14. Se $A = \begin{pmatrix} 3 & -2 \\ -4 & 3 \end{pmatrix}$, ache B, de modo que $B^2 = A$.

$$B = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \qquad B \cdot B = B^2 = A = \begin{pmatrix} 3 & -2 \\ -4 & 3 \end{pmatrix}$$

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} a^2 + bc & ab + bd \\ ac + cd & bc + d^2 \end{pmatrix} = \begin{pmatrix} 3 & -2 \\ -4 & 3 \end{pmatrix}$$

$$\begin{cases} a^2 + bc = 3 & (1) \\ ab + bd = -2 & (2) \\ ac + cd = -4 & (3) \\ bc + d^2 = 3 & (4) \end{cases}$$

De (2) e (3)

$$\begin{cases} ac + cd = -4 \\ ab + bd = -2 \end{cases}$$

$$\frac{c\cancel{(a+d)}}{b\cancel{(a+d)}} = \frac{-4}{-2} \qquad a + d \neq 0 \qquad a \neq -d$$

$$\frac{c}{b} = \frac{-4}{-2}$$

$$\frac{c}{b} = 2$$

$$\boxed{\begin{matrix} c = 2b \\ \text{e } a \neq -d \end{matrix}} \quad (5)$$

De 1 e 4

$$\begin{cases} a^2 + bc = 3 \\ bc + d^2 = 3 \end{cases} \Rightarrow a^2 + \cancel{bc} = \cancel{bc} + d^2$$

$$a^2 = d^2$$

$$a^2 - d^2 = 0$$

$$\underbrace{(a+d)}_{\neq 0}\underbrace{(a-d)}_{=0} = 0$$

5

$$a - d = 0$$

$$\boxed{a = d} \quad 6$$

$$a = d \neq 0.$$

Como $c = 2b$ e $a = d$; segue

$$B = \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} a & b \\ 2b & a \end{pmatrix}$$

$$A = B^2 = \begin{pmatrix} a & b \\ 2b & a \end{pmatrix}\begin{pmatrix} a & b \\ 2b & a \end{pmatrix} = \begin{pmatrix} 3 & -2 \\ -4 & 3 \end{pmatrix}$$

$$\begin{pmatrix} a^2 + 2b^2 & 2ab \\ 4ab & a^2 + 2b^2 \end{pmatrix} = \begin{pmatrix} 3 & -2 \\ -4 & 3 \end{pmatrix}$$

$$\begin{cases} a^2 + 2b^2 = 3 & 7 \\ 2ab = -2 & 8 \end{cases}$$

8 $2ab = -2$

$ab = -1$

$\boxed{a = -1/b}$

9

7 $a^2 + 2b^2 = 3$

Suponha $a, b \in \mathbb{Z}$

então p/ $a^2 + 2b^2 = 3$

segue:

$(a, b) = (\pm 1, \pm 1)$

MAS $a = -1/b$

então

$(a, b) = (1, -1)$

ou $(a, b) = (-1, 1)$

voltando:

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} a & b \\ 2b & a \end{pmatrix} = B \ ?$$

se $(a, b) = (1, -1)$

Então:

$$B = \begin{pmatrix} 1 & -1 \\ -2 & 1 \end{pmatrix}$$

$$B^2 = \begin{pmatrix} 1 & -1 \\ -2 & 1 \end{pmatrix} \begin{pmatrix} 1 & -1 \\ -2 & 1 \end{pmatrix}$$

$$B^2 = \begin{pmatrix} 3 & -2 \\ -4 & 3 \end{pmatrix} = A$$

ok.

se $(a, u) = (-1, 1)$

então:

$$B = \begin{pmatrix} -1 & 1 \\ 2 & -1 \end{pmatrix}$$

$$B^2 = \begin{pmatrix} -1 & 1 \\ 2 & -1 \end{pmatrix} \begin{pmatrix} -1 & 1 \\ 2 & -1 \end{pmatrix} = \begin{pmatrix} 3 & -2 \\ -4 & 3 \end{pmatrix} = A$$

ok

Então: $B = \left\{ \begin{pmatrix} -1 & 1 \\ 2 & -1 \end{pmatrix}, \begin{pmatrix} 1 & -1 \\ -2 & 1 \end{pmatrix} \right\}$