplication! So $2k^2$ is even.

...and then proofs...

Direct: $P \implies Q$

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Contrapositive: $P \implies Q$

...and then proofs...

Direct: $P \implies Q$

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$a^2 = 4k^2 = 2(2k^2)$

Integers closed under multiplication! So $2k^2$ is even.

a^2 is even.

Contrapositive: $P \implies Q$ or $\neg Q \implies \neg P$.

...and then proofs...

Direct: $P \implies Q$

Example: a is even $\implies a^2$ is even.

Approach: What is even? $a = 2k$

$a^2 = 4k^2 = 2(2k^2)$

Integers closed under multiplication! So $2k^2$ is even.

a^2 is even.

Contrapositive: $P \implies Q$ or $\neg Q \implies \neg P$.

Example: a^2 is odd $\implies a$ is odd.

...and then proofs...

Direct: $P \implies Q$

Example: a is even $\implies a^2$ is even.

Approach: What is even? $a = 2k$

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Contrapositive: a is even $\implies a^2$ is even.

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Contrapositive: $P \implies Q$ or $\neg Q \implies \neg P$.

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...and then proofs...

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Example: rogue couple does not exist.

...jumping forward..

Contradiction in induction:

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Find a place where induction step doesn't hold.

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$(3^{2n}-1 = 8d)$

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\begin{aligned}
3^{2n+2} - 1 &= 9(3^{2n}) - 1 \quad \text{(by induction hypothesis)} \\
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&= 72d + 8
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$\square$

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Connected Graph: one connected component.

Graph Algorithm: Eulerian Tour

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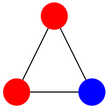

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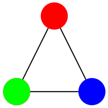

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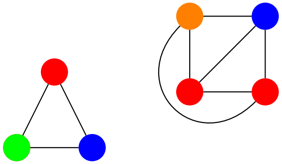

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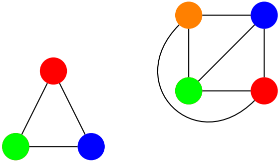

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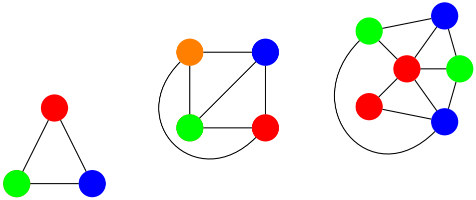

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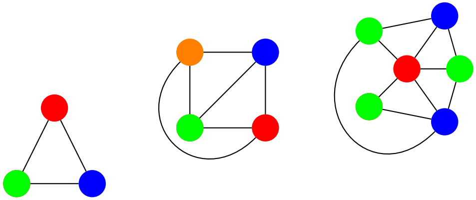

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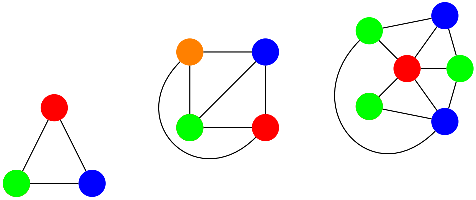

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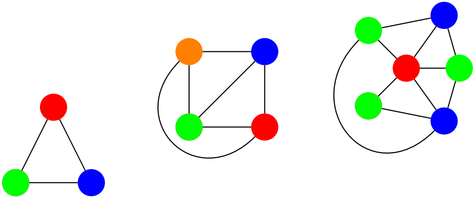

Notice that the last one, has one three colors.

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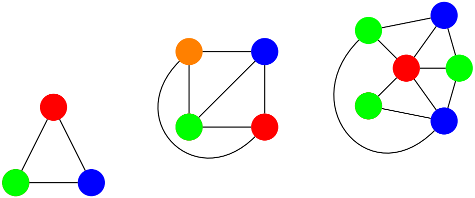

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Fewer colors than number of vertices.

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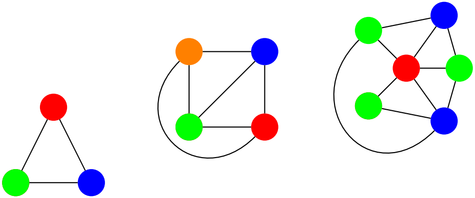

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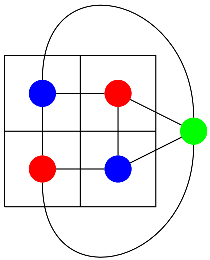
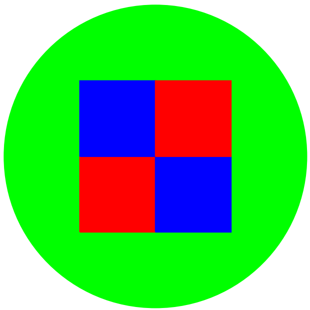
Fewer colors than max degree node.

Planar graphs and maps.

Planar graph coloring $\equiv$ map coloring.





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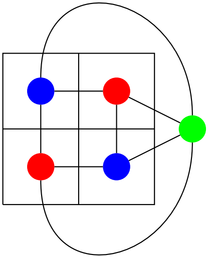

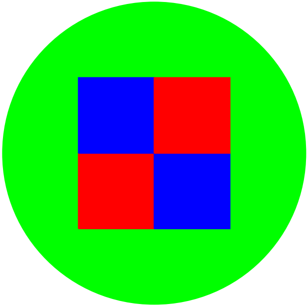

Four color theorem is about planar graphs!

Six color theorem.

Theorem: Every planar graph can be colored with six colors.

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Proof: Not Today!

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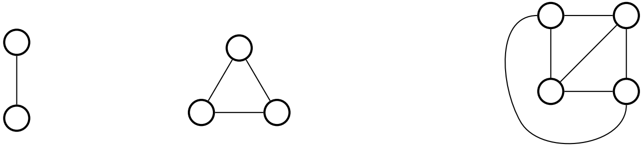
Graph Types: Complete Graph.



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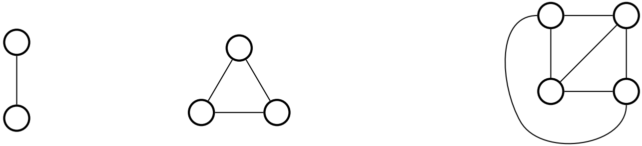

K_n, $|V| = n$

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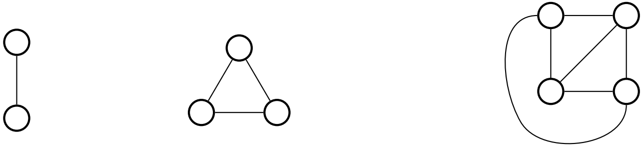

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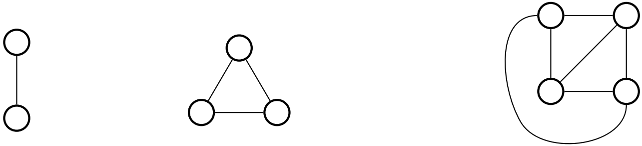

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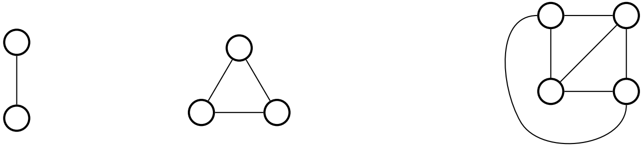

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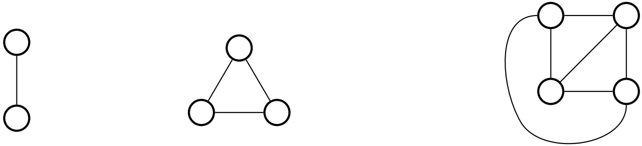

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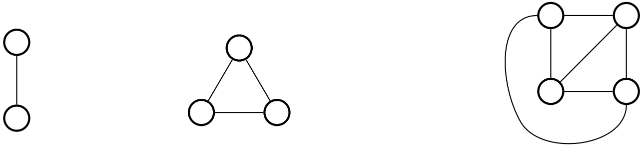

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 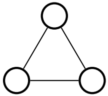 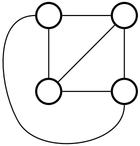

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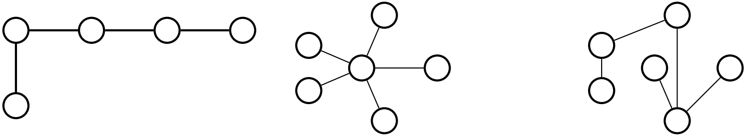
Lots of edges: $n(n-1)/2$.

Trees.



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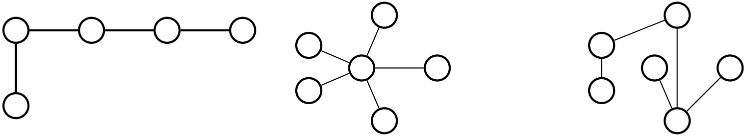

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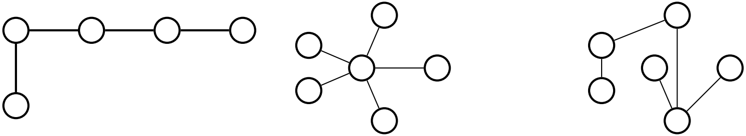

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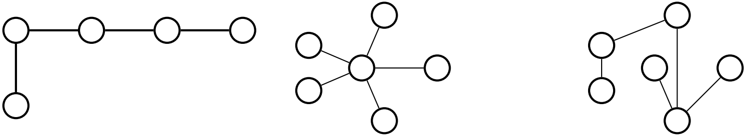

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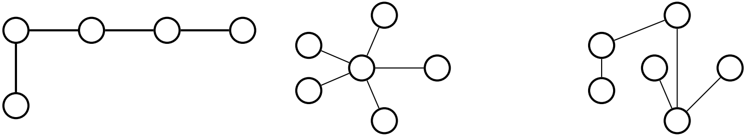
A connected graph where any edge removal disconnects it.

Trees.



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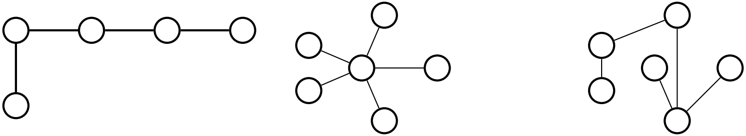

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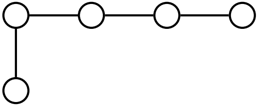 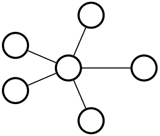 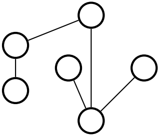

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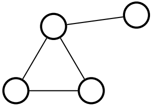
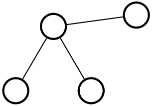
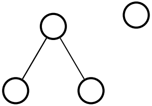
To tree or not to tree!

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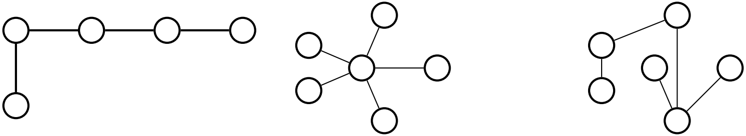

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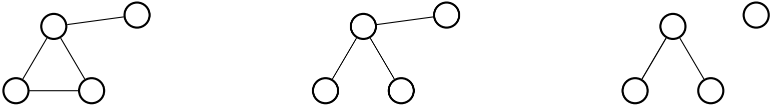

Minimally connected, minimum number of edges to connect.

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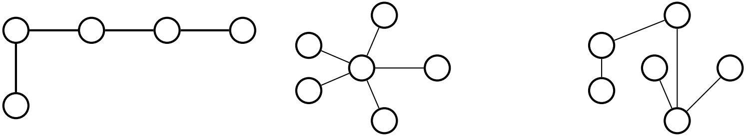

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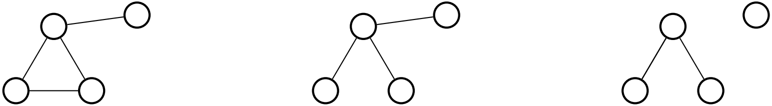

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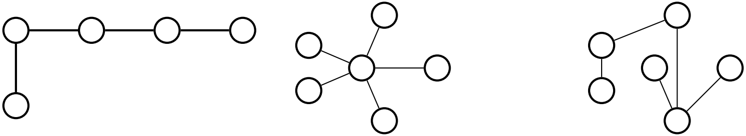

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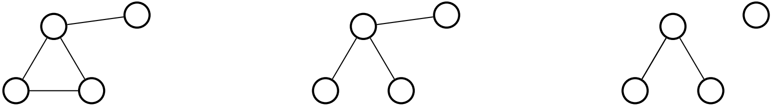

Minimally connected, minimum number of edges to connect.

Property:
Can remove a single node and break into components of size at most $|V|/2$.

Hypercube

Hypercubes.

Hypercube

Hypercubes. Really connected.

Hypercube

Hypercubes. Really connected. $|V| \log |V|$ edges!

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Wait what?

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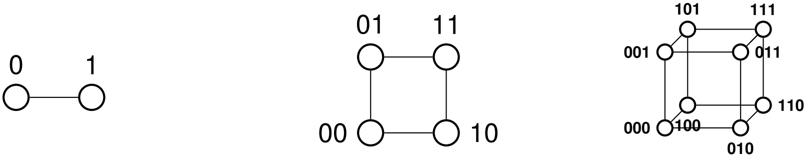

Recursive Definition.

A 0-dimensional hypercube is a node labelled with the empty string of bits.

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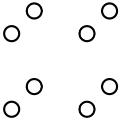

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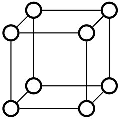

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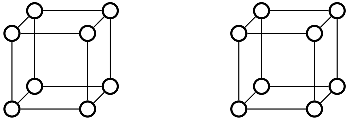

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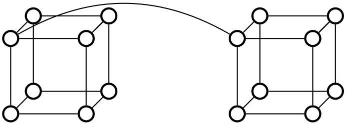

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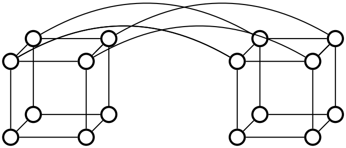

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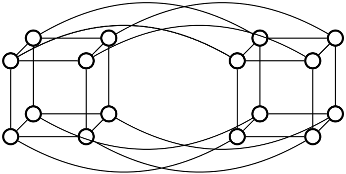

Hypercube:properties

Dense cuts: Cutting off k nodes needs $\geq k$ edges.

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FYI:

Hypercube:properties

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FYI: Also cuts represent boolean functions.

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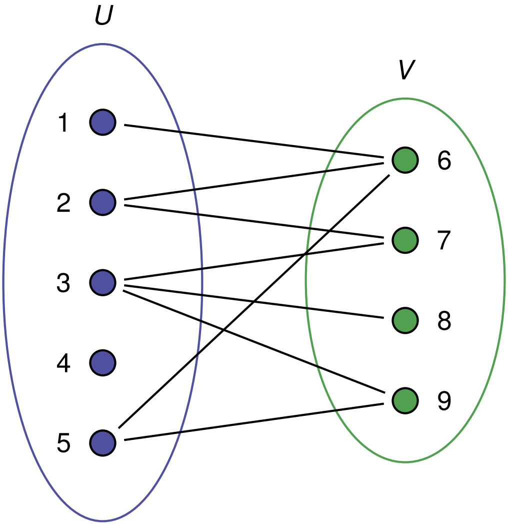
Good communication network!

Bipartite graphs



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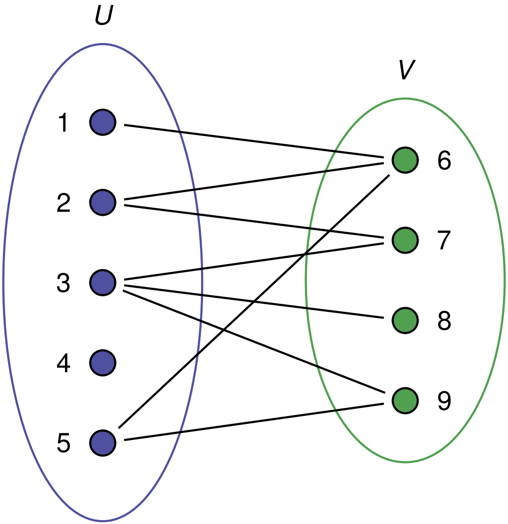

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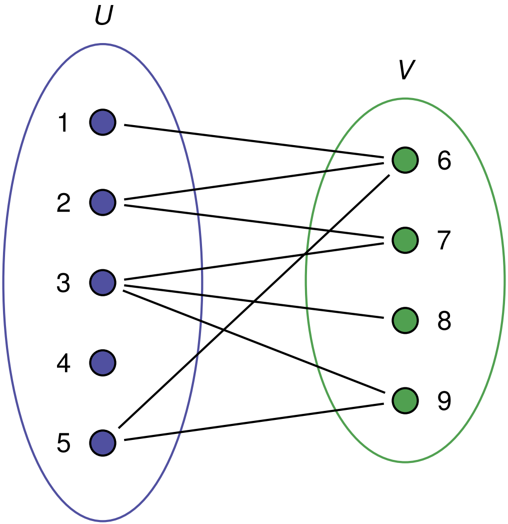

There is a cut with all the edges.

Cycles have length 4 or more edges.

Stable Marriage: a study in definitions and WOP.

n-men, n-women.

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Does stable pairing exist?

Yes for matching.

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Pairing with no rogue couples.

Does stable pairing exist?

Yes for matching.
No, for roommates problem.

TMA.

Traditional Marriage Algorithm:

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Each Day:

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Every man proposes to his favorite woman from the ones that haven't already rejected him.

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$\implies$ W ended up with someone she liked better than M.

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Stability: No rogue couple.
rogue couple (M,W)
$\Longrightarrow$ M proposed to W
$\Longrightarrow$ W ended up with someone she liked better than M.
Not rogue couple!

Optimality/Pessimal

Optimal partner if best partner in any stable pairing.

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Thm: woman pessimal.

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Thm: woman pessimal.

  Man optimal $\implies$ Woman pessimal.

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Man optimal $\implies$ Woman pessimal.
Woman optimal $\implies$ Man pessimal.

And then countability

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More than one infinities

And then countability

More than one infinities

Some things are countable

And then countability

More than one infinities

Some things are countable , like the natural numbers

And then countability

More than one infinities

Some things are countable , like the natural numbers , or the rationals...

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Why?

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Why? **Diagonalization:** Well, assume there is a list.

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Why? **Diagonalization:** Well, assume there is a list. Can construct a diagonal element x. x is not in the list! Contradiction.

HALTING

HALTING

The HALT problem:

HALTING

The HALT problem: Is there a program that can tell you if another (generic) program halts on an input?

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NO!

Why? Self reference!

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Who cares?

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Like: Will this program *P* even print ”Hello World”?

HALTING

The HALT problem: Is there a program that can tell you if another (generic) program halts on an input?

NO!

Why? Self reference!

Who cares? Using the same trick I can show that a bunch of problems are undecidable!
Like: Will this program P even print "Hello World"?
Or "Is there an input for this program P that will give an attacker admin access?

Counting!

Sample k items out of n.

	With Replacement	Without Replacement
Order matters	n^k	$\frac{n!}{(n-k)!}$
Order doesn't matter	$\binom{n+k-1}{n-1}$	$\binom{n}{k}$

Stars and bars!

Stars and bars!

Confusion yesterday: 10 hats.

Stars and bars!

Confusion yesterday: 10 hats. 7 days.

Stars and bars!

Confusion yesterday: 10 hats. 7 days. I can wear the same hat on different days (replacement).

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How many stars?

Stars and bars!

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Why is this stars and bars?

How many stars? One for each day.

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Didn't wear hats 1 and 2. Wore hat 3 for 1 day, hat 4 for 2 days, hat 5 days. Didn't wear hats 6 and 7. Hat 8 for 3 days. Didn't wear hats 9 and 10.

Combinatorial Proofs.

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What's the thing on the left? Number of subsets of size k of $\{1, 2, \ldots, n+1\}$.

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Add them up. (**Sum rule**)

Midterm format

Time: 110 minutes.

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Time: 110 minutes.

Some short answers.

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Some short answers.
  Get at ideas that you learned.

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  If something is taking too long maybe there is a trick!

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Know material well:

Midterm format

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Get at ideas that you learned.

If something is taking too long maybe there is a trick!

Know material well: fast,

Midterm format

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If something is taking too long maybe there is a trick!

Know material well: fast, correct.

Midterm format

Time: 110 minutes.

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Midterm format

Time: 110 minutes.

Some short answers.

Get at ideas that you learned.

If something is taking too long maybe there is a trick!

Know material well: fast, correct.

Know material medium: slower,

Midterm format

Time: 110 minutes.

Some short answers.

- Get at ideas that you learned.
- If something is taking too long maybe there is a trick!
  - Know material well: fast, correct.
  - Know material medium: slower, less correct.

Midterm format

Time: 110 minutes.

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Know material not so well:	

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Remember that a problem from hw and/or discussions is in the midterm! (identical or almost identical)

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So study those!

Wrapup.

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If you sent us an email about Midterm conflicts

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