

1. $H = \text{span} \left\{ \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}, \begin{pmatrix} 3 \\ 1 \\ -1 \end{pmatrix} \right\}$. $\dim H =$

2-3. A 는 5×6 행렬(matrix)이고 $Ax = 0$ 은 2개의 free variable을 가지면

2. $\dim \text{Nul } A = ?$

3. $\dim \text{Col } A = ?$

4. A 는 4×4 행렬(matrix), difference equation $x_{k+1} = Ax_k$ ($k = 0, 1, 2, \dots$), x_0 는 A 의 eigenvalue 2에 대한 eigenvector일 때, $x_5 = ?$

5. $\left\{ \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 3 \\ 7 \\ -2 \end{pmatrix}, \begin{pmatrix} 0 \\ 2 \\ -1 \end{pmatrix} \right\}$ 는 \mathbb{R}^3 의 기저(a basis)인가? 이유는?

6. $B = \left\{ \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 2 \\ -1 \end{pmatrix} \right\}$. $x = \begin{pmatrix} 4 \\ 5 \end{pmatrix} \in \mathbb{R}^2$ 의 B -coordinate를 구하시오.

7-8. $B = \left\{ \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 2 \\ -1 \end{pmatrix} \right\}$, $C = \left\{ \begin{pmatrix} 1 \\ -1 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \end{pmatrix} \right\}$

7. Find the change of basis coordinates matrix from B to C .

8. $[x]_C = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$ 일 때, $[x]_B = ?$

9. P_2 는 실수계수를 갖는 2차이하의 다항식들의 집합이다.

$T: P_2 \rightarrow P_2$, $T(a_0 + a_1t + a_2t^2) = 2a_2 + a_1t^2$, $B = \{1, t, t^2\}$. Find the B -matrix for T .

10. Find all eigenvalues of the matrix $\begin{pmatrix} 0 & -4 \\ 1 & 0 \end{pmatrix}$ and the corresponding eigenvectors.

11-14. $A = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$

11. Find all eigenvalues of A and the corresponding eigenvectors.

12. Is A similar to $B = \begin{pmatrix} 2 & 1 \\ 0 & 0 \end{pmatrix}$? Explain.

13. Diagonalize A . Explain.

14. $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$, $T(x) = Ax$. Find a basis B for \mathbb{R}^2 with the property that the B -matrix for T is a diagonal matrix. Explain.