

Lesson 3:

Sorting, Searching

3.1

3.1 Sorting Introduction

1 Sorting

- Arranging set of elements in a prescribed order
- Many algorithms available that perform sorting

5 Purpose

- Organize data to be more meaningful for display
- Some algorithms require data to be sorted

9 Example: sorting ints

- Unsorted:
 - 9, 3, 13, 1, 22
- Sorted – ascending order
 - 1, 3, 9, 13, 22
- Sorted – descending order
 - 22, 13, 9, 3, 1

17 Example: sorting people

- Person struct:

```
typedef struct Person_  
{  
    char firstName[MAX_NAME_LENGTH];  
    char lastName[MAX_NAME_LENGTH];  
} Person;
```

- Unsorted:
 1. { "Joe", "Smith" }
 2. { "Jane", "Doe" }
 3. { "Sam", "Jones" }
 4. { "Jane", "Smith" }
- Sorted – first name ascending
 1. { "Jane", "Doe" }
 2. { "Jane", "Smith" }
 3. { "Joe", "Smith" }
 4. { "Sam", "Jones" }

...OR...

 1. { "Jane", "Smith" }
 2. { "Jane", "Doe" }
 3. { "Joe", "Smith" }
 4. { "Sam", "Jones" }

- 1 ○ Sorted – first name ascending, last name descending (subsort)
- 2 ▪ { “Jane”, “Smith” }
- 3 ▪ { “Jane”, “Doe” }
- 4 ▪ { “Joe”, “Smith” }
- 5 ▪ { “Sam”, “Jones” }

3.2 Sorting Properties

Sorting classes

- Comparison sorts
 - Rely on comparing elements to place them in correct order
 - Requires comparison function to compare two elements
 - Comparison functions described in-full later
 - Fastest possible time: $O(n \lg n)$
- Linear-time sorts
 - Rely on certain characteristics in data
 - Requires that data possess characteristics needed by sort algorithm
 - Cannot always be applied
 - Fastest possible time: $O(n)$

Space requirements

- In-place
 - No additional space required during the sort
- Extra storage
 - Additional space required during the sort

Stability

- Unstable
 - Elements that compare as equal *may be reordered* during the sort
 - Example: unstable sort
 - Employee struct:


```
typedef struct Employee_
{
    int id;
    char name[MAX_NAME_LENGTH];
} Employee;
```
 - Unsorted:
 - { 1, "Ray" }
 - { 3, "Joe" }
 - { 2, "Ray" }
 - { 4, "Joe" }
 - Sorted – descending name (unstable):
 - { 2, "Ray" } ← Position swapped with ID 1
 - { 1, "Ray" }
 - { 4, "Joe" } ← Position swapped with ID 3
 - { 3, "Joe" }
- Stable
 - Elements that compare as equal *will not be reordered* during the sort

▪ Example: stable sort

• Employee struct:

```
typedef struct Employee_  
{  
    int id;  
    char name[MAX_NAME_LENGTH];  
} Employee;
```

• Unsorted:

- { 1, "Ray" }
- { 3, "Joe" }
- { 2, "Ray" }
- { 4, "Joe" }

• Sorted – descending name (unstable):

- { 1, "Ray" } ← Position unchanged with ID 2
- { 2, "Ray" }
- { 3, "Joe" } ← Position unchanged with ID 4
- { 4, "Joe" }

3.3 Sorting Applications

1 Order statistics

- 2 ○ Finding the i th smallest element in a set
- 3 ○ One simplistic approach – sort the set then select i th element

5 Binary search

- 6 ○ Efficient search method
- 7 ○ Requires data to be sorted

9 Directory listings

- 10 ○ Operating system typically sorts file listing by various criteria before displaying
- 11 ○ Sorted listing makes data easier for person to understand

13 Database systems

- 14 ○ Large amounts of data must be stored & retrieved quickly
- 15 ○ Storing data in sorted arrangement allows fast reads & writes

17 Spell checkers

- 18 ○ Programs that check spelling of words in text
- 19 ○ Dictionary stored in sorted arrangement allows efficient searches

21 Spreadsheets

- 22 ○ Important for business, financial, scientific data
- 23 ○ Data more meaningful when sorted
- 24 ○ Sorting / subsorting can occur on multiple columns

3.4 Comparison Functions

1 Required for comparison sorts

- 2 ○ Must understand comparison functions before we can look at
- 3 comparison sorts

5 Comparison function

- 6 ○ Takes pointers to two elements
- 7 ○ Returns <0 if first element should appear **before** second
- 8 ○ Returns >0 if first element should appear **after** second
- 9 ○ Returns 0 if elements are equal

11 Example: strcmp

- 12 ○ C library function “strcmp” is a comparison function for null-terminated
- 13 strings

15 Example: custom comparison functions

```
16 /*
17  * Demonstration of comparison functions.
18  *
19  * Program output:
20     Compare by ID = -1
21     Compare by ID = 0
22     Compare by name = 1
23     Compare by name = 0
24  */
25 #include <stdlib.h>
26 #include <stdio.h>
27 #include <string.h>
28
29 #define MAX_NAME_LENGTH 256
30
31 /* A type for us to compare */
32 typedef struct Employee_
33 {
34     int id;
```



```
1     char name[MAX_NAME_LENGTH];
2 } Employee;
3
4 /* Comparison function 1 */
5 int compareEmployeesById(const void *pKey1, const void *pKey2)
6 {
7     Employee *pEmp1;
8     Employee *pEmp2;
9
10    pEmp1 = (Employee *)pKey1;
11    pEmp2 = (Employee *)pKey2;
12
13    if (pEmp1->id > pEmp2->id) {
14        return 1;
15    }
16    if (pEmp1->id < pEmp2->id) {
17        return -1;
18    }
19    return 0;
20 }
21
22 /* Comparison function 2 */
23 int compareEmployeesByName(const void *pKey1, const void *pKey2)
24 {
25     Employee *pEmp1;
26     Employee *pEmp2;
27     int strcmpResult;
28
29    pEmp1 = (Employee *)pKey1;
30    pEmp2 = (Employee *)pKey2;
31
32    strcmpResult = strcmp(pEmp1->name, pEmp2->name);
33
34    if (strcmpResult > 0) {
35        return 1;
36    }
37    if (strcmpResult < 0) {
38        return -1;
39    }
40    return 0;
41 }
42
```

```
1  int main()
2  {
3      Employee emp1 = { 1, "Ray" };
4      Employee emp2 = { 2, "Joe" };
5      int (*compare)(const void *key1, const void *key2);
6
7      /* Compare by ID */
8      compare = compareEmployeesById;
9      printf("Compare by ID = %d\n", compare(&emp1, &emp2));      /* -1 */
10     printf("Compare by ID = %d\n", compare(&emp1, &emp1));      /* 0 */
11
12     /* Compare by name */
13     compare = compareEmployeesByName;
14     printf("Compare by name = %d\n", compare(&emp1, &emp2));      /* 1 */
15     printf("Compare by name = %d\n", compare(&emp1, &emp1));      /* 0 */
16
17     return EXIT_SUCCESS;
18 }
```

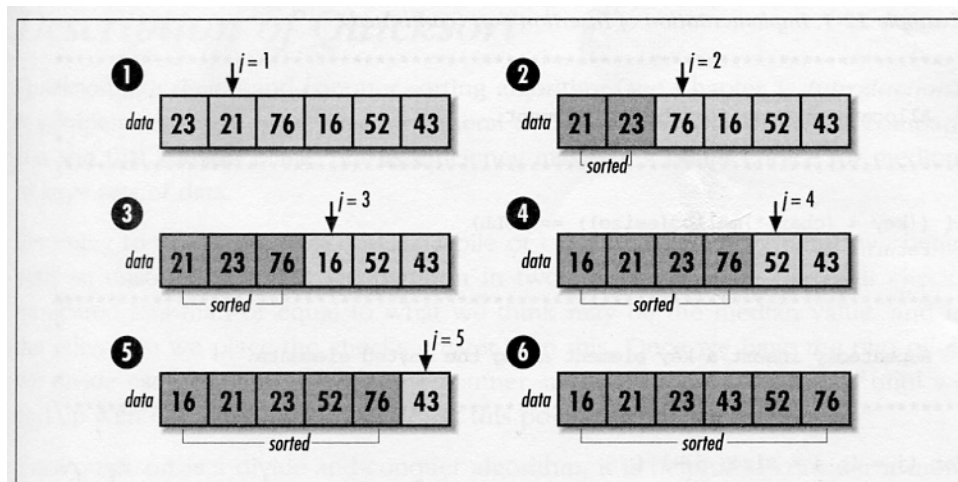
3.5 Insertion Sort

1 Overview

- 2 ○ Comparison sort
- 3 ○ One of simplest sorting algorithms
- 4 ○ Example: Sorting pile of canceled checks by hand
 - 5 ■ Start with pile of unsorted checks
 - 6 ■ One at a time, remove a check from unsorted pile and put in
 - 7 proper position in sorted pile
- 8 ○ Not the fastest sort
 - 9 ■ Inefficient speed for large sets of data
- 10 ○ At first appears would require additional space, but can represent
- 11 unsorted and sorted “piles” in-place
- 12 ○ Good at inserting element into already sorted set (see example of use)
 - 13 ■ Inserting one element requires only one scan of sorted elements
 - 14 (incremental sort) as opposed to complete run of the algorithm

Visualization

- j (looks like an i in diagram) is next element to be added to sorted sequence
- Elements to left of j are sorted
- Elements to right of j (including j) are unsorted
- Procedure:
 - Start with $j = 1$
 - Save value at j
 - Starting with element to left of j , shift each element larger than j one position to the right, stop once position for j is found
 - Assign original value at j to position
 - Repeat until end of sequence reached at which point entire sequence is sorted



Time

- $O(n^2)$
 - Note: Inserting element into already sorted set is $O(n)$

Space

- In-place

Stability

- Stable

Data structures supported

- Array
- Doubly-linked list

When to use?

- Small sets of data
- If you must use linked list data structure
- *When solving problems that involve incremental sorting:* Inserting new elements into sorted set
- Example: Hotel reservation system
 - One display in the system lists all guests, sorted by name
 - New guests inserted as they arrive (requiring only single scan for insertion point)

3.6 Insertion Sort Implementation

1 Interface

```
2 int issort(void *data, int size, int esize,  
3     int (*compare)(const void *key1, const void *key2))
```

4

5 • Parameters

- 6 ○ data – Pointer to array of data to be sorted
- 7 ○ size – Number of elements in data
- 8 ○ esize – Size of each element
- 9 ○ compare – Comparison function used to compare elements

10 • Returns

- 11 ○ 0 if sorting successful, -1 otherwise

12

13

1 Implementation

```
1  /*
2   * issort.c
3   */
4  #include <stdlib.h>
5  #include <string.h>
6
7  #include "sort.h"
8
9
10 int issort(void *data, int size, int esize,
11 int (*compare)(const void *key1, const void *key2))
12 {
13     char *a = data;
14     void *key;
15     int i,
16         j;
17
18     /* Allocate storage for the key element. */
19     if ((key = (char *)malloc(esize)) == NULL)
20         return -1;
21
22     /* Repeatedly insert a key element among the sorted elements. */
23     for (j = 1; j < size; j++) {
24         memcpy(key, &a[j * esize], esize);
25         i = j - 1;
26
27         /* Determine the position at which to insert the key element. */
28         while (i >= 0 && compare(&a[i * esize], key) > 0) {
29             memcpy(&a[(i + 1) * esize], &a[i * esize], esize);
30             i--;
31         }
32
33         memcpy(&a[(i + 1) * esize], key, esize);
34     }
35
36     /* Free the storage allocated for sorting. */
37     free(key);
38
39     return 0;
40 }
```

3.7 Quick Sort

Description

- Comparison sort
- Divide and conquer algorithm
- Widely regarded as best sort for general use
- Example: Sorting pile of canceled checks by hand
 - Start with pile of unsorted checks
 - Partition pile in two
 - In one pile place all checks less than or equal to what we think might be the median value
 - In the other pile place all checks greater than this
 - Once we have two piles, divide each of them in the same manner
 - Repeat until we end up with one check in every pile
 - At this point the checks are sorted
- Worst case performs same as insertion sort ($O(n^2)$)
 - We can make worst case so unlikely that we can count on algorithm performing in average case ($O(n \lg n)$)
 - Key to reaching average case performance is selecting partition value that results in balanced sub-groups
- Partitioning example: imbalanced – *worst-case performance*
 - Unsorted: { 15, 20, 18, 51, 36, 10, 77, 43 }
 - Partition: 10
 - Sub-groups: { 10 }, { 15, 20, 18, 51, 36, 77, 43 }
- Partitioning example: balanced – *average-case performance*
 - Unsorted: { 15, 20, 18, 51, 36, 10, 77, 43 }
 - Partition: 36
 - Sub-groups: { 15, 20, 18, 10 }, { 36, 51, 77, 43 }
- Selecting good partition
 - Selecting randomly
 - Median-of-three method
 - Randomly select 3 elements
 - Select median value as partition
 - Virtually guarantees average-case performance

- Quick sort is a randomized algorithm (because partition chosen randomly)

Visualization – partition

- Procedure:
 - Pivot value selected from median-of-three random values
 - Move k to left until value found less than pivot value
 - Move i to right until value found greater than pivot value
 - If k & i did not cross paths
 - Swap values at k & i (putting values on correct side of pivot)
 - Repeat procedure

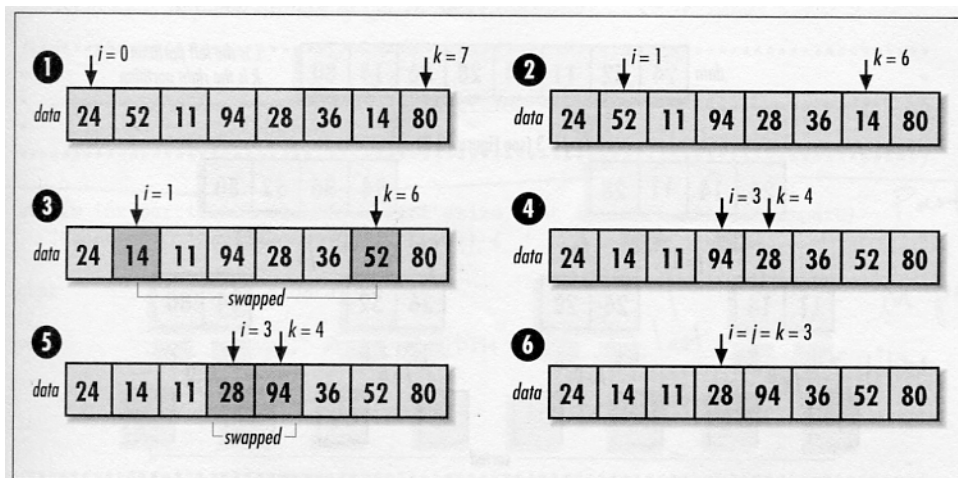


Figure 12-2. Partitioning around 28

Visualization – quicksort

- Procedure:
 - If array has more than one element
 - Partition array
 - quicksort left partition
 - quicksort right partition

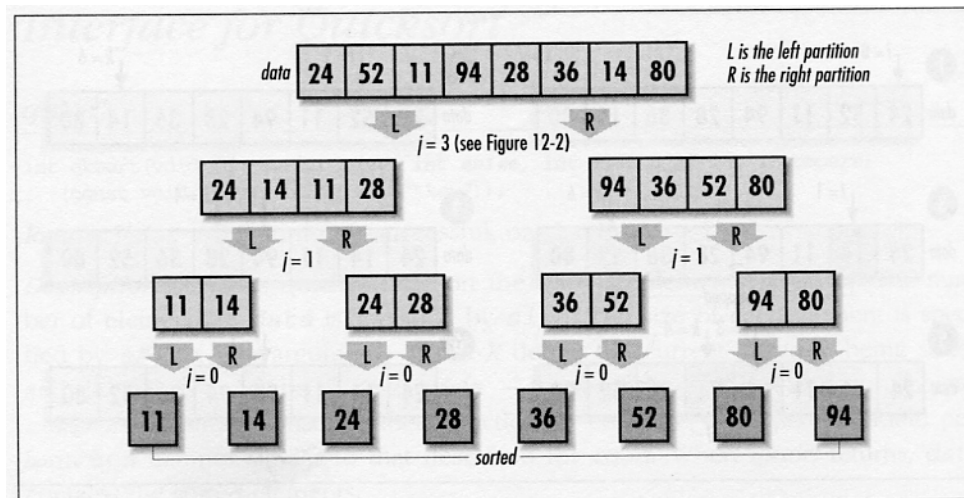


Figure 12-3. Sorting with quicksort assuming optimal partitioning

Time

- $O(n \lg n)$ – Average case
 - By selecting random partition value we can virtually guarantee average case performance

Space

- In-place

Stability

- Unstable

Data structures supported

- Array – requires random access

When to use?

- Any general sort
- Medium and large data sets
- Only restriction is that space must be provided to store all elements in memory
 - Unpredictable partition size prevents identifying space required to hold partition
 - Cannot break sort into guaranteed smaller pieces
 - Merge sort can be used when entire sequence to be sorted cannot be held in memory

3.8 Quick Sort Implementation

1 Interface

```
2 int qksort(void *data, int size, int esize, int i, int k,  
3 int (*compare)(const void *key1, const void *key2))
```

4

5 • Parameters

- 6 ○ data – Pointer to array of data to be sorted
- 7 ○ size – Number of elements in data
- 8 ○ esize – Size of each element
- 9 ○ i – left index of range in data to be sorted
- 10 ○ k – right index of range in data to be sorted
- 11 ○ compare – Comparison function used to compare elements

12 • Returns

- 13 ○ 0 if sorting successful, -1 otherwise

14

15

Implementation

```
1  /*
2  * qksort.c
3  */
4  #include <stdlib.h>
5  #include <string.h>
6
7  #include "sort.h"
8
9
10 static int compare_int(const void *int1, const void *int2)
11 {
12     /* Compare two integers (used during median-of-three partitioning). */
13     if (*(const int *)int1 > *(const int *)int2)
14         return 1;
15     else if (*(const int *)int1 < *(const int *)int2)
16         return -1;
17     else
18         return 0;
19 }
20
21 static int partition(void *data, int esize, int i, int k,
22                     int (*compare)(const void *key1, const void *key2))
23 {
24     char *a = data;
25     void *pval,
26         *temp;
27     int r[3];
28
29     /* Allocate storage for the partition value and swapping. */
30     if ((pval = malloc(esize)) == NULL)
31         return -1;
32
33     if ((temp = malloc(esize)) == NULL) {
34         free(pval);
35         return -1;
36     }
37
38     /* Use the median-of-three method to find the partition value. */
39     r[0] = (rand() % (k - i + 1)) + i; /* Get random index in range [i, k] */
40     r[1] = (rand() % (k - i + 1)) + i; /* "" */
41     r[2] = (rand() % (k - i + 1)) + i; /* "" */
```

```
1  /*
2   * BUG: The next two lines are selecting the median INDEX from the three
3   * randomly generated indices. These lines should instead be selecting
4   * the median VALUE from the values at the three randomly generated
5   * indices. This bug effectively causes a random value to be selected
6   * for the pivot rather than the median-of-three random values. This means
7   * quicksort will not be virtually guaranteed to perform in average case
8   *  $O(n \lg n)$  and instead may perform in worst case  $O(n^2)$ .
9   */
10  issort(r, 3, sizeof(int), compare_int); /* Sort random indices */
11  memcpy(pval, &a[r[1] * esize], esize); /* Set pivot = value at mid index */
12
13  /* Create two partitions around the partition value. */
14  i--;
15  k++;
16
17  while (1) {
18      /* Move left until an element is found in the wrong partition. */
19      do {
20          k--;
21      } while (compare(&a[k * esize], pval) > 0);
22
23      /* Move right until an element is found in the wrong partition. */
24      do {
25          i++;
26      } while (compare(&a[i * esize], pval) < 0);
27
28      if (i >= k) {
29          /* Stop partitioning when the left and right counters cross. */
30          break;
31      }
32      else {
33          /* Swap the elements now under the left and right counters. */
34          memcpy(temp, &a[i * esize], esize);
35          memcpy(&a[i * esize], &a[k * esize], esize);
36          memcpy(&a[k * esize], temp, esize);
37      }
38  }
39
40  /* Free the storage allocated for partitioning. */
41  free(pval);
42  free(temp);
```

```
1
2     /* Return the position dividing the two partitions. */
3     return k;
4 }
5
6 int qksort(void *data, int size, int esize, int i, int k,
7           int (*compare)(const void *key1, const void *key2))
8 {
9     int j;
10
11     /* Stop the recursion when it is not possible to partition further. */
12     if (i < k) {
13         /* Determine where to partition the elements. */
14         if ((j = partition(data, esize, i, k, compare)) < 0)
15             return -1;
16
17         /* Recursively sort the left partition. */
18         if (qksort(data, size, esize, i, j, compare) < 0)
19             return -1;
20
21         /* Recursively sort the right partition. */
22         if (qksort(data, size, esize, j + 1, k, compare) < 0)
23             return -1;
24     }
25
26     return 0;
27 }
```

3.9 Merge Sort

1 Description

- 2 ○ Comparison sort
- 3 ○ Divide and conquer algorithm
- 4 ○ In all cases performs as well as average case of quicksort
- 5 ○ Requires twice the space of the unsorted data
- 6 ○ Valuable for large sets of data
 - 7 ▪ Divides data into predictable sizes (unlike quick sort)
 - 8 ▪ Does not require all elements to be in memory at one time
- 9 ○ Example: Sorting pile of canceled checks by hand
 - 10 ▪ Start with pile of unsorted checks
 - 11 ▪ Divide the pile in half
 - 12 ▪ Divide each of the resulting piles in half
 - 13 ▪ Repeat until all piles contain a single check
 - 14 ▪ Merge the piles two-by-two so each new pile is a sorted
 - 15 combination of the two that were merged
 - 16 ▪ Repeat until we end up with one big pile again
 - 17 ▪ At this point the checks are sorted

Visualization – merge

○ Procedure:

- While ipos hasn't reached j and jpos hasn't reached the end of the list
 - Compare the values at ipos and jpos
 - Place the smaller value at the next position in the merged list and increment that index
- Add all remaining elements pointed to by ipos or jpos to the end of the merged list (only ipos or jpos will point to elements)

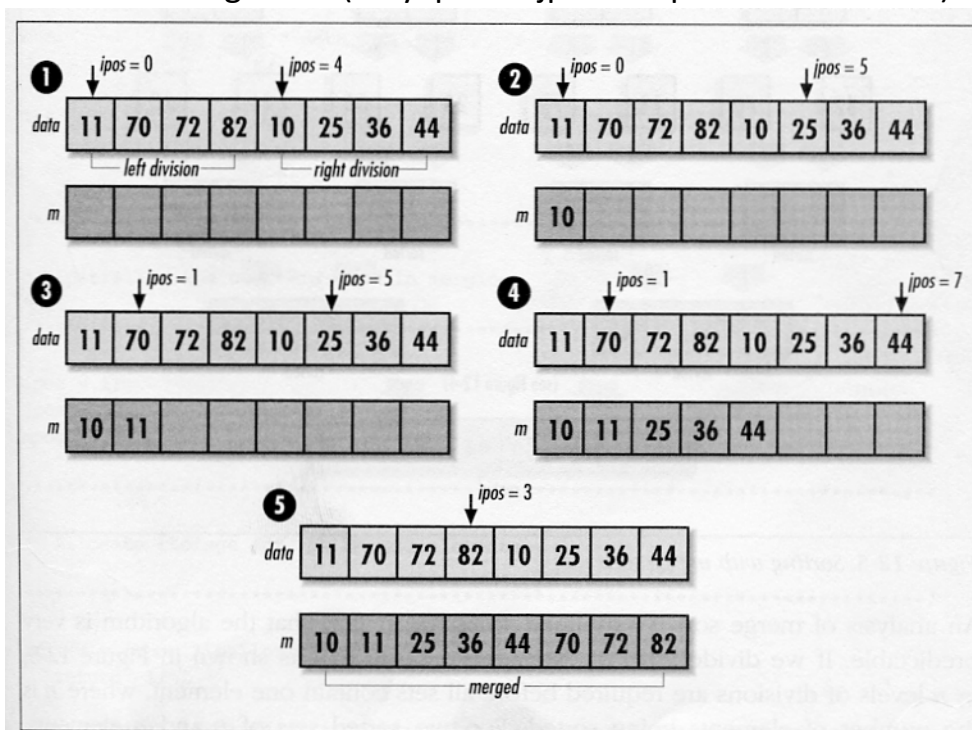


Figure 12-4. Merging two sorted sets

Visualization – mergesort

○ Procedure:

- (light gray) If the list has more than one element
 - Split the list into left and right halves
 - mergesort the left half
 - mergesort the right half
- (dark gray) merge the left and right halves

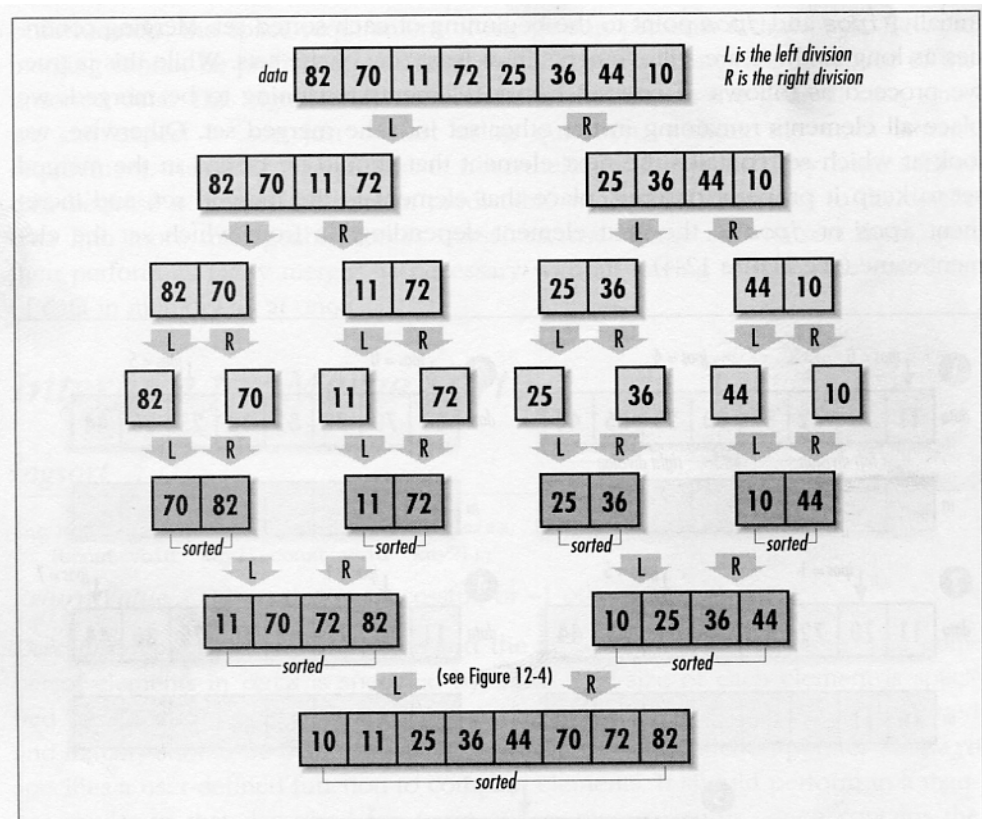


Figure 12-5. Sorting with merge sort

Time

- $O(n \lg n)$
 - $\lg n$ levels of divisions are required
 - For each of the $\lg n$ levels, we traverse all n elements to merge

Space

- Twice the space of the original list

Stability

- Stable

Data structures supported

- Array
- Singly-linked list
- Doubly-linked list

When to use?

- When dealing with very large sets; when there's not enough memory to hold all elements in memory
- When stable sort is required

3.10 Merge Sort Implementation

1 Interface

```
2 int mgsort(void *data, int size, int esize, int i, int k,  
3 int (*compare)(const void *key1, const void *key2))
```

- Parameters

- data – Pointer to array of data to be sorted
- size – Number of elements in data
- esize – Size of each element
- i – left index of range in data to be sorted; should always be 0
- k – right index of range in data to be sorted; should always be size - 1
- compare – Comparison function used to compare elements

- Returns

- 0 if sorting successful, -1 otherwise

Implementation

```
1  /*
2   * mgsort.c
3   */
4  #include <stdlib.h>
5  #include <string.h>
6
7  #include "sort.h"
8
9
10 static int merge(void *data, int esize, int i, int j, int k,
11                 int (*compare) (const void *key1, const void *key2))
12 {
13     char *a = data,
14          *m;
15
16     int ipos,
17         jpos,
18         mpos;
19
20     /* Initialize the counters used in merging. */
21     ipos = i;
22     jpos = j + 1;
23     mpos = 0;
24
25     /* Allocate storage for the merged elements. */
26     if ((m = (char *)malloc(esize * ((k - i) + 1))) == NULL)
27         return -1;
28
29     /* Continue while either division has elements to merge. */
30     while (ipos <= j || jpos <= k) {
31         if (ipos > j) {
32
33             /* The left division has no more elements to merge. */
34             while (jpos <= k) {
35                 memcpy(&m[mpos * esize], &a[jpos * esize], esize);
36                 jpos++;
37                 mpos++;
38             }
39
40             continue;
41         }
```

```
1     else if (jpos > k) {
2         /* The right division has no more elements to merge. */
3         while (ipos <= j) {
4             memcpy(&m[mpos * esize], &a[ipos * esize], esize);
5             ipos++;
6             mpos++;
7         }
8
9         continue;
10    }
11
12    /* Append the next ordered element to the merged elements. */
13    if (compare(&a[ipos * esize], &a[jpos * esize]) < 0) {
14        memcpy(&m[mpos * esize], &a[ipos * esize], esize);
15        ipos++;
16        mpos++;
17    }
18    else {
19        memcpy(&m[mpos * esize], &a[jpos * esize], esize);
20        jpos++;
21        mpos++;
22    }
23 }
24
25 /* Prepare to pass back the merged data. */
26 memcpy(&a[i * esize], m, esize * ((k - i) + 1));
27
28 /* Free the storage allocated for merging. */
29 free(m);
30
31 return 0;
32 }
33
34 int mgsort(void *data, int size, int esize, int i, int k,
35           int (*compare)(const void *key1, const void *key2))
36 {
37     int j;
38
39     /* Stop the recursion when no more divisions can be made. */
40     if (i < k) {
41         /* Determine where to divide the elements. */
42         j = (int)(((i + k - 1) / 2));
```

```
1      /* Recursively sort the two divisions. */
2      if (mgsort(data, size, esize, i, j, compare) < 0)
3          return -1;
4
5      if (mgsort(data, size, esize, j + 1, k, compare) < 0)
6          return -1;
7
8      /* Merge the two sorted divisions into a single sorted set. */
9      if (merge(data, esize, i, j, k, compare) < 0)
10         return -1;
11     }
12
13     return 0;
14 }
15 }
```

3.11 Counting Sort

1 Description

- 2 ○ Linear-time sort
- 3 ○ Counts how many times each element of a set occurs to determine how
- 4 the set should be ordered
- 5 ○ Does not compare elements in the set
- 6 ○ Improves on $O(n \lg n)$ run-time upper bound of comparison sorts
- 7 ○ Requires elements in set to be integral type
- 8 ○ We must know largest value that can occur in the set so we can allocate
- 9 space for the counts
- 10 ○ Overview
 - 11 ■ Start with unsorted set of integers (or objects that can be
 - 12 represented as integers)
 - 13 ■ Allocate array of counts initialized to zero; size of array = value of
 - 14 largest integer in unsorted set
 - 15 ■ Loop over unsorted set incrementing count of each integer
 - 16 ■ Place integers in sorted order into set using counts from array
 - 17

Visualization

○ Procedure:

- Phase I – Count (steps 1a & 1b)
 - Allocate array of counts initialized to zero; size of array = value of largest integer in unsorted set
 - Loop over unsorted set incrementing count of each integer
- Phase II – Adjust counts so they reflect last index of each value in sorted array (step 1c)
 - Loop over counts and increment each count by all previous counts
- Phase III – Place sorted values into new set (steps 2a – 2f)
 - Loop over unsorted values in reverse order
 - For each unsorted value
 - Lookup count of that value in array of counts
 - Assign unsorted value into sorted array using the count as the index
 - Decrement count by 1

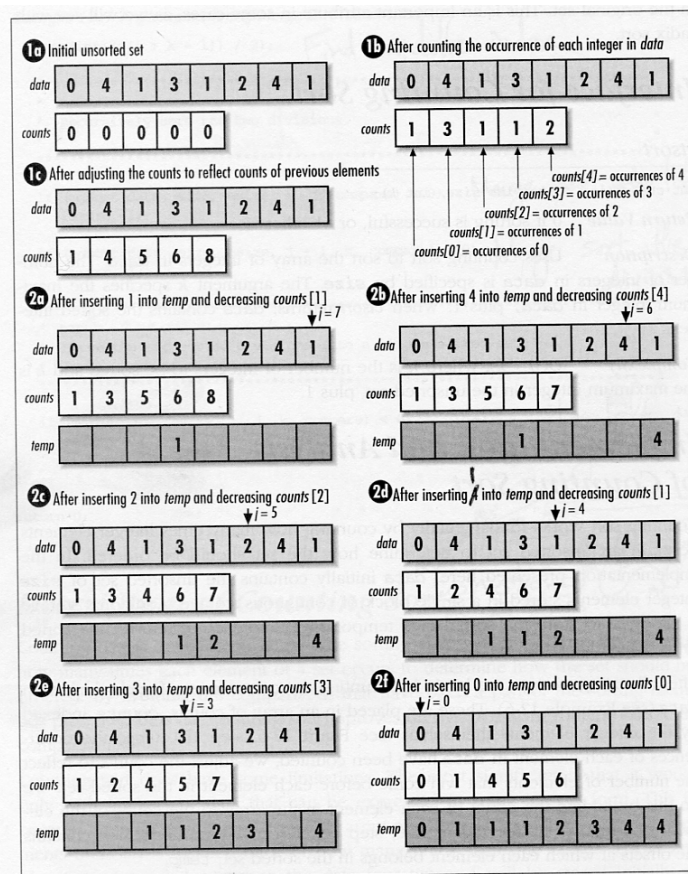


Figure 12-6. Sorting with counting sort

Time

- $O(n + k)$
 - n is the number of integers to be sorted
 - k is the maximum integer in the unsorted set plus 1

Space

- Twice the space of the original list plus space for the counts

Stability

- Stable

Data structures supported

- Array
- Singly-linked list
- Doubly-linked list

When to use?

- When sorting integral types that have a small maximum value

3.12 Counting Sort Implementation

1 Interface

```
2 int ctsort(int *data, int size, int k)
```

- 3 • Parameters
 - 4 ○ data – Pointer to array of ints to be sorted
 - 5 ○ size – Number of elements in data
 - 6 ○ k – Maximum value occurring in data
- 7 • Returns
 - 8 ○ 0 if sorting successful, -1 otherwise

9

10

1 Implementation

```
2  /*
3   * ctsort.c
4   */
5  #include <stdlib.h>
6  #include <string.h>
7
8  #include "sort.h"
9
10 int ctsort(int *data, int size, int k)
11 {
12     int *counts,
13         *temp;
14
15     int i,
16         j;
17
18     /* Allocate storage for the counts. */
19     if ((counts = (int *)malloc(k * sizeof(int))) == NULL)
20         return -1;
21
22     /* Allocate storage for the sorted elements. */
23     if ((temp = (int *)malloc(size * sizeof(int))) == NULL)
24         return -1;
25
26     /* Initialize the counts. */
27     for (i = 0; i < k; i++)
28         counts[i] = 0;
29
30     /* Count the occurrences of each element. */
31     for (j = 0; j < size; j++)
32         counts[data[j]] = counts[data[j]] + 1;
33
34     /* Adjust each count to reflect the counts before it. */
35     for (i = 1; i < k; i++)
36         counts[i] = counts[i] + counts[i - 1];
37
38     /* Use the counts to position each element where it belongs. */
39     for (j = size - 1; j >= 0; j--) {
40         temp[counts[data[j]] - 1] = data[j];
41         counts[data[j]] = counts[data[j]] - 1;
```

```
1     }  
2  
3     /* Prepare to pass back the sorted data. */  
4     memcpy(data, temp, size * sizeof(int));  
5  
6     /* Free the storage allocated for sorting. */  
7     free(counts);  
8     free(temp);  
9  
10    return 0;  
11 }
```

3.13 Radix Sort

Description

- Linear-time sort
- Sorts data in pieces called *digit*, one digit at a time, from the digit in least significant position to the most significant
- Example: Sorting radix-10 numbers
 - Unsorted: { 15, 12, 49, 16, 36, 40 }
 - After sorting least significant digit: { 40, 12, 15, 16, 36, 49 }
 - After sorting most significant digit: { 12, 15, 16, 36, 40, 49 }
- Digit sorting must use a stable sort
 - Important because once less significant digit order determined, more significant digit sorts must not cause reordering for values where more significant digits are the same
 - Example: Using unstable digit sort
 - Unsorted: { 15, 12 }
 - After sorting least significant digit: { 12, 15 }
 - After *unstable* sorting most significant digit : { 15, 12 }
- Uses counting sort to sort digits
 - Stable
 - We know largest integer any digit may be (radix – 1)
- Not limited to sorting data keyed by integers
 - Can sort any data that can be divided into integer pieces
 - Examples:
 - Strings (radix 2^8)
 - 64-bit integers (four-digit, radix 2^{16} values)

Visualization

- Procedure:
 - For each digit from least to most significant
 - Apply count sort for the digit

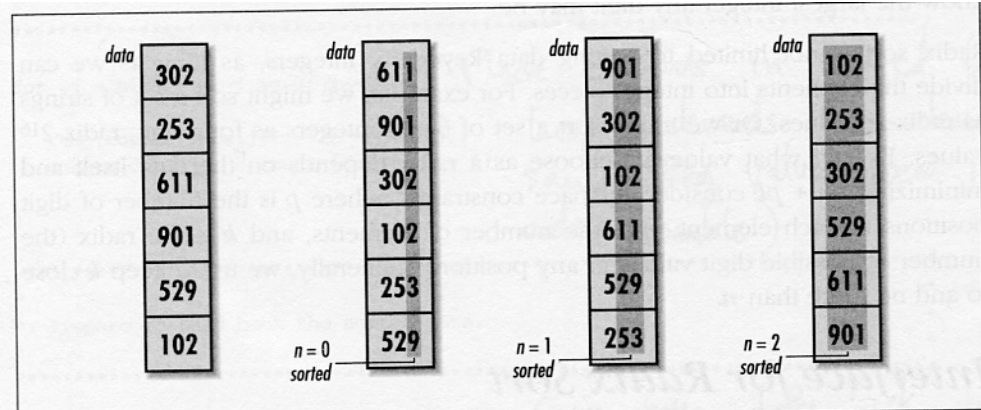


Figure 12-7. Sorting integers as radix-10 numbers with radix sort

Time

- $O(pn + pk)$
 - p is the number of digit positions in each element
 - n is the number of elements
 - k is the radix
 - Note: We try to keep k close to and no more than n

Space

- Twice the space of the original list plus space for the counts

Stability

- Stable

Data structures supported

- Array

When to use?

- When data can be split into integer pieces

3.14 Radix Sort Implementation

Interface

```
int rxsort(int *data, int size, int p, int k)
```

- Parameters
 - data – Pointer to array of ints to be sorted
 - size – Number of elements in data
 - p – Number of digit positions in each element
 - k – Radix
- Returns
 - 0 if sorting successful, -1 otherwise

Implementation

```
/*
 * rxsort.c
 */
#include <limits.h>
#include <math.h>
#include <stdlib.h>
#include <string.h>

#include "sort.h"

int rxsort(int *data, int size, int p, int k)
{
    int *counts,
        *temp;

    int index,
        pval,
        i,
        j,
        n;

    /* Allocate storage for the counts. */
    if ((counts = (int *)malloc(k * sizeof(int))) == NULL)
        return -1;
```



```
1  /* Allocate storage for the sorted elements. */
2  if ((temp = (int *)malloc(size * sizeof(int))) == NULL)
3      return -1;
4
5  /* Sort from the least significant position to the most significant. */
6  for (n = 0; n < p; n++) {
7      /* Initialize the counts. */
8      for (i = 0; i < k; i++)
9          counts[i] = 0;
10
11     /* Calculate the position value. */
12     pval = (int)pow((double)k, (double)n);
13
14     /* Count the occurrences of each digit value. */
15     for (j = 0; j < size; j++) {
16         index = (int)(data[j] / pval) % k;
17         counts[index] = counts[index] + 1;
18     }
19
20     /* Adjust each count to reflect the counts before it. */
21     for (i = 1; i < k; i++)
22         counts[i] = counts[i] + counts[i - 1];
23
24     /* Use the counts to position each element where it belongs. */
25     for (j = size - 1; j >= 0; j--) {
26         index = (int)(data[j] / pval) % k;
27         temp[counts[index] - 1] = data[j];
28         counts[index] = counts[index] - 1;
29     }
30
31     /* Prepare to pass back the data as sorted thus far. */
32     memcpy(data, temp, size * sizeof(int));
33 }
34
35 /* Free the storage allocated for sorting. */
36 free(counts);
37 free(temp);
38
39 return 0;
40 }
```

3.15 Searching Introduction

1 Overview

- 2 ○ Looking for a target value in a set

3

4 Data structure requirements

- 5 ○ Generally search requires
 - 6 ▪ Random access
 - 7 ▪ Data sorted
- 8 ○ Specialized searches exist for specific data structures
- 9 ▪ More on this later in class

3.16 Linear Search

1 Description

- 2
 - Does not require set to be sorted
- 3
 - Search from beginning of set to end
- 4
 - $O(n)$
- 5
 - Worst possible search
- 6
 - Only acceptable for very small sets

3.17 Binary Search

Description

- Requires set to be sorted
- Divide and conquer
- Procedure
 - Look at middle element
 - Return true if element matches target
 - Repeat search on lower half of elements if middle element > target
 - Repeat search on upper half of elements if middle element < target
 - Return false if no more elements left to search

Visualization

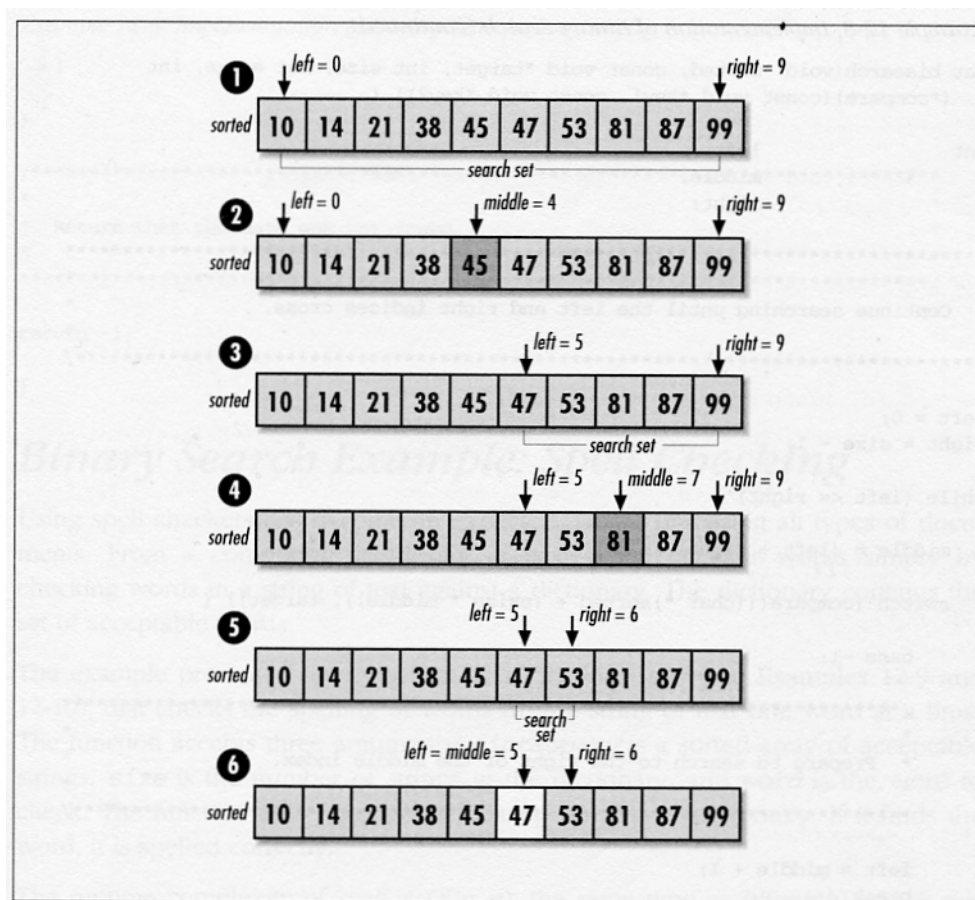


Figure 12-8. Searching for 47 using binary search

1

2 **Time**

- 3 ○ $O(\lg n)$
- 4 ▪ Each division reduces problem size by $\frac{1}{2}$

5

6 **Space**

- 7 ○ In-place

8

9 **Data structures supported**

- 10 ○ Array – requires random access

11

12 **When to use?**

- 13 ○ When fast searches are required

14

3.18 Binary Search Implementation

1 Interface

```
2 int bisearch(void *sorted, const void *target, int size, int esize,  
3 int (*compare)(const void *key1, const void *key2))
```

- Parameters

- sorted – Pointer to array of elements to be sorted, will be sorted after call completes
- target – Value being searched for
- size – Number of elements in sorted array
- esize – Size of each element in sorted array
- compare – Comparison function used to compare elements during search

- Returns

- index where target was found, -1 if target not found

1 Implementation

```
1  /*
2   * bsearch.c
3   */
4   #include <stdlib.h>
5   #include <string.h>
6
7   #include "search.h"
8
9
10  int bsearch(void *sorted, const void *target, int size, int esize,
11             int (*compare)(const void *key1, const void *key2))
12  {
13      int left,
14          middle,
15          right;
16
17      /* Continue searching until the left and right indices cross. */
18      left = 0;
19      right = size - 1;
20
21      while (left <= right) {
22          middle = (left + right) / 2;
23
24          switch (compare(((char *)sorted + (esize * middle)), target)) {
25              case -1:
26                  /* Prepare to search to the right of the middle index. */
27                  left = middle + 1;
28                  break;
29
30              case 1:
31                  /* Prepare to search to the left of the middle index. */
32                  right = middle - 1;
33                  break;
34
35              case 0:
36                  /* Return the exact index where the data has been found. */
37                  return middle;
38          }
39      }
40
41      /* Return that the data was not found. */
```

```
1  return -1;  
2  }
```