# Capítulo 4

## Seção 4.7

1. a) 
$$2x - y - 2 = 0$$
;  $y = -1$ ;  $2ax - y - a^2 - 1 = 0$ 

b) 
$$5x + y - 5 = 0$$
;  $x - y + 2 = 0$ 

c) 
$$8x + 4y + 3 = 0$$
;  $(6a - 5)x - y - 3a^2 = 0$ 

**2.** a) 
$$x + 2y - 1 = 0$$
;  $x = 0$ ;  $x + 2ay - 2a^3 + a = 0$ 

b) 
$$x - 5y + 51 = 0$$
;  $x + y - 6 = 0$ 

c) 
$$x - 2y - 4 = 0$$
;  $x - (5 - 6a)y - 18a^3 + 45a^2 - 26a = 0$ 

3. 
$$4x + 4y - 5 = 0$$
 4.  $6x + y + 3 = 0$ ;  $x - 6y + 56 = 0$ 

**5.** a) 
$$16 + 2b + h \text{ m/s}$$
 b)  $22,1 \text{ m/s}; 22,01 \text{ m/s}; 22,001 \text{ m/s}$ 

2.1 m/s; 22.01 m/s; 22.001 m/s c) 16 + 2t m/s

**6.** a) 
$$\frac{-b}{4} + c$$
 b)  $\frac{2b}{t^3}$ 

**8.** a) 
$$-8x$$
 b)  $4x-1$  c)  $\frac{-1}{(x+2)^2}$  d)  $\frac{-4}{(x+3)^2}$  e)  $\frac{-1}{(2x-1)\sqrt{2x-1}}$  f)  $\frac{1}{3\sqrt[3]{(x+3)^2}}$ 

**9.** a) 
$$\frac{(x-1)^2}{-x^2+2x-2}$$
 b)  $-\left(\frac{x-1}{2-x}\right)^2$  c)  $\frac{2}{(x-1)^4}-3$  d)  $\frac{-4}{(x-1)^2}$ 

e) 
$$\frac{4x^3 - 8x^2 + 4x - 1}{(x - 1)^2}$$
 f)  $\frac{-1 - 8x(x - 1)^2}{(x - 1)^2}$  g)  $\frac{-4x}{x - 1}$ 

**12.** a) 
$$(3/4, +\infty)$$
 b)  $(-\infty, 3/4)$ 

**13.** (2,4), 
$$y = 4x - 4$$
;  $(-2,4), y = -4x - 4$  **14.** 2,  $(2,\frac{4}{3}), (-2,4)$ 

#### Seção 4.10

**1.** 
$$f'(3^+) = 2$$
;  $f'(3^-) = -2$  **2.**  $f'(1^+) = 2$ ;  $f'(1^-) = 1$ 

**3.** 
$$f'(-2^+) = 2$$
;  $f'(-2^-) = -2$  **4.**  $f'(-1^+) = 0$ ;  $f'(-1^-) = 2$ ;  $f'(1^+) = -2$ ;  $f'(1^-) = 0$ 

**5.** 
$$f'(-2^+) = 0$$
;  $f'(-2^-) = 4$ ;  $f'(2^+) = 2$ ;  $f'(2^-) = 0$ 

d) 
$$f'(x) = \begin{cases} 2x, & \text{se } |x| < 1, \\ -2x, & \text{se } |x| > 1, D = \mathbb{R} - \{-1, 1\} \end{cases}$$

#### Seção 4.12

1. 
$$2\pi r$$

2. 
$$6x + 6$$

4. 
$$\frac{3}{2x^4}$$

5. 
$$18x^2 + 6x + 12$$

6. 
$$14x + 27$$

7. 
$$-27x^8 + 30x^4 + 4x^3$$
 8.  $\frac{-20}{(5x-3)^2}$ 

8. 
$$\frac{-20}{(5x-3)^2}$$

**10.** 
$$(s^2-1)(3s-1)(15s^2+2)+3(s^2-1)(5s^3+2s)+2s(3s-1)(5s^3+2s)$$

11. 
$$7(2a x + b)$$

12. 
$$-24u^2 + 8au + 2a$$

13. 
$$\frac{-14}{(3x-1)^2}$$

14. 
$$\frac{2}{(t+1)^2}$$

15. 
$$\frac{3t^2-6t-4}{(t-1)^2}$$

$$16. \quad \frac{-t^2+4t-2}{t^2-4t+4}$$

17. 
$$\frac{-x^2+8x-5}{(5-x^2)^2}$$

18. 
$$\frac{-24}{(2x-2)^2}$$

$$19. \quad \frac{6x^3 + 27x^2 + 36 + 12}{(x+2)^2}$$

**20.** 
$$\frac{t^2 - 2bt - a^2 + 2ab}{(t - b)^2}$$

**21.** 
$$\frac{-12}{x^5} - \frac{25}{x^6}$$

**22.** 
$$2x^3 - \frac{12}{x^7}$$

**24.** 
$$A = B = 1/2$$

25. 
$$4t + 1$$

**26.** 
$$11x + 49y + 4 = 0$$

**27.** 
$$x + 64y - 1026 = 0$$

**28.** 
$$x - y - 2\sqrt{2} + 2 = 0$$
;  $x - y + 2 + 2\sqrt{2} = 0$ 

**30.** 
$$a = 3$$
;  $b = 2$ 

## Seção 4.16

**1.** a) 
$$9x + y - 6 = 0$$
;  $x + 9y - 6 = 0$ 

b) 
$$x + (2 + a)^2 y + 4 + a = 0$$
;  $x + (4 - a)^2 y - 8 + a = 0$ 

c) 
$$x = 0$$
;  $x - \sqrt{3}y + 3 = 0$ ;  $x - \sqrt{a}y + a = 0$ 

**2.** 
$$3\sqrt{3}x - 3\sqrt{3}y - 3\sqrt{3} - 2 = 0$$
;  $3\sqrt{3}x - 3\sqrt{3}y - 3\sqrt{3} + 2 = 0$ 

b) 3 m/s; 0 m/s; 
$$-9$$
 m/s;  $-24$  m/s

**3.** a) 
$$-16 \text{ m}$$
 b)  $3 \text{ m/s}$ ;  $0 \text{ m/s}$ ;  $-9 \text{ m/s}$ ;  $-24 \text{ m/s}$  c)  $0 \text{ m/s}^2$ ;  $-6 \text{ m/s}^2$ ;  $-12 \text{ m/s}^2$ ;  $-18 \text{ m/s}^2$ 

5. 
$$100(3x^2 + 7x - 3)^9(6x + 7)$$

6. 
$$\frac{3}{a}(bx^2+ax)^2(2bx+a)$$

**6.** 
$$\frac{3}{a}(bx^2+ax)^2(2bx+a)$$
 **7.**  $(7t^2+6t)^6(3t-1)^3[12(7t^2+6t)+7(3t-1)(14t+6)]$ 

8. 
$$\frac{3(7t+1)^2(-14t^2-4t+21)}{(2t^2-3)^4}$$
 9.  $\frac{4(x+1)}{\sqrt[3]{3x^2+6x-2}}$  10.  $\frac{3x-2}{(3x-1)\sqrt{3x-1}}$  11.  $\frac{-3}{2(t-1)^{3/2}(2t+1)^{1/2}}$ 

9. 
$$\frac{4(x+1)}{\sqrt[3]{3x^2+6x-2}}$$

10. 
$$\frac{3x-2}{(3x-1)\sqrt{3x-1}}$$

11. 
$$\frac{-3}{2(t-1)^{3/2}(2t+1)^{1/2}}$$

13. 
$$2^{3x^2+6x}$$
 6  $(x+1)$  ln 2

$$6[(7s^2+6s-1)^2(7s+3)-e^{-3s}$$

**12.** 
$$-\frac{1}{3}e^{3-x}$$
 **13.**  $2^{3x^2+6x}$  6  $(x+1)$  ln 2 **14.** 6  $[(7s^2+6s-1)^2(7s+3)-e^{-3s}]$  **15.**  $e^{t/2}(1/2t^2+9/2t+5)$ 

16. 
$$\frac{2}{2x+4}\log_2 e$$

17. 
$$\frac{\log_3 e}{2(s+1)}$$

18. 
$$\frac{-x-2}{x(x+1)}$$

**16.** 
$$\frac{2}{2x+4}\log_2 e$$
 **17.**  $\frac{\log_3 e}{2(s+1)}$  **18.**  $\frac{-x-2}{x(x+1)}$  **19.**  $\frac{3(\ln a)a^{3x}-a^{3x}(6x-6)\ln b}{b^{3x^3-6x}}$ 

**20.** 
$$2t(2t+1)^{t^2-1}\ln(2t+1)+2(2t+1)^{t^2-2}(t^2-1)$$
 **21.**  $\frac{b(a+bs)^{\ln(a+bs)}\ln(a+bs)}{a+bs}$  **22.**  $\sin(\frac{\pi}{2}-u)$ 

21. 
$$\frac{b(a+bs)^{\ln(a+bs)}\ln(a+bs)}{a+bs}$$

**22.** sen 
$$(\frac{\pi}{2} - u)$$

23. 
$$4\cos\theta^2\cos 2\theta - 4\theta \sin 2\theta \sin \theta^2$$

**24.** 
$$3 \sin^2(3x^2 + 6x) \cos(3x^2 + 6x)(6x + 6)$$

**25.** 
$$6 \sec^2(2x+1) + \frac{1}{2\sqrt{x}}$$

**26.** 
$$\frac{6x\sec^2 x \tan x - 3\sec^2 x}{x^2}$$

**27.** 
$$e^{2x}(2\cos 3x - 3\sin 3x)$$

**28.** 
$$6 \theta^2 \csc^2 \theta^3 \cdot \cot \theta^3$$

29. 
$$\frac{-ab \operatorname{sen} bx}{2\sqrt{\cos bx}}$$

**30.** 
$$2u^2 \sec^2 u \, tg u + 2u \, tg^2 u$$

31. 
$$-a^{\cot\theta} \ln a \csc^2 \theta$$

$$32. \quad \frac{2 \arcsin x}{\sqrt{1-x^2}}$$

33. 
$$\frac{-3t}{\sqrt{1-9t^2}}$$
 + arc cos 3t

35. 
$$\frac{1}{2x\sqrt{x-1}}$$

36. 
$$\frac{-2t^2}{|2t+3|\sqrt{(2t+3)^2-1}} + 2t \operatorname{arc cosec} (2t+3)$$

37. 
$$\frac{x \cot ghx - \ln(senhx)}{x^2}$$

38. 
$$\frac{-(t+1)\operatorname{cosech}^{2}(t+1)^{2}}{\sqrt{\operatorname{cotgh}(t+1)^{2}}}$$

38. 
$$\frac{-(t+1)\operatorname{cosech}^2(t+1)^2}{\sqrt{\operatorname{cotgh}(t+1)^2}}$$
 39.  $\frac{3}{x^2}\left(\operatorname{cosech}\frac{3x+1}{x}\right)^3\operatorname{cotgh}\left(\frac{3x+1}{x}\right)$  40. arg cosh x

**41.** 
$$\frac{2x^2}{1-x^4}$$
 + arg cotgh  $x^2$ 

42. 
$$\frac{2x \operatorname{arg cosh} x^2}{\sqrt{x^4 - 1}}$$

**42.** 
$$\frac{2x \operatorname{arg cosh} x^2}{\sqrt{x^4 - 1}}$$
 **43.**  $\frac{10}{3} (2x^5 + 6x^{-3})^4 (5x^4 - 9x^{-4})$ 

**44.** 
$$60(3x^2+6x)^9(x+1)+\frac{2}{x^3}$$

**45.** 
$$(5x-2)^5(3x-1)^2(135x-48)$$

**46.** 
$$8(2x-5)^3 - \frac{1}{(x+1)^2} - \frac{1}{2\sqrt{x}}$$

**46.** 
$$8(2x-5)^3 - \frac{1}{(x+1)^2} - \frac{1}{2\sqrt{x}}$$
 **47.**  $-\frac{1}{3}(4t^2-5t+2)^{-4/3}(8t-5)$ 

**48.** 
$$-\frac{21}{10}x^2$$
 (3)

428

**48.** 
$$-\frac{21}{10}x^2(3x+1)^{-6/5}+7x(3x+1)^{-1/5}+\frac{3}{2}(3x+1)^{-1/2}$$

**49.** 
$$12e^{3x^2+6x+7}(x+1)$$

$$50. \ \frac{e^{\sqrt{x}}}{2\sqrt{x}}$$

51. 
$$\frac{2^{\ln 2x} \ln 2}{x}$$

52. 
$$\frac{-2t^2 e^{-t^2} - e^{-t^2} - 1}{t^2}$$

**53.** 
$$\sqrt{\frac{e^t+1}{e^t-1}} \cdot \frac{e^t}{(e^t+1)^2}$$

$$54. \quad \frac{2bx^2-a}{ax}$$

**55.** 
$$\frac{7x}{7x^2-4}$$

**56.** 
$$\frac{2}{1-x^2}$$

**57.** 
$$\left(\frac{a}{b}\right)^{\sqrt{t}} \ln\left(\frac{a}{b}\right) \cdot \frac{1}{2\sqrt{t}}$$

**58.** 
$$(e^{x^2} + 4)^{\sqrt{x}} \ln (e^{x^2} + 4) \frac{1}{2\sqrt{x}} + 2x\sqrt{x} (e^{x^2} + 4)^{\sqrt{x} - 1} e^{x^2}$$

**59.** 
$$2\cos(2x+4)$$

**60.** 
$$-2\sin(2\theta^2-3\theta+1)(4\theta-3)$$
 **61.**  $-\sin 2\alpha$ 

61. 
$$-\sin 2\alpha$$

**63.** 
$$-16(2s-3)\cot^3(2s-3)^2\csc^2(2s-3)^2$$

**64.** 
$$\frac{-2 \cos x}{\sin^3 x}$$

**65.** 
$$\frac{\cos(x+1) - \sin(x+1)}{e^x}$$

**66.** 
$$-\sin^3\frac{x}{2}\cos\frac{x}{2} + \cos^3\frac{x}{2}\sin\frac{x}{2}$$
 **67.**  $-2\tan^3\frac{x}{2}$ 

68. 
$$\frac{3+2 \sin 2x}{3x-\cos 2x} \log_2 e$$

**69.** 
$$-4 \sin 2t e^{2 \cos 2t}$$

70. 
$$\frac{-2}{\sqrt{9-4x^2}}$$

71. 
$$\frac{1}{(s+1)^2} \left( \frac{s+1}{\sqrt{4-s^2}} - \arcsin \frac{s}{2} \right)$$
 72.  $\frac{2x}{x^4 - 2x^2 + 2}$ 

72. 
$$\frac{2x}{x^4-2x^2+2}$$

**73.** 
$$2\cosh(2x-1)$$

**74.** 
$$2t \, \text{tgh}(t^2 - 1)$$

**75.** 
$$16t (4t^2 - 3) \operatorname{sech}^2 (4t^2 - 3)^2$$

76. 
$$\frac{-\operatorname{sech}(\ln x)\operatorname{tgh}(\ln x)}{x}$$

77. 
$$\frac{2 \operatorname{arg senh} x}{\sqrt{x^2 + 1}}$$

78. 
$$\frac{4x}{4-x^4}$$

**79.** 
$$\frac{-(x+1)}{x\sqrt{1-4x^2}}$$
 + arg sech 2x

**80.** a) 
$$f'(x) = \begin{cases} -1, x \le 0 \\ -e^{-x}, x > 0 \end{cases}$$

b) 
$$\frac{4}{4x-3}$$

c) 
$$f'(x) = \begin{cases} 2e^{2x-1}, & x > 1/2 \\ -2e^{1-2x}, & x < 1/2 \end{cases}$$

**82.** 
$$\frac{3+2\sqrt{3}}{6}$$

**94.** a) 
$$\frac{\pi(2k+1)}{4}, k \in \mathbb{Z}$$

b) 
$$k \pi, k \in \mathbb{Z}$$

b) reta tangente 1: 
$$(1, 2), (-1, -2)$$
 c)  $y = 2x$ ;  $y = -2x$ 

c) 
$$y = 2x$$
;  $y = -2x$ 

reta tangente 2: 
$$(-1, 2), (1, -2)$$

**96.** (a) 
$$y = 3 + \sqrt{x+4}$$
,  $x \ge -4$  (b)  $y = 4x - 20$ 

(b) 
$$y = 4x - 20$$

(c) 
$$y = \frac{1}{4}x + 5$$

#### Seção 4.21

1. 
$$y^{\nu} = 0$$

2. 
$$y''' = 6a$$

3. 
$$y^{(10)} = 0$$

**4.** 
$$y'' = \frac{-3}{(3-x^2)\sqrt{3-x^2}}$$

$$5. \quad y^{iv} = \frac{24}{(x-1)^5}$$

**6.** 
$$y''' = 8e^{2x+1}$$

7. 
$$y^{iv} = \frac{1}{e^x}$$

8. 
$$y'' = \frac{-1}{r^2}$$

$$9. \quad y^{vii} = -a^7 \cos ax$$

10. 
$$y^v = \frac{1}{16} \sin \frac{x}{2}$$

11. 
$$y''' = 2 \sec^4 x + 4 \sec^2 x \cdot tg^2 x$$
 12.  $y'' = \frac{-2x}{(1+x^2)^2}$ 

12. 
$$y'' = \frac{-2x}{(1+x^2)^2}$$

**18.** a) 
$$\frac{-x^2}{y^2}$$

$$\frac{-3x^2 - 2xy}{x^2 + 2y}$$

c) 
$$-\sqrt{\frac{y}{x}}$$

d) 
$$\frac{1-y^3}{3xy^2+4y^3+1}$$

f) 
$$\frac{y}{\sec^2 y - x}$$

$$g) \qquad \frac{1}{e^{y}-1}$$

**19.** retas tangentes: 
$$x - \sqrt{3}y + 2 = 0$$
 e  $x + \sqrt{3}y + 2 = 0$ 

retas normais: 
$$\sqrt{3}x + y - 2\sqrt{3} = 0 e \sqrt{3}x - y - 2\sqrt{3} = 0$$

**23.** a) 
$$\frac{3}{2}t$$
,  $t > 0$ 

b) 
$$-\cot 2t, t \in (0, \pi/2)$$

c) 
$$-4/3 \cot t$$
,  $t \in (\pi, 2\pi)$ 

d) 
$$- \operatorname{tg} t, t \in (-\pi/2, 0)$$
 e)  $\frac{3}{2}t^2, t \in \mathbb{R}$ 

e) 
$$\frac{3}{2}t^2, t \in \mathbb{R}$$

f) 
$$- \operatorname{tg} t, t \in (0, \pi/2) \cup (\pi/2, \pi)$$

**24.** 
$$2y + 3x - 6\sqrt{2} = 0$$

**25.** 
$$2\sqrt{3}x - 2y + \sqrt{3} = 0$$
;  $x + \sqrt{3}y - 1 = 0$ 

**26.** a) 
$$3 (\Delta x)^2$$

$$\frac{2\Delta x}{\sqrt{x + \Delta x} + \sqrt{x}} - \frac{\Delta x}{\sqrt{x}}$$

**26.** a) 
$$3(\Delta x)^2$$
 b)  $\frac{2\Delta x}{\sqrt{x + \Delta x} + \sqrt{x}} - \frac{\Delta x}{\sqrt{x}}$  c)  $\frac{-3\Delta x}{(2x + 2\Delta x - 1)(2x - 1)} + \frac{3\Delta x}{(2x - 1)^2}$ 

b) 
$$-0.118; -0.12$$

c) 
$$-0.078; -0.075$$

**29.** a) 
$$\frac{6x-4}{3x^2-4x}dx$$

b) 
$$\frac{-x}{e^x} dx$$

c) 
$$10x \cos(5x^2 + 6) dx$$

**35.** 
$$\pm 24.000 \text{ m}^2$$

# Capítulo 5

# Seção 5.3

1. a) 54 gramas/dia

b) 54,5 g

c) 24,4 gramas/dia

2. -5,444 ... °C/hora

3.  $-c/100 \text{ cm}^3/\text{kgf/cm}^3$ 

4. a) 6 horas

b) 17.500 1/hora

c) 10.000 1/hora

- **5.** a) f(t) = 4.500 + 1.550 t
- b) 1.550,00/ano

c) 25,6%

- d) Tenderá para zero.
- a) 0,8 milhares de pessoas/ano
- b) 0,068 milhares de pessoas
- 7. 1/12

8. 4,875 1/hora

- 9.  $\frac{1}{\pi}$  m/hora;  $10\pi$  horas
- 10.  $\frac{d^2}{\sqrt{3}}$  m<sup>2</sup>;  $6\sqrt{3}$  m<sup>3</sup>/s.

11. a) 
$$\frac{4\pi r^2}{3}$$

b) 
$$1,066 \, \pi \, \text{m}^3/\text{s}$$

**12.** a)  $15\sqrt{3}$  cm<sup>2</sup>/s

b) 7,5 cm/s

13. 18 unidades/min

14. 119,09 km/hora

15. 1,45 m/s

16.  $\sqrt[3]{\frac{2\pi}{3V}}$ 

- 17. (a) custo fixo
  - (b) Inicialmente o custo marginal diminui e depois passa a crescer

18. (a) 120

(b) 410

- (c) 5,44; 1,2
- E = -0,087; um pequeno aumento no preço acarretará uma diminuição muito baixa da demanda.
- **20.** (a)  $\frac{y(60 0.12y)}{15 + 60y 0.06y^2}$
- (b) 0,57; o aumento de 1% na renda, acarretará um aumento de ≅ 0,57% na demanda