In[1]:= $\$Assumptions = \{\sigma > 0\}$

Out[1]= $\{\sigma > 0\}$

D=1

In[2]:= $\rho[r\_] = \dfrac{\mathrm{Exp}\left[-\frac{r^2}{2\,\sigma^2}\right]}{\sqrt{2\,\pi\,\sigma^2}}$

Out[2]= $\dfrac{e^{-\frac{r^2}{2\,\sigma^2}}}{\sqrt{2\,\pi}\;\sqrt{\sigma^2}}$

In[3]:= $\int_0^\infty 2\,\rho[r]\,dr$

Out[3]= 1

In[4]:= $\mathrm{dsol} = \mathrm{Simplify}\left[\mathrm{DSolve}[u''[r] == \rho[r], u[r], r][[1]] \;/.\; \left\{c_1 \to -\dfrac{\sqrt{\sigma^2}}{\sqrt{2\,\pi}}, c_2 \to 0\right\}\right]$

Out[4]= $\left\{u[r] \to \dfrac{\left(-1 + e^{-\frac{r^2}{2\,\sigma^2}}\right)\sigma}{\sqrt{2\,\pi}} + \dfrac{1}{2}\,r\,\mathrm{Erf}\left[\dfrac{r}{\sqrt{2}\;\sigma}\right]\right\}$

In[5]:= $\mathrm{FullSimplify}[u[r] \;/.\; \mathrm{dsol}]$

Out[5]= $\dfrac{\left(-1 + e^{-\frac{r^2}{2\,\sigma^2}}\right)\sigma}{\sqrt{2\,\pi}} + \dfrac{1}{2}\,r\,\mathrm{Erf}\left[\dfrac{r}{\sqrt{2}\;\sigma}\right]$

In[6]:= $\mathrm{Simplify}[\mathrm{Series}[u[r] \;/.\; \mathrm{dsol}, \{r, 0, 5\}, \mathrm{Assumptions} \to r > 0]]$

Out[6]= $\dfrac{r^2}{2\,\sqrt{2\,\pi}\;\sigma} - \dfrac{r^4}{24\,\left(\sqrt{2\,\pi}\;\sigma^3\right)} + O[r]^6$

In[7]:= $\lim_{r\to 0^+} (u[r] \;/.\; \mathrm{dsol})$

Out[7]= 0

In[8]:= $\lim_{r\to 0^+} \partial_r (u[r] \;/.\; \mathrm{dsol})$

Out[8]= 0

In[9]:= $\lim_{r\to \infty} (u[r] \;/.\; \mathrm{dsol})$

Out[9]= ∞

D=2

In[10]:= $\rho[r\_] = \frac{\text{Exp}\left[-\frac{r^2}{2\sigma^2}\right]}{\sqrt{2\pi\sigma^2}^2}$

Out[10]= $\frac{e^{-\frac{r^2}{2\sigma^2}}}{2\pi\sigma^2}$

In[11]:= $\int_0^\infty 2\pi r\,\rho[r]\,dr$

Out[11]= 1

In[12]:= dsol = Simplify[

$\text{DSolve}\left[u''[r] + \frac{u'[r]}{r} == \rho[r], u[r], r\right][[1]] \text{ /. } \left\{c_1 \to \frac{1}{2\pi}, c_2 \to \frac{2\,\text{EulerGamma} - \text{Log}\left[4\sigma^4\right]}{8\pi}\right\}]$

Out[12]= $\left\{u[r] \to -\frac{-2\,\text{EulerGamma} + 2\,\text{ExpIntegralEi}\left[-\frac{r^2}{2\sigma^2}\right] - 4\,\text{Log}[r] + \text{Log}\left[4\sigma^4\right]}{8\pi}\right\}$

In[13]:= FullSimplify[u[r] /. dsol]

Out[13]= $-\frac{-\text{EulerGamma} + \text{ExpIntegralEi}\left[-\frac{r^2}{2\sigma^2}\right] + \text{Log}[2] - 2\,\text{Log}[r] + 2\,\text{Log}[\sigma]}{4\pi}$

In[14]:= Simplify[Series[u[r] /. dsol, {r, 0, 5}, Assumptions → r > 0]]

Out[14]= $\frac{r^2}{8\pi\sigma^2} - \frac{r^4}{64\left(\pi\sigma^4\right)} + O[r]^6$

In[15]:= $\text{Simplify}\left[\lim_{r\to 0^+}(u[r] \text{ /. dsol})\right]$

Out[15]= 0

In[16]:= $\lim_{r\to\infty}(u[r] \text{ /. dsol})$

Out[16]= ∞

In[17]:= $\lim_{r\to\infty}\partial_r(u[r] \text{ /. dsol})$

Out[17]= 0

D=3

In[18]:= $\rho[r\_] = \dfrac{\text{Exp}\left[-\frac{r^2}{2\,\sigma^2}\right]}{\sqrt{2\,\pi\,\sigma^2}^{\,3}}$

Out[18]= $\dfrac{e^{-\frac{r^2}{2\,\sigma^2}}}{2\,\sqrt{2}\,\pi^{3/2}\,\left(\sigma^2\right)^{3/2}}$

In[19]:= $\int_0^\infty 4\,\pi\,r^2\,\rho[r]\,dr$

Out[19]= 1

In[20]:= $\text{dsol} = \text{Simplify}\left[\text{DSolve}\left[u''[r] + \dfrac{2\,u'[r]}{r} == \rho[r], u[r], r\right][[1]] \;/.\; \{c_1 \to 0, c_2 \to 0\}\right]$

Out[20]= $\left\{u[r] \to -\dfrac{\text{Erf}\left[\frac{r}{\sqrt{2}\,\sigma}\right]}{4\,\pi\,r}\right\}$

In[21]:= FullSimplify[u[r] /. dsol]

Out[21]= $-\dfrac{\text{Erf}\left[\frac{r}{\sqrt{2}\,\sigma}\right]}{4\,\pi\,r}$

In[22]:= Simplify[Series[u[r] /. dsol, {r, 0, 5}, Assumptions → r > 0]]

Out[22]= $-\dfrac{1}{2\,\left(\sqrt{2}\,\pi^{3/2}\,\sigma\right)} + \dfrac{r^2}{12\,\sqrt{2}\,\pi^{3/2}\,\sigma^3} - \dfrac{r^4}{80\,\left(\sqrt{2}\,\pi^{3/2}\,\sigma^5\right)} + O[r]^6$

In[23]:= $\text{Simplify}\left[\lim_{r\to 0^+} (u[r] \;/.\; \text{dsol})\right]$

Out[23]= $-\dfrac{1}{2\,\sqrt{2}\,\pi^{3/2}\,\sigma}$

In[24]:= $\lim_{r\to\infty} (u[r] \;/.\; \text{dsol})$

Out[24]= 0

D=4

In[25]:= $\rho[r\_] = \dfrac{\text{Exp}\left[-\frac{r^2}{2\,\sigma^2}\right]}{\sqrt{2\,\pi\,\sigma^2}^{\,4}}$

Out[25]= $\dfrac{e^{-\frac{r^2}{2\,\sigma^2}}}{4\,\pi^2\,\sigma^4}$

In[26]:= $\int_0^\infty 2\,\pi^2\,r^3\,\rho[r]\,dr$

Out[26]= 1

In[27]:= $\texttt{dsol = Simplify}\left[\texttt{DSolve}\left[u''[r] + \frac{3\,u'[r]}{r} == \rho[r],\, u[r],\, r\right][[1]] \;/.\; \left\{c_1 \to \frac{1}{2\,\pi^2},\, c_2 \to 0\right\}\right]$

Out[27]= $\left\{u[r] \to \frac{-1 + e^{-\frac{r^2}{2\,\sigma^2}}}{4\,\pi^2\,r^2}\right\}$

In[28]:= FullSimplify[u[r] /. dsol]

Out[28]= $\frac{-1 + e^{-\frac{r^2}{2\,\sigma^2}}}{4\,\pi^2\,r^2}$

In[29]:= $\lim_{r\to 0^+} \partial_r\left(u[r]\;/.\;\texttt{dsol}\right)$

Out[29]= 0

In[30]:= $\lim_{r\to\infty} \left(u[r]\;/.\;\texttt{dsol}\right)$

Out[30]= 0