Giovani Zarella da Maia

2) $(x+3)y'' + (x+2)y' - y = 0$

$y_1 = x^2$ , $y_1' = 2x$ , $y'' = 2$

$(x+3)2 + (x+2)2x - x^2 = 0$
$2x + 6 + 2x^2 + 4x - x^2 = 0$
$x^2 + 6x + 6 = 0$

∴ Resultado $\neq 0$
não é solução

$y_3 = e^{-x}$ , $y' = -e^{-x}$ , $y = e^{-x}$
$(x+3)y'' + (x+2)y' - y = 0$

$(x+3)e^{-x} + (x+2)\cdot(-e^{-x}) - e^{-x} = 0$

$\cancel{xe^{-x}} + 3e^{-x} - \cancel{xe^{-x}} - 2e^{-x} - e^{-x} = 0$

$3e^{-x} - 3e^{-x} = 0$

∴ $y_3 = e^{-x}$ é solução pois resposta é igual a zero

3) a) $y = f(x)$, no plano $xy$, passa pelo ponto $(9,4)$ e (coef. angular da reta tangente) $3\sqrt{x}$ (derivada)

Queremos uma $f(x)$ que:

- Passe no ponto $(9,4)$
- E possui derivada igual $f'(x) = 3\sqrt{x}$

p/ achar $f(x)$, integramos a derivada

$f(x) = 3\int x^{\frac{1}{2}}\, dx \Rightarrow f(x) = x^{3/2} + K$

determinar o valor de $K$, usando $(9,4)$

$\Rightarrow 4 = 9^{3/2} + K \Rightarrow 4 = 27 + K \Rightarrow K = -23$

$$\boxed{f(x) = x^{\frac{3}{2}} - 23}$$

b) determine a $f(x)$, cuja segunda derivada é $y'' = f''(x) = 6x$, e ponto $(0,1)$

$\int 6x\, dx = 6\int x\, dx = 3x^2 + K_1$

$f'(0) = 3\cdot(0)^2 + K_1 \Rightarrow K_1 = 0$

$y' = 3x$

$\int 3x^2\, dx = 3\int x^2\, dx = x^3 + K_2$

$f(0) = 1 \Rightarrow 1 = 0 + K_2 \Rightarrow K_2 = 1$

$f(x) = x^3 + 1$

c) $S'(t)$ = velocidade instantanea no temp T

formulas:

velocidade $v(t) = v_0 + a \cdot t$ (velocidade inicial; aceleração; instante tempo)

Altura $s(t) = s_0 + v_0 \cdot t + \frac{1}{2}a \cdot t^2$

4)

b) $\frac{dy}{dx} = (x+1)^2$

$dy = (x+1)^2\, dx$

$\int dy = \int (x+1)^2\, dx$

$y = \frac{1}{3}(x+1)^3$

I $= \int (x+1)^2\, dx$ $\quad u = x+1$, $du = dx$

$= \int u^2\, du$

$= \frac{u^3}{3} + K$

$= \frac{1}{3}(x+1)^3$

C) $dx + e^{3x}\, dy = 0$

$\frac{dx}{e^{3x}} + \frac{e^{3x}\, dy}{e^{3x}} = 0 \iff e^{-3x}\, dx + dy = 0$

$\int dy = -\int e^{-3x}\, dx$

$y = \frac{1}{3}\int e^{u}$

$y = \frac{1}{3}e^{u}$

$y = \frac{1}{3}e^{-3x} + K$

f) $e^{x}\frac{dy}{dx} = 2x$

$\frac{dy}{dx} = \frac{2x}{e^{x}} \iff e^{x}\, dy = 2x\, dx \quad \frac{e^{x}\, dy}{e^{x}} = \frac{2x}{e^{x}}\, dx$

$dy = 2x\, e^{-x}\, dx \iff \int dy = 2\int e^{-x} x\, dx$

$u = x$, $du = dx$ $\quad dv = e^{-x}\, dx$, $v = -e^{-x}$ $\quad -xe^{-x} + \int e^{-x}\, dx$ $\quad -xe^{-x} - e^{-x} + K$

$y = -2x e^{-x} + 2e^{-x} + K$

G) $xy' = 4y$

$y' = \frac{4y}{x} \Rightarrow \frac{dy}{dx} = \frac{4}{x}\cdot y$

$\frac{dy}{y} = \frac{4}{x}\, dx \Rightarrow \int \frac{1}{y}\, dy = 4\int \frac{1}{x}\, dx$

$\ln|y| = 4\ln|x| + K$

$\ln|y| = \ln|x|^4 + K \Rightarrow e^{\ln|y|} = e^{\ln|x|^4 + K}$

$y = e^{\ln|x|^4}\cdot e^{K} \Rightarrow y = x^4 \cdot \underbrace{e^{K}}_{K}$

$y = x^4 C$

K) $\frac{dx}{dy} = \frac{x^2y^2}{1+x}$

$(1+x)\, dx = (x^2y^2)\, dy \Rightarrow \frac{1+x}{x^2}\, dx = y^2\, dy$

$\int \frac{1}{x^2}\, dx + \int \frac{x}{x^2}\, dx = \int y^2\, dy \Rightarrow -\frac{1}{x} + \ln|x| + K = \frac{y^3}{3}$

$3\left(-\frac{1}{x} + \ln|x| + K\right) = y^3 \Rightarrow x\cdot\left(-\frac{3}{x} + 3\ln|x| + 3K\right) = y^3 x$

$-3 + 3x\ln|x| + 3xK = xy^3 \Rightarrow xy^3 = -3 + 3x\ln|x| + Cx$

5) a) $\begin{cases} (e^{-y} + 1)\,\text{sen}\, x\, dx = (1 + \cos x)\, dy \\ y(0) = 0 \end{cases}$

$(e^{-y} + 1)\,\text{sen}\, x\, dx = (1 + \cos x)\, dy$

I $\underbrace{\frac{\text{sen}\, x}{1 + \cos x}\cdot dx}$ $= \frac{1}{e^{-y} + 1}\, dy$ $\quad$ I $u = 1 + \cos x$, $du = -\text{sen}\, x\, dx$

$-\frac{1}{u}\, du = \underbrace{\int \frac{e^{y}}{1 + e^{y}}\, dy}_{II}$ $\quad$ II $u = 1 + e^{y}$, $du = e^{y}\, dy$

$-\int \frac{1}{u}\, du = \int \frac{1}{u}\, du$

$-\ln|1 + \cos x| = \ln|1 + e^{y}| + K$

$\ln|1 + e^{y}| + K + \ln|1 + \cos x| = 0$

$\ln|(1 + e^{y})\cdot(1 + \cos x)| = K$

$(1 + e^{y})\cdot(1 + \cos x) = e^{K}$

$(1 + e^{y})(1 + \cos x) = K$

$x = 0$ e $y = 0$

$(1 + e^{0})(1 + \cos 0) = K$

$2 \cdot 2 = K$

$K = 4$

d) $\begin{cases} \frac{dy}{dt} + Ty = y \\ y(1) = 3 \end{cases}$

$\frac{dy}{dt} + Ty - y = 0$

$\frac{dy}{dt} + y(T - 1) = 0$

$y = C\cdot e^{-\int P(x)\, dx}$

$y = C\cdot e^{-\int t - 1\, dt}$

$y = C\cdot e^{-\int t - 1\, dt}$

$y = C\cdot e^{-\frac{t^2}{2} + t}$

$y = C\cdot e^{-\frac{t^2 + 2t}{2}}$

$y = \frac{3}{e^{\frac{1}{2}}}\cdot e^{-\frac{t^2}{2} + t}$

$y = 3\cdot e^{-\frac{1}{2}}\cdot e^{-\frac{t^2}{2} + t}$

$y = 3\cdot e^{-\frac{t^2}{2} + t - \frac{1}{2}}$

6) a) $y' - \frac{4}{x}y = -\frac{2}{x^3} \quad (*)$

$\mu(x) = e^{\int p(x)\,dx} = e^{\int -\frac{4}{x}\,dx} = e^{-4\int \frac{1}{x}\,dx} = e^{-4\ln x} = e^{\ln x^{-4}} = x^{-4}$

multiplicar $(*)$ por $\mu(x)$:

$x^{-4}\cdot\frac{dy}{dx} - x^{-4}\cdot\frac{4}{x}\cdot y = -\frac{2}{x^3}\cdot\frac{1}{x^4}$

$\frac{1}{x^4}\cdot\frac{dy}{dx} - \frac{1}{x^4}\cdot\frac{4}{x}\cdot y = -\frac{2}{x^7}$

$x^{-4}\left(\frac{dy}{dx}\right) - \frac{4}{x^5}\cdot y = -\frac{2}{x^7}$

$\frac{dy}{dx}\left(x^{-4}\cdot y\right) = -\frac{2}{x^7}$

Integrar dois lados:

$\int \frac{dy}{dx}\left(x^{-4}\cdot y\right)dx = -2\int \frac{1}{x^7}\,dx$

$x^{-4}\cdot y = \frac{1}{3x^6} + K$

$y = \frac{1}{3x^{\cancel{6}}}\cdot x^{\cancel{4}} + x^4\cdot K$

$y = \frac{1}{3x^2} + x^4\cdot K$

b) $y' - \frac{1}{x}y = -x \quad (*)$

$\mu(x): e^{\int -\frac{1}{x}\,dx} = e^{-\ln|x|} = e^{\ln|x|^{-1}} = \frac{1}{|x|}$

multiplicar $(*)$ p/ $\mu(x)$:

$\frac{1}{|x|}\cdot y' - \frac{1}{x}\cdot\frac{1}{|x|}\cdot y = -\frac{1}{|x|}\cdot x$

$\frac{1}{x}\cdot\frac{dy}{dx} - \frac{1}{x^2}\cdot y = -1$

$\frac{dy}{dx}\left(\frac{1}{x}\cdot y\right) = -1$

Integrar 2 lados

$\int \frac{dy}{dx}\left(\frac{1}{x}\cdot y\right)dx = \int -1\,dx$

$\frac{1}{x}\cdot y = -x + k$

$y = -x^2 + xK$

c) $y' - \frac{4}{x}y = x^5\cdot e^x \quad (*)$

$\mu(x) = e^{\int -\frac{4}{x}\,dx} = x^{-4}$

Multiplicar $(*)$ p/ $\mu(x)$

$x^{-4}\cdot y' - \frac{4}{x}\cdot\frac{1}{x^4}\,y = \frac{1}{x^4}\cdot x^5\cdot e^x$

$x^{-4}\cdot\frac{dy}{dx} - \frac{4}{x^5}y = x\cdot e^x$

$\frac{dy}{dx}\left(x^{-4}\cdot y\right) = x\cdot e^x$

Integrar

$\int \frac{dy}{dx}\left(x^{-4}\cdot y\right)dx = \int x\cdot e^x\,dx$

$x^{-4}\cdot y = e^x x - e^x + k$

$y = x^5 e^x - x^4 e^x + x^4 K$

d) $\frac{dy}{dx} + tg(x)\,y = \cos^2 x \qquad -\frac{\pi}{2} < x < \frac{\pi}{2}$

$\mu(x) = e^{\int tg(x)\,dx} = e^{-\ln|\cos x|} = e^{\ln|\cos x|^{-1}} = \cos^{-1} x$

multiplicar por $\mu(x)$

$\cos^{-1}(x)\,\frac{dy}{dx} + \cos^{-1}(x)\cdot tgx\cdot y = \cos^2 x\cdot \operatorname{arc\,cos}(x)$

$\frac{dy}{dx}\left(\operatorname{arccos}(x)\cdot y\right) = \cos^2 x\cdot \operatorname{arc\,cos} x$

E) $x\cdot\frac{dy}{dx} - y = 2x\,\ln(x) \qquad x > 0 \Rightarrow \frac{dy}{dx} - xy = 2\ln x$

$\mu(x) = e^{\int -x\,dx} = e^{-\frac{x^2}{2}}$

$e^{-\frac{x^2}{2}}\,\frac{dy}{dx} - e^{-\frac{x^2}{2}}\,xy = 2e^{-\frac{x^2}{2}}\,\ln x$

$\frac{dy}{dx}\left(e^{-\frac{x^2}{2}}\cdot y\right) = 2e^{-\frac{x^2}{2}}\,\ln|x|$

$\int \frac{dy}{dx}\left(e^{-\frac{x^2}{2}}\cdot y\right)dx = 2\int e^{-\frac{x^2}{2}}\cdot \ln|x|\,dx$

$e^{-\frac{x^2}{2}}\cdot y = 2$

7) a) $\begin{cases} y' + (1-2x)y = xe^{-x} \\ y(0) = 2 \end{cases}$

$y' + (1-2x)y = xe^{-x} \longleftrightarrow$

$\frac{dy}{dx} + (1-2x)y = xe^{-x}$

$\mu(x) = e^{\int(1-2x)\,dx} = e^{x-x^2}$

$e^{x-x^2}\frac{dy}{dx} + e^{x-x^2}\cdot(1-2x)y = xe^{-x}\cdot e^{x-x^2}$

$e^{x-x^2}\frac{dy}{dx} + e^{x-x^2}\cdot(1-2x)y = x\cdot e^{-x^2}$

$\frac{dy}{dx}\left(e^{x-x^2}\cdot y\right) = x\cdot e^{-x^2}$

$\int \frac{dy}{dx}\left(e^{x-x^2}\cdot y\right) = \int x\cdot e^{-x^2}\,dx$

$e^{x-x^2}\cdot y = -\frac{1}{2}e^{-x^2} + K$

$y = \frac{1}{e^{x-x^2}}\cdot\left(-\frac{1}{2}\cdot e^{-x^2} + K\right)$

$y = -\frac{1}{2}e^{-x} + e^{x^2-x}\cdot K$

$x = 0 \qquad y = 2$

$K = \frac{5}{2}$

9) $\begin{cases} t\cdot\frac{dy}{dt} + 2y = t^2 \\ y(2) = 3 \end{cases} \qquad \div t \longleftrightarrow \frac{dy}{dt} + \frac{2}{T}y = t$

$\mu(x) = e^{\int \frac{2}{T}\,dt} = e^{\ln T^2} = T^2$

$T^2\,\frac{dy}{dt} + \frac{2}{T}\cdot T^2\cdot y = T\cdot T^2$

$T^2\,\frac{dy}{dx} + 2Ty = T^3$

$\frac{dy}{dx}\left(T^2\cdot y\right) = T^3$

$\int \frac{dy}{dx}\left(T^2\cdot y\right) = \int T^3\,dt$

$T^2\cdot y = \frac{T^4}{4} + k$

$y = \frac{\frac{T^4}{4}}{T^2} + \frac{k}{T^2}$

$y = \frac{T^2}{4} + k\cdot T^{-2}$

$K = 8$

$y = \frac{T^2}{4} + \frac{3}{T^2}$

9) $\begin{cases} t\,\frac{dy}{dt} + 2y = t^2 \\ y(2) = 3 \end{cases}$

$\frac{dy}{dt} + \frac{2}{t}\,y = t$

$P(x) = \frac{2}{t}$

$\mu(x) = e^{\int \frac{2}{t}\,dx} = e^{2\ln|t| + c} = t^2 + c$

$\mu'(x) = 2t$

$t^2 y' + 2ty = t^3$

$\frac{d}{dt}\left(t^2 y\right) = t^3$

$t^2 y = \int t^3\,dt$

$t^2 y = \int t^3\,dt$

$t^2 y = \frac{t^4}{4} + c$

$y = \frac{t^2}{4} + \frac{c}{t^2}$

$y(2) = 3$

$\frac{4}{4} + \frac{c}{4} = 3$

$c = 8$

$y = \frac{t^2}{4} + \frac{8}{t^2}$

11) problema de Crescimento

Cresce proporcionalmente de acordo com o num de bacterias já presente, no tempo T, esse tipo de crescimento é dado pela fun exponencial:

$N(t) = N_0\cdot e^{K\cdot T}$

$N(t)$ : num bac no tempo T
$N_0$ : num inicial de bac (quando $T=0$)
$K$ : cons que represela taxa de crescimento

$N(3) = 400$ bacterias
$N(10) = 2000$ bacterias

Encontrar $N_0$.

$N(3) = N_0\cdot e^{3\cdot K} = 400$

$N(10) = N_0\cdot e^{10\cdot K} = 2000$

divir as eq Para eliminar $N_0$ e encontrar K

$\frac{\cancel{N_0}\cdot e^{10K}}{\cancel{N_0}\,e^{3K}} = \frac{2000}{400}$

$e^{10K-3K} = 5$

$e^{7K} = 5$

$7K = \ln 5$

$K = 0{,}2299$

$N_0\cdot e^{3\cdot 0{,}2299} = 400$

$N_0 = \frac{400}{e^{0{,}6897}}$

$N_0 = \frac{400}{1{,}99} \simeq 201$

12) $N(t) = N_0\cdot e^{K\cdot T}$

$N(t)$ : qtd restante no tempo t
$N_0$ : qtd inicial (1g) (quando $T=0$)
$K$ : const que represela taxa de decrescimento
$T$ : tempo que queremos encontrar

$\left\{ \underbrace{\frac{dN}{dt}}_{\text{taxa de variação de qtd do material ao longo do tempo}} = \underbrace{-K}_{\text{decrece por isso o menos}}\cdot N \right.$

(eq separavel)

meia hora $\left(\frac{t}{2}\right) = 3{,}3$ horas $\qquad N\left(\frac{t}{2}\right) = \frac{N_0}{2}$

$N\cdot\left(\frac{t}{2}\right) = N_0\cdot e^{-K\cdot\frac{t}{2}}$

$\frac{\cancel{N_0}}{2} = \cancel{N_0}\cdot e^{-K\cdot\frac{T}{2}}$

Determinar K

$\frac{1}{2} = e^{-K\cdot\frac{t}{2}}$

$\ln\left|\frac{1}{2}\right| = -K\cdot\frac{t}{2}$

$\ln|2|^{-1} = -K\cdot\frac{t}{2}$

$-\ln|2| = -K\,\frac{t}{2}$

$K = \frac{\ln|2|}{t/2}$

$\left(\frac{t}{2}\right) = 3{,}3$

$K = \frac{\ln|2|}{3{,}3} \simeq 0{,}21$

Determinar tempo p/ 90% desaparecer

se 90% desaparecer sobra 10%

$N(t) = 0{,}1\cdot N_0$

$0{,}1\cdot\cancel{N_0} = \cancel{N_0}\cdot e^{-K\cdot T}$

$0{,}1 = e^{-K\cdot T}$

$\ln 0{,}1 = -K\cdot T$

$-2{,}302 = -0{,}21\cdot t$

$t = \frac{2{,}302}{0{,}21} \simeq 10{,}96$ horas

15) $T(t)$ : temp termometro
$T_m$ : temp ambiente (5°C)

$T(1) = 20$
$T(5) = 10$

$T(t) = T_m + (T_0 - T_m)\, e^{-kt}$

$t = 1$
$20 = 5 + (T_0 - 5)\, e^{-k}$
$15 = (T_0 - 5)\, e^{-k}$

$t = 5$
$10 = 5 + (T_0 - 5)\, e^{-5k}$
$5 = (T_0 - 5)\, e^{-kt}$

dividir as duas eq para cortar $(T_0 - 5)$

$\frac{15}{5} = \frac{\cancel{(T_0 - 5)} \cdot e^{-k}}{\cancel{(T_0 - 5)} \cdot e^{-5k}} \iff 3 = e^{-k-(-5k)}$

$3 = e^{4k}$

$\ln 3 = 4k \iff k = \frac{\ln(3)}{4} = \frac{1}{4}$

Substituir $K$, para encontrar $T_0$

$15 = (T_0 - 5)\, e^{-k}$
$15 = (T_0 - 5)\, e^{-\frac{\ln(3)}{4}}$
$15 = (T_0 - 5) \cdot 3^{-\frac{1}{4}}$

$(T_0 - 5) = 3^{1/4} \cdot 15$
$T_0 - 5 = 19,74$
$T_0 \simeq 24,74$

16) $L \cdot \frac{di}{dt} + Ri = E(T)$

$L = 0,5$
$R = 50$
$E(t) = 30$
$i(0) = 0$

$0,5 \cdot \frac{di}{dt} + 50 = 30$

A) $i(t)$ ?

$0,5 \cdot \frac{di}{dt} + 50 \cdot i = 30$

$\frac{di}{dt} + 100\, i = 60$

$\mu(x) = e^{\int 100\, dt} = e^{100t}$

$e^{100t} \cdot \frac{di}{dt} + 100\, i \cdot e^{100t} = 60 \cdot e^{100t}$

$\frac{di}{dt}\left(e^{100t} \cdot i\right) = 60 \cdot e^{100t}$

$\int \frac{di}{dt}\left(e^{100t} \cdot i\right) dt = \int 60 \cdot e^{100t}\, dt$

$e^{100t} \cdot i = 60 \cdot \frac{1}{100} \cdot e^{100t} + K$

$i(t) = \frac{3}{5} + K \cdot e^{-100t}$

$i(0) = \frac{3}{5} + K \Rightarrow K\ \frac{3}{5}$

$i(t) = \frac{3}{5} + \frac{3}{5} e^{-100t}$

b) $\lim_{T \to \infty} \left(\frac{3}{5} + \frac{3}{5} e^{-100t}\right) = \lim_{T \to \infty} \left(\frac{3}{5} + \frac{3}{5} \cdot \frac{1}{e^{\infty}}\right) = \frac{3}{5}$

18)

• $A(t)$ = qtd de sal no tanque no instante $t$

$\frac{dA}{dt}$ = entrada sal − saida sal

entrada do sal $= \frac{1G}{L} \cdot \frac{4L}{M} = 4\, g/m$

saida do sal $= \frac{A(t)}{200} \frac{g}{L} \cdot \frac{4L}{min} = \frac{4\, A(t)}{200} = \frac{A(t)}{50}$

$\frac{dA}{dt} = 4 - \frac{A(t)}{50}$

$\frac{dA}{dt} + \frac{1}{50} A(t) = 4$

$\mu = e^{\int \frac{1}{50} dt} = e^{t/50}$

$e^{t/50} \frac{dA}{dt} - e^{t/50} \cdot \frac{1}{50} A(t) = 4 e^{t/50}$

$\frac{dA}{dt}\left(e^{\frac{t}{50}} \cdot A(t)\right) = 4 e^{t/50}$

$\int \frac{dA}{dt}\left(e^{\frac{t}{50}} \cdot A(t)\right) dt = \int 4 e^{t/50}\, dt$

$e^{t/50} \cdot A(t) = 4 \cdot 50 \cdot e^{t/50} + K$

$e^{t/50} \cdot A(t) = 200 \cdot e^{t/50} + K$

$A(t) = 200 + K \cdot e^{-t/50}$

$A(0) = 200 + K = 30$
$K = -170$

$A(t) = 200 - 170 \cdot e^{-t/50}$

21) $\frac{dA}{dt}$ = entrada − saida

taxa de entrada $= 0,5\, kg/L \cdot 1\, L/m = 0,5\, kg/m$

taxa de saida $= \frac{A(t)}{V(t)} \cdot 3 = \frac{A(t)}{500 - 2t} \cdot 3$

$V(t) = 500 - 2t$

Eq diferencial

$\frac{dA}{dt} = 0,5 - \frac{3 \cdot A(t)}{500 - 2t} = 0$

$\frac{3 \cdot A(t)}{500 - 2t} = 0,5$

$3 \cdot A(t) = 0,5\, (500 - 2t)$

$A(t) = \frac{250 - t}{3}$

$V(t) = 500 - 2t = 250$
$2t = 250$
$t = 125$ min

23)

a) $\frac{dx}{dt} = -\frac{10\, x(t)}{100 + 10t}$

$\frac{dx}{dt} = -\frac{10\, dt}{100 + 10t}$

$\int \frac{dx}{dt} = -10 \int \frac{1}{100 + 10t}\, dt$

$\ln|x(t)| = -\ln(100 - 10t) + c$

$\ln(100) = -\ln(100) + c$

$x(t) = 100\, e^{-1/100\, t}$

b) $\frac{dy}{dt} = 10 \cdot \frac{x(t)}{100 + 10t}$

$\frac{dy}{dt} = 10 \cdot \frac{100\, e^{-1/100\, t}}{100 + 10t}$

$y(t) = \int 10 \cdot \frac{100\, e^{-1/100\, t}}{100 + 10t}$

$y(t) = 10t \cdot e^{-\frac{1}{10} t}$

C) $\lim_{t \to \infty} \left(10 \cdot \frac{100\, e^{-1/100\, t}}{100 + 10t}\right) = 100$

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24)

a) $4y'' + y' = 0$

$y'' = r^2$
$y' = r$

$4r^2 + r = 0$
$r(4r + 1) = 0$

$r_1 = 0$
$r_2 = -1/4$

$y(t) = K_1 \cdot e^{r_1 \cdot t} + K_2\, e^{r_2 t}$

$y(t) = K_1 + K_2 \cdot e^{-t/4}$

b) $2y'' - 5y' = 0$

$2r^2 - 5r = 0$

$r(2r - 5) = 0$

$r_1 = 0$
$r_2 = 5/2$

$y(x) = K_1 + K_2\, e^{5x/2}$

G) $y'' - y' - 6y = 0$

$r^2 - r - 6 = 0$

$x_1 + x_2 = 1$
$x_1 \cdot x_2 = -6$

$x_1 = 3$
$x_2 = -2$

$y(x) = K e^{3x} + e^{-2x}$

h) $y'' - 3y' + 2y = 0$

$r^2 - 3r + 2 = 0$

$x_1 = 1 \quad x_2 = 2$

$y(x) = K e^{x} + K_2 \cdot e^{2x}$

25) a) $y'' - 4y = 0$
$y(0) = 2$
$y'(0)\ 4$

EDO 2° ordem linear e homogenea

$r^2 - 4r = 0$

$r_1 = 2$
$r_2 = -2$

$y(x) = K_1\, e^{2x} + K_2\, e^{-2x}$

aplicar $y(0) = 2$

$y(0) = K_1 \cdot e^{2 \cdot 0} + K_2 \cdot e^{-2 \cdot 0}$
$= K_1 + K_2$

aplicar $y'(0) = 4$

$y'(x) = 2K_1 \cdot e^{2x} - 2K_2 \cdot e^{-2x}$

$y'(0) = 2 \cdot K_1 \cdot e^{2 \cdot 0} - 2 \cdot K_2^{-2 \cdot 0}$

$= 2K_1 - 2K_2 = 4$

$K_1 + K_2 = 2$
$2K_1 - 2K_2 = 4$

$K_1 = 2 - C_2$

$2(2 - K_2) - 2K_2 = 4$
$4 - 2K_2 - 2K_2 = 4$
$-4K_2 = 0$
$K_2 = 0$

$K_1 = 2 - 0 \Rightarrow K_1 = 2$

$y(x) = 2e^{2x}$

26)

c) $y'' - 10y' + 25y = 30x + 3$

resolver eq homogenea

$r^2 - 10r + 25 = 0$

$r_1 = 5 \qquad r_2 = 5$

$y_h(x) = e^{5x}(c_1 + c_2)$

Solução particular

$y_p(x) = Ax + B$

$y'_p(x) = A$

$y''_p(x) = 0$

$0 - 10A + 25(Ax + b) = 30x + 3$

$-10A + 25Ax + 25b = 30x + 3$

$(25A)x + (-10A + 25B) = 30x + 3$

$\begin{cases} 25A = 30 \\ -10A + 25B = 3 \end{cases}$

$A = \frac{30}{25} = \frac{6}{5}$

$-10\frac{6}{5} + 25B = 3$

$-12 + 25B = 3$

$25B = 15$

$B = \frac{3}{5}$

$y_p(x) = \frac{6}{5}x + \frac{3}{5}$

$y(x) = y_h(x) + y_p(x)$

$y(x) = (K_1 + K_2 \cdot x)e^{5x} + \frac{6}{5}x + \frac{3}{5}$

f) $4y'' - 4y' - 3y = \cos 2x$

$4r^2 - 4r - 3 = 0$

$r_1 = \frac{3}{2} \qquad r_2 = -\frac{1}{2}$

$y_h(x) = K_1 \cdot e^{3x/2} + K_2 \cdot e^{-1/2}$

solução particular

$y_p(x) = A\cos(2x) + B\,\text{sen}(2x)$

$y'_p(x) = -2A\,\text{sen}(2x) + 2B\cos(2x)$

$y''_p(x) = -4A\cos(2x) - 4B\,\text{sen}(2x)$

$4(-4A\cos(2x) - 4B\,\text{sen}(2x)) - 4(-2A\,\text{sen}(2x) + 2B\cos(2x)) - 3(A\cos(2x) + B\,\text{sen}(2x)) = \cos(2x)$

$-16A\cos(2x) - 16B\,\text{sen}(2x) + 8A\,\text{sen}(2x) - 8B\cos(2x) - 3A\cos(2x) - 3B\,\text{sen}(2x) = \cos(2x)$

$(-16A - 8B - 3A)\cdot\cos(2x) + (-16B + 8A - 3B)\,\text{sen}(2x) = \cos(2x)$

$(-19A - 8B)\cos(2x) + (8A - 19B)\,\text{sen}(2x) = \cos(2x)$

$\begin{cases} -19A - 8B = 1 \\ 8A - 19B = 0 \end{cases}$

$B = -\frac{8}{425} \qquad A = -\frac{19}{425}$

$y(x) = K_1 \cdot e^{\frac{3x}{2}} + K_2 \cdot e^{-\frac{x}{2}} - \frac{19}{425}\cos(2x) - \frac{8}{425}\,\text{sen}(2x)$

27) a) $y'' + y = \sec x$

resolver eq homogenea

$r^2 + 1 = 0$

$r_1 = 1 \qquad r_2 = -1$

$y_h(x) = K_1\cos(x) + K_2\,\text{sen}(x)$

$y_p(x) = u_1(x)\cdot\cos x + u_2(x)\,\text{sen}(x)$

$y'_p(x) = -u_1(x)\cdot\text{sen}(x) + u_2(x)\cos(x)$

$y''_p(x) = -u'_1(x)\cdot\text{sen}(x) - u_1(x)\cos(x) + u'_2(x)\cos(x) - u_2(x)\,\text{sen}(x)$

$-u'_1(x)\,\text{sen}(x) + u'_2(x)\cdot\cos(x) = \sec(x)$

$\begin{cases} u'_1(x)\cos(x) + u'_2(x)\,\text{sen}(x) = 0 \\ -u'_1(x)\,\text{sen}(x) + u'_2(x)\cdot\cos(x) = \sec(x) \end{cases}$

$u'_1(x) = -1 \qquad u'_2(x) = 1$

$y_p(x) = (-x)\cos(x) + (x)\,\text{sen}(x)$

$y(x) = K_1 - \cos(x) + K_2\,\text{sen}(x) - x\cdot\cos x + x\cdot\text{sen}\,x$