ITP20002-01 Discrete Mathematics, 2019 Fall

Midterm Exam

Write an answer to each problem on the corresponding box in your answer sheets.

1. Use set-builder notation to define the set of all one-to-one correspondence functions from a set X to a set Y (20 points)

2. Prove that $\sqrt[3]{2}$ is irrational (16 points).

3. Prove or disprove that $|\mathcal{P}(S)| < |\mathcal{P}(\mathcal{P}(S))|$ for a countably infinite set S (25 points)

4. State in predicate logic that two functions that map from $\mathbb{Z}$ to $\mathbb{R}$, f and g, are of the same order (i.e., $f(x)$ is $\Theta(g(x))$ (15 points).

5. Prove or disprove that $(A - B) - C = A - (B - C)$ for sets A, B, and C (10 points)

6. Prove that the following premises $\forall x\,(\neg Q(x) \lor S(x))$, $\forall x\,(R(x) \to \neg S(x))$, $\forall x\,(P(x) \lor Q(x))$, and $\exists x \neg P(x)$ imply that $\exists x \neg R(x)$. Declare the corresponding rule of inferences at each step in your proof. (14 points)

ITP20002-02 Discrete Mathematics, 2019 Fall

Midterm Exam

Write an answer to each problem on the corresponding box in your answer sheets.

1. Compare the cardinalities of a range of real numbers $[0, 1)$ and $\mathcal{P}(\mathbb{N})$ (18 points)

2. Use set-builder notation to define the set of all enumerations of a set X without redundancy (20 points).

3. Answer each of the following questions (12 points)

   a. Is it possible to write a program A that receives a program p as input, and returns True if there exists an input that makes p do not halt?

   b. Is it possible to write a program B that receives a program p as input, and returns True if p always halt for every input?

   c. Assume that we have A and B. Can we give a solution of the Halting Problem by using A and B?

4. Prove that x is irrational if x^3 is irrational (16 points)

5. Show that $f(x)$ is $\Theta(h(x))$ if $f(x)$ is $\Theta(g(x))$ and $g(x)$ is $\Theta(h(x))$ (16 points)

6. Prove that $\forall x\big(P(x) \rightarrow Q(x)\big)$, $\forall x\big(\neg Q(x) \vee S(x)\big)$, $\forall x\big(R(x) \rightarrow \neg S(x)\big)$ and $\exists x R(x)$ imply that $\exists x \neg P(x)$, while declaring the corresponding rule of inferences at each step. (18 points)