Simulating Physics with Quantum Computers

Quantum is Better!!

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Contents

Introduction

- $H = -\mathcal{J}\sum_i\left(\sigma_i^x\sigma_{i+1}^x + \sigma_i^y\sigma_{i+1}^y\right) + U\sum_i \sigma_i^z\sigma_{i+1}^z + \sum_i h_i\sigma_i^z$
- $|\psi(t)\rangle = U(t)|\psi(0)\rangle = e^{-i\hat{H}t}|\psi(0)\rangle$
- $\hat{H}$: $2^N \times 2^N$!!! $\rightarrow$ *Too many eigen vectors and eigen values*
- Direct computation with quantum algorithms

Classical Computation

$$f(H)\mathbf{x}$$

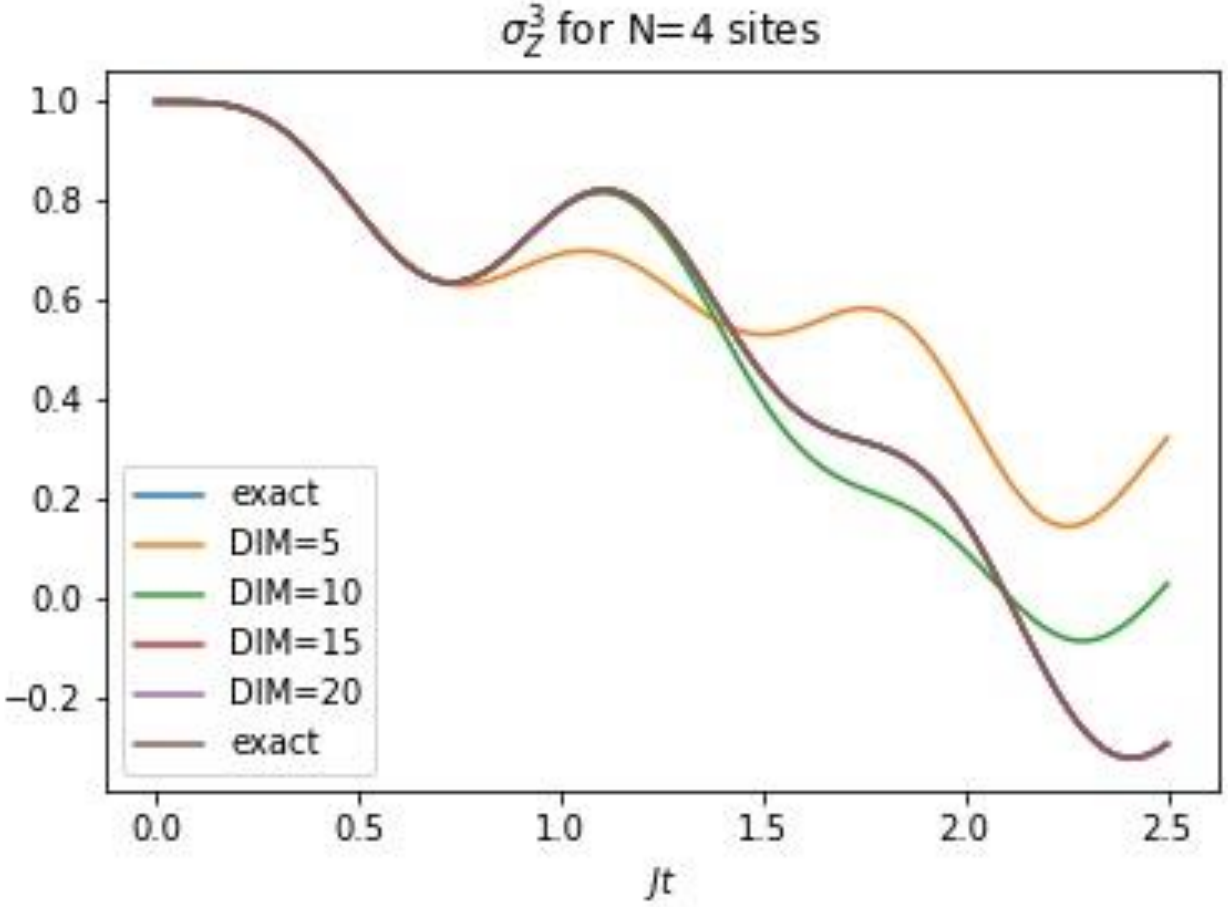
A huge H

Lanczos Algorithm

DIM = 50

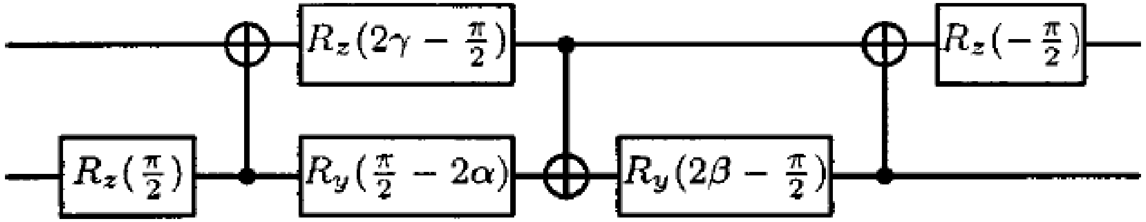
$$Q_k f(T_k) Q_k^T \mathbf{x}$$

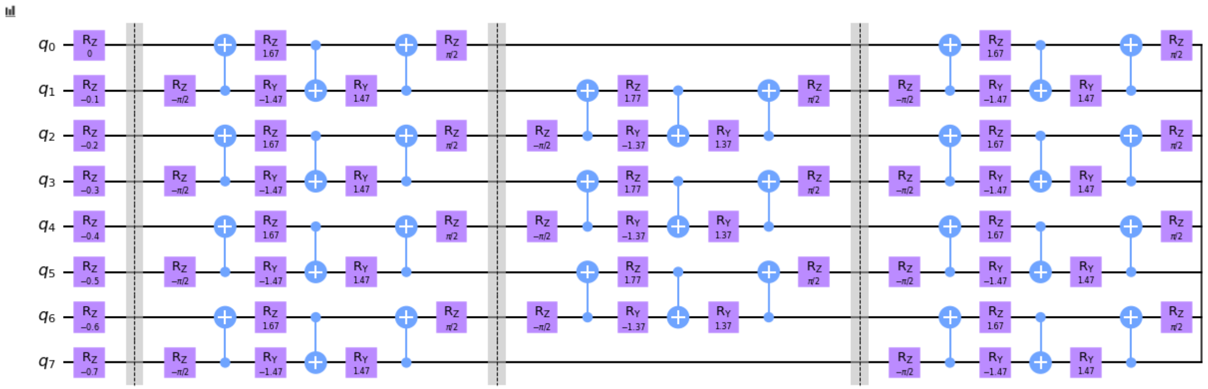
$$f(H) := e^{-iHt}$$



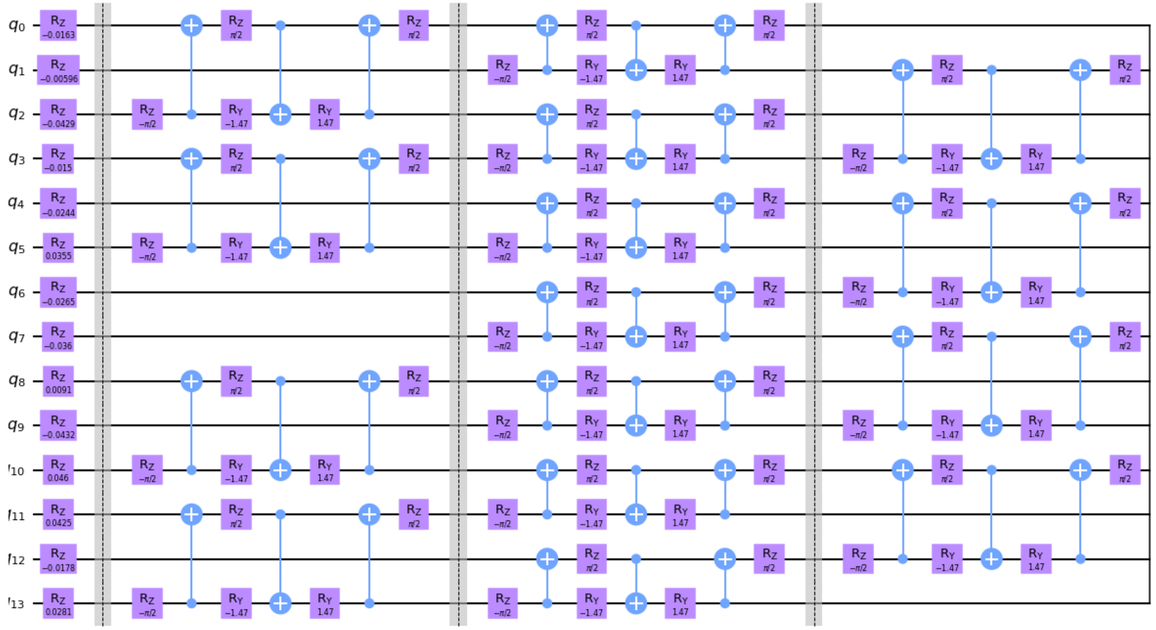
Quantum Circuit

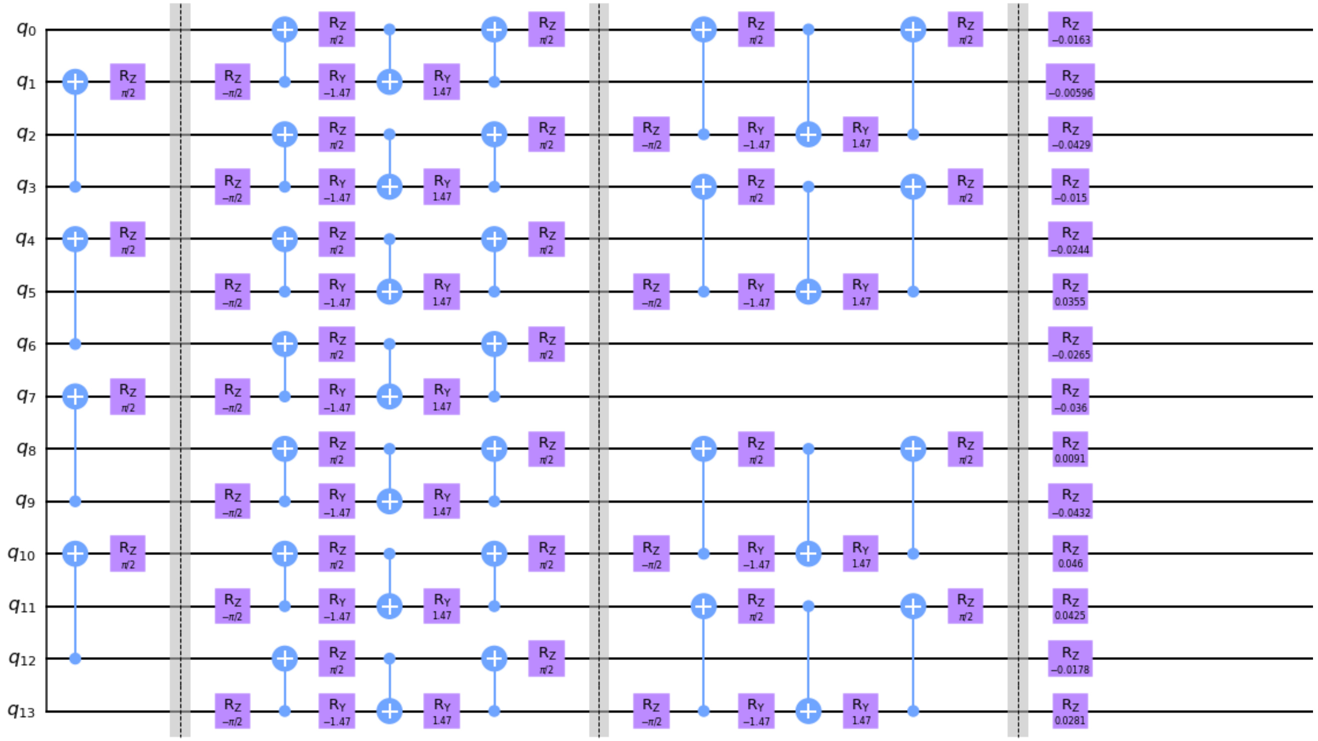
- $U_1(\delta t) = e^{i\mathcal{J}\delta t\left[\sum_{j:odd}\left(\sigma_j^x \otimes \sigma_{j+1}^x + \sigma_j^y \otimes \sigma_{j+1}^y\right) - \frac{U}{\mathcal{J}} \sum_{j:odd} \sigma_j^z \otimes \sigma_{j+1}^z\right]}$
- $U_2(\delta t) = e^{i\mathcal{J}\delta t\left[\sum_{j:even}\left(\sigma_j^x \otimes \sigma_{j+1}^x + \sigma_j^y \otimes \sigma_{j+1}^y\right) - \frac{U}{\mathcal{J}} \sum_{j:even} \sigma_j^z \otimes \sigma_{j+1}^z\right]}$
- $U_3(\delta t) = e^{i\mathcal{J}\delta t\left[-\sum_j \frac{h_j}{\mathcal{J}} \sigma_j^z\right]}$



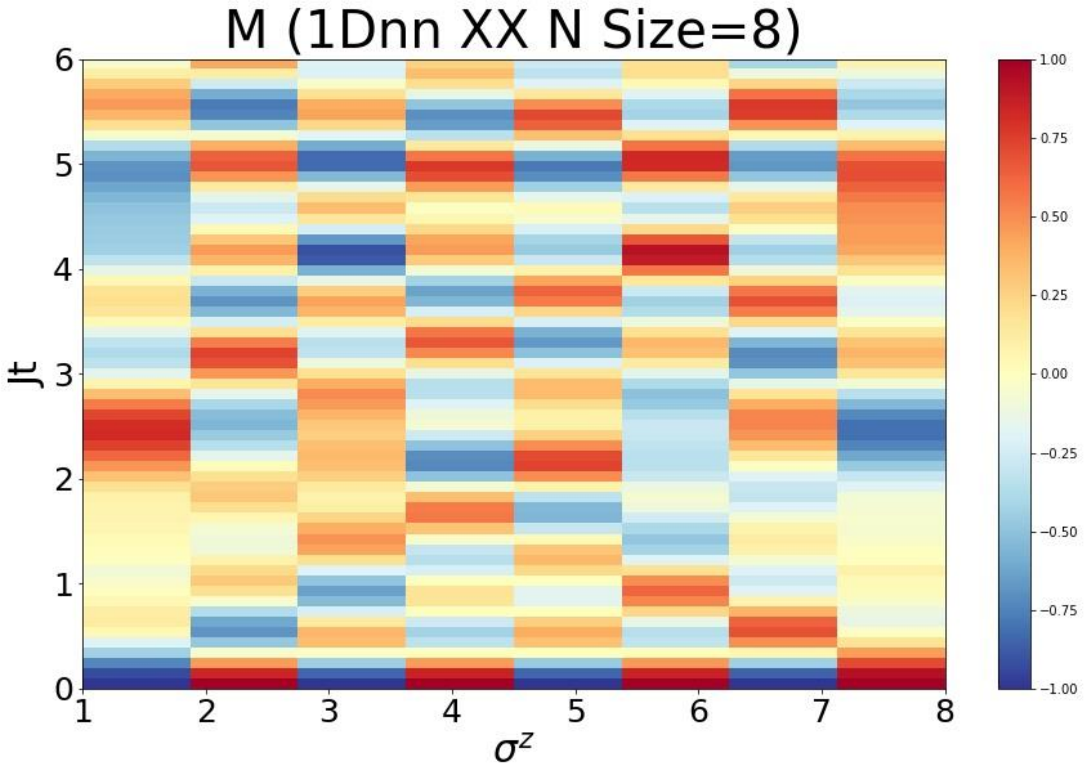


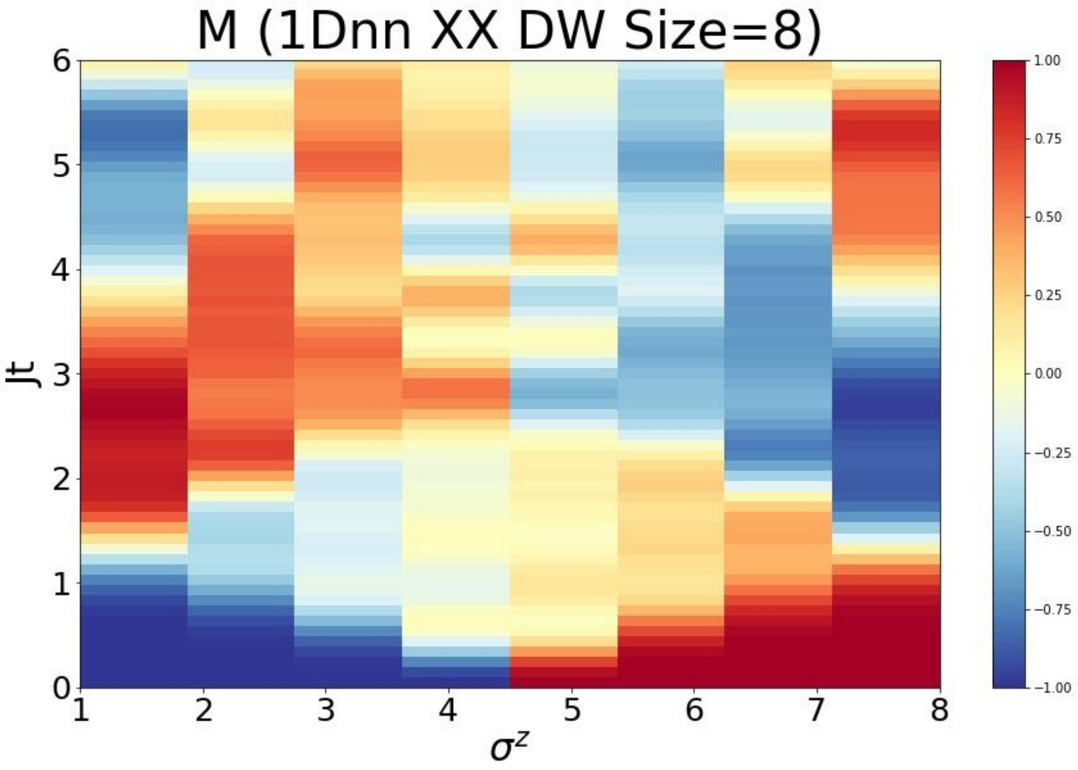
Quantum Circuit



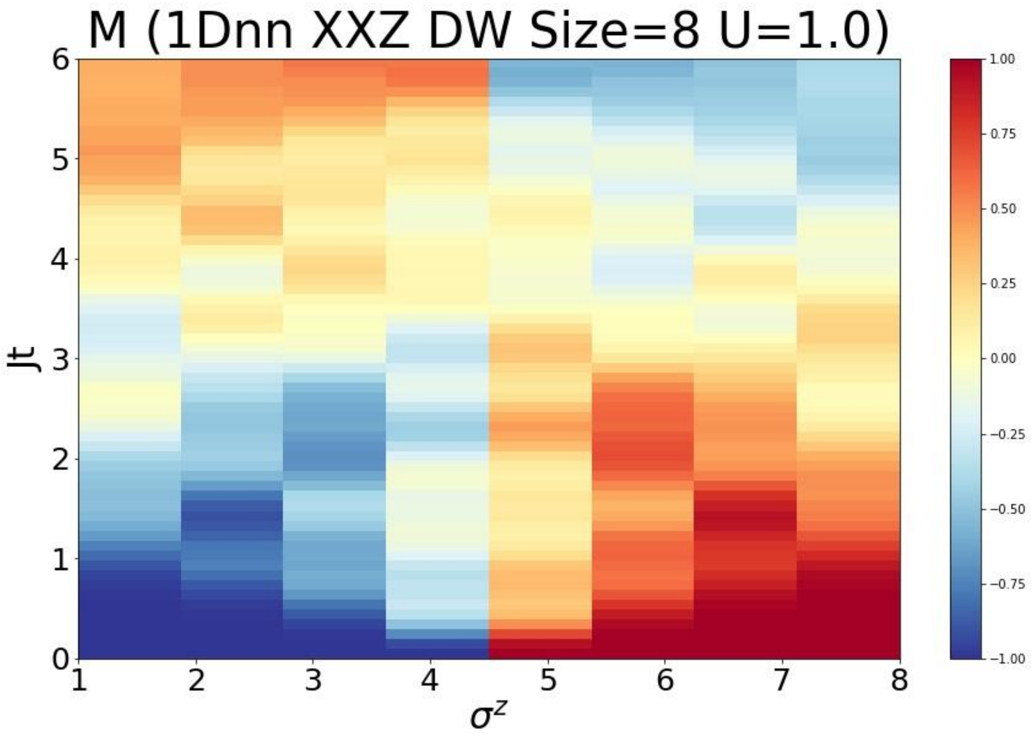


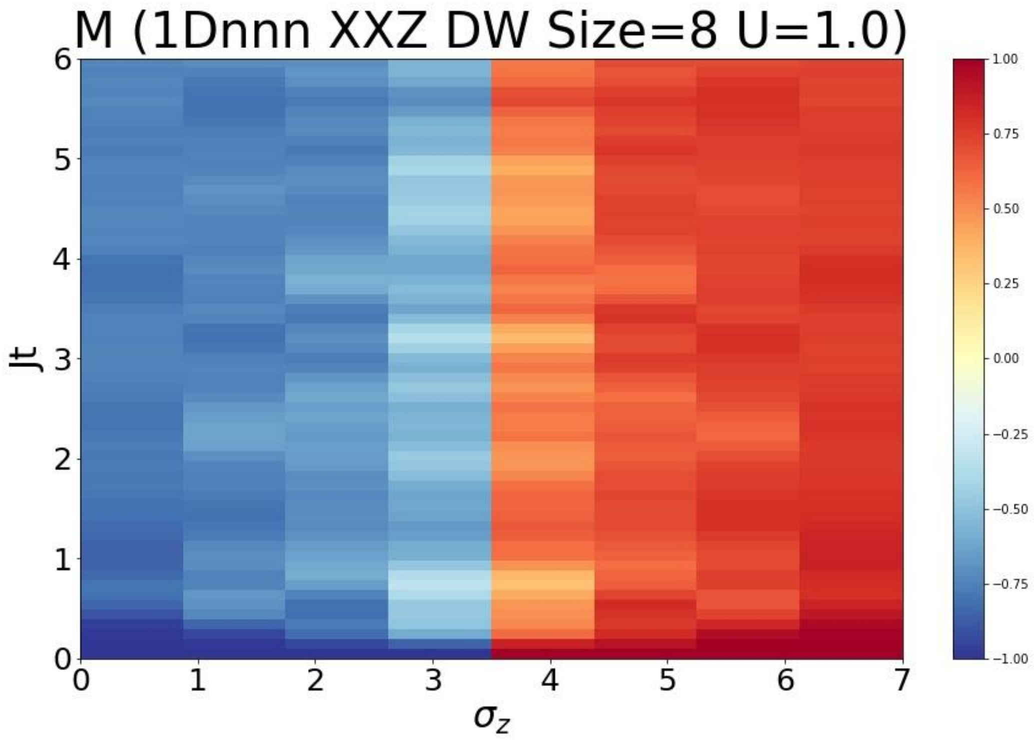
Local Magnetization



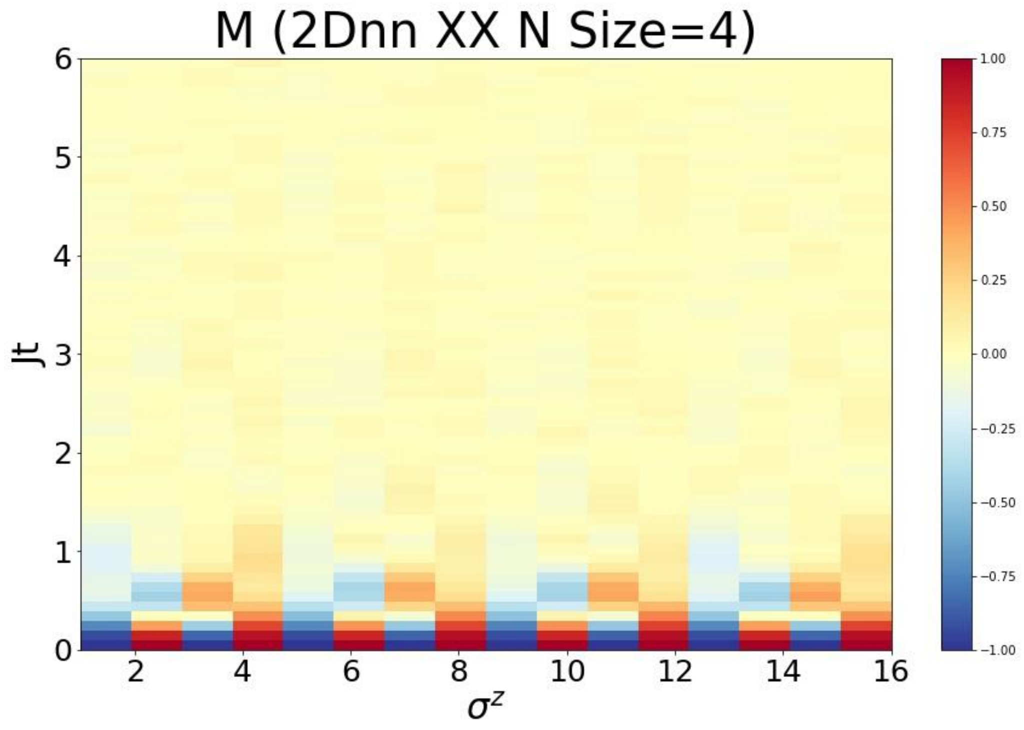


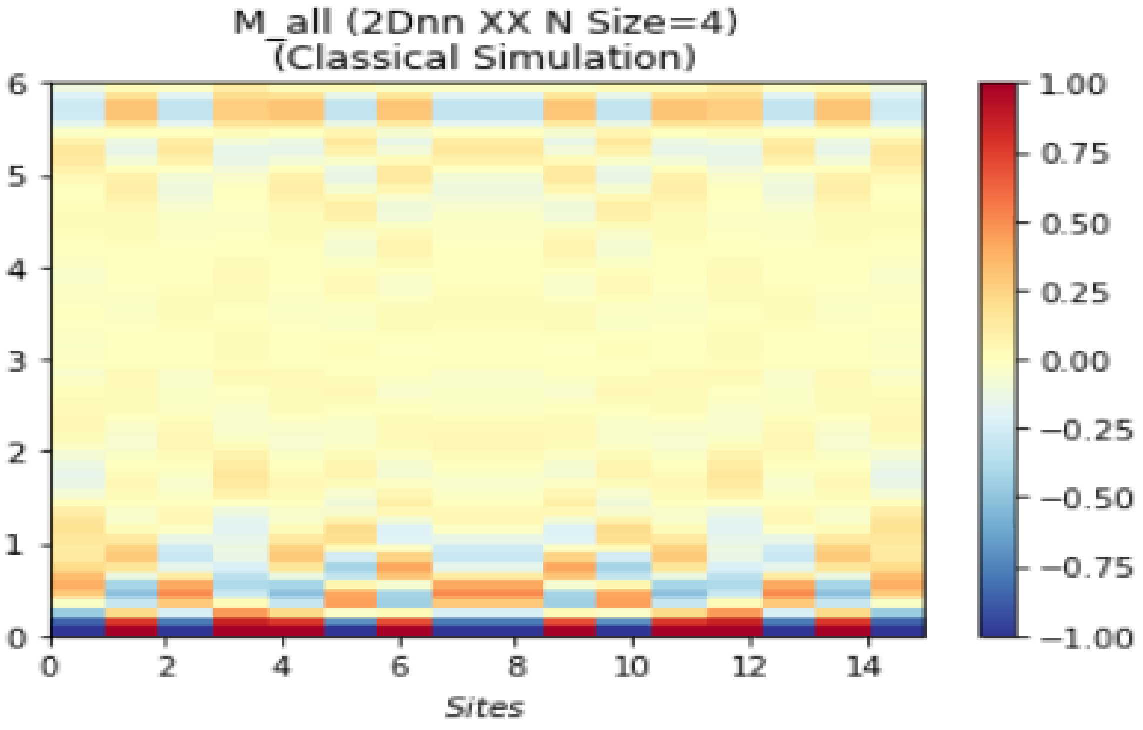
Local Magnetization



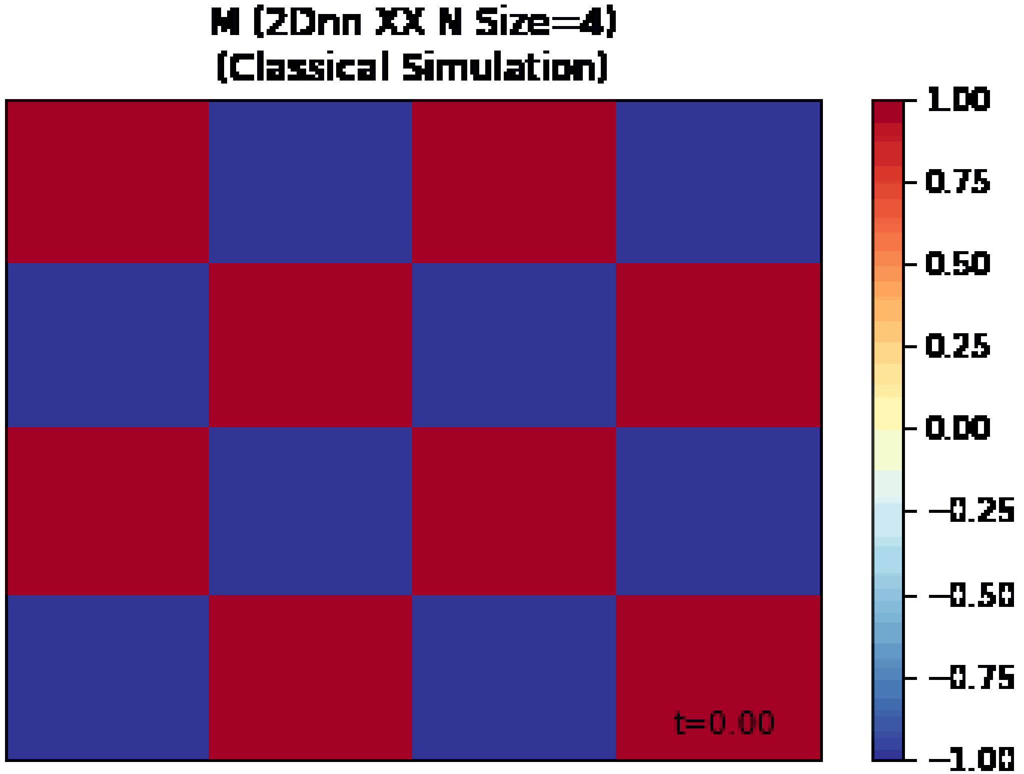


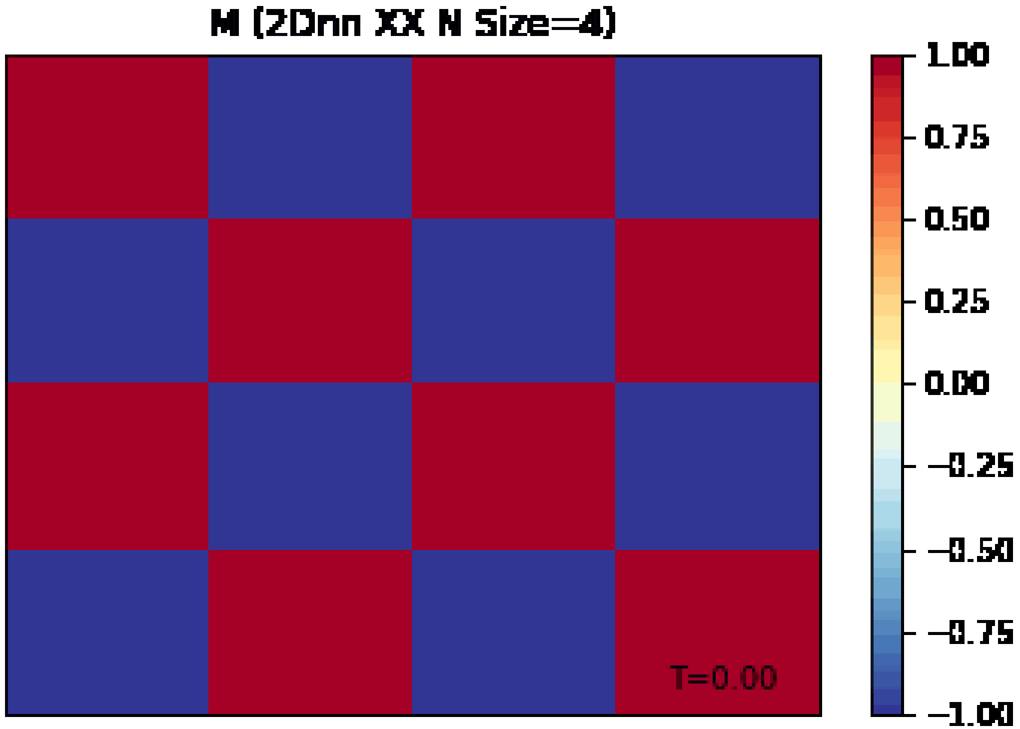
Local Magnetization



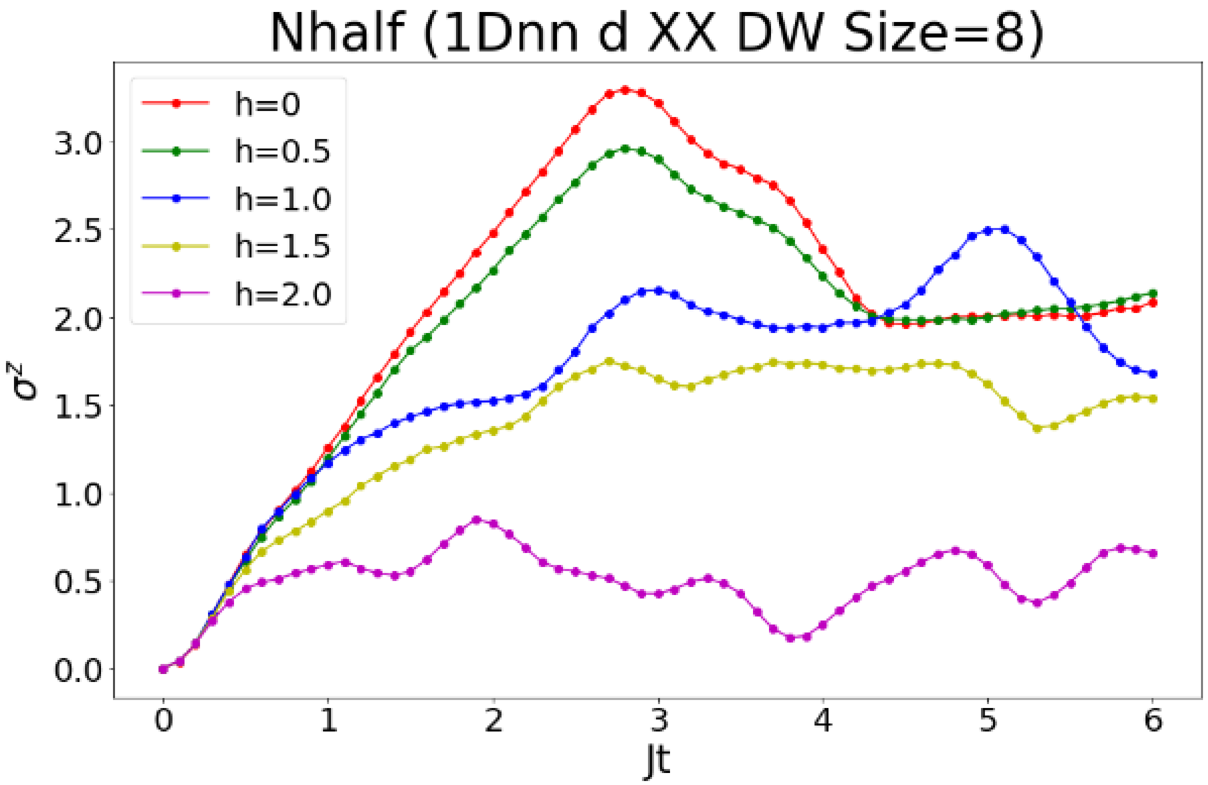


Local Magnetization

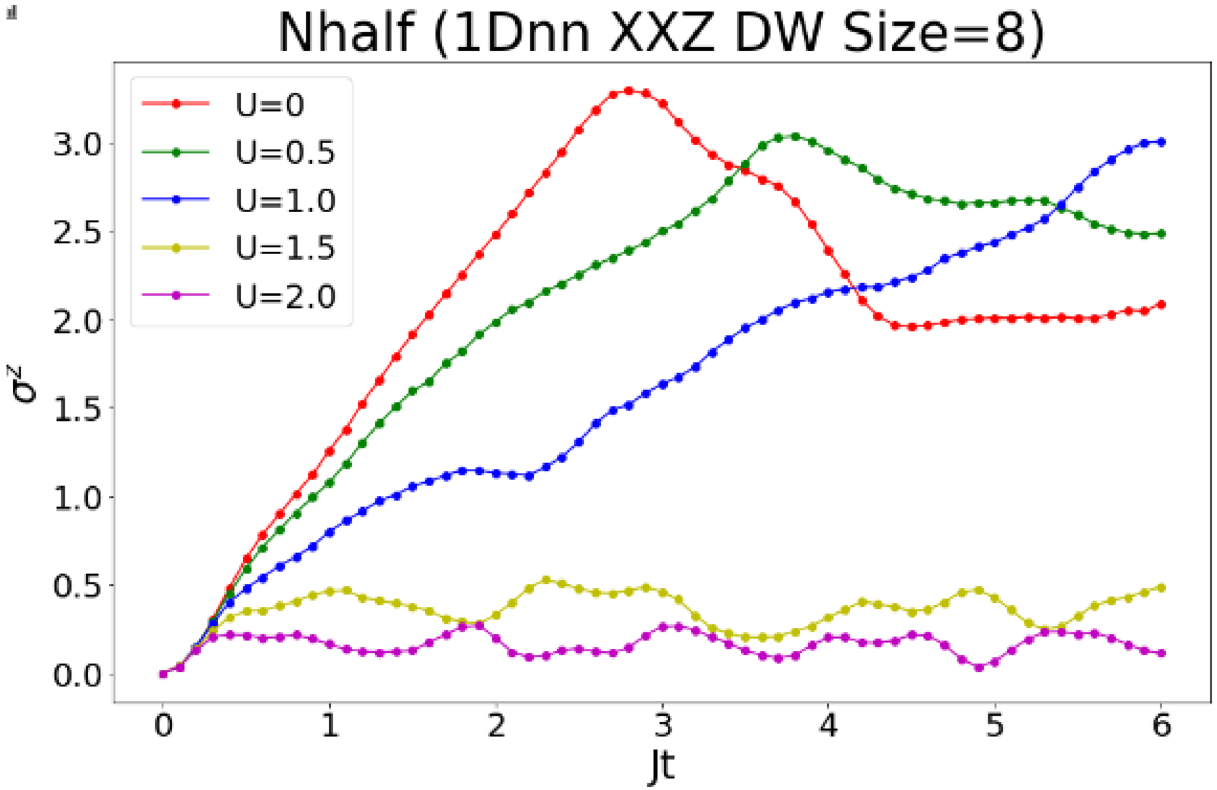




Nhalf

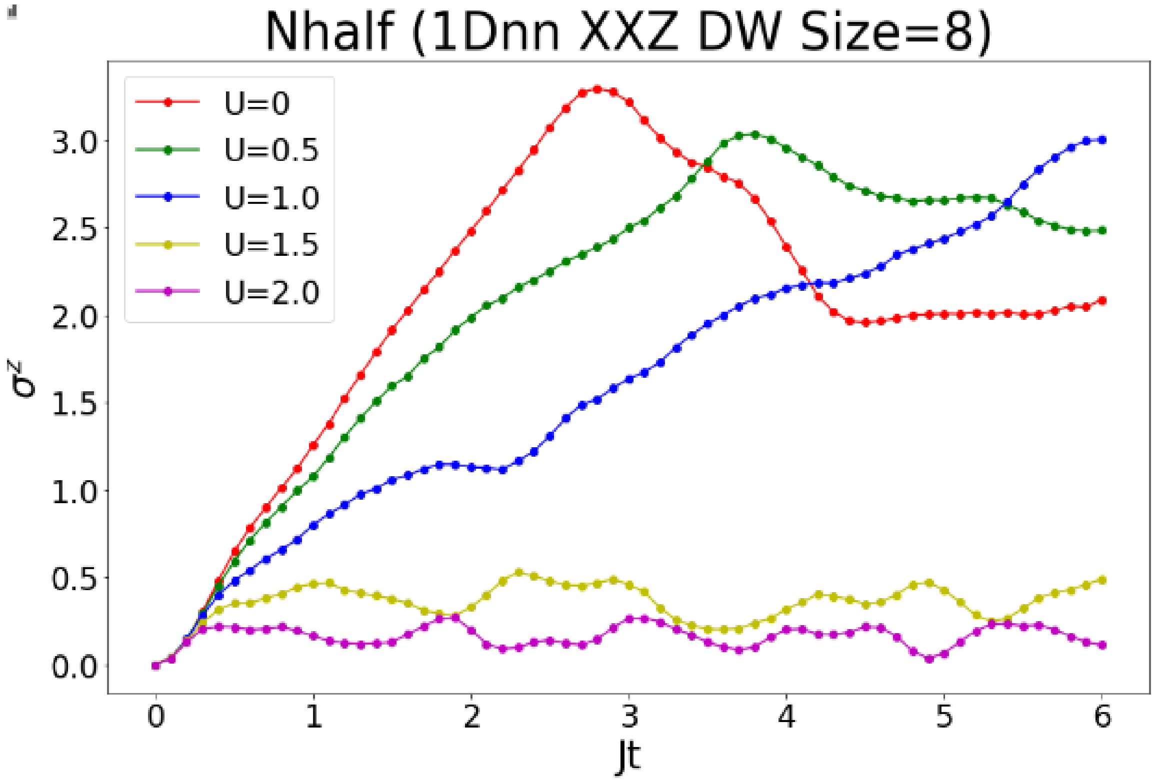


Nhalf (1Dnn d XX DW Size=8)
h=0
h=0.5
h=1.0
h=1.5
h=2.0
σ^z
0.0
0.5
1.0
1.5
2.0
2.5
3.0
0
1
2
3
4
5
6
Jt

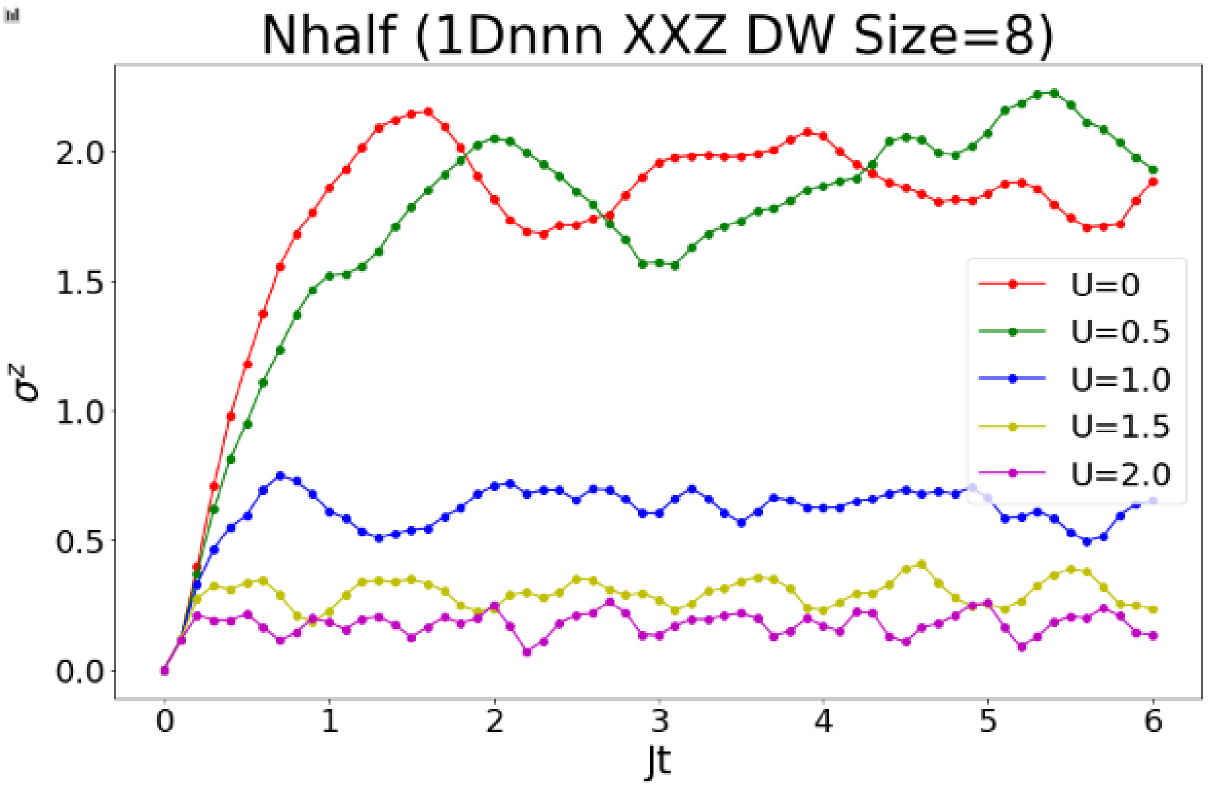


Nhalf (1Dnn XXZ DW Size=8)
U=0
U=0.5
U=1.0
U=1.5
U=2.0
σ^z
0.0
0.5
1.0
1.5
2.0
2.5
3.0
0
1
2
3
4
5
6
Jt

Nhalf

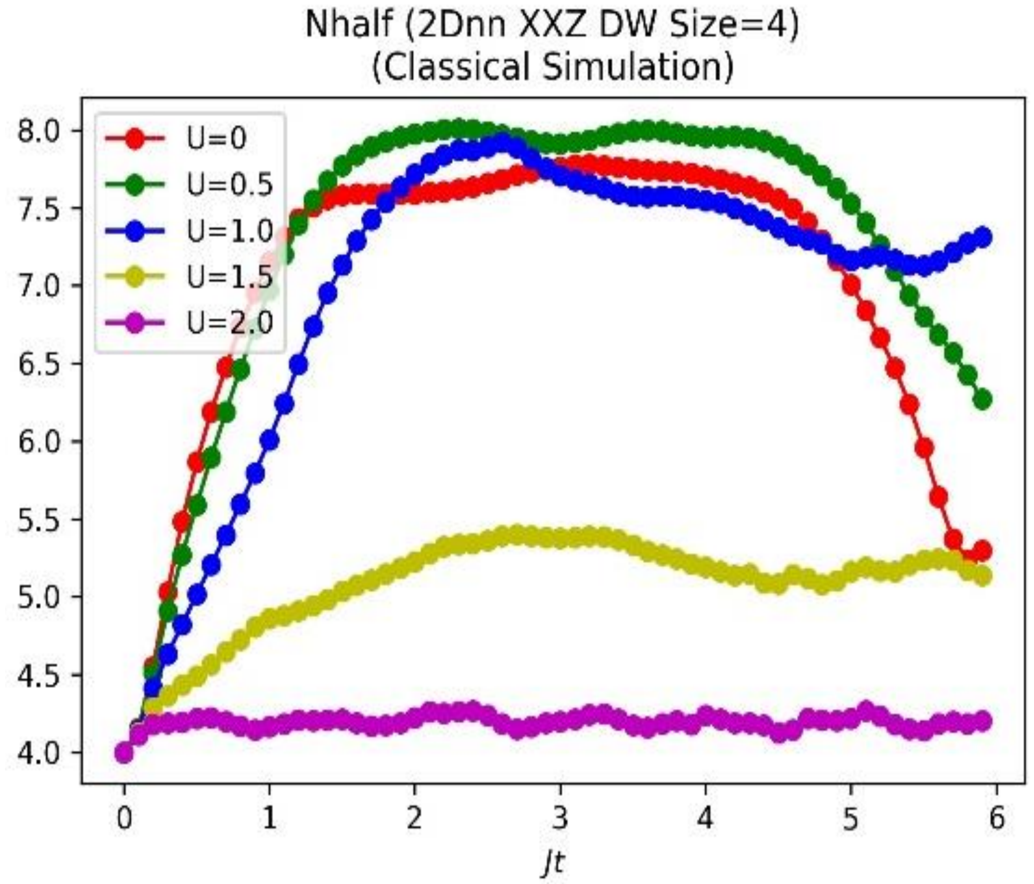


Nhalf (1Dnn XXZ DW Size=8)
U=0
U=0.5
U=1.0
U=1.5
U=2.0
σ^z
Jt

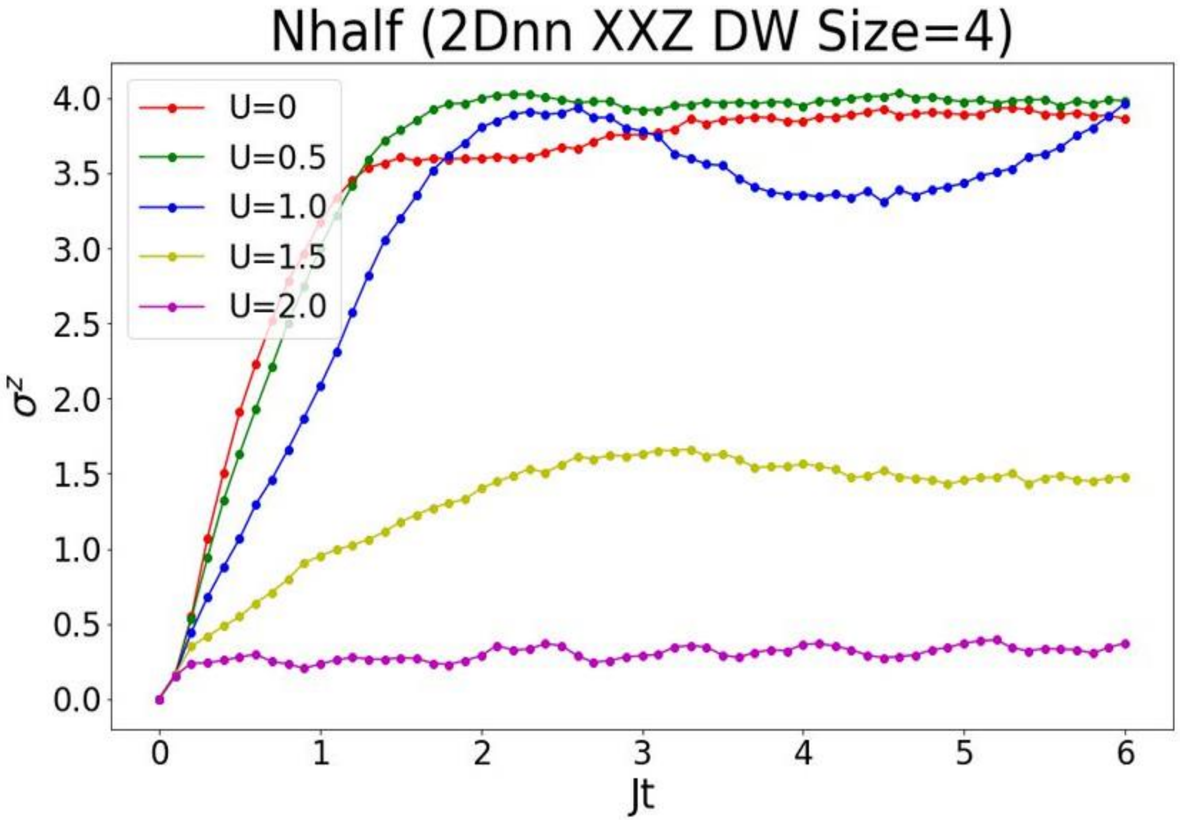


Nhalf (1Dnnn XXZ DW Size=8)
U=0
U=0.5
U=1.0
U=1.5
U=2.0
σ^z
Jt

Nhalf

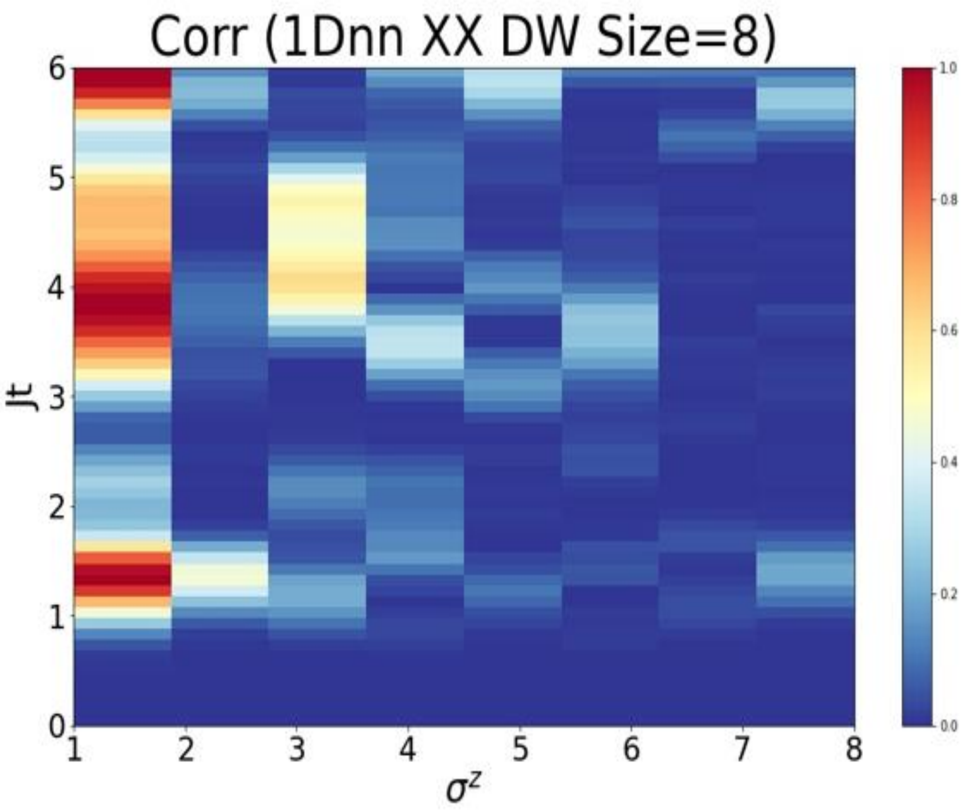


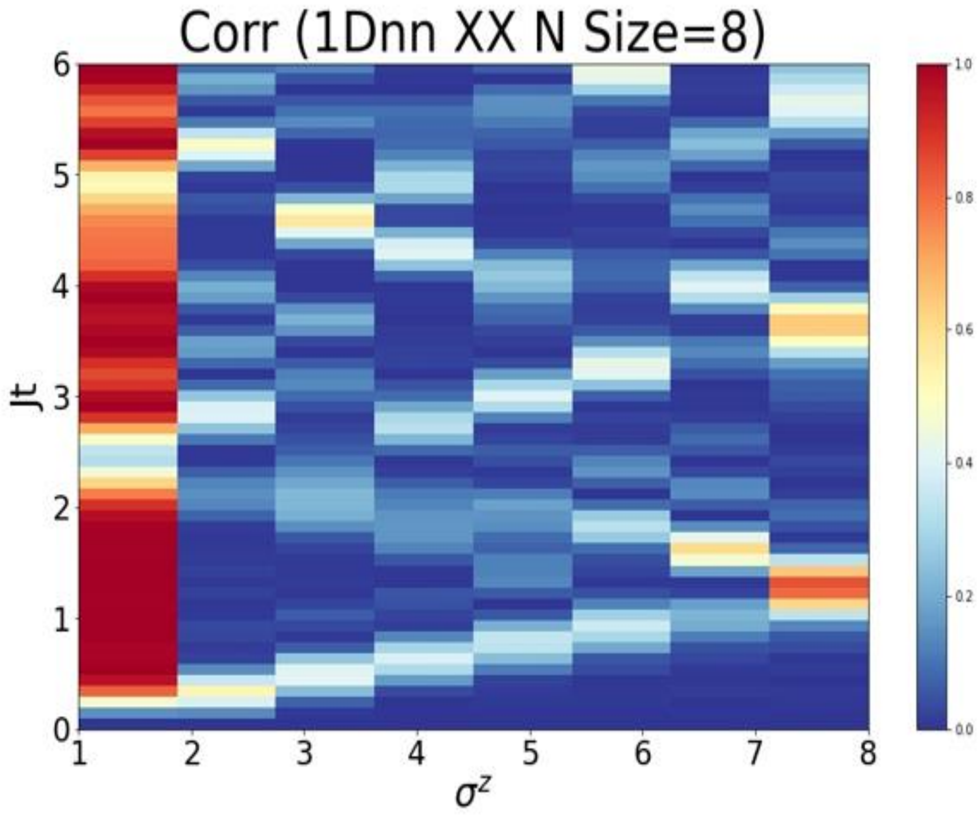
Nhalf (2Dnn XXZ DW Size=4)
(Classical Simulation)
U=0
U=0.5
U=1.0
U=1.5
U=2.0
8.0
7.5
7.0
6.5
6.0
5.5
5.0
4.5
4.0
0
1
2
3
4
5
6
Jt



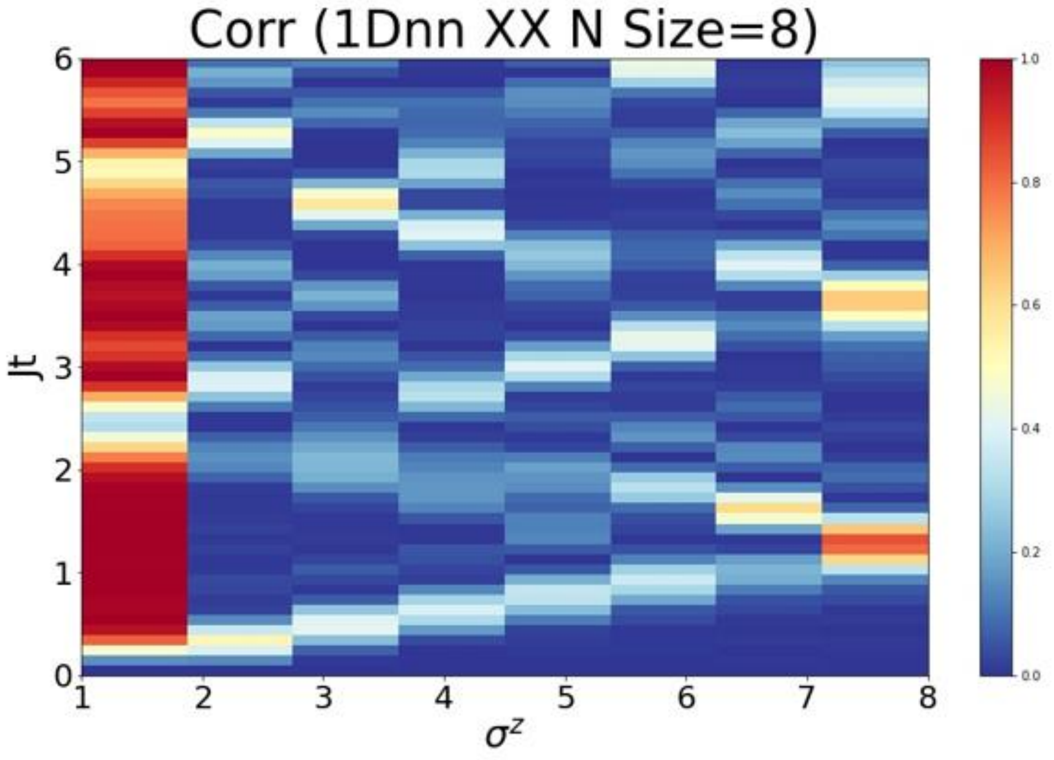
Nhalf (2Dnn XXZ DW Size=4)
U=0
U=0.5
U=1.0
U=1.5
U=2.0
σ^z
4.0
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
0
1
2
3
4
5
6
Jt

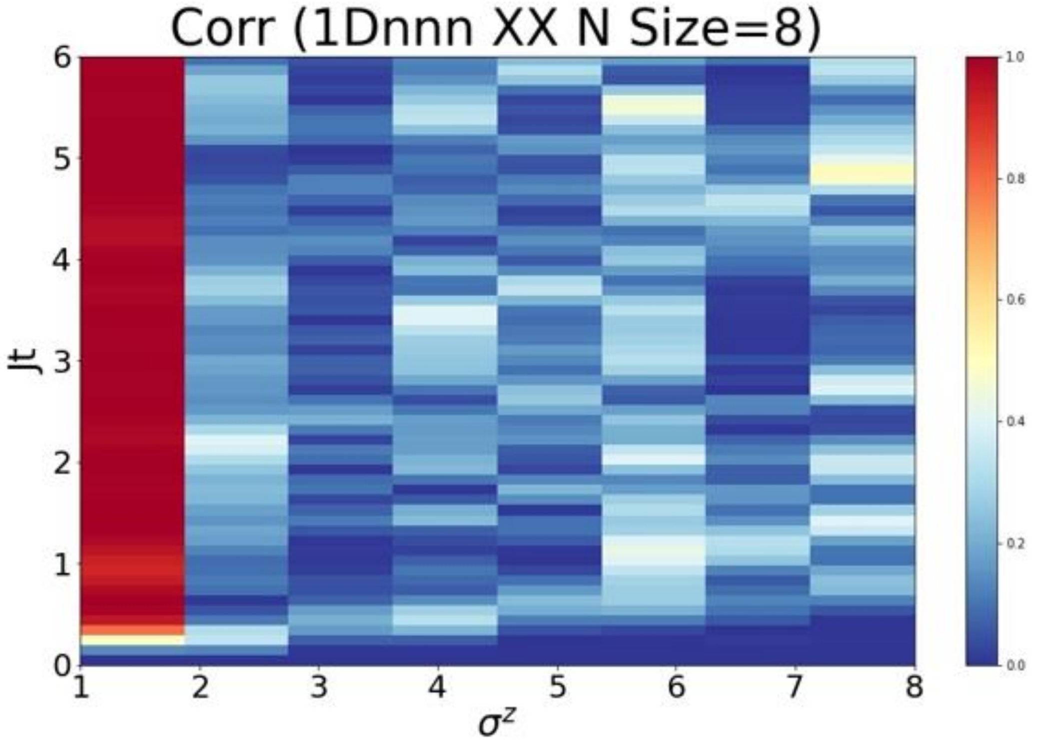
Correlation Function



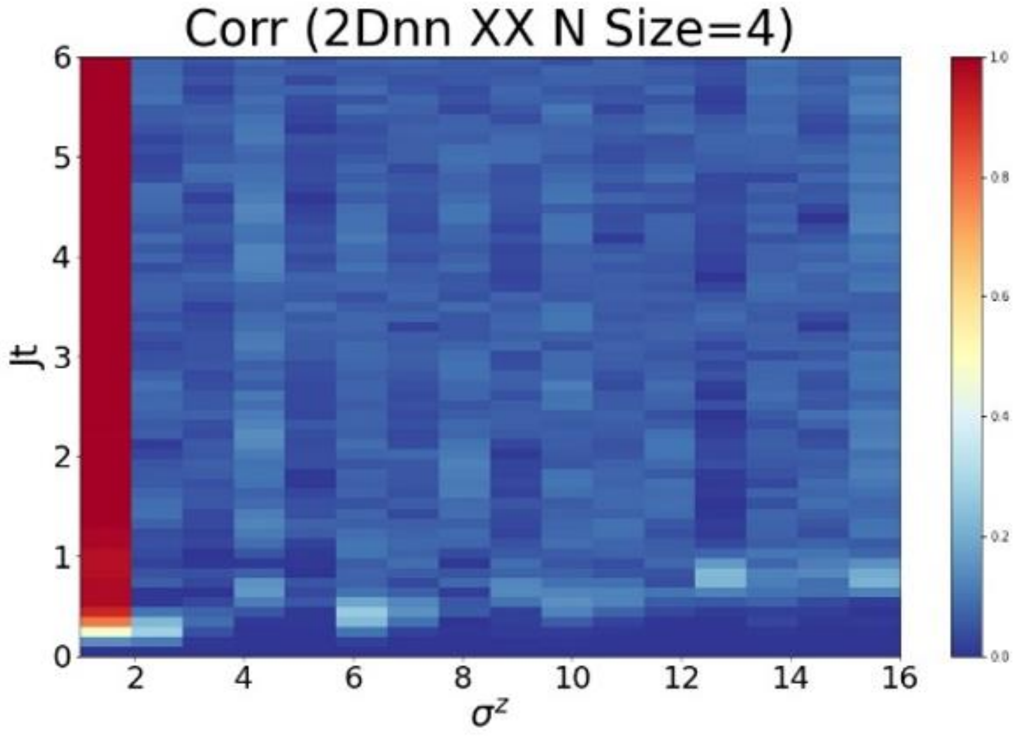


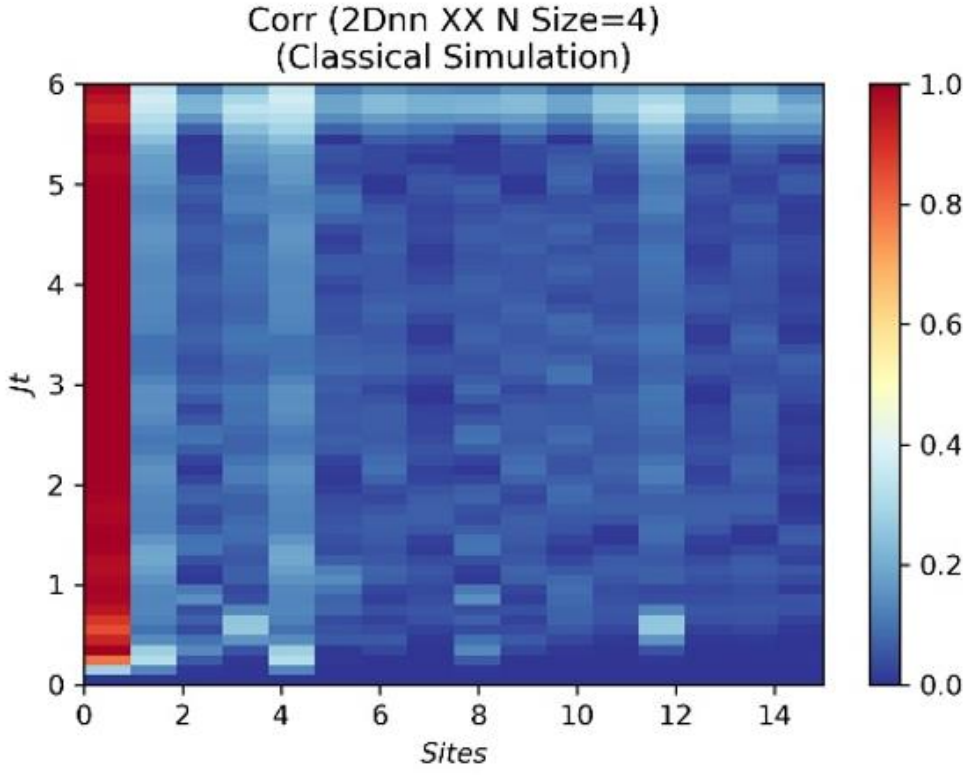
Correlation Function



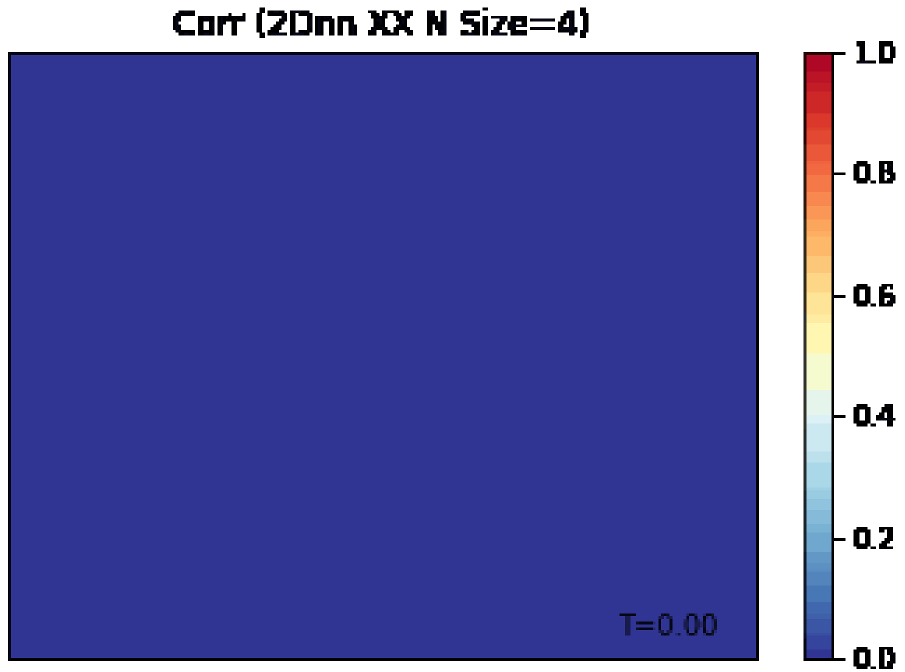


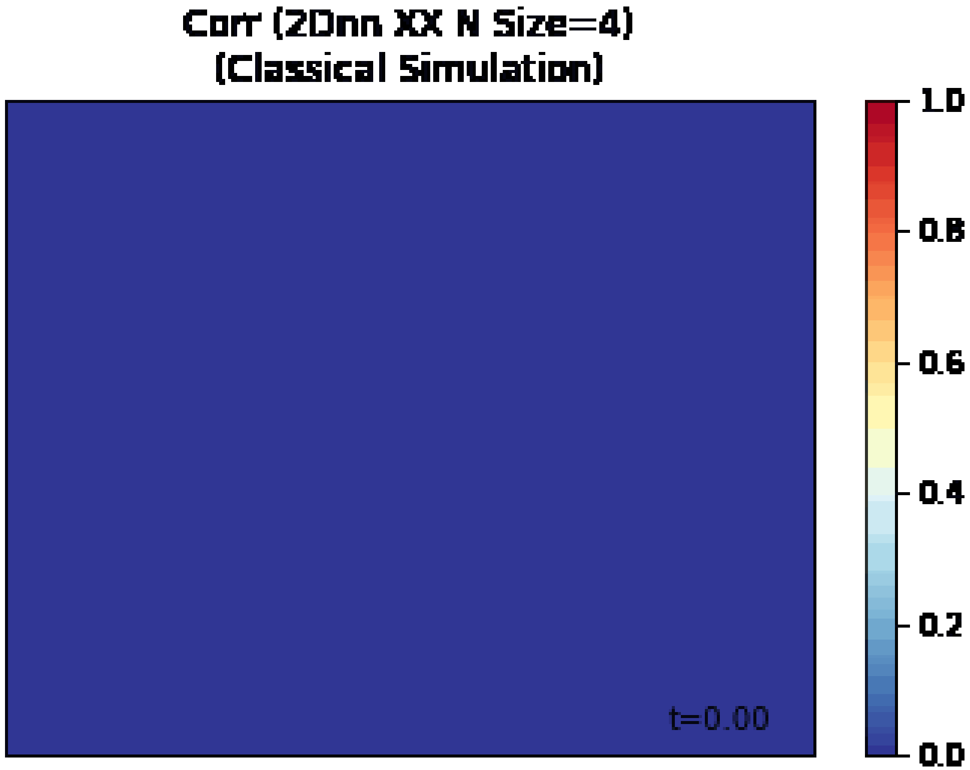
Correlation Fucntion



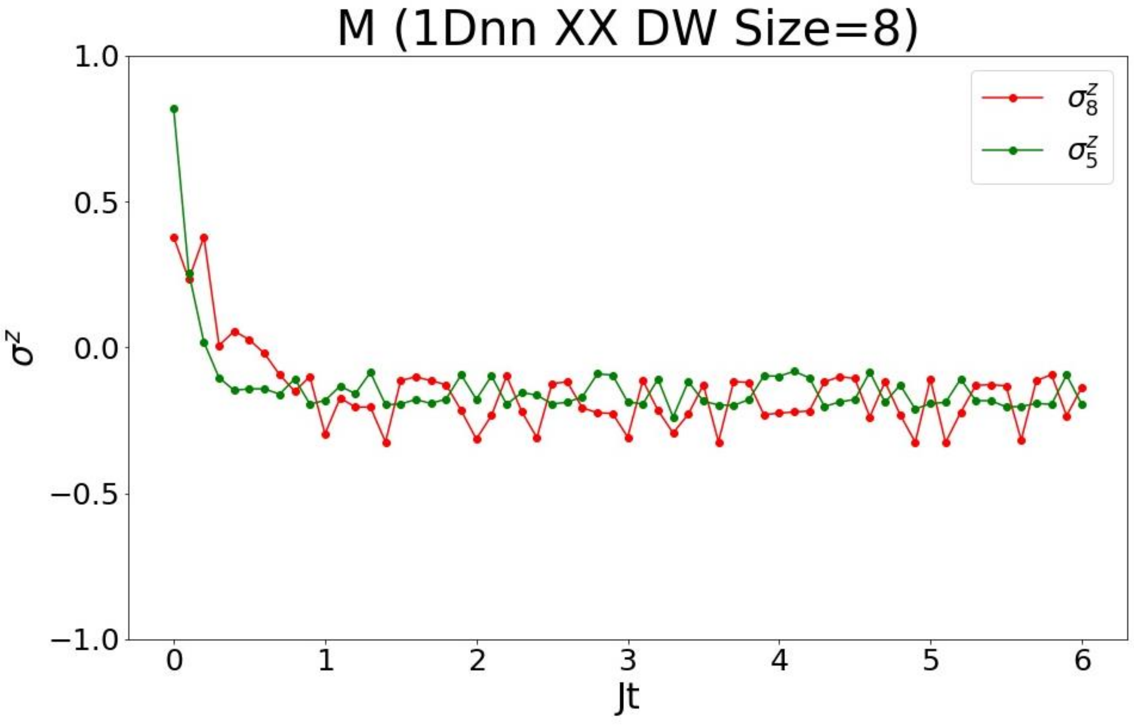


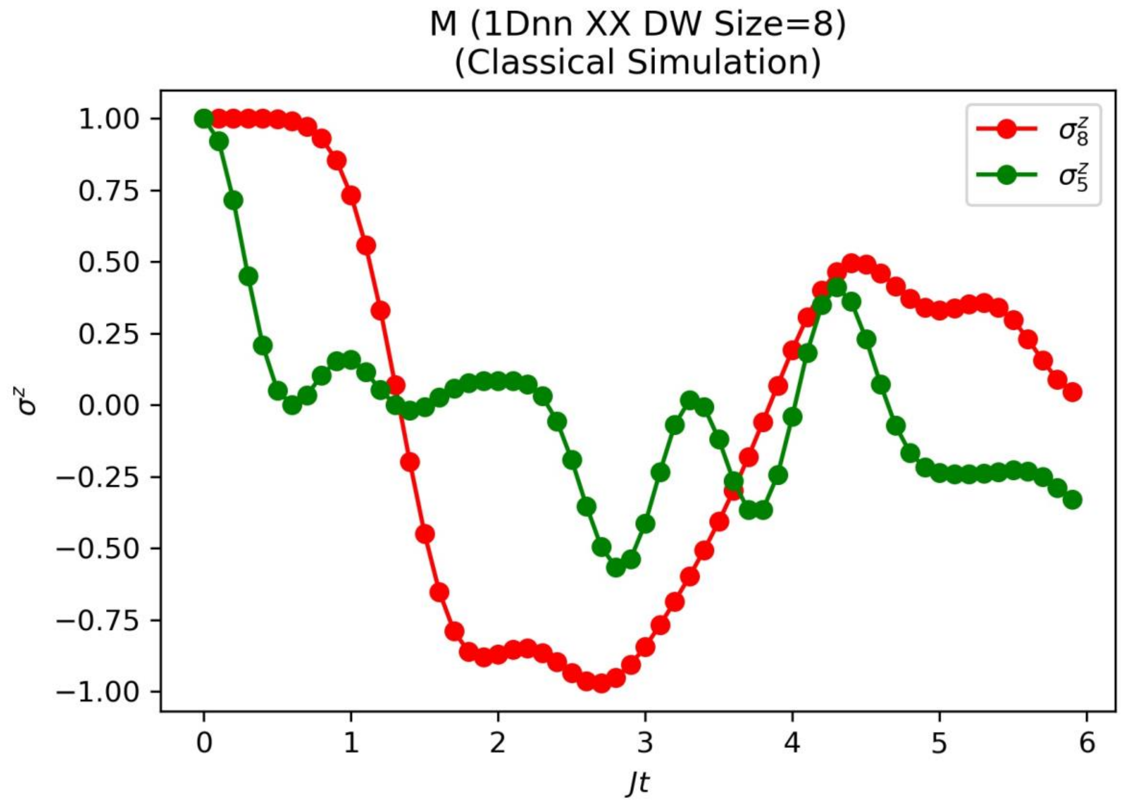
Correlation Fucntion





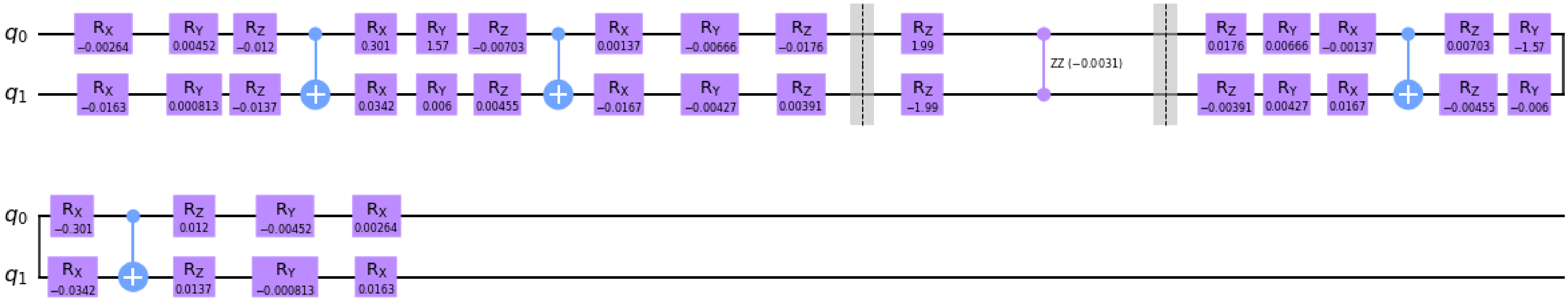
Limitations of Noisy Quantum Backend



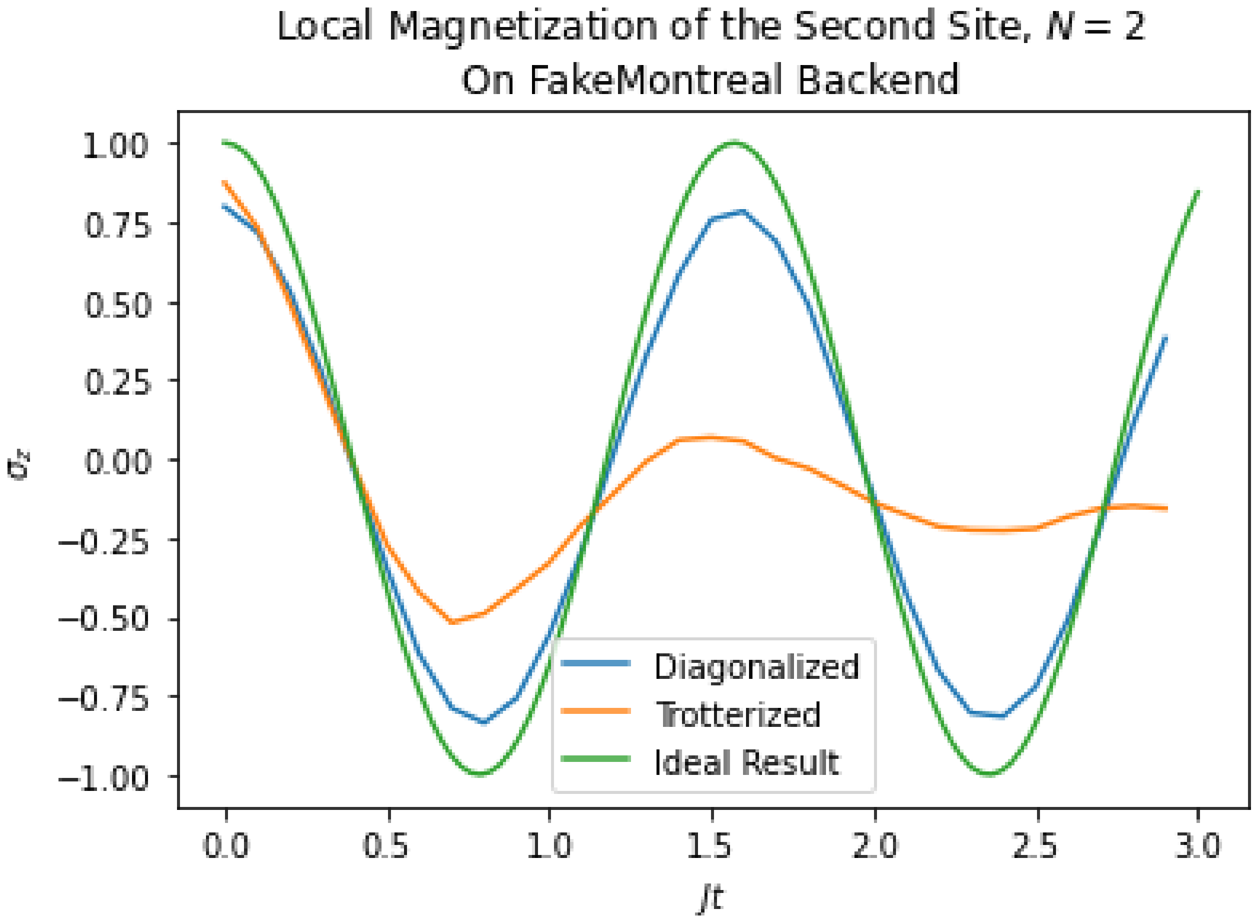


Solution: Variational Fast Forwarding

$$U(dt) \longrightarrow W(\boldsymbol{\theta})A(dt\boldsymbol{\gamma})W^{\dagger}(\boldsymbol{\theta})$$



Simulation of the Diagonalized Circuit



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