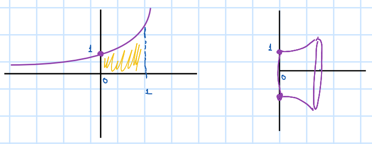
Giovani Zanella da Maia

1) $\int_a^b \pi r^2(x)\, dx$

a) $y = e^x$, $y = 0$, $x = 0$, $x = 1$

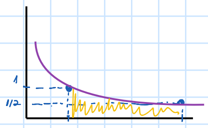


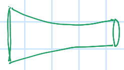
$\int_0^1 \pi (e^{x^2})\, dx = \pi \int_0^1 (e^{2x})\, dx$

$u = 2x$, $du = dx$ $= \pi \int_0^1 e^u\, du = \pi (e^{2x}) \Big|_0^1$

$= \pi (e^2 - e^0) = \pi (e^2 - 1)$ //

b) $y = \frac{1}{x}$, $x = 1$, $x = 2$, $y = 0$





$\int_1^2 \pi \left(\frac{1}{x}\right)^2 dx = \pi \int_1^2 \left(\frac{1}{x^2}\right) dx$

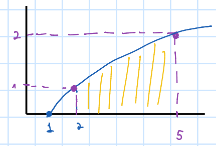
$= \pi \int_1^2 x^{-2}\, dx = \pi \left(-\frac{1}{x}\right)\Big|_1^2$

$= \pi \left(-\frac{1}{2} - \left(-\frac{1}{1}\right)\right) = \pi \left(-\frac{1}{2} + 1\right)$

$= \frac{\pi}{2}$ //

c) $y = \sqrt{x-1}$, $x = 2$, $x = 5$, $y = 0$

$x - 1 \geq 0$
$x \geq 1$



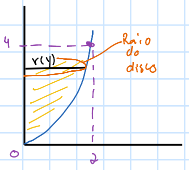
$\int_2^5 \pi (\sqrt{x-1})^2\, dx$

$\pi \int_2^5 x - 1\, dx = \pi \left(\frac{x^2}{2} - x\right)\Big|_2^5$

$\pi \left(\frac{25}{2} - 5 - \left(\frac{2^2}{2} - 2\right)\right) = \pi \frac{15}{2} = \frac{15\pi}{2}$ //

d) x =√y, x = 0, y = 4; em torno do eixo y

$y \geq 0$
$y = 4 \Rightarrow x = \sqrt{4} \Rightarrow x = 2$



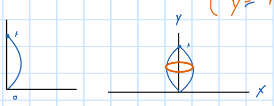
$\int_0^4 \pi (\sqrt{y})^2\, dy$

$= \pi \int_0^4 y\, dy = \pi \left(\frac{y^2}{2}\right)\Big|_0^4$

$= \pi \left(\frac{4^2}{2} - 0\right) = 8\pi$ //

E) x = y − y², x = 0; em torno do eixo y

$y(1 - y) = 0 \Rightarrow \begin{cases} y = 0 \\ y = 1 \end{cases}$



$\int_0^1 \pi (y - y^2)^2\, dy$

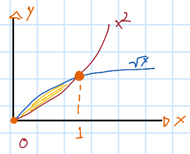
$\pi \int y^2 - 2y^3 + y^4\, dy$

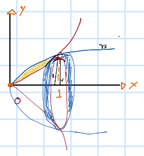
$\pi \left(\frac{y^3}{3} - \frac{y^4}{2} + \frac{y^5}{5}\right)\Big|_0^1$

$\pi \left(\frac{1}{3} - \frac{1}{2} + \frac{1}{5}\right) = \frac{\pi}{30}$

f) y = x², y = √x; em torno do eixo x

$\begin{cases} y = x^2 \\ y = \sqrt{x} \end{cases} \Leftrightarrow$ $x^2 = \sqrt{x}$
$(x^2)^2 = (\sqrt{x})^2$
$x - x^4 = 0$
$x(1 - x^3) = 0$
$x = 1,\ x = 0$





$\pi \int_0^1 (\sqrt{x})^2 - (x^2)^2\, dx$

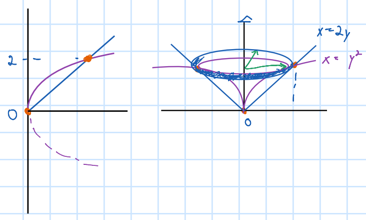
$\pi \int_0^1 x - x^4\, dx$

$\pi \left(\frac{x^2}{2} - \frac{x^5}{5}\right)\Big|_0^1$

$\pi \left(\frac{1}{2} - \frac{1}{5}\right) = \pi \left(\frac{3}{10}\right) = \frac{3\pi}{10}$

G) y² = x, x=2y; em torno do eixo y

$\begin{cases} x = y^2 \\ x = 2y \end{cases} \Leftrightarrow$ $y^2 = 2y$
$y^2 - 2y = 0$
$y(y-2) = 0$
$\begin{cases} x = 2 \\ x = 0 \end{cases}$



$\int_0^2 \pi (2y)^2 - (y^2)^2\, dy$

$\pi \int_0^2 4y^2 - y^4\, dy$

$\pi \left(\frac{4y^3}{3} - \frac{y^5}{5}\right)\Big|_0^2$

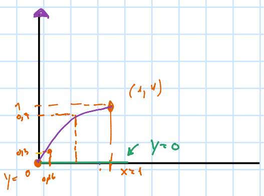
$\pi \left(\frac{4(2)^3}{3} - \frac{2^5}{5}\right)$

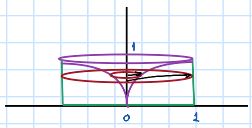
$\pi \left(\frac{32}{3} - \frac{32}{5}\right) = \pi \left(\frac{160 - 96}{15}\right)$

$= \frac{64\pi}{15}$

h) y= x^(2/3) x = 1, y = 0; em torno do eixo y

$y = x^{\frac{2}{3}} \Rightarrow x = \sqrt{y^3}$





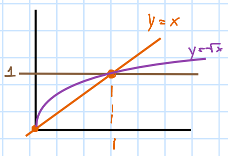
$\int_0^1 \pi \left((1)^2 - (\sqrt{y^3})^2\right) dy$

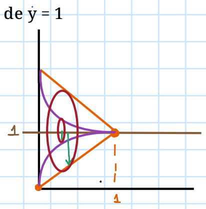
$\pi \int_0^1 1 - y^3\, dy$

$\pi \left(y - \frac{y^4}{4}\right)\Big|_0^1$

$\pi \left(1 - \frac{1}{4}\right) = \frac{3\pi}{4}$

i) y = x, y =√x; em torno de y = 1





intersecção $(1,1)(0,0)$

$\int_0^1 \pi (1 - x)^2 - (1 - \sqrt{x})^2\, dx$

$\pi \int_0^1 1 - 2x + x^2 - (1 - 2\sqrt{x} + x)$

$\pi \int_0^1 \cancel{1} - 2x + x^2 - \cancel{1} + 2\sqrt{x} - x\, dx$

$\pi \int_0^1 -3x + x^2 + 2\sqrt{x}\, dx$

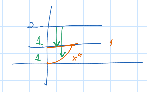
$\pi \left(-\frac{3x^2}{2} + \frac{x^3}{3} + \frac{4}{3}x^{\frac{3}{2}}\right)\Big|_0^1$

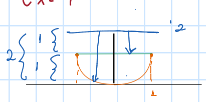
$\pi \left(-\frac{3}{2} + \frac{1}{3} + \frac{4}{3}\right) = \frac{\pi}{6}$

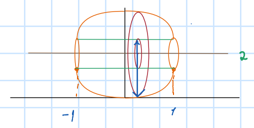
J) y = x⁴, y = 1; em torno de y = 2

$x^4 = 1 \Rightarrow x^4 - 1 = 0$

$\begin{cases} x = 1 \\ x = -1 \end{cases}$







$1 \to 2$

$x = g''(x)$

$g''(y) > 0$ direita (

$g''(y) < 0$ esquerda)

$\pi \int_{-1}^1 (2 - x^4)^2 - (2 - 1)^2\, dx$

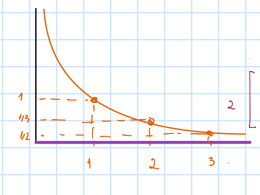
$\pi \int 4 - 4x^4 + x^8 - 1\, dx$

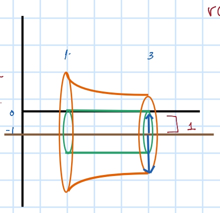
$\pi \int x^8 - 4x^4 + 3\, dx = \pi \left(\frac{x^9}{9} - \frac{4x^5}{5} + \frac{3x}{1}\right)\Big|_{-1}^1$

$\pi \left(\left(\frac{1}{9} - \frac{4}{5} + \frac{3}{1}\right) - \left(\frac{-1}{9} + \frac{4}{5} - \frac{3}{1}\right)\right)$

$\pi \left(\frac{104}{45} + \frac{104}{45}\right) = \frac{208\pi}{45}$ //

L) y = 1/x , y = 0, x = 1, x = 3; em torno de y = −1





$R(x) = \frac{1}{x} - (-1) = \frac{1}{x} + 1$

$r(x) = 0 - (-1) = 1$

$\pi \int_1^3 \left(\frac{1}{x} + 1\right)^2 - (1)^2\, dx$

$\pi \int \frac{1}{x^2} + 2\frac{1}{x}\, dx = \pi \left(-\frac{1}{x} + 2\ln|x|\right)\Big|_1^3$

$\pi \left[\left(-\frac{1}{3} + 2\ln|3|\right) + 1\right]$

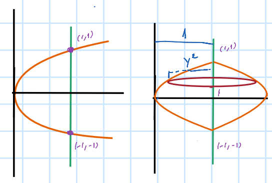
$\pi \left(\frac{2}{3} + 2\ln|3|\right) = 2\pi \left(\frac{1}{3} + \ln|3|\right)$

M) x = y², x = 1; em torno de x = 1

$x = y^2$
$x = 1$

$y^2 = 1$
$y^2 - 1 = 0$

$\begin{cases} y = 1 \\ y = -1 \end{cases}$



$= \int_{-1}^1 \pi (1 - y^2)^2\, dy$

$= \pi \int_{-1}^1 1 - 2y^2 + y^4\, dy$

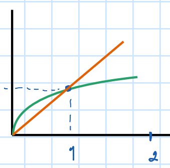
$= \pi \left(y - \frac{2y^3}{3} + \frac{y^5}{5}\right)\Big|_{-1}^1$

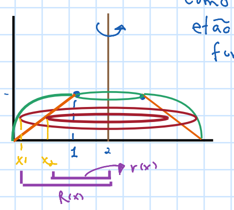
$= \pi \left(1 - \frac{2}{3} + \frac{1}{5} - \left(-1 + \frac{2}{3} - \frac{1}{5}\right)\right) =$

$= \pi \left(\frac{15 - 10 + 3}{15} - \left(\frac{-15 + 10 - 3}{15}\right)\right) = \pi \left(\frac{8}{15} - \left(-\frac{8}{15}\right)\right) = \frac{16}{15}\pi$

n) y = x, y =√x; em torno de x = 2

$x^2 = (\sqrt{x})^2$
$x^2 = x$
$x^2 - x = 0$
$x(x - 1) = 0$
$\begin{cases} x = 0 \\ x = 1 \end{cases}$





Como a rotação é em $x = 2$ então os raios estão em função de y

$y = x \Rightarrow x_2 = y$
$y = \sqrt{x} \Rightarrow x_1 = y^2$

$R(y) = 2 - x_1 = 2 - y^2$
$r(y) = 2 - x_2 = 2 - y$

$\int_0^1 \pi (2 - y^2)^2 - (2 - y)^2\, dy$

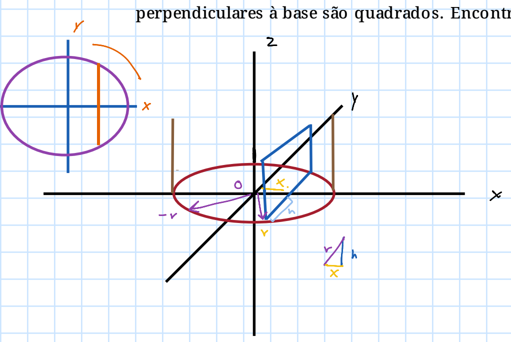
$\pi \int_0^1 4 - 4y^2 + y^4 - (4 - 4y + y^2)\, dy$

$\pi \int_0^1 y^4 - 5y^2 + 4y\, dy$

$\pi \left(\frac{y^5}{5} - \frac{5y^3}{3} + 2y\right)\Big|_0^1$

$\pi \left(\frac{1}{5} - \frac{5}{3} + 2\right) = \pi \cdot \left(\frac{3 - 25 + 30}{15}\right) = \pi \left(\frac{8}{15}\right)$ //

2) A base de um sólido S é um disco circular de raio r. Secções transversais paralelas, perpendiculares à base são quadrados. Encontre o volume de S.



$\int_{-r}^{r} A(x)\,dx$ ou $2\int_{0}^{r} A(x)\,dx$

Metodo de um comprimento(lado) de um quadrado é :

$\sqrt{r^2-x^2}$

Já o comprimento total do quadrado é :

$L = 2\sqrt{r^2-x^2}$

A área do quadrado é : $A = L^2$

$A = \left(2\sqrt{r^2-x^2}\right)^2 = 4(r^2-x^2)$

$\int_{-r}^{r} 4(r^2-x^2)\,dx$

$4\int_{-r}^{r} r^2-x^2\,dx$

$4\left(r^2(x) - \frac{x^3}{3}\right)\Big|_{-r}^{r}$

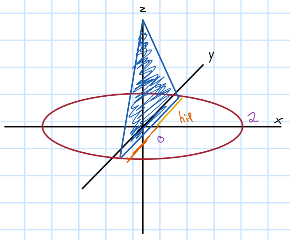
$4\left(\left(r^2(r)-\frac{r^3}{3}\right)-\left(r^2(-r)-\frac{-r^3}{3}\right)\right)$

$4\left(r^3-\frac{r^3}{3}+r^3-\frac{r^3}{3}\right)$

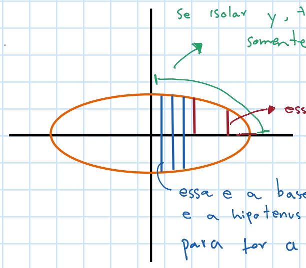
$4\left(2r^3-\frac{2r^3}{3}\right) = 4\left(\frac{6r^3-2r^3}{3}\right)$

$4\left(\frac{4r^3}{3}\right) = \frac{16r^3}{3}$ //

3) A base de um sólido S é uma região elíptica limitada pela curva $9x^2 + 4y^2 = 36$. As secções transversais perpendiculares ao eixo x são triângulos isósceles retos com hipotenusa na base. Determine o volume do sólido S.



a base e a região elíptica da curva $9x^2+4y^2=36$ se isolar y, teremos a função da curva, somente do 1º quadrante



essa altura é a função

essa e a base do triangulo e a hipotenusa, multiplicar f(x) por 2 para for a parte inferior do x

achar função da curva :

$9x^2+4y^2=36$

$4y^2 = 36-9x^2$

$y = \frac{\sqrt{36-9x^2}}{4}$

hipotenusa/base triangulo é :

$L = \frac{2\sqrt{36-9x^2}}{4} = \frac{\sqrt{36-9x^2}}{2}$

Intervalo é isolar x

$y=0 \Rightarrow 9x^2-4y^2=36$

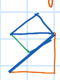
$9x^2=36 \Rightarrow x^2=\frac{36}{9}$

$x=\sqrt{\frac{36}{9}} \Rightarrow x=\frac{6}{3}=2$

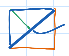
$-2<y\le 2$

$2(0\le y\le 2)$

triângulo isoceles (2 lados iguais)



hipotenusa



do triangulo conseguimos um quadrado cujo $A(x)=L^2$

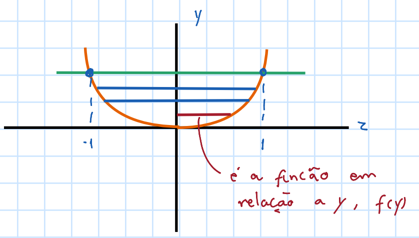
ou usar : $A(x)=\frac{b\cdot h}{2}$ com isso cateto = $\frac{hip}{\sqrt{2}}$

$\int_a^b A(x)\,dx$

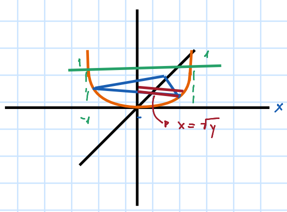
$A(x) = L^2 = \left(\frac{\sqrt{36-9x^2}}{2}\right)^2 = \frac{36-9x^2}{4}$

$2\int_0^2 \frac{36-9x^2}{4}\,dx = 2\int_0^2 \frac{36}{4}-\frac{9x^2}{4}\,dx = 2\left(\frac{36}{4}x-\frac{9}{4}\cdot\frac{x^3}{3}\right)\Big|_0^2 = 2\left(9x-\frac{3}{4}x^3\right)\Big|_0^2 = 2(18-6) = 2(12) = 24$

4) Determine o volume do solido S, cuja base e a regiao limitada por $y = x^2$ e $y = 1$. As secções transversais perpendiculares ao eixo y são triangulos equilateros.



é a função em relação a y, f(y)



$x=\sqrt{y}$

deixar em função de y

$y=x^2 \Rightarrow x=\sqrt{y}$

$L = 2\sqrt{y}$

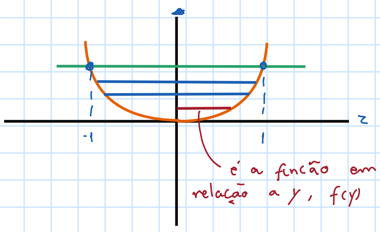
$A(y) = \frac{L^2\sqrt{3}}{4}$

$\int_0^1 \frac{(2\sqrt{y})^2\sqrt{3}}{4}\,dy$

$\frac{\sqrt{3}}{4}\int_0^1 4y\,dy = \frac{\sqrt{3}}{4}\cdot 4\int_0^1 y\,dy$

$\sqrt{3}\left(\frac{y^2}{2}\right)\Big|_0^1 = \frac{\sqrt{3}}{2}$ //

5)



é a função em relação a y, f(y)

$y=x^2$, como é perpendicular ao eixo y

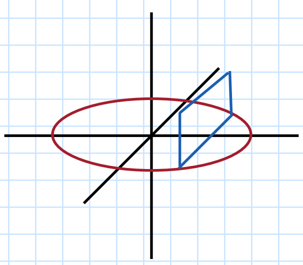
$y=x^2 \Rightarrow x=\sqrt{y}$

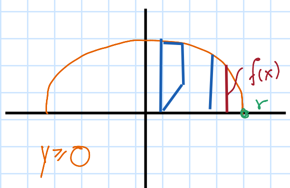
$L = 2\sqrt{y}$

$A(x) = L^2 = (2\sqrt{y})^2 = 4y$

$\int_0^1 4y\,dy = 4\left(\frac{y^2}{2}\right)\Big|_0^1 = 4\left(\frac{1}{2}\right) = \frac{4}{2} = 2$ //

6) Calcule o volume do sólido cuja base é o semicirculo $x^2 + y^2 \le r^2$, $y \ge 0$, e cujas secções perpendiculares ao eixo x são quadrados.





$f(x)$

$y\ge 0$

$x^2+y^2=r^2$

$y^2=r^2-x^2$

$y=\sqrt{r^2-x^2}$

$L=\sqrt{r^2-x^2}$

$A(x) = \left(\sqrt{r^2-x^2}\right)^2 = (r^2-x^2)$

intervalo

$-r\le x\le r$

ou

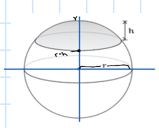
$0\le x\le r$

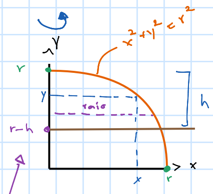
$2\int_0^r (r^2-x^2)\,dx = 2\int_0^r r^2-x^2\,dx$

$2\left(r^2x-\frac{x^3}{3}\right)\Big|_0^r = 2\left(r^3-\frac{r^3}{3}\right) = 2\left(\frac{3r^3-r^3}{3}\right)$

$2\left(\frac{2r^3}{3}\right) = \frac{4r^3}{3}$ //

(7) Calcule o volume da calota de uma esfera de raio r e altura h.





$x^2+y^2=r^2$

raio

h

• Intervalo de integração

$r-h\le y\le r$

Solução : A calota pode ser obtida pela rotação em torno do eixo y, da região do semi-circulo $x^2+y^2=r^2$, com $x\ge 0$ e $r-h\le y\le r$

• $x^2+y^2=r^2 \Rightarrow x=\sqrt{r^2-y^2}$ (raio da secção)

• Área da secção transversal

$A(y) = \pi(\text{raio})^2$

$= \pi\left(\sqrt{r^2-y^2}\right)^2$

$= \pi(r^2-y^2)$

Então o volume é :

$V = \int_{r-h}^{r} A(y)\,dy = \int_{r-h}^{r} \pi(r^2-y^2)\,dy$

$= \pi\left(r^2y-\frac{y^3}{3}\right)\Big|_{r-h}^{r}$

$= \pi\left[\left(r^3-\frac{r^3}{3}\right)-\left(r^2(r-h)-\frac{(r-h)^3}{3}\right)\right]$

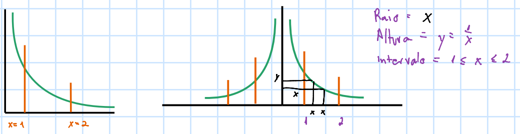
$= \pi\left[\cancel{r^3}-\cancel{\frac{r^3}{3}}-\cancel{r^3}+r^2h+\cancel{\frac{r^3}{3}}-\cancel{\frac{3r^2h}{3}}+\frac{3rh^2}{3}-\frac{h^3}{3}\right]$

$= \pi\left[rh^2-\frac{h^3}{3}\right]$

$= \pi h^2\left(r-\frac{h}{3}\right)$

8) $\int_a^b 2\pi\,(\text{raio})\,(\text{altura})$

a) y = 1/x , y = 0, x = 1, x = 2; ao redor do eixo y



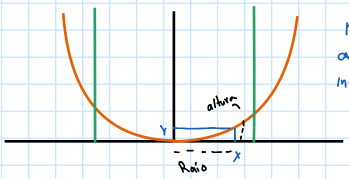
$\int_1^2 2\pi\,\cancel{(x)}\left(\frac{1}{\cancel{x}}\right)\,dx$

$2\pi \int_1^2 dx$

$2\pi\,(x)\Big|_1^2$

$2\pi\,(2-1) = 2\pi$ //

b) y = x², y = 0, x = 1; ao redor do eixo y



raio : x
altura : $y = x^2$
intervalo : $0 \le x \le 1$

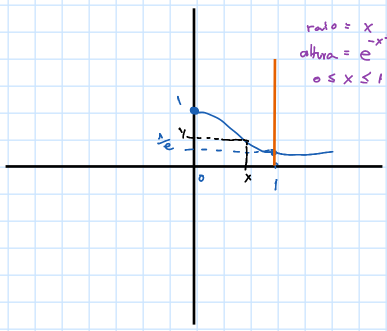
$\int_0^1 2\pi\, x \cdot x^2\, dx$

$2\pi \int_0^1 x^3\, dx$

$2\pi \left(\frac{x^4}{4}\right)\Big|_0^1$

$\cancel{2}\pi\, \frac{1}{\cancel{4}} = \frac{\pi}{2}$

c) y = e^(-x²) , y = 0, x = 0, x = 1; ao redor do eixo y



raio = x
altura = e^{-x^2}
$0 \le x \le 1$

$\int_0^1 2\pi\,(x)(e^{-x^2})\,dx$

$2\pi \int_0^1 x \cdot e^{-x^2}\,dx$

$u = -x^2$
$du = -2x\,dx$
$x\,dx = \frac{du}{-2}$

$x = 1 \Rightarrow u = -1^2 = -1$
$x = 0 \Rightarrow u = 0$

$2\pi \int_0^{-1} e^u\, \frac{du}{-2}$ $\qquad 2\pi \cdot -\frac{1}{2} \int_0^{-1} e^u\, du$

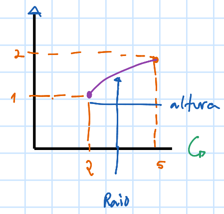
$-\pi \int_0^{-1} e^u\, du = +\pi \int_{-1}^{0} e^u\, du$

A inversão dos limites de 0 até -1, mudou para -1 até 0 de integração e inverteu o sinal da integral, deixando o π positivo na resposta final.

$\pi\,(e^{-x^2})\Big|_{-1}^{0} = +\pi\,(e^0 - e^{-(-1)^2})$

$+\pi\,(1 - \frac{1}{e})$

d) x = 1 + y², x = 0, y = 1, y = 2; ao redor do eixo x



$\int_1^2 2\pi\, y\,(1+y^2)\,dy$

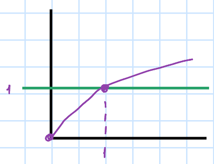
$2\pi \int_1^2 y + y^3\, dy$

$2\pi \left(\frac{y^2}{2} + \frac{y^4}{4}\right)\Big|_1^2$

$2\pi \left(\frac{4}{2} + \frac{16}{4} - \frac{1}{2} - \frac{1}{4}\right)$

$2\pi\,(2 + 4 - \frac{1}{2} - \frac{1}{4}) = \cancel{2}\pi\left(\frac{21}{\cancel{4}}\right) = \pi\left(\frac{21}{2}\right)$

E) x =√y, x = 0, y = 1; ao redor do eixo x



Raio : $\sqrt{y}$
altura : y
inter : $0 \le y \le 1$

$\int_0^1 2\pi\,(\sqrt{y})(y)\,dy$

$2\pi \int_0^1 y^{\frac{1}{2}+1}\,dy$

$2\pi \int_0^1 y^{3/2}\,dy$

$2\pi \left(\frac{2y^{\frac{5}{2}}}{5}\right)\Big|_0^1 = 2\pi\left(\frac{2}{5}\left(1^{\frac{5}{2}} - 0^{\frac{5}{2}}\right)\right) = \frac{4\pi}{5}$

9) a) $y = 1 + 6x^{\frac{3}{2}}$ $\qquad 0 \le x \le 1$

$f(x) = 1 + 6x^{\frac{3}{2}}$

$f'(x) = 9\,x^{\frac{1}{2}}$

$(f'(x))^2 = (9\sqrt{x})^2 = 81x$

$\int_0^1 \sqrt{1 + 81x}\,dx$

$u = 1 + 81x$
$du = 81\,dx \Rightarrow dx = \frac{du}{81}$

$x = 1 \Rightarrow u = 82$
$x = 0 \Rightarrow u = 1$

$\int \sqrt{u}\,\frac{du}{81}$

$\frac{1}{81}\int u^{\frac{1}{2}}\,du$

$\frac{1}{81}\left(\frac{2u^{3/2}}{3}\right)$

$\frac{1}{81}\left(\frac{2}{3}\left(u^{3/2}\right)\right)$

$\frac{2}{243}\left(u^{\frac{3}{2}}\right)\Big|_1^{82} = \frac{2}{243}\left(82^{\frac{3}{2}} - 1^{\frac{3}{2}}\right)$

$\frac{2}{243}\left(\sqrt[2]{(82)^3} - 1\right)$

$\frac{2}{243}\left(\sqrt{551368} - 1\right)$

$\frac{2}{243}\left(82\sqrt{82} - 1\right)$ //

b) $y = \frac{x^2}{2} - \frac{\ln x}{4}$ $\qquad 2 \le x \le 4$

$f'(x) = x - \frac{1}{4x}$

$(f'(x))^2 = \left(x - \frac{1}{4x}\right)^2 = x^2 - \cancel{2x}\cdot\frac{1}{4x} + \left(\frac{1}{4x}\right)^2$

$x^2 - \frac{1}{2} + \frac{1}{16x^2}$

$\int_2^4 \sqrt{1 + x^2 - \frac{1}{2} + \frac{1}{16x^2}}\,dx$

$\int_2^4 \sqrt{\frac{1}{2} + x^2 + \frac{1}{16x^2}}\,dx$

$\int_2^4 \sqrt{\frac{8x^2 + 16x^4 + 1}{16x^2}}\,dx$

$\int_2^4 \frac{\sqrt{8x^2 + 16x^4 + 1}}{\sqrt{16x^2}}\,dx$

$(8x^2 + 16x^4 + 1) = (4x^2 + 1)^2$

$\int_2^4 \frac{\cancel{\sqrt{(4x^2+1)^2}}}{4x}\,dx$

$\int_2^4 \frac{4x^2+1}{4x}\,dx$

$\int_2^4 \frac{4x^2}{4x} + \frac{1}{4x}\,dx$

$\int_2^4 x\,dx + \frac{1}{4}\int_2^4 \frac{1}{x}\,dx$

$\left(\frac{x^2}{2} + \frac{1}{4}\ln|x|\right)\Big|_2^4$

$\frac{1}{2}\left(x^2 + \frac{1}{2}\ln|x|\right)\Big|_2^4$

$\frac{1}{2}\left[\left(16 + \frac{1}{2}\ln|4|\right) - \left(4 + \frac{1}{2}\ln|2|\right)\right]$

$\frac{1}{2}\left[16 - 4 + \frac{1}{2}\ln\left|\frac{4}{2}\right|\right]$

$\frac{1}{2}\left[12 + \frac{1}{2}\ln|2|\right] = 6 + \frac{1}{4}\ln|2|$

10)

a) superfície de revolução obtida pelo giro de y = x³, 0 ≤ x ≤ 2, em torno do eixo x.

$f(x) = x^3$
$f'(x) = 3x^2$
$(f'(x))^2 = 9x^4$

$\int_0^2 2\pi \cdot x^3\sqrt{1 + 9x^4}\,dx$

$2\pi \int_0^2 x^3\sqrt{1 + 9x^4}\,dx$

$u = 1 + 9x^4$
$du = 36x^3\,dx \Rightarrow x^3\,dx = \frac{du}{36}$

$x = 2 \Rightarrow u = 145$
$x = 0 \Rightarrow u = 1$

$2\pi \int_1^{145} \sqrt{u}\,\frac{du}{36} = 2\pi\,\frac{1}{36}\int_1^{145} u^{\frac{1}{2}}\,du$

$\frac{1}{18}\pi\left(\frac{2}{3}u^{3/2}\right)\Big|_1^{145} = \frac{2}{54}\,\pi\left(\sqrt{145^3} - \sqrt{1^3}\right)$

$= \frac{1}{27}\,\pi\left(145\sqrt{145} - 1\right)$ //

b superfície de revolução obtida pelo giro de y =x/2, 0 ≤ x ≤ 4, em torno do eixo x.

$f'(x) = \frac{1}{2}$

$(f'(x))^2 = \frac{1}{4}$

$\int_0^4 \cancel{2}\pi\,\frac{x}{\cancel{2}}\sqrt{1 + \frac{1}{4}}\,dx$

$\pi\int_0^4 x\sqrt{\frac{5}{4}}\,dx$

$\pi\frac{\sqrt{5}}{2}\int_0^4 x\,dx$

$\pi\frac{\sqrt{5}}{2}\left(\frac{x^2}{2}\right)\Big|_0^4$

$= \pi\frac{\sqrt{5}}{2}\left(\frac{1}{2}(4^2 - 0)\right)$

$= \pi\cdot\frac{\sqrt{5}}{2}\cdot\frac{16}{2}$

$= \pi\,\frac{\sqrt{5}}{\cancel{2}}\,\cancel{8} = 4\sqrt{5}\,\pi$ //