ITP20002-02 Discrete Mathematics, Fall 2017

Midterm Exam

Name and Student Number: ____________________________

- Total **10 pages** including this cover page and 3 blank pages for notes (pp.8—10).
- You have **120 minutes** to complete **8 problems** (100 points).
- Write answers only on given boxes. Write them clearly. No point for illegible answers.
- Write all answers in English. Korean is allowed only for commenting your English expressions.
- Remind the following quoted from Handong CSEE Standard and put your signature below as a mean to show your agreement to keep the standard in taking this exam.

Examination
1. Examination is an educational act necessary for evaluation of the students' achievement and for encouraging the students to absorb the material in the process of preparation. 2. Student should do their best to prepare for exams in order to improve her/his own knowledge and skill, and should fully engage in the test during examination hour. 3. Accessing or providing unauthorized information, including other students' answer sheets, is regarded as cheating. The use of electronic devices, including cell phones and computers, without permission is strictly prohibited. 4. Entering or leaving the classroom during the examination before the finish time without permission is regarded as cheating.

I agree to uphold Handong Honor Code and Handong CSEE Standard in taking this exam.

Signature: ____________________________________

1. Define the satisfiability and the validity of a propositional formula (10 points)

2. Give a result of converting the following propositional formula into a DIMACS representation (8 points)

$$(\neg p \vee \neg q) \rightarrow (r \leftrightarrow \neg q)$$

3. Show that the power set of a countably infinite set is uncountable (16 points).

4. Give a tautology of the following six rules of inference (18 points)

Modus ponens	
Modus tollens	
Resolution	
Simplification	
Addition	
Disjunctive syllogism	

5. Explain what is a theorem and also explain what is a proof (9 points)

6. Use mathematical induction to show that $\neg(\boldsymbol{p_1} \vee \boldsymbol{p_2} \vee \ldots \vee \boldsymbol{p_n})$ is equivalent to $\neg\boldsymbol{p_1} \wedge \neg\boldsymbol{p_2} \wedge \ldots . \wedge \neg\boldsymbol{p_n}$ (14 points).

7. Give two versions of proofs – one as a proof by contradiction, and the other as a proof by contraposition – that show that a positive integer n is odd if n^3+5 is odd (10 points)

(a) a proof by contradiction

(b) a proof by contraposition

8. Give an answer to each of the following questions (15 points)

(a) For two finite sets *A* and *B* of the same universe *U*, list the followings in order of increasing size:

$$|A|, \qquad |A \cup B|, \qquad |A \cap B|, \qquad |\emptyset|, \qquad |U|$$

(b) For two finite sets A and B, list the followings in order of increasing size:

$$|A - B|, \qquad |A| + |B|, \qquad |A \cup B|, \qquad |\emptyset|, \qquad |A \oplus B|$$

(c) Let f be a function from $\{1, 2, 3, 4\}$ to $\{a, b, c, d\}$, and let g be a function from $\{a, b, c, d\}$ to $\{1, 2, 3, 4\}$. Suppose that $f(1) = d, f(2) = c, f(3) = a, f(4) = b$, and $g(a) = 2, g(b) = 1, g(c) = 3, g(d) = 2$. Find the inverse of f and also the inverse of g.

(d) Specify the condition that a relation $R \subset A \times B$ represents a function from a finite set A to a finite set B.

For your note

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