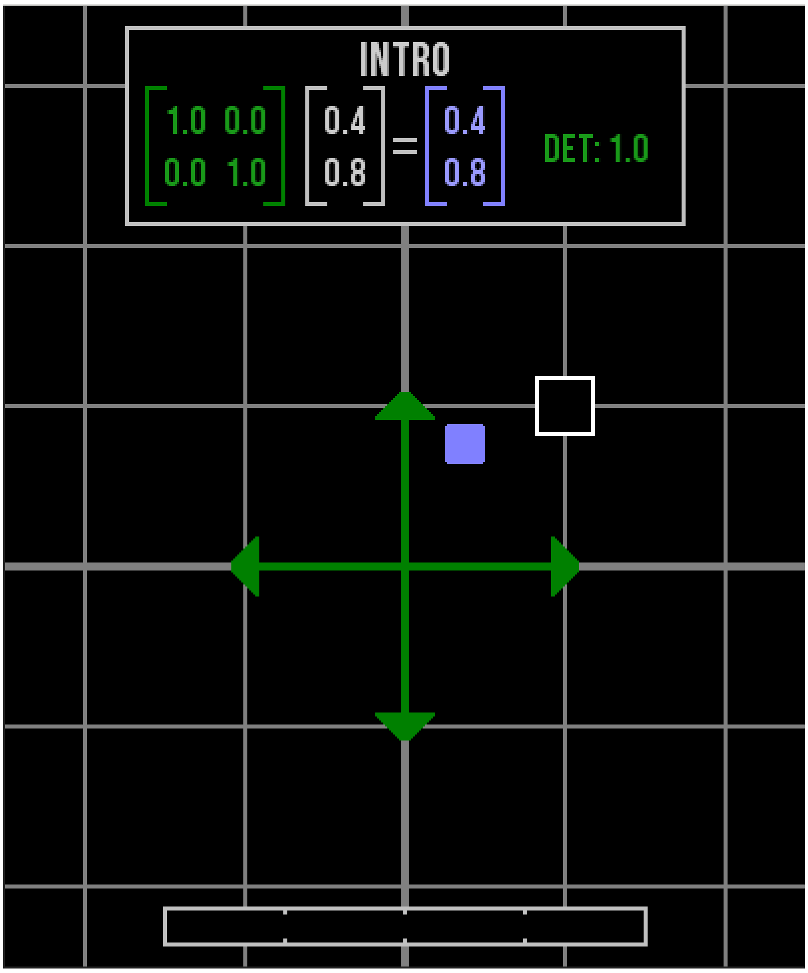
Overall:



- The green box with four numbers is the *matrix*. It describes how the input vector becomes the output vector.
- The off-white box next to it is the *input vector*. It's where your mouse is.
- The blue box after the equals sign is the *output vector*. It's where the blue dot is.
- The green number after DET is the *determinant*. It's a fact about the matrix: if you draw a shape with the input vector, the shape drawn by the output vector will be DET times the size.
- The green arrows are how I chose to represent the *eigenvectors* and *eigenvalues* of the matrix. In this picture, there are two eigenvectors: one vertical and one horizontal. Each eigenvector here has an eigenvalue of 1, represented by the arrows being one unit away from the *origin* at the centre of the grid.

Intro:

- We start with the Identity matrix $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$: a transformation that changes nothing.
- If you put the input vector somewhere in the path of an eigenvector, the output vector will also be along the path of that eigenvector. That's what makes an eigenvector special.
- If you hold the left mouse button, you can draw shapes. Shapes drawn by the output vector (larger blue sparkles) are DET times the size of the shape drawn by the input vector (smaller white sparkles).
- When a determinant is negative, if you draw a shape while moving clockwise, the output shape will be drawn moving counterclockwise.

Scale:

- A 2x2 scale matrix is a matrix of the form $\begin{bmatrix} a & 0 \\ 0 & a \end{bmatrix}$. It multiplies distance from the origin by *a*, while changing nothing else.
- The identity matrix is also a scale matrix, scaling input by a factor of 1.
- For a scale matrix, all possible vectors are eigenvectors. I represent this by having them form an asterisk, computer code for “anything” and “multiply”. A more faithful depiction would have an infinite number of eigenvectors pointing in every possible direction from the origin.
- If you hold Space, time slows down, and you can see more clearly how the eigenvalues, eigenvectors, and determinant all change as the matrix does.

Rotation:

- A 2x2 rotation matrix is a matrix of the form $\begin{bmatrix} \cos(theta) & -\sin(theta) \\ \sin(theta) & \cos(theta) \end{bmatrix}$. It rotates input theta degrees counterclockwise about the origin.
- The identity matrix is also a rotation matrix, rotating input zero degrees.
- $\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$ is both a scale and a rotation matrix, because a rotation of 180 degrees is the same as multiplying by -1.
- For a matrix of the format $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$, moving the input one unit to the right moves the output *a* units to the right and *c* units up. Similarly, moving the input one unit up moves the input *b* units to the right and *d* units up.

Diagonal:

- A 2x2 diagonal matrix is a matrix of the form $\begin{bmatrix} a & 0 \\ 0 & d \end{bmatrix}$. It multiplies horizontal distance from the origin by *a*, and multiplies vertical distance from the origin by *d*.
- The identity matrix is also a diagonal matrix, scaling vertical and horizontal input by a factor of 1.
- These matrices are called ‘diagonal’ because their nonzero numbers are all on the leading diagonal, which goes from the top left corner of the matrix to the bottom right.
- All scale matrices are diagonal, but not all diagonal matrices are scale matrices.

Lower Triangular:

- A 2x2 lower triangular matrix is a matrix of the form $\begin{bmatrix} a & 0 \\ c & d \end{bmatrix}$. It has one eigenvector pointing vertically. It isn’t a very important matrix type compared to others in this game.
- The identity matrix is also a lower triangular matrix.
- Some 2x2 matrices have only one eigenvector. The third matrix in this level is an example.
- These matrices are called “lower triangular” for a similar reason to why diagonal matrices are called “diagonal”.

Singular:

- A 2x2 singular matrix is one which maps every input to a line or point.
- The identity matrix is NOT a singular matrix. Take that, identity matrix!
- A singular matrix has a determinant of zero; a matrix with a determinant of zero is singular.
- A singular matrix always has at least one eigenvector with an eigenvalue of zero.

Symmetric:

- A 2x2 symmetric matrix is a matrix of the form $\begin{bmatrix} a & b \\ b & d \end{bmatrix}$. It has eigenvectors at right angles to each other. Its transformation can be seen as “rotate, apply a diagonal matrix, rotate back”: this makes it easy to decompose and invert, which is why it keeps showing up in highschool exam questions.
- The identity matrix is a symmetric matrix: it rotates zero degrees, applies itself, then rotates minus zero degrees.
- These matrices are called “symmetric” because they’re symmetric about the leading diagonal: i.e. the number above the leading diagonal is the same as the number below.
- The eigenvectors don’t have to be 45 degrees off from from the axes: the only reason all my examples are like that is that the arrows looked awful at any other orientation.

Finale:

- A 2x2 finale matrix is a matrix present in this finale.
- The identity matrix is a finale matrix because it’s in the finale.
- You might want to see if you can remember what the types of matrices that show up in the finale are called.
- A 2x2 zero matrix is a matrix of the form $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$. It’s a special kind of singular matrix which sends all input to the origin.