

## Problem Set #4

ECON 833, Prof. Jason DeBacker  
Due Thursday, September 25, 2:30 p.m.

This problem requires you to define a statistical objective function in Python and then use SciPy to find the parameter values that minimize that function.

The Stata data file `PSID_data.dta` provides you with the PSID used by [Heathcoate, Perri, and Violante \(Review of Economic Dynamics, 2010\)](#). The raw data are available [here](#). I've modified this only to deflate the labor incomes of heads and spouses (`hlabinc` and `wlabinc`) to 2005\$.

You need to do the following:

1. Write a function to take the “raw” data and cleans it (i.e., it should accept and return a DataFrame object where the input is the raw and the output has the correct sample selection and necessary variables). Specifically:
  - Select only male heads of household who are between 25 and 60 years of age and earn wages > \$7/hr.
  - Create indicator and continuous variables as necessary (see model below).
2. Write 2 unit tests for your data cleaning function. These should check that:
  - (a) There are no observations that do not meet the selection criteria above.
  - (b) Indicator variables are coded correctly (e.g., no values other than 0 or 1, sum to one across categories, etc.).
3. Estimate the following model via a Maximum Likelihood Estimator separately for  $t = 1971, 1980, 1990, 2000$ :
$$\ln(w_{i,t}) = \alpha + \beta_1 Educ_{i,t} + \beta_2 Age_{i,t} + \beta_3 Age_{i,t}^2 + \beta_4 Black_{i,t} + \beta_5 Hispanic_{i,t} + \beta_6 OtherRace_{i,t} + \varepsilon_{i,t},$$
where:
  - $w_{i,t}$  = wage of individual  $i$  in survey year  $t$
  - $Educ_{i,t}$  = education in years
  - $Age_{i,t}$  = age in years
  - $Black_{i,t}, Hispanic_{i,t}, OtherRace_{i,t}$  = dummy variables for race = Black, Hispanic, Not  $\in \{\text{White, Black, Hispanic}\}$ .
4. Interpret the coefficient  $\beta_1$ . How do the returns to education change over time in these data?
5. Write a unit test that checks that your MLE is correct. You can do this by comparing your MLE to the results from an OLS regression (see hint below).

Please submit your answers as a set of `.py` files with your code (and a separate file containing your unit tests) and a pdf (compiled in TeX) in which you put your answers. Please organize the code and your documentation clearly and include docstrings (using Google or Numpy standards) for any function definitions. Please write out the equation for the likelihood function that you estimate in your PDF. Include as an image in your PDF a screenshot of the successful test results shown when you run `pytest` in your terminal. You will submit your problem set by pushing the files to your GitHub repository that you created from forking the repository for this class. You will put this in the path `/CompEcon_Fall2025/ProblemSets/ProblemSet4` on the `ProblemSets` branch of your fork of the class repository.

The following variable definitions are provided for your benefit (the [PSID provides further documentation](#)):

- `hlabinc` = annual labor income of the head
- `hannhrs` = annual hours of the head
- `hsex` = gender of the head (1=Male, 2=Female)
- `hrace` = race of the head (1=White, 2=Black, 3=Native American, 4=Asian/Pacific Islander, 5=Hispanic, 6,7=Other)
- `age` = age of the head
- `hyrsed` = years of education of the head

HINT: This model is linear. So you can check your MLE against an OLS estimator to confirm your results. See [this QuantEcon notebook](#) for a short tutorial on linear regressions in Python. Also, you may want a bounded optimizer e.g., try the “L-BFGS-B” and “SLSQP” methods in `scipy.optimize.minimize()` .