D=1

In[1]:=
```
xs = {{0}, {1}}
ws = {0, 0}
```

Out[1]= {{0}, {1}}

Out[2]= {0, 0}

In[3]:=
```
bc = Total[xs] / Length[xs]
```

Out[3]= $\left\{\frac{1}{2}\right\}$

In[4]:=
```
cc = {c1}
```

Out[4]= {c1}

In[5]:=
```
csol = Solve[
   Table[Total[(xs[[i]] - cc)^2 - ws[[i]]] == cr2, {i, Length[xs]}], {c1, cr2}][[1]]
```

Out[5]= $\left\{c1 \to \frac{1}{2}, cr2 \to \frac{1}{4}\right\}$

In[6]:=
```
wsol = Solve[Table[cc[[i]] == bc[[i]], {i, Length[cc]}] /. csol][[1]]
```

Out[6]= {}

In[7]:=
```
csol /. wsol
```

Out[7]= $\left\{c1 \to \frac{1}{2}, cr2 \to \frac{1}{4}\right\}$

D=2

In[8]:=
```
xs = {{0, 0}, {0, 1}, {1, 1}}
ws = {0, w2, 0}
```

Out[8]= {{0, 0}, {0, 1}, {1, 1}}

Out[9]= {0, w2, 0}

In[10]:=
```
bc = Total[xs] / Length[xs]
```

Out[10]= $\left\{\frac{1}{3}, \frac{2}{3}\right\}$

In[11]:=
```
cc = {c1, c2}
```

Out[11]= {c1, c2}

In[12]:=
```
csol = Solve[
    Table[Total[(xs[[i]] - cc)^2 - ws[[i]]] == cr2, {i, Length[xs]}], {c1, c2, cr2}][[1]]
```

Out[12]= $\left\{c1 \to \frac{1}{2}\left(1 + 2\,w2\right),\ c2 \to \frac{1}{2}\left(1 - 2\,w2\right),\ cr2 \to \frac{1}{2}\left(1 + 4\,w2^2\right)\right\}$

In[13]:=
```
wsol = Solve[Table[cc[[i]] == bc[[i]], {i, Length[cc]}] /. csol][[1]]
```

Out[13]= $\left\{w2 \to -\frac{1}{6}\right\}$

In[14]:=
```
csol /. wsol
```

Out[14]= $\left\{c1 \to \frac{1}{3},\ c2 \to \frac{2}{3},\ cr2 \to \frac{5}{9}\right\}$

D=3

In[15]:=
```
xs = {{0, 0, 0}, {0, 0, 1}, {0, 1, 1}, {1, 1, 1}}
ws = {0, w2, w3, 0}
```

Out[15]= {{0, 0, 0}, {0, 0, 1}, {0, 1, 1}, {1, 1, 1}}

Out[16]= {0, w2, w3, 0}

In[17]:=
```
bc = Total[xs] / Length[xs]
```

Out[17]= $\left\{\frac{1}{4}, \frac{1}{2}, \frac{3}{4}\right\}$

In[18]:=
```
cc = {c1, c2, c3}
```

Out[18]= {c1, c2, c3}

In[19]:=
```
csol = Solve[Table[Total[(xs[[i]] - cc)^2 - ws[[i]]] == cr2, {i, Length[xs]}],
    {c1, c2, c3, cr2}][[1]]
```

Out[19]= $\left\{c1 \to \frac{1}{2}\left(1 + 3\,w3\right),\ c2 \to \frac{1}{2}\left(1 + 3\,w2 - 3\,w3\right),\right.$
$\left.c3 \to \frac{1}{2}\left(1 - 3\,w2\right),\ cr2 \to \frac{3}{4}\left(1 + 6\,w2^2 - 6\,w2\,w3 + 6\,w3^2\right)\right\}$

In[20]:=
```
wsol = Solve[Table[cc[[i]] == bc[[i]], {i, Length[cc]}] /. csol][[1]]
```

Out[20]= $\left\{w2 \to -\frac{1}{6},\ w3 \to -\frac{1}{6}\right\}$

In[21]:=
```
csol /. wsol
```

Out[21]= $\left\{c1 \to \frac{1}{4},\ c2 \to \frac{1}{2},\ c3 \to \frac{3}{4},\ cr2 \to \frac{7}{8}\right\}$

D=4

In[22]:=
```
xs = {{0, 0, 0, 0}, {0, 0, 0, 1}, {0, 0, 1, 1}, {0, 1, 1, 1}, {1, 1, 1, 1}}
ws = {0, w2, w3, w4, 0}
```

Out[22]= $\{\{0, 0, 0, 0\}, \{0, 0, 0, 1\}, \{0, 0, 1, 1\}, \{0, 1, 1, 1\}, \{1, 1, 1, 1\}\}$

Out[23]= $\{0, w2, w3, w4, 0\}$

In[24]:=
```
bc = Total[xs] / Length[xs]
```

Out[24]= $\left\{\frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \frac{4}{5}\right\}$

In[25]:=
```
cc = {c1, c2, c3, c4}
```

Out[25]= $\{c1, c2, c3, c4\}$

In[26]:=
```
csol = Solve[Table[Total[(xs[[i]] - cc)^2 - ws[[i]]] == cr2, {i, Length[xs]}],
    {c1, c2, c3, c4, cr2}][[1]]
```

Out[26]= $\left\{c1 \to \frac{1}{2}(1 + 4\,w4),\ c2 \to \frac{1}{2}(1 + 4\,w3 - 4\,w4),\ c3 \to \frac{1}{2}(1 + 4\,w2 - 4\,w3),\right.$
$\left. c4 \to \frac{1}{2}(1 - 4\,w2),\ cr2 \to 1 + 8\,w2^2 - 8\,w2\,w3 + 8\,w3^2 - 8\,w3\,w4 + 8\,w4^2\right\}$

In[27]:=
```
wsol = Solve[Table[cc[[i]] == bc[[i]], {i, Length[cc]}] /. csol][[1]]
```

Out[27]= $\left\{w2 \to -\frac{3}{20},\ w3 \to -\frac{1}{5},\ w4 \to -\frac{3}{20}\right\}$

In[28]:=
```
csol /. wsol
```

Out[28]= $\left\{c1 \to \frac{1}{5},\ c2 \to \frac{2}{5},\ c3 \to \frac{3}{5},\ c4 \to \frac{4}{5},\ cr2 \to \frac{6}{5}\right\}$

D=5

In[29]:=
```
xs = {{0, 0, 0, 0, 0}, {0, 0, 0, 0, 1},
   {0, 0, 0, 1, 1}, {0, 0, 1, 1, 1}, {0, 1, 1, 1, 1}, {1, 1, 1, 1, 1}}
ws = {0, w2, w3, w4, w5, 0}
```

Out[29]= $\{\{0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 1\}, \{0, 0, 0, 1, 1\},$
$\{0, 0, 1, 1, 1\}, \{0, 1, 1, 1, 1\}, \{1, 1, 1, 1, 1\}\}$

Out[30]= $\{0, w2, w3, w4, w5, 0\}$

In[31]:=
```
bc = Total[xs] / Length[xs]
```

Out[31]= $\left\{\frac{1}{6}, \frac{1}{3}, \frac{1}{2}, \frac{2}{3}, \frac{5}{6}\right\}$

In[32]:=
```
cc = {c1, c2, c3, c4, c5}
```

Out[32]=
$$\{c1, c2, c3, c4, c5\}$$

In[33]:=
```
csol = Solve[Table[Total[(xs[[i]] - cc)^2 - ws[[i]]] == cr2, {i, Length[xs]}],
    {c1, c2, c3, c4, c5, cr2}][[1]]
```

Out[33]=
$$\left\{c1 \to \frac{1}{2}(1 + 5\,w5),\ c2 \to \frac{1}{2}(1 + 5\,w4 - 5\,w5),\ c3 \to \frac{1}{2}(1 + 5\,w3 - 5\,w4),\ c4 \to \frac{1}{2}(1 + 5\,w2 - 5\,w3),\right.$$
$$\left. c5 \to \frac{1}{2}(1 - 5\,w2),\ cr2 \to \frac{5}{4}(1 + 10\,w2^2 - 10\,w2\,w3 + 10\,w3^2 - 10\,w3\,w4 + 10\,w4^2 - 10\,w4\,w5 + 10\,w5^2)\right\}$$

In[34]:=
```
wsol = Solve[Table[cc[[i]] == bc[[i]], {i, Length[cc]}] /. csol][[1]]
```

Out[34]=
$$\left\{w2 \to -\frac{2}{15},\ w3 \to -\frac{1}{5},\ w4 \to -\frac{1}{5},\ w5 \to -\frac{2}{15}\right\}$$

In[35]:=
```
csol /. wsol
```

Out[35]=
$$\left\{c1 \to \frac{1}{6},\ c2 \to \frac{1}{3},\ c3 \to \frac{1}{2},\ c4 \to \frac{2}{3},\ c5 \to \frac{5}{6},\ cr2 \to \frac{55}{36}\right\}$$