14) A) $r = \theta \qquad 0 \leq \theta \leq \pi$

Esboço gráfico

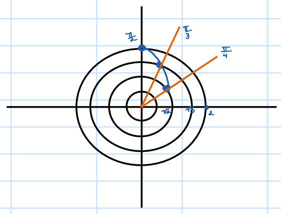
$A = \frac{1}{2}\int_0^{\pi} (\theta)^2 \, d\theta$

$\frac{1}{2}\left(\frac{\theta^3}{3}\right)\Big|_0^{\pi}$

$\frac{1}{2}\left(\frac{\pi^3}{3}\right) = \frac{\pi^3}{6}$

B) $r = 2\,\text{sen}\,\theta \qquad \frac{\pi}{4} \leq \theta \leq \frac{\pi}{2}$

	θ	$r = 2\,\text{sen}\,\theta$
45	$\pi/4$	$\sqrt{2}$
60	$\pi/3$	$\sqrt{3}$
90	$\pi/2$	2



$\frac{7\pi}{2} = \frac{4\pi}{2} + \frac{3\pi}{2}$, 2π

$\frac{1}{2}\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} (2\,\text{sen}\,\theta)^2 \, d\theta$

$\cancel{\frac{1}{2}}\int \cancel{4}\,\text{sen}^2\theta \, d\theta = 2\int \text{sen}^2\theta \, d\theta$

$\cancel{2}\int \frac{1-\cos(2\theta)}{\cancel{2}} \, d\theta = \int 1 - \cos 2\theta \, d\theta$

$\left(\theta - \frac{1}{2}\,\text{sen}\,2\theta\right)\Big|_{\frac{\pi}{4}}^{\frac{\pi}{2}} = \left(\frac{\pi}{2} - \frac{1}{2}\cancel{\text{sen}\left(2\frac{\pi}{2}\right)}^{0} - \frac{\pi}{4} + \frac{1}{2}\cancel{\text{sen}\left(2\frac{\pi}{4}\right)}^{1}\right)$

$\left(\frac{\pi}{2} - \frac{\pi}{4} + \frac{1}{2}\right) = \frac{\pi}{4} + \frac{1}{2} = \frac{\pi+2}{4}$ //

C) $r = 4 + 2\cos\theta$

Teste simetria:

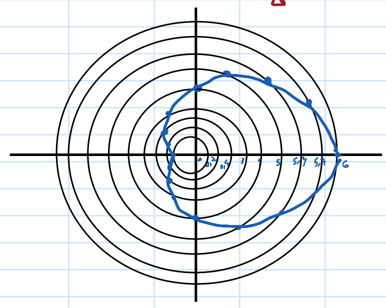
Eixo Polar: $f(-\theta) = 4 + 2\cos(-\theta) = 4 + 2\cos(\theta)$

$\theta = \frac{\pi}{2}$: $f(\pi - \theta) = 4 + 2\cos(\pi - \theta)$
$= 4 + 2[\cos(\pi)\cos(\theta) + \cancel{\text{sen}(\pi)}^{0}\,\text{sen}(\theta)]$
$= 4 + 2(-\cos\theta)$
$= 4 - 2\cos\theta$

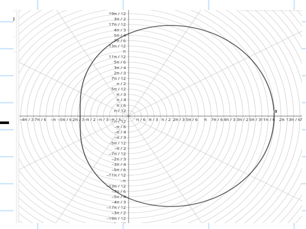
não é simétrico ao eixo $\theta = \frac{\pi}{2}$

θ	$r = 4 + 2\cos\theta$
0	6 //
$\pi/6$	$4 + \cancel{2}\cdot\frac{\sqrt{3}}{\cancel{2}} = 4 + \sqrt{3} \simeq 5{,}7$
$\pi/4$	$4 + \cancel{2}\frac{\sqrt{2}}{\cancel{2}} = 4 + \sqrt{2} \simeq 5{,}4$
$\pi/3$	$4 + 1 = 5$
$\pi/2$	4
$2\pi/3$	$4 + 2\cos\left(\frac{2\pi}{3}\right) = 3$
$3\pi/4$	$4 + 2\cos\left(\frac{3\pi}{4}\right) = 4 - \sqrt{2}$
$5\pi/6$	$4 + 2\cos\left(\frac{5\pi}{6}\right) = 4 - \sqrt{3}$
π	2

Realidade



Expectativa:



$\frac{1+\cos(2\theta)}{2}$

$2\cdot\frac{1}{2}\int_0^{\pi} (4 + 2\cos\theta)^2 \, d\theta = \cancel{2}\cdot\cancel{\frac{1}{2}}\int_0^{\pi} 16 + 4\cos^2\theta \, d\theta = \int_0^{\pi} 16\, d\theta + 4\int_0^{\pi}\cos^2\theta \, d\theta = \left(16\theta + \cancel{4}\left(\frac{1}{\cancel{2}}\left(\theta + \frac{1}{2}\,\text{sen}(2\theta)\right)\right)\right)\Big|_0^{\pi} = \left(16\theta + 2\theta + \text{sen}(2\theta)\right)\Big|_0^{\pi} = \left(16\pi + 2\pi + \cancel{\text{sen}(2\pi)}^{0}\right) = 18\pi$ //

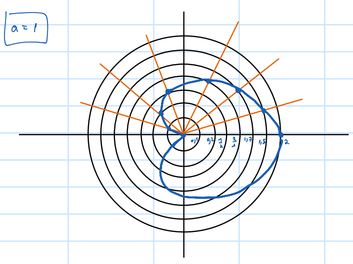
d) cardioide $r = a(1 + \cos\theta)$, sendo $a > 0$

Eixo Polar: $f(-\theta) = a(1 + \cos(-\theta)) = f(\theta)$

$\theta = \frac{\pi}{2}$: $f(\pi - \theta) = a(1 + \cos(\pi - \theta))$
$= a(1 + [\cos(\pi)\cdot\cos(\theta) + \cancel{\text{sen}(\pi)}^{0}\cdot\text{sen}(\theta)])$
$= a(1 - \cos\theta)$

θ	$r = a(1 + \cos\theta)$	
0	$2a$	
$\pi/6$	$a\left(\frac{2+\sqrt{3}}{2}\right) \simeq$	$1{,}8$
$\pi/4$	$a\left(\frac{2+\sqrt{2}}{2}\right) \simeq$	$1{,}7$
$\pi/3$	$a\left(\frac{3}{2}\right) \simeq$	
$\pi/2$	a	
$2\pi/3$	$a\cdot\frac{1}{2}$	
$3\pi/4$	$a\cdot\left(\frac{2-\sqrt{2}}{2}\right) \simeq$	$0{,}2$
$5\pi/6$	$a\left(\frac{2-\sqrt{3}}{2}\right) \simeq$	$0{,}1$
π	0	

$\boxed{a = 1}$



$2\cdot\frac{1}{2}\int_0^{\pi} [a(1 + \cos\theta)]^2 \, d\theta$

$\int_0^{\pi} a^2(1 + \cos\theta)^2 \, d\theta$

$a^2\int_0^{\pi} 1 + 2\cos\theta + \cos^2\theta \, d\theta$

$a^2\left[\int_0^{\pi} 1\, d\theta + 2\int_0^{\pi}\cos\theta\, d\theta + \int_0^{\pi}\cos^2\theta\, d\theta\right]$

$a^2\left[\theta + 2\,\text{sen}\,\theta + \frac{1}{2}\left(\theta + \frac{\text{sen}\,2\theta}{2}\right)\right]\Big|_0^{\pi}$

$a^2\left[\pi + 2\cancel{\text{sen}(\pi)}^{0} + \frac{1}{2}\left(\pi + \cancel{\frac{\text{sen}(2\pi)}{2}}^{0}\right)\right]$

$a^2\left[\pi + \frac{\pi}{2}\right] = \frac{3\pi}{2}a^2$ //

E) região de uma pétala da rosácea de 4 pétalas $r = \cos(2\theta)$

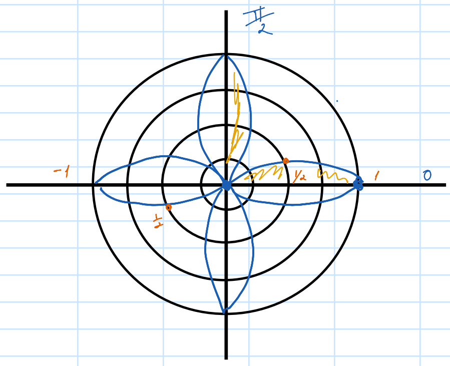
teste de simetria:

eixo polar: $f(\theta) = \cos(-2\theta) = f(\theta)$

$\theta = \frac{\pi}{2} \Rightarrow f(\pi - \theta) = \cos(2\pi - 2\theta)$
$= [\cancel{\cos(2\pi)}^{1}\cdot\cos(2\theta) - \cancel{\text{sen}(2\pi)}^{0}\cdot\text{sen}(2\theta)]$

$= \cos(2\theta)$

θ	$r = \cos(2\theta)$
0	1
$\frac{\pi}{6}$	$\frac{1}{2}$
$\frac{\pi}{4}$	0
$\frac{\pi}{3}$	$-1/2$
$\frac{\pi}{2}$	-1



$\frac{1}{2}\int_0^{\frac{\pi}{2}} \cos^2(2\theta) \, d\theta$

$\frac{1}{2}\int_0^{\frac{\pi}{2}} \frac{1 + \cos(4\theta)}{2} \, d\theta$

$\frac{1}{2}\int_0^{\frac{\pi}{2}} 1 + \cos 4\theta \, d\theta$

$\frac{1}{2}\left(\frac{1}{2}\left(\theta + \frac{\text{sen}(4\theta)}{4}\right)\right)\Big|_0^{\frac{\pi}{2}}$

$\frac{1}{2}\left(\frac{\pi}{4} + \cancel{\frac{\text{sen}(2\pi)}{4}}^{0}\right) = \frac{\pi}{8}$

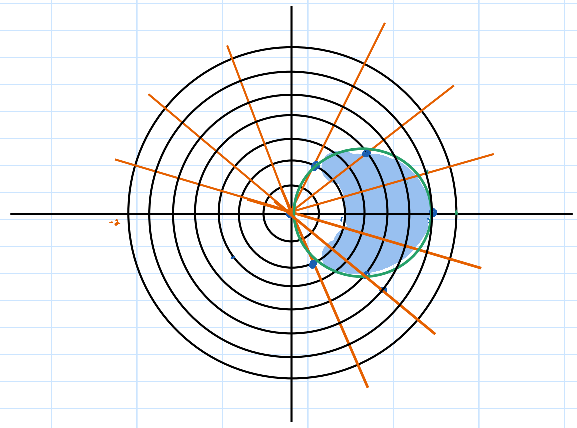
f) região interna do círculo $r = 2\cos\theta$ e externa ao $r = 1$

teste simetria:

Eixo polar: $f(-\theta) = 2\cos(-\theta) = 2\cos\theta$

$\theta = \frac{\pi}{2}$: $f(\pi - \theta) = 2\cos(\pi - \theta)$
$= 2[\cos(\pi)\cos\theta - \cancel{\text{sen}(\pi)}^{0}\cdot\text{sen}(\theta)]$
$= 2[-\cos\theta]$

θ	$r = 2\cos\theta$	
0	2	
$\pi/6$	$\sqrt{3}$	$1{,}7$
$\pi/4$	$\sqrt{2}$	$1{,}41$
$\pi/3$	1	
$\pi/2$	0	
$2\pi/3$	-1	
$3\pi/4$	$-\sqrt{2}$	
$5\pi/6$	$-\sqrt{3}$	
π	-2	



Intersecção

$2\cos\theta = 1$
$\cos\theta = \frac{1}{2}$

$\arccos\frac{1}{2} = \theta$
$60° = \theta$
$\frac{\pi}{3} = \theta$

$D = 0 \leq r \leq \frac{\pi}{3}$

$\int_0^{\frac{\pi}{3}} (2\cos\theta)^2 - 1 \, d\theta$

$\int_0^{\frac{\pi}{3}} 4\cos^2\theta \, d\theta - 1\, d\theta$

$\left[\cancel{4}\left(\frac{1}{\cancel{2}}\left(\theta + \frac{\text{sen}(\theta)}{2}\right) - \theta\right)\right]\Big|_0^{\frac{\pi}{3}}$

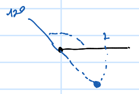
$\left[2\theta + \text{sen}\,\theta - \theta\right]\Big|_0^{\frac{\pi}{3}}$

$\left[\frac{2\pi}{3} + \text{sen}\left(\frac{\pi}{3}\right) - \frac{\pi}{3}\right]$

$\left[\frac{\sqrt{3}}{2} + \frac{\pi}{3}\right]$ //

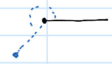
11) a) $\left(-2, \frac{2}{3}\pi\right)$

$$\begin{cases} x = -2 \cos\left(\frac{2}{3}\pi\right) = 1 \\ y = -2 \operatorname{sen}\left(\frac{2}{3}\pi\right) = -\sqrt{3} \end{cases}$$



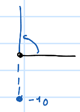
b) $\left(3, \frac{13\pi}{4}\right)$

$$\begin{cases} x = 3 \cdot -\cos(45) = 3\left(-\frac{\sqrt{2}}{2}\right) \\ y = 3 \cdot -\operatorname{sen}(45) = 3\left(-\frac{\sqrt{2}}{2}\right) \end{cases}$$



c) $\left(-10, \frac{\pi}{2}\right)$

$$\begin{cases} x = -10 \cos\left(\frac{\pi}{2}\right) = 0 \\ y = -10 \operatorname{sen}\left(\frac{\pi}{2}\right) = -10 \end{cases}$$



d) $\left(-10, \frac{3\pi}{2}\right)$

$$\begin{cases} x = -10 \cos\left(\overset{270}{\frac{3\pi}{2}}\right) = 0 \\ y = -10 \operatorname{sen}\left(\frac{3\pi}{2}\right) = 10 \end{cases}$$