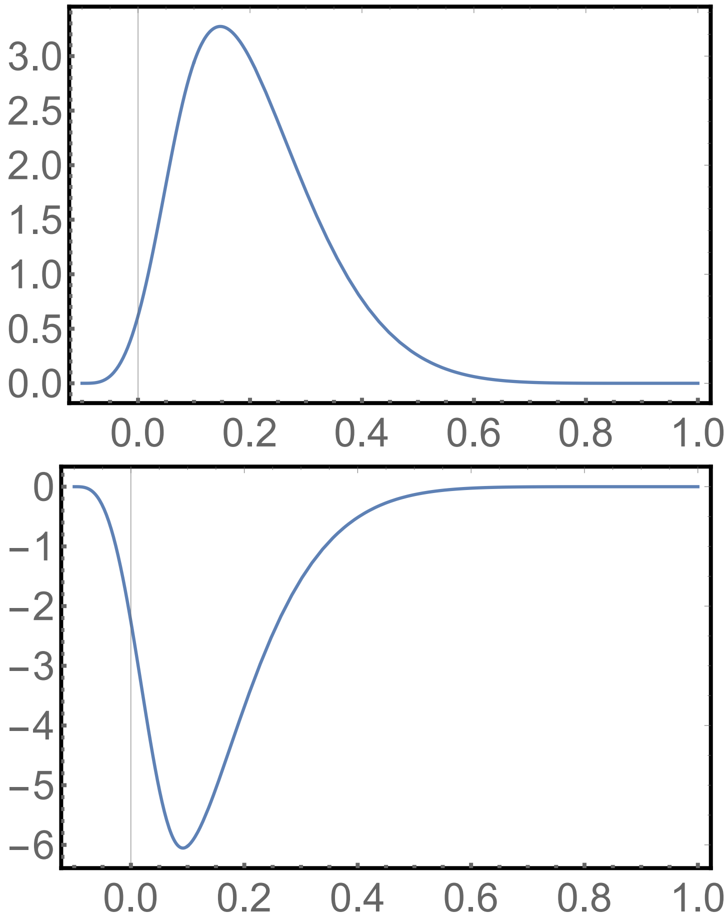

Figure 1: H and E from diagram b

The smooth curve in baryon loop diagram like
exists because in baryon loop we do not include the D term in convolution calculation like the meson loop.

The convolution formula for baryon loop

$$
\begin{aligned}
&\int_x^1 d(1-y)\frac{1}{1-y}f(y,\xi,t)H_q(\frac{x}{1-y},\frac{\xi}{1-y},t), \quad 1-y>x>\xi \\
&\int_\xi^1 d(1-y)\frac{1}{1-y}f(y,\xi,t)H_q(\frac{x}{1-y},\frac{\xi}{1-y},t), \quad 1-y>\xi>x \\
&\int_{-\xi}^{\xi} d(1-y)\frac{1}{2\xi}f(y,\xi,t)\frac{1}{\pi}\frac{\xi}{1-y}\int_{s_0}^{\infty} ds\frac{Im\Phi(\frac{\frac{x}{\xi}+1}{2},\frac{\frac{1-y}{\xi}+1}{2},s)}{s-t+i\epsilon} \quad \xi>\{1-y,|x|\} \\
&\int_{-x}^1 d(1-y)\frac{1}{1-y}f(y,\xi,t)H_q(\frac{x}{1-y},\frac{\xi}{1-y},t), \quad -\xi>x>-1
\end{aligned}
$$

and for meson loop

$$
\begin{aligned}
&\int_x^1 dy\frac{1}{y}f(y,\xi,t)H_q(\frac{x}{y},\frac{\xi}{y},t), \quad y>x>\xi \\
&\int_\xi^1 dy\frac{1}{y}f(y,\xi,t)H_q(\frac{x}{y},\frac{\xi}{y},t), \quad y>\xi>x \\
&\int_{-\xi}^{\xi} dy\frac{1}{2\xi}f(y,\xi,t)\frac{\xi}{\pi y}\int_{s_0}^{\infty} ds\frac{Im\Phi(\frac{\frac{x}{\xi}+1}{2},\frac{\frac{y}{\xi}+1}{2},s)}{s-t+i\epsilon} \quad \xi>\{y,|x|\} \\
&\int_{-x}^1 dy\frac{1}{y}f(y,\xi,t)H_q(\frac{x}{y},\frac{\xi}{y},t), \quad -\xi>x>-1
\end{aligned}
$$

The input GPD H_q and GDA Φ are both in so called double distribution representation and they have this form

$$
\begin{aligned}
H(x,\xi,t) &= \int_{-1}^1 d\beta\int_{-1+|\beta|}^{1-|\beta|} d\alpha\delta(x-\beta-\alpha\xi)h(\beta,\alpha,t) \\
E(x,\xi,t) &= \int_{-1}^1 d\beta\int_{-1+|\beta|}^{1-|\beta|} d\alpha\delta(x-\beta-\alpha\xi)e(\beta,\alpha,t) \\
\Phi_1(z,\eta,s) &= 2(2\eta-1)\int_{-1}^1 d\beta\int_{-1+|\beta|}^{1-|\beta|} d\alpha\delta(2z-1-(2\eta-1)\beta-\alpha)h_0(\beta,\alpha)H(\beta,0,s)
\end{aligned}
$$

In double distribution representation, the first moment of GPD do not have D term which is observed by experiment, and a easy way to solve this is add a extra term in formula above which is

$$
\begin{aligned}
H(x,\xi,t) &= \int_{-1}^1 d\beta\int_{-1+|\beta|}^{1-|\beta|} d\alpha\delta(x-\beta-\alpha\xi)h(\beta,\alpha,t)+D(\frac{x}{\xi},t)\theta(\xi-|x|) \\
\Phi_1(z,\eta,s) &= 2(2\eta-1)\int_{-1}^1 d\beta\int_{-1+|\beta|}^{1-|\beta|} d\alpha\delta(2z-1-(2\eta-1)\beta-\alpha)h_0(\beta,\alpha)H(\beta,0,s) \\
&\quad + D(1-2z,t)\theta(1-|1-2z|)
\end{aligned}
$$

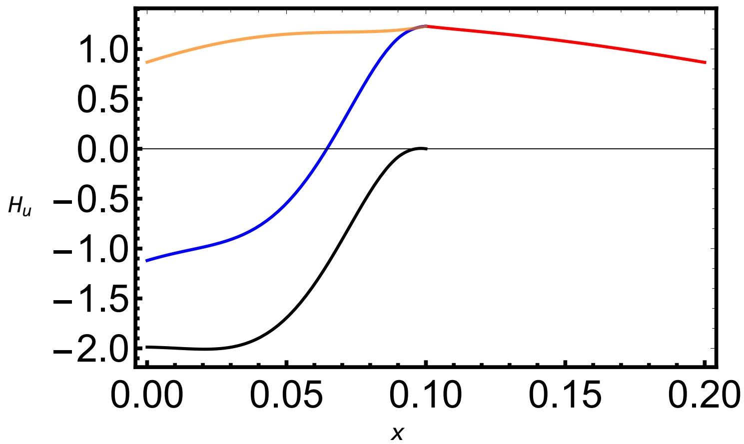

Figure 2: original result of $H_u(x,\xi,t)$ from diagram a, where $\xi = 0.1$ $t = -1$

The problem is in GDA case

$$\int_{-\xi}^{\xi} d(1-y)\frac{1}{2\xi}f(y,\xi,t)\frac{1}{\pi}\frac{\xi}{1-y}\int_{s_0}^{\infty} ds\frac{Im\Phi(\frac{\frac{x}{\xi}+1}{2},\frac{\frac{1-y}{\xi}+1}{2},s)}{s-t+i\epsilon}$$

when we consider the D-term part

$$\int_{-\xi}^{\xi} d(1-y)\frac{1}{2}f(y,\xi,t)\frac{1}{1-y}D(\frac{x}{\xi},t)$$
$$\int_{-\xi}^{\xi} dy\frac{1}{2}f(y,\xi,t)\frac{1}{y}D(\frac{x}{\xi},t)$$

Obviously we have a pole in $y = 0$and$\bar{y} = 1-y = 0$. In meson case, $\int_{-\xi}^{\xi} dy\frac{1}{2}f(y,\xi,t)\frac{1}{y}D(\frac{x}{\xi},t)$, the splitting function is always 0 at $y = 0$ which cancel the pole. In baryon loop case, we can not cancel the divergence.

In early calculation, I do not consider the D-term in baryon loop, and this will make the result of GPD has a smooth curve. It can be checked by the result of diagram a. The different lines in Fig.2 and Fig.3 represent the different input in convolution calculation. For orange line, the input is GPD, for black line, the input is GDA and the blue line is the combine of GPD and GDA. The red line is result with GPD input in DGLAP region.

The D-term in baryon loop may act like $\int_{-\xi}^{\xi} dy\frac{1}{y} = 0$, where the divergence around $1-y = 0$ cancel with each other and we can get a converged result. The integral which contain divergence is

$$\int_{-\xi}^{\xi} d(1-y)f(y,\xi,t)\frac{1}{1-y}D(\frac{x}{\xi},t)$$

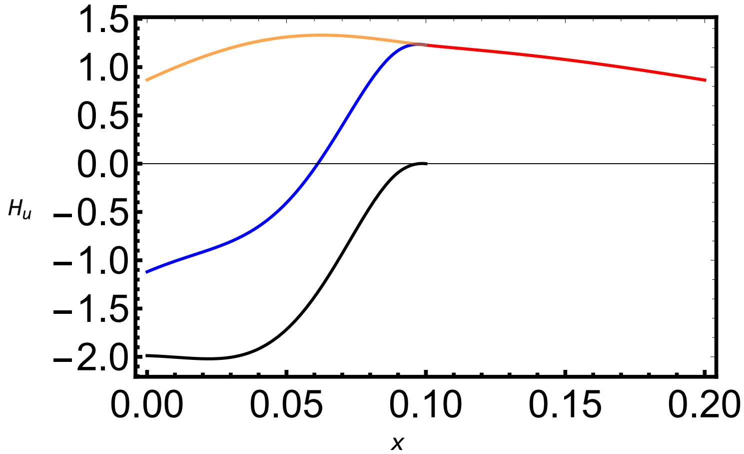

Figure 3: same result as Fig.2 only without D term in ERBL region

The function

$$F(y,x,\xi,t) = f(y,\xi,t)\frac{1}{1-y}D(\frac{x}{\xi},t)$$

gives a curve like where we chose $D(x,t) = \frac{15x(1-x^2)D(t)}{4}$, $D(t) = \frac{\alpha}{(1-\frac{t}{\Lambda^2})^2}$ from lattice result.

Change the integral in this way

$$\int_{-\xi}^{\xi} d(1-y)f(y,\xi,t)\frac{1}{1-y}D(\frac{x}{\xi},t)$$

$$= \int_{1-\xi}^{1+\xi} d(y)f(y,\xi,t)\frac{1}{1-y}D(\frac{x}{\xi},t)$$

$$= \int_{1-\xi}^{1-a} d(y)f(y,\xi,t)\frac{1}{1-y}D(\frac{x}{\xi},t) + \int_{1-a}^{1+a} d(y)f(y,\xi,t)\frac{1}{1-y}D(\frac{x}{\xi},t) + \int_{1+a}^{1+\xi} d(y)f(y,\xi,t)\frac{1}{1-y}D(\frac{x}{\xi},t)$$

We only calculate the first and second integral to avoid the pole

$$\operatorname*{Lim}_{a\to 0} \int_{1-\xi}^{1-a} d(y)f(y,\xi,t)\frac{1}{1-y}D(\frac{x}{\xi},t) + \int_{1+a}^{1+\xi} d(y)f(y,\xi,t)\frac{1}{1-y}D(\frac{x}{\xi},t)$$

$$= \int_{1-\xi}^{1+\xi} d(y)f(y,\xi,t)\frac{1}{1-y}D(\frac{x}{\xi},t)$$

And if the divergence around $1-y=0$ cancel we have $\int_{1-a}^{1+a} d(y)f(y,\xi,t)\frac{1}{1-y}D(\frac{x}{\xi},t) = 0$ when a is small enough which is equal to the integral above is same with different a.

$$I(a) = \int_{1-\xi}^{1-a} d(y)f(y,\xi,t)\frac{1}{1-y}D(\frac{x}{\xi},t) + \int_{1+a}^{1+\xi} d(y)f(y,\xi,t)\frac{1}{1-y}D(\frac{x}{\xi},t)$$

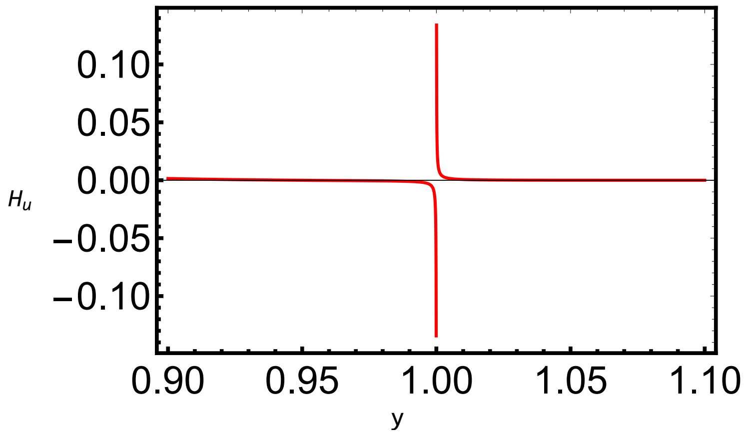

Figure 4: $F(y, -0.01, 0.1, -1)$from diagram b

$$I(0.01) = 0.00002284065045$$
$$I(10^{-4}) = 0.00001941004441$$
$$I(10^{-6}) = 0.0000193752819$$
$$I(10^{-8}) = 0.0000193749343$$
$$I(10^{-10}) = 0.0000193749308$$

This integral $\int_{1-\xi}^{1-a} d(y) f(y,\xi,t) \frac{1}{1-y} D(\frac{x}{\xi},t)$ is relatively small compared to the normal $\frac{1}{x}$ integral, like $\int_{-0.1}^{-a} dx \frac{1}{x}$. For example

$$I_1(a) = \int_{1-\xi}^{1-a} d(y) f(y,\xi,t) \frac{1}{1-y} D(\frac{x}{\xi},t)$$

$$I_1(0.01) = 0.00001580367639$$
$$I_1(10^{-4}) = -0.00002524554336$$
$$I_1(10^{-6}) = -0.00006466894493$$
$$I_1(10^{-8}) = -0.0001040750182$$
$$I_1(10^{-10}) = -0.0001434809205$$

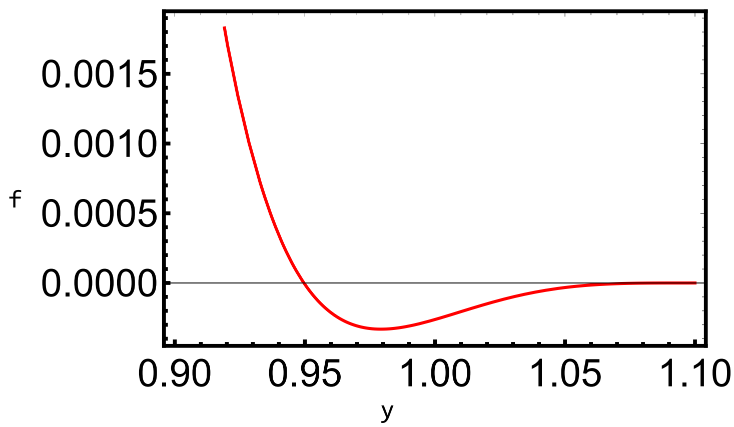

Figure 5: $f(y, 0.1, -1)$

and the $\int_{-0.1}^{-a} dx \frac{1}{x}$ is

$$\int_{-0.1}^{-a} dx \frac{1}{x}|_{a=0.01} = -2.30259$$
$$\int_{-0.1}^{-a} dx \frac{1}{x}|_{a=10^{-4}} = -6.90776$$
$$\int_{-0.1}^{-a} dx \frac{1}{x}|_{a=10^{-6}} = -11.5129$$
$$\int_{-0.1}^{-a} dx \frac{1}{x}|_{a=10^{-8}} = -16.1181$$
$$\int_{-0.1}^{-a} dx \frac{1}{x}|_{a=10^{-10}} = -20.7233$$

The reason is the numerator in $I_1(a)$ which is $f(y, \xi, t)D(\frac{x}{\xi}, t)$ is very small, the curve of $f(y, 0.1, -1)$ is while $D(\frac{-0.01}{0.1}, -1) = 0.0326303$

and this makes $I_1(a)$ is more like $\int_{-0.1}^{-a} dx \frac{0.00001}{x}$. And the numerical integral in mathematica can not give a divergent result even for $\int_{-0.1}^{-a} dx \frac{1}{x}$, which will lost precision when $a < 10^{-60}$ and for $a > 10^{-60}$, the result of $\int_{-0.1}^{-a} dx \frac{1}{x} > -135.853$

The result of GPD from diagram b if we add D-term is

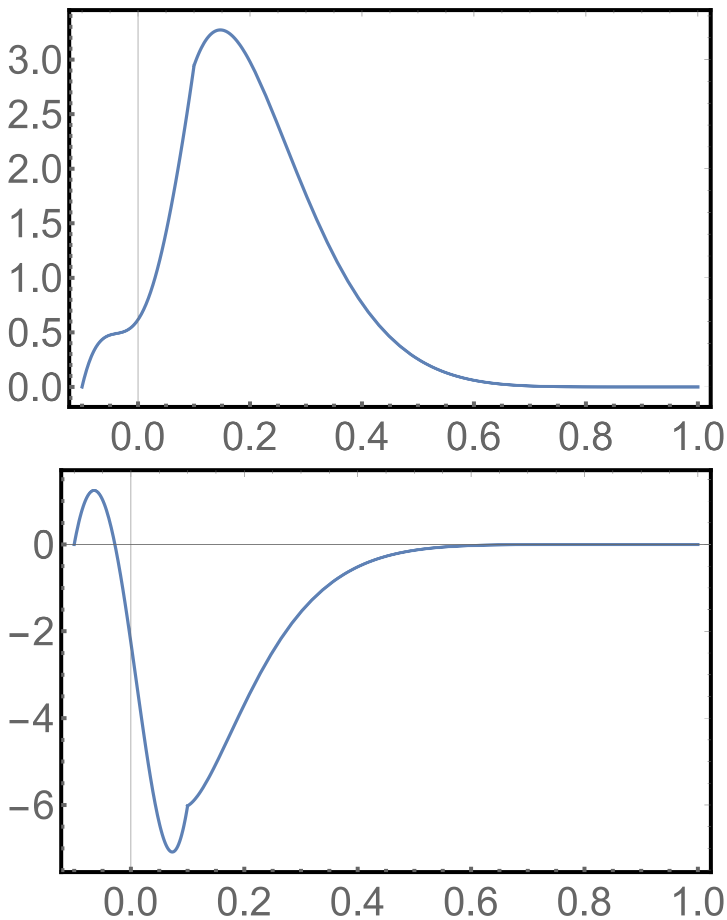

Figure 6: H and E from diagram b

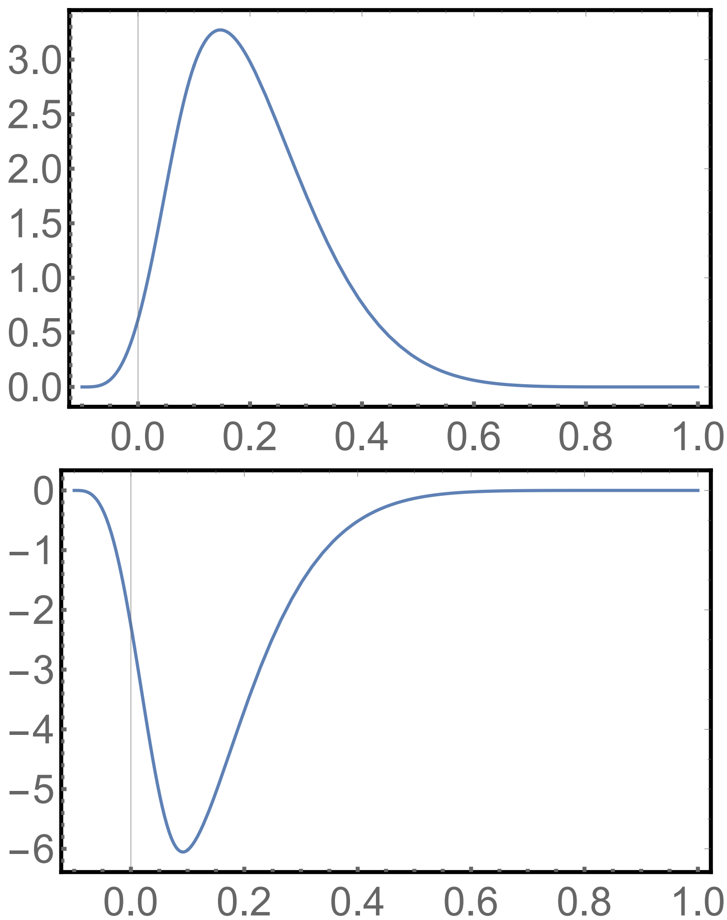

Figure 7: H and E from diagram b