$X_i \sim Be(p) \Rightarrow (p, pq)$ $\quad E(X_i) = p$

$\bar{X} = \frac{\sum X_i}{n} \Rightarrow E(\bar{X}) = \frac{1}{n}\sum E(X_i) = \frac{np}{n} = p$

$V(\bar{X}) = V\left(\frac{\sum X_i}{n}\right) = \frac{1}{n^2}\sum V(X_i) = \frac{npq}{n^2} = \frac{pq}{n}$

$1.96\sqrt{\frac{1}{4n}}$

$E(X \pm Y) = E(X) \pm E(Y)$

$E(\hat{p}) = p$: unbiased estimator

$X \sim (\mu, \sigma^2) \to \bar{X} \sim \left(\mu, \frac{\sigma^2}{n}\right)$

$\hat{p} = \bar{X} \overset{CLT}{\sim} N\left(p, \frac{pq}{n}\right)$

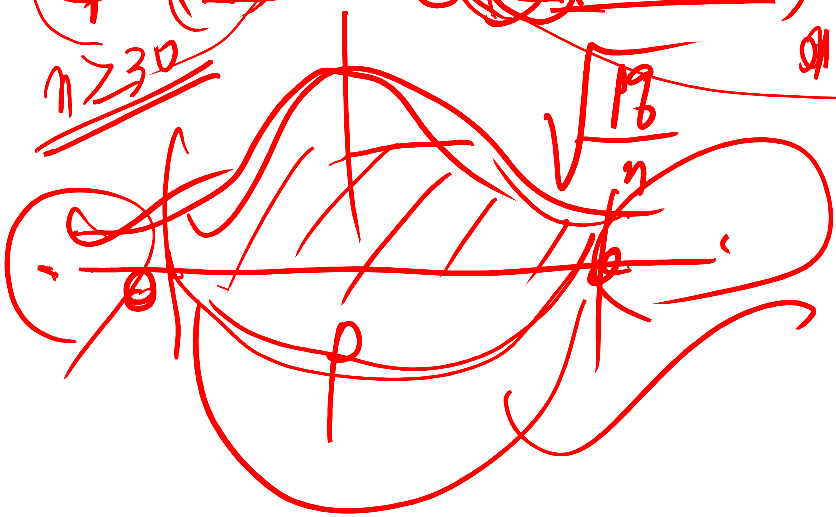
$n \geq 30$

p : 95% $(1-\alpha)$ $\quad \hat{p} \pm z_{\alpha/2}\sqrt{\frac{pq}{n}}$ $\quad \frac{pq}{n}$, $p = \frac{1}{2}$

$np > 5, nq > 5$

$P(X > c) \approx P(X_N > c)$

$N(np, npq)$



95%

± 1.96

90%

± 1.27

$\bar{X} \pm 1.96\frac{S}{\sqrt{n}}$ $\quad \sigma^2 \to S^2$

$X \sim N(\mu, \sigma^2)$ $\quad \bar{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$

ANOVA

σ^2

$X \sim (\mu, \sigma^2) \xrightarrow[CLT]{n \geq 30} \bar{X} \overset{\cdot}{\sim} N\left(\mu, \frac{\sigma^2}{n}\right)$

binomial case

$np > 5, \; nq > 5$

$X \overset{\cdot}{\sim} N(np, npq) \Rightarrow \bar{X} = \hat{p} \overset{\cdot}{\sim} N\left(p, \frac{pq}{n}\right)$

신뢰구간 $X \sim N(\mu, \sigma^2)$: μ 에 대한 $(1-\alpha)$ C.I.

$\bar{X} \pm z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$

$\mu \to (L, U)$

$\Rightarrow \hat{p} = \bar{X} \overset{\cdot}{\sim} N\left(p, \frac{pq}{n}\right) \quad (np > 5, \; nq > 5)$ — $X \sim b(n, p)$

95%.

$\hat{p} \pm 1.96 \sqrt{\frac{pq}{n}} \Rightarrow \boxed{\hat{p} \pm 1.96 \sqrt{\frac{1}{4n}}}$

$P(X > c) \approx P(X_N > c)$

$\downarrow$

(np, npq)

$\hat{p}$: % $\quad 100\hat{p} \pm 1.96 \frac{100}{\sqrt{4n}} \Leftarrow ?$ $\quad (\hat{p} = 0.1, 0.2, 0.8, 0.9)$

- CLT → $\bar{X} \sim N$
- $\pm 1.96\ s.e.(\hat{\theta})$

I

II

$X_i \sim Be(p)$

$\bar{X} \sim N(p, \frac{pq}{n})$

$\bar{x}(\hat{p}) \pm 3.1\% p$

$\bar{X} \sim N(100p, \frac{10^4 pq}{n})$

$(\hat{p} = 45\%) \sim N(100p, \frac{10^4 pq}{n})$

$\hat{p} = 0.1 \quad 0.2$ $\quad 0.8, \ 0.9$

0.5

$X_i \sim N(0, 4)$

$(n = 5, 10, 15, 30, 50)$

$\bar{X} \sim N(0, \frac{4}{n})$

$\bar{x} \pm 1.96 \sqrt{\frac{4}{n}}$

X_i, $1 \sim n$

$\bar{X}$

$\frac{n}{4}$

$\bar{x}_1 \in (L, U)$ 1

0

$\vdots$

$\bar{x}_m$ 0

$\frac{0.95m}{m}$

m

$X \sim N\left(100p, \frac{10^4 pq}{n}\right)$ $(p, q) \in (0,1)$

95%

$p = 0.1$

$\bar{X} \sim N\left(10, \frac{900}{n}\right)$ ①

$\Rightarrow \bar{x}_i \in$

$(10 \pm 3.1\% p)$

$\pm 3.1\% p$

n 次

0.95

$I_1 \; \dots \; I_m$

empirical confidence level

$p = 0.2$

$n = 30$ 50

0.5 0.8, 0.9

$p = 0.1$ $p = 0.2$

$\hat{p} \pm 3.1\% \; p$

$|\hat{p} - p| \le 3.1$

```
n <- 5
m <- 1000
c <- 0
sdn <- sqrt(4/n)
for (i in 1:m)
{
  x <- rnorm(1, 0, sd=sdn)
  if abs(x) > se
    c = c+1
}
tc <- m - c
cL <- tc/m
```

$N(0, 4)$

$\pm 1.96 \frac{\sigma}{\sqrt{n}}$

```
se = 1.96 x 2/sqrt(n)
```

```
x[i] <- rnorm(1, 0, sdn)
xbar <- mean(X)
XSD <- SD(X)/sqrt(n)
```

$p \in \hat{p} \pm 3.1\%$

$\hat{p} - 3.1 < p < \hat{p} + 3.1$

HW

① $X_i \sim N(0, \sigma^2)$ $\sigma^2 = 4, 49$

$n = 5, 10, 15, 30, 50$

$m = 1000, 10000$

emp conf. level : 95%

② $\bar{X}_i \overset{\cdot}{\sim} N(p, \frac{pq}{n})$ p: 비율 s.e. $1.96\sqrt{\frac{pq}{n}} \Rightarrow 1.96\sqrt{\frac{10000}{4n}}$

$\hat{p}$

$p = 0.1, 0.2, 0.5, 0.8, 0.9$

$n = 30, 50,$ $m = 1000, 10000$

$\frac{1.96}{2\sqrt{n}}$

(… 0.5)

$p = 50 = q$

최대

③ $X_i \sim U(0, 10)$ $U(0, 5)$ $\Rightarrow$ $E(X_i)$ $V(X_i)$?

$\bar{X}$: $n = 30, 50, 100$

$m = 1000, 10000$ $U(0, 10)$

④ $X_i \sim \mathcal{E}(\lambda)$ $\lambda = 1, 5$

$n = 30, 50, 100$

$m = 1000, 10000$

due 결과 정리

$X \rightarrow$ (waiting time)

exponential di

$$f(x \mid \lambda) = \lambda e^{-\lambda x} \quad x > 0,$$

$$E(X) = \int_0^\infty \lambda x e^{-\lambda x}\, dx = -x e^{-\lambda x}\Big|_0^\infty + \int e^{-\lambda x}\, dx = -\frac{1}{\lambda} e^{-\lambda x}\Big|_0^\infty = \frac{1}{\lambda}$$

$$\frac{1}{\lambda} e^{-x/\lambda} \quad E(X) = \lambda$$

$$E(X) = \cancel{\lambda}\ \tfrac{1}{\lambda}$$

$$V(X) = 1/\lambda^2$$

$$(UV)' = U'V + UV'$$

$$\int (uv)' = \int u'v + \int uv'$$

$$\int u'v = \int (uv)' - \int uv' = uv - \int uv'$$

statmethods.net

R in Action

- 구간추정 → 신뢰구간 + CLT
- 검정 → [α / power]

- one-, two-sample problems

(analysis of variance)

- ~~1 sample t-test 2 sam~~
  - ANOVA (1-way, 2-way)
  - Regression (simple, multiple)
  - χ^2-test

(p-value)

$H_0: \mu = \mu_0$ vs $H_1: \mu \neq \mu_0$

$\mu \leftarrow \bar{X}$: unbiased (불편)

m.v.

s.e.

$1-\alpha$

$\mu \in (\bar{X} - s.e. \times z_{\alpha/2}, \bar{X} + s.e. \, z_{\alpha/2})$

$\hat{\sigma}^2 = \frac{1}{n-1}\sum(X_i - \bar{X})^2$ 无偏估计

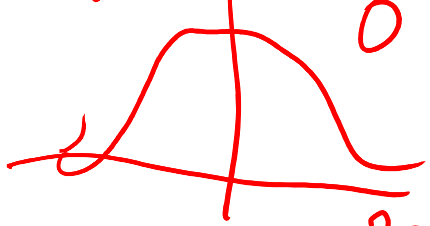
$\bar{X} \pm t(n-1, \frac{\alpha}{2})\frac{S}{\sqrt{n}}$ ① μ

$\frac{\bar{X}-\mu}{\sigma/\sqrt{n}} \sim N(0,1)$

$\frac{\bar{X}-\mu}{S/\sqrt{n}} \sim t(n-1)$

$E(S^2) = \sigma^2$

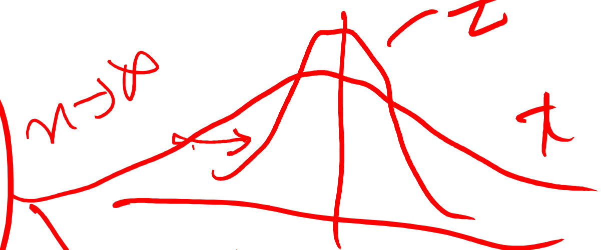
t-分布



$H_0: \mu = \mu_0$ vs $H_1: \mu \neq \mu_0$ 双边

$(L \quad U)$ μ_0

$\mu \geq \mu_0$, $\mu \leq \mu_0$ 单边 one-sided



$n \geq 30$

$t \sim N(0,1)$