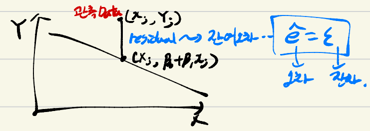
$Ax=b$로 지금까지 익혔던걸 통계모델로 다루기.

$\Rightarrow X\beta = Y$ ~> 선형회귀식 인듯(?)



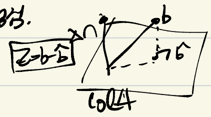

$y = \beta_0 + \beta_1 X$ 라고 할때,

$\beta_0, \beta_1$은 회귀계수

이때 $Y = X\beta$ 라고 할 때,

$$\begin{bmatrix} y_1 \\ \vdots \\ y_n \end{bmatrix} = \begin{bmatrix} 1 & x_{11} & \cdots & x_{1k} \\ 1 & x_{21} & \cdots & x_{2k} \\ \vdots & \vdots & & \\ 1 & x_{n1} & \cdots & x_{nk} \end{bmatrix} \begin{bmatrix} \beta_0 \\ \beta_1 \\ \vdots \\ \beta_k \end{bmatrix}$$ ~> 다중선형회귀식

$y = X\beta$ 그 자체의 해는 구할 수는 없다 ~> $y - \hat{y}$의 오차를 최소화하는 게 핵심.




① $\boxed{A\hat{x} = b}$

$A^TAx = A^Tb$

② A가 직교기저이면

$A\hat{x} = \hat{b} = a_1 x_1 + a_2 x_2 = \hat{b}$

$= \boxed{\dfrac{b \cdot a_1}{a_1 \cdot a_1}} a_1 + \boxed{\dfrac{b \cdot a_2}{a_2 \cdot a_2}} a_2$

③ $\boxed{Rx = Q^Tb}$

$\boxed{ex1}$ 점 (2,1), (5,2), (7,3), (8,3)에 가장 근접한 최소제곱직선 $y = \beta_0 + \beta_1 X$ 구하기.

- $y = X\beta$
- $\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 1 & x_1 \\ 1 & x_2 \end{bmatrix} \begin{bmatrix} \beta_0 \\ \beta_1 \end{bmatrix}$
- $X = \begin{bmatrix} 1 & 2 \\ 1 & 5 \\ 1 & 7 \\ 1 & 8 \end{bmatrix}$, $Y = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 3 \end{bmatrix}$

$A\hat{x} = \hat{b}$ ~> $A^TAx = A^Tb$ 인것처럼

$X\beta = Y$ ~> $X^TX\beta = X^TY$

$X^TX$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 2 & 5 & 7 & 8 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 1 & 5 \\ 1 & 7 \\ 1 & 8 \end{bmatrix} = \begin{bmatrix} 4 & 22 \\ 22 & 142 \end{bmatrix}$$

$X^TY$

$$\underset{2\times4}{\begin{bmatrix} 1 & 1 & 1 & 1 \\ 2 & 5 & 7 & 8 \end{bmatrix}} \underset{4\times1}{\begin{bmatrix} 1 \\ 2 \\ 3 \\ 3 \end{bmatrix}} = \begin{bmatrix} 9 \\ 57 \end{bmatrix}$$

$2+10+21+24 = 12+45 = 57$

$$\left[\begin{array}{cc|c} 4 & 22 & 9 \\ 22 & 142 & 57 \end{array}\right] \sim \begin{bmatrix} 2/7 \\ 5/14 \end{bmatrix}$$

연습문제1) $y = \beta_0 + \beta_1 X$를 구하라.

(0,1), (1,1), (2,2), (3,2)

$y = X\beta$ 라 할때,

$$\begin{bmatrix} 1 \\ 1 \\ 2 \\ 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 2 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} \beta_0 \\ \beta_1 \end{bmatrix}$$

$X^TX\beta = X^Ty$

① $X^TX = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 2 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 4 & 6 \\ 6 & 14 \end{bmatrix}$

② $X^Ty = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 2 \\ 2 \end{bmatrix} = \begin{bmatrix} 6 \\ 11 \end{bmatrix}$

$\boxed{y = \beta_0 + \beta_1 X}$

$$\left[\begin{array}{cc|c} 4 & 6 & 6 \\ 6 & 14 & 11 \end{array}\right] \sim \left[\begin{array}{cc|c} 2 & 3 & 3 \\ 6 & 14 & 11 \end{array}\right]$$

$$\sim \left[\begin{array}{cc|c} 2 & 3 & 3 \\ 0 & \cancel{5}\ 1 & \cancel{2}\ \frac{2}{5} \end{array}\right]$$

$$= \left[\begin{array}{cc|c} \cancel{2}\ 1 & 0 & \frac{9}{10} \\ 0 & 1 & 2/5 \end{array}\right]$$

$y = \frac{2}{5}x_1 + \frac{9}{10}$