1. 합성함수의 미분법(Chain rule)

$z = f(x(t), y(t))$

이때, $\frac{dz}{dt} = \frac{\partial z}{\partial x}\cdot\frac{dx}{dt} + \frac{\partial z}{\partial y}\cdot\frac{dy}{dt}$

$\therefore dz = f_x \cdot dx + f_y \cdot dy$

ex) $f(x_1, x_2, x_3) = x_1^2 + x_2^2 + x_3^2$

$\frac{df}{dt} = \frac{\partial f}{\partial x_1}\cdot\frac{dx_1}{dt} + \frac{\partial f}{\partial x_2}\cdot\frac{dx_2}{dt} + \frac{\partial f}{\partial x_3}\cdot\frac{dx_3}{dt}$

이때, $X(t) = (\cos t, \sin t, t)$ ⋯ $2x_1\cdot(-\sin t) + 2x_2\cdot(\cos t) + 2x_3\cdot 1$

$= 2(\cos t)(-\sin t) + 2(\sin t)(\cos t) + 2t$

$= 2t.$

ex) $f(x_1, x_2) = x_1^3 + 2x_1x_2$

$\because x_1(r,\theta) = r\cos\theta, \quad x_2(r,\theta) = r\sin\theta$

$\frac{\partial f}{\partial r} = \frac{\partial f}{\partial x_1}\frac{\partial x_1}{\partial r} + \frac{\partial f}{\partial x_2}\frac{\partial x_2}{\partial r}$

$= (3x_1^2 + 2x_2)\cdot\cos\theta + (2x_1)\sin\theta$

$= (3r^2\cos^2\theta + 2r\sin\theta)\cos\theta + (2r\cos\theta)\sin\theta$

$= 3r^2\cos^3\theta + 4r\sin\theta\cos\theta$

$r = \sqrt{x_1^2 + x_2^2}$

$\cos\theta = \frac{x_1}{\sqrt{x_1^2 + x_2^2}}$

$= \frac{3(x_1^2+x_2^2)\cdot x_1^3}{\sqrt{x_1^2+x_2^2}\,(x_1^2+x_2^2)} + \frac{4\sqrt{x_1^2+x_2^2}\cdot x_1\cdot x_2}{(\sqrt{x_1^2+x_2^2})^2}$

$= \frac{3x_1^3 + 4x_1x_2}{\sqrt{x_1^2+x_2^2}}$

방향미분 ~> gradient : 점 Point 함수 f의 gradient.

$\Rightarrow X'(0) = \left(\frac{dx_1}{dt}(0), \dots, \frac{dx_n}{dt}(0)\right) = \vec{v}$

$\vec{v}$ 에 대해, $D_{\vec{v}} f(P) = \nabla f(P) \cdot V$

($\nabla f(P)$: gradient, V: 내적)

Point 함수값이 가장 크게 변하는 방향

ex) $f(x,y,z) = x^3 - xy^2 - z$

$P = (1,1,0), \quad V = (2,-3,6)$

이때 $D_v f(P) = \nabla f(P)\cdot V$ (각각 편미분)

$= (3x^2 - y^2, -2xy, -1)\cdot V$

$= (2,-2,-1)\cdot(2,-3,6)$

$= 4 + 6 - 6 = 4.$