

Calculus3 week 3 Problem set

(토요일 자정까지 제출)

1. Use Green's theorem to find the counterclockwise circulation for vector field \vec{F} along the curve C (that is, work done by \vec{F} over the curve C).

(a) $\vec{F} = (x^2 + 4y)\vec{i} + (x + y^2)\vec{j}$, C: The square bounded by $x=0$, $x=1$, $y=0$, $y=1$.

(b) $\vec{F} = (x + y)\vec{i} - (x^2 + y^2)\vec{j}$ C: The triangle bounded by $y=0$, $x=1$, $y=x$.

2. Use a parametrization to express the area of the surfaces given below as a double integral. Evaluate the integral.

(a) Plane inside cylinder: the portion of the plane $x+z=0$ inside the cylinder $x^2 + y^2 = 4$

(b) Parabolic cap: the cap cut from the paraboloid $z = 2 - x^2 - y^2$ by the cone $z = \sqrt{x^2 + y^2}$

3. Find the flux $\iint_S \vec{F} \cdot \vec{n} d\sigma$ across the surface in the given direction by direct computation.

(a) $\vec{F} = x\vec{i} + y\vec{j} + z\vec{k}$ across the sphere $x^2 + y^2 + z^2 = 4$ in the direction away from the origin.

(b) $\vec{F} = xy\vec{i} - z\vec{k}$ outward through the cone $z = \sqrt{x^2 + y^2}$, $0 \leq z \leq 1$.

4. Use the surface integral in Stokes' theorem to calculate the circulation of the vector field $\vec{F} = x^2\vec{i} + 2x\vec{j} + z^2\vec{k}$ around the curve C: the circle $4x^2 + y^2 = 4$ in the xy-plane, counterclockwise when viewed from above.

5. Use divergence theorem to find the outward flux of $\vec{F} = x^2\vec{i} + y^2\vec{j} + z^2\vec{k}$ across the boundary of the region D: the cube cut from the first octant by the planes $x=1$, $y=1$, $z=1$.