

**Problem 1:** (35) On Gradient Descent (GD). Find the global minimum of  $f(t) = 50 * \sin(t) + t^2$  over  $-10 \leq t \leq 10$ . This problem intends to give you a hands-on experience on how gradient descent works and how it can get trapped at the local minima.

- a) (5) Plot this function. Visualize the multiple local minima and the global minimum.
- b) (30) Implement gradient descent in Python to find the local minimum.
  - i) (10) Pick a starting point at  $t=7$ . What's the minimum? Show the convergence path. Experimenting with different learning rate.
  - ii) (10) Pick a starting point at  $t=1$ . What's the minimum? Show the convergence path. Experimenting with different learning rate.
  - iii) (10) Comment on the results from the above experiments

**Problem 2:** (30) On Perceptron.

- a) (15) Use Perceptron to implement the OR logic. Show output from each iteration (that is, the two inputs, the targeted output, and the Perceptron output) with the maximum number of iterations being 10.
- b) (15) Use Perceptron to implement the XOR logic. Show output from each iteration (that is, the two inputs, the targeted output, and the Perceptron output) with the maximum number of iterations being 10.

**Problem 3:** (35) Comparison between FLD, PCA, and Perceptron.

Note that FLD and PCA are dimensionality reduction methods that only output a projection direction. Additional classification methods need to be applied to find the decision boundary. Suppose the minimum (Euclidean) distance (MD) classifier is used. On the other hand, Perceptron is a linear classifier that outputs the decision boundary directly. On the same figure, plot the four samples of the AND gate (5 pts), and compare the decision boundary from FLD+MD (10 pts), PCA+MD (10 pts), and Perceptron (10 pts). You can use whichever language that you feel comfortable (pencil & paper or Python).