Calculus 3 week 1 problem set

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*학습지원장학금의 수여 대상이 되려면 각 문제에 대한 풀이를 상세하게 작성하여 12월 26일 밤 11시까지 제출. Blind copy한 풀이는 받지 않습니다.

1. 다음 함수의 편미분 $\frac{\partial f}{\partial x}$, $\frac{\partial f}{\partial y}$을 구하시오.

(a) $f(x,y) = x^2 - xy + y^2$

(b) $f(x,y) = \sqrt{x^2 + y^2}$

(c) $f(x,y) = \frac{x}{x^2 + y^2}$

(d) $f(x,y) = e^{-x}\sin(x+y)$

(e) $f(x,y) = \ln(x^2 + 5y^2)$

2. Express $\frac{dw}{dt}$ as a function of t using chain rule

(a) $w = x^2 + y^2$, $x = \cos t$, $y = \sin t$

(b) $w = \ln(x^2 + y^2 + z^2)$, $x = e^{t,}$ $y = \cosh t$, $z = \sinh t$

3. Express $\frac{\partial z}{\partial u}$, $\frac{\partial z}{\partial v}$ as a function of u and v and evaluate at $(u,v) = (2, \pi/4)$

$z = e^x \ln y$, $x = u\cos v, y = u\sin v$.

4. For $f(x,y,z) = x^2 - y^2 + 5z^2$ find gradient ∇f at (1,1,1).

5. Find the directional derivative of given function at P in the direction of v.

(a) $f(x,y) = xy - y^2$, P=(1,1), $v = 4i + 3j$

(b) $f(x,y,z) = e^x \cos yz$, P=(0,π,0.5), $v = 2i + j - 3k$

6. Find local max, local min and saddle point of given functions

$f(x,y) = x^2 + 2xy$

7. Find the absolute max and min of $f(x,y) = x^2 - xy + y^2$ over the triangle in the first quadrant bounded by the lines $x = 0, y = 4, y = x$.

8. Find the points on the ellipse $x^2 + 2y^2 = 1$ where $f(x,y) = xy$ has its extreme

values. (Use Lagrange multiplier method)

9. Find the max and min of $f(x,y,z)=x-2y+5z$ on the sphere $x^2+y^2+z^2=16$ (Use Lagrange multiplier method)

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(a) $f(x,y)=x^2-xy+y^2$

(b) $f(x,y)=\sqrt{x^2+y^2}$

(c) $f(x,y)=\dfrac{x}{x^2+y^2}$

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(e) $f(x,y)=\ln(x^2+5y^2)$

(a) $\dfrac{\partial f}{\partial x}=2x-y$, $\dfrac{\partial f}{\partial y}=-x+2y$

(b) $\dfrac{\partial f}{\partial x}=\dfrac{\cancel{2}x}{\cancel{2}\sqrt{x^2+y^2}}$, $\dfrac{\partial f}{\partial y}=\dfrac{\cancel{2}y}{\cancel{2}\sqrt{x^2+y^2}}$

(c) $\dfrac{df}{dx}=\dfrac{f(x)}{g(x)}=\dfrac{f'(x)g(x)-f(x)g'(x)}{g(x)^2}=\dfrac{(x^2+y^2)-x(2x)}{(x^2+y^2)^2}$

$=\dfrac{y^2-x^2}{(x^2+y^2)^2}$

$\dfrac{df}{dy}=\dfrac{0\cdot(x^2+y^2)-x(2y)}{(x^2+y^2)^2}$

$=\dfrac{-2xy}{(x^2+y^2)^2}$

(d) $f(x,y)=e^{-x}\sin(x+y)=f(x)g(x)$

$\dfrac{df}{dx}=f'(x)g(x)+f(x)g'(x)$

$=-e^{-x}\sin(x+y)+e^{-x}\cdot\cos(x+y)$

$\dfrac{df}{dy}=e^{-x}\cos(x+y)$

(e) $\ln(x^2+5y^2) \rightsquigarrow \dfrac{df}{dx}=\dfrac{2x}{x^2+5y^2}$

$\dfrac{df}{dy}=\dfrac{10y}{x^2+5y^2}$