INSTITUTO FEDERAL
Catarinense
Campus Blumenau

Assunto: Integrais indefinidas, integrais definidas, áreas e técnicas de integração
Professor: Fabricio Alves Oliveira

Essa lista deverá ser resolvida de forma manuscrita e entregue no dia da primeira prova.

(1) Calcule as integrais indefinidas:

(a) $\int (6x^2 - 2x + 1)\, dx$ (b) $\int \frac{x^2+1}{x^2}\, dx$ (c) $\int \frac{x^2}{x^2+1}\, dx$

(d) $\int \left(\frac{e^t}{2} + \sqrt{t} + \frac{1}{t}\right) dt$ (e) $\int \frac{\operatorname{sen} x}{\cos^2 x}\, dx$ (f) $\int \cos\theta \operatorname{tg}\theta\, d\theta$

(g) $\int (2^t - \sqrt{2}e^t + \cosh t)\, dt$ (h) $\int \sqrt{\frac{9}{1-x^2}}\, dx$ (i) $\int \frac{\ln x}{x \ln x^2}\, dx$

(2) Encontrar uma primitiva da função $f(x) = \frac{1}{x^2} + 1$ que se anule no ponto $x = 2$.

(3) Calcule as seguintes integrais usando o método da substituição:

(a) $\int (2x^2 + 2x - 3)^{10}(2x+1)\, dx$ (b) $\int \frac{x}{\sqrt[5]{x^2-1}}\, dx$ (c) $\int 5x\sqrt{4-3x^2}\, dx$

(d) $\int \sqrt{x^2 + 2x^4}\, dx$ (e) $\int (e^{2t}+2)^{\frac{1}{3}} e^{2t}\, dt$ (f) $\int \frac{e^t}{e^t+4}\, dt$

(g) $\int \operatorname{tg} x \sec^2 x\, dx$ (h) $\int \operatorname{sen}^4 x \cos x\, dx$ (i) $\int \frac{\operatorname{sen} x}{\cos^5 x}\, dx$

(j) $\int \frac{\operatorname{arcsen} x}{2\sqrt{1-x^2}}\, dx$ (k) $\int \frac{1}{t^2 - 4t + 4}\, dt$ (l) $\int \frac{\ln^2 x}{x}\, dx$

(m) $\int \frac{\cos\sqrt{x}}{\sqrt{x}}\, dx$ (n) $\int \frac{3}{x \ln^2 3x}\, dx$ (o) $\int \operatorname{cotg} u\, du$

(p) $\int \operatorname{sen} 5x\, dx$ (q) $\int \cos^2 x\, dx$ (r) $\int \operatorname{tg}^3 x \cos x\, dx$

(4) Calcule as integrais utilizando o método de integração por partes:

(a) $\int x \operatorname{sen} 5x\, dx$ (b) $\int \ln(1-x)\, dx$ (c) $\int t e^{4t}\, dt$

(d) $\int (x+1)\cos(2x)\, dx$ (e) $\int x \ln 3x\, dx$ (f) $\int \cos^3 x\, dx$

(g) $\int e^x \cos\left(\frac{x}{2}\right) dx$ (h) $\int \sqrt{x} \ln x\, dx$ (i) $\int \arccos x\, dx$

(j) $\int x \sec^2 x\, dx$ (k) $\int (x^2 - 5x)\, e^x\, dx$ (l) $\int e^{2x} \cos 3x\, dx$

(m) $\int x^3 e^{x^2}\, dx$ (n) $\int x^3 \cos x^2\, dx$ (o) $\int e^{-x} \cos 2x\, dx$

(p) $\int x^2 \operatorname{sen} x\, dx$

(**5**) Calcule as integrais definidas:

(a) $\displaystyle\int_{-1}^{2} x(1+x^3)\ dx$ (b) $\displaystyle\int_{-3}^{0} x^2-4x+7\ dx$ (c) $\displaystyle\int_{1}^{2} \frac{1}{x^6}\ dx$

(d) $\displaystyle\int_{4}^{9} 2t\sqrt{t}\ dt$ (e) $\displaystyle\int_{0}^{1} \frac{1}{\sqrt{3y+1}}\ dy$ (f) $\displaystyle\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \operatorname{sen} x\cos x\ dx$

(g) $\displaystyle\int_{-2}^{5} |2t-4|\ dt$ (h) $\displaystyle\int_{0}^{\frac{\pi}{2}} \operatorname{sen}^2 x\ dx$ (i) $\displaystyle\int_{0}^{\frac{\pi}{2}} \frac{\cos x}{(1+\operatorname{sen} x)^5}\ dx$

(j) $\displaystyle\int_{1}^{2} x\ln x\ dx$ (k) $\displaystyle\int_{\frac{\pi}{2}}^{\pi} x\operatorname{sen} x\ dx$ (l) $\displaystyle\int_{0}^{1} xe^{-x}\ dx$

(**6**) Nos itens abaixo, fazer um esboço e determinar a área da região limitada pelas curvas:

(a) $x=\frac{1}{2}, x=\sqrt{y}$ e $y=-x+2$

(b) $y^2=2x$ e $x^2=2y$

(c) $y=5-x^2$ e $y=x+3$

(d) $y=\cosh x, y=\operatorname{senh} x, x=-1$ e $x=1$

(e) $y=\operatorname{arcsen} x, y=\frac{\pi}{2}$ e $x=0$

(f) $x+y=3$ e $y+x^2=3$

(g) $y=e^x, x=0, x=1$ e $y=0$

(h) $x=y^3$ e $x=y$

(i) $y=\ln x, y=0$ e $x=4$

(j) $y=\operatorname{sen} x$ e $y=-\operatorname{sen} x$, com $x\in[0,2\pi]$

(**7**) Calcule as integrais utilizando substituição trigonométrica:

(a) $\displaystyle\int \sqrt{1-4x^2}\ dx$ (b) $\displaystyle\int \frac{1}{\sqrt{4-x^2}}\ dx$ (c) $\displaystyle\int \frac{1}{\sqrt{4+x^2}}\ dx$

(d) $\displaystyle\int \frac{1}{4+x^2}\ dx$ (e) $\displaystyle\int \frac{x}{\sqrt{1-x^2}}\ dx$ (f) $\displaystyle\int \sqrt{3-4x^2}\ dx$

(g) $\displaystyle\int \frac{x^2}{\sqrt{1-x^2}}\ dx$ (h) $\displaystyle\int x^2\sqrt{1-x^2}\ dx$ (i) $\displaystyle\int \frac{1}{x\sqrt{1+x^2}}\ dx$

(j) $\displaystyle\int \sqrt{9-(x-1)^2}\ dx$ (k) $\displaystyle\int \sqrt{-x^2+2x+2}\ dx$ (l) $\displaystyle\int \sqrt{-x^2+2x+3}\ dx$

(**8**) Calcule as integrais abaixo, utilizando o método das frações parciais:

(a) $\displaystyle\int \frac{1}{x^2-4}\ dx$ (b) $\displaystyle\int \frac{x}{x^2-5x+6}\ dx$ (c) $\displaystyle\int \frac{2x+1}{x^2-1}\ dx$

(d) $\displaystyle\int \frac{x^2+3x+1}{x^2-2x-3}\ dx$ (e) $\displaystyle\int \frac{x^2+1}{(x-2)^3}\ dx$ (f) $\displaystyle\int \frac{x^3+x+1}{x^2-2x+1}\ dx$

(g) $\displaystyle\int \frac{2x-3}{(x-1)^3}\ dx$ (h) $\displaystyle\int \frac{x+1}{x(x-2)(x+3)}\ dx$ (i) $\displaystyle\int \frac{x+5}{x^3-4x^2+4x}\ dx$

(j) $\displaystyle\int \frac{x^5+3}{x^3-4x}\ dx$ (k) $\displaystyle\int \frac{4x^2+17x+13}{(x-1)(x^2+6x+10)}\ dx$ (l) $\displaystyle\int \frac{x+2}{x^3+2x^2+5x}\ dx$

Respostas

(**1**)

(a) $2x^3 - x^2 + x + k$ (b) $x - \dfrac{1}{x} + k$ (c) $x - \operatorname{arctg} x + k$

(d) $\dfrac{1}{2}e^t + \dfrac{2}{3}t^{3/2} + \ln|t| + k$ (e) $\sec x + k$ (f) $-\cos\theta + k$

(g) $\dfrac{2^t}{\ln 2} - \sqrt{2}e^t + \operatorname{senh} t + k$ (h) $3\operatorname{arcsen} x + k$ (i) $\frac{1}{2}\ln|x| + k$

(**2**) $-\frac{1}{x} + x - \frac{3}{2}$

(**3**)

(a) $\frac{1}{22}(2x^2+2x+3)^{11} + k$ (b) $\frac{5}{8}(x^2-1)^{\frac{4}{5}} + k$ (c) $-\frac{5}{9}(4-3x^2)^{\frac{3}{2}} + k$

(d) $\frac{1}{6}(1+2x^2)^{\frac{3}{2}} + k$ (e) $\frac{3}{8}(e^{2t}+2)^{\frac{4}{3}} + k$ (f) $\ln(e^t+4) + k$

(g) $\frac{\operatorname{tg}^2 x}{2} + k$ (h) $\frac{\operatorname{sen}^5 x}{5} + k$ (i) $\frac{1}{4}\sec^4 x + k$

(j) $\frac{1}{4}(\operatorname{arcsen} x)^2 + k$ (k) $\frac{1}{2-t} + k$ (l) $\frac{\ln^3 x}{3} + k$

(m) $2\operatorname{sen}\sqrt{x} + k$ (n) $-\frac{3}{\ln 3x} + k$ (o) $\ln|\operatorname{sen} u| + k$

(p) $-\frac{1}{5}\cos 5x + k$ (q) $\frac{x}{2} + \frac{\operatorname{sen} 2x}{4} + k$ (r) $\sec x + \cos x + k$

(**4**)

(a) $-\frac{x}{5}\cos 5x + \frac{1}{25}\operatorname{sen} 5x + k$ (b) $(x-1)\ln(1-x) - x + k$ (c) $\frac{e^{4t}}{4}(t - \frac{1}{4}) + k$

(d) $\frac{x+1}{2}\operatorname{sen} 2x + \frac{1}{4}\cos 2x + k$ (e) $\frac{x^2}{2}(\ln 3x - \frac{1}{2}) + k$ (f) $\cos^2 x \operatorname{sen} x + \frac{2}{3}\operatorname{sen}^3 x + k$

(g) $\frac{2}{5}e^x\left(\operatorname{sen}\frac{x}{2} + 2\cos\frac{x}{2}\right) + k$ (h) $\frac{2}{3}x\sqrt{x}\ln x - \frac{4}{9}x\sqrt{x} + k$ (i) $x\arccos x - \sqrt{1-x^2} + k$

(j) $x\operatorname{tg} x + \ln|\cos x| + k$ (k) $\left(x^2 - 7x + 7\right)e^x + k$ (l) $\dfrac{e^{2x}}{13}(3\operatorname{sen} 3x + 2\cos 3x) + k$

(m) $\dfrac{1}{2}(x^2-1)e^{x^2} + k$ (n) $\dfrac{1}{2}(x^2\operatorname{sen} x^2 + \cos x^2) + k$ (o) $\dfrac{e^{-x}}{5}(2\operatorname{sen} 2x - \cos 2x) + k$

(p) $-x^2\cos x + 2x\operatorname{sen} x + 2\cos x + k$

(**5**)

(a) $\frac{81}{10}$ (b) 48 (c) $\frac{31}{160}$

(d) $\frac{844}{5}$ (e) $\frac{2}{3}$ (f) 0

(g) 25 (h) $\frac{\pi}{4}$ (i) $\frac{15}{64}$

(j) $2\ln 2 - \frac{3}{4}$ (k) $\pi - 1$ (l) $\frac{e-2}{e}$

(**6**)

(a) $\frac{1}{3}$ (b) $\frac{4}{3}$ (c) $\frac{9}{2}$

(d) $e - \frac{1}{e}$ (e) 1 (f) $\frac{1}{6}$

(g) $e - 1$ (h) $\frac{1}{2}$ (i) $8\ln 2 - 3$

(j) 8

(**7**)

(a) $\frac{1}{4}\left(\text{arcsen}\, 2x + 2x\sqrt{1-4x^2}\right) + k$

(b) $\text{arcsen}\, \frac{x}{2} + k$

(c) $\ln(x + \sqrt{4+x^2}) + k$

(d) $\frac{1}{2}\text{arctg}\, \frac{x}{2} + k$

(e) $-\sqrt{1-x^2} + k$

(f) $\frac{3}{4}\left(\text{arcsen}\, \frac{2x}{\sqrt{3}} + \frac{2x}{3}\sqrt{3-4x^2}\right) + k$

(g) $\frac{1}{2}\left(\text{arcsen}\, x - x\sqrt{1-x^2}\right) + k$

(h) $\frac{1}{8}\left[\text{arcsen}\, x - x\sqrt{1-x^2}(1-2x^2)\right] + k$

(i) $\ln\left|\frac{x}{1+\sqrt{1+x^2}}\right| + k$

(j) $\frac{9}{2}\text{arcsen}\, \left(\frac{x-1}{3}\right) + \frac{(x-1)\sqrt{9-(x-1)^2}}{2} + k$

(k) Dica: observe que $-x^2 + 2x + 2 = 3 - (x-1)^2$

(l) $2\text{arcsen}\, \left(\frac{x-1}{2}\right) + \left(\frac{x-1}{2}\right)\sqrt{4-(x-1)^2} + k$

(**8**)

(a) $\frac{1}{4}\ln\left|\frac{x-2}{x+2}\right| + k$

(b) $-2\ln|x-2| + 3\ln|x-3| + k$

(c) $\ln|x^2-1| + \frac{1}{2}\ln\left|\frac{x-1}{x+1}\right| + k$

(d) $x + \frac{1}{4}\ln|x+1| + \frac{19}{4}\ln|x-3| + k$

(e) $\ln|x-2| - \frac{4}{x-2} - \frac{5}{2(x-2)^2} + k$

(f) $\frac{x^2}{2} + 2x + 4\ln|x-1| - \frac{3}{x-1} + k$

(g) $\frac{-2}{x-1} + \frac{1}{2(x-1)^2} + k$

(h) $-\frac{1}{6}\ln|x| + \frac{3}{10}\ln|x-2| - \frac{2}{15}\ln|x+3| + k$

(i) $\frac{5}{4}\ln|x| - \frac{5}{4}\ln|x-2| - \frac{7}{2(x-2)} + k$

(j) $\frac{x^3}{4} + 4x - \frac{3}{4}\ln|x| + \frac{35}{8}\ln|x-2| - \frac{29}{8}\ln|x+2| + k$

(k) $2\ln|x-1| + \ln|x^2+6x+10| + \text{arctg}\,(x+3) + k$

(l) $\frac{2}{5}\ln|x| - \frac{1}{5}\ln|x^2+2x+5| + \frac{3}{10}\text{arctg}\,\left(\frac{x+1}{2}\right) + k$