chapter 2 연습문제 21500468 이규상

2.1 $f_{1,2}(x,y) = 10xy^2 I(0<x<y<1)$

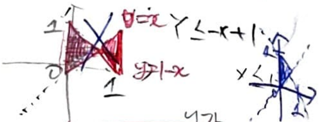
(a) $P(Y \le \frac{1}{2})$

$\int_{\frac{1}{2}}^{1}\int_0^y 10xy^2\,dx\,dy = \int_{\frac{1}{2}}^{1} 10y^2 \int_0^y x\,dx\,dy$

$= \int_0^{\frac{1}{2}} 10y^2 \cdot \frac{1}{2}y^2\,dy = \int_{\frac{1}{2}} 5y^4\,dy = y^5 \Big|_0^{\frac{1}{2}} = \left(\frac{1}{2}\right)^5$

(b) $P(X+Y \le 1)$

$\boxed{풀이}$ $Y \le 1-X$



$\Rightarrow \int_0^1 \int^{y \wedge 1}$

y가 2개로 파악하는 게 키포인트

$y : \frac{1}{2} \sim 1$일 때 $x : 0 \sim 1-y$

$y : 0 \sim \frac{1}{2}$일 때 $x : 0 \sim y$

$\int_0^{\frac{1}{2}}\int_0^y 10xy^2\,dx\,dy + \int_{\frac{1}{2}}^{1}\int_0^{1-y} 10xy^2\,dx\,dy$

$= \int_0^{\frac{1}{2}} 5y^2 y^2\,dy + \int_{\frac{1}{2}}^{1} 5y^2(1-y)^2\,dy$ 치환 $1-y=t$ $y=1-t$

$= y^5 \Big|_0^{\frac{1}{2}} - \int_{\frac{1}{2}}^{0} 5(1-t)^2 t^2\,dt$ $0 \cdots t \cdots \frac{1}{2}$

$(t^2-2t+1)t^2 \cdots t^4 - 2t^3 + t^2$ $-dy = dt$

$\left(\frac{1}{2}\right)^5 - 5\left[\frac{1}{5}t^5 - \frac{1}{2}t^4 + \frac{1}{3}t^3\right]_0^{\frac{1}{2}}$

$\left(\frac{1}{2}\right)^5 - 5\left[\frac{1}{5}\left(\frac{1}{2}\right)^5 - \left(\frac{1}{2}\right)^5 + \frac{1}{3}\left(\frac{1}{2}\right)^3\right]$

$= \left(\frac{1}{2}\right)^5 - \left(\frac{1}{2}\right)^5 + 5\left(\frac{1}{2}\right)^5 - \frac{5}{3}\left(\frac{1}{2}\right)^3$

$\frac{5}{32} - \frac{5}{24}$ (?)

$\Rightarrow$ pass

2.2) $f_{1,2}(x,y) = cx^2y$ 일 때

1) 상수 c값

$\int_0^1\int_0^y cx^2y\,dx\,dy = c\int_0^1 y \cdot \frac{1}{3}y^3\,dy$

$\frac{c}{3}\cdot\frac{1}{5} = \frac{c}{15} = 1$ $\quad$ $c = 15$

2) $P(Y \le \frac{1}{2})$

$\int_0^{\frac{1}{2}}\int_0^y 15x^2y\,dx\,dy$

$\int_0^{\frac{1}{2}} 15y \cdot \frac{1}{3}y^3 = \int_0^{\frac{1}{2}} 5y^4\,dy = \left(\frac{1}{2}\right)^5 = \frac{1}{32}$

$\frac{1}{2} < X < Y$ … debug 이 중요.

3) $P(Y \le 2X)$

y가 0~1일 때 x는 $\frac{y}{2} \sim y$

$\int_0^1 \int_{\frac{y}{2}}^{y} 15x^2y\,dx\,dy$

$\int_0^1 15y \cdot \frac{1}{3}x^3 \Big|_{\frac{y}{2}}^{y}$

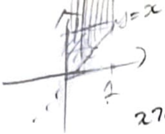
$\int_0^1 y\left[y^3 - \left(\frac{y}{2}\right)^3\right]$

$\int_0^1 y\left[y^3 - \frac{1}{8}y^3\right] = \int_0^1 y\,\frac{7}{8}y^3$

$= \frac{7}{8}\cdot\frac{1}{5} = \frac{7}{40}$

2.3

a) $P(X\le1)$



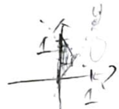
x가 0~1일때

y는 x~∞

$\int_0^1\int_x^\infty e^{-y}\,dy\,dx$

$\int_0^1 -e^{-y}\Big|_x^\infty = \int_0^1 e^{-x} = \boxed{e^{-1}-1}$

b) $P(X+Y\le1)$



$\int_0^{\frac12}\int_0^y e^{-y}\,dx\,dy +$

$\int_{\frac12}^1\int_0^{1-y} e^{-y}\,dx\,dy$

① y가 $\frac12$~1일때 x는 0~1-y

② y가 0~$\frac12$일때 x는 0~y 이다

↱ 부호계산 실수

$\int_0^{\frac12} e^{-y}y\,dy + \int_{\frac12}^1 +e^{-y}(1-y)\,dy$

$-e^{-y}+ye^{-y}$

$= y\cdot(-e^{-y})\Big|_0^{\frac12} + \int_0^{\frac12} -e^{-y}\,dy$

$e^{-y}\Big|_{\frac12}^1 - ye^{-y} + \int_{\frac12}^1 +e^{-y}$

$\frac12(-e^{-\frac12}) - e^{-y}\Big|_0^{\frac12}$

$-\cancel{e}+\cancel{e}+e^{-1}+\cancel{\frac12 e^{-\frac12}}$

$+e^{-1}-e^{-1}$

$-\frac12 e^{-\frac12} - e^{-\frac12}+1$

$= -2e^{-\frac12}+e^{-1}+1$

$\lim_{x\to\infty} x^2e^{-x} = \frac{x^2}{e^x}$

이방법이 더 쉬움

책

$\int_0^{\frac12}\int_x^{1-x} e^{-y}\,dy\,dx$

$(1-y)e^{-y}\Big|_{\frac12}$

$-\frac12 e^{-\frac12}$

$= \int_0^{\frac12} -e^{-y}\Big|_x^{1-x} = \int_0^{\frac12}\left(-e^{x-1}+e^{-x}\right)dx$

$-e^{x-1} - e^{-x}\Big|_0^{\frac12}$

$= -e^{-\frac12} - e^{-\frac12} + e^{-1} + 1$

$= \boxed{-2e^{-\frac12}+e^{-1}+1}$

결합확률밀도함수 $f_{1,2}(x,y) = 2I(0<x<y<1)$

(a) $\int_x^1 2\,dy = 2y\Big|_x^1 = (2-2x)I(0<x<1)$

(b) $f_{1,2}(x,y) = e^{-y}I(0<x<y<\infty)$

$\Rightarrow f_1(x) = \int_x^\infty e^{-y}\,dy = -e^{-y}\Big|_x^\infty$

$= e^{-x}I(x>0)$

2.5) 결합누적 → 결합확률밀도 $f(x,y)=\frac{\partial^2}{\partial x\partial y}F(x,y)$

(a) $\cancel{F_{1,2}}(x,y) = \begin{cases} x^2(1-e^{-2y}) & 0\le x<1,\ y\ge0 \\ (1-e^{-2y}) & x\ge1,\ y\ge0 \\ 0 & y<0 \end{cases}$

미분실수

① x,y 각각 편미분 ⇒ $2(1-e^{-2y})x = 2x-2xe^{-2y}$

$\frac{d}{dy}-2xe^{-2y} = 2x\,2e^{-2y}\,I(0\le x\le1,\ y\ge0)$

② 0 ③ 0

(b) $f_{1,2}(x,y) \Rightarrow \cancel{2e^{-2x}} - 2e^{-x-y}$ ①

$2(e^{-x-y}$

$= 2e^{-x-y}I(0<x<y)$

2.6) $f_{1,2}(x,y) = xe^{-y}I(0<x<y<\infty)$ 일때

Cov(X,Y)값

$\Rightarrow E[(X-E[X])(Y-E[Y])]$

$= E[XY]-E[X]E[Y]$

① E[XY] 값 $= 8$

$\int_0^\infty\int_x^\infty xy\,xe^{-y}\,dy\,dx$

$= \int_0^\infty x^2\int_x^\infty ye^{-y}\,dy\,dx = \int_0^\infty x^2(-1-y)e^{-y}\Big|_x^\infty dx$

$= \int_0^\infty x^2(1+1x)e^{-x} = \int_0^\infty x^2e^{-x}+x^3e^{-x}\,dx$

$4\left(-x^2e^{-x}-2(1+x)e^{-x}\right) - x^3e^{-x}\Big|_0^\infty$

$2\cdot1\cdot1$

$-x^3e^{-x}+3\int x^2e^{-x}$

$\int x^2e^{-x}$

$-x^2e^{-x}+\int 2xe^{-x}$

$-x^2e^{-x}-2(1+x)$

$\int ye^{-y} = y\cdot(-e^{-y}) + \int e^{-y}$

$-e^{-y}$

$(y+1)(-e^{-y}) = (-1-y)e^{-y}$

$E[X]\,E[Y]$

(1) $E(X) \to = \int_0^\infty \int_x^\infty x e^{-y}\, dy\, dx$

$= \int_0^\infty x^2 \left. -e^{-y} \right|_x^\infty dx = \int_0^\infty e^{-x} x^2\, dx$

$= -x^2 e^{-x} - 2(1+x)e^{-x} \Big|_0^\infty$

$= -[-2\cdot 1\cdot 1] = 2$

(2) $E[Y] = \int_0^\infty \int_0^y y \cdot x e^{-y}\, dx\, dy$

$= \int_0^\infty y e^{-y} \cdot \frac{1}{2} y^2\, dy = \frac{1}{2}\int_0^\infty y^3 e^{-y}\, dy$

$= \frac{1}{2}\left[4\left[-y^2 e^{-y} - 2(1+y)e^{-y}\right] - y^3 e^{-y} \right]_0^\infty$

$= \frac{1}{2}\cdot 4\,(- -2(1+0)) = 4$

$\therefore E[XY] - E[X]E[Y] = 8 - 8 = 0$

7.1) $f_{1,2}(x,y) = e^{-y} I(0<x<y<\infty)$ 일 때

(a) X, Y 결합적률생성함수 $e^{(t_2-1)y}$

$M(t_1,t_2) = \int_0^\infty \int_x^\infty e^{t_1 x + t_2 y} e^{-y}\, dy\, dx$

$= \int_0^\infty e^{t_1 x} \left[\frac{1}{t_2-1} e^{(t_2-1)y} \Big|_x^\infty \right]$

$= \int_0^\infty e^{t_1 x} \cdot \frac{1}{1-t_2} e^{(t_2-1)x}$ $\quad \boxed{t_2 - 1 < 0}$ $\quad \boxed{t_1+t_2-1 < 0}$

$= \frac{1}{1-t_2} \int_0^\infty e^{(t_1+t_2-1)x}\, dx = \dfrac{\dfrac{1}{t_1+t_2-1} e^{t_1+t_2-1}\Big|_0^\infty}{(1-t_2)}$

$= \frac{1}{1-t_2} \cdot \frac{1}{1-t_1-t_2}, \quad t_2 < 1,\ t_1+t_2 < 1$

b) X, Y 상관계수 $\quad \boxed{C(t_1,t_2)}$

$\mathrm{Corr}(X,Y) = \dfrac{\mathrm{Cov}(X,Y)}{\sqrt{V(X)}\cdot\sqrt{V(Y)}}$

$\log\left(\frac{1}{1-t_2} \cdot \frac{1}{1-t_1-t_2}\right) = -\log(1-t_2) - \log(1-(t_1+t_2))$

$= \sum_{n=1}^\infty \frac{1}{n}(t_2)^n + \sum_{n=1}^\infty \frac{1}{n}(t_1+t_2)^n = \boxed{\sum_{r=0}^\infty \sum_{s=0}^\infty \frac{C(r,s)}{r!\,s!} t_1^r t_2^s, \ r+s\ge 1}$

$= (t_2 + \frac{1}{2}t_2^2 + \cdots) + (t_1 + t_2 + \frac{1}{2}(t_1+t_2)^2 + \cdots)$

$C(1,1) = \frac{1}{2}\cdot 2 t_1 t_2 = 1 = \mathrm{Cov}(X,Y)$

$\frac{C(2,0)}{2!} t_1^2 = \frac{1}{2} t_1^2 \Rightarrow V(X) = 1$

$\frac{C(0,2)}{2!} t_2^2 = \frac{1}{2} t_2^2 + \frac{1}{2} t_2^2 \Rightarrow V(Y) = 2$

$= \frac{2}{2} t_2^2$

$\therefore \mathrm{Corr}(X,Y) = \frac{1}{\sqrt{1}\cdot\sqrt{2}} = \frac{1}{\sqrt{2}}$

28) $f_{1,2}(x,y) = 10xy^2 I(0<x<y<1)$

(a) $f_2(y) = \int_0^y 10xy^2\, dx = 10y^2 \cdot y^2$

$= 10y^4 I(0<y<1)$

(b) $f(x|y) = \frac{f(x,y)}{f(y)} = \frac{10xy^2}{10y^4} = xy^{-2} I(0<x<y<1)$

(c) $E[X|Y] = \int_0^y x y^{-2}\, dx = y^{-2}\cdot\frac{1}{2}x^2\Big|_0^y = \frac{1}{2}$

(d) $V[X|Y] = E[X^2|Y] - \{E[X|Y]\}^2$

$= \int_0^y x^2 y^{-2}\, dx - \left(\frac{1}{2}\right)^2 = y^{-2}\cdot\frac{1}{3}x^3\Big|_0^y$

$= \frac{1}{3}y - \frac{1}{4}$

$\log(1+A) = A - \frac{A^2}{2} + \frac{A^3}{3}$

$-\log(1-A) = \sum_{n=1}^\infty \frac{1}{n} A^n$

2.9) $f_{1,2}(x,y) = e^{-y} I(0<x<y<\infty)$

(a) Y의 주변확률밀도함수

$\int_0^y e^{-y} dx = \boxed{e^{-y} y}$ $I(y>0)$

(b) $f_{1|2}(x|y) = \frac{f_{1,2}(x,y)}{f_2(y)} = \frac{e^{-y}}{e^{-y} \cdot y} = y^{-1} I(0<x<y)$

(c) $E[X|Y]$

$\int_0^y x \frac{1}{y} dx = \frac{1}{y} \frac{1}{2} x^2 \Big|_0^y = \frac{1}{2} y$

(d) $V[X|Y]$

$= \int_0^y x^2 y^{-1} dx$

$= y^{-1} \cdot \frac{1}{3} x^3 \Big|_0^y = \frac{1}{3} y^2$

$\rightarrow \left(\frac{1}{3} y^2\right) - \left(\frac{1}{2} y\right)^2$

$= \frac{1}{12} y^2$

(d) $V(E(X|Y))$, $E[V(X|Y)]$

$V(\frac{1}{2} y) = \boxed{\frac{1}{4} V(y)} = \frac{1}{2}$

$\hookrightarrow E[\frac{1}{3} y^2]$

$= \boxed{\frac{1}{12} E[y^2]}$

$= \frac{1}{12} \times 6 = \frac{1}{2}$

구하긴 해가지고

$f(y) = e^{-y} y \; I(y>0)$ 이므로

$E[Y] = \int_0^\infty y \cdot e^{-y} y \, dy = \int_0^\infty y^2 e^{-y} dy = -y^2 e^{-y} - 2(1+y) e^{-y} \Big|_0^\infty$

$= -(-2(1)) = 2$

$E[Y^2] = \int_0^\infty y^3 e^{-y} dy = 3[-y^2 e^{-y} - 2(1+y) e^{-y}] - y^3 e^{-y} \Big|_0^\infty$

계산실수

$= 3 \cdot 2 \cdot 1 = 6$

$V[Y] = 6 - 2^2 = 2$

(e) $f_1(x)$

$\int_x^\infty e^{-y} dy = -e^{-y} \Big|_x^\infty = e^{-x} I_{(x>0)}$

$V(x) = E[x^2] - \{E[x]\}^2$ 이므로

$E[x] = \int_0^\infty x \cdot e^{-x} dx = (-x-1) e^{-x} \Big|_0^\infty$

$= -(0-1) = 1$

$E[x^2] = \int_0^\infty x^2 e^{-x} dx = (-x^2 - 2x - 2) e^{-x} \Big|_0^\infty$

$= -(-0-0-2) = 2$

$\therefore V(x) = 2 - 1^2 = 1$

$\boxed{2.10}$ $f_{1,2}(x,y) = 2 I(0<x<y<1)$ $V(E(X|Y))$ $E(V(X|Y))$를 구하라.

$f(y) = \int_0^y 2 \, dx = 2y \; I(0<y<1)$

$f(x|y) = \frac{f(x,y)}{f(y)} = \frac{2}{2y} = \frac{1}{y} \boxed{(0<x<y)}$

$E[X|Y] = \int_0^y x \cdot \frac{1}{y} dx = \frac{1}{y} \cdot \frac{1}{2} y^2 = \frac{1}{2} y \; I(y<1)$

$V(X|Y) = E(X^2|Y) - \{E[X|Y]\}^2$ 이므로

$\int_0^y x^2 \frac{1}{y} dx = \frac{1}{y} \cdot \frac{1}{3} y^3 = \boxed{\frac{1}{3} y^2}$

$\therefore V(X|Y) = \frac{1}{3} y^2 - \left(\frac{1}{2} y\right)^2$

$= \boxed{\frac{1}{12} y^2 \; I(y<1)}$

$V(E(X|Y)) = V(\frac{1}{2} y) = \frac{1}{4} V(y)$ 를 구하자

이때 $V(Y) = E(Y^2) - \{E(Y)\}^2$ 이므로

$E[Y] = \int_0^1 y \, 2y \, dy = \frac{2}{3} y^3 \Big|_0^1 = \frac{2}{3}$

$E[Y^2] = \int_0^1 y^2 \, 2y \, dy = \frac{2}{4} y^4 \Big|_0^1 = \frac{1}{2}$

$\therefore V(Y) = \frac{1}{2} - \left(\frac{2}{3}\right)^2 = \frac{1}{2} - \frac{4}{9} = \frac{1}{18}$

$\therefore V(E(X|Y)) = \frac{1}{4} V(Y) = \frac{1}{4} \times \frac{1}{18} = \frac{1}{72}$

$(\ln x)' = \frac{1}{x}$

$\int y e^{-y} = (-y-1) e^{-y}$

$\int y^2 e^{-y} = -y^2 e^{-y} - 2(1+y) e^{-y}$

$= (-y^2 - 2y - 2) e^{-y}$

$\int y^3 e^{-y} = -y^3 e^{-y} + 3(-y^2 - 2y - 2) e^{-y}$

$= -y^3 e^{-y} + 3 \int y^2 e^{-y}$

$= E(X) - E(X) = 0$

$E(V(X|Y)) = E[\frac{1}{12}y^2]$

$= \frac{1}{12}E[Y^2]$

$= \frac{1}{12}\cdot\frac{1}{2} = \frac{1}{24}$

(b) pass

$\therefore \frac{1}{18} + \frac{1}{4}\times\frac{1}{18} + \frac{1}{2}\times\frac{1}{36} = \frac{6}{72} = \frac{1}{12}$

(정리) $V(Y) + E(V(X|Y))$

$\frac{1}{18} + \frac{1}{12}E[Y^2] = \frac{1}{18} + \frac{1}{12}\cdot\frac{1}{2} = \frac{4+3}{72} = \frac{7}{72}$

2.11) $Var(Y+X-E(X|Y)) = E((X-E(X|Y))+Y)^2$

다시

$= E[[X-E(X|Y)]^2 + 2(X-E(X|Y))Y + Y^2]$

(a) $f_{1,2}(x,y) = 2I(0<x<y<1)$

먼저 $E(X|Y)$를 구해보자 $\Rightarrow \frac{1}{2}y$

$\therefore Var(Y+X-\frac{1}{2}Y) = V(X+\frac{1}{2}Y)$ $= E[V(X|Y)] + E[Y^2]$

$\Rightarrow V(X) + V(\frac{1}{2}Y) + Cov(X, \frac{1}{2}Y)$

$= V(X) + \frac{1}{4}V(Y) + \frac{1}{2}Cov(X,Y)$

① $V(Y) = \frac{1}{18}$ (2.10번 참고)

② $V(X)$를 구해보자..

$f(x) = \int_x^1 2\,dy = 2(1-x)I_{(0<x<1)}$

$E(X) = \int_0^1 x(2-2x)dx = \int_0^1 -2x^2+2x\,dx = -\frac{2}{3}x^3+x^2\Big|_0^1$

$= -\frac{2}{3}+1 = \frac{1}{3}$

$E(X^2) = \int_0^1 x^2(2-2x)dx = \int_0^1 2x^2-2x^3dx = -\frac{2}{4}x^4+\frac{2}{3}x^3$

$= -\frac{2}{4}+\frac{2}{3}$

$= \frac{2}{12} = \frac{1}{6}$

$\therefore V(X) = \frac{1}{6} - (\frac{1}{3})^2 = \frac{1}{6}-\frac{1}{9} = \frac{1}{18}$

③ $Cov(X,Y) = E(XY) - E[X]E[Y]$ 이므로

$E[XY] = \int_0^1\int_x^1 2xy\,dy\,dx = \int_0^1 2x\cdot\frac{1}{2}y^2\Big|_x^1 dx$

$= \int_0^1 x-x^3\,dx$

$= \frac{1}{2}-\frac{1}{4} = \frac{1}{4}$

$E[X] = \frac{1}{3}$, $E[Y] = \frac{2}{3}$ 이므로 $Cov(X,Y) = \frac{1}{4} - \frac{1}{3}\cdot\frac{2}{3}$

$= \frac{9-8}{36} = \frac{1}{36}$

$Var(Y+X-E(X|Y))$

$\Rightarrow Var(Y) + Var(X-E(X|Y))$

$+ ~~2Cov(Y, X-E(X|Y))~~$

$= Var(Y) + E((X-E(X|Y))^2) + [E(X-E(X|Y))]^2$, $E(X-E(X|Y)) = 0$

$= Var(Y) + E(V(X|Y))$

$= \frac{1}{18} + E(\frac{1}{12}y^2)$

$= \frac{1}{18} + \frac{1}{12}E(Y^2) = \frac{1}{18} + \frac{1}{24}$

$E(Y-E(Y|X)) = E(Y) - E(E(Y|X))$

증명

$Z = Y-E(Y|X)$ $\therefore E(Z|X) = E(Y|X) - E(E(Y|X)|X)$

$= E(Y|X) - E(Y|X) = 0$

$E(Z) = E(E(Z|X)) = 0$ 이다.

$\therefore Cov(Z, t(X)) = E(Z\cdot t(X)) - E(Z)\cdot E(t(X))$

$= E(Z\cdot t(X)) = E(Y-E(Y|X))t(X)$

$= E(E(Z\cdot t(X)|X))$

$= E(t(X)\cdot E(Z|X)) = 0$

$Cov(Y-E(Y|X), r(X)) = 0$

$E((X-E(X|Y))^2) = E(Var(X|Y))$

$= E(E((X-E(X|Y))^2|Y))$

$E(X-E(X|Y)) = 0$ $\therefore E(X) - E(E(X|Y))$

$= E(X) - E(X) = 0$

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(2.11 (b))

$V(Y+X-E(X|Y))$

$= V(Y) + V(X-E(X|Y)) + 2\,Cov(Y, X-E(X|Y))$ $\Rightarrow 0$

$= V(Y) + E(X-E(X|Y))^2 - \{E(X-E(X|Y))\}^2$

이때 $E(X-E(X|Y)) = E(X) - E(E(X|Y))$
$= E(X) - E(X) = 0$

$V(X-E(X|Y)) = E(X-E(X|Y))^2$
$= E(E(X-E(X|Y))|Y)^2$
$= E(V(X|Y))$

∴ 본식은 $V(Y) + E(V(X|Y))$로 정리된다

$f_{1,2}(x,y) = 10xy^2 I(0<x<y<1)$ 이므로,

$f_2(y) = \int_0^y 10xy^2\,dx = 10y^2 \cdot \frac{1}{2}y^2 = 5y^4 I(0<y<1)$

이때 $E(Y) = \int_0^1 y \cdot 5y^4\,dy = \frac{5}{6}$

$E(Y^2) = \int_0^1 y^2\, 5y^4\,dy = \frac{5}{7}$

$V(Y) = \frac{5}{7} - \left(\frac{5}{6}\right)^2 = \frac{5}{7} - \frac{25}{36} = \frac{5}{252}$

$V(X|Y)$

$f(x|y) = \frac{f(x,y)}{f(y)} = \frac{10xy^2 I(0<x<y<1)}{5y^4 I(0<y<1)} = \frac{2x}{y^2} I(0<x<y<1)$

$E(X|Y) = \int_0^y x \cdot \frac{2x}{y^2}\,dx = \frac{1}{y^2} \times \frac{2}{3}y^3 = \frac{2}{3}y\, I(0<y<1)$

$E(X^2|Y) = \int_0^y x^2 \frac{2x}{y^2}\,dx = \frac{1}{y^2} \cdot \frac{2}{4}y^4 = \frac{1}{2}y^2 I(0<y<1)$

$\therefore V(X|Y) = \frac{1}{2}y^2 - \left(\frac{2}{3}y\right)^2 = \frac{1}{2}y^2 - \frac{4}{9}y^2 = \frac{1}{18}y^2$

$\therefore V(Y) + E(V(X|Y))$

$\frac{5}{252} + \frac{1}{18}E(Y^2) = \frac{5}{252} + \frac{1}{18} \cdot \frac{5}{7}$

$= \frac{5}{252} + \frac{5}{126}$

$= \frac{15}{252}$

2.12 pass

2.13 (a) 독립 (b) 종속

$f_{1,2}(x,y) = 2e^{-x-y} I(0<x<y)$

$f_1(x) = \int_x^\infty 2e^{-x-y}\,dy = 2e^{-x} + e^{-x}$
$= 2e^{-2x}$

$E(X) = \int_0^\infty 2xe^{-2x} = 2\left[-\frac{1}{2}xe^{-2x} + \int \frac{1}{2}e^{-2x}\right]$

$2\left[-\frac{1}{2}xe^{-2x} - \frac{1}{4}e^{-2x}\Big|_0^\infty\right]$

$= 2 \times \frac{1}{4} = \frac{1}{2}$

$f_2(y) = \int_0^y 2e^{-x-y}\,dx$

$= 2e^{-y} \cdot -e^{-x}\Big|_0^y$

$= 2e^{-y} \cdot -(e^{-y}-1)$

$= -2e^{-y}(e^{-y}-1)$

$= 2e^{-y} - 2e^{-2y}$

$E[Y] = \int_0^\infty y(2e^{-y} - 2e^{-2y})$

$= 2\int_0^\infty ye^{-y}\,dy$!!

$- \int_0^\infty 2ye^{-2y}$

$= 2 - \frac{1}{2} = \frac{3}{2}$

$E[XY]$

$= \int_0^\infty \int_x^\infty xy\,2e^{-x-y}\,dy\,dx$

$= \int_0^\infty 2xe^{-x}\left[\int_x^\infty ye^{-y}\,dy\right]dx$

$(-y-1)e^{-y}\Big|_x^\infty$

$\int_0^\infty 2xe^{-x}(x+1)e^{-x} = \int_0^\infty 2x(x+1)e^{-2x}$

$\int_0^\infty 2x^2e^{-2x} + 2xe^{-2x}$ ①

$2x^2e^{-2x} + 2xe^{-2x}$

$\frac{1}{2}\int 2x^2e^{-2x} + \frac{1}{2}\int 2xe^{-2x}$

$\frac{1}{4}$

$\int_0^\infty x^n e^{-x}\,dx = n!$

기억

2 $E(XY) - E(X)E(Y) = 1 - \frac{1}{2} \times \frac{3}{2} = 1 - \frac{3}{4} = \frac{1}{4}$

=> 이것보다 차라리 mgf를 구해볼까?

c) $f_{1,2}(x,y) = \frac{25}{104}(1+x^4y^4) I$ ... 공식

$$M(t_1,t_2) = \int_{-1}^{1}\int_{-1}^{1} e^{t_1x+t_2y} \times \frac{25}{104}(1+x^4y^4)\,dx\,dy$$

$$\frac{25}{104}\int_{-1}^{1}\int_{-1}^{1} e^{t_1x+t_2y} + e^{t_1x+t_2y}x^4y^4$$

$$\frac{1}{t_1} e^{t_1x+t_2y} +$$

=> $\boxed{Cov(X,Y) = 0}$

(d) pass

2.14

$$M(t_1,t_2) = \int_0^\infty \int_x^\infty e^{t_1x+t_2y} \cdot 6e^{-x-2y}\,dy\,dx$$

$$= \int_0^\infty 6e^{t_1x-x} \boxed{\int_x^\infty e^{t_2y-2y}\,dy}\,dx$$

$$= \int_0^\infty 6e^{t_1x-x} \cdot \frac{1}{t_2-2} e^{(t_2-2)y} \Big|_x^\infty$$

$$= \frac{1}{t_2-2}\int_0^\infty 6e^{t_1x-x} \cdot e^{(t_2-2)x}$$

$t_2 - 2 < 0$

$t_1 + t_2 - 3 < 0$

$$\cancel{\frac{6}{t_2-2}}\int_0^\infty e^{(t_1+t_2-3)x}\,dx \Rightarrow \frac{6}{2-t_2} \times \frac{1}{3-t_1-t_2} e^{(t_1+t_2-3)x}$$

(a)

$$mgf(t_1,t_2) = \frac{6}{2-t_2} \cdot \frac{1}{3-t_1-t_2}$$

(b)

X+Y의 분산 .. $V(X+Y) = V(X) + V(Y) + 2Cov(X,Y)$

나누는것도 있어미....

$$\log\frac{2}{2-t_2} + \log\frac{3}{3-t_1-t_2} = -\log\left(1-\frac{t_2}{2}\right) - \log\left(1-\frac{t_1+t_2}{3}\right)$$

$$\sum_{n=1}^\infty \frac{1}{n}\left(\frac{t_2}{2}\right)^n + \sum_{n=1}^\infty \frac{1}{n}\left(\frac{t_1+t_2}{3}\right)^n = \sum_{r=0}^\infty\sum_{s=0}^\infty \frac{C(r,s)}{r!s!} t_1^r t_2^s \quad (r+s \geq 1)$$

$\frac{1}{2}\left(\frac{t_2}{2}\right)^2$

$1\left(\frac{t_1+t_2}{3}\right) + \frac{1}{2}\left(\frac{t_1+t_2}{3}\right)^2$

$= \frac{C(2,0)}{2!} t_1^2 = \frac{1}{2} \times \frac{1}{9} t_1^2 \cdot C(2,0) = \frac{1}{9} = V(X)$

$\frac{C(0,2)}{2!} t_2^2 = \frac{1}{2} \cdot \frac{1}{4} t_2^2 + \frac{1}{2} \cdot \frac{1}{9} t_2^2 \Rightarrow V(Y) = \frac{13}{36}$

$Cov(X,Y) \Rightarrow \frac{1}{2}\left(\frac{t_1+t_2}{3}\right)^2 \; C(1,1) = \frac{1}{2} \times \frac{1}{9} \times 2 = Cov(X,Y) = \frac{1}{9}$

$\therefore \frac{1}{9} + \frac{13}{36} + 2 \times \frac{2}{9} = \frac{4+13+16}{36} = \frac{33}{36}$

(c) X와 $\boxed{Z = Y - X}$ 의 결합 적률생성함수 구하기 $= \frac{11}{12}$

$$f_{1,2}(x,y) = 6e^{-x-2y} I(0<x<y<\infty)$$

$\boxed{Z = Y - X}$

$\boxed{\text{풀정리}}$ ① 구간정하기 => 그래프파악

② $\log(1+A) = A - \frac{A^2}{2} + \frac{A^3}{3} - \frac{A^4}{4} \cdots$

③ $-\log(1-A) = \sum_{n=1}^\infty \frac{1}{n} A^n$

④ $M(t_1,t_2) = \sum_{r=0}^\infty \sum_{s=0}^\infty \frac{C(r,s)}{r!s!} t_1^r t_2^s \quad (r+s \geq 1)$

⑤ $\int y e^{-y}\,dy = (-y-1)e^{-y}$

$\int y^2 e^{-y}\,dy = (-y^2-2y-2)e^{-y}$

$\int y^3 e^{-y}\,dy = -y^3 e^{-y} + 3(-y^2-2y-2)e^{-y}$

⑥ $Cov(Y - E(Y|X), r(X)) = 0$

$= \boxed{E(r(X)Z)} - E(Z) \cdot E(r(X))$

이것만 보고 cov로 읽기

$0 \because E(Z) = E(Y - E(Y|X))$

$= E(\underbrace{E(Y - E(Y|X)|X)}_{0})$

$= 0$

$\Rightarrow E((Y - E(Y|X))r(X)) = Cov(Y - E(Y|X), r(X)) = 0$

⑦ $E((X - E(X|Y))^2)$

$= E(E((X - E(X|Y))^2|Y))$

$= E(V(X|Y))$

⑧ $E(Y - E(Y|X)) = 0$ ... $E(Y) - E(E(Y|X))$

$= E(Y) - E(Y)$

$= 0$

$\cancel{E(E[Y - E(Y|X)]|X)}$

2.15) $f_{1,2,3}(x_1,x_2,x_3)=48x_1x_2x_3 I_{(0<x_1<x_2<x_3<1)}$

(a) $f_{1,2|3}(x_1,x_2|z)$

$x_1=x,\ x_2=yz$

$\Rightarrow f_3(z)=\int_0^z\int_0^y 48x_1yz\,dx_1dy$

$=\int_0^z 48yz\int_0^y x\,dx\,dy=\int_0^z 48yz\cdot\frac{1}{2}y^2dy$

$=24z\int_0^z y^3dy$

$=24z\cdot\frac{1}{4}y^4\Big|_0^z$

$=\boxed{6z^5}$

$\therefore f_{1,2|3}(x_1,x_2|z)=\frac{48x_1x_2z}{6z^5}=\boxed{\frac{8x_1x_2}{z^4}}I_{(0<x_1<x_2<z)}$

(b) 풀기 · 범위가 묶여있음

$t_2+t_3-5<0$

$=\frac{90}{3-t_3}\int_0^\infty e^{(t_1-1)x}\cdot-\frac{1}{t_2+t_3-5}e^{(t_2+t_3-5)x}$

$=\frac{90}{(3-t_3)}\cdot\frac{1}{(5-t_2-t_3)}\cdot\int_0^\infty e^{(t_1+t_2+t_3-6)x}dx$

$=\frac{90}{(3-t_3)(5-t_2-t_3)(6-t_1-t_2-t_3)}$ $\quad t_3-3<0,\ t_2+t_3-5<0,\ t_1+t_2+t_3-6<0$

(b) $X=(X_1,X_2,X_3)'$의 분산행렬 $V(X)$를 구하여

위의 식을 바탕으로 cgf를 구해보자.

방법상 $-\log(1-\frac{t_3}{3})-\log(1-\frac{t_2+t_3}{5})-\log(1-\frac{t_1+t_2+t_3}{6})$

$-\log(1-A)=\sum_{n=1}^{\infty}\frac{1}{n}A^n$ 이므로

$(\frac{t_3}{3}+\frac{1}{2}(\frac{t_3}{3})^2+\cdots)$

$+(\frac{t_2+t_3}{5}+\frac{1}{2}(\frac{t_2+t_3}{5})^2+\cdots)$

$+(\frac{t_1+t_2+t_3}{6}+\frac{1}{2}(\frac{t_1+t_2+t_3}{6})^2+\cdots)$

2.16) $f_{1,2,3}(x,y,z)=90e^{-(x+2y+3z)}\cdot I_{(0<x<y<z<\infty)}$

(a) mgf $f_{1,2,3}(t_1,t_2,t_3)$를 구하라.

$E[e^{t_1x+t_2y+t_3z}]=\int_0^\infty\int_x^\infty\int_y^\infty e^{t_1x+t_2y+t_3z}\cdot 90e^{-(x+2y+3z)}dz\,dy\,dx$

$=90\int_0^\infty\int_x^\infty e^{(t_1-1)x+(t_2-2)y}dy\,dx\int_y^\infty e^{(t_3-3)z}dz$

$\Rightarrow\frac{1}{t_3-3}e^{(t_3-3)z}\Big|_y^\infty$

$\Rightarrow\frac{1}{3-t_3}e^{(t_3-3)y}$ $\quad t_3-3<0$

$=\frac{90}{3-t_3}\int_0^\infty e^{(t_1-1)x}\int_x^\infty e^{(t_2+t_3-5)y}dy\,dx$

$=\frac{90}{3-t_3}\int_0^\infty e^{(t_1-1)x}\cdot\frac{1}{t_2+t_3-5}e^{(t_2+t_3-5)y}\Big|_x^\infty$

$\boxed{\sum_{r_1}\sum_{r_2}\sum_{r_3}\frac{C(r_1,r_2,r_3)}{r_1!r_2!r_3!}t_1^{r_1}t_2^{r_2}t_3^{r_3}}$

$\frac{C(1,0,0)}{1}t_1=\frac{1}{6}$

$\frac{C(0,1,0)}{1}t_2=(\frac{1}{6}+\frac{1}{5})$

$\frac{C(0,0,1)}{1}t_3=(\frac{1}{6}+\frac{1}{5}+\frac{1}{3})$

$\frac{C(2,0,0)}{2!}t_1^2=\frac{1}{2}(\frac{1}{36})t_1^2$

$\frac{C(0,2,0)}{2!}t_2^2=\frac{1}{2}(\frac{1}{36}+\frac{1}{25})t_2^2$

$\frac{C(0,0,2)}{2!}t_3^2=\frac{1}{2}(\frac{1}{36}+\frac{1}{25}+\frac{1}{9})t_3^2$

$\frac{C(1,1,0)}{1!1!}t_1t_2=\frac{1}{2}\times\frac{2}{36}t_1t_2$

$C(1,0,1)t_1t_3=\frac{1}{2}\cdot\frac{2}{36}t_1t_3$

$C(0,1,1)t_2t_3=\frac{1}{2}\cdot\frac{2}{36}+\frac{1}{2}\cdot\frac{2}{25}$

$\frac{5}{36}+\frac{?}{25}$ $\quad\frac{161}{900}$

$\frac{1}{900}\times\begin{bmatrix}25&25&25\\25&61&61\\25&61&161\end{bmatrix}$

(c) $V(X_1+X_2+X_3)$ => 계산해보기

$E[X_1+X_2+X_3]^2 - \left[E[X_1+X_2+X_3]\right]^2$

$= E[X_1^2]+E[X_2^2]+E[X_3^2]$

$+2E[X_1X_2]+2E[X_1X_3]+2E[X_2X_3]$

$+\left(E[X_1]+E[X_2]+E[X_3]\right)^2$

$E[X_1]^2+E[X_2]^2+E[X_3]^2$

$+2E[X_1]E[X_2]+2E[X_2]E[X_3]$

$+2E[X_1]E[X_3]$

=>

$2[E[X_1X_2]-E[X_1]E[X_2]]$

$+2[E[X_2X_3]-E[X_2]E[X_3]]$

$+2[E[X_1X_3]-E[X_1]E[X_3]]$

$+[E[X_1^2]-E[X_1]^2]$

$+[E[X_2^2]-E[X_2]^2]$

$+[E[X_3^2]-E[X_3]^2]$

$= V(X_1)+V(X_2)+V(X_3)+2Cov(X_1,X_2)$
$+2Cov(X_2,X_3)$
$+2Cov(X_3,X_1)$

$= \frac{25+61+161+2\cdot 25+2\cdot 61+2\cdot 25}{900}$

$$\begin{array}{r} 125 \\ 183 \\ +161 \\ \hline 469 \end{array}$$

$= \frac{469}{900}$.

$M^{(n)}(0)=E[X^n]$

(d) X_1, X_2-X_1, X_3-X_2가 독립인가?

$Y_1=X_1$
$Y_2=X_2-X_1$
$Y_3=X_3-X_2$

$E[e^{s_1Y_1+s_2Y_2+s_3Y_3}]$

$=E[e^{s_1X_1+s_2(X_2-X_1)+s_3(X_3-X_2)}]$

$=E[e^{(s_1-s_2)X_1+(s_2-s_3)X_2+s_3X_3}]$

$t_1 \quad t_2 \quad t_3$

이전 => 90

$(3-t_3)(5-t_2-t_3)(6-t_1-t_2-t_3)$

$\frac{1}{(3-s_3)(5-s_2)(6-s_1)}$

$s_1-s_2=t_1$
$\to s_1<6$
$s_2-s_3=t_2$
$\to s_2<5$
$s_3=t_3$
$\to s_3<3$

$t_2+t_3-5<0$

$s_2-s_3+s_3-5<0$

$t_1+t_2+t_3-6<0$

s_1

=> 독립

=> Y의 mgf랑 X의 mgf를 같다고 두는구나..

2.17)

(a) $f_{2,3|1}(x_2,x_3|x)$를 구하여라.

$f(x)=\int_x^\infty\int_y^\infty 90e^{-(x+2y+3z)}\,dz\,dy$

$=\int_x^\infty 90e^{-x-2y}\boxed{\int_y^\infty e^{-3z}\,dz}\,dy$

$=\int_x^\infty 90e^{-x-2y}\left[-\frac{1}{3}e^{-3z}\right]_y^\infty dy$

$=\int_x^\infty 90e^{-x-2y}\times\frac{1}{3}e^{-3y}\,dy = 30e^{-x}\int_x^\infty e^{-5y}\,dy$

$=30e^{-x}\cdot -\frac{1}{5}e^{-5y}\Big|_x^\infty$

$=30e^{-x}\cdot\frac{1}{5}e^{-5x}=\boxed{6e^{-6x}}$

$I_{(0<x<\infty)}$

$\therefore f_{2,3|1}(x_2,x_3|x)$

$=\frac{90e^{-(x+2y+3z)}}{6e^{-6x}} = 15e^{5x-2y-3z}\,I_{(x<y<z<\infty)}$

2.17) (b)

$Y=(X_2, X_3)^t$ 라 할 때, $E(Y|X_1)$, $Var(Y|X_1)$ 을 구하라.

$$V\begin{pmatrix} X_1 \\ X_1 \end{pmatrix} = \begin{bmatrix} V(X_1) & Cov(X_2,X_1) \\ Cov(X_1) & V(X_2) \end{bmatrix}$$

$$Mgf_{(2,3|1)}(t_2,t_3) = \int_x^\infty \int_y^\infty 15 e^{5x-2y-3z} I_{(x<y<z<\infty)} \, dz\,dy$$

$$= 15e^{5x}\int_x^\infty e^{(t_2-2)y} \boxed{\int_y^\infty e^{(t_3-3)z}\,dz}\,dy$$

$\frac{1}{3-t_3} e^{(t_3-3)y}$ $\quad t_3-3<0$

$$= \frac{15e^{5x}}{3-t_3}\int_x^\infty e^{(t_3+t_2-5)y}\,dy$$

$$= \frac{15}{(3-t_3)(5-t_3-t_2)} e^{(t_3+t_2)x} \quad \boxed{t_3-t_2-5<0}$$

$f(x_1) = 6e^{-6x} I_{(0<x<\infty)}$

$$E[X_1] = \int_0^\infty x\,6e^{-6x}\,dx$$

계산실수가 좀 많았음.

$$= 6\int_0^\infty x e^{-6x}\,dx$$

$$= 6\left[-\frac{1}{6}xe^{-6x}\Big|_0^\infty + \frac{1}{6}\int_0^\infty e^{-6x}\,dx\right]$$

$$= 6\left[\frac{1}{36}\right] = \frac{1}{6}$$

$$E[X_1^2] = \int_0^\infty x^2\,6e^{-6x}\,dx$$

$$= 6\int_0^\infty x^2 e^{-6x}\,dx$$

$$= 6\left[-\frac{1}{6}x^2e^{-6x}\Big|_0^\infty - \int_0^\infty 2x \cdot -\frac{1}{6}e^{-6x}\,dx\right]$$

$$= 6\left[\frac{1}{3}\int_0^\infty xe^{-6x}\,dx\right]$$

$$= \frac{1}{3}\times\frac{1}{6} = \frac{1}{18}$$

$\therefore E[X_1^2]-E[X_1]^2$

$$= \frac{1}{18}-\frac{1}{36} = \frac{1}{36}$$

이때 cgf를 구하자 $-\log\left(1-\frac{t_3}{3}\right) - \log\left(1-\frac{t_3+t_2}{5}\right) + (t_3+t_2)x$

$$= \left(\frac{t_3}{3} + \frac{1}{2}\cdot\frac{1}{9}t_3^2 + \cdots\right) + \left(\frac{t_2+t_3}{5} + \frac{(t_2+t_3)^2}{2\,25} + \cdots\right)$$

$$E[Y|X_1] = \begin{pmatrix} \frac{1}{5}+X_1 \\ \frac{1}{3}+\frac{1}{5}+X_1 \end{pmatrix}$$

$\cdots t_2$ 1일 때

$\cdots t_3$ 1일 때

$\frac{c}{2!}t_2^2 = \frac{1}{2}\frac{1}{25}t_2^2$

$\frac{c}{2!}t_3^2 = \frac{1}{2}\left(\frac{1}{9}+\frac{1}{25}\right)t_3^2$

$= \frac{34}{225}$

$$V[Y|X_1] = \begin{pmatrix} \frac{1}{25} & \frac{1}{25} \\ \frac{1}{25} & \frac{34}{225} \end{pmatrix}$$

$Cov(X_2,X_3) = \frac{1}{2}\cdot\frac{2}{25}t_2t_3$

$V[E(Y|X_1)]$, $E[V(Y|X)]$

$$\Rightarrow V\begin{pmatrix} \frac{1}{5}+X_1 \\ \frac{8}{15}+X_1 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} \frac{1}{25} & \frac{1}{25} \\ \frac{1}{25} & \frac{34}{225} \end{pmatrix}$$

$$\therefore V[E(Y|X_1)] = \begin{pmatrix} \frac{1}{36} & \frac{1}{36} \\ \frac{1}{36} & \frac{1}{36} \end{pmatrix}$$

$$\int xe^{-x} = (-x-1)e^{-x}$$

$$\int xe^{-3x} = -\frac{1}{3}e^{-3x} + \int\frac{1}{3}e^{-3x}$$

$$= -\frac{1}{3}e^{-3x} + \frac{1}{9}e^{-3x}$$

$$= -\frac{4}{9}e^{-3x}$$

$$\int ye^{-5y}$$

$$= -\frac{1}{5}ye^{-5y} + \frac{1}{5}\int e^{-5y}$$

$$= -\frac{1}{5}e^{-5y} - \frac{1}{25}e^{-5y}$$

$$= -\frac{6}{25}e^{-5y}$$

2.17) (b)

$Y=(X_2, X_3)^t$ 라 할 때, $E(Y|X_1)$, $Var(Y|X_1)$ 을 구하라.

$V\begin{pmatrix} X_1 \\ X_1 \end{pmatrix} = \begin{bmatrix} V(X_1) & Cov(X_1,X_1) \\ Cov(X_1) & V(X_1) \end{bmatrix}$

$Mgf_{(2,3|1)}(t_2,t_3) = \int_x^\infty \int_y^\infty e^{t_2 y + t_3 z} \cdot 15 e^{5x - 2y - 3z} I_{(x<y<z<\infty)} \, dz\,dy$

$= 15e^{5x} \int_x^\infty e^{(t_2-2)y} \boxed{\int_y^\infty e^{(t_3-3)z}\,dz}\,dy$

$\frac{1}{3-t_3} e^{(t_3-3)y}$ $\quad t_3 - 3 < 0$

$= \frac{15e^{5x}}{3-t_3} \int_x^\infty e^{(t_3+t_2-5)y}\,dy$

$= \frac{15}{(3-t_3)(5-t_3-t_2)} e^{(t_3+t_2)x}$ $\quad t_3 - t_2 - 5 < 0$

$f(x_1) = 6e^{-6x} I_{(0<x<\infty)}$

$E[X_1] = \int_0^\infty x\,6e^{-6x}\,dx$

계산법 가 중요했음.

$= 6\int_0^\infty x e^{-6x}\,dx$

$= 6\left[\cancel{-\frac{1}{6}xe^{-6x}\Big|_0^\infty} + \frac{1}{6}\int_0^\infty e^{-6x}\,dx\right]$

$= 6\left[\frac{1}{36}\right] = \frac{1}{6}$

$E[X_1^2] = \int_0^\infty x^2\,6e^{-6x}\,dx$

$= 6\int_0^\infty x^2 e^{-6x}\,dx$

$= 6\left[\cancel{-\frac{1}{6}x^2 e^{-6x}}\Big|_0^\infty - \int_0^\infty 2x \cdot -\frac{1}{6}e^{-6x}\,dx\right]$

$= 6\left[\frac{1}{3}\int_0^\infty x e^{-6x}\,dx\right]$

$= \frac{1}{3} \times \frac{1}{6} = \frac{1}{18}$

$\therefore E[X_1^2] - E[X_1]^2$

$= \frac{1}{18} - \frac{1}{36} = \frac{1}{36}$

$\therefore V[E(Y|X_1)] = \begin{pmatrix} \frac{1}{36} & \frac{1}{36} \\ \frac{1}{36} & \frac{1}{36} \end{pmatrix}$

이때 cgf를 구하자 $-\log\left(1-\frac{t_3}{3}\right) - \log\left(1 - \frac{t_3+t_2}{5}\right)$ $\quad (t_3+t_2)x$

$= \left(\frac{t_3}{3} + \frac{1}{2}\cdot\frac{1}{9}t_3^2 + \cdots\right) + \left(\frac{t_2+t_3}{5} + \frac{(t_2+t_3)^2}{2\,25} + \cdots\right)$

$E[Y|X_1] = \begin{pmatrix} \frac{1}{5} + X_1 \\ \frac{1}{3}+\frac{1}{5} + X_1 \end{pmatrix}$ ⋯ t_2 일때 ⋯ t_3 일때

$\frac{c}{2!}t_2^2 = \frac{1}{2}\frac{1}{25}t_2^2$

$\frac{c}{2!}t_3^2 = \frac{1}{2}\left(\frac{1}{9}+\frac{1}{25}\right)t_3^2$

$= \frac{34}{225}$

$V[Y|X_1] = \begin{pmatrix} \frac{1}{25} & \frac{1}{25} \\ \frac{1}{25} & \frac{34}{225} \end{pmatrix}$

$Cov(X_2, X_3) = \frac{1}{2}\cdot\frac{2}{25}t_2 t_3$

$V[E(Y|X_1)]$, $E[V(Y|X)]$

$\Rightarrow V\begin{pmatrix} \frac{1}{5}+X_1 \\ \frac{8}{15}+X_1 \end{pmatrix}$ $\quad \Rightarrow \begin{pmatrix} \frac{1}{25} & \frac{1}{25} \\ \frac{1}{25} & \frac{34}{225} \end{pmatrix}$

$\int xe^{-x} = (-x-1)e^{-x}$

$\int xe^{-3x} = -\frac{1}{3}e^{-3x} + \int \frac{1}{3}e^{-3x}$

$= -\frac{1}{3}e^{-3x} - \frac{1}{9}e^{-3x}$

$= -\frac{4}{9}e^{-3x}$

$\int ye^{-5y}$

$= -\frac{1}{5}e^{-5y} + \frac{1}{5}\int e^{-5y}$

$= -\frac{1}{5}e^{-5y} - \frac{1}{25}e^{-5y}$

$= -\frac{6}{25}e^{-5y}$