Linear Algebra 2017/2 Mid 3 Sol

1) $\dim H = 2$

2) $\dim \mathrm{Nul}\, A = 2$

3) $\dim \mathrm{Col}\, A = 4$

4) $x_5 = 2^5 x_0 = 32 x_0$

5) $\begin{pmatrix} 3 \\ 7 \\ -2 \end{pmatrix} = a\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + b\begin{pmatrix} 0 \\ 2 \\ -1 \end{pmatrix}$

$a=3,\ b=2$

$\therefore$ dependent. $\rightarrow$ basis X

6) $\begin{bmatrix} 4 \\ 5 \end{bmatrix} = a\begin{bmatrix} 1 \\ 0 \end{bmatrix} + b\begin{bmatrix} 2 \\ -1 \end{bmatrix} = \begin{bmatrix} a+2b \\ -b \end{bmatrix}$

$b=-5,\ a=14$

$\therefore [x]_B = \begin{bmatrix} 14 \\ -5 \end{bmatrix}$

7) $\underset{C\leftarrow B}{P} = P_C^{-1} P_B$

$P_C^{-1} = \begin{bmatrix} 1 & 1 \\ -1 & 2 \end{bmatrix}^{-1} = \frac{1}{3}\begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix}$

$\therefore \underset{C\leftarrow B}{P} = \frac{1}{3}\begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} \frac{2}{3} & \frac{5}{3} \\ \frac{1}{3} & \frac{1}{3} \end{bmatrix}$

8) $[x]_C = \underset{C\leftarrow B}{P}\, [x]_B$

$[x]_B = \underset{C\leftarrow B}{P}^{-1} [x]_C$

$= \begin{bmatrix} \frac{2}{3} & \frac{5}{3} \\ \frac{1}{3} & \frac{1}{3} \end{bmatrix}^{-1} \begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 0 \end{bmatrix}$

9) $[T]_B \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} 2a_2 \\ 0 \\ a_1 \end{bmatrix}$

$\therefore [T]_B = \begin{bmatrix} 0 & 0 & 2 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$

10) $A = \begin{bmatrix} 0 & -4 \\ 1 & 0 \end{bmatrix}$

$|A-\lambda I| = \left|\begin{bmatrix} -\lambda & -4 \\ 1 & -\lambda \end{bmatrix}\right| \rightarrow \lambda^2+4=0$

$\lambda = \pm 2i$

① $\lambda = 2i$

$[A-2iI] = \begin{bmatrix} -2i & -4 \\ 1 & -2i \end{bmatrix} \sim \begin{bmatrix} i & 2 \\ 0 & 0 \end{bmatrix}$

$x = \begin{bmatrix} 2i \\ 1 \end{bmatrix} x_2$, x_2 is free v.

$\therefore V_1 = \begin{bmatrix} 2i \\ 1 \end{bmatrix}$

② $\lambda = -2i \rightarrow V_2 = \begin{bmatrix} -2i \\ 1 \end{bmatrix}$ ($\because$ Conjugate)

11) $\det(A-\lambda I) = (1-\lambda)^2 - 1 = 0$

$\lambda(\lambda-2)=0$

$\lambda = 0$ or 2

① $\lambda = 0 \rightarrow V_1 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$

② $\lambda = 2 \rightarrow V_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

12) A similar to B ●.

$\det(A-\lambda I) = \det(B-\lambda I)$, $A = QBQ^{-1}$, $B = Q^{-1}AQ$ 만족 $Q = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$ or $\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$

13) $AP = PD$

$AP = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} -1 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 2 \\ 0 & 2 \end{bmatrix}$

$PD = \begin{bmatrix} -1 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 0 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 0 & 2 \\ 0 & 2 \end{bmatrix}$

14) $T(x) = Ax$

$A = PDP^{-1}$, $D = P^{-1}AP$: B-matrix for T

$\therefore B = \left\{ \begin{bmatrix} -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right\}$