IFC-BLUMENAU–21/03/24–EEB0908 ÁLGEBRA LINEAR–24/1–PROF. CECHETTO-PROVA 1-P1

Nome Estevão Goerll Nascimento Nº 4

- Cada questão vale 1 ponto.
- Proibido o uso de qualquer material de consulta.
- Liberado o uso de calculadora comum e/ou cientifica, desde que não sejam calculadoras gráficas, que calculem matrizes, determinantes, sistemas, tenha wifi, seja celular.
- Todas as questões devem ser feitas mostrando os raciocínios e cálculos envolvidos.
- Frações devem ser simplificadas e não podem ser escritas em forma de decimais aproximados.

6,9

1. Dadas as matrizes:

$$A=\begin{pmatrix}\frac{1}{3} & \frac{1}{2}\\ \frac{1}{2} & \frac{1}{3}\end{pmatrix}\quad B=\begin{pmatrix}5 & -1\\ 2 & 4\end{pmatrix}\quad C=\begin{pmatrix}-2 & 2\\ 0 & -4\end{pmatrix}\quad D=\begin{pmatrix}2 & -3\\ -2 & 0\end{pmatrix}\quad E=\begin{pmatrix}\frac{1}{2} & -\frac{1}{2}\\ -2 & 0\end{pmatrix}$$

Calcule o valor de x tal que:

$$x = det(A^{-1}) + det(BC) + det(CD+E)$$

$$BC = \begin{pmatrix}5 & -1\\ 2 & 4\end{pmatrix}\cdot\begin{pmatrix}-2 & 2\\ 0 & -4\end{pmatrix} = \begin{pmatrix}-10+0 & 10+4\\ -4+0 & 4-16\end{pmatrix} = \begin{pmatrix}-10 & 14\\ -4 & -12\end{pmatrix}$$

$$det(BC) = 120 + 56 = 176$$

$$CD+E = \begin{pmatrix}-2 & 2\\ 0 & -4\end{pmatrix}\cdot\begin{pmatrix}2 & -3\\ -2 & 0\end{pmatrix} + \begin{pmatrix}\frac{1}{2} & -\frac{1}{2}\\ -2 & 0\end{pmatrix}$$

$$= \begin{pmatrix}-4-4 & 6+0\\ 0+8 & 0+0\end{pmatrix} + \begin{pmatrix}\frac{1}{2} & -\frac{1}{2}\\ -2 & 0\end{pmatrix} = \begin{pmatrix}-\frac{15}{2} & \frac{11}{2}\\ 6 & 0\end{pmatrix}$$

$$det(CD+E) = 0 - \frac{66}{2} = -33$$

1,0

A^{-1}

$$\begin{pmatrix}\frac{1}{3} & \frac{1}{2}\\ \frac{1}{2} & \frac{1}{3}\end{pmatrix}\cdot\begin{pmatrix}a & b\\ c & d\end{pmatrix} = \begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$$

$\frac{1}{3}a + \frac{1}{2}c = 1 \qquad \frac{1}{3}b + \frac{1}{2}d = 0$

$\frac{1}{2}a + \frac{1}{3}c = 0 \qquad \frac{1}{2}b + \frac{1}{3}d = 1$

$3\cdot(\frac{1}{3}a + \frac{1}{2}c) - 2\cdot(\frac{1}{2}a + \frac{1}{3}c) = 3 - 0$

$\cancel{a} + \frac{3}{2}c - \cancel{a} - \frac{2}{3}c = 3$

$\frac{9}{6}c - \frac{4}{6}c = 3 \qquad \frac{5}{6}c = 3 \qquad c = \frac{3}{1}\cdot\frac{6}{5} = \frac{18}{5}$

$\frac{1}{3}a = 1 - \frac{1}{2}c$

$\frac{1}{3}a = -\frac{4}{5}$

$a = -\frac{12}{5}$

(continua no verso)

$3 \cdot \left(\frac{1}{3}b + \frac{1}{2}d\right) - 2 \cdot \left(\frac{1}{2}b + \frac{1}{3}d\right) = -2$

$\cancel{b} + \frac{3}{2}d - \cancel{b} - \frac{2}{3}d = -2$ $\qquad b + \frac{2}{3}d = 2$

$\frac{9}{6}d - \frac{4}{6}d = -2$ $\qquad b = 2 + \frac{24}{15}$

$\frac{5}{6}d = -2$ $\qquad b = \frac{18}{5}$

$d = -2 \cdot \frac{6}{5} = -\frac{12}{5}$

$$A^{-1} = \begin{pmatrix} -\frac{12}{5} & \frac{18}{5} \\ \frac{18}{5} & -\frac{12}{5} \end{pmatrix}$$

$\det(A^{-1}) = \frac{144}{25} - \frac{324}{25} = -\frac{180}{25} = -\frac{36}{5}$

$X = -\frac{36}{5} + 176 + (-33) = -\frac{36}{5} + 143 = \frac{679}{5}$

$X = \frac{679}{5}$

2. Uma matriz quadrada A se diz ortogonal se A é inversível e $A^{-1} = A^t$. Determine os números reais x, y e z tais que a matriz B seja ortogonal. Determine todas as soluções.

$$B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\ x & y & z \end{bmatrix}$$

se $B^{-1} = B^T$, $B \cdot B^T = I$

$$B^T = \begin{pmatrix} 1 & 0 & x \\ 0 & \frac{\sqrt{2}}{2} & y \\ 0 & \frac{\sqrt{2}}{2} & z \end{pmatrix}$$

$$B \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\ x & y & z \end{pmatrix} \cdot B^T \begin{pmatrix} 1 & 0 & x \\ 0 & \frac{\sqrt{2}}{2} & y \\ 0 & \frac{\sqrt{2}}{2} & z \end{pmatrix} = I \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$X + z.0 + y.0 = 0$

$X.0 + \frac{\sqrt{2}}{2} y + \frac{\sqrt{2}}{2} z = 0$

$x^2 + y^2 + z^2 = 1$

$x + y.0 + z.0 = 0$

$X = 0$

$x^2 + y^2 + z^2 = 1$

$y^2 + z^2 = 1$

$\frac{\sqrt{2}}{2} y + \frac{\sqrt{2}}{2} z = 0$

$\frac{\sqrt{2}}{2}(y + z) = 0$

$y + z = 0$

$y = -z$

substituindo em $y^2 + z^2 = 1$

$y^2 + (-y)^2 = 1$

$2y^2 = 1$

$y^2 = \frac{1}{2}$

$y = \frac{\sqrt{1}}{\sqrt{2}} = \frac{1}{\sqrt{2}}$

soluções:

$X = 0$, $y = \frac{1}{\sqrt{2}}$, $z = -\frac{1}{\sqrt{2}}$

ou

$X = 0$, $y = -\frac{1}{\sqrt{2}}$, $z = \frac{1}{\sqrt{2}}$

9,5

3. Resolva o sistema matricial para definir as matrizes X e Y.

$$\begin{cases} X + Y + \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 4 & 0 \\ \frac{1}{2} & -1 \end{bmatrix} + \begin{bmatrix} 0 & 2 \\ \frac{3}{2} & 4 \end{bmatrix} \\ X - Y = \begin{bmatrix} 6 & -3 \\ 4 & 0 \end{bmatrix} - \begin{bmatrix} 2 & -2 \\ 2 & 1 \end{bmatrix} \end{cases}$$

$$X + Y + \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 4 & 2 \\ 2 & 3 \end{bmatrix}$$

$$X + Y = \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}$$

$$X - Y = \begin{bmatrix} 4 & -1 \\ 2 & -1 \end{bmatrix}$$

$$(X+Y) - (X-Y) = 2Y = \begin{bmatrix} 0 & 3 \\ 0 & 4 \end{bmatrix}$$

$$Y = \begin{bmatrix} 0 & \frac{3}{2} \\ 0 & 2 \end{bmatrix}$$

$$X + \overset{Y}{\begin{bmatrix} 0 & \frac{3}{2} \\ 0 & \frac{4}{2} \end{bmatrix}} = \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} \rightarrow X = \begin{bmatrix} 3 & -\frac{1}{2} \\ 1 & 0 \end{bmatrix}$$

4. Escalone completamente a matriz:

$$B = \begin{bmatrix} 0 & -1 & -1 & 1 \\ 2 & 2 & 2 & 2 \\ 2 & 4 & 1 & -2 \\ 3 & 1 & -2 & 2 \end{bmatrix} \xrightarrow{L1 \leftrightarrow L2} \begin{bmatrix} 2 & 2 & 2 & 2 \\ 0 & -1 & -1 & 1 \\ 2 & 4 & 1 & -2 \\ 3 & 1 & -2 & 2 \end{bmatrix}$$

$L3 = L3 - L1$
$L4 = L4 - \frac{3}{2}L1$

$$\longrightarrow \begin{bmatrix} 2 & 2 & 2 & 2 \\ 0 & -1 & -1 & 1 \\ 0 & 2 & -1 & -4 \\ 0 & -\frac{5}{2} & -5 & -1 \end{bmatrix}$$

$L1 = L1 \div 2$
$L1 = L1 + L2$

$$\longrightarrow \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & -1 & -1 & 1 \\ 0 & 2 & -1 & -4 \\ 0 & -\frac{5}{2} & -5 & -1 \end{bmatrix}$$

$L3 = L3 + 2.L2$

$$\longrightarrow \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & -1 & -1 & 1 \\ 0 & 0 & -3 & -2 \\ 0 & -\frac{5}{2} & -5 & -1 \end{bmatrix} \xrightarrow{L4 = L4 - \frac{5}{2}L2} \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & -1 & -1 & 1 \\ 0 & 0 & -3 & 2 \\ 0 & 0 & -\frac{5}{2} & -\frac{7}{2} \end{bmatrix}$$

$L2 = L2 \cdot -1$

$$\longrightarrow \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 1 & -1 \\ 0 & 0 & -3 & 2 \\ 0 & 0 & -\frac{5}{2} & -\frac{7}{2} \end{bmatrix}$$

5. Calcule, caso exista, a matriz inversa da matriz abaixo.

$$\begin{bmatrix} -\frac{1}{8} & \frac{3}{8} & -\frac{1}{8} \\ -\frac{1}{4} & 0 & \frac{1}{4} \\ \frac{1}{2} & -\frac{1}{4} & 0 \end{bmatrix}$$

Se não existir a inversa, justifique.

$$\left[\begin{array}{ccc|cc} -\frac{1}{8} & \frac{3}{8} & -\frac{1}{8} & -\frac{1}{8} & \frac{3}{8} \\ -\frac{1}{4} & 0 & \frac{1}{4} & -\frac{1}{4} & 0 \\ \frac{1}{2} & -\frac{1}{4} & 0 & \frac{1}{2} & -\frac{1}{4} \end{array}\right]$$

$\det = \frac{3}{64} - \frac{1}{128} - \frac{1}{128}$

$\det = \frac{6}{128} - \frac{2}{128} = \frac{4}{128}$ portanto tem inversa

$$\left[\begin{array}{ccc|ccc} -\frac{1}{8} & \frac{3}{8} & -\frac{1}{8} & 1 & 0 & 0 \\ -\frac{1}{4} & 0 & \frac{1}{4} & 0 & 1 & 0 \\ \frac{1}{2} & -\frac{1}{4} & 0 & 0 & 0 & 1 \end{array}\right] \quad \begin{array}{l} L1 = L1 \cdot 8 \\ L2 = L2 \cdot 4 \\ L3 = L3 \cdot 4 \end{array} \longrightarrow \left[\begin{array}{ccc|ccc} -1 & 3 & -1 & 8 & 0 & 0 \\ -1 & 0 & 1 & 0 & 4 & 0 \\ 2 & -1 & 0 & 0 & 0 & 4 \end{array}\right]$$

95

$$\begin{array}{l} L2 = L2 + L1 \cdot -1 \\ L3 = L3 + L1 \cdot 2 \end{array} \longrightarrow \left[\begin{array}{ccc|ccc} -1 & 3 & -1 & 8 & 0 & 0 \\ 0 & -3 & 2 & -8 & 4 & 0 \\ 0 & 5 & -2 & 16 & 0 & 4 \end{array}\right] \quad \begin{array}{l} L1 = L1 + L2 \\ L3 = L3 + L2 \cdot \frac{5}{3} \end{array} \longrightarrow \left[\begin{array}{ccc|ccc} -1 & 0 & 1 & 0 & 4 & 0 \\ 0 & -3 & 2 & -8 & 4 & 0 \\ 0 & 0 & \frac{4}{3} & \frac{8}{3} & \frac{20}{3} & 4 \end{array}\right]$$

$$\begin{array}{l} L1 = L1 + -\frac{3}{4} \cdot L3 \\ L2 = L2 + -\frac{3}{2} \cdot L3 \end{array} \longrightarrow \left[\begin{array}{ccc|ccc} -1 & 0 & 0 & -2 & -1 & -3 \\ 0 & -3 & 0 & -12 & -6 & -6 \\ 0 & 0 & -\frac{4}{3} & \frac{8}{3} & \frac{20}{3} & 4 \end{array}\right] \quad \begin{array}{l} L1 = L1 \cdot \frac{1}{1} \\ L2 = L2 \cdot -\frac{1}{3} \\ L3 = L3 \cdot -\frac{3}{4} \end{array} \longrightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 2 & 1 & 3 \\ 0 & 1 & 0 & 4 & 2 & 2 \\ 0 & 0 & 1 & -2 & -5 & -3 \end{array}\right]$$

$$A^{-1} = \begin{bmatrix} 2 & 1 & 3 \\ 4 & 2 & 2 \\ -2 & -5 & -3 \end{bmatrix}$$

6. Calcule o determinante da matriz abaixo:

$$C=\begin{bmatrix}-2 & -3 & -1 & -2\\ -1 & 0 & 1 & -2\\ -3 & -1 & -4 & 1\\ -2 & 2 & -3 & -1\end{bmatrix}$$

1. $L2 \leftrightarrow L1$
2. $L1 = L1 \cdot -1$

$$\longrightarrow \begin{bmatrix}1 & 0 & -1 & 2\\ -2 & -3 & -1 & -2\\ -3 & -1 & -4 & 1\\ -2 & 2 & -3 & -1\end{bmatrix}$$

Chió

$$\longrightarrow \begin{bmatrix}-3 & -3 & 2\\ -1 & -4 & 7\\ 2 & -8 & 3\end{bmatrix}\begin{matrix}-3 & -3\\ -1 & -4\\ 2 & -8\end{matrix}$$

$-1 \cdot -1 \cdot \det = 63 + (-42) + 10 - (-28) - 105 - 9$

$\det = -55$

7. Calcule o valor de X em $3X + 2A = B^t + 2X$, se:

$$A = \begin{pmatrix} 1 & 0 & 1 \\ -2 & 1 & 3 \\ -1 & 0 & 0 \end{pmatrix} \quad \text{e} \quad B = \begin{pmatrix} 5 & -1 & 4 \\ 1 & 2 & 3 \\ -3 & -4 & 1 \end{pmatrix}$$

$$3X + \begin{bmatrix} 2 & 0 & 2 \\ -4 & 2 & 6 \\ -2 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 5 & 1 & -3 \\ -1 & 2 & -4 \\ 4 & 3 & 1 \end{bmatrix} + 2X$$

$$X = \begin{bmatrix} 5 & 1 & -3 \\ -1 & 2 & -4 \\ 4 & 3 & 1 \end{bmatrix} - \begin{bmatrix} 2 & 0 & 2 \\ -4 & 2 & 6 \\ -2 & 0 & 0 \end{bmatrix}$$

$$X = \begin{bmatrix} 3 & 1 & -5 \\ 3 & 0 & -10 \\ 6 & 3 & 1 \end{bmatrix}$$

1,0

8. Resolva a desigualdade no conjunto dos números reais:

$$\begin{vmatrix} 6 & 1 & -5 \\ x & 0 & 1 \\ 1 & -3 & 2 \end{vmatrix} \geq \begin{vmatrix} 1 & 2 & -1 \\ 0 & x & 4 \\ 0 & 0 & -6 \end{vmatrix}$$

$$\left|\begin{array}{ccc|cc} 6 & 1 & -5 & 6 & 1 \\ x & 0 & 1 & x & 0 \\ 1 & -3 & 2 & 1 & -3 \end{array}\right| \qquad \left|\begin{array}{ccc|cc} 1 & 2 & -1 & 1 & 2 \\ 0 & x & 4 & 0 & x \\ 0 & 0 & -6 & 0 & 0 \end{array}\right|$$

$1 + 15x + 18 - 2x = 13x + 19$

$-6x$

$13x + 19 \geq -6x$

$13x + 19 + 6x \geq 0$

$19x + 19 \geq 0$

$19x + 19 = 0$

$19x = -19$

$x = -1$

$19x+19$ $-$ -1 $+$

$S = \{x \in \mathbb{R} \mid x \geq -1\}$

9. Calcule o valor de x no conjunto dos números reais, sabendo que o valor do determinante abaixo é −79

$$\begin{vmatrix} 2^x & 0 & 1 & 2 \\ 1 & 2 & 1 & -3 \\ 0 & 0 & -1 & 1 \\ -3 & -1 & 2 & 0 \end{vmatrix} \quad L1 \leftrightarrow L2 \quad \longrightarrow \begin{bmatrix} 1 & 2 & 1 & -3 \\ 2^x & 0 & 1 & 2 \\ 0 & 0 & -1 & 1 \\ -3 & -1 & 2 & 0 \end{bmatrix}$$

Chió

$$\longrightarrow \begin{bmatrix} -2^{x+1} & 1-2^x & 2+3.2^x \\ 0 & -1 & 1 \\ 8 & 5 & -9 \end{bmatrix} \begin{matrix} -2^{x+1} & 1-2^x \\ 0 & -1 \\ 5 & 5 \end{matrix}$$

$9.(-2^{x+1}) + 5.(1-2^x) - (-5.(2+3.2^x) - 5.(-2^{x+1})$

$\cancel{4.(-2^{x+1}) + 5.(1-2^x) + 5.(2+3.2^x)}$

$\cancel{4.-2^{x+1} + 5 - 5.2^x + 10 + 15.2^x}$

$\cancel{4.-2^{x+1} + 15 + 10.2^x}$

$\cancel{4.(-2^x.2) + 15 + 10.2^x}$

$\cancel{8.-2^x + 10.2^x + 15 = 79}$

$\cancel{8.-2^x + 10.2^x = 64}$

$\cancel{4.-2^x + 5.2^x = 32}$

$2^2.-2^x + 5.2^x = 2^5.$

10. Calcule o valor do determinante:

$$\begin{vmatrix} 2 & 2 & 3 & -4 & 2 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 4 & 0 & 2 & 1 \\ 0 & -5 & 5 & 1 & 4 \\ 0 & 1 & 0 & -1 & 2 \end{vmatrix}$$

Utilizando La Place

$\det(A) = 0 . C_{21} + 1 . C_{22} + 0 . C_{23} + 0 . C_{24} + 0 . C_{25}$

$\det(A) = 1 . C_{22}$

$\det(A) = 1 . -1^{4} . \begin{vmatrix} 2 & 3 & -4 & 2 \\ 0 & 0 & 2 & 1 \\ 0 & 5 & 1 & 4 \\ 0 & 0 & -1 & 2 \end{vmatrix}$

$\det(A) = 2 . -1 . \begin{vmatrix} 2 & 3 & 2 \\ 0 & 5 & 4 \\ 0 & 0 & 2 \end{vmatrix} + 1 . 1 . \begin{vmatrix} 2 & 3 & -4 \\ 0 & 5 & 1 \\ 0 & 0 & -1 \end{vmatrix}$

$\det(A) = -2 . 20 + (-10)$

$\det(A) = -50$

10