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# Lista 1 - calculo 2

1)

a) $\int (6x^2 - 2x + 1)\,dx \Rightarrow \int 6x^2\,dx - \int 2x\,dx + \int 1\,dx$

$\Rightarrow \frac{6x^3}{3} - \frac{2x^2}{2} + x + K$

$\Rightarrow 2x^3 - x^2 + x + K$ //

b) $\int \frac{x+1}{x^2}\,dx \Rightarrow \int \frac{x^2}{x^2} + \frac{1}{x^2}\,dx$

$\Rightarrow \int 1\,dx + \int x^{-2}\,dx$

$\Rightarrow x + \frac{x^{-1}}{-1} + K$

$\Rightarrow x - \frac{1}{x} + K$ //

c) $\int \underbrace{\left(\frac{x^2}{x^2+1}\right)}_{f(x)} dx \Rightarrow$

$\cancel{x^2} \,|\, x^2+1$

$\cancel{-x^2} - 1 \quad 1$

$-1$

$f(x) = (x^2+1)\cdot 1 - 1$

$\frac{x^2}{x^2+1} = (x^2+1) - 1$

$= \frac{x^2+1}{x^2+1} - \frac{1}{x^2+1}$

$= 1 - \frac{1}{x^2+1}$ //

$\Rightarrow \int 1 - \frac{1}{x^2+1}\,dx$

$\Rightarrow \int 1\,dx - \int \frac{1}{x^2+1}\,dx$

$\Rightarrow x \cdot \text{arc } tg\ x + K$ //

d) $\int \frac{e^{-}}{2} + \sqrt{t} + \frac{1}{t})\,dt \Rightarrow$

$\Rightarrow \int \frac{e^t}{2}\,dt + \int t^{\frac{1}{2}}\,dt + \int \frac{1}{t}\,dt$

$\Rightarrow \frac{1}{2}\cdot\int e^t\,dt + \int t^{\frac{1}{2}}\,dt + \int \frac{1}{t}\,dx$

$\Rightarrow \frac{1}{2}\cdot e^t + \frac{2\sqrt[2]{t^3}}{3} + \ln|t| + K$ //

e) $\int \frac{\text{sen } x}{\cos^2 x}\,dx \Rightarrow \int \frac{\text{sen } x}{\cos x} \cdot \frac{1}{\cos x}\,dx$

$\Rightarrow \int tg\ x \cdot \sec x\,dx$

$\Rightarrow \sec x + K$

f) $\int \cos x \cdot tg\ x\,dx$

$\Rightarrow \int \cancel{\cos x} \cdot \frac{\text{sen } x}{\cancel{\cos x}}\,dx$

$\Rightarrow \int \text{sen } x\,dx$

$\Rightarrow -\cos x + K$ //

g) $\int 2^x - \sqrt{2}\,e^x + \cosh x\,dx$

$\Rightarrow \int 2^x\,dx - \int 2^{\frac{1}{2}}\cdot e^x\,dx + \int \cosh x\,dx$

$\Rightarrow \frac{2^x}{\ln 2} - 2^{\frac{1}{2}}\cdot e^x + \text{senH } x + K$ //

H) $\int \sqrt{\frac{9}{1-x^2}}\,dx$

$\Rightarrow \int \frac{\sqrt{9}}{\sqrt{1-x^2}}\,dx$

$\Rightarrow \int \frac{3}{\sqrt{1-x^2}}\,dx$

$\Rightarrow \int \frac{3}{1}\cdot\frac{1}{\sqrt{1-x^2}}\,dx$

$\Rightarrow 3\int \frac{1}{\sqrt{1-x^2}}\,dx$

$\Rightarrow 3\cdot \text{arc sen } x + 1$

i) $\int \frac{\ln x}{x \cdot \ln x^2}\,dx$

$\Rightarrow \int \frac{\cancel{\ln x}}{2x \cdot \cancel{\ln x}}\,dx$

$\Rightarrow \int \frac{1}{2x}\,dx$

$\Rightarrow \frac{1}{2}\int \frac{1}{x}\,dx$

$\Rightarrow \frac{1}{2}\cdot \ln|x| + K$ //

2) $\int \frac{1}{x^2} + 1$

$\int x^{-2} + 1\,dx$

$\int x^{-2}\,dx + \int 1\,dx$

$-\frac{1}{x} + x + k$

$x = 2$

$-\frac{1}{2} + 2 + K = 0$

$\frac{3}{2} + K = 0$

$K = -\frac{3}{2}$

5)

3)

a) $\int (2x^2+2x-3)^{10}(4x+1)\,dx$

$u = 2x^2+2x-3$

$\frac{du}{dx} = \frac{4x+2}{1}$

$\frac{(4x+2)dx}{2} = \frac{du}{2}$

$\boxed{\frac{(4x+2)dx}{2} = \frac{du}{2}}$

$\Rightarrow \int u^{10}\,\frac{du}{2}$

$\Rightarrow \int u^{10}\,\frac{1}{2}\,du$

$\Rightarrow \frac{1}{2}\int u^{10}\,du$

$\Rightarrow \frac{1}{2}\cdot\frac{u^{11}}{11} + K$

$\Rightarrow \frac{1}{2}\cdot\frac{(2x^2+2x-3)^{11}}{11} + K$

$\Rightarrow \frac{(2x^2+2x-3)^{11}}{22} + K$

B) $\int \frac{x}{\sqrt[5]{x^2-1}}\,dx$

$u = x^2-1$

$\frac{du}{dx} = 2x$

$2x\,dx = du$

$x\,dx = \frac{du}{2}$

$\Rightarrow \int \frac{1}{\sqrt[5]{x^2-1}}\cdot x\,dx$

$\Rightarrow \int \frac{1}{\sqrt[5]{u}}\,\frac{du}{2}$

$\Rightarrow \frac{1}{2}\cdot\int u^{-\frac{1}{5}}\,du$

$\Rightarrow \frac{1}{2}\cdot\frac{5u^{4/5}}{4} + K$

$\Rightarrow \frac{5u^{\frac{4}{5}}}{8} + K$

$\Rightarrow \frac{5(x^2-1)^{\frac{4}{5}}}{8} + K$

c) $\int 5x\sqrt{4-3x^2}\,dx$

$u = 4-3x^2$

$\frac{du}{dx} = -6x$

$-6x\,dx = du$

$x\,dx = \frac{-du}{6}$

$\Rightarrow \int 5\sqrt{4-3x^2}\,x\,dx$

$\Rightarrow \int 5\sqrt{u}\;-\frac{1}{6}\,du$

$\Rightarrow -\frac{5}{6}\int u^{\frac{1}{2}}\,du$

$\Rightarrow -\frac{5}{6}\cdot\frac{2u^{3/2}}{3} + K$

$\Rightarrow \frac{-10(4-3x^2)^{\frac{3}{2}}}{18} + K$

$\Rightarrow -\frac{5}{9}(4-3x^2)^{\frac{3}{2}} + K$

D) $\int \sqrt{x^2+2x^4}\,dx$

$u = 1+2x^2$

$\frac{du}{dx} = \frac{4x}{4}$

$4x\,dx = du$

$x\,dx = \frac{du}{4}$

$\Rightarrow \int \sqrt{\cancel{x}\,x^{\cancel{2}}(1+2x^2)}\,dx$

$\Rightarrow \int x\cdot\sqrt{1+2x^2}\,dx$

$\Rightarrow \int \sqrt{1+2x^2}\cdot x\,dx$

$\Rightarrow \int \sqrt{u}\cdot\frac{du}{4}$

$\Rightarrow \frac{1}{4}\cdot\int u^{\frac{1}{2}}\,du$

$\Rightarrow \frac{1}{4}\cdot\frac{2u^{3/2}}{3} + K$

$\Rightarrow \frac{2(1+2x^2)^{3/2}}{12} + K$

$\Rightarrow \frac{(1+2x^2)^{3/2}}{6} + K$

Me fodi

e) $\int (e^{2x}+2)^{\frac{1}{3}}\cdot e^{2x}\,dx$

$u = e^{2x}+2$

$\frac{du}{dx} = 2e^{2x}$

$2e^{2x}\,dx = du$

$e^{2x}\,dx = \frac{du}{2}$

$\Rightarrow \int (u)^{\frac{1}{3}}\cdot e^{2x}\,dx$

$\Rightarrow \int u^{\frac{1}{3}}\cdot\frac{1}{2}\,du$

$\Rightarrow \frac{1}{2}\int u^{\frac{1}{3}}\,du$

$\Rightarrow \frac{1}{2}\cdot\frac{u^{\frac{4}{3}}}{4/3} + K$

$\Rightarrow \frac{1}{2}\cdot\frac{3u^{4/3}}{4} + K$

$\Rightarrow \frac{3(e^{2x}+2)^{\frac{4}{3}}}{8} + K$

f) $\int \frac{e^x}{e^x+4}\,dx$

$u = e^x+4$

$\frac{du}{dx} = e^x$

$dx\cdot e^x = du$

$dx = \frac{du}{e^x}$

$\Rightarrow \int \frac{e^x}{u}\,\frac{du}{e^x}$

$\Rightarrow \int \frac{e^x}{u}\cdot\frac{1}{e^x}\,du$

$\Rightarrow \int \frac{1}{u}\,du$

$\ln|u| + K$

$\Rightarrow \ln|e^x+4| + K$ //

Me fodi

g) $\int tg\,x\cdot\sec^2 x\,dx$

$u = tg\,x$

$\frac{du}{dx} = \sec^2 x$

$dx\cdot\sec^2 x = du$

$dx = \frac{du}{\sec^2 x}$

$\Rightarrow \int u\cdot\sec^2 x\cdot\frac{1}{\sec^2 x}\,du$

$\Rightarrow \int u\,du$

$\Rightarrow \frac{u^2}{2} + K$

$\Rightarrow \frac{tg^2 x}{2} + K$ //

H) $\int sen^4 x\,\cos x\,dx$

$u = sen\,x$

$\frac{du}{dx} = \cos x$

$dx\cdot\cos x = du$

$dx = \frac{du}{\cos}$

$\Rightarrow \int u^4\cdot\cancel{\cos x}\cdot\frac{1}{\cancel{\cos x}}\,du$

$\Rightarrow \int u^4\,du$

$\Rightarrow \frac{u^5}{5} + K$

$\Rightarrow \frac{sen^5 x}{5} + K$

i) $\int \frac{sen\,x}{\cos^5 x}\,dx$

$u = \cos x$

$\frac{du}{dx} = -sen\,x$

$-sen\,x\,dx = du$

$dx = \frac{du}{-sen\,x}$

$\Rightarrow \int \frac{\cancel{sen\,x}}{u^5}\cdot\frac{-1}{\cancel{sen\,x}}\cdot du$

$\Rightarrow \int \frac{-1}{u^5}\,du$

$\Rightarrow \int -u^{-5}\,du$

$\Rightarrow -\frac{u^{-4}}{-4} + K$

$\Rightarrow +\frac{\cos x^{-4}}{4} + K$

obs: $\frac{1}{\cos x} = \cos x^{-1}$

$\Rightarrow \boxed{\frac{1}{\cos^4 x} = \sec^4}$

$\Rightarrow \frac{1}{4}\cdot\sec^4 x + K$ //

J) $\int \frac{arc\,sen\,x}{2\sqrt{1-x^2}}\,dx$

$u = arc\,sen\,x$

$\frac{du}{dx} = \frac{1}{\sqrt{1-x^2}}$

$dx = du\cdot\sqrt{1-x^2}$

$\Rightarrow \int \frac{1}{2}\,arc\,sen\,x\cdot\frac{1}{\sqrt{1-x^2}}\,dx$

$\Rightarrow \frac{1}{2}\int arc\,sen\,x\cdot\frac{1}{\sqrt{1-x^2}}\,dx$

$\Rightarrow \frac{1}{2}\int u\cdot\frac{1}{\cancel{\sqrt{1-x^2}}}\cdot\cancel{\sqrt{1-x^2}}\,du$

$\Rightarrow \frac{1}{2}\int u\,du$

$\Rightarrow \frac{1}{2}\cdot\frac{u^2}{2} + K$

$\Rightarrow \frac{(arc\,sen\,x)^2}{4} + K$

K) $\int \frac{1}{x^2-4x+4}\,dx$

$u = x-2$

$\frac{du}{dx} = 1$

$du = dx$

$\Rightarrow \int \frac{1}{(x-2)(x-2)}\,dx$

$\Rightarrow \int \frac{1}{u^2}\,du$

$\Rightarrow \int u^{-2}\,du$

$\Rightarrow \frac{u^{-1}}{-1} + K$

$\Rightarrow -(x-2)^{-1} + K$

$\Rightarrow \frac{1}{-x+2} + K$ //

L) $\int \frac{\ln^2 x}{x}\,dx$

$u = \ln x$

$\frac{du}{dx} = \frac{1}{x}$

$dx = x\,du$

$\Rightarrow \int \frac{u^2}{\cancel{x}}\cdot\cancel{x}\,du$

$\Rightarrow \int u^2\,du$

$\Rightarrow \frac{u^3}{3} + K$

$\Rightarrow \frac{(\ln x)^3}{3} + K$

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m) $\int \frac{\cos\sqrt{x}}{\sqrt{x}}$ $\Rightarrow \int \cos u \cdot \frac{1}{\sqrt{x}} \cdot 2\sqrt{x}\, du$

$u = \sqrt{x} = x^{\frac{1}{2}}$ $\Rightarrow \int \cos u \cdot 2 \; du$

$\frac{du}{dx} \to \frac{1}{2\sqrt{x}}$ $\Rightarrow 2\int \cos u \, du$

$dx = 2\sqrt{x}\, du$ $\Rightarrow 2 \operatorname{sen} u + k$

$\Rightarrow 2 \operatorname{sen} \sqrt{x} + K$ //

n) $\int \frac{3}{x \cdot \ln^2 3x} dx$ $\Rightarrow 3 \int \frac{1}{x} \cdot \frac{1}{\ln^2 3x}\, dx$

$\Rightarrow 3 \int$

$u = \ln 3x$

$\frac{du}{dx} = \frac{1}{x}$

$dx = x \cdot du$

o) $\int \cot g\, x \; dx = \int \frac{\cos x}{\operatorname{sen} x} dx \Rightarrow$

$\Rightarrow \int \cos x \cdot \frac{1}{\operatorname{sen} x} dx$

$u = \operatorname{sen} x$ $\Rightarrow \int \cancel{\cos x} \cdot \frac{1}{u} \cdot \frac{1}{\cancel{\cos}} du$

$\frac{du}{dx} = \cos x$ $\Rightarrow \int \frac{1}{u} du$

$\Rightarrow \ln |u| + k$

$dx = \frac{du}{\cos}$ $\ln |\operatorname{sen} x| + K$ //

p) $\int \operatorname{sen} 5x \; dx$ $\Rightarrow \int \operatorname{sen} u \; \frac{du}{5}$

$u = 5x$ $\Rightarrow \frac{1}{5} \int \operatorname{sen} u \, du$

$\frac{du}{dx} = 5$ $\Rightarrow \frac{1}{5} - \cos u$

$dx = \frac{du}{5}$ $\Rightarrow -\frac{1}{5}\cos 5x + k$

q) $\int \cos^2 x \; dx$ $\Rightarrow \int \frac{1+\cos 2x}{2} dx$

$u = 2x$ $\Rightarrow \int \frac{1}{2} + \int \frac{\cos 2x}{2} dx$

$\frac{du}{dx} = 2$ $\Rightarrow \int \frac{1}{2} dx + \int \frac{1}{2} \cos u \cdot \frac{du}{2}$

$dx = \frac{du}{2}$ $\Rightarrow \frac{1}{2}x + \frac{1}{4}\int \cos u$

$\Rightarrow \frac{1}{2}x + \frac{1}{4} \operatorname{sen} u + K$

$\Rightarrow \frac{x}{2} + \frac{\operatorname{sen} 2x}{4} + K$ //

r) $\int tg^3 x \cdot \cos x \; dx \Rightarrow \int \frac{\operatorname{sen}^3 x}{\cancel{\cos^3 x}} \cdot \cancel{\cos x}\, \cancel{dx}$

$\boxed{u = \cos x \quad du = -\operatorname{sen} x \; dx}$

$\Rightarrow \int \frac{\operatorname{sen}^3 x}{\cos^2 x} dx$

$\Rightarrow \int \frac{\operatorname{sen}^2 x}{\cos^2 x} \cdot \operatorname{sen} x \cdot dx$

$u = 2x$

$du = 2x\, dx$

$\Rightarrow \int \frac{\operatorname{sen}^2 x}{u^2} - du$

$\operatorname{sen}^2 x + \cos^2 x = 1$

$\Rightarrow -\int \frac{1-\cos^2 x}{u^2} du$

$\Rightarrow -\int \frac{1-u^2}{u^2} du$

$\Rightarrow -\int \frac{1}{u^2} - \frac{u^2}{u^2} du$

$\Rightarrow -\int \frac{1}{u^2} - 1 \; du$

$\Rightarrow -\int \frac{1}{u^2} du - \int 1\, du$

$\Rightarrow -\left(-\frac{1}{u}\right) - u + K$

$\Rightarrow \frac{1}{u} - u + K$

$\Rightarrow \frac{1}{\cos x} - \cos x + K$

$\Rightarrow \sec x + \cos x + K$ //

4) a) $\int \underset{u}{x} \cdot \underset{dv}{\operatorname{sen} 5x \; dx}$ $\Rightarrow u \cdot v - \int v \; du$

$u = x$ $\quad dv = \operatorname{sen}(5x)\, dx$ $\Rightarrow x \cdot \frac{1}{5} - \cos(5x) - \int \frac{1}{5} - \cos 5x \; dx$

$du = dx$ $\quad v = \int \operatorname{sen}(5x)\, dx$ $\Rightarrow \frac{x}{5} - \cos(5x) + \frac{1}{5}\int \cos(5x)\, dx$

$= \int \operatorname{sen} u \; \frac{du}{5}$ $\Rightarrow \frac{x}{5} - \cos(5x) - \frac{1}{5} \cdot \frac{1}{5} \cdot \operatorname{sen} 5x + K$

$= \frac{1}{5} \int \operatorname{sen} u \; du$ $\Rightarrow -\frac{x}{5} \cdot \cos(5x) - \frac{1}{25} \operatorname{sen}(5x) + K$ //

$v = (\frac{1}{5} - \cos 5x)$

b) $\int \ln(1-x)\, dx$ $\Rightarrow \int \ln u - du \Rightarrow -\int \ln u \; du$

$u = 1-x$ $\Rightarrow -\; u \cdot \ln u + \int \cancel{u} \cdot \frac{1}{\cancel{u}} du$

$du = -dx \Rightarrow dx = -du$ $\Rightarrow -u \ln u + \int du$

$\int = \ln u \qquad dv = du$ $\Rightarrow -u \ln u + u + k$

$d\int = \frac{1}{u} du \qquad v = u$ $\Rightarrow -(1-x)\ln(1-x) + (1-x) + K$

$\Rightarrow (x-1)\ln|1-x| + 1 - x + K$ //

c) $\int t e^{4t} \; dt$ $\Rightarrow t \cdot \frac{e^{4t}}{4} - \int \frac{1}{4} e^{4t} \; dt$

$u = t \quad dv = \int e^{4t} dt$ (I) $\Rightarrow t \cdot \frac{e^{4t}}{4} - \frac{1}{4} \int e^{4t} \; dt$ (II)

$du = 1$ $\Rightarrow t \cdot \frac{e^{4t}}{4} - \frac{1}{4} \cdot \int e^u \; \frac{du}{4}$

(I) $u = 4t$ $\quad dv = \int e^u \frac{du}{4}$ $\Rightarrow t \cdot \frac{e^{4t}}{4} - \frac{1}{4} \cdot \frac{1}{4} \int e^u \; du$

$du = 4\, dx$ $\quad \frac{1}{4} \int e^u du$ $\Rightarrow t \cdot \frac{e^{4t}}{4} - \frac{1}{4} \cdot \frac{1}{4}\, e^{4t} + K$

$dx = \frac{du}{4}$ $\quad \frac{1}{4} \cdot e^u + K$ $\Rightarrow t \cdot \frac{e^{4t}}{4} - \frac{1}{16} e^{4t} + K$

$\boxed{v = \frac{1}{4} e^{4t} + K}$ $\Rightarrow t \cdot \frac{e^{4t}}{4} - \frac{e^{4t}}{16} + k$

(II) mudança variável $\Rightarrow \frac{e^{4t}}{4}\left(t - \frac{1}{4}\right) + K$ //

$U = 4t$

$du = 4\, dt \Rightarrow dt = \frac{du}{4}$

d) $\int \underset{u}{(x+1)} \cdot \underset{dv}{\cos(2x)}\, dx$ $\Rightarrow (x+1) \cdot \left(\frac{1}{2} \operatorname{sen}(2x)\right) - \frac{1}{2} \int \operatorname{sen}(2x)\, dx$ (I)

$u = x+1$ $\quad dv = \int \cos(2x)\, dx$ $\Rightarrow (x+1) \cdot \left(\frac{1}{2}\operatorname{sen}(2x)\right) - \frac{1}{2} \int \operatorname{sen} u \; \frac{du}{2}$

$du = 1$ $\quad u = 2x$ $\quad \int \cos u \; \frac{du}{2}$ $\Rightarrow (x+1) \cdot \left(\frac{1}{2}\operatorname{sen}(2x)\right) - \frac{1}{2} \cdot \frac{1}{2} \int \operatorname{sen} u \; du$

$du = 2\, dx$

$\boxed{dx = \frac{du}{2}}$ $\quad \frac{1}{2} \int \cos u \; du$ $\Rightarrow (x+1) \cdot \left(\frac{1}{2}\operatorname{sen}(2x)\right) - \frac{1}{4} \cdot -\cos 2x + K$

$V = \frac{1}{2} \operatorname{sen} 2x + K$ $\Rightarrow \frac{x+1}{2} \cdot \operatorname{sen}(2x) + \frac{1}{4} \cos(2x) + K$ //

$\boxed{\text{(I)} \; u = 2x \quad du = 2\, dx \quad dx = \frac{du}{2}}$

E) $\int x \cdot \ln 3x \; dx$ $\Rightarrow x \cdot (\ln(3x) \cdot x - x) - \int (\ln(3x) \cdot x - x)\, dx$

$u = x$ $\quad dv = \int \ln(3x)\, dx$ (I) $\Rightarrow x^2 (\ln(3x) - 1) - \int x \cdot (\ln(3x) - 1) \; dx$

$du = 1$

(I) $\boxed{u = 3x \quad du = 3\, dx \quad dx = \frac{du}{3}}$ $\quad v = \int \ln u \; \frac{du}{3}$

$= \frac{1}{3} \int \ln(u)\; du$ (II)

Reescrevendo:

$\int x \cdot \ln(3x) = x^2 (\ln(3x) - 1) - \int x \cdot (\ln(3x) - 1)\, dx$

$\int x \cdot \ln(3x) = x^2(\ln(3x) - 1) - \int x \ln(3x) - \int x\, dx$

$2 \int x \cdot \ln(3x) = x^2(\ln(3x) - 1) - \frac{x^2}{2} + k$

(II) $\boxed{u = \ln t \quad dv = \int dt \quad du = \frac{1}{t} \quad v = T}$ $= \frac{1}{3} \cdot (\ln(u) \cdot u - \int \cancel{u} \cdot \frac{1}{\cancel{u}} du)$

$\frac{1}{3}(\ln(u) \cdot u - \int du)$

$\frac{1}{3}(\ln u \cdot u - u)$

$\frac{1}{3} \cdot (\ln(3x) \cdot 3x - 3x)$

$\frac{1}{\cancel{3}}\cancel{3}(\ln(3x) \cdot x - x)$

$V = \ln(3x) \cdot x - x$

$= x^2 \ln(3x) - x^2 - \frac{x^2}{2} + K$

$= \frac{1}{2}\left(x^2 \ln(3x) - (x^2 - \frac{x^2}{2})\right) + K$

$= \frac{1}{2}\left(x^2 \ln(3x) - \frac{x^2}{2}\right) + K$

$= \frac{x^2}{2}\left(\ln(3x) - \frac{1}{2}\right) + K$ //

- Log
- Inver. trig
- Algebrica $(3x^2)$
- Trigonometrica
- Exponencial

f) $\int \cos^3 x \; dx \Rightarrow \int \cos^2 x \cdot \cos x \; dx$

$u = \cos^2 x$ $\quad dv = \cos x \, dx$

$du = -2\cos x \cdot \operatorname{sen} x \, dx$ $\quad v = \operatorname{sen} x$

$\Rightarrow \cos^2 x \cdot \operatorname{sen} x - \int \operatorname{sen} x \cdot -2\cos x \cdot \operatorname{sen} x \, dx$

$\Rightarrow \cos^2 x \operatorname{sen} x + 2 \cdot \int \operatorname{sen}^2 \cdot \cos x \, dx$ (I)

$\Rightarrow \cos^2 x \operatorname{sen} x + 2 \cdot \left( \int u^2 \, du \right)$

$\Rightarrow \cos^2 x \cdot \operatorname{sen} x + 2 \cdot \frac{u^3}{3} + K$

$\Rightarrow \cos^2 x \cdot \operatorname{sen} x + \frac{2}{3} \operatorname{sen}^3 x + K$ //

(I) $\int \operatorname{sen}^2 \cdot \cos x \; dx$

$u = \operatorname{sen} x$

$du = \cos x \, dx$

$\int u^2 \cdot du$

G) $\int e^x \cdot \cos\left(\frac{x}{2}\right) dx$

$u = \cos\left(\frac{x}{2}\right)$ $\quad dv = e^x dx$

$du = -\frac{\operatorname{sen}\left(\frac{x}{2}\right)}{2} dx$ $\quad v = e^x$

$\Rightarrow \cos\left(\frac{x}{2}\right) \cdot e^x - \int e^x \cdot \left(-\frac{1}{2} \cdot \operatorname{sen}\left(\frac{x}{2}\right)\right) dx$

$\Rightarrow \cos\left(\frac{x}{2}\right) \cdot e^x + \frac{1}{2} \int e^x \operatorname{sen}\left(\frac{x}{2}\right) dx$ (I)

$\Rightarrow \cos\left(\frac{x}{2}\right) \cdot e^x + \frac{1}{2}\left( \operatorname{sen}\left(\frac{x}{2}\right) \cdot e^x - \int e^x \frac{1}{2} \cos\left(\frac{x}{2}\right) dx \right)$

$\Rightarrow \cos\left(\frac{x}{2}\right) e^x + \frac{1}{2} \operatorname{sen}\left(\frac{x}{2}\right) \cdot e^x - \frac{1}{4} \int e^x \cos\left(\frac{x}{2}\right) dx$

(I) $u = \operatorname{sen}\left(\frac{x}{2}\right)$ $\quad dv = e^x dx$

$du = \frac{\cos\left(\frac{x}{2}\right)}{2} dx$ $\quad v = e^x$

$\int e^x \cdot \cos\left(\frac{x}{2}\right) dx = \cos\left(\frac{x}{2}\right) e^x + \frac{1}{2} \operatorname{sen}\left(\frac{x}{2}\right) \cdot e^x - \frac{1}{4} \int e^x \cos\left(\frac{x}{2}\right) dx$

$\int e^x \cdot \cos\left(\frac{x}{2}\right) dx + \frac{1}{4} \int e^x \cdot \cos\left(\frac{x}{2}\right) dx = \cos\left(\frac{x}{2}\right) e^x + \frac{1}{2} \operatorname{sen}\left(\frac{x}{2}\right) \cdot e^x$

$\left(\frac{5}{4}\right) \int e^x \cdot \cos\left(\frac{x}{2}\right) dx = \cos\left(\frac{x}{2}\right) e^x + \frac{1}{2} \operatorname{sen}\left(\frac{x}{2}\right) \cdot e^x + K$

$\int e^x \cdot \cos\left(\frac{x}{2}\right) dx = \frac{4}{5}\left[\cos\left(\frac{x}{2}\right) e^x + \frac{1}{2} \operatorname{sen}\left(\frac{x}{2}\right) \cdot e^x\right] + K$

$= \frac{4}{5} \cos\left(\frac{x}{2}\right) e^x + \frac{2}{5} \cdot \operatorname{sen}\left(\frac{x}{2}\right) \cdot e^x + K$

$= \frac{2}{5} e^x \left[2\cos\left(\frac{x}{2}\right) + \operatorname{sen}\left(\frac{x}{2}\right) \cdot e^x\right] + K$ //

h) $\int \sqrt{x} \; \ln x \; dx$

$u = \ln x$ $\quad dv = \int \sqrt{x} \, dx$

$du = \frac{1}{x} dx$ $\quad v = \frac{2}{3} \sqrt{x^3}$

(I) $u = \sqrt{x^3} = x^{\frac{3}{2}}$ $\quad dv = \frac{1}{x} dx$

$du = \frac{3}{2} x^{\frac{1}{2}} dx$ $\quad v = \ln|x|$

$= \ln x \cdot \frac{2}{3}\sqrt{x^3} - \int \frac{2}{3}\sqrt{x^3} \cdot \frac{1}{x} dx$

$= \frac{2}{3}\sqrt{x^3} \ln x - \frac{2}{3} \int \sqrt{x^3} \cdot \frac{1}{x} \, dx$ (I)

$= \frac{2}{3}\sqrt{x^3} \ln x - \frac{2}{3}\left( \sqrt{x^3} \cdot \ln|x| - \int \ln|x| \cdot \frac{3}{2} x^{\frac{1}{2}} dx \right)$

$= \frac{2}{3}\sqrt{x^3} \ln x - \frac{2}{3}\sqrt{x^3} \ln|x| + \frac{2}{3} \cdot \frac{3}{2} \int \ln|x| \cdot \sqrt{x} \, dx$

$\int \sqrt{x} \; \ln x \; dx = \frac{2}{3}\sqrt{x^3} \ln x - \frac{2}{3}\sqrt{x^3} \ln|x| + \int \ln|x| \cdot \sqrt{x} \, dx$

sem módulo

módulo

$= \ln x \cdot \frac{2}{3}\sqrt{x^3} - \frac{2}{3} \int x^{\frac{1}{2}} dx$

$= \ln x \cdot \frac{2}{3}\sqrt{x^3} - \frac{2}{3}\left(\frac{2}{3} x^{\frac{3}{2}}\right)$

$= \frac{2}{3} \ln x \cdot \sqrt{x^3} - \frac{4}{3}\sqrt{x^3} + K$

$\frac{2}{3} x \sqrt{x} \cdot \ln x - \frac{4}{3} x \sqrt{x} + K$

I) $\int \arccos x \; dx$

$u = \arccos x$ $\quad dv = dx$

$du = \frac{-1}{\sqrt{1-x^2}} dx$ $\quad v = x$

(I) $u = 1 - x^2$

$du = -2x \, dx \Rightarrow x\,dx = \frac{du}{-2}$

$= \arccos x \cdot x - \int x \cdot -\frac{1}{\sqrt{1-x^2}} dx$

$= \arccos x \cdot x + \int \frac{1}{\sqrt{u}} \frac{du}{-2}$

$= \arccos x \cdot x - \frac{1}{2} \int \frac{1}{\sqrt{u}} \, du$

$= \arccos x \cdot x - \frac{1}{2} \int u^{-\frac{1}{2}} \, du$

$= \arccos x \cdot x - \cancel{\frac{1}{2}} \cancel{2} u^{\frac{1}{2}} + K$

$= \arccos x \cdot x - u^{\frac{1}{2}} + K$

$= \arccos x \cdot x - (1-x^2)^{\frac{1}{2}} + K$

$= \arccos x \cdot x - \sqrt{1-x^2} + K$ //

J) $\int x \cdot \sec^2 x \; dx = x \cdot \operatorname{tg} x - \int \operatorname{tg} x \; dx$

$u = x$ $\quad dv = \sec^2 dx$

$du = dx$ $\quad v = \operatorname{tg} x$

$x \cdot \operatorname{tg} x + \ln|\cos x| + K$

K) $\int (x^2 - 5x) e^x \; dx$

$u = x^2 - 5x$ $\quad dv = e^x dx$

$du = 2x - 5 \, dx$ $\quad v = e^x$

(I) $u = 2x - 5$ $\quad dv = e^x dx$

$du = 2 \, dx$ $\quad v = e^x$

$= (x^2 - 5x) \cdot e^x - \int e^x \cdot (2x - 5) \, dx$ (I)

$= (x^2 - 5x) \cdot e^x - \left[ (2x-5) \cdot e^x - \int e^x \cdot 2 \, dx \right]$

$= (x^2 - 5x) \cdot e^x - (2x - 5) \cdot e^x + 2e^x + K$

$= e^x \left[ (x^2 - 5x) - (2x - 5) + 2 \right] + K$

$= e^x \left[ x^2 - 7x + 7 \right] + K$ //

L) $\int e^{2x} \cos 3x \; dx$

$u = \cos 3x$ $\quad dv = e^{2x} dx$

$du = -\operatorname{sen} 3x \cdot 3$ $\quad v = \frac{e^{2x}}{2}$

(I) $u = \operatorname{sen} 3x$ $\quad dv = e^{2x} dx$

$du = \cos 3x \cdot 3$ $\quad v = \frac{e^{2x}}{2}$

$= \frac{e^{2x}}{2} \cdot \cos 3x - \int \frac{e^{2x}}{2} \cdot -\operatorname{sen} 3x \cdot 3 \; dx$

$= \frac{e^{2x}}{2} \cdot \cos 3x + \frac{1}{2} \cdot 3 \int e^{2x} \cdot \operatorname{sen} 3x \, dx$ (I)

$= \frac{e^{2x}}{2} \cdot \cos 3x + \frac{3}{2}\left( \frac{e^{2x}}{2} \cdot \operatorname{sen} 3x - \frac{1}{2} \cdot 3 \int e^{2x} \cdot \cos 3x \, dx \right.$

$= \frac{e^{2x}}{2} \cdot \cos 3x + \frac{3}{2} \cdot \frac{e^{2x}}{2} \cdot \operatorname{sen} 3x - \frac{3}{2} \cdot \frac{3}{2} \int e^{2x} \cdot \cos 3x \, dx$

$\int e^{2x} \cos 3x \; dx = \frac{1}{2} e^{2x} \cdot \cos 3x + \frac{3}{4} e^{2x} \cdot \operatorname{sen} 3x - \frac{9}{4} \int e^{2x} \cdot \cos 3x \, dx$

$\frac{13}{4} \int e^{2x} \cdot \cos 3x \, dx = \frac{1}{2} e^{2x} \cdot \cos 3x + \frac{3}{4} e^{2x} \cdot \operatorname{sen} 3x$

$= \frac{4}{13}\left[ \frac{1}{2} e^{2x} \cdot \cos 3x + \frac{3}{4} e^{2x} \cdot \operatorname{sen} 3x \right] + K$

$= \frac{2}{13} e^{2x} \cdot \cos 3x + \frac{3}{13} e^{2x} \cdot \operatorname{sen} 3x + K$

$= \frac{e^{2x}}{13}\left[ 2\cos 3x + 3 \operatorname{sen} 3x \right] + K$ //

M) $\int x^3 \, e^{x^2} dx$

$u = x^2$ $\quad dv = e^{x^2} x \, dx$

$du = 2x \, dx$

$u = x^2$ $\quad du = 2x\,dx$

$dx = \frac{du}{2x}$

$\int e^u \cdot \cancel{x} \cdot \frac{1}{2\cancel{x}} du$

$\frac{1}{2} \int e^u du$

$v = \frac{1}{2} e^{x^2}$

(I) $u = x^2$

$du = 2x \, dx$

$= \int x^2 \cdot x \; e^{x^2} dx$

$= x^2 \cdot \frac{1}{2} e^{x^2} - \frac{1}{2} \int e^{x^2} \cdot 2x \, dx$ (I)

$= x^2 \cdot \frac{1}{2} e^{x^2} - \frac{1}{2} \int e^u \, du$

$= x^2 \cdot \frac{1}{2} e^{x^2} - \frac{1}{2} e^{x^2} + K$

$= \frac{e^{x^2}}{2}\left[ x^2 - 1 \right] + K$ //

N) $\int x^3 \cdot \cos x^2 \; dx$

$u = x^2$

$du = 2x \, dx \rightarrow x\,dx = \frac{du}{2}$

(I)

$z = u$ $\quad dv = \cos u \, du$

$dz = 1$ $\quad v = \operatorname{sen} u$

$= \int x^2 \cdot x \cdot \cos x^2 \; dx$

$= \int u \cdot \cos u \; \frac{du}{2}$

$= \frac{1}{2} \int u \cdot \cos u \; du$

$= \frac{1}{2}\left[ u \cdot \operatorname{sen} u - \int \operatorname{sen} u \; du \right]$

$= \frac{1}{2}\left[ u \cdot \operatorname{sen} u + \cos u \right] + K$

$= \frac{1}{2}\left[ x^2 \cdot \operatorname{sen} x^2 + \cos x^2 \right] + K$ //

O) $\int e^{-x} \cdot \cos(2x) \, dx$

$u = \cos 2x$ $\quad dv = e^{-x} dx$

$du = -\operatorname{sen} 2x \cdot 2 \, dx$ $\quad = \int e^{u} \cdot du$

$= -\int e^{u} du$

$v = -e^{-x}$

(I) $u = \operatorname{sen}(2x)$ $\quad dv = e^{-x} dx$

$du = \cos 2x \cdot 2$ $\quad v = -e^{-x}$

$= \cos 2x \cdot -e^{-x} - \int -e^{-x} \cdot -\operatorname{sen} 2x \cdot 2 \, dx$

$= \cos 2x \cdot -e^{-x} - 2 \int e^{-x} \cdot \operatorname{sen}(2x) \, dx$

$= \cos 2x \cdot -e^{-x} - 2 \cdot \left[ -e^{-x} \cdot \operatorname{sen}(2x) + \int e^{-x} \cdot \cos 2x \cdot 2 \right]$

$= \cos 2x \cdot -e^{-x} - 2 \cdot \left[ -e^{-x} \cdot \operatorname{sen}(2x) + 2 \int e^{-x} \cdot \cos 2x \right]$

$= \cos 2x \cdot -e^{-x} + 2e^{-x} \cdot \operatorname{sen}(2x) - 4 \int e^{-x} \cdot \cos 2x$

$\int e^{-x} \cos(2x) \, dx = \cos 2x \cdot -e^{-x} + 2e^{-x} \cdot \operatorname{sen}(2x) - 4 \int e^{-x} \cdot \cos 2x$

$5 \int e^{-x} \cdot \cos 2x = \cos 2x \cdot -e^{-x} + 2e^{-x} \cdot \operatorname{sen}(2x)$

$= \frac{1}{5}\left[ \cos 2x \cdot -e^{-x} + 2e^{-x} \cdot \operatorname{sen}(2x) \right] + K$

$= \frac{1}{5}\left[ \cos 2x \cdot -e^{-x} + 2e^{-x} \cdot \operatorname{sen}(2x) \right] + K$

$= \frac{1}{5}\left( e^{-x} \left[ \cos 2x \cdot -1 + 2 \cdot \operatorname{sen}(2x) \right] \right)$

$= \frac{e^{-x}}{5} \left( -\cos(2x) + 2 \operatorname{sen}(2x) \right)$ //

P) $\int x^2 \operatorname{sen} x \, dx = x^2 \cdot -\cos x + \int \cos x \cdot 2x \, dx$ (I)

$u = x^2$ $\quad dv = \operatorname{sen} x \, dx$
$du = 2x \, dx$ $\quad v = -\cos x$

$= x^2 \cdot -\cos x + \left[ 2x \cdot \operatorname{sen} x - \int \operatorname{sen} x \cdot 2 dx \right]$

$= -x^2 \cdot \cos x + 2x \cdot \operatorname{sen} x + 2 \cos x + K$ //

$u = 2x$ $\quad dv = \cos x \, dx$
$du = 2dx$ $\quad v = \operatorname{sen} x$

5) a) $\int_{-1}^{2} x(1+x^3) \, dx$

$= \int x + x^4 \, dx$

$= \int x \, dx + \int x^4 \, dx$

$= \frac{x^2}{2} + \frac{x^5}{5} + K$

$f(2) = \frac{2^2}{2} + \frac{2^5}{5} + K = \frac{42}{5}$

$f(-1) = \frac{1}{2} - \frac{1}{5} + K = \frac{3}{10}$

$f(2) - f(-1) = \frac{42}{5} - \frac{3}{10} = \frac{420 - 15}{50} = \frac{405 : 5}{50 : 5} = \frac{81}{10}$

Truque Estevão $\left(\frac{42}{5}\right) \cdot 2 = \frac{84}{10} - \frac{3}{10} = \frac{81}{10}$

b) $\int_{-3}^{0} x^2 - 4x + 7 \, dx$

$= \int x^2 dx - 4 \int x \, dx + \int 7 \, dx$

$= \frac{x^3}{3} - \cancel{4} \frac{x^2}{\cancel{2}} + 7x + K$

$= \frac{x^3}{3} - 2x^2 + 7x + K$

$f(0) = \frac{0^3}{3} - 2 \cdot 0 + 7 \cdot 0 = 0$

$f(-3) = -\frac{27}{3} - 2(-3)^2 - 21 = -48$

$f(0) - f(-3) = 0 - (-48) = 48$

c) $\int_1^2 \frac{1}{x^6} \, dx$

$= \int x^{-6} dx$

$= \frac{-x^{-5}}{5} + K$

$= -\frac{1}{5x^5} + K$

$f(2) = -\frac{1}{5(2)^5} = -\frac{1}{160}$

$f(1) = -\frac{1}{5(1)^5} = -\frac{1}{5}$

$f(2) - f(1) = -\frac{1}{160} + \frac{1}{5}$

$= \frac{-1 + 32}{160} = \frac{31}{160}$ //

d) $\int_4^9 2x\sqrt{x} \, dx$

$= 2 \int x \cdot x^{\frac{1}{2}} \, dx$

$= 2 \int x^{\frac{3}{2}} \, dx$

$= 2 \cdot \frac{x^{\frac{5}{2}}}{(5/2)} + K$

$= \frac{4 x^{\frac{5}{2}}}{5} + K$

$f(9) = \frac{4\sqrt[2]{(9)^5}}{5} = \frac{4\sqrt{59049}}{5} = \frac{4 \cdot (243)}{5} = \frac{972}{5}$

$f(4) = \frac{4\sqrt{(4)^5}}{5} = \frac{4\sqrt{1024}}{5} = \frac{4 \cdot 32}{5} = \frac{128}{5}$

$f(9) - f(4) = \frac{972}{5} - \frac{128}{5} = \frac{972 - 128}{5} = \frac{844}{5}$

e) $\int \frac{1}{\sqrt{3x+1}} \, dx$

$u = 3x + 1$
$du = 3 \, dx \Rightarrow dx = \frac{du}{3}$

$= \int \frac{1}{\sqrt{u}} \cdot \frac{1}{3} du$

$= \frac{1}{3} \int u^{-\frac{1}{2}} \, du$

$= \frac{1}{3} \, 2u^{\frac{1}{2}} \, du$

$\frac{1}{3} \cdot 2\sqrt{3x+1} + K$

$\frac{2}{3} \sqrt{3x+1} + K$

$f(1) = \frac{2}{3}\sqrt{4} = \frac{4}{3}$

$f(0) = \frac{2}{3}\sqrt{1} = \frac{2}{3}$

$f(1) - f(0) = \frac{4}{3} - \frac{2}{3} = \frac{2}{3}$ //

f) $\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \operatorname{sen} x \cdot \cos x \, dx = \operatorname{sen} x \cdot \operatorname{sen} x - \int \operatorname{sen} x \cdot \cos x \, dx$

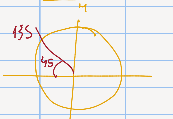
$u = \operatorname{sen} x$ $\quad dv = \cos x \, dx$
$du = \cos x \, dx$ $\quad v = \operatorname{sen} x$

$2 \int \operatorname{sen} x \cdot \cos x \, dx = \operatorname{sen} x \cdot \operatorname{sen} x$

$= \frac{1}{2} (\operatorname{sen} x \cdot \operatorname{sen} x)$

$\frac{3\pi}{4} = \frac{180}{\pi}$

$\frac{3 \cdot 180}{4} = 135$



$f\left(\frac{3\pi}{4}\right) = \frac{1}{2}\left(\operatorname{sen}\left(\frac{3\pi}{4}\right) \cdot \operatorname{sen}\left(\frac{3\pi}{4}\right)\right) \Rightarrow \frac{1}{2}(\operatorname{sen}(45) \cdot \operatorname{sen}(45))$

$= \frac{1}{2}\left(\frac{\sqrt{2}}{2} \cdot \frac{\sqrt{2}}{2}\right) = \frac{1}{\cancel{2}} \frac{\cancel{2}}{4} = \frac{1}{4}$

$\frac{\pi}{4} = \frac{180}{\pi} =$

$\frac{180}{4} = 45$

$f\left(\frac{\pi}{4}\right) = \frac{1}{2}\left(\operatorname{sen}\left(\frac{\pi}{4}\right) \cdot \operatorname{sen}\left(\frac{\pi}{4}\right)\right) = \frac{1}{2}(\operatorname{sen}(45) \cdot \operatorname{sen}(45)) = \frac{1}{4}$

$f\left(\frac{3\pi}{4}\right) - f\left(\frac{\pi}{4}\right) = \frac{1}{4} - \frac{1}{4} = 0$ //

G) $\int_{-2}^{5} |2x - 4| \, dx = I$

$|2x-4| \begin{cases} 2x - 4, & \text{se } x \geq 2 \\ -2x + 4, & \text{se } x < 2 \end{cases}$

$I = \int_{-2}^{2} (-2x+4) \, dx + \int_2^5 (2x-4) \, dx$

$= \underbrace{(-x^2 + 4x)\Big|_{-2}^{2}}_{\mathrm{I}} + \underbrace{(x^2 - 4x)\Big|_2^5}_{\mathrm{II}}$

(I) $[-(2)^2 + 4(2) - (-(-2)^2 + 4(-2))]$
$-4 + 8 + 4 + 8 = 16$

(II) $[25 - 20 - (4 - 8)]$
$5 - 4 + 8 = 9$

$\mathrm{I} + \mathrm{II} = 16 + 9 = 25$ //

h) $\int_0^{\frac{\pi}{2}} \operatorname{sen}^2 x \, dx = \int \frac{1 - \cos 2x}{2} dx$

$= \int \frac{1}{2} - \frac{\cos 2x}{2} \, dx$

$= \int \frac{1}{2} dx - \int \frac{\cos 2x}{2} \, dx$

$= \frac{x}{2} - \left(\frac{1}{2} \int \cos 2x \, dx\right)$

$= \frac{x}{2} - \left(\frac{1}{2} \int \cos u \, \frac{du}{2}\right)$

$= \frac{x}{2} - \left(\frac{1}{4} \cdot \operatorname{sen} 2x\right)$ //

$f\left(\frac{\pi}{2}\right) = \frac{\left(\frac{\pi}{2}\right)}{2} - \left(\frac{1}{4} \cdot \overset{0}{\operatorname{sen}\left(\cancel{2} \cdot \frac{\pi}{\cancel{2}}\right)}\right) = \frac{\pi}{4}$

$f(0) = \frac{0}{2} - \frac{1}{4} \cdot \operatorname{sen}(2 \cdot 0) = 0$

$f\left(\frac{\pi}{2}\right) - f(0) = \frac{\pi}{4} - 0 = \frac{\pi}{4}$

i) $\int_0^{\frac{\pi}{2}} \frac{\cos x}{(1 + \operatorname{sen} x)^5} \, dx$

$= \int \frac{\cos x}{(1 + \operatorname{sen} x)^5} \, dx$

$= \int \frac{1}{(1 + \operatorname{sen} x)^5} \cdot \cos x \, dx$

$= \int \frac{1}{u^5} \, du$

$u = 1 + \operatorname{sen} x$
$du = \cos x \cdot dx$

$= \int u^{-5} \, du$

$= -\frac{u^{-4}}{4}$

$= -\frac{(1 + \operatorname{sen} x)^{-4}}{4}$ //

$f\left(\frac{\pi}{2}\right) = -\frac{\left(1 + \operatorname{sen} \frac{\pi}{2}\right)^{-4}}{4} = -\frac{\frac{1}{2^4}}{4} = -\frac{1}{16} \cdot \frac{1}{4} = -\frac{1}{64}$

$f(0) = -\frac{(1 + \operatorname{sen} 0)^{-4}}{4} = -\frac{1^{-4}}{4} = -\frac{\frac{1}{1^4}}{4} = -\frac{1}{4}$

$f\left(\frac{\pi}{2}\right) - f(0) = -\frac{1}{64} + \frac{1}{4} = \frac{15}{64}$ //

J) $\int_1^2 x \cdot \ln x \, dx$

$u = \ln x$ $\quad dv = x \, dx$
$du = \frac{1}{x} dx$ $\quad v = \frac{x^2}{2}$

$= \frac{x^2}{2} \cdot \ln|x| - \int \frac{x^{\cancel{2}}}{2} \cdot \frac{1}{\cancel{x}} \, dx$

$= \frac{x^2}{2} \cdot \ln|x| - \frac{1}{2} \int x \, dx$

$= \frac{x^2}{2} \cdot \ln|x| - \frac{1}{2} \cdot \frac{x^2}{2}$

$= \frac{x^2}{2} \left(\ln|x| - \frac{1}{2}\right)$

$f(2) = \frac{2^2}{2}\left(\ln|2| - \frac{1}{2}\right)$

$= 2 \ln|2| - 1$

$f(1) = \frac{1}{2}\left(0 - \frac{1}{2}\right) = -\frac{1}{4}$

$f(2) - f(1) = 2 \ln|2| - 1 + \frac{1}{4}$

$= 2 \ln|2| + \frac{3}{4}$

K) $\int_{\frac{\pi}{2}}^{\pi} x \cdot \operatorname{sen} x \, dx$

$= x \cdot -\cos x - \int -\cos x \, dx$

$= -x \cdot \cos x + \int \cos x \, dx$

$= -x \cdot \cos x + \operatorname{sen} x + K$ //

$u = x$ $\quad dv = \operatorname{sen} x \, dx$
$du = dx$ $\quad v = -\cos x$

$f(\pi) = -\pi \cdot \cos \pi + \operatorname{sen} \pi$
$-\pi \cdot -1 = \pi$

$f\left(\frac{\pi}{2}\right) = -\frac{\pi}{2} \cdot \cos\left(\frac{\pi}{2}\right) + \operatorname{sen}\left(\frac{\pi}{2}\right)$
$= -\frac{\pi}{2} \cdot 0 + 1 = +1$

$f(\pi) - f\left(\frac{\pi}{2}\right) = \pi - (1) = \pi - 1$

L) $\int_0^1 x \cdot e^{-x} \, dx$

$u = x$ $\quad dv = e^{-x} dx$
$du = dx$ $\quad v = -e^{-x}$

$= -e^{-x} \cdot x - \int -e^{-x} \, dx$

$= -e^{-x} \cdot x - e^{-x}$

$= -e^{-x} \cdot (x + 1)$ //

$f(1) = -e^{-1} \cdot (1+1) = -\frac{2}{e}$

$f(0) = -e^{0} \cdot (0+1) = -1$

$f(1) - f(0) = -\frac{2}{e} + 1$

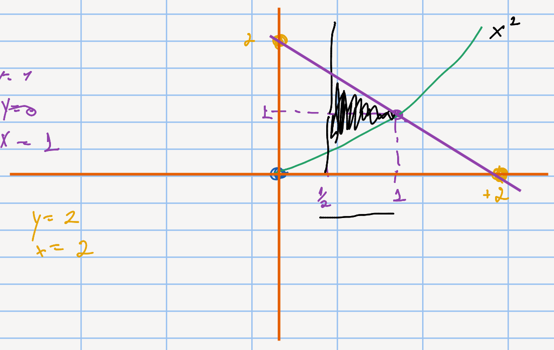
$= \frac{-2 + e}{e}$

6) $x = \frac{1}{2}$, $x = \sqrt{y}$, $y = -x + 2$

1) $y = x^2$
2) $y = -x + 2$

1) $x = 1$, $y = 1^2$, $y = 1$
$x = 0$, $y = 0^2$, $y = 0$
$y = 1$, $1 = x^2$, $x = 1$

2) $x = 0$, $y = 0 + 2$, $y = 2$
$y = 0$, $0 = -x + 2$, $x = 2$



$y^2 - 2x = 0$

$$\int_{1/2}^{1} (-x+2) - (x^2)\, dx$$

$$\int -x^2 - x + 2\, dx$$

$$-\frac{x^3}{3} - \frac{x^2}{2} + 2x$$

$f(1) = -\frac{1}{3} - \frac{1}{2} + 2 = \frac{-2-3}{6} = \frac{-5}{6} + 2 = \frac{-5+12}{6} = \frac{7}{6}$

$f(\frac{1}{2}) = -\frac{(\frac{1}{8})}{3} - \frac{(\frac{1}{4})}{2} + 1 = -\frac{1}{24} - \frac{1}{8} + 1 = \frac{5}{6}$

$\frac{7}{6} - \frac{5}{6} = \frac{2}{6} = \frac{1}{3}$

b) $\underbrace{y^2 = 2x}_{\text{I}}$ $\underbrace{x^2 = 2y}_{\text{II}}$

- isolar alguma variável x ou y de qualquer equação

I $y^2 = 2x \Rightarrow x = \frac{y^2}{2}$

- substituir o valor encontrado na segunda equação.

II $x^2 = 2y \Rightarrow \left(\frac{y^2}{2}\right)^2 = 2y$

$\Rightarrow \frac{y^4}{4} = 2y \Rightarrow y^4 = 8y$

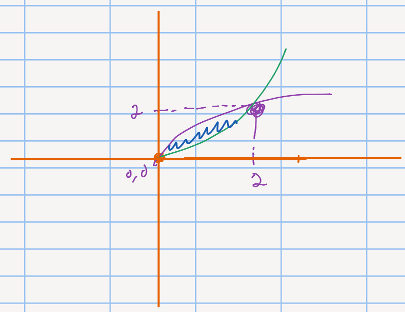
$\Rightarrow y^4 - 8y = 0 \Rightarrow y(y^3 - 8) = 0$

$y = 0$
ou
$y^3 - 8 = 0 \Rightarrow y^3 = +8$

$\Rightarrow \sqrt[3]{y^3} = \sqrt[3]{8} \Rightarrow y = 2$,

I $y = 0 \Rightarrow 0^2 = 2x \Rightarrow x = 0$
$y = 2 \Rightarrow 2^2 = 2x \Rightarrow x = 2$

II $x = 0$, $0^2 = 2y \Rightarrow y = 0$
$x = 2$, $2^2 = 2y \Rightarrow y = 2$



$$\int_0^2 \left(\sqrt{2x} - \frac{x^2}{2}\right) dx$$

$$\int \sqrt{2x}\, dx - \int \frac{x^2}{2}\, dx$$

$$\int (2x)^{\frac{1}{2}} dx - \frac{1}{2}\int x^2\, dx$$

$$\sqrt{2}\int x^{\frac{1}{2}} dx -$$

$$\underbrace{\sqrt{2}\left(\frac{2x^{3/2}}{3}\right)\Big|_0^2}_{\text{I}} - \underbrace{\frac{1}{2}\left(\frac{x^3}{3}\right)\Big|_0^2}_{\text{II}}$$

I $= \sqrt{2} \cdot \frac{2}{3} \cdot 2\sqrt{2} = \frac{8}{3}$

II $\frac{1}{2} \cdot \frac{8}{3} = \frac{8}{6} = \frac{4}{3}$

$\boxed{\text{I} - \text{II}} = \frac{8}{3} - \frac{4}{3} = \frac{4}{3}$ //

c) $y = -x^2 + 5$ $y = x + 3$

$\begin{cases} y = -x^2 + 5 \\ y = x + 3 \end{cases}$

$-x^2 + 5 = x + 3$
$-x^2 - x + 5 - 3 = 0$
$-x^2 - x + 2 = 0$

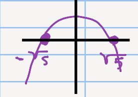
$\Delta = (-1)^2 - 4 \cdot (-1) \cdot (2)$
$\Delta = 1 + 8 = 9$

$x = \frac{-1 \pm \sqrt{9}}{-2}$

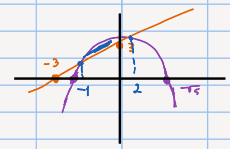
$x = \frac{-1+3}{-2} = -1$

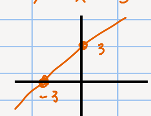
$x = \frac{-1-3}{-2} = 2$

$y = -x^2 + 5$
$x_1 = \sqrt{5}$ $x_2 = -\sqrt{5}$



$y = x + 3$





$$\int_{-1}^{2} (-x^2+5) - (x+3)\, dx$$

$$\int (-x^2+5)\, dx - \int (x+3)\, dx$$

$$-\int x^2 dx + \int 5\, dx - \int x\, dx + \int 3\, dx$$

$$-\frac{x^3}{3} + 5x - \frac{x^2}{2} - 3x$$

$$\left(-\frac{x^3}{3} - \frac{x^2}{2} + 2x\right)\Big|_{-1}^{2}$$

$f(2) = \left(-\frac{8}{3} - \frac{4}{2} + 2 \cdot 2\right) = -\frac{8}{3} - 2 + 4$
$= -\frac{8}{3} + 2 = \frac{-8+6}{3} = \frac{-2}{3}$

$f(-1) = +\frac{1}{3} - \frac{1}{2} - 2 = \frac{2-3}{6} - \frac{2}{1} = \frac{1}{6} - \frac{2}{1} = -\frac{13}{6}$

$-\frac{2}{3} + \frac{13}{6} = \frac{-4+13}{6} = \frac{9}{6}$

F) $x + y = 3$ $y + x^2 = 3$

- pontos de intersecção

$\begin{cases} y = 3 - x \\ y = 3 - x^2 \end{cases}$

$3 - x = 3 - x^2$
$3 - 3 - x + x^2 = 0$
$x^2 - x = 0$

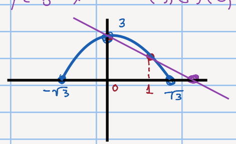
$\Delta = 1$ $x_1 = \frac{1+1}{2} = 1$

$x = \frac{1 \pm \sqrt{1}}{2}$ $x_2 = \frac{1-1}{2} = 0$

- Esboço gráfico

$y = 3 - x^2 = 0$
$x^2 = 3 \Rightarrow x = \pm\sqrt{3}$
$y_v = \frac{-\Delta}{4 \cdot a} = \frac{-12}{4 \cdot (-1)} = 3$

$y = 3 - x$ $(3,0)\ (0,3)$



$\Rightarrow \int_0^1 (-x^2+3) - (3-x)\, dx$

$\Rightarrow \int -x^2 + 3\, dx - \int 3 - x\, dx$

$\Rightarrow \left(-\frac{x^3}{3} + 3x\right)\Big|_0^1 - \left(-\frac{x^2}{2} + 3x\right)\Big|_0^1$

$\Rightarrow \underbrace{\left(-\frac{x^3}{3} + 3x\right)\Big|_0^1}_{f(x)} \underbrace{\left(+\frac{x^2}{2} - 3x\right)\Big|_0^1}_{g(x)}$

$\Rightarrow \frac{8}{3} - \frac{5}{2} = \frac{16-15}{6} = \frac{1}{6}$ //

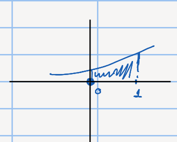
$f(1) = -\frac{1}{3} + 3 = \frac{-1+9}{3} = \frac{8}{3}$

$f(0) = 0$

$g(1) = \frac{1}{2} - 3 = -\frac{5}{2}$

$g(0) = 0$

g) $y = e^x$ $x = 0$ $x = 1$ $y = 0$



$\int_0^1 e^x\, dx = (e^x)\Big|_0^1$

$e^1 - e^0 = e - 1$ //

h) $x = y^3$ $x = y$

$\begin{cases} x = y^3 \\ x = y \end{cases}$

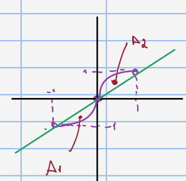
$y^3 = y$
$y^3 - y = 0$
$y(y^2 - 1) = 0$

$y = 0$
ou
$y^2 - 1 = 0$
$y^2 = 1$
$y = \pm 1$

$x = y^3$

$y = 0 \Rightarrow x = 0$
$y = 1 \Rightarrow x = 1$
$y = -1 \Rightarrow x = -1$

- Esboço gráfico



obs: o gráfico está sendo projetado em relação a "y", então a integral fica o que ta na direita menos esquerda

1°) modo $A = A_1 + A_2$

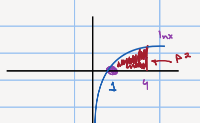
$A_1 = -\int_{-1}^{0} y^3 - y\, dy$
$= -\left(\frac{y^4}{4} - \frac{y^2}{2}\right)\Big|_{-1}^{0}$
$f(-1) = -\frac{1}{4} + \frac{1}{2} = \frac{1}{4}$

$A_2 = \int_0^1 y - y^3\, dy$
$= \left(\frac{y^2}{2} - \frac{y^4}{4}\right)\Big|_0^1$
$f(1) = \frac{1}{2} - \frac{1}{4} = \frac{1}{4}$

$A = \frac{1}{4} + \frac{1}{4} = \frac{2}{4} = \frac{1}{2}$ //

i) $y = \ln x$ $y = 0$ $x = 4$

$\ln 0 = 1$



$\int_1^4 \ln x\, dx$
$u = \ln x$ $dv = dx$
$du = \frac{1}{x}$ $v = x$
$\ln x \cdot x - \int x \cdot \frac{1}{x}\, dx$
$(\ln x \cdot x - x)\Big|_1^4$

$f(1) = \ln 1 \cdot 1 - 1 = -1$

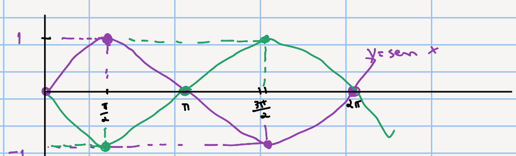
$f(4) = \ln(4) \cdot 4 - 4 = \ln(2^2) \cdot 4 - 4$
$= 8 \ln 2 - 4$

$f(4) - f(1) = 8 \ln 2 - 4 - 1$

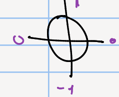
$= 8 \ln(2) - 4 - (-1)$

$= 8 \ln(2) - 3$ //

I) $y = \operatorname{sen} x$ $y = -\operatorname{sen} x$ com $x \in [0, 2\pi]$



$\Rightarrow 2\int_0^{\pi} \operatorname{sen} x - (-\operatorname{sen} x)\, dx$



$\Rightarrow 2\int 2 \operatorname{sen} x\, dx$

$\Rightarrow 4 \int \operatorname{sen} x\, dx$

$\Rightarrow 4 \cdot \overbrace{(-\cos x)}^{f(x)}\Big|_0^{\pi}$

$f(0) = -\cos 0 = -1$

$f(\pi) = -\cos \pi = -(-1) = 1$

$4[f(\pi) - f(0)] = 4[1 - (-1)] = 2 \cdot 4 = 8$ //

7) a) $\int \sqrt{1-4x^2}\,dx$

1º passo: deixar (2x) como variavel

$u = 2x$
$du = 2\,dx$

• 1º Caso: $\sqrt{a^2 - x^2}$

$u = 1 \cdot \operatorname{sen}\theta$
$du = 1 \cdot \cos\theta\, d\theta$
$\sqrt{1^2 - u^2} = 1 \cdot \cos\theta$

$= \int \sqrt{1^2 - (2x)^2}\,dx$

$= \int \sqrt{1^2 - u^2} \cdot \frac{du}{2}$

agora está no formato adequado para usar o 1º caso.

$= \frac{1}{2}\int \sqrt{1^2 - u^2}\,du$

$= \frac{1}{2}\int \cos\theta \cdot \cos\theta\, d\theta$

$= \frac{1}{2}\int \cos^2\theta\, d\theta$

$= \frac{1}{2}\int \frac{1+\cos(2\cdot\theta)}{2}\,d\theta$

$= \frac{1}{2}\left[\int \frac{1}{2}\,d + \frac{1}{2}\int \cos\overbrace{(2\cdot\theta)}^{w}\,d\theta\right]$

$= \frac{1}{2}\left(\frac{\theta}{2} + \frac{1}{2}\left[-\frac{1}{2}\operatorname{sen}(2\theta)\right]\right)$

$= \frac{1}{2}\left[\frac{\theta}{2} - \frac{1}{4}\operatorname{sen}(2\theta)\right] + K$

$= \frac{1}{2}\left[\frac{\theta}{2} - \frac{\operatorname{sen}(2\theta)}{4}\right] + K$

• formula ângulo duplo: $\operatorname{sen}(2x) = 2\operatorname{sen}x \cdot \cos x$

$= \frac{1}{2}\left[\frac{\theta}{2} - \frac{\cancel{2}\operatorname{sen}\theta \cdot \cos\theta}{\cancel{4}}\right] + K$

$= \frac{1}{2}\left[\frac{\theta}{2} - \frac{\operatorname{sen}\theta \cdot \cos\theta}{2}\right] + K$

• Precisamos voltar para U

$u = \operatorname{sen}\theta \Rightarrow \operatorname{sen}\theta = u$
$\cos\theta = \sqrt{1^2 - u^2}$

Para descobrir $\theta$, precisamos isolar $\operatorname{sen}\theta$, depois aplicar a inversa em ambos os lados da equação

Você aplica a função inversa do seno, arcsin, para encontrar o valor de θ porque ela "desfaz" o efeito da função seno. A função seno toma um ângulo θ e devolve um valor (o seno desse ângulo). Quando vc tem esse valor e quer descobrir qual era o angulo original da função, aplica a função inversa arcsin que retorna esse angulo que reproduziu o determinado valor do sen

$\operatorname{sen}\theta = u$
$\operatorname{arc\,sen}(\operatorname{sen}\theta) = \operatorname{arc\,sen}(u)$

$\theta = \operatorname{arc\,sen} u$

$= \frac{1}{2}\left[\frac{\operatorname{arc\,sen} u}{2} - \frac{u\cdot\sqrt{1^2-u^2}}{2}\right] K$

• agora precisamos voltar para $x$

$= \frac{1}{2}\left[\frac{\operatorname{arc\,sen} 2x}{2} - \frac{2x\sqrt{1-(2x)^2}}{2}\right] + K$

$= \frac{1}{4}\left[\operatorname{arc\,sen} 2x - 2x\sqrt{1-4x^2}\right] + K$

b) $\int \frac{1}{\sqrt{4-x^2}}\,dx$

$x = 2\operatorname{sen}\theta$
$dx = 2\cos\theta\, d\theta$
$\sqrt{2^2 - x^2} = 2\cos\theta$

$= \int \frac{1}{\sqrt{2^2 - x^2}}\,dx$

$= \int \frac{1}{\cancel{2\cos\theta}} \cdot \cancel{2\cos\theta}\, d\theta$

$= \int d\theta$

$= \theta + K$

$= \operatorname{arc\,sen}\left(\frac{x}{2}\right) + K$

c) $\int \frac{1}{\sqrt{4+x^2}}\,dx$

$x = 2\cdot tg\,\theta$
$dx = 2\cdot\sec^2\theta\, d\theta$
$\sqrt{2^2+x^2} = 2\sec\theta$

$\theta = \operatorname{arc\,tg}\frac{x}{2}$

$= \int \frac{1}{\sqrt{2^2+x^2}}\,dx$

$= \int \frac{1}{\cancel{2}\cancel{\sec\theta}} \cdot \cancel{2}\sec^{\cancel{2}}\theta\, d\theta$

$= \int \sec\theta\, d\theta$

$= \ln|\sec\theta + tg\,\theta| + K$

$= \ln\left|\frac{\sqrt{4+x^2}}{2} + \frac{x}{2}\right| + K$

$= \ln\left|\frac{1}{2}\left(\sqrt{4+x^2} + x\right)\right| + K$

D) $\int \frac{1}{4+x^2}\,dx$

$x = 2\cdot tg\,\theta$
$dx = 2\sec^2\theta\, d\theta$

$a^2 + x^2 = 2\sec\theta$

$= \int \frac{1}{2^2+x^2}\,dx$

$= \int \frac{1}{4+(2\cdot tg\,\theta)^2} \cdot 2\sec^2\theta\, d\theta$

$= \int \frac{2\cdot\sec^2\theta}{4+(2\,tg\,\theta)^2}\,d\theta$

$= \int \frac{2\cdot\sec^2\theta}{4+4\,tg^2\theta}\,d\theta$

$= \int \frac{2\cdot\sec^2\theta}{4(1+tg^2\theta)}\,d\theta$

$= \int \frac{2\cdot\cancel{\sec^2\theta}}{4\cdot\cancel{\sec^2\theta}}\,d\theta$

$= \int \frac{2}{4}\,d\theta \Rightarrow \frac{1}{2}\theta + K$

$= \frac{1}{2}\operatorname{arctg}\frac{x}{2} + K$ //

E) $\int \frac{x}{\sqrt{1-x^2}}\,dx$

$x = 1\operatorname{sen}\theta$
$dx = 1\cos\theta\, d\theta$

$\sqrt{1^2-x^2} = \cos\theta$

$= \int \frac{x}{\sqrt{1^2-x^2}}\,dx$

$= \int \frac{\operatorname{sen}\theta}{\cancel{\cos\theta}} \cdot \cancel{\cos\theta}\, d\theta$

$= \int \operatorname{sen}\theta\, d\theta$

$= -\cos\theta + K$

$= -\sqrt{1-x^2} + K$

f) $\int \sqrt{3-4x^2}\,dx$

$\sqrt{(\sqrt{3})^2 - \underbrace{(2x)^2}_{x}}$

$a = \sqrt{3}$

$2x = a\operatorname{sen}\theta$
$(2x)' = (\sqrt{3}\cdot\operatorname{sen}\theta)'$

$2\,dx = \sqrt{3}\cos\theta$
$dx = \frac{\sqrt{3}}{2}\cos\theta$

$\sqrt{(\sqrt{3})^2 - (2x)^2} = \sqrt{3}\cdot\cos\theta$

$= \int \sqrt{3}\cdot\cos\theta \cdot \frac{\sqrt{3}}{2}\cos\theta\, d\theta$

$= \frac{3}{2}\int \cos^2\theta\, d\theta$

$= \frac{3}{2}\left(\frac{\theta}{2} + \frac{\operatorname{sen}\theta\cdot\cos\theta}{2}\right) + K$

$\frac{x}{2} + \frac{\operatorname{sen}x \cdot \cos x}{2}$

$\frac{3}{4}(\theta + \operatorname{sen}\theta \cdot \cos\theta) + K$

$\frac{3}{4}\left(\operatorname{arc\,sen}\left(\frac{2x}{\sqrt{3}}\right) + \frac{2x}{\sqrt{3}} \cdot \frac{\sqrt{3-x^2}}{\sqrt{3}}\right)$

$\frac{3}{4}\left(\operatorname{arc\,sen}\left(\frac{2x}{\sqrt{3}}\right) + \frac{2x\sqrt{3-x^2}}{3}\right) + K$ //

G) $\int \frac{x^2}{\sqrt{1-x^2}}\,dx$

$x = \operatorname{sen}\theta$
$dx = 1\cdot\cos\theta\, d\theta$
$\sqrt{1^2-x^2} = \cos\theta$

$= \int \frac{x^2}{\sqrt{1^2-x^2}}\,dx$

$= \int \frac{\operatorname{sen}^2\theta}{\cancel{\cos\theta}} \cdot \cancel{\cos\theta}\, d\theta$

$= \int \operatorname{sen}^2\theta\, d\theta$

$= \int \frac{1}{2} - \frac{\cos 2x}{2}\,dx$

$= \int \frac{1}{2}\,d\theta - \frac{1}{2}\int \cos 2\theta\, d\theta$

$= \frac{\theta}{2} - \frac{1}{2}\int \cos u\, \frac{du}{2}$

$= \frac{\theta}{2} - \frac{1}{4}\int \cos u\, du$

$= \frac{\theta}{2} - \frac{1}{4}\operatorname{sen} 2\theta + K$

$= \frac{\theta}{2} - \frac{1}{4}(2\operatorname{sen}\theta \cdot \cos\theta) + K$

$= \frac{\operatorname{arc\,sen} x}{2} - \frac{1}{2}\cdot x \cdot \sqrt{1-x^2} + K$

$\frac{1}{2}\left(\operatorname{arc\,sen} x - x\sqrt{1-x^2}\right) + K$ //

$\frac{x}{2} + \frac{\operatorname{sen}x \cdot \cos x}{2}$

h) $\int x^2 \cdot \sqrt{1-x^2}\,dx$

• Subs trigonometrica

$x = \operatorname{sen}\theta$
$dx = \cos\theta\, d\theta$

$\sqrt{1^2-x^2} = \cos\theta$

$= \int \operatorname{sen}^2\theta \cdot \cos\theta\, \cos\theta\, d\theta$

$= \int \operatorname{sen}^2\theta \cdot \cos^2\theta\, d\theta$

$= \int \left(\frac{1-\cos(2\theta)}{2}\right) \cdot \left(\frac{1+\cos(2\theta)}{2}\right) d\theta$

$= \int \frac{1-\cos^2(2\theta)}{4}\,d\theta$

$= \frac{1}{4}\int (1-\cos^2(2\theta))\,d\theta$

$= \frac{1}{4}\left[\int 1\,d\theta - \underbrace{\int \cos^2(2\theta)\,d\theta}_{} - \mathrm{I}\right]$

$\cos^2 x = \frac{1+\cos(2x)}{2}$

$\cos^2(2\theta) = \frac{1+\cos(2\cdot(2\theta))}{2}$

$= \frac{1}{4}\left[\theta - \frac{1}{2}\int 1 + \cos(4\theta)\,d\theta\right]$

$= \frac{1}{4}\left[\theta - \frac{1}{2}\left[\theta + \int \cos u\, \frac{du}{4}\right]\right]$

$= \frac{1}{4}\left[\theta - \frac{1}{2}\left[\theta + \frac{1}{4}\operatorname{sen}4\theta\right]\right]$

$= \frac{1}{4}\left[\theta - \frac{1}{2}\left[\theta + \frac{1}{4}\operatorname{sen}4\theta\right]\right] + K$

$\frac{1}{4}\left[\operatorname{arc\,sen} x - \frac{1}{2}\left[\operatorname{arc\,sen} x + \cancel{\frac{1}{4}}\left(\operatorname{sen}(4\theta)\right)\right]\right]$

$\operatorname{sen} 2x = 2\operatorname{sen} x \cdot \cos x$

$2x = 4\theta \qquad = 2\operatorname{sen}2\theta \cdot \cos 2\theta$
$x = 2\theta$

$\frac{1}{4}\left[\operatorname{arc\,sen} x - \frac{1}{2}\left[\operatorname{arc\,sen} x + \frac{1}{4}(2\operatorname{sen}2\theta \cdot \cos 2\theta)\right]\right]$

$x = \operatorname{sen}\theta$
$dx = \cos\theta\, d\theta$
$\sqrt{1^2-x^2} = \cos\theta$

$\operatorname{sen}2\theta = 2\cdot\operatorname{sen}\theta \cdot \cos\theta$

$\cos 2\theta = 1 - 2\operatorname{sen}^2 x$

$\frac{1}{4}\left[\operatorname{arc\,sen}x - \frac{1}{2}\left[\operatorname{arc\,sen} x + \left[\frac{1}{4}\, 2\cdot(2\operatorname{sen}\theta \cdot \cos\theta) \cdot (1-2\operatorname{sen}^2\theta)\right]\right.\right.$

$\frac{1}{4}\left[\operatorname{arc\,sen}x - \frac{1}{2}\left[\operatorname{arc\,sen} x + \left[\cancel{\frac{1}{4}}(\cancel{4}\operatorname{sen}\theta \cdot \cos\theta) \cdot (1-2\operatorname{sen}^2\theta)\right]\right.\right.$

$\frac{1}{4}\left[\operatorname{arc\,sen}x - \frac{1}{2}\operatorname{arsen}x - \frac{1}{2}\left[(x \cdot \sqrt{1-x^2}) \cdot (1-2x^2)\right]\right.$

$\frac{1}{4}\left[\frac{1}{2}\operatorname{arc\,sen}x - \frac{1}{2}(x\sqrt{1-x^2})(1-2x^2)\right]$

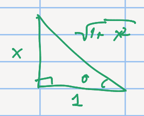
$\frac{1}{4}\cdot\frac{1}{2}\left[\operatorname{arc\,sen}x - (x\sqrt{1-x^2})(1-2x^2)\right.$ //

i) $\int \frac{1}{x\sqrt{1+x^2}}\,dx$

$x = tg\,\theta$
$dx = sec^2\theta \; d\theta$
$\sqrt{1+x^2} = sec\,\theta$

$tg = \frac{sen}{cos}$

$cossec = \frac{1}{sen\,\theta}$




$sen = \frac{x}{\sqrt{1+x^2}}$

$cos = \frac{1}{\sqrt{1+x^2}}$

$tg\,\theta = \frac{\left(\frac{x}{\sqrt{1+x^2}}\right)}{\left(\frac{1}{\sqrt{1+x^2}}\right)} = \frac{x}{\sqrt{1+x^2}} \cdot \frac{\sqrt{1+x^2}}{1} = x$

$= \int \frac{1}{tg\theta \cdot sec\theta} \cdot sec^2\theta \, d\theta$

$= \int \frac{1}{tg\theta} \cdot sec\,\theta \; d\theta$

$\frac{\frac{1}{sen}}{\frac{1}{cos}} = \frac{cos}{sen} \cdot \frac{1}{cos}$

$\int \frac{1}{sen} \, d\theta$

$\int cossec\;\theta \, d\theta$

$ln\,|cossec\,\theta - cotg\,\theta| + K$

$ln\left|\frac{\sqrt{1+x^2}}{x} - \frac{1}{x}\right|$ //

J) $\int \sqrt{9-(x-1)^2}\;dx$

$x-1 = 3\,sen\,\theta$
$dx = 3\cos\theta\; d\theta$

$\sqrt{3^2-(x-1)^2} = 3\cos\theta$

$\int \sqrt{3^2-(x-1)^2}\;dx$

$\int 3\cdot cos\;\theta \cdot 3\cos\theta\; d\theta$

$9\int cos^2\theta\; d\theta$

$9\left(\frac{\theta}{2} + \frac{sen\,\theta \cdot cos\,\theta}{2}\right) + K$

$9 \cdot \frac{1}{2}\,(\theta + sen\theta \cdot cos\,\theta) + K$

$\frac{9}{2}\left[arc\,sen\left(\frac{x-1}{3}\right) + \left(\frac{x-1}{3}\right)\cdot\left(\frac{\sqrt{9-(x-1)^2}}{3}\right)\right] + K$

$\frac{9}{2}\,arc\,sen\left(\frac{x-1}{3}\right) + \frac{(x-1)\cdot\sqrt{9-(x-1)^2}}{2}$ //

K) $\int \sqrt{-x^2+2x+2}\;dx$

fatorar sinal

$-(x^2-2x)+2$

$\left(\frac{-2}{2}\right)^2 = 1$

$-(x^2-2x+1-1)+2$

$-((x-1)^2-1)+2$

$-(x-1)^2+3$

$x-1 = \sqrt{3}\;sen\;\theta$
$dx = \sqrt{3}\;cos\;\theta$

$\sqrt{(\sqrt{3})^2-(x-1)^2} = \sqrt{3}\cdot cos\;\theta$

$\int \sqrt{3-(x-1)^2}$

$\int \sqrt{3}\cdot cos\,\theta \cdot \sqrt{3}\cdot cos\,\theta \quad \theta$

$3\int cos^2\theta \quad d\theta$

$3\left(\frac{\theta}{2} + \frac{sen\theta\,cos\theta}{2}\right) + K$

$\frac{3}{2}(\theta + sen\,\theta \cdot cos\,\theta) + K$

$\frac{3}{2}\left[arc\,sen\left(\frac{x-1}{\sqrt{3}}\right) + \left(\frac{x-1}{\sqrt{3}}\right)\cdot\left(\frac{\sqrt{3-(x-1)^2}}{\sqrt{3}}\right)\right] + K$

$\frac{3}{2}\left[arc\,sen\left(\frac{x-1}{\sqrt{3}}\right) + \frac{(x-1)\cdot\sqrt{3-(x-1)^2}}{3}\right] + K$

L) $\int \sqrt{-x^2+2x+3}\;dx$

$\left(\frac{2}{2}\right)^2 = 1 \qquad (a+d)^2+e$

$-(x^2-2x)+3$

$-(x^2-2x+1-1)+3$

$-(x-1)^2+1+3$

$-(x-1)^2+4$

$x-1 = 2\,sen\,\theta$
$dx = 2\cos\theta\; d\theta$

$\sqrt{a^2-x^2} = 2\cos\theta$

$\int \sqrt{4-(x-1)^2}$

$\int \sqrt{(2)^2-(x-1)^2}$

$\int 2\cdot cos\,\theta \cdot 2\cos\theta\; d\theta$

$4\int cos^2\theta \quad d\theta$

$4\left(\frac{\theta}{2} + \frac{sen\,\theta\cdot cos\,\theta}{2}\right)$

$2\,arc\,sen\left(\frac{x-1}{2}\right) + \left(\frac{x-1}{2}\right)\cdot\frac{\sqrt{4-(x-1)^2}}{2}$

$2\,arc\,sen\left(\frac{x-1}{2}\right) + \frac{(x-1)\sqrt{4-(x-1)^2}}{2}$ //

8) a) $\int \frac{1}{x^2-4}\,dx = \int \frac{1}{(x-2)(x+2)}$

$\frac{1}{(x-2)(x+2)} = \frac{A}{(x-2)} + \frac{B}{(x+2)} = \frac{A(x+2)+B(x-2)}{(x-2)(x+2)}$

• Como os denominadores são iguais, logo:

$1 = A(x+2) + B(x-2)$

$x = 2 \Rightarrow 1 = 4A \Rightarrow A = \frac{1}{4}$

$x = -2 \Rightarrow 1 = B\cdot -4 \Rightarrow B = -\frac{1}{4}$

$= \int \frac{\frac{1}{4}}{(x-2)} + \frac{-\frac{1}{4}}{(x+2)} =$

$= \frac{1}{4}\cdot\int \frac{1}{(x-2)}\,dx - \frac{1}{4}\int \frac{1}{(x+2)}\,dx$

$= \frac{1}{4}\cdot\int \frac{1}{u}\,dx - \frac{1}{4}\int \frac{1}{u}\,du$ $\quad u = x-2$, $du = dx$

$= \frac{1}{4}\cdot \ln|x-2| - \frac{1}{4}\int \ln|x+2|$

$= \frac{1}{4}\left(\ln|x-2| - \ln|x+2|\right)$

$= \frac{1}{4}\ln\left|\frac{x-2}{x+2}\right| + K$

b) $\int \frac{x}{x^2-5x+6}\,dx$

$x_1 + x_2 = \frac{-b}{a} = \frac{-(-5)}{1} = 5$

$x_1 \cdot x_2 = \frac{c}{a} = \frac{6}{1} = 6$

$x_1 = 2 \quad x_2 = 3$

$= \frac{x}{(x-2)(x-3)} = \frac{A}{(x-2)} + \frac{B}{(x-3)} = \frac{A(x-3)+B(x-2)}{(x-2)(x-3)}$

$x = A(x-3) + B(x-2)$

$x = Ax - 3A + Bx - 2B$

$x = (A+B)x - 3A - 2B$

$\begin{cases} A + B = 1 \\ -3A - 2B = 0 \end{cases}$

$A = 1 - B$

$-3(1-B) - 2B = 0$

$-3 + 3B - 2B = 0$

$B = 3$

$A = 1 - 3 = -2$

$= \int \frac{-2}{(x-2)} + \frac{3}{(x-3)}\,dx$

$= -2\int \frac{1}{(x-2)}\,dx + 3\int \frac{1}{(x-3)}\,dx$

$u = x-2$, $du = dx$ $\quad w = x-3$, $dw = dx$

$= -2\int \frac{1}{u}\,du + 3\int \frac{1}{w}\,dw$

$= -2\ln|x-2| + 3\ln|x-3| + K$

c) $\int \frac{2x+1}{x^2-1}\,dx$

$x^2 - 1 = 0$

$x_1 = -1 \quad x_2 = 1$

$\frac{2x+1}{(x-1)(x+1)} = \frac{A}{(x+1)} + \frac{B}{(x-1)} = \frac{A(x-1)+B(x+1)}{(x+1)(x-1)}$

$2x+1 = A(x-1) + B(x+1)$

$2x+1 = Ax - A + Bx + B$

$2x+1 = (A+B)x - A + B$

$\begin{cases} A + B = 2 \\ -A + B = 1 \end{cases}$

$2B = 3 \Rightarrow B = \frac{3}{2}$

$A + \frac{3}{2} = 2 \Rightarrow A = \frac{1}{2}$

$\int \frac{\frac{1}{2}}{(x+1)} + \frac{\frac{3}{2}}{(x-1)}\,dx = \frac{1}{2}\int \frac{1}{x+1} + \frac{3}{2}\int \frac{1}{(x-1)}\,dx$

$u = x+1$, $du = dx$ $\quad w = x-1$, $dw = dx$

$= \frac{1}{2}\int \frac{1}{u}\,du + \frac{3}{2}\int \frac{1}{w}\,dw = \frac{1}{2}\ln|u| + \frac{3}{2}\ln|w| + K$

$= \frac{1}{2}\ln|x+1| + \frac{3}{2}\ln|x-1| + K$

d) $\int \frac{x^2+3x+1}{x^2-2x-3}\,dx$

$x^2 - 2x - 3$

$x_1 = -1 \quad x_2 = 3$

$\begin{array}{r|l} x^2+3x+1 & x^2-2x-3 \\ -x^2+2x+3 & 1 \\ \hline 5x+4 & \end{array}$

$f(x) = (x^2-2x-3)\cdot 1 + (5x+4)$

$\Rightarrow x^2+3x+1 = (x^2-2x-3)\cdot 1 + (5x+4)$

• dividimos ambos os lados por $x^2-2x-3$

$\Rightarrow \frac{x^2+3x+1}{x^2-2x-3} = \frac{(x^2-2x-3)\cdot 1}{x^2-2x-3} + \frac{(5x+4)}{x^2-2x-3}$

$= 1 + \frac{(5x+4)}{x^2-2x-3}$, assim transformamos

a $\int \frac{x^2+3x+1}{x^2-2x-3}\,dx$ em $\int 1 + \frac{5x+4}{x^2-2x-3}\,dx$

$\Rightarrow \int 1\,dx + \underbrace{\int \frac{5x+4}{x^2-2x-3}\,dx}_{\mathrm{I}}$

(I) $\Rightarrow \frac{5x+4}{(x+1)(x-3)} = \frac{A}{(x+1)} + \frac{B}{(x-3)}$

$\Rightarrow \frac{A(x-3)+B(x+1)}{(x+1)(x-3)}$

$\begin{cases} A + B = 5 \\ -3A + B = 4 \end{cases}$

$\Rightarrow 5x+4 = (A+B)x - 3A + B$

$B = 5 - A$

$B = 5 - \frac{1}{4}$

$B = \frac{19}{4}$

$-3A + 5 - A = 4$

$-4A = -1$

$A = \frac{1}{4}$

$\Rightarrow \int \frac{\frac{1}{4}}{(x+1)} + \frac{\frac{19}{4}}{(x-3)}\,dx$

$\Rightarrow \frac{1}{4}\int \frac{1}{x+1}\,dx + \frac{19}{4}\int \frac{1}{x-3}\,dx$

$\Rightarrow \frac{1}{4}\int \frac{1}{u}\,du + \frac{19}{4}\int \frac{1}{w}\,dw$

$\Rightarrow \frac{1}{4}\ln|x+1| + \frac{19}{4}\ln|x-3| + K$

$\int 1\,dx + \mathrm{(I)}$

$x + \frac{1}{4}\ln|x+1| + \frac{19}{4}\ln|x-3| + K$ //

e) $\int \frac{x^2+1}{(x-2)^3}\,dx$

$\frac{x^2+1}{(x-2)^3} = \frac{A}{(x-2)} + \frac{B}{(x-2)^2} + \frac{C}{(x-2)^3}$

$\frac{A(x-2)^2 + B(x-2) + C}{(x-2)^3}$

$x^2 + 1 = A(x-2)^2 + B(x-2) + C$

$= A(x^2 - 4x + 4) + Bx - 2B + C$

$= Ax^2 - 4Ax + 4A + Bx - 2B + C$

$= Ax^2 + x(-4A+B) + 4A - 2B + C$

$\begin{cases} \mathrm{I}\ Ax^2 = \\ \mathrm{II}\ -4A + B = 0 \\ \mathrm{III}\ 4A - 2B + C = 1 \end{cases}$

(II) $-4\cdot 1 + B = 0$ $\quad \boxed{B = 4}$

(III) $4\cdot 1 - 2\cdot 4 + C = 1$

$-4 + C = 1$

$C = 5$

$\Rightarrow \int \frac{1}{(x-2)} + \frac{4}{(x-2)^2} + \frac{5}{(x-2)^3}\,dx$

$\Rightarrow \int \frac{1}{(x-2)}\,dx + 4\int \frac{1}{(x-2)^2}\,dx + 5\int \frac{1}{(x-2)^3}\,dx$

$\Rightarrow u = x-2$, $du = dx$ $\quad w = x-2$, $dw = dx$ $\quad z = x-2$, $dz = dx$

$\Rightarrow \int \frac{1}{u}\,du + 4\int \frac{1}{w^2}\,dw + 5\int \frac{1}{z^3}\,dz$

$\Rightarrow \ln|x-2| + 4\cdot -\frac{1}{x-2} + \frac{5}{2(x-2)^2} + K$

$\Rightarrow \ln|x-2| - \frac{4}{x-2} + \frac{5}{2(x-2)^2} + K$ //

f) $\int \frac{x^3+x+1}{x^2-2x+1}\,dx$

$\frac{x^3+x+1}{x^2-2x+1} = \frac{(x^2-2x+1)\cdot(x+2) + (4x-1)}{x^2-2x+1}$

$\Rightarrow \int x+2\,dx + \underbrace{\int \frac{4x-1}{x^2-2x+1}\,dx}_{\mathrm{II}}$

$\Rightarrow \frac{x^2}{2} + 2x + 4\ln|x-1| - \frac{3}{x-1} + K$ //

$\begin{array}{r|l} x^3+x+1 & x^2-2x+1 \\ -x^3+2x^2-x & x+2 \\ \hline 2x^2+1 & \\ -2x^2+4x-2 & \\ \hline 4x-1 & \end{array}$

$f(x) = (x^2-2x+1)\cdot(x+2) + (4x-1)$

(II) $\frac{4x-1}{(x-1)^2} = \frac{A}{(x-1)} + \frac{B}{(x-1)^2}$

$4x - 1 = Ax - A + B$

$\begin{cases} A = 4 \\ -A + B = -1 \end{cases}$ $\quad A = 4$, $B = 3$

$4\int \frac{1}{(x-1)} + 3\int \frac{1}{(x-1)^2}$

$4\ln|x-1| - \frac{3}{x-1}$

G) $\int \frac{2x-3}{(x-1)^3}\,dx$

$\frac{A}{(x-1)} + \frac{B}{(x-1)^2} + \frac{C}{(x-1)^3} = \frac{A(x-1)^2 + B(x-1) + C}{(x-1)^3}$

$2x - 3 = Ax^2 - 2Ax + A + Bx - B + C$

$= Ax^2 + x(2A+B) + A - B + C$

$\begin{cases} A = 0 \\ 2A + B = 2 \\ A - B + C = -3 \end{cases}$ $\quad A = 0$, $B = 2$, $C = -1$

$\Rightarrow \int \frac{0}{(x-1)} + \frac{2}{(x-1)^2} - \frac{1}{(x-1)^3}\,dx$

$\Rightarrow 2\int \frac{1}{(x-1)^2}\,dx - \int \frac{1}{(x-1)^3}\,dx$

$u = x-1$, $du = dx$

$\Rightarrow 2\int u^{-2}\,du - \int u^{-3}\,du$

$-\frac{2}{x-1} - \frac{1}{2(x-1)^2} + K$ //

h) $\int \frac{x+1}{x(x-2)(x+3)}\,dx$

$\frac{A}{x} + \frac{B}{x-2} + \frac{C}{x+3} \Rightarrow \frac{A(x-2)(x+3) + B\cdot x\cdot(x+3) + Cx(x-2)}{x(x-2)(x+3)}$

$x + 1 = A(x-2)(x+3) + B\cdot x\cdot(x+3) + Cx(x-2)$

$x = 1 \Rightarrow A = -\frac{1}{6}$

$y = 2 \Rightarrow B = \frac{3}{10}$

$x = -3 \Rightarrow C = -2/15$

$\Rightarrow \int \frac{\left(-\frac{1}{6}\right)}{x} + \frac{\left(\frac{3}{10}\right)}{x-2} + \frac{\left(-\frac{2}{15}\right)}{x+3}\,dx$

$\Rightarrow -\frac{1}{6}\int \frac{1}{x}\,dx + \frac{3}{10}\int \frac{1}{x-2} - \frac{2}{15}\int \frac{1}{x+3}\,dx$

$\Rightarrow -\frac{1}{6}\ln|x| + \frac{3}{10}\ln|x-2| - \frac{2}{15}\ln|x+3| + K$ //

i) $\int \frac{x+5}{x^3-4x^2+4x}\,dx$

$x(x^2-4x+4)=0$

$x=0$ ou $x^2-4x+4=0$

$x_1=0 \quad x_2=2 \quad x_3=2$

$\Rightarrow \int \frac{x+5}{x(x-2)^2} \quad \frac{A}{x}+\frac{B}{x-2}+\frac{C}{(x-2)^2}$

$\Rightarrow \frac{A(x-2)^2+Bx(x-2)+Cx}{x(x-2)^2}$

$x+5 = A(x-2)^2 + Bx(x-2) + Cx$
$= Ax^2-4Ax+4A+Bx^2-2Bx+Cx$
$= x^2(A+B)+x(-4A-2B+C)+4A$

$\begin{cases} A+B=0 \\ -4A-2B+C=1 \\ 4A=5 \end{cases}$

$B=-A=-\frac{5}{4}$

$A=\frac{5}{4}$

$C=\frac{-7}{4}$

$\Rightarrow \int \frac{\frac{5}{4}}{x}+\frac{-\frac{5}{4}}{x-2}+\frac{-\frac{7}{4}}{(x-2)^2}$

$\Rightarrow \frac{5}{4}\int\frac{1}{x}\,dx-\frac{5}{4}\int\frac{1}{x-2}\,dx-\frac{7}{4}\int\frac{1}{(x-2)^2}$

$\Rightarrow \frac{5}{4}\ln|x|-\frac{5}{4}\ln|x-2|-\frac{7}{8(x-2)^2}$ //

J) $\int \frac{x^5+3}{x^3-4x}\,dx$

$x^5+3 \;\; \underline{|\,x^3-4x}$
$-x^5+4x^3 \quad x^2+4$
$4x^3+3$
$-4x^3+16x$
$16x+3$

$\frac{x^5+3}{x^3-4x} = \frac{(x^3-4x)(x^2+4)}{x^3-4x}+\frac{16x+3}{x^3-4x}$

$= x^2+4 + \frac{16x+3}{x^3-4x}$

$\Rightarrow \int (x^2+4) + \frac{16x+3}{x^3-4x}\,dx$

$\Rightarrow \int x^2+4\,dx + \underbrace{\int \frac{16x+3}{x^3-4x}\,dx}$

$\Rightarrow \frac{x^3}{3}+4x + \text{I}$

$\boxed{\frac{x^3}{3}+4x+\frac{3}{4}\ln x-\frac{29}{8}\ln|x+2|+\frac{35}{8}\ln|x-2|+K}$ //

I $= \int \frac{16x+3}{x^3-4x}\,dx =$

$x^3-4x$
$x(x^2-4)=0$

$x_1=0 \quad x_2=-2 \quad x_3=2$

$\frac{16+3}{x(x+2)(x-2)} = \frac{A}{x}+\frac{B}{(x+2)}+\frac{C}{(x-2)}$

$\Rightarrow \frac{A(x+2)(x-2)+Bx(x-2)+Cx(x+2)}{x(x-2)(x+2)}$

$x^5+3 = A(x+2)(x-2)+Bx(x-2)+Cx(x+2)$

$x=2$:
$35=8C \Rightarrow \boxed{C=\frac{35}{8}}$

$x=-2$:
$-29=8B \Rightarrow \boxed{B=\frac{-29}{8}}$

método Raízes

• para achar $A$ =
$= Ax^2-4A+Bx^2-B2x+Cx^2+2Cx$
$= x^2(A+B+C)+x(-2B+2C)+4A$

$\begin{cases} A+B+C=0 \\ -2B+2C=0 \\ 4A=3 \end{cases}$ $\quad \boxed{A=\frac{3}{4}}$

$\Rightarrow \int \frac{\frac{3}{4}}{x}+\frac{-\frac{29}{8}}{(x+2)}+\frac{\frac{35}{8}}{(x-2)}$

$\Rightarrow \frac{3}{4}\int\frac{1}{x}\,dx-\frac{29}{8}\int\frac{1}{x+2}\,dx+\frac{35}{8}\int\frac{1}{x-2}$

$= \frac{3}{4}\ln x-\frac{29}{8}\ln|x+2|+\frac{35}{8}\ln|x-2|+K$

K) $\int \frac{4x^2+17x+13}{(x-1)(x^2+6x+10)}\,dx$

$\frac{A}{(x-1)}+\frac{Bx+C}{x^2+6x+10} = \frac{A(x^2+6x+10)+(Bx+C)(x-1)}{(x-1)(x^2+6x+10)}$

$4x^2+17x+13 = A(x^2+6x+10)+(Bx+C)(x-1)$
$= Ax^2+6Ax+10A+Bx^2+Cx-Bx-C$

$\begin{cases} A+B=4 \\ 6A-B+C=17 \\ 10A-C=13 \end{cases}$

$B=4-A$
$B=4-2$
$\boxed{B=2}$

$6A-(4-A)+C=17$
$6A-4+A+C=17$
$7A+C=21$

$10A-C=13$
$C=-13+10A$
$C=-13+20$
$\boxed{C=7}$

$7A+(-13+10A)=21$
$17A=34$
$A=2$

$\int \frac{2}{(x-1)}+\frac{2x+7}{(x^2+6x+10)} =$

$\Rightarrow 2\int\frac{1}{x-1}\,dx+\int\frac{2x+7}{x^2+6x+10}\,dx$

completar quadrado

$ax^2+bx+c = (x+d)^2+e$, então:

$x^2+6x+10 =$

• $\frac{b}{2}=\frac{6}{2}=3$, depois elevar esse valor ao quadrado

• $3^2=9$, agora somamos e subtraímos 9

Logo: $x^2+6x+9-9+10$

agrupar os três primeiros termos

$(x^2+6x+9)+1$

$(x+3)^2+1$

$\Rightarrow 2\int\frac{1}{x-1}\,dx+\int\frac{2x+7}{(x+3)^2+1}\,dx$

$u=x+3 \quad x=u-3$
$du=dx$

$\Rightarrow 2\ln|x-1|+\int\frac{2(u-3)+7}{u^2+1}\,du$

$\Rightarrow 2\ln|x-1|+\int\frac{2u+1}{u^2+1}\,du$

$\Rightarrow 2\ln|x-1|+\underbrace{\int\frac{2u}{u^2+1}\,du}_{\text{II}}+\int\frac{1}{u^2+1}$

II $\quad v=u^2+1$
$dv=2u\,du$

$\Rightarrow 2\ln|x-1|+\int\frac{1}{u^2+1}\cdot 2u\,du+\int\frac{1}{u^2+1}$

$\Rightarrow 2\ln|x-1|+\int\frac{1}{v}\,dv+\text{arctg}\,u$

$\Rightarrow 2\ln|x-1|+\ln|(x^2+6x+10)|+\text{arc tg}\,(x+3)+K$ //

L) $\int \frac{x+2}{x^3+2x^2+5x}\,dx$

$\int \frac{x+2}{x(x^2+2x+5)}\,dx$

$\frac{A}{x}+\frac{Bx+C}{(x^2+2x+5)}$

$\frac{x+2}{x(x^2+2x+5)} = \frac{A(x^2+2x+5)+x(Bx+C)}{x(x^2+2x+5)}$

$x+2 = Ax^2+2Ax+5A+Bx^2+Cx$

$\begin{cases} A+B=0 \\ 2A+C=1 \\ 5A=2 \end{cases}$ $\quad A=\frac{2}{5} \quad C=\frac{1}{5} \quad B=-\frac{2}{5}$

$\int \frac{\left(\frac{2}{5}\right)}{x}+\frac{\left(-\frac{2}{5}\right)x+\left(\frac{1}{5}\right)}{(x^2+2x+5)}\,dx$

$\frac{2}{5}\int\frac{1}{x}\,dx+\int\frac{\left(-\frac{2}{5}\right)x+\left(\frac{1}{5}\right)}{x^2+2x+5}\,dx$

$\frac{2}{5}\ln|x| \quad -\frac{2}{5}\underbrace{\int\frac{x}{x^2+2x+5}\,dx}_{\text{I}}+\frac{1}{5}\boxed{\int\frac{1}{x^2+2x+5}\,dx}$ II

$\frac{2}{5}\ln|x|-\frac{2}{5}\left(\frac{1}{2}\ln|(x+1)^2+4|-\frac{1}{2}\text{arc tg}\left(\frac{x+1}{2}\right)\right)+\frac{1}{5}\left(\frac{1}{2}\text{arc tg}\left(\frac{x+1}{2}\right)\right)$

$\frac{2}{5}\ln|x|-\frac{1}{5}\ln|(x+1)^2+4|+\frac{1}{5}\text{arc tg}\left(\frac{x+1}{2}\right)+\frac{1}{10}\text{arc tg}\left(\frac{x+1}{2}\right)$

$\frac{2}{5}\ln x-\frac{1}{6}\ln|x^2+2x+5|+\frac{3}{10}\text{arc tg}\left(\frac{x+1}{2}\right)$ //

II

$\int\frac{1}{(x+1)^2+4} \quad u=x+1, \; du=dx$

$\int\frac{1}{u^2+2^2} = \frac{1}{2}\,\text{arc tg}\left(\frac{u}{2}\right)$

$= \frac{1}{2}\,\text{arc tg}\left(\frac{x+1}{2}\right)$

I

completar quadrado

$\frac{2}{2}=1$
$(x^2+2x+1)-1+5$
$(x+1)^2+4$

$\int\frac{x}{(x+1)^2+4}\,dx \leftarrow u=x+1, \; du=dx \quad x=u-1$

$\int\frac{u-1}{u^2+4}\,du$

$\int\frac{u}{u^2+4}\,du-\int\frac{1}{u^2+4}\,du$

primitiva imediata (20)

$-\int\frac{1}{u^2+2^2}\,du$

$\int\frac{\cancel{u}}{w}\cdot\frac{dw}{2\cancel{u}}$

$-\frac{1}{u}\cdot\text{arc tg}\left(\frac{2}{u}\right)$

$w=u^2+4$
$dw=2u\,du$
$du=\frac{dw}{2u}$

$\frac{1}{2}\int\frac{1}{w}\,dw$

$\frac{1}{2}\ln(u^2+4)-\frac{1}{2}\cdot\text{arc tg}\left(\frac{u}{2}\right)$