IFC-BLUMENAU–21/03/24–EEB0908 ÁLGEBRA LINEAR–24/1–PROF. CECHETTO-PROVA 1-P1

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- Cada questão vale 1 ponto.
- Proibido o uso de qualquer material de consulta.
- Liberado o uso de calculadora comum e/ou cientifica, desde que não sejam calculadoras gráficas, que calculem matrizes, determinantes, sistemas, tenha wifi, seja celular.
- Todas as questões devem ser feitas mostrando os raciocínios e cálculos envolvidos.
- Frações devem ser simplificadas e não podem ser escritas em forma de decimais aproximados.

-4 -4 6

1. Dadas as matrizes:

$$A = \begin{pmatrix} \frac{1}{3} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{3} \end{pmatrix} \quad B = \begin{pmatrix} 5 & -1 \\ 2 & 4 \end{pmatrix} \quad C = \begin{pmatrix} -2 & 2 \\ 0 & -4 \end{pmatrix} \quad D = \begin{pmatrix} 2 & -3 \\ -2 & 0 \end{pmatrix} \quad E = \begin{pmatrix} \frac{1}{2} & -\frac{1}{2} \\ -2 & 0 \end{pmatrix}$$

Calcule o valor de x tal que:

$$x = \underbrace{det(A^{-1})}_{\text{metodo variaveis}} + det(BC) + det(CD + E)$$

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$$X = det \begin{bmatrix} -\frac{12}{5} & \frac{18}{5} \\ \frac{18}{5} & -\frac{12}{5} \end{bmatrix} + det \begin{pmatrix} -16 & 14 \\ -4 & -12 \end{pmatrix} + det \begin{pmatrix} -\frac{15}{2} & \frac{11}{2} \\ 6 & 0 \end{pmatrix}$$

$-\frac{36}{5}$

$$det \begin{bmatrix} -\frac{12}{5} & \frac{18}{5} \\ \frac{18}{5} & -\frac{12}{5} \end{bmatrix} +$$

$$= -\frac{36}{5} + \cancel{\frac{248}{1}} - \frac{33}{2} = \frac{1032}{5}$$

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2. Uma matriz quadrada A se diz ortogonal se A é inversível e $A^{-1} = A^t$. Determine os números reais x, y e z tais que a matriz B seja ortogonal. Determine todas as soluções.

$$B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\ x & y & z \end{bmatrix}$$

2×2

3. Resolva o sistema matricial para definir as matrizes X e Y.

$$\begin{cases} X + Y + \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 4 & 0 \\ \frac{1}{2} & -1 \end{bmatrix} + \begin{bmatrix} 0 & 2 \\ \frac{3}{2} & 4 \end{bmatrix} = \begin{vmatrix} 4 & 2 \\ 2 & 3 \end{vmatrix} \\ X - Y = \begin{bmatrix} 6 & -3 \\ 4 & 0 \end{bmatrix} - \begin{bmatrix} 2 & -2 \\ 2 & 1 \end{bmatrix} = \begin{vmatrix} 4 & -1 \\ 2 & -1 \end{vmatrix} \end{cases}$$

$$\begin{cases} X + Y = \begin{vmatrix} 4 & 2 \\ 2 & 3 \end{vmatrix} - \begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} \\ X - Y = \begin{vmatrix} 4 & -1 \\ 2 & -1 \end{vmatrix} \end{cases} = \begin{cases} X + \cancel{Y} = \begin{vmatrix} 3 & 1 \\ 1 & 2 \end{vmatrix} \\ X - \cancel{Y} = \begin{vmatrix} 4 & -1 \\ 2 & -1 \end{vmatrix} \end{cases}$$

$$2X = \begin{vmatrix} 3 & 1 \\ 1 & 2 \end{vmatrix} + \begin{vmatrix} 4 & -1 \\ 2 & -1 \end{vmatrix} \Rightarrow 2X = \begin{vmatrix} 7 & 0 \\ 3 & 1 \end{vmatrix} =$$

$$X = \begin{vmatrix} 7/2 & 0 \\ 3/2 & 1/2 \end{vmatrix}$$

$$Y = \begin{vmatrix} 3 & 1 \\ 1 & 2 \end{vmatrix} - X \Rightarrow Y = \begin{vmatrix} 3 & 1 \\ 1 & 2 \end{vmatrix} - \begin{vmatrix} 7/2 & 0 \\ 3/2 & 1/2 \end{vmatrix}$$

$$Y = \begin{vmatrix} -1/2 & 1 \\ -1/2 & 3/2 \end{vmatrix}$$

$$2 - \frac{1}{2} = \frac{4-1}{2} = \frac{3}{2}$$

4. Escalone completamente a matriz:

$$B = \begin{bmatrix} 0 & -1 & -1 & 1 \\ 2 & 2 & 2 & 2 \\ 2 & 4 & 1 & -2 \\ 3 & 1 & -2 & 2 \end{bmatrix}$$

$L_1 = L_2 + L_1$

$L_2 = L_3 - L_2$

$L_3 = L_3 - L_2$

$$\begin{vmatrix} 2 & 1 & 1 & 1 \\ 0 & 2 & -1 & -4 \\ 0 & 2 & -1 & -4 \\ 3 & 1 & -2 & 2 \end{vmatrix}$$

$L_1 = L_3 - L_1$

$$\begin{vmatrix} 1 & 0 & -3 & 1 \\ 0 & 2 & -1 & -4 \\ 0 & 2 & -1 & -4 \\ 3 & 1 & -2 & \end{vmatrix}$$

5. Calcule, caso exista, a matriz inversa da matriz abaixo.

$$\begin{bmatrix} -\frac{1}{8} & \frac{3}{8} & -\frac{1}{8} \\ -\frac{1}{4} & 0 & \frac{1}{4} \\ \frac{1}{2} & -\frac{1}{4} & 0 \end{bmatrix}$$

Se não existir a inversa, justifique.

$$\left|\begin{array}{ccc|cc} -1/8 & 3/8 & -\frac{1}{8} & -1/8 & 3/8 \\ -1/4 & 0 & 1/4 & -1/4 & 0 \\ 1/2 & -1/4 & 0 & 1/2 & -1/4 \end{array}\right|$$

0 0 0

$$0 + \left(\frac{3}{8}\cdot\frac{1}{4}\cdot\frac{1}{2}\right) + \left(-\frac{1}{8}\cdot -\frac{1}{4}\cdot -\frac{1}{4}\right) - \left(-\frac{1}{4}\cdot\frac{1}{4}\cdot -\frac{1}{8}\right)$$

$$\frac{3}{64} - \frac{1}{128} - \frac{1}{128} = \frac{1}{32}$$

$$\left[\begin{array}{ccc|ccc} -\frac{1}{8} & \frac{3}{8} & -\frac{1}{8} & 1 & 0 & 0 \\ -1/4 & 0 & \frac{1}{4} & 0 & 1 & 0 \\ 1/2 & -\frac{1}{4} & 0 & 0 & 0 & 1 \end{array}\right]$$

$L1 = L1 \cdot (-8)$

$L2 = L2 \cdot 4$ $\Rightarrow$

$L3 = L3 \cdot 2$

$$\left[\begin{array}{ccc|ccc} 1 & -3 & 1 & -8 & 0 & 0 \\ -1 & 0 & 1 & 0 & 4 & 0 \\ 1 & -2 & 0 & 0 & 0 & 2 \end{array}\right]$$

$L2 = L3 + L2$

$L3 = L3 - L1$

$$\left[\begin{array}{ccc|ccc} 1 & -3 & 1 & -8 & 0 & 0 \\ 0 & -\frac{1}{2} & -1 & 0 & -4 & 2 \\ 0 & \frac{5}{2} & -1 & 8 & 0 & 2 \end{array}\right] \Rightarrow \left[\begin{array}{ccc|ccc} 1 & -3 & 1 & -8 & 0 & 0 \\ 0 & 1 & 2 & 0 & 8 & -4 \\ 0 & 5/2 & -1 & 8 & 0 & 2 \end{array}\right]$$

$L2 = L2 \cdot (-2)$

6. Calcule o determinante da matriz abaixo:

Chió

$$C=\begin{bmatrix} -2 & -3 & -1 & -2 \\ -1 & 0 & 1 & -2 \\ -3 & -1 & -4 & 1 \\ -2 & 2 & -3 & -1 \end{bmatrix}$$

$L1 \leftrightarrow L2 \cdot (-1)$

$$\begin{vmatrix} 1 & 0 & -1 & 2 \\ -2 & -3 & -1 & -2 \\ -3 & -1 & -4 & 1 \\ -2 & 2 & -3 & -1 \end{vmatrix}$$

$$\begin{vmatrix} -3-0 & -1-(-2\cdot -1) & -2-(-2\cdot 2) \\ -1-0 & -4-(-3\cdot -1) & 1-(-3\cdot 2) \\ 2-0 & -3-(-2\cdot -1) & -1-(-2\cdot 2) \end{vmatrix}$$

$$\begin{vmatrix} -3 & -3 & 2 \\ -1 & -7 & 7 \\ 2 & -5 & 3 \end{vmatrix}\begin{matrix} -3 & -3 \\ -1 & -7 \\ 2 & -5 \end{matrix}$$

$(-3\cdot -7\cdot 3)+(-3\cdot 7\cdot 2)+(2\cdot -1=5)$

$-(2\cdot 2\cdot -7)-(15\cdot 7)-9$

-25

$= (63-42+10)+(28-105-9) = -55$

7. Calcule o valor de X em $3X + 2A = B^t + 2X$, se:

$$A = \begin{pmatrix} 1 & 0 & 1 \\ -2 & 1 & 3 \\ -1 & 0 & 0 \end{pmatrix} \quad \text{e} \quad B = \begin{pmatrix} 5 & -1 & 4 \\ 1 & 2 & 3 \\ -3 & -4 & 1 \end{pmatrix}$$

$$3x + 2A = B^t + 2x$$

$$3x - 2x = B^t - 2A$$

$$x = B^t - 2A$$

$$-2 \cdot \begin{pmatrix} 1 & 0 & 1 \\ -2 & 1 & 3 \\ -1 & 0 & 0 \end{pmatrix}$$

$$2 \cdot \begin{pmatrix} -1 & 0 & -1 \\ 2 & -1 & 3 \\ 1 & 0 & 0 \end{pmatrix} = \begin{pmatrix} -2 & 0 & -2 \\ 4 & -2 & 6 \\ 8 & 0 & 0 \end{pmatrix}$$

$$B^t = \begin{pmatrix} 5 & 1 & -3 \\ -1 & 2 & -4 \\ 4 & 3 & 1 \end{pmatrix}$$

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$$x = \begin{pmatrix} 5 & 1 & -1 \\ -1 & 2 & -4 \\ 4 & 3 & 1 \end{pmatrix} + \begin{pmatrix} -2 & 0 & -2 \\ 4 & -2 & 6 \\ 8 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 3 & 1 & -\cancel{3} \\ 3 & 0 & \cancel{2} \\ \cancel{12} & 3 & 1 \end{pmatrix} //$$

8. Resolva a desigualdade no conjunto dos números reais:

$$\begin{vmatrix} 6 & 1 & -5 \\ x & 0 & 1 \\ 1 & -3 & 2 \end{vmatrix} \geq \begin{vmatrix} 1 & 2 & -1 \\ 0 & x & 4 \\ 0 & 0 & -6 \end{vmatrix}$$

$$\begin{vmatrix} 6 & 1 & -5 \\ x & 0 & 1 \\ 1 & -3 & 2 \end{vmatrix} - \begin{vmatrix} 1 & 2 & -1 \\ 0 & x & 4 \\ 0 & 0 & -6 \end{vmatrix} \geqslant 0 \Rightarrow \begin{vmatrix} 6 & 1 & -5 \\ x & 0 & 1 \\ 1 & -3 & 2 \end{vmatrix} + \begin{vmatrix} -1 & -2 & 1 \\ 0 & -x & -4 \\ 0 & 0 & 6 \end{vmatrix} \geqslant 0$$

$$\begin{vmatrix} 5 & 1 & -4 & 5 & 1 \\ x & -x & -3 & x & -x \\ 1 & -3 & 8 & 1 & -3 \end{vmatrix} \geqslant 0 \Rightarrow \begin{aligned} &(5 \cdot (-x) \cdot 8) + (-3) + (x + 12) \\ &- (-4 \cdot (-x)) - (5 \cdot -9) - 8x \end{aligned}$$

$$\Rightarrow -40x - 3 + 12x - 4x + 45 - 8x \geqslant 0$$

$$\Rightarrow -40x + 42 \geqslant 0$$

$$-40x \geqslant -42 \quad \cdot(-1)$$

$$40x \leqslant 42$$

$$x \leqslant \frac{42}{40} \Rightarrow \frac{21}{20}$$

$$\boxed{x \leqslant \frac{21}{20}}$$

9. Calcule o valor de x no conjunto dos números reais, sabendo que o valor do determinante abaixo é -79

$$\begin{vmatrix} 2^x & 0 & 1 & 2 \\ 1 & 2 & 1 & -3 \\ 0 & 0 & -1 & 1 \\ -3 & -1 & 2 & 0 \end{vmatrix}$$

$C \quad (-1)^{1+2} \cdot |A| \qquad 2 \cdot C_{22}$

10. Calcule o valor do determinante:

$$\begin{vmatrix} 2 & 2 & 3 & -4 & 2 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 4 & 0 & 2 & 1 \\ 0 & -5 & 5 & 1 & 4 \\ 0 & -1 & 0 & -1 & 2 \end{vmatrix}$$

$2 \cdot C_{11} + \cancel{0 \cdot C_{21}} + \cancel{0 \cdot C_{31}} + \cancel{0 \cdot C_{41}} + \cancel{0 \cdot C_{45}}$

$2 \cdot (-1)^{1+1} \cdot \begin{vmatrix} 1 & 0 & 0 & 0 \\ 4 & 0 & 2 & 1 \\ -5 & 5 & 1 & 4 \\ -1 & 0 & -1 & 2 \end{vmatrix} \Rightarrow \begin{vmatrix} 2 & 0 & 0 & 0 \\ 8 & 0 & 4 & 2 \\ -10 & 10 & 2 & 8 \\ -2 & 0 & -2 & 4 \end{vmatrix} \Rightarrow 2 \cdot C_{11} + \cancel{0 \cdot C_{12}} + \cancel{0 \cdot C_{13}} + \cancel{0 \cdot C_{14}}$

$2 \cdot (-1)^{1+1} \begin{vmatrix} 0 & 4 & 2 \\ 10 & 2 & 8 \\ 0 & -2 & 4 \end{vmatrix} = \begin{vmatrix} 0 & 8 & 4 \\ 20 & 4 & 16 \\ 0 & -4 & 8 \end{vmatrix} \Rightarrow \begin{vmatrix} 0 & 8 & 4 \\ 20 & 4 & 16 \\ 0 & -4 & 8 \end{vmatrix} \begin{matrix} 0 & 8 \\ 20 & 4 \\ 0 & -4 \end{matrix}$

$0 \quad 0 \quad 0 \quad 0 \quad (-320)$

$-320 - 1280 = \boxed{-1600}$