⟨5.4 로지스틱 회귀⟩

· $f(z) = \frac{e^{z}}{(1+e^{z})^2}$, $F(z) = 1 - \frac{1}{(1+e^{z})}$

pdf   cdf

· From ⟨theorem 4.3.3⟩ $F(x) \sim U(0,1)$

어떤 cdf

이때 $F(x) = 1 - \frac{1}{1+e^{x}} \equiv u$

$$\Rightarrow F^{-1}(u) = \underbrace{\log \frac{u}{1-u}}_{x} \sim \boxed{L(0,1)}$$

$\boxed{F^{-1}(u) = x \text{ 인데}}$ $= \ln \frac{u}{1-u}$

$1-u = \frac{1}{1+e^{x}}$

$(1+e^{x}) = \frac{1}{1-u}$

$e^{x} = \frac{1}{1-u} - \frac{1-u}{1-u}$

$= \frac{u}{1-u}$

$x = \ln \frac{u}{1-u}$

$$\sigma x + \mu = \sigma \log \frac{u}{1-u} + \mu \sim L(\mu, \sigma)$$

② $U \sim U(0,1)$ 이면 $X \sim \exp(1)$ → $f(x) = e^{-x}$, $F(x) = 1 - e^{-x} \sim U(0,1)$

$F(x) = 1 - e^{-x} = u$ ∴ $F^{-1}(u) = x =$

$1-u = e^{-x}$

$-\ln(1-u) = x$

$F^{-1}(u) = -\ln(1-u)$

$\sim \exp(1)$

$\therefore \sigma(-\ln(1-u)) \sim \exp(\sigma)$

③ $\Phi(x) \sim U(0,1)$

$\Phi^{-1}(u) \sim N(0,1)$

$\sigma \Phi^{-1}(u) + \mu \sim N(\mu, \sigma^2)$

P150 $\boxed{\text{Ex 4.1.5}}$ $\boxed{X = \sigma Z + \mu}$,

$$\mathrm{pdf}_X(x) = f(z) \left| \frac{dx}{dz} \right|^{-1},$$

$$= \boxed{\frac{1}{\sigma} f\left(\frac{x-\mu}{\sigma}\right)}$$

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* uniform (0,1) 을 만드는 방법

① run-time library

② pseudo-random number generator

* Box-muller transformation

$X_1, X_2 \sim U(0,1)$ indep

$Y_1 = \sqrt{-2\ln X_1} \cdot \cos(2\pi X_2)$

$Y_2 = \sqrt{-2\ln X_1} \cdot \sin(2\pi X_2)$

$X_i = (b-a)U_i + a \sim U(a,b)$

$$= \operatorname*{plim}_{n\to\infty} \frac{b-a}{n} \sum_{i=1}^{n} g(X_i) = \int_a^b g(x)\,dx$$

증명 ⇒ $(b-a) \operatorname*{plim}_{n\to\infty} \frac{1}{n} \sum g(X_i)$

$\boxed{\text{LLN}}$ $(b-a) E(g(X_i))$

$\Rightarrow (b-a) \cdot \underbrace{\frac{1}{b-a}}_{f(x)} \int_a^b \boxed{g(x)dx}$

$= \int_a^b g(x)dx$

⇒ Monte Carlo Integration

$y_i = (b-a) g((b-a)u_i + a)$