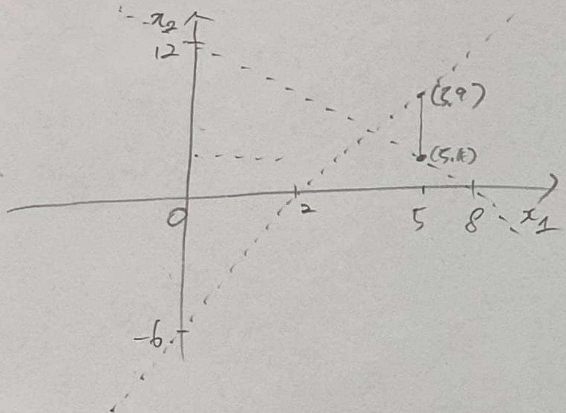
21500468 〈최적화기법 과제〉 이규석

Problem #4

(a)

Minimize $Z = 4x_1 + 5x_2$ (5, 4)

Subject to
$$\begin{cases} 3x_1 + 2x_2 \le 24 & \cdots\ x_2 \le -\frac{3}{2}x_1 + 12 \\ x_1 \ge 5 & \\ 3x_1 - x_2 \le 6 & \cdots\ x_2 \ge 3x_1 - 6 \\ x_1, x_2 \ge 0 & \end{cases}$$



⇒ 영역이 정의되지 않습니다.
⇒ infeasible

$-\frac{3}{2}x_1 = -12$

$x_1 = -12 \times -\frac{2}{3}$

$= 8.$

(b)

Minimize $Z = x_1 - 4x_2$ ⇝ $x_1 - 4x_2 = Z$

$\frac{x_1 - Z}{4} = x_2$

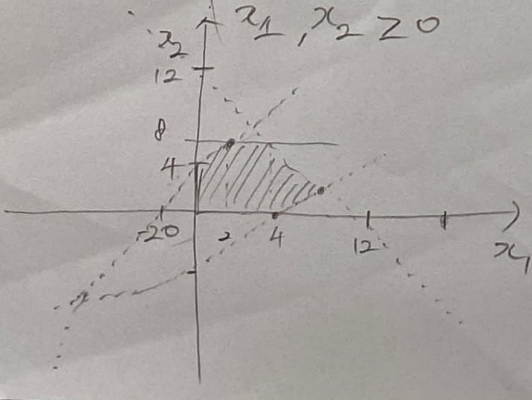
Subject to
$$\begin{cases} x_1 + x_2 \le 12 & \\ -2x_1 + x_2 \le 4 & \cdots\ x_2 \le 2x_1 + 4 \\ x_2 \le 8 & \\ x_1 - 3x_2 \le 4 & \\ x_1, x_2 \ge 0 & \end{cases}$$

$3x_1 = 9$

$-3x_2 \le -x_1 + 4$

$x_2 \ge \frac{1}{3}x_1 - \frac{4}{3}$

$x_2 \le -x_1 + 12$



$Z = x_1 - 4x_2$

$4x_2 = x_1 - Z$

$x_2 = \frac{1}{4}x_1 - \frac{1}{4}Z$

Z값이 최소

$-\frac{1}{4}Z$값이 가장 최대

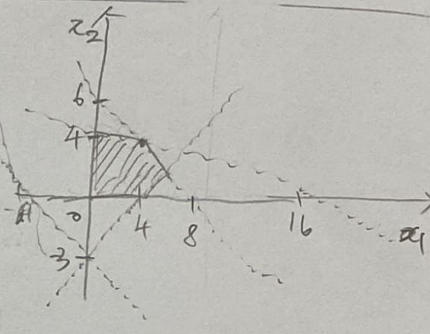
(2, 8)에서

$$\boxed{\begin{aligned} Z &= 2 - 32 \\ &= -30 \end{aligned}}$$ 최솟값이 된다.

(c)

Maximize $Z = 6x_1 + 8x_2$

Subject to
$x_1 + 4x_2 \le 16$
$3x_1 + 4x_2 \le 24$
$3x_1 - 4x_2 \le 12$
$x_1, x_2 \ge 0$



① $x_2 \le \frac{-x_1 + 16}{4}$ ② $x_2 \le \frac{-3x_1 + 24}{4}$

③ $\frac{3x_1 - 12}{4} \le x_2$

$Z = 6x_1 + 8x_2$

$Z - 6x_1 = 8x_2$ → $x_2 = \frac{-6x + Z}{8}$

$-2x_1 = -8$

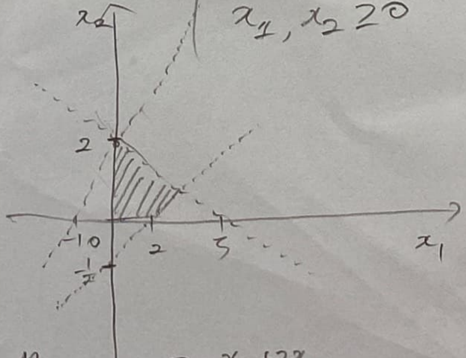
$x_1 = 4$, $x_2 = 3$ 일때 →

$Z = 24 + 24$

$= 48$

(d) Maximize $Z = x_1 + 2x_2$

Subject to
$$\begin{cases} -2x_1 + x_2 \le 2 & \cdots\ x_2 \le 2x_1 + 2 \\ 2x_1 + 5x_2 \ge 10 & \cdots\ x_2 \ge \frac{-2x_1 + 10}{5} \\ x_1 - 4x_2 \le 2 & \cdots\ x_2 \ge \frac{x_1 - 2}{4} \\ x_1, x_2 \ge 0 & \end{cases}$$



Maximize $Z = x_1 + 2x_2$

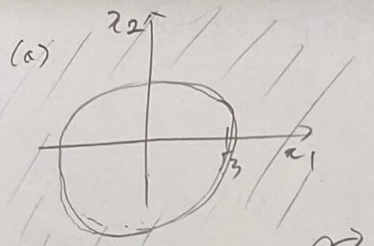
$x_2 = \frac{Z - x_1}{2}$

(0, 2) 일때

$\boxed{Z = 4}$

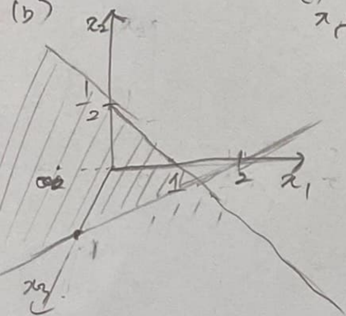
problem 5

(a)



⇝ Non-convex

(b)



① $x_1+2x_2 \le 1$

$2x_2 \le -x_1+1$

$x_2 \le \frac{-x_1+1}{2}$

② $x_1-2x_3 \le 2$

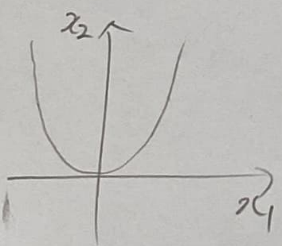
$x_1-2 \le 2x_3$

$\frac{x_1-2}{2} \le x_3$

⇝ Convex

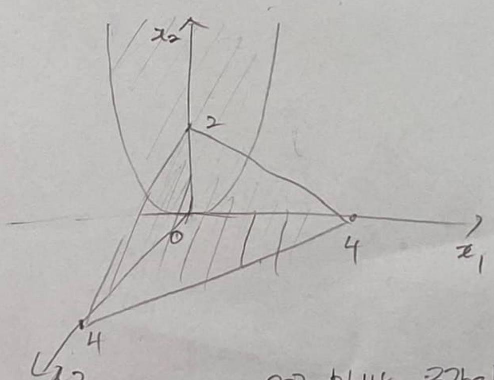
(c) $-3x_1^2+x_2=0$

$x_2=3x_1^2$



⇝ convex

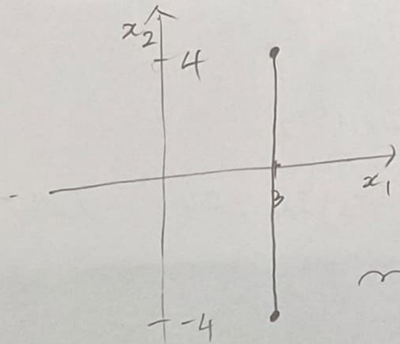
(d) $x_2 \ge x_1^2$, $x_1+2x_2+x_3 \le 4$



⇝ 만나는 구간이 (0,2,0) 밖에 없으므로

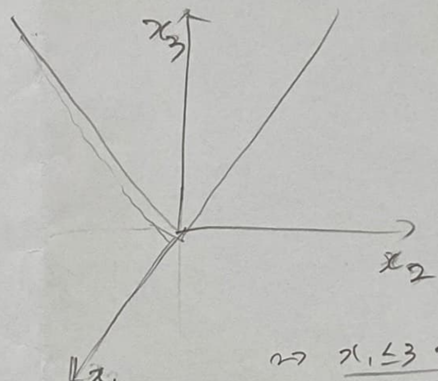
⇝ Non-convex

e) $x_1=3$, $|x_2| \le 4$



⇝ convex

f) $x_3=|x_2|$, $x_1 \le 3$



⇝ $x_1 \le 3$ 인 것과 $x_3=|x_2|$ 는 $\boxed{\text{독립사건}}$

$P(AB)=P(A)\cdot P(B)$ 니까

$P(A)$와 $P(B) \ne 0$ 이므로

$P(AB) \ne 0$ 이기에

convex 이다.

Assignment #2

minimize $z = -2x_1 - 3x_2$

Subject to $\frac{2}{9}x_1 + \frac{1}{4}x_2 \le 1$, ... ①

$\frac{1}{7}x_1 + \frac{1}{3}x_2 \le 1$ ... ②

$x_1, x_2 \ge 0$ (integer)

$9x_2 \le -8x_1 + 36$

① $8x_1 + 9x_2 \le 36$ $\times 3$ $\quad 24x_1 + 27x_2 \le 108$

$3x_1 + 7x_2 \le 21$ $\times 8$ $\quad - \; 24x_1 + 56x_2 \le 168$

$7x_2 \le -3x_1 + 21$

$-29x_2 \le -60$

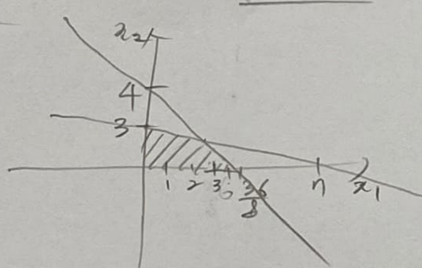
$\boxed{x_2 \ge \frac{60}{29}}$

$56x_1 + 63x_2 \le 252$

$- \; 27x_1 + 63x_2 \le 189$

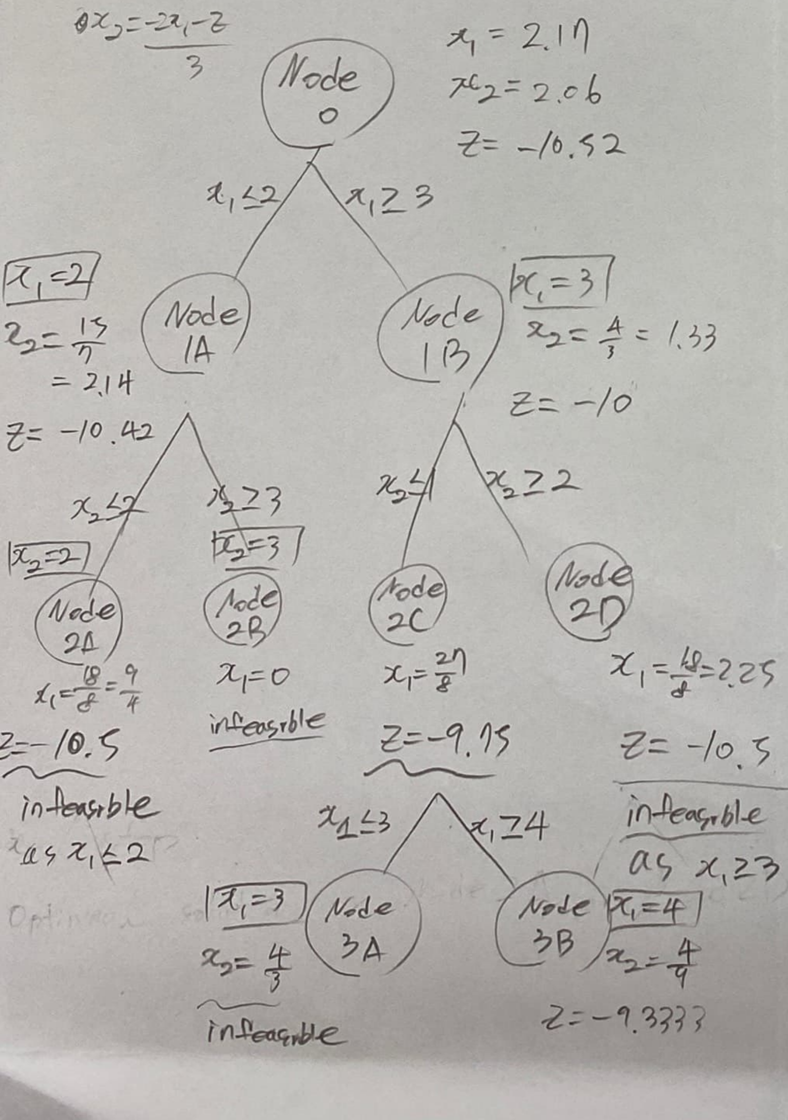
$29x_1 \le 63$

$\boxed{x_1 \le \frac{63}{29}}$



$z = -2x_1 - 3x_2$

$3x_2 = \frac{-2x_1 - z}{3}$



Minimize된 값은 Node 1A

$x_1 = 2$이고 $x_2 = \frac{15}{7} = 2.14$로 2로 이해

$z = -2 \times 2 - 3 \times 2$

$= -10$.

Problem 2

(a) BFS와 FS의 차이는 BFS는 음수가 아닌 조건을 만족하여 해를 구하는 반면, FS는 그렇지 않다.

(b) OBFS는 BFS 조건에서 optimal한 값을 찾는 것을 의미한다.

(c) P는 Poly-nomial을 의미하지만, NP는 다항식으로 결정되지 않는 것을 의미한다.

(d) Linear Programming으로 모형을 만들어서 범위(제약조건)에 해당하는 영역에서 integer 값을 찾는 것이 효율적이기 때문이다.