IFC - álgebra linear - 2023.2 - avaliação 3 de 3 – 01/12/23 – Prof. Cechetto – 2ª CHAMADA

NOME: [redacted]

As questões devem ser feitas da forma completa, com seus raciocínios, não colocar somente resposta final.

Simplificar frações e racionalizar raízes, evitar ao máximo o uso de aproximações.

Pode ser usada calculadora cientifica que não são calculadoras gráficas e/ou programáveis.

Permitido o uso do caderno com qualquer informação.

Cada QUESTÃO vale 1,25.

---

1. Verifique se a relação $T: \mathbb{R}^3 \to \mathbb{R}^2$ definida por $T(x, y, z) = (3x + 2y\ ,\ x - 5y - z)$ é uma transformação linear.

~~0) $T(0,0,0) = (3.0+2.0,\ 0-5.0-0)$ ok~~

~~$Ta(u+v) = aT(u)+T(v)$~~

~~$Ta(x$~~

0

2. Seja a transformação linear $T: \mathbb{R}^3 \to \mathbb{R}^2$ tal que: $T(1,0,0) = (-2,8)$, $T(0,1,0) = \left(\frac{1}{2}, \frac{1}{3}\right)$
$T(0,0,1) = (-3,5)$. Calcule $T(6,-1,10)$

$(6,-1,10) = 6\cdot(1,0,0) - 1(0,1,0) + 10\cdot(0,0,1) =$

$T(6,-1,10) = 6T(1,0,0) - 1T(0,1,0) + 10T(0,0,1) =$

$T(6,-1,10) = 6\cdot(-2,8) - \left(\frac{1}{2}, \frac{1}{3}\right) + 10(-3,5) =$

$T(6,-1,10) = (-12,48) - \left(\frac{1}{2}, \frac{1}{3}\right) + (-30,50) =$

$T(6,-1,10) = \left(-12 - \frac{1}{2} - 30, 48 - \frac{1}{3} + 50\right)$

$T(6,-1,10) = \left(\frac{-24-1-60}{2}, \frac{144-1+150}{3}\right) =$

10

$T(6,-1,10) = \left(-\frac{85}{2}, \frac{293}{3}\right)$

$(x,y,z) =$

$T(6,-1,10) = 6T(1,0,0) - T(0,1,0) + 10T(0,0,1)$

$6(-2,8) - \left(\frac{1}{2}, \frac{1}{3}\right) + 10(-3,5)$

$\left(-12 - \frac{1}{2} - 30, 48 - \frac{1}{3} + 50\right)$

$\left(-\frac{85}{2}, \frac{293}{3}\right)$

3. Dado o operador limear: $T(x,y,z) = \left(-12x - \frac{1}{2}y - 3z\ , -15x + 2y + 3z\ , 16x + 2y + z\right)$

a) Calcule o Núcleo

b) Calcule a imagem

c) determine as dimensões do nucleo e da imagem

a) $\left(\begin{array}{ccc|c} -12 & -\frac{1}{2} & -3 & 0 \\ -15 & 2 & 3 & 0 \\ 16 & 2 & 1 & 0 \end{array}\right) \xrightarrow{C_1 \leftrightarrow C_3}$ $\begin{array}{ccc} z & y & x \end{array}$ $\left(\begin{array}{ccc|c} -3 & -\frac{1}{2} & -12 & 0 \\ 3 & 2 & -15 & 0 \\ 1 & 2 & 16 & 0 \end{array}\right) \xrightarrow{L_1 = L_1 \cdot (-\frac{1}{3})}$

$\left(\begin{array}{ccc|c} 1 & \frac{1}{6} & 4 & 0 \\ -3 & 2 & -15 & 0 \\ 1 & 2 & 16 & 0 \end{array}\right)$ $\begin{array}{ccc} z & y & x \end{array}$ $\xrightarrow[L_3 = L_3 \cdot (\frac{1}{2})]{L_2 = L_2 \cdot \frac{1}{3}}$ $\left(\begin{array}{ccc|c} 1 & \frac{1}{6} & 4 & 0 \\ 1 & \frac{2}{3} & -5 & 0 \\ \frac{1}{2} & 1 & 8 & 0 \end{array}\right) \xrightarrow{L_2 = L_2 - L_1}$

$\left(\begin{array}{ccc|c} 1 & \frac{1}{6} & 4 & 0 \\ 0 & -\frac{1}{2} & -9 & 0 \\ \frac{1}{2} & 1 & 8 & 0 \end{array}\right) \xrightarrow[L_3 = L_3 \cdot 2]{L_2 = L_2 \cdot (-2)}$ $\left(\begin{array}{ccc|c} 1 & \frac{1}{6} & 4 & 0 \\ 0 & 1 & 18 & 0 \\ 1 & 2 & 16 & 0 \end{array}\right) \xrightarrow{L_3 = L_3 - L_1}$

$\left(\begin{array}{ccc|c} 1 & \frac{1}{6} & 4 & 0 \\ 0 & 1 & 18 & 0 \\ 0 & -\frac{11}{6} & 14 & 0 \end{array}\right) \xrightarrow{L_3 = L_3 + (\frac{11}{6}) L_2}$ $\left(\begin{array}{ccc|c} 1 & \frac{1}{6} & 4 & 0 \\ 0 & 1 & 18 & 0 \\ 0 & 0 & \frac{44}{3} & 0 \end{array}\right) \longrightarrow$ $L_3 =$

$\frac{44}{3}x = 0$

$\boxed{x = 0}$

$18x + y = 0$

$0 + y = 0$

$y = 0$

$z + \frac{1}{6}y + 4x = 0$

$z = 0$

o núcleo contém apenas o vetor nulo

$\dim(N(T)) = 0$

4. Dados os operadores lineares:
$T_1(x,y,z) = (-x\,, 2y - z\,, 16x + 2y)$ e $T_2(x,y,z) = (2x - 5y\;\,, -z\,, x + 2y)$
Calcule a composta: $\quad T_1 o T_2$

$$\begin{pmatrix} -1 & 0 & 0 \\ 0 & 2 & -1 \\ 16 & 2 & 0 \end{pmatrix} \cdot \begin{pmatrix} 2 & -5 & 0 \\ 0 & 0 & -1 \\ 1 & 2 & 0 \end{pmatrix} = \begin{pmatrix} -2 & 5 & 0 \\ 16 & -2 & -2 \\ 32 & 80 & -2 \end{pmatrix}$$

10

…ação da 3 => b)

$$\left(\begin{array}{ccc|c} -\frac{1}{2} & -3 & & a \\ 2 & 3 & & b \\ 2 & 1 & & c \end{array}\right) \xrightarrow{C_1 \Leftrightarrow C_3} \begin{array}{ccc} z & y & x \end{array} \left(\begin{array}{ccc|c} 3 & -\frac{1}{2} & -12 & a \\ 3 & 2 & -15 & b \\ 1 & 2 & 16 & c \end{array}\right) \xrightarrow[L_2 = L_2 \cdot \left(\frac{1}{3}\right)]{L_1 = L_1 \cdot \left(\frac{1}{3}\right)} \begin{pmatrix} 1 & \frac{1}{6} & 4 \\ 1 & \frac{2}{3} & -5 \\ 1 & 2 & 16 \end{pmatrix}$$

$$\xrightarrow{L_2 - L_1} \left(\begin{array}{ccc|c} 1 & \frac{1}{6} & 4 & -\frac{a}{3} \\ 0 & -\frac{1}{2} & -9 & \frac{b+a}{3} \\ 1 & 2 & 16 & c \end{array}\right) \xrightarrow[L_3 = L_3 - L_1]{L_2 = L_2 \cdot (-2)} \left(\begin{array}{ccc|c} 1 & \frac{1}{6} & 4 & -\frac{a}{3} \\ 0 & 1 & 18 & \frac{-2a-2b}{3} \\ 0 & \frac{11}{6} & 12 & c + \frac{a}{3} \end{array}\right) \xrightarrow{L_3 = \frac{11}{6} \cdot L_2}$$

$$\left(\begin{array}{ccc|c} 1 & \frac{1}{6} & 4 & -\frac{a}{3} \\ 0 & 1 & 18 & \frac{-2a-2b}{3} \\ 0 & 0 & 33 & \frac{-33c + 11a}{18} \end{array}\right) \xrightarrow{L_3 = L_3 \cdot \left(\frac{1}{33}\right)} \left(\begin{array}{ccc|c} 1 & \frac{1}{6} & 4 & -\frac{a}{3} \\ 0 & 1 & 18 & -2a-2b \\ 0 & 0 & 1 & \frac{33c + 11a}{594} \end{array}\right)$$

$x = \dfrac{33c + 11a}{594}$

Use transformação inear para rotacionar por um ângulo de $\frac{\pi}{3}$ o vetor $(4\sqrt{3},\ 2\sqrt{3},\ 8\sqrt{3})$ no sentido anti-horário em torno do eixo z.

$$\begin{pmatrix} \cos\frac{\pi}{3} & -\operatorname{sen}\frac{\pi}{3} & 0 \\ \operatorname{sen}\frac{\pi}{3} & \cos\frac{\pi}{3} & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 4\sqrt{3} \\ 2\sqrt{3} \\ 8\sqrt{3} \end{pmatrix} =$$

$$\begin{pmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} & 0 \\ \frac{\sqrt{3}}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 4\sqrt{3} \\ 2\sqrt{3} \\ 8\sqrt{3} \end{pmatrix} = \begin{pmatrix} \frac{1}{2}\cdot(4\sqrt{3}) - \frac{\sqrt{3}}{2}\cdot 2\sqrt{3} \\ \frac{\sqrt{3}}{2}\cdot(4\sqrt{3}) + \frac{1}{2}\cdot 2\sqrt{3} \\ 8\sqrt{3} \end{pmatrix}$$

10

$$\begin{pmatrix} \frac{4\sqrt{3}}{2} - 3 \\ 6 + \sqrt{3} \\ 8\sqrt{3} \end{pmatrix} =$$

Continuação da 6:

$$\left(\begin{array}{ccc|ccc} 1 & -2 & 0 & 1 & 0 & 0 \\ 0 & -1 & -\frac{1}{2} & 0 & 1 & 0 \\ 0 & 0 & 1 & \frac{8}{3} & 0 & \frac{4}{3} \end{array}\right) \xrightarrow{L_2 : L_2\cdot(-1)} \left(\begin{array}{ccc|ccc} 1 & -2 & 0 & 1 & 0 & 0 \\ 0 & +1 & \frac{1}{2} & 0 & -1 & 0 \\ 0 & 0 & 1 & \frac{8}{3} & 0 & \frac{4}{3} \end{array}\right) \xrightarrow{L_2 = L_2 - \frac{1}{2}}$$

$$\left(\begin{array}{ccc|ccc} 1 & -2 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & -\frac{1}{2} & 0 \\ 0 & 0 & 1 & \frac{8}{3} & 0 & \frac{4}{3} \end{array}\right) \xrightarrow{L_1 = L_1 + 2\cdot L_2} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & -2 & 0 \\ 0 & 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & \frac{8}{3} & 0 & \frac{4}{3} \end{array}\right)$$

$$T^{-1}(x, y, z) = \left(x - 2y,\ -y,\ \frac{8}{3}x + \frac{4}{3}z\right)$$

6. Verifique se o operador linear $T(x,y,z) = \left(x - 2y\ ,\ y - \frac{z}{2}\ , x - y + \frac{z}{4}\right)$ é um isomorfismo, se for, calcule o operador inverso T^{-1} (não necessariamente nesta ordem).

$$[T] = \begin{pmatrix} 1 & -2 & 0 \\ 0 & 1 & -\frac{1}{2} \\ 1 & -1 & \frac{1}{4} \end{pmatrix} \Rightarrow \left(\begin{array}{ccc|c|c} 1 & -2 & 0 & 0 & a \\ 0 & 1 & -\frac{1}{2} & 0 & b \\ 1 & -1 & \frac{1}{4} & 0 & c \end{array}\right) \xrightarrow{L_3 = L_3 - L_1}$$

$$\left(\begin{array}{ccc|c|c} 1 & -2 & 0 & 0 & a \\ 0 & 1 & -\frac{1}{2} & 0 & b \\ 0 & -3 & \frac{3}{4} & 0 & c-a \end{array}\right) \xrightarrow[L_3 = L_3 + 3 \cdot L_1]{} \left(\begin{array}{ccc|c|c} 1 & -2 & 0 & 0 & a \\ 0 & 1 & -\frac{1}{2} & 0 & b \\ 0 & 0 & \frac{3}{4} & 0 & \frac{3}{4} + 3(c-a) \end{array}\right)$$

$$L_3\left(\frac{4}{3}\right) \rightarrow \left(\begin{array}{ccc|c|c} 1 & -2 & 0 & 0 & a \\ 0 & 1 & -\frac{1}{2} & 0 & b \\ 0 & 0 & 1 & 0 & 14 \cdot (c-a) \end{array}\right)$$

x y 3

1,2

Logo: $N(T) \cdot \{(0,0,0)\}$

$\dim N(T) = 0$

$\text{Dim}|T(N(T))| = 0$

$\text{Im}(T) = R^3$

onde:

$\dim(N(T)) + \dim(\text{Im}(T)) = \dim(V)$

$0 + 3 = 3$

$D(V) = D(W)$ e $N(T) = \vec{0}$

$\Downarrow$

Bijetora

$$\left(\begin{array}{cc|ccc} 2 & 0 & 1 & 0 & 0 \\ -\frac{1}{2} & 0 & 1 & 0 & \\ 1 & \frac{1}{4} & 0 & 0 & 1 \end{array}\right) \xrightarrow{L_3 = L_3 - L_1}$$

$$\left(\begin{array}{cc|ccc} 2 & 0 & 1 & 0 & 0 \\ 1 & -\frac{1}{2} & 0 & 1 & 0 \\ 3 & \frac{3}{4} & -1 & 0 & 1 \end{array}\right) \xrightarrow{L_3 = L_3 + 3 \cdot L_1}$$

$$\left(\begin{array}{c|ccc} 0 & 1 & 0 & 0 \\ -\frac{1}{2} & 0 & 1 & 0 \\ \frac{3}{4} & 2 & 0 & 1 \end{array}\right) \xrightarrow{L_3 = L_3 \cdot \frac{4}{3}} \left(\begin{array}{ccc|ccc} 1 & -2 & 0 & 1 & 0 & 0 \\ 0 & 1 & -\frac{1}{2} & 0 & 1 & 0 \\ 0 & 0 & 1 & \frac{8}{3} & 0 & \frac{4}{3} \end{array}\right)$$

[illegible] verso da folha

Determine os autovalores e autovetores associados a:
$T(x,y,z) = (-5x - y + z\,, -y + \frac{1}{2}z\,, 3z\,)$, se existirem.

$$[T] = \begin{pmatrix} -5 & -1 & 1 \\ 0 & -1 & \frac{1}{2} \\ 0 & 0 & 3 \end{pmatrix} \Rightarrow A - \lambda I = \begin{pmatrix} -5-\lambda & -1 & 1 \\ 0 & -1-\lambda & \frac{1}{2} \\ 0 & 0 & 3-\lambda \end{pmatrix}$$

$$\det(A - \lambda I) = 0 \Rightarrow \begin{bmatrix} -5-\lambda & -1 & 1 \\ 0 & -1-\lambda & \frac{1}{2} \\ 0 & 0 & 3-\lambda \end{bmatrix}$$

$(-5-\lambda)\cdot(-1-\lambda)\cdot(3-\lambda)$

$-5-\lambda = 0$
$-\lambda = 5 \cdot (-1)$
$\boxed{\lambda = -5}$ λ_1

$-1-\lambda = 0$
$-\lambda = 1 \cdot (-1)$
$\boxed{\lambda = -1}$ λ_2

$3-\lambda = 0$
$-\lambda = -3$
$\boxed{\lambda = +3}$ λ_3

$P\lambda_1 = -5$

$$\begin{pmatrix} 0 & -1 & 1 \\ 0 & -6 & \frac{1}{2} \\ 0 & 0 & -2 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{cases} -y + z = 0 \\ -6y + \frac{1}{2}z = 0 \\ -2z = 0 \Rightarrow z = 0 \end{cases}$$

$-5y = 0 \Rightarrow y = 0 \Rightarrow y = 0$

$z = y$

$V\lambda_1 = (0, 0, 1)$

$\lambda_1 = (z, z, z) = z(1, 1$

$P\lambda_2 = -1$

$$\begin{pmatrix} -6 & -1 & 1 \\ 0 & -1 & \frac{1}{2} \\ 0 & 0 & 2 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{cases} -6x - y + z = 0 \\ -y + \frac{1}{2}z = 0 \\ 2z = 0 \Rightarrow \boxed{z = 0} \end{cases}$$

$\boxed{-y = 0}$

8. Verifique se em $T(x, y, z) = (x - y\,, x + 2y + z\,, x - 7y + 4z)$ o vetor $u = (1\,, 2\,, 8)$ é autovetor associado a transformação T. Em caso afirmativo, calcule o autovalor associado.