COSC522 – Homework #1 (Due 09/02)

Problem 1 (65): In a 1-D, 3-class problem, the density function of each class can be adequately represented by univariate Gaussians, with $\mu_1 = 4, \sigma_1 = 2, \mu_2 = 6, \sigma_2 = 3, \mu_3 = 5, \sigma_3 = 2$.

(1) (15) Sketch the three density functions on the same figure using pencil and paper (i.e., without Python or MATLAB or other software package). Assume equal prior probability, predict how many decision regions there would be. (Grading: pay attention to the relative position, relative height, relative width of the Gaussians)

(2) (35) Assume equal prior probability,

a. (5) If x=4.7, which class does x belong to? Use the MAP method. Show detailed steps.

b. (15) Find the exact decision boundary using analytical methods instead of the sketch.

c. (15) Solve for the overall probability of error. Provide details.

(3) (15) Assume that $P(\omega_1) = 0.6, P(\omega_2) = 0.2, P(\omega_3) = 0.2$.

a. (10) Use a software package to draw the posterior probability.

b. (5) Redo question (2)a.

Problem 2 (35): You are doing a test on disease diagnosis. If the patient has the disease, then he/she is classified as positive. Let class 1 be the actual negative cases with a uniform pdf on the interval [0, 1]. Let class 2 be the actual positive case with a uniform pdf on the interval [0.95, 3.95].

(1) (5) Plot the pdfs on the same figure.

(2) (15) Suppose equal prior probability. Given a decision boundary of 0.97,

a. What is the probability for false-negative? That is, the patient actually has the disease but the classifier says the patience is healthy.

b. What is the probability for false-positive? That is, the patient is healthy but our classifier says otherwise. Note that the loss for false-negative should be much higher than that for false-positive.

(3) (15) Is 0.97 the optimal decision boundary in Baysian sense? If not, what is the optimal decision boundary that minimizes the overall probability of error? How to adjust the prior probabilities to further reduce the overall probability of error?