



**INSTITUTO FEDERAL**  
Catarinense  
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**Assunto:** Integrais indefinidas, integrais definidas, áreas e técnicas de integração  
**Professor:** Fabricio Alves Oliveira

Essa lista deverá ser resolvida de forma manuscrita e entregue no dia da primeira prova.

(1) Calcule as integrais indefinidas:

(a)  $\int (6x^2 - 2x + 1) dx$

(b)  $\int \frac{x^2 + 1}{x^2} dx$

(c)  $\int \frac{x^2}{x^2 + 1} dx$

(d)  $\int \left( \frac{e^t}{2} + \sqrt{t} + \frac{1}{t} \right) dt$

(e)  $\int \frac{\sin x}{\cos^2 x} dx$

(f)  $\int \cos \theta \operatorname{tg} \theta d\theta$

(g)  $\int (2^t - \sqrt{2}e^t + \cosh t) dt$

(h)  $\int \sqrt{\frac{9}{1-x^2}} dx$

(i)  $\int \frac{\ln x}{x \ln x^2} dx$

(2) Encontrar uma primitiva da função  $f(x) = \frac{1}{x^2} + 1$  que se anule no ponto  $x = 2$ .

(3) Calcule as seguintes integrais usando o método da substituição:

(a)  $\int (2x^2 + 2x - 3)^{10} (2x + 1) dx$

(b)  $\int \frac{x}{\sqrt[5]{x^2 - 1}} dx$

(c)  $\int 5x \sqrt{4 - 3x^2} dx$

(d)  $\int \sqrt{x^2 + 2x^4} dx$

(e)  $\int (e^{2t} + 2)^{\frac{1}{3}} e^{2t} dt$

(f)  $\int \frac{e^t}{e^t + 4} dt$

(g)  $\int \operatorname{tg} x \sec^2 x dx$

(h)  $\int \sin^4 x \cos x dx$

(i)  $\int \frac{\sin x}{\cos^5 x} dx$

(j)  $\int \frac{\arcsen x}{2\sqrt{1-x^2}} dx$

(k)  $\int \frac{1}{t^2 - 4t + 4} dt$

(l)  $\int \frac{\ln^2 x}{x} dx$

(m)  $\int \frac{\cos \sqrt{x}}{\sqrt{x}} dx$

(n)  $\int \frac{3}{x \ln^2 3x} dx$

(o)  $\int \cotg u du$

(p)  $\int \sin 5x dx$

(q)  $\int \cos^2 x dx$

(r)  $\int \operatorname{tg}^3 x \cos x dx$

(4) Calcule as integrais utilizando o método de integração por partes:

(a)  $\int x \sin 5x dx$

(b)  $\int \ln(1-x) dx$

(c)  $\int t e^{4t} dt$

(d)  $\int (x+1) \cos(2x) dx$

(e)  $\int x \ln 3x dx$

(f)  $\int \cos^3 x dx$

(g)  $\int e^x \cos\left(\frac{x}{2}\right) dx$

(h)  $\int \sqrt{x} \ln x dx$

(i)  $\int \arccos x dx$

(j)  $\int x \sec^2 x dx$

(k)  $\int (x^2 - 5x) e^x dx$

(l)  $\int e^{2x} \cos 3x dx$

(m)  $\int x^3 e^{x^2} dx$

(n)  $\int x^3 \cos x^2 dx$

(o)  $\int e^{-x} \cos 2x dx$

(p)  $\int x^2 \sin x dx$

(5) Calcule as integrais definidas:

$$(a) \int_{-1}^2 x(1+x^3) dx$$

$$(b) \int_{-3}^0 x^2 - 4x + 7 dx$$

$$(c) \int_1^2 \frac{1}{x^6} dx$$

$$(d) \int_4^9 2t\sqrt{t} dt$$

$$(e) \int_0^1 \frac{1}{\sqrt{3y+1}} dy$$

$$(f) \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \sin x \cos x dx$$

$$(g) \int_{-2}^5 |2t-4| dt$$

$$(h) \int_0^{\frac{\pi}{2}} \sin^2 x dx$$

$$(i) \int_0^{\frac{\pi}{2}} \frac{\cos x}{(1+\sin x)^5} dx$$

$$(j) \int_1^2 x \ln x dx$$

$$(k) \int_{\frac{\pi}{2}}^{\pi} x \sin x dx$$

$$(l) \int_0^1 x e^{-x} dx$$

(6) Nos itens abaixo, fazer um esboço e determinar a área da região limitada pelas curvas:

$$(a) x = \frac{1}{2}, x = \sqrt{y} \text{ e } y = -x + 2$$

$$(f) x + y = 3 \text{ e } y + x^2 = 3$$

$$(b) y^2 = 2x \text{ e } x^2 = 2y$$

$$(g) y = e^x, x = 0, x = 1 \text{ e } y = 0$$

$$(c) y = 5 - x^2 \text{ e } y = x + 3$$

$$(h) x = y^3 \text{ e } x = y$$

$$(d) y = \cosh x, y = \sinh x, x = -1 \text{ e } x = 1$$

$$(i) y = \ln x, y = 0 \text{ e } x = 4$$

$$(e) y = \arcsen x, y = \frac{\pi}{2} \text{ e } x = 0$$

$$(j) y = \sin x \text{ e } y = -\sin x, \text{ com } x \in [0, 2\pi]$$

(7) Calcule as integrais utilizando substituição trigonométrica:

$$(a) \int \sqrt{1-4x^2} dx$$

$$(b) \int \frac{1}{\sqrt{4-x^2}} dx$$

$$(c) \int \frac{1}{\sqrt{4+x^2}} dx$$

$$(d) \int \frac{1}{4+x^2} dx$$

$$(e) \int \frac{x}{\sqrt{1-x^2}} dx$$

$$(f) \int \sqrt{3-4x^2} dx$$

$$(g) \int \frac{x^2}{\sqrt{1-x^2}} dx$$

$$(h) \int x^2 \sqrt{1-x^2} dx$$

$$(i) \int \frac{1}{x\sqrt{1+x^2}} dx$$

$$(j) \int \sqrt{9-(x-1)^2} dx$$

$$(k) \int \sqrt{-x^2+2x+2} dx$$

$$(l) \int \sqrt{-x^2+2x+3} dx$$

(8) Calcule as integrais abaixo, utilizando o método das frações parciais:

$$(a) \int \frac{1}{x^2-4} dx$$

$$(b) \int \frac{x}{x^2-5x+6} dx$$

$$(c) \int \frac{2x+1}{x^2-1} dx$$

$$(d) \int \frac{x^2+3x+1}{x^2-2x-3} dx$$

$$(e) \int \frac{x^2+1}{(x-2)^3} dx$$

$$(f) \int \frac{x^3+x+1}{x^2-2x+1} dx$$

$$(g) \int \frac{2x-3}{(x-1)^3} dx$$

$$(h) \int \frac{x+1}{x(x-2)(x+3)} dx$$

$$(i) \int \frac{x+5}{x^3-4x^2+4x} dx$$

$$(j) \int \frac{x^5+3}{x^3-4x} dx$$

$$(k) \int \frac{4x^2+17x+13}{(x-1)(x^2+6x+10)} dx$$

$$(l) \int \frac{x+2}{x^3+2x^2+5x} dx$$

## Respostas

(1)

(a)  $2x^3 - x^2 + x + k$

(b)  $x - \frac{1}{x} + k$

(c)  $x - \operatorname{arctg} x + k$

(d)  $\frac{1}{2}e^t + \frac{2}{3}t^{3/2} + \ln|t| + k$

(e)  $\sec x + k$

(f)  $-\cos \theta + k$

(g)  $\frac{2^t}{\ln 2} - \sqrt{2}e^t + \sinh t + k$

(h)  $3\operatorname{arcsen} x + k$

(i)  $\frac{1}{2} \ln|x| + k$

(2)  $-\frac{1}{x} + x - \frac{3}{2}$

(3)

(a)  $\frac{1}{22}(2x^2 + 2x + 3)^{11} + k$

(b)  $\frac{5}{8}(x^2 - 1)^{\frac{4}{5}} + k$

(c)  $-\frac{5}{9}(4 - 3x^2)^{\frac{3}{2}} + k$

(d)  $\frac{1}{6}(1 + 2x^2)^{\frac{3}{2}} + k$

(e)  $\frac{3}{8}(e^{2t} + 2)^{\frac{4}{3}} + k$

(f)  $\ln(e^t + 4) + k$

(g)  $\frac{\operatorname{tg}^2 x}{2} + k$

(h)  $\frac{\operatorname{sen}^5 x}{5} + k$

(i)  $\frac{1}{4} \sec^4 x + k$

(j)  $\frac{1}{4}(\operatorname{arcsen} x)^2 + k$

(k)  $\frac{1}{2-x} + k$

(l)  $\frac{\ln^3 x}{3} + k$

(m)  $2\operatorname{sen} \sqrt{x} + k$

(n)  $-\frac{3}{\ln 3x} + k$

(o)  $\ln|\operatorname{sen} x| + k$

(p)  $-\frac{1}{5} \cos 5x + k$

(q)  $\frac{x}{2} + \frac{\operatorname{sen} 2x}{4} + k$

(r)  $\sec x + \cos x + k$

(4)

(a)  $-\frac{x}{5} \cos 5x + \frac{1}{25} \operatorname{sen} 5x + k$

(b)  $(x - 1) \ln|1 - x| - x + k$

(c)  $\frac{e^{4t}}{4}(t - \frac{1}{4}) + k$

(d)  $\frac{x+1}{2} \operatorname{sen} 2x + \frac{1}{4} \cos 2x + k$

(e)  $\frac{x^2}{2}(\ln 3x - \frac{1}{2}) + k$

(f)  $\cos^2 x \operatorname{sen} x + \frac{2}{3} \operatorname{sen}^3 x + k$

(g)  $\frac{2}{5}e^x (\operatorname{sen} \frac{x}{2} + 2 \cos \frac{x}{2}) + k$

(h)  $\frac{2}{3}x\sqrt{x} \ln x - \frac{4}{9}x\sqrt{x} + k$

(i)  $x \arccos x - \sqrt{1 - x^2} + k$

(j)  $x \operatorname{tg} x + \ln|\cos x| + k$

(k)  $(x^2 - 7x + 7)e^x + k$

(l)  $\frac{e^{2x}}{13}(3\operatorname{sen} 3x + 2 \cos 3x) + k$

(m)  $\frac{1}{2}(x^2 - 1)e^{x^2} + k$

(n)  $\frac{1}{2}(x^2 \operatorname{sen} x^2 + \cos x^2) + k$

(o)  $\frac{e^{-x}}{5}(2\operatorname{sen} 2x - \cos 2x) + k$

(p)  $-x^2 \cos x + 2x \operatorname{sen} x + 2 \cos x + k$

(5)

(a)  $\frac{81}{10}$

(b) 48

(c)  $\frac{31}{160}$

(d)  $\frac{844}{5}$

(e)  $\frac{2}{3}$

(f) 0

(g) 25

(h)  $\frac{\pi}{4}$

(i)  $\frac{15}{64}$

(j)  $2 \ln 2 - \frac{3}{4}$

(k)  $\pi - 1$

(l)  $\frac{e-2}{e}$

(6)

(a)  $\frac{1}{3}$

(b)  $\frac{4}{3}$

(c)  $\frac{9}{2}$

(d)  $e - \frac{1}{e}$

(e) 1

(f)  $\frac{1}{e}$

(g)  $e - 1$

(h)  $\frac{1}{2}$

(i)  $8 \ln 2 - 3$

(j) 8

(7)

(a)  $\frac{1}{4} \left( \arcsen 2x + 2x\sqrt{1-4x^2} \right) + k$

(b)  $\arcsen \frac{x}{2} + k$

(c)  $\ln(x + \sqrt{4+x^2}) + k$

(d)  $\frac{1}{2} \operatorname{arctg} \frac{x}{2} + k$

(e)  $-\sqrt{1-x^2} + k$

(f)  $\frac{3}{4} \left( \arcsen \frac{2x}{\sqrt{3}} + \frac{2x}{3} \sqrt{3-4x^2} \right) + k$

(g)  $\frac{1}{2} \left( \arcsen x - x\sqrt{1-x^2} \right) + k$

(h)  $\frac{1}{8} \left[ \arcsen x - x\sqrt{1-x^2}(1-2x^2) \right] + k$

(i)  $\ln \left| \frac{x}{1+\sqrt{1+x^2}} \right| + k$

(j)  $\frac{9}{2} \arcsen \left( \frac{x-1}{3} \right) + \frac{(x-1)\sqrt{9-(x-1)^2}}{2} + k$

(k) Dica: observe que  $-x^2 + 2x + 2 = 3 - (x-1)^2$

(l)  $2 \arcsen \left( \frac{x-1}{2} \right) + \left( \frac{x-1}{2} \right) \sqrt{4-(x-1)^2} + k$

(8)

(a)  $\frac{1}{4} \ln \left| \frac{x-2}{x+2} \right| + k$

(b)  $-2 \ln |x-2| + 3 \ln |x-3| + k$

(c)  $\ln |x^2-1| + \frac{1}{2} \ln \left| \frac{x-1}{x+1} \right| + k$

(d)  $x + \frac{1}{4} \ln |x+1| + \frac{19}{4} \ln |x-3| + k$

(e)  $\ln |x-2| - \frac{4}{x-2} - \frac{5}{2(x-2)^2} + k$

(f)  $\frac{x^2}{2} + 2x + 4 \ln |x-1| - \frac{3}{x-1} + k$

(g)  $\frac{-2}{x-1} + \frac{1}{2(x-1)^2} + k$

(h)  $-\frac{1}{6} \ln |x| + \frac{3}{10} \ln |x-2| - \frac{2}{15} \ln |x+3| + k$

(i)  $\frac{5}{4} \ln |x| - \frac{5}{4} \ln |x-2| - \frac{7}{2(x-2)} + k$

(j)  $\frac{x^3}{4} + 4x - \frac{3}{4} \ln |x| + \frac{35}{8} \ln |x-2| - \frac{29}{8} \ln |x+2| + k$

(k)  $2 \ln |x-1| + \ln |x^2+6x+10| + \operatorname{arctg} (x+3) + k$

(l)  $\frac{2}{5} \ln |x| - \frac{1}{5} \ln |x^2+2x+5| + \frac{3}{10} \operatorname{arctg} \left( \frac{x+1}{2} \right) + k$