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$\operatorname{sen} a \cos b = \frac{1}{2}[\operatorname{sen}(a+b) + \operatorname{sen}(a-b)]$

$\cos a \cos b = \frac{1}{2}[\cos(a+b) + \cos(a-b)]$

$\operatorname{sen} a \operatorname{sen} b = \frac{1}{2}[\cos(a-b) - \cos(a+b)]$

1)

a) $\int \operatorname{sen} 3x \cdot \operatorname{sen} 5x \, dx$

I $u = 2x$, $du = 2\,dx$, $dx = \frac{1}{2}du$

II $u = 8x$, $dx = \frac{1}{8}dx$

$= \int \frac{1}{2}[\cos(3x-5x) - \cos(3x+5x)]\,dx$

$= \frac{1}{2}\left[\int \cos(-2x)\,dx - \int \cos(8x)\,dx\right]$

//Não entendi, mas a função cos é par logo $\cos(-\theta) = \cos(\theta)$

$= \frac{1}{2}\left[\int \cos(2x)\,dx - \int \cos(8x)\,dx\right]$

$= \frac{1}{2}\left[\frac{1}{2}\operatorname{sen}(2x) - \frac{1}{8}\operatorname{sen}(8x)\right]$

$= \frac{\operatorname{sen}(2x)}{4} - \frac{\operatorname{sen}(8x)}{4} + K$ //

b) $\int \cos 2x \cdot \cos x \, dx$

$= \frac{1}{2}\int [\cos(2x+x) + \cos(2x-x)]\,dx$

$= \frac{1}{2}\left[\int \cos(3x)\,dx + \int \cos(x)\,dx\right]$

$= \frac{1}{2}\left[\frac{1}{3}\operatorname{sen} 3x + \operatorname{sen} x\right]$

$= \frac{\operatorname{sen} 3x}{6} + \frac{\operatorname{sen} x}{2} + K$ //

C) $\int \operatorname{sen}^5 x \, dx$

$u = \cos x$, $du = -\operatorname{sen} x\,dx$

$= \int \operatorname{sen}^2 x \cdot \operatorname{sen}^2 x \cdot \operatorname{sen} x\,dx$

$= \int (1-\cos^2 x)\cdot(1-\cos^2 x)\cdot \operatorname{sen} x\,dx$

$= \int (1-u^2)(1-u^2)\cdot -du$

$= \int -1 + u^2 + u^2 + u^4\,du$

$= -x + \frac{2u^3}{3} + \frac{u^5}{5} + K$

$= -x + \frac{2\cos^3 x}{3} + \frac{\cos^x x}{5} + K$

d) $\int \cos^2 5x \, dx$

$= \int \frac{1}{2} + \frac{\cos 10x}{2}\,dx$

$= \int \frac{1}{2}dx + \frac{1}{2}\int \cos 10x\,dx$

$= \int \frac{1}{2}dx + \frac{1}{2}\cdot\frac{1}{10}\int \cos u\,du$

$= \frac{x}{2} + \frac{1}{20}\cdot \operatorname{sen} 10x + K$

$= \frac{x}{2} + \frac{\operatorname{sen} 10x}{20} + K$ //

E) $\int \operatorname{sen}^2 x \cdot \cos^4 x \, dx$

$= \int \left(\frac{1}{2} - \frac{\cos 2x}{2}\right)\cdot\left(\frac{1}{2} + \frac{\cos(2x)}{2}\right)^2 dx$

$= \int \left(\frac{1}{2} - \frac{\cos 2x}{2}\right)\cdot\left(\frac{(1+\cos(2x))^2}{4}\right)dx$

$= \int \left(\frac{1}{2} - \frac{\cos 2x}{2}\right)\cdot\left(\frac{1 + 2\cos(2x) + \cos^2(2x)}{4}\right)dx$

$= \int \frac{1}{2}(1-\cos 2x)\cdot\frac{1}{4}(1 + 2\cos(2x) + \cos^2(2x))\,dx$

$= \frac{1}{8}\int (1-\cos 2x)(1 + 2\cos(2x) + \cos^2(2x))\,dx$

$= \frac{1}{8}\int 1 + \cancel{2\cos(2x)} + \cancel{\cos^2(2x)} - \cancel{\cos(2x)} - \cancel{2\cos^2(2x)} - \cos^3(2x)\,dx$

$= \frac{1}{8}\int 1 + \cos(2x) - \cos^2(2x) - \cos^3(2x)\,dx$

$= \frac{1}{8}\left[\int 1\,dx + \int \cos(2x)\,dx - \int \cos^2(2x)\,dx - \int \cos^3(2x)\,dx\right]$

= Ate - aqui ta certo, ta faltando

f) $\int \operatorname{sen}^2(2x)\cdot\cos^2(3x)\,dx$

$= \int \left(\frac{1-\cos(4x)}{2}\right)\left(\frac{1+\cos(6x)}{2}\right)dx$

$= \int \frac{1}{2}(1-\cos(4x))\,\frac{1}{2}(1+\cos(6x))\,dx$

$= \frac{1}{4}\int (1-\cos(4x))(1+\cos(6x))$

$= \frac{1}{4}\int 1 + \cos(6x) - \cos(4x) - \cos(6x)\cdot\cos(4x)\,dx$

$= \frac{1}{4}\left[\int 1\,dx + \int \cos(6x)\,dx - \int \cos(4x)\,dx - \underbrace{\int \cos(6x)\cdot\cos(4x)\,dx}_{\mathrm{I}}\right]$

$= \frac{1}{4}\left[x + \frac{1}{6}\operatorname{sen}(6x) - \frac{1}{4}\operatorname{sen}(4x) - \frac{1}{20}\operatorname{sen}(10x) + \frac{1}{4}\operatorname{sen}(2x)\right]$

$= \frac{x}{4} + \frac{\operatorname{sen}(6x)}{24} - \frac{\operatorname{sen}(4x)}{16} - \frac{\operatorname{sen}(10x)}{80} + \frac{\operatorname{sen}(2x)}{16} + K$

I $= \int \cos(6x)\cdot\cos(4x)\,dx$

$= \frac{1}{2}\left(\int \cos(10x) + \cos 2x\,dx\right)$

$= \frac{1}{2}\left(\int \cos(10x) + \int \cos 2x\,dx\right)$

$= \frac{1}{2}\left[\frac{1}{10}\operatorname{sen}(10x) + \frac{1}{2}\operatorname{sen}(2x)\right] + K$

$\frac{1}{20}\cdot\operatorname{sen}(10x) + \frac{1}{4}\operatorname{sen}(2x)$

G) $\int \cos x \cdot \cos^2 4x\,dx$

$= \int \cos x \cdot \left(\frac{1+\cos(8x)}{2}\right)dx$

$= \int \cos x \cdot \left(\frac{1}{2} + \frac{\cos(8x)}{2}\right)dx$

$= \int \frac{1}{2}\cos x + \frac{1}{2}\cos(8x)\cdot\cos x\,dx$

$= \frac{1}{2}\int \cos x + \frac{1}{2}\int \cos(8x)\cdot\cos x\,dx$

$= \frac{1}{2}\int \cos x + \frac{1}{2}\int \frac{1}{2}[\cos(9x) + \cos(7x)]\,dx$

$= \frac{1}{2}\cdot\operatorname{sen} x + \frac{1}{4}\left[\int \cos(9x)\,dx + \int \cos(7x)\,dx\right]$

$= \frac{1}{2}\cdot\operatorname{sen} x + \frac{1}{4}\left[\frac{1}{9}\operatorname{sen}(9x) + \frac{1}{7}\operatorname{sen}(7x)\right] + K$

$= \frac{1}{2}\cdot\operatorname{sen} x + \frac{1}{36}\operatorname{sen}(9x) + \frac{1}{28}\operatorname{sen}(7x) + K$ //

h) $\int tg^5 x \cdot \sec^2 x\,dx$

$= \int tg^4 x\cdot tg\,x\cdot \sec^2 x\,dx$

$= \int (\sec^2 x - 1)^2\cdot \sec^2 x\cdot tg\,x\,dx$

$= \int (\sec^4 x - 2\sec^2 x + 1)\cdot \sec^2\cdot tg\,x\,dx$

$u = \sec x$ $\quad du = \sec x\cdot tg\,x\,dx$

$= \int (u^4 - 2u^2 + 1)\cdot u\,du$

$= \int u^5 - 2u^3 + u\,du$

$= \frac{u^6}{6} - \frac{u^4}{2} + \frac{u^2}{2}$

i) $\int tg^3(2x)\cdot\sec(2x)\,dx$

$= \int tg^2(2x)\cdot tg(2x)\cdot\sec(2x)\,dx$

$= \int (\sec^2(2x) - 1)\cdot tg(2x)\cdot\sec(2x)\,dx$

$u = \sec 2x$ $\quad du = \sec 2x\cdot tg\,2x\cdot 2\,dx$

$= \int (u^2-1)\cdot \cancel{tg(2x)\cdot\sec(2x)}\,\frac{du}{\cancel{\sec(2x)\cdot tg(2x)}\cdot 2}$

$= \frac{1}{2}\int u^2 - 1\,du$

$= \frac{1}{2}\left[\frac{u^3}{3} - u\right] + K$

$= \frac{\sec^3(2x)}{6} - \frac{\sec(2x)}{2} + K$ //

J) $\int tg^6 x\,dx$

$= \int tg^4 x\,(\sec^2 x - 1)\,dx$

$= \int tg^4 x\cdot\sec^2 x - tg^4 x\,dx$

$= \underbrace{\int tg^4\cdot\sec^2 x\,dx}_{\mathrm{I}} - \int tg^4 x\,dx$

$u = tg\,x$ $\quad du = \sec^2 x\,dx$

$= \int u^4\,du - \int (\sec^2 x - 1)^2\,dx$

$= \frac{u^5}{5} - \int \sec^4 x - 2\sec^2 x + 1\,dx$

$= \frac{tg^5 x}{5} - \left(\int \sec^4 x\,dx - 2\underbrace{\int \sec^2 x\,dx}_{\text{Primitiva}} + \int 1\,dx\right)$

$= \frac{tg^5 x}{5} - \cancel{tg\,x} - \frac{tg^3 x}{3} + 2\cancel{tg\,x} - x$

$= \frac{tg^5 x}{5} - \frac{tg^3 x}{3} + tg\,x - x + K$ //

$\int (1 + tg^2 x)\cdot\sec^2 x\,dx$

$u = tg\,x$ $\quad du = \sec^2 x\,dx$

$\int 1 + u^2\,du$

$\int 1\,du + \int u^2\,du$

$u + \frac{u^3}{3} + K$

$tg\,x + \frac{tg^3 x}{3}$

K) $\int \sec^5 x\,dx$

$u = \sec x$, $du = \sec x\cdot tg\,x$

$du = \sec^2\cdot\sec x\,dx$

$v = \int \sec^2 x\cdot\sec^2 x\,dx$

$* = \int (1 + tg^2 x)\cdot\sec^2 x\,dx$

$u = tg\,x$, $du = \sec^2 x\,dx$

$= \int 1 + u^2\,du$

$= u + \frac{u^3}{3}$

$= tg\,x + \frac{tg^3 x}{3} + K$

$= \int \sec^2 x\cdot\sec^2 x\cdot\sec x\,dx$

$= \sec x\cdot tg\,x + \frac{tg^3 x}{3} - \int \left(tg\,x + \frac{tg^3 x}{3}\right)\cdot\sec x\cdot tg\,x\,dx$

$-\frac{1}{3}\int tg^2 x\cdot\sec x + tg^4 x\cdot\sec x\,dx$

$-\frac{1}{3}\underbrace{\int tg^2 x\cdot\sec x\,dx}_{\mathrm{I}} + \underbrace{\int tg^2\cdot tg^2\cdot\sec x\,dx}$

$= \sec x\cdot tg\,x + \frac{tg^3 x}{3} - \frac{1}{3}\left[\frac{1}{2}(\sec x\cdot tg\,x - \ln|\sec x + tg\,x|) + \frac{1}{3}(\sec^3 x - \sec x\cdot tg\,x)\right]$

$= \sec x\cdot tg\,x + \frac{tg^3 x}{3} - \frac{1}{6}\sec x\cdot tg\,x + \frac{1}{6}\ln|\sec x + tg\,x| - \frac{1}{9}\sec^3 x + \frac{1}{9}\sec x\cdot tg\,x$

$-\frac{1}{18}\sec x\cdot tg\,x + \frac{1}{6}\ln|\sec x + tg\,x| - \frac{\sec^3 x}{9}$

$= \int (\sec^2 x - 1)^2\sec x\,dx$

$= \int (\sec^4 x - 2\sec^2 x + 1)\sec x\,dx$

$= \int \sec^5 x - 2\sec^3 x + \sec x\,dx$

$= \int \sec^5 x\,dx - 2\int \sec^3 x\,dx + \int \sec x$

I $\int (\sec^2 x - 1)\sec x$

$\int \sec^3 x - \int \sec x$

$\Delta\ \int \sec^2 x\cdot\sec - \ln|\sec x + tg\,x|$

$u = \sec x$ $\quad dv = \sec^2 x$

$du = \sec x\cdot tg\,x$ $\quad v = tg\,x$

$\Delta = \sec x\cdot tg\,x - \int tg^2 x\cdot\sec x\,dx$

I $= \int tg^2 x\cdot\sec x = \Delta - \ln|\sec x + tg\,x|$

$2\int tg^2 x\cdot\sec x = \sec x\cdot tg\,x - \ln|\sec x + tg\,x|$

$= \frac{1}{2}(\sec x\cdot tg\,x - \ln|\sec x + tg\,x|)$

l) $\int \text{cossec}^4(x) \cdot \text{cotg}^6(x)\, dx$

$= \int \csc^2(x) \cdot \csc^2(x) \cdot \text{cotg}^6(x)\, dx$

$= \int (1 + \text{cotg}^2 x) \cdot \text{cotg}^6 x \cdot \csc^2(x)\, dx$

$= \int (\text{cotg}^6 x + \text{cotg}^8 x)\, \csc^2(x)\, dx$

$u = \text{cotg}\, x$
$du = -\csc^2 x\, dx$

$= \int u^6 + u^8 \; -du$

$= -\frac{u^7}{7} - \frac{u^9}{9} + K$

$= -\frac{\text{cotg}^7 x}{7} - \frac{\text{cotg}^9 x}{9} + K$

m) $\int \text{cotg}^5 x\, dx$

$\Rightarrow \int \text{ctg}^3 \cdot \text{ctg}^2$

$\Rightarrow \int \text{ctg}^3 x \cdot (\csc^2 x - 1)\, dx$

$\Rightarrow \int \text{ctg}^3 x \cdot \csc^2 x - \text{ctg}^3 x\, dx$

$\Rightarrow \int \text{ctg}^3 \cdot \csc^2 x\, dx - \int \text{ctg}^3 x\, dx$

$u = \text{ctg}\, x$ $\quad du = -\csc^2 x$

$\Rightarrow \int u^3 \; -du - \left(\int (\csc^2 x - 1) \cdot \text{ctg}\, x\, dt\right)$

$\Rightarrow -\frac{\text{ctg}^4 x}{4} - \left(\int \text{ctg}\, x \cdot \csc^2 x\, dx - \int \text{ctg}\, x\, dx\right)$

$u = \text{ctg}\, x$
$du = -\csc^2 x\, dx$
Primitiva

$\Rightarrow -\frac{\text{ctg}^4 x}{4} - \left(\int u \; -du - \ln|\text{sen}\, x|\right)$

$\Rightarrow -\frac{\text{ctg}^4 x}{4} + \frac{\text{ctg}^2 x}{2} + \ln|\text{sen}\, x|$

n) $\int \text{cossec}^4 x\, dx$

$\int (1 + \text{cotg}^2 x) \cdot \csc^2 x\, dx$

$u = \text{cotg}(x)$ $\quad du = -\text{cossec}^2 x\, dx$

$\int 1 + u^2 \; -du$

$-\int du - \int u^2\, du$

$-u - \frac{u^3}{3}$

$-\text{cotg}\, x - \frac{\text{cotg}^3 x}{3} + K$

2)

a) $\int \text{sen}^n x\, dx = -\frac{1}{n} \text{sen}^{n-1} \cdot \cos x + \frac{n-1}{n} \cdot \int \text{sen}^{n-2}\, dx$

$\int \text{sen}^n x\, dx = \int \text{sen}^{n-1} \cdot \text{sen}\, x\, dx$

$u = \text{sen}^{n-1} x$ $\quad dv\ \text{sen}\, x$
$du = (n-1) \cdot \text{sen}^{n-2} \cdot \cos x$ $\quad v\ -\cos x$

$= \text{sen}^{n-1} x \cdot -\cos x + \int \cos^2 x \cdot (n-1) \cdot \text{sen}^{n-2}$

$= \text{sen}^{n-1} x \cdot -\cos x + \int (1 - \text{sen}^2 x)\, (n-1)\, \text{sen}^{n-2} x$

$= \text{sen}^{n-1} x \cdot -\cos x + \int (n-1)\, \text{sen}^{n-2} x - \text{sen}^n x\, (n-1)$

$\int \text{sen}^n x\, dx = \text{sen}^{n-1} x \cdot -\cos x + (n-1) \int \text{sen}^{n-2} x\, dx - (n-1) \int \text{sen}^n x\, dx$

$\int \text{sen}^n x\, dx + (n-1) \int \text{sen}^n x\, dx = \text{sen}^{n-1} x \cdot -\cos x + (n-1) \int \text{sen}^{n-2} x\, dx$

$n \int \text{sen}^n x\, dx = \text{sen}^{n-1} x \cdot -\cos x + (n-1) \int \text{sen}^{n-2} x\, dx$

$\int \text{sen}^n x\, dx = \frac{1}{n}\left(\text{sen}^{n-1} x \cdot -\cos x + (n-1) \int \text{sen}^{n-2} x\, dx\right)$

$\int \text{sen}^n x\, dx = -\frac{1}{n} \text{sen}^{n-1}(x) \cdot \cos(x) + \frac{n-1}{n} \int \text{sen}^{n-2} x\, dx$ //

b) $\int \text{tg}^n x\, dx = \frac{\text{tg}^{n-1} x}{n-1} - \int \text{tg}^{n-2}(x)\, dx$

$\int \text{tg}^n x\, dx = \int \text{tg}^{n-2}(x) \cdot \text{tg}^2 x\, dx$

$= \int \text{tg}^{n-2}(x)\, (\sec^2(x) - 1)\, dx$

$= \int \text{tg}^{n-2}(x) \cdot \sec^2(x)\, dx - \int \text{tg}^{n-2}(x)\, dx$

$= u = \text{tg}(x)$ $\quad du = \sec^2 x\, dx$

$= \int u^{n-2}\, du - \int \text{tg}^{n-2}(x)\, dx$

$= \frac{u^{n-1}}{n-1} - \int \text{tg}^{n-2}(x)\, dx$

$= \frac{\text{tg}^{n-1}(x)}{n-1} - \int \text{tg}^{n-2}(x)\, dx$ //

C) $\int \text{tg}^n(x) \cdot \sec^2(x)\, dy = \frac{\text{tg}^{n+1}(x)}{n+1} + K, \quad n \neq -1$

$\int \text{tg}^n(x) \cdot \sec^2(x)\, dx = \int u^n\, du = \frac{u^{n+1}}{n+1} + K$

$u = \text{tg}(x)$
$du = \sec^2(x)\, dx$

$= \frac{\text{tg}^{n+1}(x)}{n+1} + K, \quad n \neq -1$ //

D) $\int \sec^{n+1}(x) \cdot \text{tg}\, x\, dx = \frac{\sec^{n+1}(x)}{n+1} + K, \quad n \neq -1$

$\int \sec^{n+1}(x) \cdot \text{tg}\, x\, dx = \int \sec^n(x) \cdot \sec(x) \cdot \text{tg}(x)\, dx = \int u^n\, du$

$u = \sec x$
$du = \sec x\ \text{tg}\, x\, dx$

$= \frac{u^{n+1}}{n+1} + K = \frac{\sec^{n+1}(x)}{n+1} + K$ //

e) $\int \text{cotg}^n(x)\, \text{cossec}^2(x)\, dx = \frac{\text{cotg}^{n+1}}{n+1} + K, \quad n \neq -1$

$\int \text{cotg}^n(x)\, \text{cossec}^2(x)\, dx = -\int u^n\, du = -\frac{u^{n+1}}{n+1} + K$

$u = \text{cotg}(x)$
$du = -\text{cossec}^2 x\, dx$

$= -\frac{\text{cotg}^{n+1}(x)}{n+1} + K$ //

f) $\int \text{cossec}^n x\ \text{cossec}(x) \cdot \text{cotg}(x)\, dx = -\frac{\text{cossec}^{n+1}(x)}{n+1} + K, \quad n \neq -1$

$\int \text{cossec}^n x\ \text{cossec}(x) \cdot \text{cotg}(x)\, dx = -\int u^n\, du = -\frac{u^{n+1}}{n+1} + K$

$u = \text{cossec}(x)$
$du = -\text{cossec}(x) \cdot \text{cotg}(x)\, dx$

$= -\frac{\text{cossec}^{n+1}(x)}{n+1} + K, \quad n \neq -1$ //

3) a) $\int \frac{\text{sen}\, x}{1 + \text{sen}\, x}\, dx$

$= \int \frac{\left(\frac{2u}{1+u^2}\right)}{1 + \left(\frac{1+u^2}{1-u^2}\right)} \cdot \frac{2}{1+u^2}\, du = \int \frac{\left(\frac{2u}{1+u^2}\right)}{1 + \left(\frac{1+u^2}{1-u^2}\right)} \cdot \frac{2}{1+u^2}\, du = \int \frac{\left(\frac{2u}{1+u^2}\right)}{\left(\frac{2}{1-u^2}\right)} \cdot \frac{2}{1+u^2}\, du = \int \frac{2u}{(1+u^2)} \cdot \frac{(1-u^2)}{\cancel{2}} \cdot \frac{\cancel{2}}{(1+u^2)}\, du = \int \frac{2u(1-u^2)}{(1+u^2)^2}\, du$

$w = 1 + u^2$ $\quad dw = 2u\, du$
$u^2 = w - 1$

$= \int \frac{1 - (w-1)}{w^2}\, dw = \int \frac{2 - w}{w^2}\, dw = \int \frac{2}{w^2} - \frac{1}{w}\, dw = 2\int w^{-2}\, dw - \ln|w| = 2 \cdot \frac{-1}{w} - \ln|w| + K = -\frac{2}{1+u^2} - \ln|1+u^2| + K$

$= -\frac{2}{1 + \text{tg}^2\left(\frac{x}{2}\right)} - \ln\left|1 + \text{tg}^2\left(\frac{x}{2}\right)\right| + K$ //

B) $\int \frac{1}{\text{sen}\, x + \cos x}\, dx$

$= \int \frac{1}{\left(\frac{2u}{1+u^2}\right) + \left(\frac{1-u^2}{1+u^2}\right)} \cdot \frac{2}{(1+u^2)}\, du$

$= \int \frac{1}{\left(\frac{2u + 1 - u^2}{1+u^2}\right)} \cdot \frac{2}{(1+u^2)}\, du$

$= -\int \frac{\cancel{1+u^2}}{u^2 - 2u - 1} \cdot \frac{2}{\cancel{1+u^2}}\, du$

$= -2 \int \frac{1}{\underline{u^2 - 2u - 1}}\, du$

tem raiz, porém o $x_{1,2}$ fica todo zoado, isso se da pois a raiz do delta $\sqrt{\Delta}$ não e exata Ex) $\sqrt{8}$

Completar quadrado: $\left(\frac{2}{2}\right)^2 = 1$
$(u^2 - 2u)$ separar (-1)
$(u^2 - 2u) + 1 - 1(-1) = 0$ $\quad \Delta = 0$ $\quad (u-1)^2 - 2$
$(u^2 - 2u + 1) - 2 = 0$ $\quad x = 1$

$= -2 \int \frac{1}{(u-1)^2 - 2}\, du$ $\quad$ $w = u - 1$, $dw = du$

$= -2 \int \frac{1}{w^2 - 2}\, dw$

$= \frac{1}{\sqrt{2}\,2} \ln \int \frac{1}{(w - \sqrt{2})(w + \sqrt{2})}\, dw$ $\quad$ $w^2 - 2 = 0$, $w^2 = 2$, $w = \pm\sqrt{2}$

Frações Parciais

$\frac{A}{(w - \sqrt{2})} + \frac{B}{(w + \sqrt{2})} = \frac{Aw + A\sqrt{2} + Bw - B\sqrt{2}}{(w - \sqrt{2})(w + \sqrt{2})}$

$= \frac{w(A+B) + \sqrt{2}(A-B)}{(w - \sqrt{2})(w + \sqrt{2})} = \frac{0w + 1}{(w - \sqrt{2})(w + \sqrt{2})}$

$w(A+B) = 0$
$A + B = 0$
$\sqrt{2}(A - B) = 1$
$A - B = \frac{1}{\sqrt{2}}$
$\begin{cases} A + B = 0 \\ A - B = \frac{1}{\sqrt{2}} \end{cases}$

$2A = \frac{1}{\sqrt{2}}$
$A = \frac{1}{2\sqrt{2}}$
$\frac{1}{2\sqrt{2}} + B = 0$
$B = -\frac{1}{2\sqrt{2}}$

$= -\cancel{2} \int \frac{\frac{1}{\cancel{2}\sqrt{2}}}{(w - \sqrt{2})} - \frac{\frac{1}{\cancel{2}\sqrt{2}}}{(w + \sqrt{2})}\, dw$

$= -\frac{1}{\sqrt{2}} \int \frac{1}{(w - \sqrt{2})}\, dw + \frac{1}{\sqrt{2}} \int \frac{1}{(w + \sqrt{2})}\, dw$

$= -\frac{1}{\sqrt{2}} \int \frac{1}{v}\, dv + \frac{1}{\sqrt{2}} \int \frac{1}{J}\, dw$

$= -\frac{1}{\sqrt{2}} \ln|w - \sqrt{2}| + \frac{1}{\sqrt{2}} \ln|w + \sqrt{2}| + K$

$= \frac{2}{\sqrt{2}} \ln\left|\frac{w + \sqrt{2}}{w - \sqrt{2}}\right| + K$

$= \frac{2}{\sqrt{2}} \ln\left|\frac{u - 1 + \sqrt{2}}{u - 1 - \sqrt{2}}\right| + K$

$= \frac{2}{\sqrt{2}} \ln\left|\frac{\text{tg}\left(\frac{x}{2}\right) - 1 + \sqrt{2}}{\text{tg}\left(\frac{x}{2}\right) - 1 - \sqrt{2}}\right| + K$ //

4) a) $\int_1^{\infty} \frac{1}{(3x+1)^2}\, dx = \lim_{t \to \infty} \left(\int_1^{T} \frac{1}{(3x+1)^2}\, dx \right)$

$\int_1^{T} \frac{1}{(3x+1)^2}\, dx =$ $\quad u = 3x+1$, $du = 3\,dx$ $\quad x = 1$, $x = T$ $\quad u = 3(1)+1 = 4$, $u = 3T+1$

$= \lim_{T\to\infty} \left(\int_4^{3T+1} \frac{1}{u^2} \frac{du}{3} \right) = \frac{1}{3} \lim_{T\to\infty} \left(\int_4^{3T+1} u^{-2}\, du \right)$

Calcular a integral:

$= \frac{1}{3} \int u^{-2}\, du = \frac{1}{3} \lim_{T\to\infty} \left(-\frac{1}{u}\right) \Big|_4^{3t+1}$

$= \frac{1}{3} \left[\lim_{T\to\infty} \left(-\frac{1}{3t+1} - \left(-\frac{1}{4}\right) \right) \right] = \frac{1}{3}\left(0 + \frac{1}{4}\right) = \frac{1}{12}$

Converge em $\frac{1}{12}$

b) $\int_{-\infty}^{0} \frac{1}{2x-5}\, dx = \lim_{T\to-\infty} \Big(\underbrace{\int_T^0 \frac{1}{2x-5}\, dx}_{\text{I}} \Big)$

I $\int_T^0 \frac{1}{2x-5}\, dx$ $\quad u = 2x-5$, $dx = \frac{1}{2} du$ $\quad x = 0 \Rightarrow u = 2\cdot 0 - 5 = -5$, $x = T \Rightarrow u = 2t - 5$

$= \frac{1}{2} \int_{2t-5}^{-5} \frac{1}{u}\, du = \frac{1}{2} (\ln|u|) \Big|_{2t-5}^{-5}$

$= \frac{1}{2} \left[\lim_{T\to-\infty} \left(\ln|u| \Big|_{2T-5}^{-5} \right) \right]$

$= \frac{1}{2} \left[\lim_{T\to-\infty} \left(\ln|-5| - \ln|2T-5| \right) \right]$

$\therefore$ a integral é divergente

C) $\int_{-\infty}^{-1} \frac{1}{\sqrt{2-x}}\, dx = \lim_{a\to-\infty} \left(\int_a^{-1} \frac{1}{\sqrt{2-x}}\, dx \right)$

$\int \frac{1}{\sqrt{2-x}}\, dx =$ $\quad u = 2-x$, $du = -dx$ $\quad -\int \frac{1}{\sqrt{u}}\, du = -\int u^{-\frac{1}{2}}\, du$

$= -2 \cdot u^{\frac{1}{2}} = \left(-2\sqrt{2-x} \right) \Big|_a^{-1}$

$\lim_{a\to-\infty} \left(-2\sqrt{2-(-1)} + 2\sqrt{2-(a)} \right)$

$\lim_{a\to-\infty} \left(-2\sqrt{3} + 2\sqrt{2-a} \right)$

d) $\int_0^{\infty} \frac{x}{(x^2+2)^2}\, dx = \lim_{b\to\infty} \left(\int_0^b \frac{x}{(x^2+2)^2}\, dx \right)$

$\int \frac{x}{(x^2+2)^2}\, dx$ $\quad u = x^2+2$, $du = 2x\, dx$ $\quad = \frac{1}{2} \int \frac{1}{u^2}\, du = \frac{1}{2} \int u^{-2}\, du$

$= \frac{1}{2} \cdot \left(-\frac{1}{u}\right) = -\frac{1}{2u} = -\frac{1}{2x^2+4}$

$\lim_{b\to\infty} \left(\left(-\frac{1}{2x^2+4}\right) \Big|_0^b \right) = \lim_{b\to\infty} \left(-\cancelto{0}{\frac{1}{2(b)^2+4}} - \left(-\frac{1}{2\cdot 0+4}\right) \right) = 0 + \frac{1}{4} = \frac{1}{4}$ //

e) $\int_4^{\infty} e^{-\frac{x}{2}}\, d = \lim_{b\to\infty} \left(\int_4^b e^{-\frac{x}{2}}\, dx \right)$

$\int e^{-\frac{x}{2}}\, dx$ $\quad u = -\frac{x}{2}$, $du = -\frac{1}{2} dx \Rightarrow dx = -2du$ $\quad = -2\int e^u\, du$

$= -2e^u = -2e^{-\frac{x}{2}}$

$\lim_{b\to\infty} \left(\left(-2e^{-\frac{x}{2}}\right) \Big|_4^b \right) = \lim_{b\to\infty} \left(\left(\cancelto{0}{-2e^{-\frac{b}{2}}}\right) - \left(-2e^{-\frac{4}{2}}\right) \right) = \left(0 + 2e^{-2}\right)$

$\therefore$ a integral é convergente em $2e^{-2}$

f) $\int_{-\infty}^{-1} e^{-2x}\, dx = \lim_{a\to-\infty} \left(\int_a^{-1} e^{-2x}\, dx \right)$

$= \int e^{-2x}\, dx =$ $\quad u = -2x$, $du = -2dx$ $\quad = -\frac{1}{2} \int e^u\, d$

$= -\frac{1}{2} e^{-2x}$

$\lim_{a\to-\infty} \left(\left(-\frac{1}{2} e^{-2x}\right) \Big|_a^{-1} \right) = \lim_{a\to-\infty} \left(\left(-\frac{1}{2} e^{-2(-1)}\right) - \left(\cancelto{\infty}{-\frac{1}{2} e^{-2(-\infty)}}\right) \right)$

$= \lim_{a\to-\infty} \left(-\frac{e^2}{2} + \frac{1}{2} e^{-2a} \right)$

$\therefore$ é divergente

G) $\int_{-\infty}^{\infty} \underbrace{\frac{x}{1+x^2}}_{\text{I}}\, dx = \int_{-\infty}^{0} \frac{x}{1+x^2}\, dx + \int_0^{\infty} \frac{x}{1+x^2}\, dx$

I $= \int \frac{x}{1+x^2}\, dx = \int \frac{1}{1+x^2} \cdot x \cdot dx$ $\quad u = 1+x^2$, $du = 2x\, dx$

$\frac{1}{2} \int \frac{1}{u}\, d = \frac{1}{2} \ln|1+x^2|$

$\lim_{a\to-\infty} \left(\left(\frac{1}{2} \ln|1+x^2|\right) \Big|_a^0 \right) = \lim_{a\to-\infty} \left(\frac{1}{2}\ln|1| - \frac{1}{2}\ln|1+a^2| \right) = \lim_{a\to-\infty} \left(0 - \frac{1}{2} \ln|1+a^2| \right)$

$\lim_{b\to\infty} \left(\left(\frac{1}{2} \ln|1+x^2|\right) \Big|_0^b \right) = \lim_{b\to\infty} \left(\frac{1}{2}\ln|1+b^2| - \frac{1}{2}\ln|1| \right) = \lim_{b\to\infty} \left(\frac{1}{2} \ln|1+b^2| \right)$

h) $\int_{-\infty}^{\infty} (2-x^4)\, dx = \int_a^0 (2-x^2)\, dx + \int_0^b (2-x^2)\, dx$

$\int 2-x^4\, dx = \int 2\, dx - \int x^4\, dx = 2x - \frac{x^5}{5} + K$

$\lim_{a\to-\infty} \left(\left(2x - \frac{x^5}{5}\right) \Big|_a^0 \right) = \lim_{a\to-\infty} \left(0 - \left(2a - \frac{a^5}{5}\right) \right) = \lim_{a\to-\infty} \left(-2a + \frac{a^5}{5} \right)$

É divergente

i) $\int_{-\infty}^{\infty} xe^{-x^2}\, dx = \int_{-\infty}^{0} xe^{-x^2}\, dx + \int_0^{\infty} xe^{-x^2}\, dx$

$\int x \cdot e^{-x^2}\, dx$ $\quad u = -x^2$, $du = -2x\, dx$, $-\frac{1}{2} du = x\, dx$ $\quad = -\frac{1}{2} \int e^u\, du = -\frac{1}{2} e^{-x^2} + K$

$\lim_{a\to-\infty} \left(\left(-\frac{1}{2} e^{-x^2}\right) \Big|_a^0 \right) = \lim_{a\to-\infty} \left(-\frac{1}{2} e^0 - \left(-\frac{1}{2} e^a\right) \right) = \lim_{a\to-\infty} \left(-\frac{1}{2} + \cancelto{0}{\frac{1}{2} e^{a^2}} \right) = -\frac{1}{2}$

$\lim_{b\to\infty} \left(\left(-\frac{1}{2} e^{-x^2}\right) \Big|_0^b \right) = \lim_{b\to\infty} \left(\cancelto{0}{-\frac{1}{2} e^{-b^2}} - \left(-\frac{1}{2}\right) \right) = +\frac{1}{2}$

$\int_{-\infty}^{0} xe^{-x^2}\, dx + \int_0^{\infty} xe^{-x^2}, x = -\frac{1}{2} + \frac{1}{2} = 0$ //

J) $\int_0^3 \frac{1}{\sqrt{x}}\, dx = \lim_{a\to0^+} \left(\int_a^3 \frac{1}{\sqrt{x}}\, dx \right)$

$\int \frac{1}{\sqrt{x}}\, dx = \int x^{-\frac{1}{2}}\, dx = 2x^{\frac{1}{2}}$

$\lim_{a\to0^+} \left(\left(2\sqrt{x}\right) \Big|_a^3 \right) = \lim_{a\to0^+} \left(2\sqrt{3} - \cancelto{0}{2\sqrt{a}} \right) = 2\sqrt{3}$

Convergente

K) $\int_0^3 \frac{1}{x\sqrt{x}}\, dx = \lim_{a\to0^+} \left(\int_a^3 \frac{1}{x\sqrt{x}}\, dx \right)$

$\int \frac{1}{x\sqrt{x}}\, dx =$ $\quad x^1 \cdot x^{\frac{1}{2}} = x^{\frac{3}{2}}$ $\quad = \int \frac{1}{x^{3/2}}\, dx =$

$\int x^{-\frac{3}{2}}\, dx = \frac{x^{-\frac{1}{2}}}{-\frac{1}{2}} = \frac{-2}{\sqrt{x}} + K$

$\lim_{a\to0^+} \left(\left(-\frac{2}{\sqrt{x}}\right) \Big|_a^3 \right) = \lim_{a\to0^+} \left(\left(-\frac{2}{\sqrt{3}}\right) - \left(\cancelto{\infty}{-\frac{2}{\sqrt{0^+}}}\right) \right) = -\frac{2}{\sqrt{3}} + \infty = \infty$ //

$\frac{2}{0{,}01}$

Divergente

L) $\int_{-1}^{0} \frac{1}{x^2}\, dx = \lim_{b\to0^-} \left(\int_{-1}^{b} \frac{1}{x^2}\, dx \right)$

$\int x^{-2}\, dx = -\frac{1}{x} + K$

$\lim_{b\to0^-} \left(\left(-\frac{1}{x}\right) \Big|_{-1}^{b} \right) = \lim_{b\to0^-} \left(\left(\cancelto{\infty}{-\frac{1}{b}}\right) - \left(-\frac{1}{-1}\right) \right) = \infty - 1 = \infty$

m) $\int_1^9 \frac{1}{\sqrt[3]{x-9}}\, dx = \lim_{b\to9^-} \left(\int_1^b \frac{1}{\sqrt[3]{x-9}}\, dx \right)$

$\int \frac{1}{\sqrt[3]{x-9}}\, dx =$ $\quad u = x-9$, $du = dx$ $\quad = \int \frac{1}{\sqrt[3]{u}}\, du = \int u^{-\frac{1}{3}}\, d$

$\frac{u^{2/3}}{(2/3)} = = \frac{3}{2} \sqrt[3]{(x-9)^2}$

$\frac{3}{2}\sqrt[3]{(-8)^2}$

$-\frac{3}{2} \cdot 4 = -6$

$\lim_{b\to9^-} \left(\left(\frac{3}{2}(x-9)^{\frac{2}{3}}\right) \Big|_1^b \right) = \lim_{b\to9^-} \left(\cancelto{0}{\frac{3}{2}(b-9)^{\frac{2}{3}}} - \frac{3}{2}(1-9)^{\frac{2}{3}} \right) = -6$

n) $\int_{-2}^{3} \frac{1}{x^4}\, dx$ $\quad$ Se existe um valor c entre a e b $a < c < b$, e f(b) não existe

$= \int_{-2}^{0} \frac{1}{x^4}\, dx + \int_0^3 \frac{1}{x^4}\, dx = \underbrace{\lim_{b\to0^-} \left(\int_{-2}^{b} \frac{1}{x^4}\, dx \right)}_{\text{I}} + \underbrace{\lim_{a\to0^+} \left(\int_a^3 \frac{1}{x^4}\, dx \right)}_{\text{II}}$

$\int \frac{1}{x^4}\, dx = \int x^{-4}\, dx = \frac{x^{-3}}{-3} + K = -\frac{1}{3x^3} + K$

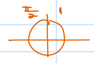
I $\lim_{b\to0^-} \left(\left(-\frac{1}{3x^3}\right) \Big|_{-2}^{b} \right) = \lim_{b\to0^-} \left(\left(\cancelto{\infty}{-\frac{1}{3b^3}}\right) - \left(-\frac{1}{3(-2)^3}\right) \right) = \infty$

II $\lim_{a\to0^+} \left(\left(-\frac{1}{3x^3}\right) \Big|_a^3 \right) = \lim_{a\to0^+} \left(\left(-\frac{1}{3(3)^3}\right) - \left(-\cancelto{\infty}{\frac{1}{3(a)^3}}\right) \right) = \infty$

É divergente

0) $\int_0^1 \frac{1}{\sqrt{1-x^2}}\,dx = \lim_{b\to 1^-}\left(\int_0^b \frac{1}{\sqrt{1-x^2}}\,dx\right) = \lim_{b\to 1^-}\left((\arcsin x)\Big|_0^b\right) = \lim_{b\to 1^-}\left(\cancelto{\frac{\pi}{2}}{\arcsin(b)} - \cancelto{0}{\arcsin(0)}\right) = \frac{\pi}{2}$

arc sen(1), estamos procurando um angulo (θ) onde sen = 1



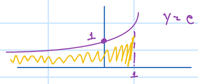
5) a) $R = \{(x,y)\in\mathbb{R}^2 \; ; \; x \le (1,0) \le y \le e^x\}$ $\quad x=1 \quad y=0$

$x \le 1 \qquad -\infty$ ate 1

$0 \le y \le e^x$, parte positiva do eixo y

$\int_a^b f(x)\,dx = \int_{-\infty}^1 e^x\,dx = \lim_{a\to-\infty}\left(\int_a^1 e^x\,dx\right)$

$= \lim_{a\to-\infty}\left((e^x)\Big|_a^1\right) = \lim_{a\to-\infty}\left(e^1 - \cancelto{0}{e^a}\right) = e$



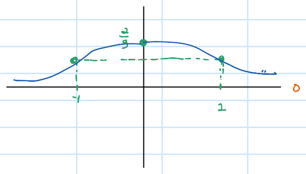
b) $R = \{(x,y)\in\mathbb{R}^2 \; ; \; 0 \le y \le \frac{2}{x^2+9}\}$

todo eixo x $\; -\infty$ até $+\infty$

$0 \le y \le \frac{2}{x^2+9}$, parte positiva do eixo y

$\int_a^b f(x)\,dx = \int_{-\infty}^{\infty} \frac{2}{x^2+9}\,dx$

$\int \frac{2}{x^2+9}\,dx = 2\int \frac{1}{x^2+3^2}\,dx = 2\cdot\frac{1}{3}\operatorname{arctg}\left(\frac{x}{3}\right) + K$



$\lim_{x\to\infty}\left(\frac{2}{x^2+9}\right) = 0$

$\lim_{x\to-\infty}\left(\frac{2}{x^2+9}\right) = 0$

$= \int_{-\infty}^0 f(x)\,dx + \int_0^\infty f(x)\,dx$

$= \lim_{a\to-\infty}\left[\left(\frac{2}{3}\operatorname{arctg}\left(\frac{x}{3}\right)\right)\Big|_a^0\right] + \lim_{b\to\infty}\left[\left(\frac{2}{3}\operatorname{arctg}\left(\frac{x}{3}\right)\right)\Big|_0^b\right]$

$= \frac{2}{3}\lim_{a\to-\infty}\left[\cancelto{0}{\operatorname{arctg}\left(\frac{0}{3}\right)} - \operatorname{arctg}\left(\frac{a}{3}\right)\right] + \frac{2}{3}\lim_{b\to\infty}\left[\operatorname{arctg}\left(\frac{b}{3}\right) - \cancelto{0}{\operatorname{arctg}\left(\frac{0}{3}\right)}\right]$

$= \frac{2}{3}\left(+\frac{\pi}{2}\right) - \frac{2}{3}\left(\frac{\pi}{2}\right)$

$= \frac{\pi}{3} + \frac{\pi}{3} = \frac{2\pi}{3}$

c) $R = \{(x,y)\in\mathbb{R}^2 \; ; \; -2 \le x \le 0 \; , \; 0 \le y \le \frac{1}{\sqrt{x+2}}\}$

$\int_{-2}^0 \frac{1}{\sqrt{x+2}}\,dx = \lim_{a\to-2^+}\left(\int_a^0 \frac{1}{\sqrt{x+2}}\,dx\right) = \lim_{a\to-2^+}\left(\left(2\sqrt{x+2}\right)\Big|_a^0\right) =$

$\lim_{a\to-2^+}\left(\left(2\sqrt{0+2}\right) - \left(2\sqrt{a+2}\right)\right) = 2\sqrt{2}$

$\int \frac{1}{\sqrt{x+2}}\,dx = \int \frac{1}{\sqrt{u}}\,du = \int u^{-\frac{1}{2}}\,du = 2\sqrt{x+2} + K$

$f(x) = \frac{1}{\sqrt{x+2}}$ $\qquad$ $\boxed{-2 \text{ até } 0}$

$x = -1 \Rightarrow y = 1$

$x = 0 \Rightarrow y = \frac{1}{\sqrt{2}}$

$\lim_{x\to-2}\left(\frac{1}{\sqrt{x+2}}\right) = \frac{1}{0} = \infty$

6) a) $\int_1^\infty \frac{1}{\sqrt{1+x^6}}\,dx$

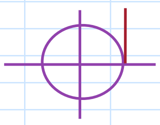
Esse lado tem que ser igual $f(x)$

$x^6 \le 1 + x^6 = \sqrt{x^6} \le \sqrt{1+x^6}$

queremos o inverso disso :

$x^3 \le \sqrt{1+x^6} \Rightarrow \frac{1}{x^3} \ge \frac{1}{\sqrt{1+x^6}}$

$\frac{\cancel{x}}{x^{\cancel{3}}} \ge \frac{x}{\sqrt{1+x^6}} = \frac{1}{x^2} \ge \frac{x}{\sqrt{1+x^6}}$



b) $\int_1^\infty \frac{\cos^2 x}{1+x^2}\,dx \qquad 0 \le \cos^2 x \le 1$

$\cos^2 x \le 1^2 = \frac{\cos^2 x}{1+x^2} \le \frac{1}{1+x^2}$

$\int_1^\infty g(x)\,dx = \int_1^\infty \frac{1}{1+x^2}\,dx = \lim_{b\to\infty}\int_1^b \frac{1}{1+x^2}$

$= \lim_{b\to\infty}\left[\left(\operatorname{artg}(x)\right)\Big|_1^b\right] = \lim_{b\to\infty}\left[\operatorname{arctg}(b) - \operatorname{arctg}(1)\right]$

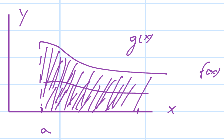
$= \frac{\pi}{2} - \frac{\pi}{4} = \frac{2\pi - \pi}{4} = \frac{\pi}{4}$

Converge

$tg\left(\frac{\pi}{4}\right) = 1$

$g(x) \ge f(x)$

se $\int_a^\infty g(x)\,dx$ converge $\quad \int_a^\infty f(x)\,dx$ converge

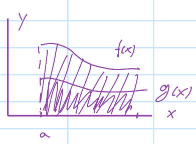


se $g(x)$ for convergente / finita

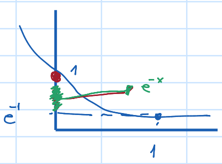
se $g(x)$ produz um valor finito então um cara menor que ele tambem produz um valor finito

$g(x) \le f(x)$

$\int_a^\infty g(x)\,dx$ diverge então um cara maior $\int_a^\infty f(x)\,dx$ tambem diverge



c) $\int_1^\infty \frac{2+e^{-x}}{x}\,dx$



$e^{-1} \le e^{-x} \le 1$

$e^{-x} \le 1$

$\frac{e^{-x}+2}{x} \le \frac{1+2}{x}$

$\frac{e^{-x}+2}{x} \le \frac{3}{x}$

$3\int_1^\infty \frac{1}{x}\,dx = 3\left(\lim_{b\to\infty}\left(\int_1^b \frac{1}{x}\,dx\right)\right)$

$= 3\left(\lim_{b\to\infty}\left(\ln|x|\right)\Big|_1^b\right)$

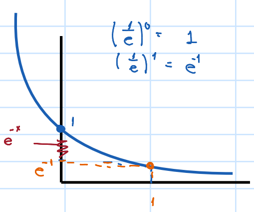
$= 3\left(\lim_{b\to\infty}\left(\ln b - \ln 1\right)\right) = \infty$

se um $\overbrace{\text{cara menor}}^{g(x)}$ diverge, $\overbrace{\text{o maior}}^{f(x)}$ diverge

$f(x) \le g(x)$ $\quad \neq$

Gabarito diverge

d) $\int_0^1 \frac{e^{-x}}{\sqrt{x}}\,dx$



$\left(\frac{1}{e}\right)^0 = 1$

$\left(\frac{1}{e}\right)^1 = e^{-1}$

$e^{-1} \le e^{-x} \le 1$

$e^{-x} \le 1$

$\frac{e^{-x}}{\sqrt{x}} \le \frac{1}{\sqrt{x}}$

$\frac{e^{-x}}{\sqrt{x}} \le \frac{1}{x^{\frac{1}{2}}}$

$f(x) \le g(x)$

$\int_0^1 \frac{1}{x^{1/2}}\,dx$

$\int_0^1 x^{-\frac{1}{2}}\,dx$

$\left(2x^{\frac{1}{2}}\right)\Big|_0^1 = 2$

$\therefore$ Converge

e) $\int_1^{\frac{\pi}{2}} \frac{1}{x\,\operatorname{sen} x}\,dx$

$-1 \le \operatorname{sen} x \le 1$

$x \cdot \operatorname{sen} x \le x$

$\frac{1}{x\,\operatorname{sen} x} \ge \frac{1}{x}$

$f(x) \ge g(x)$

$\int_1^{\frac{\pi}{2}} \frac{1}{x}\,dx = \int_1^0 \frac{1}{x}\,dx + \int_0^{\frac{\pi}{2}} \frac{1}{x}\,dx$

$= \lim_{b\to 0^-}\left(\int_1^b \frac{1}{x}\right) + \lim_{a\to 0^+}\left(\int_a^{\frac{\pi}{2}} \frac{1}{x}\,dx\right)$

$\int \frac{1}{x}\,dx = \ln|x| + K$

$\lim_{b\to 0^-}\left(\ln|x|\right)\Big|_1^b + \lim_{a\to 0^+}\left(\ln|x|\right)\Big|_a^{\frac{\pi}{2}}$

$\lim_{b\to 0^-}\left(\ln|b| - \ln|1|\right) + \lim\left(\ln\left|\frac{\pi}{2}\right| - \ln|a|\right)$

$\lim_{b\to 0^-}\left(\ln|b| - 0\right) + \lim_{a\to 0^+}\left(\ln\left|\frac{\pi}{2}\right| - \ln|a|\right)$

$g(x) < f(x)$

e $g(x)$ diverge

logo $f(x)$ diverge

f) $\int_1^\infty \frac{1}{x+e^{2x}}$

$= \lim_{x\to\infty}\left(\frac{1}{x+e^{\infty}}\right) = \frac{1}{e^{2x}}$

$\frac{1}{x+e^{\infty}} \le \frac{1}{e^{2x}}$

$\int_1^\infty \frac{1}{e^{2x}}\,dx$

$\lim_{b\to\infty}\left(\int_1^b \frac{1}{e^{2x}}\,dx\right)$

$\int e^{-2x}\,dx = -\frac{1}{2}\int e^u\,du = -\frac{1}{2}\left(e^{-2x}\right)\Big|_1^\infty$

$u = -2x$

$dx = -\frac{1}{2}\,du$

$= \lim_{b\to\infty}\left(-\frac{1}{2}\left[\cancelto{0}{e^{-2b}} - e^{-2}\right]\right) = \frac{1}{2e^2}$