

Lista 1 - Método numérico

$$\begin{array}{r} 39 \quad |2 \\ 11 \quad 19 \quad |2 \\ 11 \quad 9 \quad |2 \\ 1 \quad 4 \quad |2 \\ 0, \quad 2 \quad |2 \\ 100111 \end{array}$$

$$\begin{array}{r} 1500 \quad |2 \\ 011 \quad 750 \quad |2 \\ 011 \quad 375 \quad |2 \\ 11 \quad 197 \quad |2 \\ 11 \quad 93 \quad |2 \\ 11 \quad 46 \quad |2 \\ 011 \quad 23 \end{array}$$

$$\begin{array}{r} 23 \quad |2 \\ 11 \quad 11 \quad |2 \\ 11 \quad 5 \quad |2 \\ 11 \quad 2 \quad |2 \\ 011 \quad 14 \\ 10110 \end{array}$$

$$\begin{array}{r} 65.023 \quad |2 \\ 111 \quad 32511 \quad |2 \\ 111 \quad 16255 \quad |2 \\ 111 \quad 8127 \quad |2 \\ 111 \quad 4063 \\ 11100 \end{array}$$

$$\begin{array}{r} 253 \quad |2 \\ 011 \quad 126 \quad |2 \\ 011 \quad 63 \quad |2 \\ 1 \quad 31 \end{array}$$

$$\begin{array}{r} 31 \quad |2 \\ 11 \quad 15 \quad |2 \\ 11 \quad 7 \quad |2 \\ 11 \quad 3 \quad |2 \\ 11 \quad 1 \quad |2 \\ 11 \quad 0 \end{array}$$

$$2) a) (10111101)^2 = 1 \cdot 2^0 + 1 \cdot 2^1 + 1 \cdot 2^3 + 1 \cdot 2^5 + 1 \cdot 2^6 + 1 \cdot 2^7 = 1 + 4 + 8 + 16 + 32 + 128 = 189$$

$$b) (0.1101)^2 = 1 \cdot 2^{-1} + 1 \cdot 2^{-2} + 1 \cdot 2^{-4} = \frac{1}{2} + \frac{1}{4} + \frac{1}{16} = \frac{9}{16} + \frac{4}{16} + \frac{1}{16} = 0,8125$$

$$c) (1011.01)^2 = 1 \cdot 2^4 + 1 \cdot 2^3 + 1 \cdot 2^1 + 1 \cdot 2^0 + 1 \cdot 2^{-2} = 16 + 8 + 2 + 1 + \frac{1}{4} = 27 + \frac{1}{4} = 27,25$$

$$3) f(10,3,-4,4)$$

$$a) x_1 = 1,25 = 0,125 \times 10^1$$

$$c) -238,15 = -0,23815 \times 10^3$$

$$e) 0,000007 = 0,7 \times 10^{-5}$$

arredondamento: $-0,238 \times 10^3$

exponte menor que o esperado

truncamento: $-0,238 \times 10^3$

$$f) 719235,82 = 0,71923582 \times 10^6$$

arredondamento: $0,272 \times 10^6$

truncamento: $0,271 \times 10^6$

$$4) f(10,4,-5,5)$$

$$a) 0,1000 \times 10^{-5} = \frac{1}{10} \times \frac{1}{10^5} = \frac{1}{10^6} = 0,000001$$

$$b) 73,758 = 0,73758 \times 10^2$$

$$\text{arredondamento: } 0,7376 \times 10^2$$

$$\text{truncamento: } 0,7375 \times 10^2$$

$$c) \text{Se } a = 42450 \text{ e } b = 3$$

$$a = 0,42450 \times 10^5 \quad 0,4250 \times 10^5 + 0,00003 \times 10^5$$

$$b = 0,3 \times 10^4 \quad 0,42503 \times 10^5$$

$$5) \text{Números representados no sistema } f(2,3,-1,2)$$

$$\text{base } = 2$$

$$\text{exponente } = \{-1, \dots, 2\}$$

$$\text{números positivos } p^{e+1} = 8$$

$$0,000 \times 2^{-1}$$

$$0,001 \times 2^0$$

$$0,010 \times 2^1$$

$$0,011 \times 2^2$$

$$0,100 \times 2^3$$

$$0,101 \times 2^4$$

$$0,110 \times 2^5$$

$$0,111 \times 2^6$$

$$0,1000 \times 2^7$$

$$0,1001 \times 2^8$$

$$0,0100 \times 2^9$$

$$0,0101 \times 2^{10}$$

$$0,0110 \times 2^{11}$$

$$0,0111 \times 2^{12}$$

$$0,10000 \times 2^{13}$$

$$0,10001 \times 2^{14}$$

$$0,01000 \times 2^{15}$$

$$0,01001 \times 2^{16}$$

$$0,01010 \times 2^{17}$$

$$0,01011 \times 2^{18}$$

$$0,01100 \times 2^{19}$$

$$0,01101 \times 2^{20}$$

$$0,01110 \times 2^{21}$$

$$0,01111 \times 2^{22}$$

$$0,100000 \times 2^{23}$$

$$0,100001 \times 2^{24}$$

$$0,010000 \times 2^{25}$$

$$0,010001 \times 2^{26}$$

$$0,010010 \times 2^{27}$$

$$0,010100 \times 2^{28}$$

$$0,010110 \times 2^{29}$$

$$0,011000 \times 2^{30}$$

$$0,011010 \times 2^{31}$$

$$0,011100 \times 2^{32}$$

$$0,011110 \times 2^{33}$$

$$0,011111 \times 2^{34}$$

$$0,1000000 \times 2^{35}$$

$$0,1000001 \times 2^{36}$$

$$0,0100000 \times 2^{37}$$

$$0,0100001 \times 2^{38}$$

$$0,0100010 \times 2^{39}$$

$$0,0100011 \times 2^{40}$$

$$0,0100100 \times 2^{41}$$

$$0,0100101 \times 2^{42}$$

$$0,0100110 \times 2^{43}$$

$$0,0100111 \times 2^{44}$$

$$0,0101000 \times 2^{45}$$

$$0,0101001 \times 2^{46}$$

$$0,0101010 \times 2^{47}$$

$$0,0101011 \times 2^{48}$$

$$0,0101100 \times 2^{49}$$

$$0,0101101 \times 2^{50}$$

$$0,0101110 \times 2^{51}$$

$$0,0101111 \times 2^{52}$$

$$0,10000000 \times 2^{53}$$

$$0,10000001 \times 2^{54}$$

$$0,01000000 \times 2^{55}$$

$$0,01000001 \times 2^{56}$$

$$0,01000010 \times 2^{57}$$

$$0,01000011 \times 2^{58}$$

$$0,01000100 \times 2^{59}$$

$$0,01000101 \times 2^{60}$$

$$0,01000110 \times 2^{61}$$

$$0,01000111 \times 2^{62}$$

$$0,01001000 \times 2^{63}$$

$$0,01001001 \times 2^{64}$$

$$0,01001010 \times 2^{65}$$

$$0,01001011 \times 2^{66}$$

$$0,01001100 \times 2^{67}$$

$$0,01001101 \times 2^{68}$$

$$0,01001110 \times 2^{69}$$

$$0,01001111 \times 2^{70}$$

$$0,01010000 \times 2^{71}$$

$$0,01010001 \times 2^{72}$$

$$0,01010010 \times 2^{73}$$

$$0,01010011 \times 2^{74}$$

$$0,01010100 \times 2^{75}$$

$$0,01010101 \times 2^{76}$$

$$0,01010110 \times 2^{77}$$

$$0,01010111 \times 2^{78}$$

$$0,01011000 \times 2^{79}$$

$$0,01011001 \times 2^{80}$$

$$0,01011010 \times 2^{81}$$

$$0,01011011 \times 2^{82}$$

$$0,01011100 \times 2^{83}$$

$$0,01011101 \times 2^{84}$$

$$0,01011110 \times 2^{85}$$

$$0,01011111 \times 2^{86}$$

$$0,100000000 \times 2^{87}$$

$$0,100000001 \times 2^{88}$$

$$0,010000000 \times 2^{89}$$

$$0,010000001 \times 2^{90}$$

7)

a) $x = 0,0020$ $\bar{x} = 0,0021$

absoluto = $|0,0020 - 0,0021| = 0,001$

relativo = $\frac{0,001}{0,0020} = 0,05$

b) $x = 53\,000$ $\bar{x} = 52\,940$

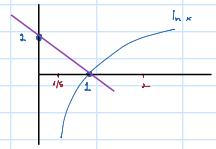
absoluto = $|53\,000 - 52\,940| = 600$

relativo = $\frac{600}{53\,000} = 0,001132$

8)

a) $\ln x - 2x + 2 = 0$

$\ln x = 2x - 2$



[$\frac{1}{2}, 2]$

b) $e^x - \sin x = 0$

[-4, -3]

c) $\ln x + 2 - 2^x = 0$

$\ln x = 2^x - 2$

2 - 2 = 0



[$\frac{1}{2}, 2]$

d) $2 \cos x + x = 0$

$\cos x = -\frac{x}{2}$

[-2, -1]

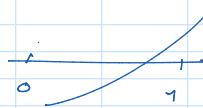
9) Método bisssecção e aproxime a menor raiz positiva com erro menor que $\epsilon = 10^{-1} \approx 0,1$

a) $2 \cos x - \frac{e^x}{2} = 0$

escolher um intervalo

$x=0 \Rightarrow 2 \cos 0 - \frac{e^0}{2} = \frac{3}{2} = 1,5$

$x=1 \Rightarrow 2 \cos 1 - \frac{e^1}{2} = -0,2784$



b) $3 \ln x - \frac{x^2}{2} = 0$

$x=1 \Rightarrow 3 \ln 1 - \frac{1}{2} = -\frac{1}{2}$

$x=2 \Rightarrow 3 \ln 2 - \frac{4}{2} = 0,07$

K	a	b	$x_k = \frac{a+b}{2}$	signat f(a)	signat f(x_k)	signat f(b)	b-a
0	0	2	0,5	+	+	-	2
1	0,5	1	0,75	+	+	-	0,5
2	0,75	1	0,875	+	-	-	0,125
3	0,75	0,875	0,8125	+	+	-	0,125
4	0,8125	0,875	0,843750	+	+	-	0,0625

fun. fixa = 0
[a, x_k] troca s_B
+ -
f(x_k) < 0 [b, x_k] fixa A
+ +

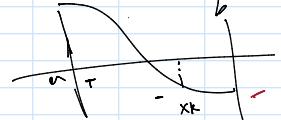
10)

a) $2 \cos x - \frac{e^x}{2} = 0$ $\epsilon = 0,5 \times 10^{-6}$

$K > \frac{\log(1-\epsilon) - \log(0,5 \times 10^{-6})}{\log(2)} = K > 6,3010 \approx 20,93$

b) $3 \ln x - \frac{x^2}{2} = 0$ [$1,2$]

$K > \frac{\log(2-1) - \log(0,5 \times 10^{-6})}{\log(3)} = K = 24$

 \therefore o num de interacções $\leq K = 24$

11) $f(x) = x^2 + x - \frac{1}{4}$ [-0,4, 0,4] erro absoluto = $0,15 \cdot 10^{-2}$

achar as funções $\varphi(x)$:

$\varphi(x) = \frac{1}{4} - x^2$ $\varphi'(x) = -2x$

teste de convergência $\varphi(x)$:→ Continua: $\varphi(x)$ e $\varphi'(x)$ são funções polinomiais logo são continuas em todo \mathbb{R}

$| \varphi'(x) | < 1 \Leftrightarrow | -2x | < 1$

$| -2x | < 1 \Rightarrow 2x < 1 \Rightarrow x < \frac{1}{2} \approx 0,5 < 1$

OK

→ Aproximação inicial $x_0 \in I$

$x_0 = 0$

$\varphi(x) = \frac{1}{4} - x^2$

$x_k = 0$

K	$\varphi(x_k)$	$ x_k - x_{k-1} $
0	0	Valor inicial
1	0,25	$ 0,25 - 0 = 0,25$
2	0,1875	$ 0,1875 - 0,25 = 0,0625$
3	0,1484375	$ 0,1484375 - 0,1875 = 0,0390625$
4	0,12384...	$ 0,12384... - 0,1484375 = 0,02458...$
5	0,1084492544	$ 0,1084492544 - 0,12384... = 0,01539...$

OK

12) Para mostra que $\varphi(x) = \frac{x}{x}$ não irá convergir/aproximar quando $v = \sqrt{a}$

Condições de convergência

- i) $\varphi(x)$ e $\varphi'(x)$ são continuas
- ii) $|\varphi'(x)| < m < 1$
- iii) $x_0 \in I$

derivada de $f(x)$
 $f'(x) = -\frac{a}{x^2}$

avalie a derivada no ponto fixo

$\varphi'(v) = -\frac{a}{(v)^2} = -\frac{a}{|v|} = -1$

ii) $|\varphi'(x)| < 1$

$|\varphi'(v)| < 1$

$| -1 | < 1$

 \therefore a p de intervalo

não converge para

$| -1 | < 1$

definir ponto fixo

$x = f(x)$

$x = f(x) = \frac{x}{x}$

$x^2 = a$

$x = \pm \sqrt{a}$

$$13) f(x) = x^3 - 0,5x^2 - 2,5x - 1,5$$

ii) verifica a convergência no $I = (2,3)$

$$|\varphi'(x)| < 1$$

$$\text{a) } f(2) = 8 - 4 - 5 - 1,5 = -0,5$$

$$f(3) = 27 - 4,5 - 7,5 - 1,5 = 13,5$$

$$\varphi'(2) = \frac{1}{3}(0,5 \cdot (2)^2 + 2,5 \cdot 2 - 1,5)^{-\frac{2}{3}} \cdot (2 \cdot 2,5)$$

$$\text{b) } x^3 - 0,5x^2 - 2,5x - 1,5$$

$$x^3 = 0,5x^2 + 2,5x - 1,5$$

$$\varphi(x) = x = \sqrt[3]{0,5x^2 + 2,5x - 1,5}$$

$$\text{c) } \varphi(x) = \frac{1}{3}(0,5x^2 + 2,5x - 1,5)^{-\frac{2}{3}} \cdot (x + 2,5)$$

$$14) f(x) = e^x - 4x^2 \in I \text{ a raiz no } I = (0,1)$$

$$x_0 = 0,5, \quad \varepsilon = 10^{-4}$$

0,0001

$$x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)}$$

$$\text{a) } \varphi(x) = \frac{1}{2}e^{\frac{x}{2}}$$

$$\text{b) } f(x) = e^x - 4x^2$$

k	$\varphi(x_k)$	$ x_k - x_{k-1} < \varepsilon$ (1 interação)
0	0,5	
1	0,4201	0,4201
2	0,42926	0,42925
3	0,42953	0,42952
4	0,42956	0,42955
5	0,42954	0,42953
6	0,42953	0,42952
7	0,42953	0,42951
8	0,42953	0,42950

$$x_0 = 0,5$$

$$f'(x) = e^x - 8x$$

k	x_k	$ f(x_k) < \varepsilon$
1	0,5	0,64872
2	0,42926	0,235
3	0,42953	0,0093
4	0,42953	0,00001

$$15) \text{ a) } \lim x = 0 \quad x_0 = 3 \quad 0,00000005$$

$$x_{k+1} = x_k - \frac{\sin x}{\cos x} = x_k - \varphi(x_k)$$

k	x_k	$ f(x_k) < \varepsilon$
0	3	0,14112
1	3,1425	0,00009
2	3,14159	0

$$\text{b) } \lim x \rightarrow 1 = 0$$

k	x_k	$ f(x_k) < \varepsilon$
0	3	
1	3,07091	
2	3,10626	
3	3,1233	
4	3,13255	
5	3,13917	
6	3,13938	
7	3,14048	
8	3,14103	
9	3,14131	
10	3,14145	
11	3,14152	
12	3,14155	
13	3,14157	
14	3,14159	$0 < \varepsilon$