Calculus 3 week2 problem set

Taylor approximation, multiple integral, polar and spherical coordinates, substitution rule, vector field and line integral.

1. Find quadratic Taylor polynomial for $f(x,y)=\sin x \sin y$ at the origin.

2. Evaluate following integrals

(a) $\int_0^1 \int_1^2 ye^{2x+y}dxdy$

(b) $\iint_T x^2+y^2 dxdy$ where T is a triangular region with vertices (0,0), (1,0), (0,1).

c) $\iint_D x^2+y^2 dxdy$ where $D=\{(\mathrm{x},\mathrm{y}):\mathrm{x}^2+\mathrm{y}^2 \le 1\}$

3. Find the volume of the region bounded by the paraboloid $z=8-x^2-y^2$ and $z=x^2+y^2$ using triple integral.

4. (spherical coordinates) Find the volume of the region between the sphere $\rho=\cos\phi$ and the hemisphere $\rho=2, z\ge 0$ using integral in spherical coordinates.

5. Evaluate $\iint_R (2x^2-xy-y^2)dxdy$ for the region R in the first quadrant bounded by the lines $y=-2x+4$, $y=-2x+7$, $y=x-2$, $y=x+1$. Use transform $u=y+2x$, $v=y-x$ and apply substitution rule.

6. Find the work done by $\vec{F}=xyi+yj-yzk$ over the curve $\vec{r}(t)=ti+t^2j+tk$, $(0\le t\le 1)$.